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A tail of labor supply and a tale of monetary policy*

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Abstract

We study the interaction between monetary policy and labor supply decisions at the household level. We uncover evidence of heterogeneous responses and a strong income effect on labor supply in the left tail of the income distribution, following a monetary policy shock in the US and the UK. That is, while aggregate hours and labor earnings decline, employed individuals at the bottom of the income distribution increase their hours worked in response to an interest rate hike. Moreover, their response is stronger in magnitude relative to other income groups. We rationalize this using a two-agent New-Keynesian (TANK) model where our empirical findings can be replicated with a lower intertemporal elasticity of substitution for the Hand-to-Mouth households. This setup has important implications for the impact of inequality on the transmission of monetary policy. We unveil a novel *dampening* effect on aggregate demand generated by the Hand-to-Mouth substitution of leisure for consumption following a negative income shock. Therefore we show that the impact of inequality on the transmission mechanism of monetary policy is highly dependent on the different layers of heterogeneity on the household side and the different combinations of nominal and real frictions. More inequality does not necessarily generate a stronger response of aggregate demand after a monetary policy shock.

Keywords: Monetary policy, Household Survey, FAVARs, TANK, Hand to Mouth.

JEL classification: E52, E32, C10

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1 Introduction

Monetary policy actions have profound consequences on households' decisions. Unexpected changes in the interest rate have a direct effect on consumption and savings plans; when the real rate falls, households have an incentive to save less or borrow more and consume more. In the modern macroeconomic paradigm, this inter-temporal choice is encoded in the Euler equation. Changes in consumption plans induced by variation in rates influence also the intra-temporal allocation between consumption and leisure, and the household supply of labor depends on how each individual can substitute consumption with working time and/or compensate with different sources of income. Hence, monetary policy might have a substantial impact on the labor supply decisions of economic agents.

The vast majority of the macro literature, however, assumes no or negligible income effects on the labor supply at the individual level. The latter is typically motivated by the evidence on elasticities of idiosyncratic (lottery-induced) variations in income on labor supply.[1] This led several studies to abstract from income effects on the labor supply and to assume implicitly that aggregate shocks affecting disposable income have little impact on the supply of hours worked at the consumer level both in representative and heterogeneous agents frameworks.[2] As a consequence, we know little about the effects of monetary policy on the household's labor supply decisions both empirically and theoretically. Moreover, the consequences of monetary policy actions on the labor market dynamics are not only of interest in academic cycles. Policymakers have expressed considerable interest in labor market outcomes across the whole spectrum of the population and in particular in low- and moderate-income communities.[3]

The scope of this paper is to study the interaction between monetary policy and labor supply decisions at the household level. First we aim to offer novel empirical evidence on the effect of monetary policy on the labor supply at a more granular, disaggregate level. To do this we study the effects of unexpected shifts in the monetary policy stance on the labor supply of households at different income levels using survey data for the U.S. and the U.K. In both countries, we find evidence of a strong income effect on labor supply for households on the left tail of the income distribution. That is, we observe an increase in

[1] E.g. Cesarini, Lindqvist, Notowidigdo and Östling (2017) use Swedish administrative data to estimate wealth effects and as a side product to compute labor supply elasticities and income effects using these estimates. Based on these computations, they conclude that income effects are modest. A recent study of Golosov, Graber, Mogstad and Novgorodsky (2021) using US data questions this results and reveals that the earnings responses to lottery winnings are sizable and far from negligible in the U.S. Moreover, Powell (2016) finds that the 2008 tax rebate encouraged households to work fewer hours while Domeij and Flodén (2006) show that ignoring borrowing constraints generates a substantial downward bias in the estimation of the intertemporal labor supply elasticity.

[2] E.g. Galí, Smets and Wouters (2012); Dyrda and Pedroni (2022); Wolf (2021); Auclert and Rognlie (2020); amongst others.

[3] E.g. in a Jackson Hole speech on August 27, 2020, J. Powell said in unveiling the new Fed strategy that "our revised statement emphasizes that maximum employment is a broad-based and inclusive goal. This change reflects our appreciation for the benefits of a strong labor market, particularly for many in low- and moderate-income communities".

real labor income for these households in response to an interest rate hike. This increase is entirely due to an increase in their hours worked, whereas aggregate hours and wages across the whole distribution decline. This channel operates through an intensive margin, i.e., individuals who remain in the labor market work more hours. Moreover, their response is more sensitive to interest rate variations compared to other percentiles of income in the population. As the labor supplied by low- and moderate-income households represents both a non-negligible share of the volatility and a relevant proportion of hours worked in the aggregate, this response is also quantitatively relevant from a macro perspective. We show that these results are robust across different household surveys, identification strategies, are not primarily driven by fixed characteristics like housing tenure nor are the result of labor demand forces or labor force composition effects.

The second contribution of the paper is to study the implications of the behavior of the left tail of labor supply after a monetary policy shock for the tale of the monetary policy transmission mechanism. Our empirical findings can be rationalized with a New Keynesian model featuring two agents (TANK), an unconstrained household[4] and a *poor* Hand to Mouth (HtM)[5] household; in these models, the Euler equation does not hold for the HtM household because, for example, they have no access to financial markets or are constrained by a borrowing limit. A strong enough income effect combined with heterogeneous intertemporal elasticity of substitution (IES) across agents are the key ingredients to replicate our empirical evidence. These assumptions are in line with available estimates of the IES across households in the US and UK (Vissing-Jørgensen 2002, Attanasio, Banks and Tanner 2002) and can be microfounded with *ex-ante* homogeneous Stone-Geary preferences that generate *ex-post* heterogeneous IES increasing in the household's level of consumption in steady state.[6]

This setup has interesting implications for the effect of income inequality on the transmission of monetary policy. While inequality amplifies the transmission of monetary policy with homogeneous IES, as in e.g. Bilbiie (2008), we unveil a novel *dampening* channel that makes the monetary policy propagation weaker when the IES of HtM agents is lower than that of unconstrained agents. The value of the elasticity of HtM consumption to aggregate income, the key driver of amplification (Bilbiie (2020), Auclert (2019), Patterson (2021)), is now directly affected by the ratio of the two agents' IES and the proportion of HtM. We show that this elasticity is increasing in the HtMs' IES. Therefore the stronger the income effect on HtMs' labor supply is the stronger the dampening effect on monetary policy would be. This happens because heterogeneous IES amplifies the heterogeneity in the marginal

[4]By *unconstrained* we mean a household that has access to complete financial markets and therefore satisfies the Permanent Income Hypothesis.

[5]We follow Kaplan, Violante and Weidner (2014) and denote these *poor* HtM to highlight that our focus is on the proportion of household with very little illiquid wealth and income and not on the *wealthy* HtM.

[6]Another way to generate similar results would be to follow the literature on *consumption commitments* (Chetty and Szeidl (2007)) and have non-homothetic preferences defined over two types of consumption goods: one freely adjustable and one costly to adjust. This would limit the ability of poor households to adjust consumption pushing those with a job to increase their labor supply.

rate of substitution between consumption and leisure across agents. This allows HtM more flexibility in response to a decline in their income. They increase their labor supply, effectively substituting leisure for consumption which translates into a lower impact of monetary policy on aggregate demand.

Finally, we show how this dampening channel interacts with the presence of sticky wages and capital accumulation. We find that the impact of inequality on the transmission mechanism of monetary policy is highly dependent on the calibration of the model and the different combinations of nominal and real frictions. Therefore more inequality does not necessarily generate a stronger response of macroeconomic aggregates after a monetary policy shock.

One caveat of our structural analysis is that we abstract from considering the extensive margin although it is likely to be qualitatively important. The main driver of this choice is to highlight the effects of this new channel we uncover and ensure comparability with the key papers in the literature. We leave the exploration of extensive margin to future research.

The paper is organized as follows. The next subsection discusses the existing literature. Section 2 describes the data and the empirical strategy and presents our empirical evidence. Section 3 presents a structural model that can account for this evidence and investigates the implication for the transmission of monetary policy. Finally, section 4 provides some concluding remarks.

1.1 Related Literature

This paper relates to the recent literature on monetary policy and households heterogeneity.

Empirically, the bulk of this literature has focused on measuring heterogeneity in marginal propensity to consume (MPC) after a monetary policy shock using survey data. Cloyne, Ferreira and Surico (2020) show that the aggregate response of consumption to interest rate changes in the U.S. and the U.K. is driven by households with a mortgage pointing at the key role of wealth inequality in the transmission of monetary policy. Auclert (2019) decomposes the aggregate consumption response to monetary policy in the U.S. into different channels to evaluate the role played by redistribution in the presence of heterogeneity in MPCs. Unlike these papers, we focus on the transmission of monetary policy shocks to labor supply decisions at the household level and abstract from the composition of their balance sheet.

Kehoe, Lopez, Pastorino and Salgado (2020) and Amir-Ahmadi, Matthes and Wang (2021) study the responses of hours worked and unemployment, respectively, of different demographic groups in the US to monetary shocks. In the former they find that age and college education make the household labor less cyclically sensitive. In the latter, they estimate large differences in the response of unemployment across different demographic groups. Unlike these studies, we take a complementary approach and sort the household surveyed population according to income/earning bins (as opposed to age, education, mortgage, or preassigned groups), from low- to high-income units.

Several papers have looked at administrative data to study the heterogeneous effects of monetary policy on labor market quantities, mostly in Scandinavian countries. Holm, Paul and Tischbirek (2021) using Norwegian data study the heterogeneous effects of monetary policy shocks along with the liquid-asset distribution. Andersen, Johannesen, Jørgensen and Peydró (2021) using Danish data study the effect of monetary policy on labor income and show that it is increasing on the income level. While they find that the salary income of low-income individuals does not respond significantly to monetary policy shocks, Amberg, Jansson, Klein and Rogantini-Picco (2022) find an opposite result for Sweden. Moreover, they find that a monetary policy tightening decreases the labor income for low-income households. Importantly, differently from us, all of them are capturing the combined effect of monetary policy on the extensive and intensive margin of labor supply. Moreover, unlike us, these papers consider only labor income and cannot distinguish between hourly wages and the number of hours worked. This implies that their estimates may reflect quantity effects as well as price effects: salary income goes up because workers are employed more hours or because the hourly wage rate goes up. Our pseudo-panel data allow us to distinguish between the price and quantity variations. Moreover, for the quantity variation, we try to get a sense of the intensive vs extensive margin of variation. Another big difference we have with these studies is the frequency. They all use annual data. We use quarterly and monthly series which allows us to exploit a longer times series dimension to identify the transmission of MP shocks. Finally, there are institutional and economic differences, a small open economy with a generous welfare system vs a large relatively closed economy with a limited welfare role, which makes the quantitative and qualitative comparisons difficult.

From a theoretical standpoint, we contribute to the literature which studies micro-level heterogeneity in the New Keynesian model. To date, this literature has focused on how household-level heterogeneity affects the consumption channel of monetary policy while either abstracting from or not focusing on labor supply heterogeneity; see, for example, Bilbiie (2008); Bilbiie (2021); McKay, Nakamura and Steinsson (2016); Kaplan, Moll and Violante (2018)) or Auclert (2019). Athreya, Owens and Schwartzman (2017) show how labor supply decisions are crucial to determine the direction and size of the output effects of fiscal transfers. They highlight the crucial role of the marginal propensities to work across the population in heterogeneous agents incomplete markets models. To the best of our knowledge, we are the first to look at this with regard to monetary policy.

The aggregate demand amplification effect of income inequality has been studied by (Bilbiie 2008, 2021), Auclert (2019), Patterson (2021) and Bilbiie, Känzig and Surico (2022) amongst others. Relative to these studies, we show that allowing for hours and IES heterogeneity unveils a novel dampening channel. We characterize this channel analytically in combination with the standard amplification one, and then use a richer TANK model to quantify the contribution of different assumptions to the transmission of monetary policy.

Our results crucially depend on the empirical evidence on IES heterogeneity. Most studies highlight the role of income in driving heterogeneity in IES. This is because poor consumers, whose consumption bundle contains a large share of necessities, tend to substitute less consumption intertemporally than rich households (Andreolli and Surico (2021)).[7] Attanasio and Browning (1995) document that the IES is increasing in the household level of consumption. Attanasio et al. (2002) and Vissing-Jørgensen (2002) show that the IES for stockholders and bondholders in the UK and the US is an order of magnitude higher than the one of those who do not hold these assets. Calvet, Campbell, Gomes and Sodini (2021) find a bi-modal distribution of the IES across households in Sweden.

Finally, our results on the behavior of the left tail of labor supply are also related to the evidence on households *consumption commitments* and *inflation inequality*. Regarding the former Chetty and Szeidl (2007) document regular payments that cannot be easily adjusted and limit households' ability to adjust the consumption margin.[8] Our story here is related as the presence of these commitments is likely to be pushing poor households with a job to increase their labor supply following a negative income shock. On the latter Jaravel (2021) discusses how heterogeneity in consumption baskets generates a dampening effect of monetary policy on aggregate demand.[9]

2 Monetary policy and labor market outcomes along the income distribution

In this section, we describe the data sources and construction, the empirical strategy to identify monetary policy shocks, and we present our empirical evidence about the transmission of these shocks to labor supply decisions at the household level. Our evidence is constructed using the information on income, earnings and hours worked at the household/individual level for two developed economies with independent monetary policy authorities, the UK and the US.

We find that, in both countries, the households at the left tail of the income distribution typically increase the weekly amount of hours worked after a monetary policy tightening. Since the unemployment rate increases across all percentiles of the income distribution, the increase in hours worked operates through the intensive margin. Therefore, those individuals who remain in the labor market after a monetary policy tightening tend to supply more hours of work. Moreover, we find that labor market outcomes (unemployment and hours worked) are more sensitive on the left tail of the income distribution.

[7]Therefore if subsistence represents an important part of the poor's consumption, these agents have limited discretion for intertemporal substitution in consumption (Blundell, Browning and Meghir (1994), Attanasio and Browning (1995)).

[8]Households allocate a significant part of their expenses to goods and services that are costly to adjust like mortgage/rental payments, insurance payments, or mobile phone plans.

[9]Households with lower marginal propensities to consume spend more on sectors with higher price rigidity. Moreover, after a monetary policy shock, prices fall more in sectors employing the poor which increases relative labor demand for the poor through changes in consumer demand. Therefore monetary policy shocks reduce labor earnings more for the low MPC agents leading to another dampening channel.

2.1 Household level data

For the US, we consider two sources of household/individual-level data, the Current Population Survey (CPS), sponsored jointly by the U.S. Census Bureau and the U.S. Bureau of Labor Statistics (BLS), and the Consumer Expenditure Survey (CE or CEX), also collected by the BLS. The former is the primary source of labor force statistics for the population of the United States; the latter collects information on expenditure and income to study the buying habits of U.S. consumers. Both surveys contain useful information about household labor supply decisions and income/earnings distribution. In particular, the CPS survey is conducted at a monthly frequency on a sample of about 60,000 U.S. households; it contains detailed information about the demographic characteristics of the household, labor market attitudes, and labor earnings. The CEX data comes at a lower frequency on a smaller sample of U.S. households but contains more detailed information about their income sources.

Therefore, from the CPS we can use individual-level data on hours worked and hourly wages but we can only sort individuals based on labor earnings and not gross income.[10] Instead, from the CEX we can use labor income and hours worked but we can sort households based on their gross income. This means that results from the two surveys are complementary but cannot be directly comparable.

Unless otherwise specified, we apply a filter to both surveys. In particular, we drop respondents that lie in the top and bottom first percentile of the earnings (e.g. CPS) or income (e.g. CEX) distribution or are aged less than 18 or more than 66. Moreover, at each point in time (e.g. the month for the CPS or the quarter for the CEX), we construct different income/earning percentile groups, e.g. quintiles and deciles. For example, quintile bins refer to five percentile groups of total income or earnings. We refer to less than or equal to the 20^{th} percentile with P_{20}, greater than 20^{th} and less than equal to 40^{th} percentile with P_{20-40} and so on. However, the main results are obtained with a specification where we split the population into two groups, $P_{\leq J}$ and $P_{>J}$ where J ranges from 5 to 95 with increments of 5. For example when $J = 5$, the two groups consist of respondents who fall below that 5th percentile of hourly earnings or income ($P_{\leq 5}$) and those that lie above the 5th percentile ($P_{>5}$).

For the CPS, individual-level data is obtained for each month from 1994 to 2019. For earnings, we use the outgoing rotation group.[11] Our measure of hourly earnings corresponds to the amount earned per hour in the current job reported by respondents who are paid hourly. For respondents that are paid weekly, we construct earnings by dividing the usual weekly earnings by usual weekly hours in the main job. Earnings are deflated using CPI. We use actual hours worked in the main job as our main measure of weekly hours. We

[10] Gross income data are not available at monthly frequency in the CPS.

[11] Households are interviewed in the CPS for four months and then again for four months after an eight-month break. In the fourth and eighth months of interviews, when households are about to rotate out of the interviews, they are asked additional questions regarding earnings. For more details on the outgoing rotation group, see https://cps.ipums.org/cps/outgoing_rotation_notes.shtml

calculate average earnings and average weekly hours for each group using the survey weights. Repeating this across all months in the sample provides a time series of average earnings and average weekly hours by percentile group. We also construct unemployment rates at various percentiles of the earnings distribution. To do that, we use mincer-type regressions to impute hourly earnings for those individuals that are unemployed. In particular, we regress earnings on level of education, a measure of experience (age minus year of schooling minus six), and individual characteristics including race, sex, and industry of occupation. The fitted values from this regression are used to obtain imputed earnings for unemployed individuals.[12]

The demographic characteristics vary substantially across bins. As we move from the left to the right of the earnings distribution, respondents tend to be older and better educated; they are more likely to work longer hours, own their homes, be white and male, and less likely to be employed in a manual job. In the appendix A.2 we provide more details on data constructions and on the demographic characteristics of individuals for different deciles of the earning distribution, e.g. see Figure A.3.

In the CEX survey, data is constructed at the Consumer Unit (CU) level. In this survey gross income for CU i and time t includes various components, which are grouped into four categories (1) the amount of wage and salary income before taxes received over the past 12 months, (2) income from farm and non-farm business, (3) financial income and (4) income from social security.[13] All income components are deflated using CPI. Total hours are defined as the sum of hours worked by each individual in the CU over the past year. In each year of the sample, we retain CUs from the second or the fifth interview when income data is updated. CUs are then assigned to each quarter of the year by their date of interview (see Cloyne and Surico (2016)), and then sorted into bins by gross income as discussed previously. Average wages, the average of the remaining components of income, and average hours are computed in each of these groups using survey weights. These calculations are repeated for each year in the sample from 1984 to 2018 delivering a quarterly time-series for each group.

The demographic characteristics of CU along the income distribution constructed from the CEX survey are similar to those obtained with the CPS survey. As we move from the left to the right, respondents are more likely to be male, white, college graduates, homeowners with a mortgage and work longer hours. Relative to the CPS survey, the CEX survey allows us to characterize the US population more precisely in terms of financial assets and sources of income. In particular, the $P_{\leq 20}$ percentile has on average less than 2% of its total income in the form of liquid assets (e.g. checking and savings accounts, bond, stocks, and other securities). Moreover and more importantly, social security transfers are a very important source as they comprise around 40% of total income. In the appendix A.1 we provide more

[12]The coefficients of the regression are shocked and used to produce predicted values. These are assigned to unemployed individuals using predicted mean matching. We produce 5 replicates, with the final imputed data taken to be the mean across these replications. We then compute the distribution of earnings (including those unemployed with imputed income) and for each percentile, we compute the fraction of unemployed people. These measures will be useful to isolate the extensive and intensive margins of labor market fluctuations.

[13]See appendix A.1 for the definition of these components.

details on data constructions and on the demographic characteristics for each bin, e.g. see Figure A.1.

The analog surveys for the UK are represented by the Labour Force Survey (LFS) and the Food and Expenditure Survey (FES) respectively, where the former collects information to construct the labor force statistics, and the latter expenditure and income to study household consumption patterns in the UK. The same pros and cons of each survey discussed for the US ones apply here as well. Similarly, we apply the filters described above to UK data – observations that lie in the top and bottom percentiles of the income/earnings distribution are dropped and only individuals older than 18 and younger than 66 are included in the sample. Moreover, we study the same percentile groups as described for the US.

From the LFS we obtain quarterly data for actual hours worked, hourly earnings, and the unemployment rate for the UK for the period 1994 to 2019. As in the case of the US, we use mincer regressions to impute earnings for unemployed individuals. Unemployment rates in earnings quintiles are then constructed by using both employed and unemployed individuals to calculate the earnings distribution. The characteristics of the UK earnings distribution are very similar to that of the US. Low earners in the UK tend to be younger, less educated, and employed in manual labor. They are less likely to be male and to own a home and their average weekly hours are shorter. In the appendix A.4 we provide more details on data constructions and on the demographic characteristics for different deciles of the earning distribution, e.g. see Figure A.6.

The second source of household-level data on income and hours worked for the UK is obtained from the Food and Expenditure Survey (FES). As in the case of the US, we obtain data on the components of gross income: wages, income from self-employment, financial and investment income, and income from social security. Total hours are the sum of hours worked by each member of the household. The households are assigned to quarters each year according to the date of the interview. They are then sorted into bins based on percentile groups of income. Average wages, average hours, and the average of other components of income are then constructed (using survey weights) for each quarter from 1979 to 2016.

The inspection of the UK households' demographic and income characteristics offers a similar picture depicted for the US. The head of the household is predominately male in the high-income quintiles while the low-income one is mostly female. Non-white heads of households can be found mostly in the low-income percentiles but they are in small proportions throughout the distribution. In terms of housing status and personal indebtedness, more than 80% in the 5^{th} quintile hold properties through mortgages while this is less than 20% in the low-income quintile, where around 50% of households are renters. Wage is the most important component of income for all quintiles and contributes to more than 80% of total income in high-income ones. In the UK as well, the main source of income for low-income households is social security benefits.[14] Finally, the weekly amount of hours worked rises to

[14]Here we decompose income only for employed households during the surveys, otherwise the contribution of

the income level with the total in richer households larger than those at the left tail of the distribution. In the appendix A.3 we provide more details on data constructions and on the demographic characteristics for each bin, e.g. see Figure A.4.

Appendix A.5 compares the aggregate series of hours worked for the two countries as constructed by the statistical offices and by aggregating our survey-based data. We show that the latter captures the main movements in aggregate data fairly well for each survey in each country.

2.2 Empirical Model

To estimate the impact of monetary policy shocks on the labor supply for different slices of the population we use a factor augmented VAR (FAVAR) model for the US and the UK, respectively. The model is defined by the VAR:

$$Y_t = c + \sum_{j=1}^{P} \beta_j Y_{t-j} + u_t \tag{1}$$

where $Y_t = \begin{pmatrix} R_t \\ \hat{F}_t \end{pmatrix}$ where R_t denotes a policy interest rate and $\hat{F}_t$ represents factors that summarize information in a panel of macroeconomic and financial series and the survey-based data on income and hours, described above. The factors are estimated using the non-stationary factor model of Barigozzi, Lippi and Luciani (2021). A key advantage of this approach is that it allows us to use the data in levels. This is convenient as our interest centers on the impact of policy on the level of wages/labor earnings and hours in the percentile groups. Denote X_t as the $(M \times 1)$ data matrix that contains the panel of macroeconomic and financial series that summarize information about the economy, and also includes the average hours and average income components deflated by CPI in various income/earnings percentile groups. The observation equation of the FAVAR is defined as:

$$X_t = c + b\tau + \Lambda F_t + \xi_t \tag{2}$$

where c is an intercept, τ denotes a time-trend, F_t are the R non-stationary factors, Λ is a $M \times R$ matrix of factor loadings and ξ_t are idiosyncratic components that are allowed to $I(1)$ or $I(0)$. Note that the idiosyncratic components corresponding to the survey-based data can be interpreted as shocks that are specific to those groups and also capture possible measurement errors. In contrast, the shocks to equation (1) represent macroeconomic or common shocks. The response to these common shocks is relevant for our investigation. This ability to estimate the impact of macroeconomic shocks while accounting for idiosyncratic disturbances is a key advantage of the FAVAR over a VAR where these two sources of fluctuations may be conflated (see De Giorgi and Gambetti (2017)). Moreover, expanding the cross-sectional dimension of the VAR with factors is important also for identification

benefits to total income is higher than 40% in the 1st income quintile.

purposes as it reduces the problem of information deficiency (see e.g. Forni and Gambetti (2014)) and shock deformation (see e.g. Canova and Ferroni (2022)).

For the US, the macro and financial data in X_t is obtained from the FRED-QD and FRED-MD database. The quarterly data runs from 1984Q1 to 2018Q4 and contains 238 series covering real activity, employment, inflation, money, credit, spreads, and asset prices. The monthly database contains 137 times series covering the same broad categories as the quarterly database, is trimmed to start in 1994m1, i.e. the first observation of hours worked constructed in the CPS; the last observation is 2019m12. For the UK, our data set contains 75 macroeconomic and financial variables over the period 1979Q1 to 2019Q4. Data from the FES survey is available over this sample. However, the LFS survey-based hours and earnings can only be obtained from 1994Q1 onwards. Note that we can expand the number of macroeconomic and financial variables to 100 over this shorter sample.[15]

To identify a monetary policy shock, we use an external instrument approach (see e.g. Stock and Watson (2008) and Mertens and Ravn (2013)). The residuals u_t are related to structural shocks ε_t via:

$$u_t = A_0 \varepsilon_t \tag{3}$$

where $cov\left(u_t\right) = \Sigma = A_0 A_0'$. We denote the shock of interest as ε_{1t} and the remaining disturbances as ε_{-t}. Identification of ε_{1t} is based an instrument m_t that satisfies the relevance and exogeniety conditions: $cov\left(m_t, \varepsilon_{1t}\right) = \alpha \neq 0$ and $cov\left(m_t, \varepsilon_{-t}\right) = 0$. As discussed in Appendix C, these conditions can be combined with the covariance restrictions to obtain an estimate of the relevant column of the the contemporanous impact matrix A_0.

For the US, our benchmark instrument to identify the monetary policy shock is taken from Gertler and Karadi (2015). This instrument is built using intra-day data on three months ahead federal funds futures. Gertler and Karadi (2015) calculate the change in this instrument during a tight window around FOMC meetings. The high frequency of the data makes it likely that changes in the futures over the window reflect unexpected changes in monetary policy. Following Gertler and Karadi (2015) R_t is assumed to be the one-year government bond yield. When using quarterly data and variables from the CEX survey, the quarterly sum of the Gertler and Karadi (2015) shock is used.

For the UK, the 1-year rate is used as the policy rate and we employ two instruments to identify the shock. When using the longer sample of data and variables from the FES survey, the shock is identified using the measure proposed by Cesa-Bianchi, Thwaites and Vicondoa (2020). Cesa-Bianchi et al. (2020) build a measure using high-frequency data on three-months sterling futures. The monetary policy surprise is calculated as the change in this rate over a tight window around meetings of the monetary policy committee (MPC).

However, the Cesa-Bianchi et al. (2020) instrument fails to identify the policy shock when the start of the sample is shifted to 1994Q1. In particular, the sign of the response of

[15] As discussed below, a larger number of factors are required to identify the policy shock over this shorter sample period. Therefore, we make use of this expanded dataset for this sample.

real activity variables is anomalous with a contractionary shock associated with a persistent increase in GDP. Therefore, for the model that uses LFS data, the shock series proposed by Gerko and Rey (2017) is used for identification. Gerko and Rey (2017) also use three-month sterling futures to build their proxy. In contrast to Cesa-Bianchi et al. (2020), their shock is constructed by calculating the change in these future rates around the release of the minutes of the MPC minutes.[16]

The number of factors in the FAVAR models are chosen via the information criteria of Bai and Ng (2002). The PC_P criteria suggest the presence of 11 factors for the US. For the UK, the FAVAR with the number of factors chosen via the PC_P criteria suggests a large price puzzle. We, therefore, follow Bernanke, Boivin and Eliasz (2005) and add additional factors to ameliorate this problem. This procedure leads to models with 13 and 15 factors, respectively, for the samples starting in 1979 and 1994. The lag length is chosen via the Akaike information criterion.

The parameters of the VAR model in (1) and (2) are estimated using a Gibbs sampling algorithm that is described in Appendix B. We employ 51,000 iterations, retaining every 10^{th} draw after a burn-in period of 1000.[17]

2.3 Response of aggregate and dis-aggregate variables

Figure 1 shows the response of some key aggregate variables to a contractionary monetary policy shock in the US and the UK. From top to bottom, we present the response of the one-year interest rate, real GDP, the consumer price index (CPI), the stock market index, and unemployment rate for the two countries respectively; the last two rows report the labor income and hours worked responses of households with income below the 20^{th} percentile of the income distributions obtained using the CES survey for the US and FES survey for the UK.

The peak decline in GDP ranges from 0.8 to 1.3 percent in the UK and the US, respectively, with the response in the former estimated to be faster. This is most likely due to different estimated persistence in the parameters of the interest rate equation. The fall in US GDP coincides with a rise in the unemployment rate of 0.5 percent. The rise in the UK unemployment rate is of a similar magnitude but occurs earlier after the policy shock. CPI declines in both countries by about 15 quarters after the shock. Stock market prices react negatively in both countries.

Several other economy-wide variables display interesting dynamics after a monetary policy shock.[18] In particular, the main components of aggregate demand, i.e. consumption and investment, and standard measures of industrial production, aggregate and sectorial,

[16]Over this sample, large announced changes in monetary policy are concentrated around the great financial crisis. As discussed in Gerko and Rey (2017) this makes it likely that changes around MPC meetings are conflated by other shocks.

[17]The prior distributions for the VAR parameters are standard and described in Appendix B.

[18]The full set of IRFs for all the variables included in the FAVAR model can be consulted here.

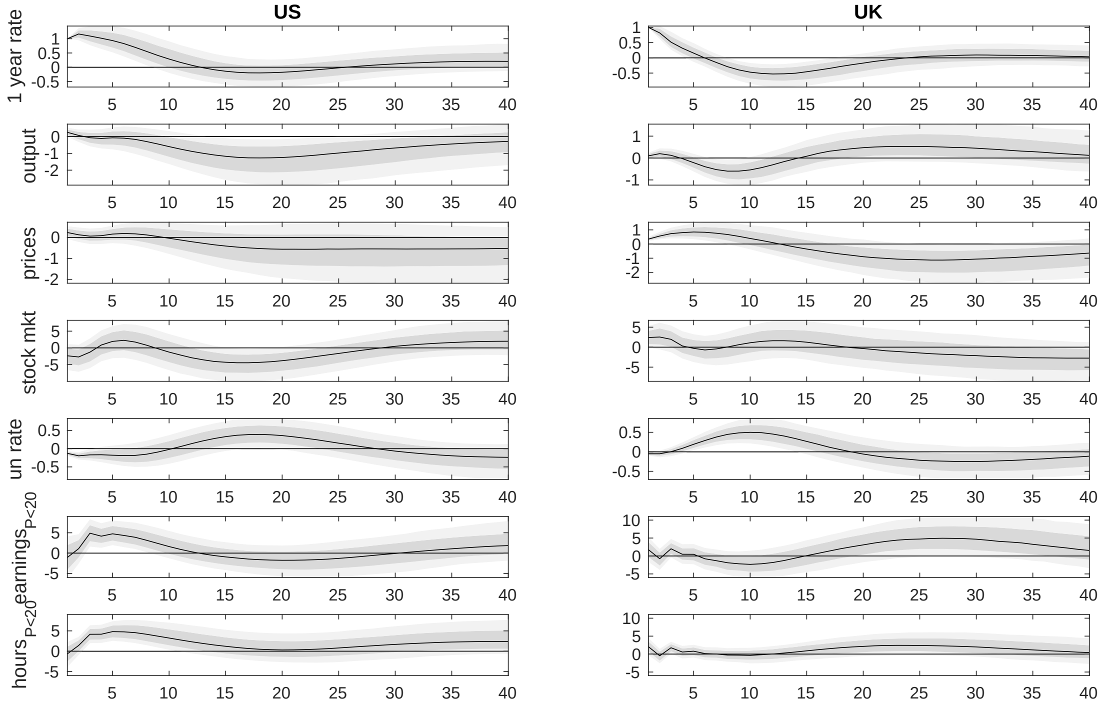

Figure 1: Responses of selected variables to a contractionary monetary policy shock in the US (CEX - left panels) and in the UK (FES - right panels) normalized to a one percent increase in the one year interest rate.

all decline. Producers and consumer price indexes contract. A number of the labor market indicators deteriorate after the rate hike: employment declines in the aggregate and various industries; real wages and compensations shrink; and aggregate measures of hours worked fall significantly after 12 quarters.[19] Standard monetary aggregates decline and liquidity become scarcer. Household home financing costs rise. Corporate credit costs, e.g. the US excess bond premium and the UK corporate bond spread, rise in the short and medium-term. Overall, these estimated dynamics are broadly consistent with a monetary-induced negative demand shock.

The last two rows of Figure 1 display the response of households in the first quintile of the income distribution; interestingly, in both countries, the labor supply increases after the negative demand shock. This is a result that does not hold for the aggregate measure of hours worked. As we discuss in the next section, this pattern, i.e. the increase in the labor supply on the left side of the income distribution, holds also when we consider the data from CPS in the US or LFS in the UK and when we consider different slices of the income distribution.

Finally, the increase in hours worked induces an increase in the labor income; the latter,

[19]The response of the aggregate hours worked computed by summing CUs level hours worked resembles remarkably well the response of aggregate hours from the statistical offices. This is not surprising given how close the two measures look in Appendix A.5.

however, does not rise more than hours worked suggesting a moderate decline in hourly wages and therefore pointing towards an income effect driving the response of hours in the left tail. These effects are shown in Figures D.1 and D.2 in the appendix. In these figures, one can appreciate that the responses of the households in the 20 percentile of the income distribution are significantly more volatile than the rest of the population, both in the US and in the UK.

So far we have arbitrarily chosen a cutoff percentile at 20%. In the next section, we look across the whole income/earnings distribution to check precisely where this threshold lies.

2.4 Income effect on Labor Supply

In this section, we present the results by running a set of empirical models for the US and the UK. We split the population into two groups, with average income below or above a certain percentile J, $P_{\leq J}$ and $P_{>J}$ respectively. And we repeat the estimation of the FAVAR for $J = 5, 10, ..., 95$.

In Figure 2 (3) we show the response of hours worked by households/individuals below a certain income/earning threshold ($P_{\leq J}$) to a monetary policy tightening in the US (UK) for different thresholds (J), different horizons and different surveys.[20] In particular, the northwest corner plot displays on the y-axis the response of hours worked one year after the shock and on the x-axis the group that belongs to the income bin $P_{\leq J}$. As we move eastward, we plot the same response two and three years after the shock, respectively. As we move southward, we consider the response of hours worked and the unemployment rate constructed from the CPS. Dark and light gray areas indicate the 68% and 90% Bayesian confidence sets.

The top panels confirm the results presented in the previous section, where we identified a portion of the population that increases the labor supply after a monetary-induced negative demand shock.[21] Moreover, they also suggest that the cutoff percentile does not necessarily lie in the first quintile and that possibly a larger fraction of US households display a strong income effect on their labor supply. Even if uncertainty gets larger, these effects tend to be fairly persistent and the increase in labor supply is significant three years after the shock. As we move to the right of the income distribution the average household with an income below -say- the median does not respond immediately to the shock; after a few years, however, their labor supply becomes procyclical and declines.

The individual responses obtained using the CPS database are qualitatively similar to the findings of the CEX database, see the second row of Figure 2. Magnitudes are different; the uncertainty surrounding these estimates seems larger; and the exact cutoff point is identified

[20] Appendix D.2 presents the full three-dimensional IRFs for hours worked labor income, hourly wages, and consumption for both countries.

[21] In the CEX we observe a non-monotoning pattern. For the group below the 5% ($P_{\leq J}$) we actually see a large decline in hours. However, this effect is short-lived (disappears after one year) and it is not present in the results from the CPS.

at different percentiles. It is important to notice, however, that estimates across surveys are not directly comparable in terms of percentiles and magnitudes; e.g. in one survey we sort *individuals* labor earning and in the other we sort *households* by gross income. That being said, a statistically significant share of the US low-income population tends to increase its labor supply after a monetary policy tightening.

The strength of the income effect on the labor supply could be the result of two mechanisms: more people at the bottom of the earning distribution enter the labor market (extensive margin), or employed people work longer (intensive margin). Since the unemployment rate for low- and moderate-income individuals tend to increase after a monetary policy, see the last row of Figure 2, we can rule out large movements in the extensive margin. Hence, after a monetary-induced negative demand shock low-income individuals who remain in the labor market tend to work longer hours.

Moreover, the other important piece of evidence consistent across surveys is that labor market outcomes (unemployment and hours worked) are more sensitive on the left tail of the income distribution.

Using the estimated responses of hours and unemployment in the CPS we can also perform some *back of the envelope* calculation of the net effect on aggregate hours of the extensive and intensive margin. For the $P_{<15}$ in 2019 we estimate that the intensive margin effect we uncover reduces the drop in hours generated by the extensive margin by 50% following a monetary policy shock.[22]

Figure 3 reports the same statistics for the UK. The top panels report the responses of hours worked at different horizons, one, two, and three years after the shock, for households below a certain income threshold using the FES database. In this case, the pattern is monotonic, i.e. the lower the income of the respondent the stronger and more positive is the response of hours worked. The picture is similar when using the LFS database.[23]

Appendix D.2 shows in detail for both countries/surveys the IRFs across the whole distribution for hours, labor earnings, and hourly wages.[24]

In sum, our empirical evidence suggests that after a monetary-induced negative demand shock low-income individuals who remain in the labor market have a strong income effect on their labor supply. The estimates of the exact magnitude of the labor supply increase and the precise threshold are difficult to pin down and depend on the survey considered. However, as

[22]In 2019 in the CPS sample there are 15142 employed individuals in the bin 5-15% and their average hours worked is 32 hours per week. At the one-year horizon, we estimate hours of $P_{<15}$ to increase by 1.5% and unemployment to increase by 3%. This implies that hours decline by 100% for 3% of the sample and we get that the extensive margin effect is 15142x0.03x(-32)=-14536. On the other hand, hours increase by 1.5% for the remaining 97% of the sample so that 15142x0.97x31x0.015=+7050 is the intensive margin effect. The same calculation for the UK produces very similar results.

[23]For the UK in the LFS, it is only possible to calculate the response of unemployment up to $P_{<50}$. For higher wage percentiles the recorded number of unemployed is close to zero in many quarters and a time series cannot be created. This is the reason why the X-axis in figure 3 stops to 50.

[24]As mentioned in the CEX/FES we have info on hours and earnings so we can implicitly construct hourly wages. In the CPS/LFS we have info on hours and hourly wages so we can implicitly construct labor earnings.

we mentioned, these surveys are designed by construction to measure different patterns and behaviors. While the CPS and LFS represent the preferential reference to construct labor market statistics, they are less accurate in measuring gross income; in particular, they ignore social security income which represents a large fraction of total income for low earners. This has an effect on characterizing the income distribution, and hence on the threshold of the response of labor supply.

Nevertheless, this conditional moment deserves some attention as low- and moderate-income households tend to explain non-negligible fractions of the variance of total hours worked and this fraction is larger than the analog for consumption. For example, in the CPS survey, the cross-sectional variance of total hours worked explained by the bottom 15% of the labor income distribution ($P_{<15}$) is around 18 percent. Incidentally in the CEX survey, the portion explained by the first quartile ($P_{<25}$) is also 18 percent; the portion of consumption variance explained by the same group is smaller at 12 percent. For the UK the numbers are similar.[25] The labor supplied by the left tail of the income distribution represents also a relevant proportion of total hours worked in the economy. For the US, in the CPS, the average proportion of hours worked by the bottom 15% of workers across the labor income distribution is 15%. These numbers highlight an important difference between our work and the literature on consumption heterogeneity. The latter has stressed the importance of the *wealthy* HtM households for matching the average marginal propensity to consume, empirically and theoretically (see e.g. Kaplan et al. (2014)). Here, our empirical analysis suggests that the labor supply heterogeneity does not arise for *wealthy* HtM rather for the *poor* HtM households who appear to be crucial in driving labor supply heterogeneity following a monetary policy shock. Finally, we wish to emphasize that the labor market outcomes for low- and moderate- income households are significantly more volatile than the rest of the population; this holds true both in the US and in the UK and for all the surveys considered.

2.5 Robustness

Before looking at our empirical evidence through the lenses of a theoretical model, we wish to briefly discuss several additional robustness exercises to complement our empirical evidence. To save space, figures and details are presented in Appendix E and we discuss here only the findings.

We start by considering alternative identification approaches. In particular, for the US we use three alternative schemes for identifying the monetary policy shock. First, we utilize the instrument proposed by Miranda-Agrippino and Ricco (2021) who refine the measure used in Gertler and Karadi (2015) by purging the component that represents a signal regarding the central bank's information about the state of the economy. Second, we use the shock

[25] The proportion of variance of total hours explained by the first quartile ($P_{<25}$) in the LFS is 18% which increases to 19% in the FES survey.

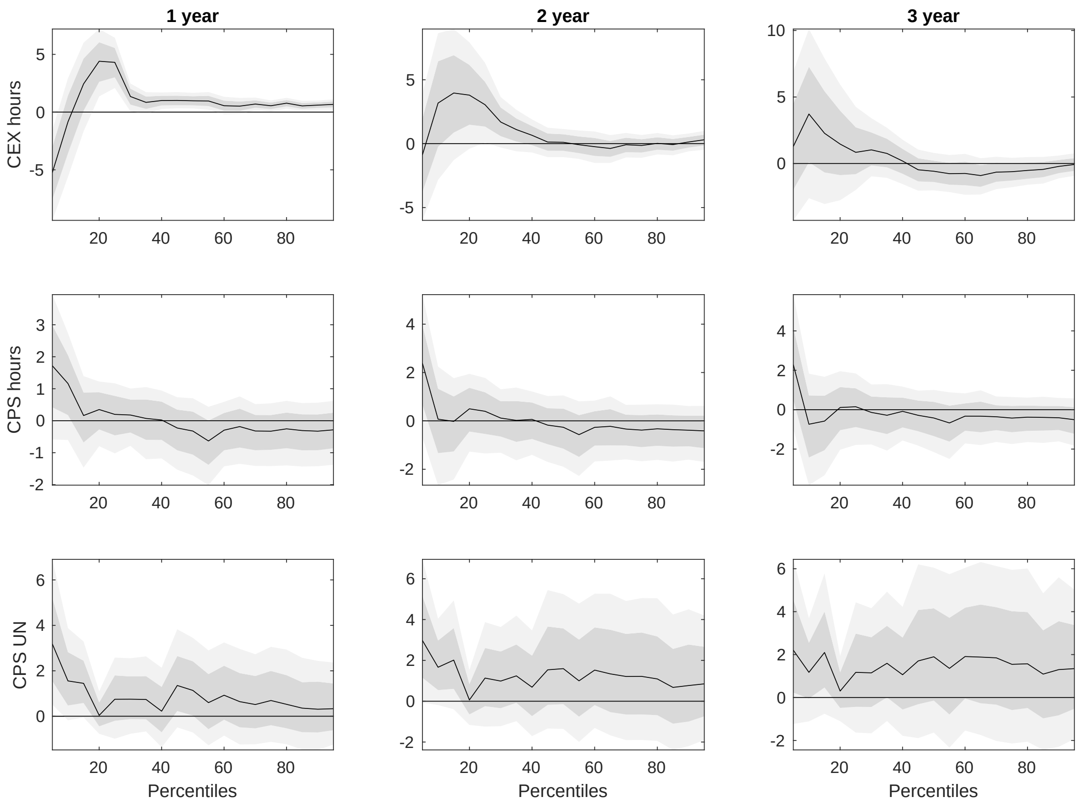

Figure 2: Impulse response of hours worked at different percentiles of the income distribution and across surveys in the US.

measure of Romer and Romer (2004) as the instrument. Finally, we identify the monetary policy shock via sign restrictions, where a contractionary shock increases the policy rate and reduces GDP, price indices, and real broad money. The response of labor income and hours supports the benchmark results. In particular, the response of both of these variables is largest and positive for group P_{20}.

For the UK, we obtain results that are qualitatively similar to the benchmark when we use two alternative identification schemes. First, we employ the monetary policy shock of Cloyne and Hurtgen (2016) as an instrument. Cloyne and Hurtgen (2016) apply the procedure of Romer and Romer (2004) to UK data. Second, we identify the monetary policy shock in the UK by using the sign restrictions on real activity, prices, and money supply described above for the US. The estimated impulse responses suggest that hours rise for households at the left tail of the income distribution.[26]

For both countries, we have also looked at general demand and supply shocks. Identifying demand shocks via sign restrictions we obtain results that look qualitatively similar for the response of hours worked.[27] Looking instead at supply shocks we find that hours at the

[26]For the case of sign restrictions, we actually find that hours raise across the whole income distribution but the increase in the left tail is larger than in the rest of the population.

[27]Results available upon request.

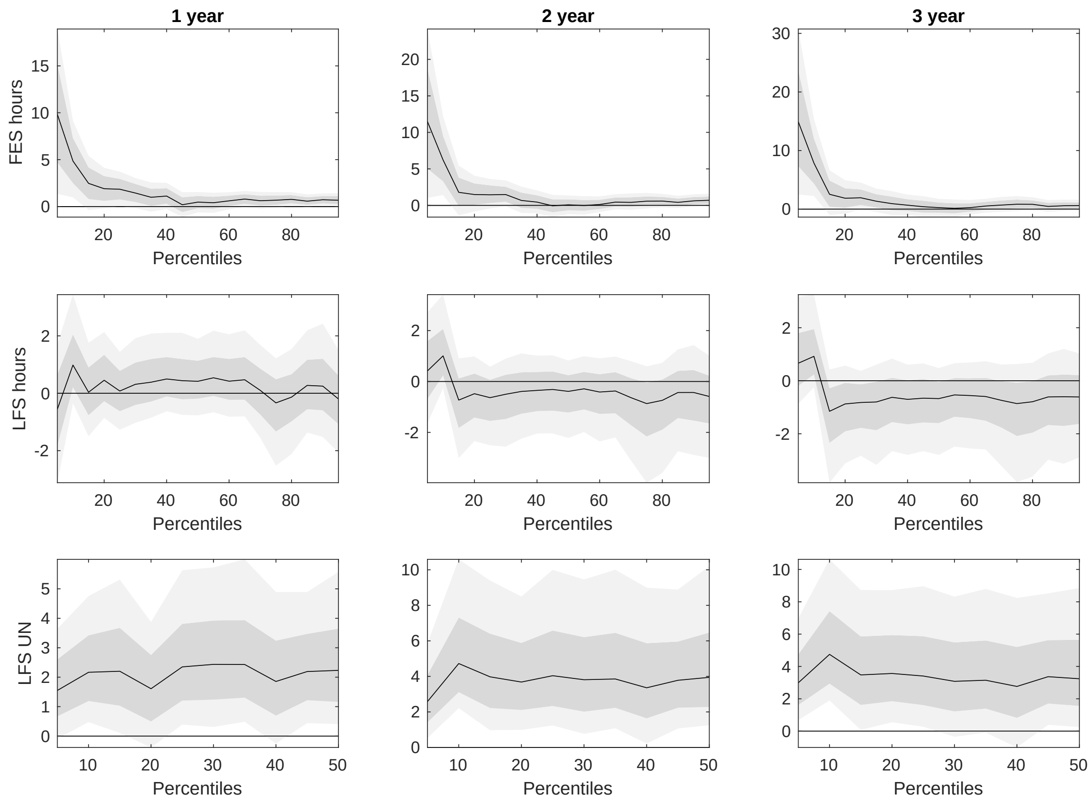

Figure 3: Impulse response of hours worked at different percentiles of the income distribution and across surveys in the UK.

bottom of the income distribution exhibit a large procyclical response to supply shocks.[28] These responses to supply shocks help link our findings with the bulk of the literature that finds that earnings of poor people are procyclical. We report novel evidence that these are countercyclical following Monetary policy shocks, however, it is well known that these shocks represent a small fraction of business cycle variation.

Furthermore, we conduct a series of robustness analyses by looking at different household characteristics. We check if the results on hours are driven by housing tenure. Results for the US suggest that labor income/hours indeed rise on impact for mortgagors. However, the increase is short-lived and becomes negative after a few quarters suggesting that having a mortgage is not the primary driver. In the UK we do not observe any clear pattern in hours worked driven by housing tenure.

To check that what we uncover is mainly a labor supply story and is not driven by labor demand we look at the response of hours by industry of occupation using the CPS/LFS. We find substantial heterogeneity across industries in both countries but no particular pattern that might point out a labor demand story driving our results.[29] We then look at the

[28]See appendix E for details.

[29]For the US, we find that hours worked increase substantially in industries producing durable goods, and utilities where wages appear to drop substantially. We take this as further evidence of income effect on labor supply. Other

response of hours worked by the occupation of the reference person in the household (CEX). Results show that occupations, where hours increase, tend to lie towards the left tail of the income distribution.

Our results on the supply of hours worked could also be affected by a composition effect of people switching jobs and/or going in and out of the labor force in between survey data collection. In the CEX, households that receive no wages or work zero hours in the past year have missing value and this means that they are not included in the averages we construct. In other words, we include only individuals that have worked over the last year. But, as the reference period covers a year, it could include spells of unemployment, job changes, or other changes in job status. As we show in Appendix A.5 the data on hours aggregated from the CEX track quite well data from national accounts. Therefore this composition effect, if present, should not be large enough to affect significantly our results. As a double-check, however, we use the occupation information to restrict the sample. We remove observations where the household reference person does not respond to the occupation question or the response does not fit in the 5 options given in the survey.[30] The hours' response of the left tail in the US does not appear to be driven by the *unemployed*.

Finally, we tested for asymmetries in the responses of hours worked to an interest rate cut or hike in the US. We could not find evidence for the latter.[31]

3 Monetary policy and labor supply in TANK

In this section, we turn to a theoretical model to rationalize the empirical results. We first do so in a stylized setup where we can obtain closed-form expressions. We then explore the implications for the monetary policy transmission in the simple setup and extensions with different layers of frictions and different degrees of heterogeneity.

To match our empirical evidence in the most parsimonious way we consider a TANK model. The TANK economy is populated by a fraction of standard unconstrained agents[32] and by a fraction of agents not on the Euler equation, typically labeled as Hand-to-Mouth (HtM) households. Usually, for the latter type of agents, the *permanent income hypothesis* does not hold either because they have no access to financial markets (Bilbiie (2008)) or because of the presence of borrowing constraints (Bilbiie, Monacelli and Perotti (2013)). In

sectors where hours increase are finance, professional services, and the public sector. In those sectors, however, wages either do not move or increase following an interest rate hike. In the UK instead, we only observe a positive and persistent increase in hours worked in Agriculture and Energy sector where this seems to be driven by an increase in wages.

[30] These are: Managerial & professional specialty; Technical, sales, and administrative support; services; Farming, forestry, and fishing; Precision production, craft, and repair, mechanics, mining; Precision production, craft, and repair, mechanics, mining; and Machine operator, fabricators, transport, and laborers.

[31] Results are available upon request.

[32] In response to a rate hike, the unconstrained household reduces both their consumption because of the intertemporal substitution channel and their labor supply because the return from saving is higher than the return from working.

what follows, we focus mainly on the former setup.[33] This two-agent structure lines up well with our empirical evidence where the working-age population in the US and the UK can be split into two groups according to the labor supply response to a monetary policy shock.

We build on the TANK model of Bilbiie (2008)[34] and allow for the IES to be increasing in the level of household consumption in steady state in line with available evidence (Attanasio and Browning (1995), Attanasio et al. (2002), Vissing-Jørgensen (2002)). In his seminal work Bilbiie (2008) showed that the response of the labor supply of HtM depends crucially on the IES parameter which captures the relative strength of the income and substitution effect. As long as the intertemporal elasticity of substitution of HtM is sufficiently low ($\sigma < 1$), the income effect dominates and their labor supply is negatively correlated with wages/consumption matching our first piece of evidence. Moreover, the heterogeneity in IES parameters helps generate larger volatility of labor supply for HtM households, allowing to capture our second empirical finding.[35]

This setup has interesting implications for the effectiveness of monetary policy transmission. In particular, with homogeneous IES, Bilbiie (2008) showed that as long as the Investment-Saving curve is negatively sloped[36] the presence of HtM agents generate *amplification* of monetary policy. Amplification arises because the elasticity of HtM consumption to aggregate income (χ) is larger than 1. That is, consumption of the HtM moves more than 1 to 1 with aggregate income (*countercycical inequality*).

When we allow for different IES across household types, however, we find a novel and additional channel that works in the opposite direction. IES heterogeneity amplifies the differences in marginal rates of substitution between consumption and leisure across agents. We show that this novel dimension of heterogeneity reduces the value of χ. Crucially the value of this elasticity depends on the relative magnitude of the IES of the two types of agents. The lower the IES of the HtM relative to the unconstrained agents, the lower χ and the less effective is the monetary policy transmission. Moreover, we study how different layers of heterogeneity on the household side and different forms of real and nominal frictions can alter the amplification mechanism of monetary policy at increasing levels of inequality. We find that models with different frictions and/or different degrees of heterogeneity generate substantially different degrees of *amplification* of monetary policy, from a low of 0.04 (dampening) for the case of sticky wages and heterogeneous hours and IES to a high of 3.83

[33] In appendix H we show similar results for a model with borrowers and savers as in Bilbiie et al. (2013).

[34] Given the focus on aggregate demand dynamics some of the TANK literature assumes an aggregate wage schedule by imposing equal labor supply response between agents (see Bilbiie et al. (2022) and Debortoli and Galí (2017)). The seminal work by Bilbiie (2008), and his subsequent papers (Bilbiie 2020, 2021), presents the model with heterogeneous labor supplies. However, these papers do not focus on the response of hours worked to monetary policy shocks.

[35] Bilbiie et al. (2022) discuss in their online appendix C.7 that with heterogeneity in labor supply with homogeneous IES the standard TANK model generates hours of savers that are much more responsive to monetary policy than the ones of the HtM.

[36] Following Bilbiie (2008) we denote this region as the Standard Aggregate Demand Logic (SADL) region. This is the region of the parameter space where the proportion of HtM is not high enough to make the elasticity of aggregate consumption to the interest rate positive.

for the case of flexible wages and homogeneous hours and IES. Therefore, more inequality does not necessarily generate amplification of monetary policy.

It is important to stress that our analysis abstracts from the extensive margin in the labor market. However, monetary policy has a first-order effect on the employment dynamics and, at least for the US (cf. figures 2-3), the unequal incidence of unemployment outcomes on the population can amplify aggregate shocks, as shown in Patterson (2021) and Broer, Kramer and Mitman (2021). While our quantitative exercise lacks a joint treatment of the extensive and intensive margin of the labor market, we are the first to offer an assessment of the implications of the latter combined with various real and nominal frictions for the relationship between inequality and monetary policy shocks.

3.1 A Simple TANK model with heterogeneous IES

In this section, we outline the TANK model a la Bilbiie (2008) with IES heterogeneity. The aim is to isolate the effect of the IES heterogeneity most simply and transparently. While this heterogeneity might appear completely ad-hoc here, we micro-found this assumption in section 3.3; in particular, the ex-post IES heterogeneity can be generated by ex-ante identical Stone-Geary type preferences where the IES is increasing in the steady state level of consumption of different households.

The economy consists of three sectors: households, firms, and a central bank. The household sector is populated by two different types: *savers* S and *hand-to-mouth (HtM)* H. A share λ of households are HtM who work and consume all of their income. The remaining $1-\lambda$ are savers who hold bonds and shares in monopolistic firms and get firm profits. Savers price all assets and get all returns, thus there is limited asset market participation. The saver's problem is the same as the standard permanent income hypothesis agent. We assume that both agents enjoy consumption and leisure and have the same $CRRA$ utility function. The firm sector is standard; intermediate goods producers face nominal rigidities in the form of quadratic adjustment costs a la Rotemberg when adjusting their prices. To simplify the analysis and without loss of generality, we follow Bilbiie (2020) and assume a production subsidy that induces marginal cost pricing in steady state. The central bank chooses the nominal interest rate by responding to inflation expectations.

The key equations of the log-linearized model are reported in Table 1, where all variables with '^' are expressed in log deviation form steady state and $\hat{x}_{t+1|t}$ represent the time t conditional expectation of variable $\hat{x}$ at time $t+1$. Real quantities are in terms of the consumption good and - unless otherwise stated - denoted by lower case letters while nominal variables are denoted by capital letters.[37]

We assume that the typical market clearing and resource constraint conditions hold, i.e.

[37] Appendix F presents details about the model derivations from first principles, steady state and log-linearization. Here we note that, as a result of our simplification assumptions, output and aggregate hours in the economy are equal to aggregate consumption, and their steady state is normalized to 1 without loss of generality.

Log-linearized Conditions		
1:	Aggregate Hours	$(1-\lambda)\hat{H}_t^S + \lambda \hat{H}_t^H = \hat{c}_t$
2:	Aggregate Consumption	$(1-\lambda)\hat{c}_t^S + \lambda \hat{c}_t^H = \hat{c}_t$
3:	Euler Savers	$\hat{c}_t^S = \hat{c}_{t+1\|t}^S - \sigma_S \left(\hat{R}_t - \hat{\Pi}_{t+1\|t}\right)$
4:	Labor Supply Savers	$\varphi \hat{H}_t^S = \hat{w}_t - \frac{1}{\sigma_S}\hat{c}_t^S$
5:	Labor Supply HtM	$\varphi \hat{H}_t^H = \hat{w}_t - \frac{1}{\sigma_H}\hat{c}_t^H$
6:	Budget constraint HtM	$\hat{c}_t^H = \hat{H}_t^H + \hat{w}_t$
7:	Phillips Curve	$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1\|t} + \kappa \hat{w}_t$
8:	Taylor Rule	$\hat{R}_t = \hat{\Pi}_{t+1\|t} + \epsilon_t^m$

Table 1: Log linearized conditions of the simple TANK model with hours and IES heterogeneity.

production utilizes labor input from S and H and equals aggregate demand (equations 1 and 2). Aggregate hours and real consumption are defined as $\hat{H}_t$ and $\hat{c}_t$ respectively. All variables with subscripts S or H correspond to savers and HtM. The third and fourth two equations characterize the behavior for the S agent, the Euler equation, and the labor supply equation respectively; σ_j (with $j = H, S$) controls the intertemporal elasticity of substitution (and risk aversion) and is heterogenous while φ is the inverse of the Frisch elasticity of the labor supply which is homogenous.[38] $\hat{R}_t$ is the nominal interest rate and $\hat{\Pi}_{t+1|t}$ the expected inflation rate. The following two equations describe the behavior of the HtM household, which is bounded to consume only its labor income proceedings (equation 6) and optimally substitute between consumption and leisure/working time (equation 5). Here $\hat{w}_t$ stands for real wages. We have the standard New Keynesian Phillips curve (equation 7) where κ is the slope of the NKPC linking the real with the nominal side of the economy. To allow for an analytical solution, we assume that the only source of fluctuation in this economy are monetary policy shocks (ϵ_t^m) and that the central bank responds one-to-one to inflation expectations as in Bilbiie (2008) and McKay et al. (2016), therefore effectively choosing the real rate. This assumption makes the real interest rate equal to the monetary policy shock and simplifies substantially the analysis.

In summary, here the only substantial departure from the standard TANK model of

[38] Relaxing this assumption won't affect the results.

Bilbiie (2008) is that we allow for IES to be different across agents.[39] This departure allows us to match jointly the two empirical findings of the previous section, namely that after a rate hike the number of hours worked by low-income households increase and this increase is larger in absolute terms relative to other households. We discuss this in the next section.

3.1.1 Matching the empirical evidence

Savers reduce their labor supply decision after a monetary policy tightening.[40] On the contrary, it is immediate to see that labor supply decisions of the HtM are countercyclical to aggregate quantities as long as the income effect dominates. This can be appreciated by solving the HtM budget constraint and intratemporal optimality condition in terms of the aggregate real wage,

$$\hat{H}_t^H = \frac{\sigma_H - 1}{\sigma_H \varphi + 1}\hat{w}_t; \quad \hat{c}_t^H = \frac{\sigma_H(\varphi + 1)}{\sigma_H \varphi + 1}\hat{w}_t. \tag{4}$$

Notice that heterogeneity in the Frish elasticity would not be able to generate a countercyclical behavior of hours worked. In what follows we restrict to the *standard aggregate demand logic* (SADL) region of the model (Bilbiie (2008)) where $\lambda < \frac{1}{1+\varphi}$.[41] A monetary induced demand shock that pushes real wages down produces an increase in the HtM hours worked when $\sigma_H < 1$. More formally, we can state the following proposition.

Proposition 1 *Under SADL ($\lambda < \frac{1}{1+\varphi}$) and with a sufficiently low IES of the HtM households (e.g. $\sigma_H < 1$), a rate hike induces a decline in total hours worked and an increase in the HtM labor supply.*

The proof is in appendix F.4. In response to a rate hike, unconstrained agents reduce both their consumption because of the intertemporal substitution channel and of the higher return from saving relative to the return from working. This effect is strong enough to generate a fall in demand which induces a decline in aggregate wages. For the HtM household, the only source of income is labor. With a low IES ($\sigma_H < 1$), the negative income effect dominates and labor supply must increase to counteract the negative demand shock. This generates a countercyclical behavior of the HtM labor supply. Note that when income and substitution effects are of the same opposite magnitudes (i.e. $\sigma_H = 1$ - log utility) the HtM are insensitive to changes in the monetary policy stance as originally showed by Bilbiie (2008). Similarly, borrowing limits can also affect the margin of adjustment of HtM labor and consumption decisions and boost the income effect on HtM labor supply.[42] In a model

[39] Different from the original TANK in Bilbiie (2008) we also abstract from dividend's redistribution, productivity shock and assume a production subsidy that induces marginal cost pricing instead of fixed costs. See Appendix F.

[40] To single out the optimal behavior of savers, assume that the share of HtM is negligible, i.e. $\lambda \to 0$. In this limiting case, we have that $\hat{H}_t^S = \hat{H}_t = \hat{c}_t = \hat{c}_t^S$. By combining the Taylor rule with the Euler equation and solving the latter forward we get that $\hat{c}_t^S = -\sigma_S \epsilon_t^m$. Therefore, the consumption and labor supply decisions of savers' households are procyclical conditional on a monetary policy shock.

[41] This is the region of the parameter space where the proportion of HtM is not high enough to make the slope of the aggregate demand function positive.

[42] While this alternative model specification is isomorphic to our baseline specification in terms of conditional moments, it does not seem to fully align with the data and empirical evidence of previous sections. According to the

with borrowers and savers similar to Bilbiie et al. (2013) one can show that a rate hike induces an increase in the borrowers labor supply when $\sigma_H < \frac{1+\bar{D}\kappa}{\gamma}$ where $\bar{D}$ is the debt limit and $\gamma = 1 + \bar{D}(\beta - 1) < 1$.[43]

Finally, the heterogeneity of the IES is important as it allows to capture the different sensitivity of the households labor supply decision to aggregate conditions. As the IES of HtM becomes smaller, HtM labor supply becomes more sensitive to changes in the real wage schedule (see equation (4)); for low enough values of σ_H, the income effect eventually gets so strong that generates an absolute increase in hours worked larger than one of the unconstrained agents. Since aggregate hours worked decline in the SADL equilibrium, the fraction of HtM households ought not to be too large. These arguments are depicted in Figure 4 where we display the absolute ratio between the HtM and savers hours worked after a monetary policy shock for different values of the intertemporal elasticity of substitution of the HtM (σ_H) and their relative share in the economy (λ).[44] We calibrate the IES of the saver following the estimates of Vissing-Jørgensen (2002) for bondholders where $\sigma_S = 0.8$ and assume a Frisch elasticity of labor supply, $1/\varphi$, equal to one so that the SADL region extends for $\lambda \in [0, 0.5)$.[45] Cool (warm) colors indicate that the hours worked of HtM households is smaller (larger) than the (absolute value of) hours worked of saver households. As we move to the northwest corner (low λ and low σ_H), we generate a relatively more sensitive response of HtM households and move closer to our empirical evidence.

3.2 Monetary policy amplification or dampening?

What are the implications for monetary policy effectiveness when the proportion of HtM agents changes and IES are different across households? Following Bilbiie (2020) we define $\chi = \frac{\hat{c}_t^H}{\hat{c}_t}$ as the elasticity of HtM consumption to aggregate income.[46] Bilbiie (2008) showed that limited asset market participation amplifies the effect of monetary policy in the SADL region with homogeneous preferences if $\chi > 1$. In our setup, which abstracts from fiscal redistribution of profits, we have

$$\chi = \frac{\frac{\sigma_H}{\sigma_S}(\varphi + 1)(\sigma_S\varphi + 1)}{\lambda\left(\frac{\sigma_H}{\sigma_S} - 1\right)(\varphi + 1) + \sigma_H\varphi + 1}. \tag{5}$$

CEX and FES surveys households at the bottom 20 percent of the income distribution are mostly renters (about 60 percent in the US and 50 percent in the UK) or homeowners and only a small fraction of them are mortgagors (about 10 percent in the US and 20 percent in the UK). This does not rule out necessarily the possibility that they are on the borrowing limit, it is well known that a large fraction of low-income families in both countries tend to finance durable consumption with debt and might use payday loans.

[43]See appendix H for details.

[44]The hours worked responses of the two agents to a monetary policy shock are derived in the proof of proposition 1 (Appendix F.4).

[45]The rest of the parameters values are $\beta = 0.99$ and $\kappa = 0.1406$. The value of k is obtained by assuming an elasticity of substitution between goods variety equal to 6 and an average price stickiness of 3.5 quarters.

[46]Recall that in this set up $\hat{c}_t$ is equal to aggregate income.

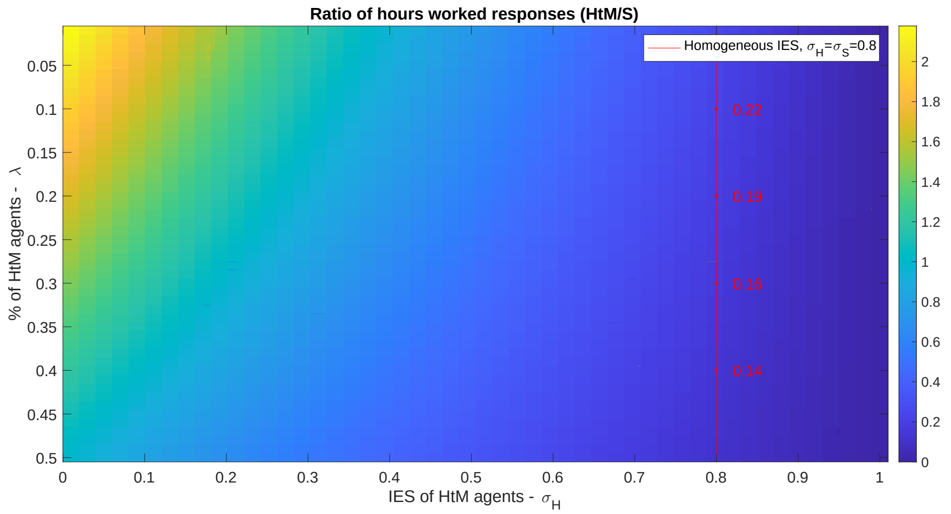

Figure 4: Relative (absolute) magnitude of the response of HtM and savers hours worked to a monetary policy shock for different values of σ_H and λ. Values larger than one indicate larger volatility of HtM labor supply relative to Savers. The gray vertical line indicates the relative ratio when the IES equals 0.8 for both agents.

With homogeneous IES ($\sigma_S = \sigma_H$), the latter collapses to $\chi = 1 + \varphi > 1$ (as in Bilbiie (2020)).[47] In general, however, the elasticity of HtM consumption to aggregate income depends on the relative size of the IES's. Importantly, χ is not independent from the proportion of HtM agents λ. It appears to be increasing (decreasing) in λ if $\frac{\sigma_H}{\sigma_S} < 1$ (> 1). This elasticity is a crucial parameter as it affects the slope of the aggregate Euler equation in the model:[48]

$$\hat{c}_t = \hat{c}_{t+1|t} - \sigma_S \frac{1-\lambda}{1-\chi\lambda}(\hat{R}_t - \hat{\Pi}_{t+1|t}). \tag{6}$$

Let's review first what this implies in the homogeneous IES case where $\chi = 1 + \varphi > 1$ and does not depend on λ. An increase in inequality (higher λ) increases the elasticity of consumption to the real interest rate. When the interest rate increases, savers substitute consumption intertemporally which generates a fall in demand. As a result real wages fall. Since poor HtM income is made only by wages they have to consume less pushing demand further down. Note that, in this case, the sign of the slope of the Euler equation is independent of σ_H. From (4) when $\sigma_H = \sigma_S = 1$ the consumption of HtM is one to one procyclical with wages $\hat{c}_t = \hat{w}_t$. Otherwise with $\sigma_S = \sigma_H < 1$ this relationship is not one to one ($\hat{c}^H_t = \frac{1+\varphi}{1/\sigma_H + \varphi}\hat{w}_t$) but it remains procyclical and still amplifies the effect of monetary policy shocks. This can be seen by looking at the slope of (6) or by substituting in the Taylor

[47]Bilbiie (2020) has $\chi = 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right)$. Here we abstract from profits redistribution which implies $\tau^D = 0$.

[48]Following Bilbiie (2020) this can be derived using the equation relating the consumption of savers with aggregate consumption, $\hat{c}^S_t = \frac{1-\chi\lambda}{(1-\lambda)\sigma}c_t$, and their Euler equations $\hat{c}^S_t = \hat{c}^S_{t+1|t} - \sigma\left(\hat{R}_t - \hat{\Pi}_{t+1|t}\right)$.

rule and solving it forward $\hat{c}_t = -\sigma_S \frac{1-\lambda}{1-(\varphi+1)\lambda}\,\epsilon_t^m$. Under SADL an increase in λ magnifies the effect of ϵ_t^m on aggregate consumption independently of what happens to the labor supply of HtM. How is this possible even when the labor supply of HtM is countercyclical? This is due to a general equilibrium effect that makes unconstrained agents' labor supply more procyclical. The less the number of non-HtM agents in the economy (higher λ) the larger the slice of firm profits that each of them receives. As profits are countercyclical this induces them to work even less after a monetary policy tightening.

The case of heterogeneous IES generates quite different implications. By plugging (5) into (6) we obtain:

$$\hat{c}_t = \hat{c}_{t+1|t} - \sigma_S \underbrace{\frac{1-\lambda}{1-(1+\varphi)\lambda}}_{(+)\text{ when }\lambda\uparrow} \times \underbrace{\frac{\lambda\left(\frac{\sigma_H}{\sigma_S}-1\right)(\varphi+1)+\sigma_H\varphi+1}{\sigma_H\varphi+1}}_{(-)\text{ when }\lambda\uparrow\text{ if }\frac{\sigma_H}{\sigma_S}<1} \times (\hat{R}_t - \hat{\Pi}_{t+1|t}). \tag{7}$$

The slope of Equation (7) has the same amplification channel as in (6) plus a new one which implies dampening if $\frac{\sigma_H}{\sigma_S} < 1$.[49] The dampening effect arises as a consequence of χ being decreasing in λ if $\frac{\sigma_H}{\sigma_S} < 1$. The source of this dampening can be traced to the heterogeneity in the marginal rate of substitution (MRS) between hours and consumption. Intuitively, heterogeneity in the marginal rate of substitution gives HtM an extra tool to use in response to a decline in their income. They can increase their labor supply, substituting leisure for consumption. Technically, when households have the same IES, the intra-temporal optimality condition between hours worked and consumption (for a given wage schedule) is equivalent at the *aggregate* and at the household level, i.e. $\varphi\hat{H}_t + \frac{\hat{c}_t}{\sigma} = \hat{w}_t$ and $\varphi\hat{H}_t^j + \frac{\hat{c}_t^j}{\sigma} = \hat{w}_t$. So, in response to a change in the real wage schedule the MRS adjusts in the same proportion for the aggregate as well as for the two agent's types. With different IES, this is no longer the case. When we combine[50] the two intratemporal optimality condition we now get:

$$\varphi\hat{H}_t + \frac{\hat{c}_t}{\sigma_S} + \lambda\left(1-\frac{\sigma_H}{\sigma_S}\right)\frac{\hat{c}_t^H}{\sigma_H} = \hat{w}_t.$$

When the real wage drops, HtM households reduce their consumption as they do not have other sources of income. Thus, for the same real wage decline, aggregate quantities (i.e. consumption and hours worked) decline less relative to the case where the IES of households are the same. Moreover, the smaller σ_H, the stronger this channel. The following proposition formalizes this and shows under which parameterizations of λ and σ_H this dampening channel dominates.

Proposition 2 *Under SADL ($\lambda < \frac{1}{1+\varphi}$) and when $\sigma_H < \sigma_S$, we have that if $\sigma_H < \sigma_H^\star$ an increase in λ reduces (increases) the aggregate impact of monetary policy shocks if $\lambda < \lambda^\star$*

[49] Note that the SADL region stays the same as in the homogeneous IES case. The slope still changes sign with $\lambda > \frac{1}{1+\varphi}$.

[50] Taking the weighted average of $\varphi\hat{H}_t^j + \frac{\hat{c}_t^j}{\sigma_j} = \hat{w}_t$ using λ and $1-\lambda$ as weights.

($\lambda > \lambda^\star$). Where

$$\lambda^\star = \frac{1}{1+\varphi} - \frac{\sqrt{\sigma_H\,\varphi\,(\sigma_S-\sigma_H)\,(\sigma_S\,\varphi+1)}}{(\sigma_S-\sigma_H)(1+\varphi)}$$
$$\sigma_H^\star = \frac{\sigma_S}{\sigma_S\,\varphi^2+\varphi+1}.$$

The proof is in appendix F.5. Note that $\lambda^\star$ and $\sigma_H^\star$ are decreasing in φ. Proposition 2 can be used to check if the conditions under which monetary policy dampening arises can be empirically and quantitatively plausible. For example, if we calibrate $\varphi = 1$ and $\sigma_S = 0.8$ as above we get $\sigma_H^\star = 0.285$. By definition then $\lambda^\star = 0$. Setting instead $\sigma_H = 0.1$, in line with the evidence in Vissing-Jørgensen (2002) for non-bondholders, we get $\lambda^\star = 0.2465$. This shows that dampening is likely to arise for plausible parameterizations of this simple model where HtM are less than 25% of the population.

Next, we show this graphically. Figure 5 plots the impact response of aggregate consumption for different values of λ in the SADL region and selected calibrations of σ_H keeping $\varphi = 1$ and $\sigma_S = 0.8$.[51] If HtM's IES is sufficiently low, we have the nonmonotonic effect of inequality on the transmission of monetary policy on aggregate quantities described by proposition 2. In the RANK benchmark ($\lambda = 0$) a 1% rate hike has an impact on aggregate consumption equal to -0.8%. In a TANK economy with 20% HtM agents with low intertemporal elasticity of substitution, say $\sigma_H = 0.1$, the same 1% hike in the nominal interest rate reduces aggregate consumption by -0.72% (Figure 5). Therefore we have a 10% reduction in the effectiveness of the monetary policy.

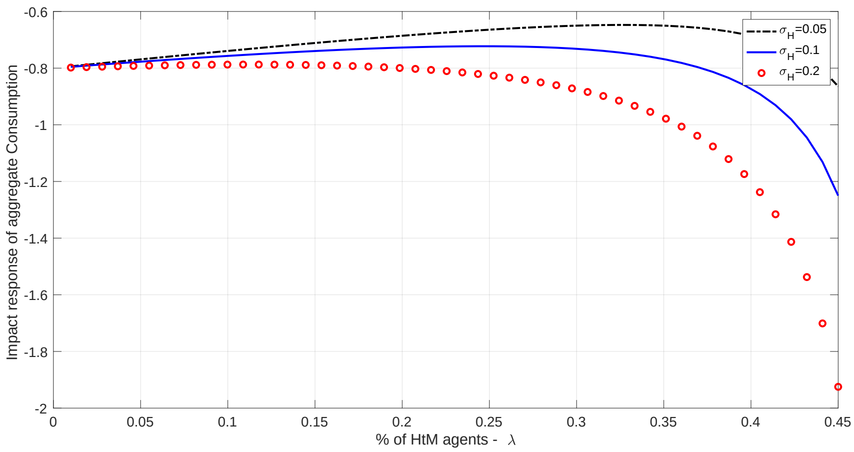

Figure 5: Impact response of Hours/Consumption to a 1% tightening with $\sigma_S = 0.8$ and $\varphi = 1$ for selected values of σ_H.

[51]The rest of the calibration is the same as in in footnote 45.

3.3 Heterogeneity, frictions and monetary policy amplification

In this section, we want to study how different layers of heterogeneity, frictions, and factors in production modify the transmission channel of monetary policy. For example, Bilbiie et al. (2022) showed that the complementarity between capital and income inequality in heterogeneous agent models amplifies substantially the effect of monetary policy. Therefore is important to check if our result carries over in more quantitative versions of the TANK model. In particular, we look at the role played by sticky wages, capital accumulation with investment adjustment costs, heterogeneity in the supply of hours worked, and heterogeneity in the degree of intertemporal substitution.

The model used in this section builds on Bilbiie et al. (2022) allowing for heterogeneity in labor supply.[52] Moreover, we now propose a microfundation of the IES heterogeneity presented so far. We do so by assuming Stone-Geary preferences that generate IES increasing in the level of consumption in steady-state. While Appendix G describes the model in detail, here we focus on the micro foundation of heterogeneous IES.

We assume that both agents ($j = H, S$) have the following preferences:

$$U(c_t^j, H_t^j) = \frac{(c_t^j - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^j \frac{(H_t^j)^{1+\varphi}}{1+\varphi}, \tag{8}$$

where c_t^j is real consumption, H_t^j hours worked, $\bar{c}$ is subsistence level of consumption, σ^{-1} is the curvature of the utility in consumption, ν the weight of hours in utility and φ^{-1} the Frish elasticity of labor supply. For a given steady state level of consumption $c^j \geq \bar{c}$ the IES is increasing in c^j and given by:

$$-\frac{U'_{c^j}}{c^j U''_{c^j}} = \sigma\left(1 - \frac{\bar{c}}{c^j}\right). \tag{9}$$

From (9) and indicating as before σ_H and σ_S the IES of the two agents, we have:

$$\sigma_H = \sigma\left(1 - \frac{\bar{c}}{c^H}\right) \tag{10}$$

$$\sigma_S = \sigma\left(1 - \frac{\bar{c}}{c^S}\right). \tag{11}$$

It follows that, in line with the data, with $c^S > c^H$ in steady state we get $\sigma_H < \sigma_S$. Therefore here we remove the steady state subsidies that induced 0 profits in steady state so far to make the algebra simpler.[53] Therefore consumption inequality in steady-state depends on profits and the different sources of income from capital and labor for the two agents. Consumption of savers is larger in steady-state because they have positive income from profits and capital but also because they work longer hours. We calibrate the steady-state of hours using individual actual hours of work from the CPS so that savers work 33%

[52] Differently from Bilbiie et al. (2022) we also abstract from idiosyncratic risk and fiscal transfers.

[53] Results are robust to keeping the subsidies in place or introducing fixed costs in production that induce 0 profits in steady state.

	Flexible wages	Sticky wages	Flexible wages + capital	Sticky wages + capital
Homogeneous Hours	3.25	1.18	3.83	1.43
Heterogeneous Hours	2.03	0.18	2.36	0.20
Heterogeneous Hours & IES	0.51	0.04	0.59	0.04

Table 2: Value of χ across model's specifications. $\varphi = 1$

and HtM 27.5% of their available time.[54] We keep the two IES following the estimates of Vissing-Jørgensen (2002) for bondholders vs nonbondholders: $\sigma_S = 0.8$ and $\sigma_H = 0.1$. Then we set $\sigma = \frac{c^H\sigma_H - c^S\sigma_S}{c^H - c^S}$ consistent with (10)-(11). Note that these two equations, together with the calibration of σ_H and σ_S also pin down $\bar{c}$. We assume the Frisch elasticity of labor supply $\frac{1}{\varphi} = 1$ and $\lambda = 0.155$ consistent with the average proportion of hours worked by the bottom 15% of workers across the labor income distribution in the CPS.[55] Finally, to combine sticky wages with heterogeneous labor supply in our two agents economy we follow Ascari, Colciago and Rossi (2017) and assume that only the savers are unionized while the HtM supply their labor taking as given the wage chosen by the savers union.[56] The rest of the calibration follows from the standard calibration of medium-scale DSGE models and is summarised in Appendix G.

Table 2 reports the value of the elasticity of HtM consumption to aggregate income, χ, the summary statistics of the degree of amplification of monetary policy at increasing level of inequality across different model specifications. The first row reports the version of the model with homogeneous hours (ie. $H_t^S = H_t^H = H_t$ as in Bilbiie et al. (2022)).[57] The second row corresponds to the model with heterogeneous hours with homogeneous IES (as in Bilbiie (2008)) where we set $\sigma_H = \sigma_S = 0.8$ while the last row the model with both heterogeneous hours and IES. The columns of the table correspond to the different ingredients we add to each model, namely sticky wages and capital accumulation subject to investment adjustment costs. Given the quantitative nature of this exercise, we now assume in all model specifications a contemporaneous monetary policy rule satisfying the Taylor principle where the Central Bank responds more than one-to-one to today's inflation.

The first row confirms that the presence of capital accumulation increases χ while sticky wages reduce it in the homogeneous hours model in line with the results in Bilbiie et al.

[54] We use P_{15} as cut off between HtM and Savers along the labor income distribution consistent with the results in Figure 2. The average weekly hours per worker in the CPS are 33 for $P_{<15}$ and 40 for $P_{>15}$. More details about CPS data are presented in Appendix A.2.

[55] Computing a similar average in the CEX for the proportion of hours worked the bottom 25% of the income distribution we obtain 11.5%. Note that by doing this we are implicitly assuming that the relationship between labor/income distribution and the response of labor supply to monetary shock has been stable over time. Unfortunately, data limitations do not allow us to see if this has been changing over time.

[56] Ascari et al. (2017) present this set up in the technical appendix accompanying their paper. See Appendix **??** for the list of equilibrium conditions of the model with sticky wages.

[57] In this case equations 4 and 5 in the model in table 1 are replaced with the aggregate labor supply relation $\varphi\hat{H}_t = \hat{w}_t - \frac{1}{\sigma}\hat{c}_t$. Combining it with the budget constraint of the HtM gives $\chi = 1 + \varphi + \sigma^{-1}$ for the flexible wages no capital case.

(2022). Importantly, however, χ remains in the amplification region (> 1) across all specifications. In the heterogeneous hours' case instead, we can see how both sticky wages and heterogeneous IES can easily bring $\chi < 1$ and substantially reduce the effect of monetary policy on the aggregate economy. And this result is independent of the presence of capital accumulation.

These results show that the dampening impact of sticky wages is significantly magnified in TANK models with heterogeneous hours. To the best of our knowledge, we are the first to show this result.[58] Importantly when we combine sticky wages with heterogeneous hours and IES the value of χ gets even smaller. Finally, the amplification induced by the presence of capital accumulation appears to be more muted compared to the homogeneous hours' case. This can be seen either moving from left to right in row two or moving down in columns 3 and 4 of table 2.

In Appendix G we show that the dampening channels identified here are robust to alternative calibrations of key parameters like the Frisch elasticity of labor supply.

In all, we find that different layers of heterogeneity on the household side and different forms of real and nominal frictions can alter substantially the amplification mechanism of monetary policy at increasing levels of inequality. More inequality does not necessarily generate a stronger response of aggregate demand after a monetary policy shock. In accordance with previous findings in the literature, it does so when wages are flexible and when there is capital income and capital accumulation. However, heterogeneous household labor supply, different intertemporal elasticities of substitution across agents, and sticky wages can significantly dampen the propagation of monetary policy. As a result, the *multiplier* of inequality on monetary policy can vary substantially, from a low of 0.04 for the case of sticky wages and heterogeneous hours and IES to a high of 3.83 for the case of flexible wages and homogeneous hours and IES.

4 Conclusion

In this paper, we study the interaction between monetary policy and labor supply decisions at the household level. Our first contribution is to establish some new empirical facts. Using survey data and a state of the art FAVAR we find that in the US and the UK the response of hours worked to monetary policy shocks across the income distribution is heterogeneous. While aggregate hours decline, the labor supply of poor households, conditionally on keeping the job, actually increases. This increase in hours at the left tail of the income distribution also pushes up their labor income pointing towards a strong income effect on labor supply for low- and moderate-income households. We also uncover that the hours' response at the left tail of the income distribution not only exhibits a different sign compared to the rest of

[58] Ascari et al. (2017) have shown the impact of sticky wages in substantially reducing the size of the parameter space generating an inverted aggregate demand logic but did not look at monetary policy amplification/dampening while Bilbiie et al. (2022) focus on the case with homogeneous hours.

the economy. It is also much larger in magnitude. We report a series of robustness checks showing that these results are not primarily driven by the extensive margin, labor demand nor household fixed characteristics, or demographics. Given that the labor supplied by low- and moderate-income households represents both a non-negligible share of the volatility and a relevant proportion of hours worked in the aggregate, these results appear to be quantitatively relevant from a macro perspective.

The second contribution of the paper is to study the implications of the behavior of the left tail of labor supply after a monetary policy shock for the tale of the monetary policy transmission mechanism. We use a two-agents New-Keynesian model where we model the left tail of the income distribution as *poor* Hand-to-Mouth (HtM). We show that our empirical findings can be replicated with a lower intertemporal elasticity of substitution (IES) for the Hand-to-Mouth households, and the latter can be micro-founded with Stone-Geary preferences that generate IES increasing in the agent's level of consumption. This setup generates interesting implications for the effectiveness of monetary policy transmission. In particular, we uncover a novel *dampening* channel on aggregate demand through which inequality affects monetary policy. Allowing for heterogeneous labor supplies and IES introduces a substantial heterogeneity in the marginal rate of substitution between consumption and leisure across agents. Therefore the HtM that are able to stay in the labor market have an extra tool to use in response to a decline in their income. They can increase their labor supply, substituting leisure for consumption. Finally, we show how this dampening channel interacts with the presence of sticky wages and capital accumulation. We find that the impact of inequality on the transmission mechanism of monetary policy is highly dependent on the calibration of the model and the different combinations of nominal and real frictions. Therefore more inequality does not necessarily generate a stronger response of aggregate demand after a monetary policy shock.

It is important to note that our structural analysis abstracts from the impact of monetary policy on the extensive margin. And this is likely to be quantitatively important. However, our aim here is to highlight the effects of the new dampening channel we uncover and ensure comparability with similar models in the literature. Therefore allowing for the interaction of intensive and extensive margin of labor supply and quantifying their implications for the impact of inequality on the transmission of monetary policy is left for future research.

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A Household level data

A.1 US: CEX Survey

In the CEX survey, data at the Consumer Unit (CU) level is constructed using the Consumer Expenditure Survey (CES). The component of income that is the focus of our study is labor income and hours worked. Labor income for CU $i$and time t ($L_{i,t}$) are the amount of wage and salary income before taxes received over the past 12 months. The remaining components of income are grouped into: (1) income from farm and non-farm business, (2) financial income and (3) income from social security.[59] Total hours are defined as the sum of hours worked by each individual in the CU over the past year.

As described below, we impute missing values in the CES income data in the pre-2004 period. This ensures that the data is consistent with the post-2004 data where imputation is carried out by the Bureau of Labor Statistics (BLS) prior to data release. In each year of the sample, we retain CUs from the second or the fifth interview when income data is updated. We drop observations that lie in the top or bottom 1 percent of total income. We retain CUs where the age of reference person is greater or equal to 18 and less than equal to 66.

CUs are then assigned to each quarter of the year by their date of interview (see Cloyne and Surico (2016)). They are then sorted into bins by gross income. The bins refer to five percentile groups of total income: (1) less than or equal to the 20^{th} percentile (P_{20}), (2) greater than 20^{th} and less than equal to 40^{th} percentile (P_{20-40}), (3) greater than 40^{th} and less than equal to 60^{th} percentile (P_{40-60}) (4) greater than $60^{th}$and less than equal to 80^{th} percentile (P_{60-80}) and (5) greater than 80^{th} percentile (P_{80}). Average wages, the average of the remaining components of income and average hours are computed in each of these groups using survey weights. These calculations are repeated for each year in the sample (1984-2018) delivering a quarterly time-series for each group.

In terms of CES variables wages are defined as the variable FSALARYX. Salary income from other sources is defined as the sum of income or loss from farm (FFRMINCX) and non-farm business (FNONFRMX) received by CU members over the past year. Financial income is the sum of interest on saving accounts/bonds (INTEARNX), income from dividends royalties, estates, or trusts (FININCX), net income or loss from roomers or boarders (INCLOSSA) and net income or loss from other rental units (INCLOSSB). Income from social security is defined as the sum of social security and railroad retirement income (FRRETIRX), supplemental security income (FSSIX), unemployment compensation (UNEMPLX), workers' compensation and veterans' payments (COMPENSX), public assistance or welfare

[59] Finanical income = Interest on saving or bonds + Amount of regular income from dividends royalties, estates, or trusts + Amount of income from pensions or annuities from private companies + Amount of net income or loss from roomers or boarders +Amount of net income or loss from other rental units. Social Security income = Amount of Social Security and Railroad Retirement income + Unemployment Compensations + Workers Compensation and veterans' payments + public assistance or welfare including job grants plus food stamps.

(WELFAREX) and food stamps (JFDSTMPA).

Total hours for each member of the household are defined as weekly hours worked (INC_HRSQ) times the number of weeks worked over the past year (INCWEEKQ). These are then summed across CU members.

Figure A.1 displays demographic and financial characteristics of the CUs for each income percentile in the US. See Figure A.2 for the evolution over time of the different income components. The CUs in the 1^{st} quintile of income are mostly females, less than 20% are college graduates and are non-white survey participants. Most of these households are renters and only 10% own a house through a mortgage. A 20% is out-right homeowners, usually to inherited properties. In terms of financial assets, the 20^{th} percentile has on average less than 2% of its total income in the form of liquid assets (e.g. checking and savings accounts, bond, stocks and other securities). While labor income is the most important source of total income, it is still the lowest percentage compared to other percentiles (only 50%). Social security transfers are a very important source as they comprise around 40% of total income. On the other tail of the distribution, the 5^{th} quintile consists mostly of white males, relatively

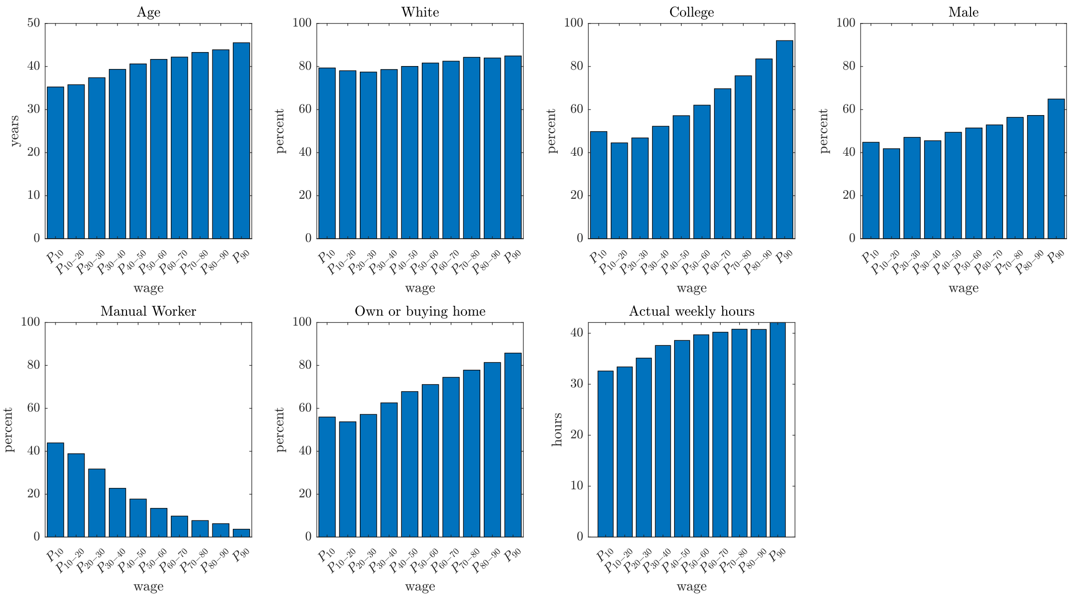


Figure A.1: Components of income in the US by percentile group.

older to the other bins, who are college graduates. Around 70% in this bin owns a property through mortgage while less than 10% rents one. Their liquid assets comprise 20% of their total income which is the highest to all other bins. This percentage may appear relatively low for this income group but as noted by Coibion, Gorodnichenko, Kueng and Silvia (2017) for the US, the contribution of financial income is fairly small throughout the sample because the CEX does not include reliable measures of household wealth. Therefore more than 80% of total income in this quintile comes from wages while this group scores the highest amount of hours worked per week by the reference person.

Figure A.2 displays the different components of income as a proportion of the total and their evolution over time. They show that their proportion has been relatively constant over time.

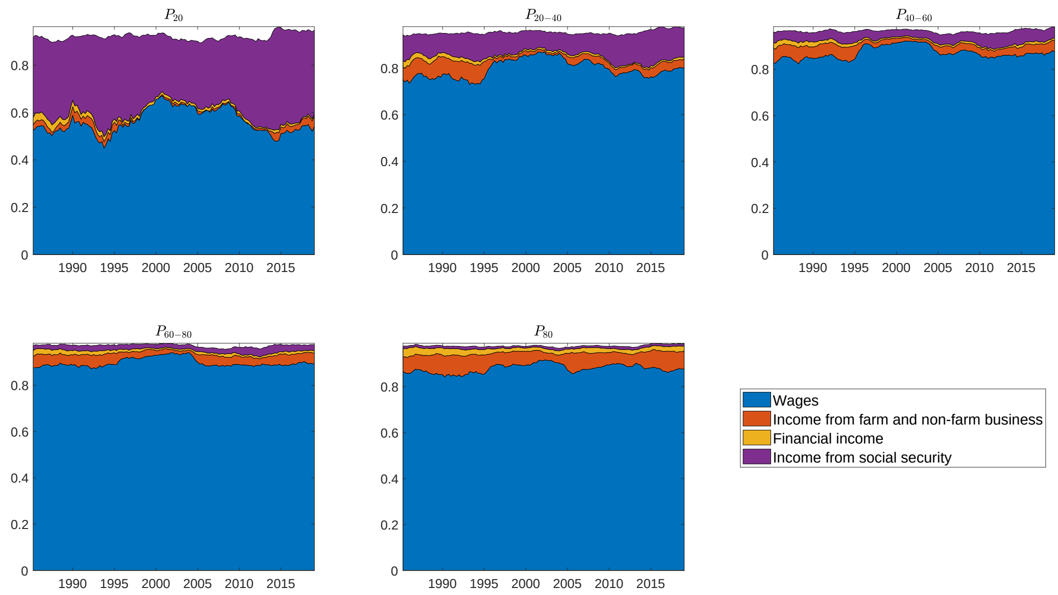


Figure A.2: Components of income in the US by percentile group. The data is smoothed using a 4 quarter moving average.

Imputation of income data After 2004, income data in the CES is imputed. In order to ensure that pre-2004 data is consistent with this approach, we follow papers such as Coibion et al. (2017) and impute components of total income where there are missing values due to an invalid non-response (see also the description here). We first estimate a logistic regression where the dependent variable is coded as 1 if the CU receives income from a particular source and zero otherwise. The independent variables are the same as those used for the imputation procedure described below. The logistic regression provides a probability that the CU received this income component. If this probability exceeds a random draw from the uniform distribution for a valid non-response/zero response, this observation is changed to an invalid response and is imputed in the next step. The approach to imputation is based on Coibion et al. (2017) who closely replicate the procedure used by CES. Imputed data is obtained as fitted values from a regression (using sampling weights) of the income component of valid reporters on age, the square of age, month of interview, sex, race, education, number of weeks worked, family size, occupation, region, marital status, total consumption expenditure, number of persions below 18, number of persons below 18, number of persons above 64 and the number of earners. Note that the median is subtracted from the dependent variable before the procedure and added back after the fitted values are obtained. The coefficients are shocked and used to produce predicted values. These are assigned to missing values using predicted mean matching. We produce 5 replicates, with the final imputed data taken to be the mean across these replications. Any income variables that do not allow negative numbers are bottom coded at 0.

This procedure is applied to the data pooled over samples of 5 years. In the regressions described above, we use a quadratic time-trend to account for growth over time. The regressions use survey weights.

A.2 US: CPS Survey

For the US, individual level data is obtained from the current population survey (CPS) for each month from 1994 to 2019. For earnings we use the outgoing rotation group.[60] Our measure of hourly earnings is constructed by using the variable HOURWAGE, the amount earned per hour in current job reported by respondents who are paid hourly. For respondents that are paid weekly, we construct earnings by dividing the variable EARNWEEK (usual weekly earnings) by UHRSWORK1 (usual weekly hours in main job). We use actual hours worked in the main job (AHRSWORK1) as our main measure of weekly hours. We drop respondents that lie in the top and bottom percentile of the earnings distribution or are aged less than 18 or more than 66.

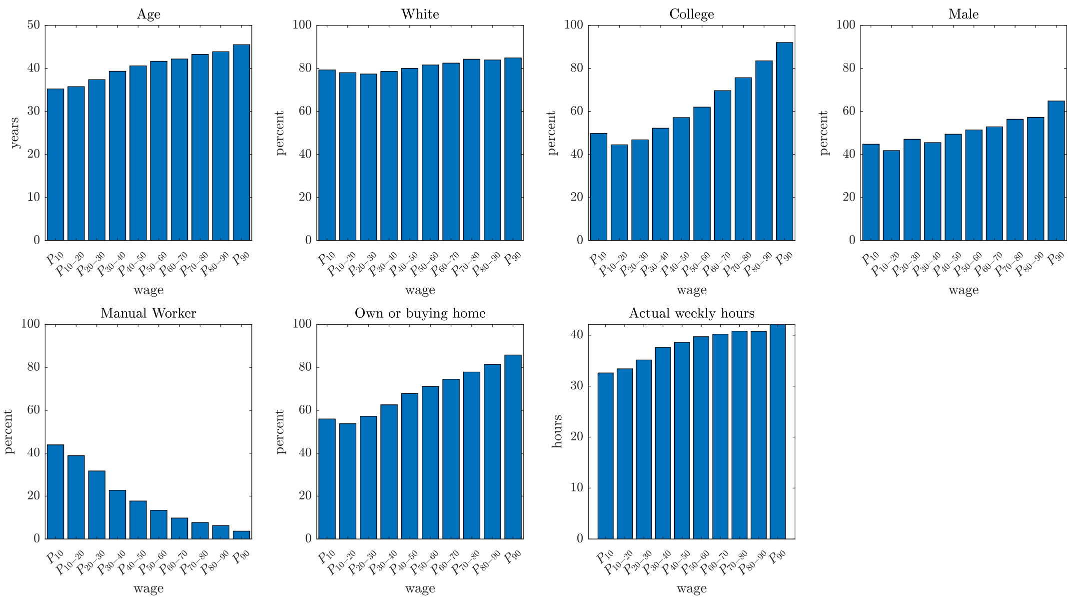


Figure A.3: The earnings distribution for the US in 2010.

For each month, we construct the earnings percentile groups $P_{\leq J}$ and $P_{>J}$ where Jranges from 5 to 95 with increments of 5. For example when $J = 5$, the two groups consist of respondents the fall below that 5th percentile of hourly earnings ($P_{\leq 5}$) and those that lie

[60]Households are interviewed in the CPS for four months and then again for four months after a eight month break. In the fourth and eighth month of interviews, when households are about to rotate out of the interviews, they are asked additional questions regarding earnings. For more details on the outgoing rotation group, see https://cps.ipums.org/cps/outgoing_rotation_notes.shtml

above the 5th percentile ($P_{>5}$). We calculate average earnings and average weekly hours for each group using the survey weights. Repeating this across all months in the sample provides a time series of average earnings and average weekly hours by percentile group. Figure A.3 provides information regarding the characterstics of the earnings distribution. Respondents in the right tail of the distribution tend to be older, better educated, are likely to work longer hours, own their home, be white and Male and less likely to be employed in a manual job.

A.3 UK: FES Survey

Data for household level hours worked and income is obtained from the Food and Expenditure survey (FES). In 2001 FES merged with the National Food Survey and became the Expenditure and Food Survey (EFS) and with the Living Costs and Food Survey (LCFS) in 2008. Wages are given by the code P008. Income from other sources is the sum of wage from subsidiary employment (P011) plus income subsidiary self-employment (P037) plus income from self-employment (P047). Financial income is the sum of income from investment (P048) and income from pensions annuities (P049). Income from social security is the code P031. Finally, hours are defined by the code A220. These variables are summed across the members of each household.

As in the case of the US, we obtain data on the components of gross income: wages, income from self employment, financial and investment income and income from social security. Total hours are the sum of hours worked by each member of the household.

We drop observations that lie in the top or bottom 1 percent of total income and retain households where the age of reference person is greater or equal to 18 and less than equal to 66. The households are assigned to quarters in each year according to the date of interview. They are then sorted into bins based on percentile groups of income as defined for the US above. Average wages, average hours and the average of other components of income are then constructed (using survey weights) for each quarter from 1979Q1 to 2016Q4.[61]

A similar picture is depicted for the UK households' demographic and financial characteristics as it can be seen in Figure A.4. The head of the household is predominately male in the high income quintiles while in the low income ones is mostly female. Non-white heads of households can be found mostly in the low income percentiles but they are in small proportions throughout the distribution. This can be due to issues of under representation of minorities in the FES survey (van de Ven (2011)). The level of education is not available for the head of the household so we proxied this by the occupational class held by the head. Impressively, more than 50% of the respondents in the 5th quintile hold managerial positions while less than 5% are managers or employers in the 1^{st} income percentile. In terms of housing status and personal indebtedness, more than 80% in the 5^{th} quintile hold properties

[61] After 1998, sampling weights are included in the FES data set. Before 1998, weights (grossing factors) are constructed by using household characteristics (see Table 3.1 in Banks, Tanner and Webb (1997)).

through mortgages while this is less than 20% in the low income quintile, where around 50% of households are renters. Wage is the most important component of income for all quintiles and contributes to more than 80% of total income in high income ones. A main source of income for low-income households is social security benefits.[62] Finally, the weekly amount of hours worked rises to the income level with the total in richer households larger than those at the left tail of the distribution.

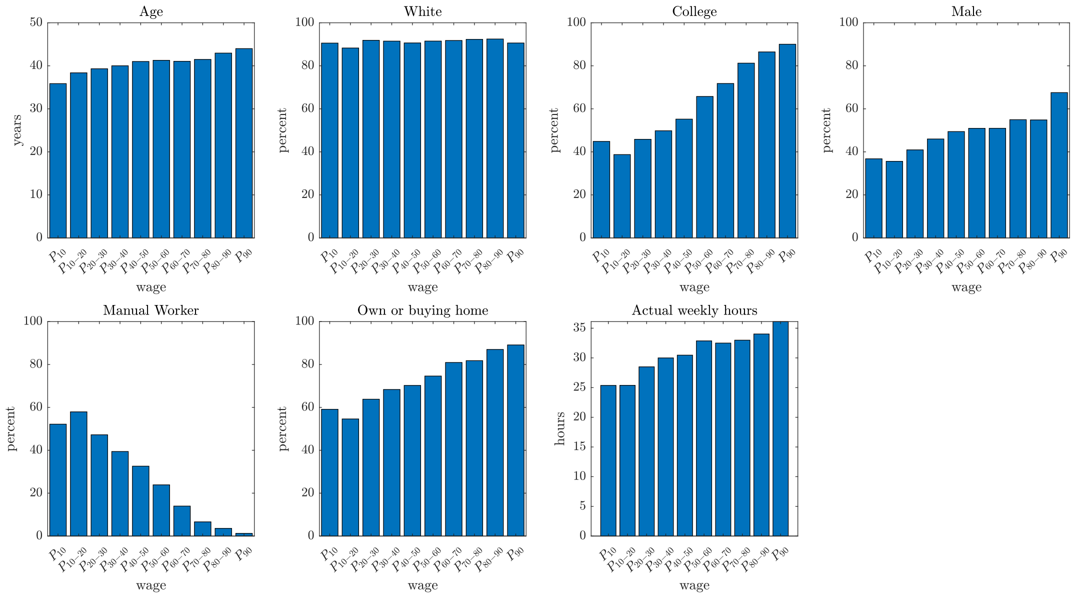


Figure A.4: Components of income in the UK by percentile group.

Figure A.5 displays the different components of income as a proportion of the total and their evolution over time. They show that their proportion has been relatively constant over time for households at the right tail of the income distribution.

A.4 UK: LFS Survey

We obtain quarterly data for the UK for the period 1994 to 2019 from the Labour Force Survey (LFS). Our measure of hourly earnings is the variable HOURPAY. We use actual hours worked for each individual (TTACHR). We apply the filters described above to UK data – observations that lie in the top and bottom percentiles of the wage distribution are dropped and individuals older than 18 and younger than 66 are included in the sample. As in the case of the US, we construct a pseudo panel using the earnings percentile groups $P_{\leq J}$ and $P_{>J}$.

Figure A.6 shows that the characteristics of the UK earnings distribution are very similar to that of the US. Low earners in the UK tend to be younger, less educated, employed in

[62] Here we decompose income only for employed households during the surveys, otherwise the contribution of benefits to total income is higher than 40% in the 1st income quintile.

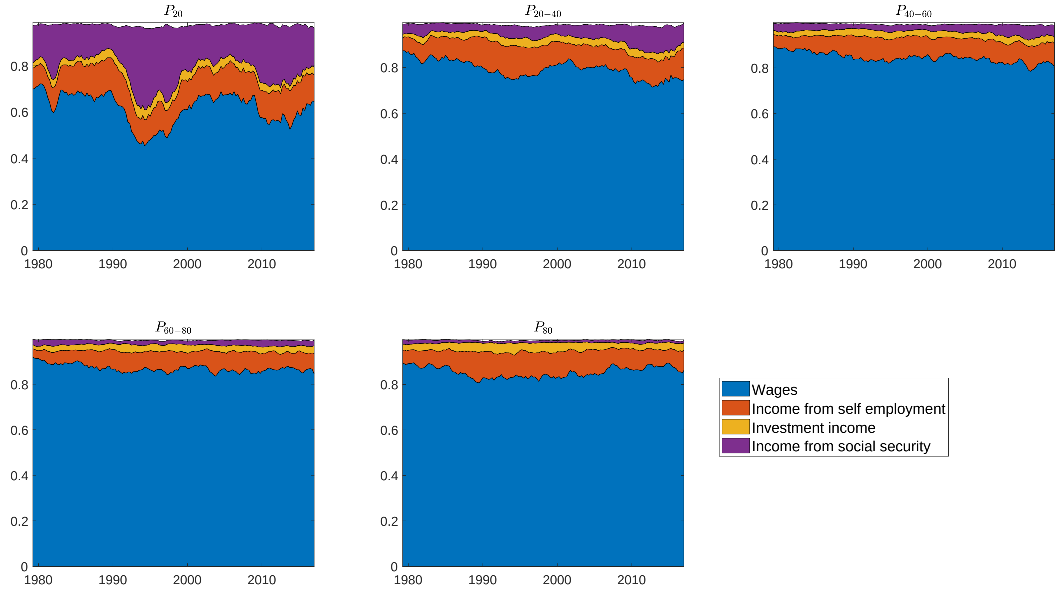


Figure A.5: Components of income in the UK by percentile group. The data is smoothed using a 4 quarter moving average.

manual labour. They are less likely to be male and to own a home and their average weekly hours are shorter.

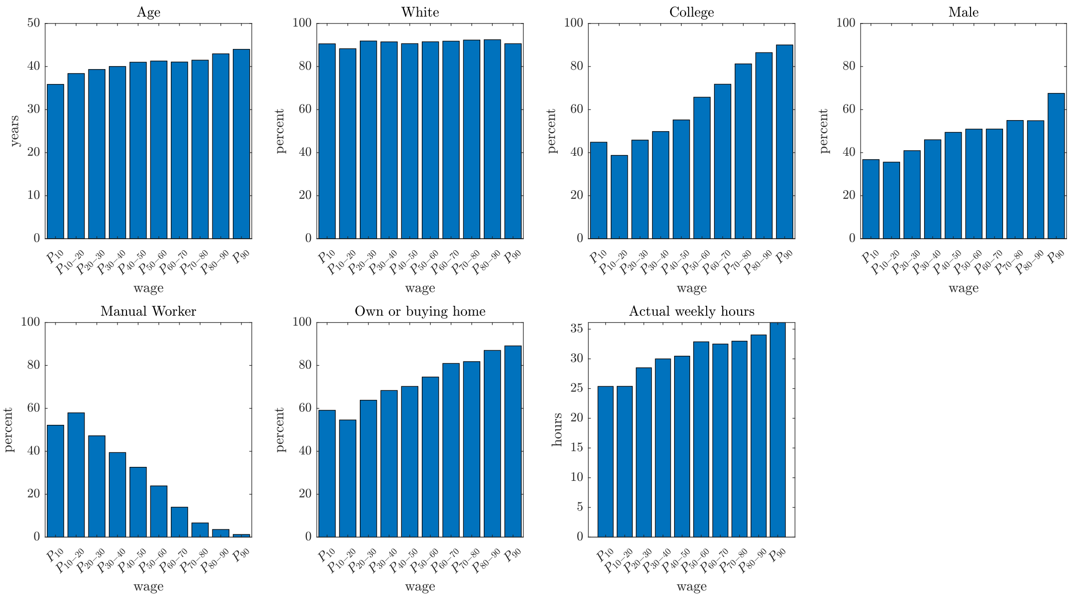


Figure A.6: The earnings distribution for the UK in 2010.

A.5 Comparison with aggregate data

The top panel of Figure A.7 compares aggregate actual hours from the CPS to a measure of monthly hours from the Bureau of Labour statistics (average weekly hours of production and non-supervisory employees). Note that the data is standardised.[63] The CPS data captures the main movements in the aggregate data fairly well. The bottom panel of the figure shows a comparison of the aggregate usual hours constructed from the CEX and total hours in the non-farm business sector from the Bureau of Labour statistics. Again, the survey-based aggregate is a good approximation of the official aggregate data.

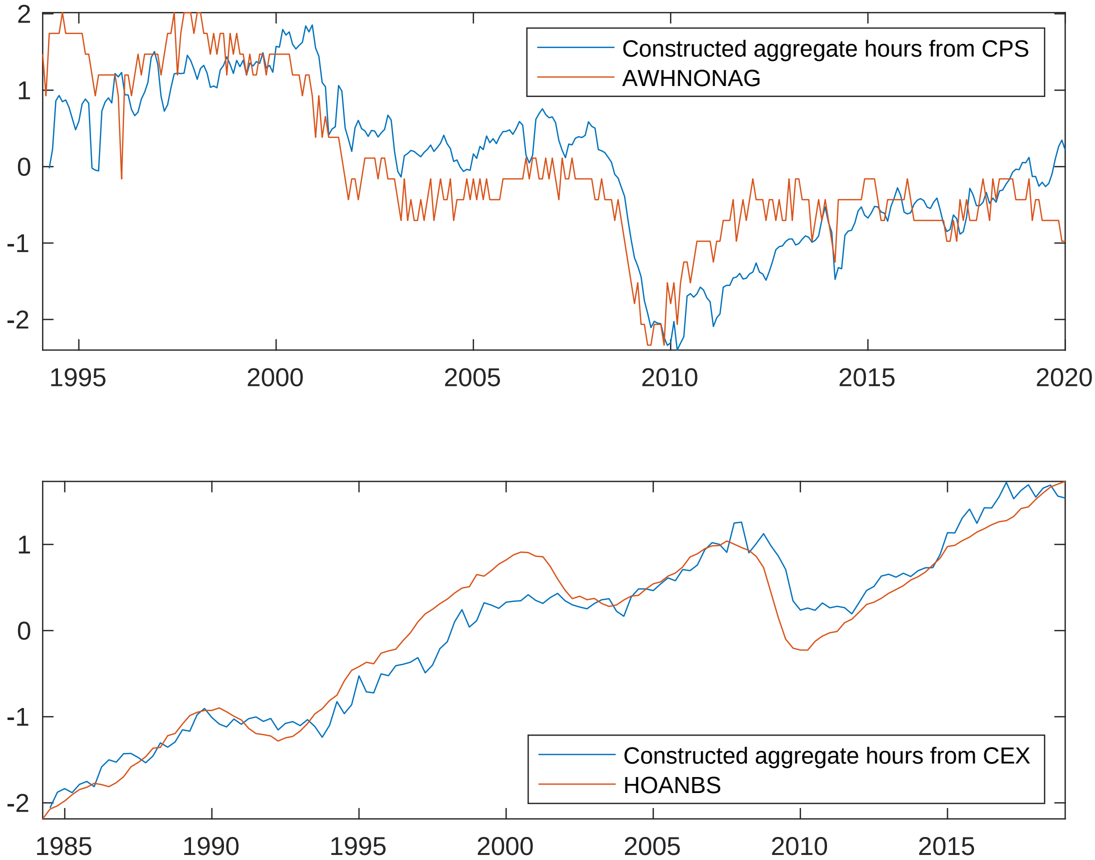


Figure A.7: Comparison of survey based total hours (blue) with aggregate (orange), US.

Figure A.8 shows this comparison for the UK. The top panel shows that aggregate hours constructed using our LFS sample reasonably closely with the aggregate (code YBUY) published by the Office of national statistics (ONS), with a correlation of 0.9. The bottom panel of the Figure shows that the downward trend in the FES based measure of hours matches the aggregate data (YBUY spliced with YBUS divided by MGRZ in the pre-1992 period).

[63] As our sample is affected by the filters we apply we do not expect to match the level of the aggregate data. The comparison in this section focusses on the trends and cylical movements in the data.

However, it is evident that the correlation deteriorates at the end of the sample. This is probably because the FES survey asks about usual hours while the ONS aggregate measures actual hours worked.

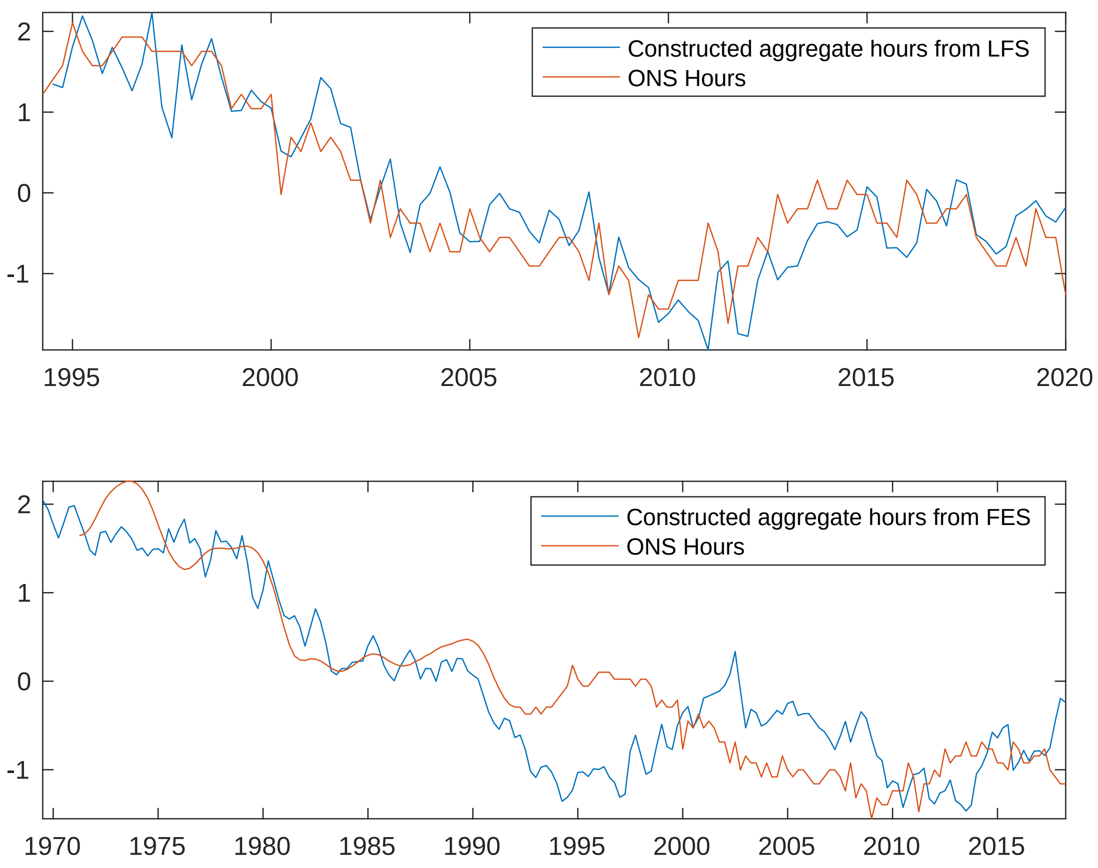


Figure A.8: Comparison of survey based total hours (blue) with aggregate (orange), US K.

B Priors and estimation of the FAVAR

The FAVAR model is defined by the following equations:

$$X_t = c + b\tau + \Lambda F_t + \xi_t \tag{B.1}$$

$$\begin{aligned} Y_t &= c + \sum_{j=1}^{P} \beta_j Y_{t-j} + u_t && \text{(B.2)} \\ cov\left(u_t\right) &= \Sigma = A_0 A_{0'} && \text{(B.3)} \end{aligned}$$

Where $Y_t = \underbrace{\begin{pmatrix} R_t \\ F_t \end{pmatrix}}_{N\times 1}$ and $R_t$denotes the 1 year interest rate. As described in Barigozzi, Lippi and Luciani (2016), the factors can be consistently estimated using a principal components (PC) estimator. In particular, the factor loadings are estimated via PC analysis of the first differenced data ΔX_t. With these in hand, the factors are estimated as $\hat{F}_t = \hat{\Lambda}'_S \left(X_t - \hat{c}_S - \hat{b}_S \tau\right)$.

Given the estimated factors, the VAR in equations B.2 is estimated using a Gibbs sampling algorithm.

B.1 Priors

Denote the var coefficients as $B = vec\left([\beta_1, \beta_2, .., \beta_P, c]\right)$. We employ a Minnesota type prior for the VAR parameters. This involves a normal prior for B and an inverse Wishart prior for Σ set using dummy observations. The prior for B is $N\left(\tilde{b}_0, H\right)$ while the prior for $\Sigma\tilde{\ } IW(\Sigma_0, T_0)$. The parameter controlling the tightness of the prior is set to 0.2.

B.2 Gibbs sampling algorithm

The algorithm samples from the following conditional posterior distributions:

1. $G\left(B|\Sigma\right)$. Denote the RHS variables in the VAR as X_t and $\hat{B} = vec\left(\left(X'_t X_t\right)^{-1}\left(X_t Y_t\right)\right)$. The conditional posterior is normal with mean

$$M = \left(H^{-1} + \Sigma^{-1} \otimes X'_t X_t\right)^{-1}\left(H^{-1}\tilde{b}_0 + \Sigma^{-1} \otimes X'_t X_t \hat{B}\right)$$

and variance:

$$V = \left(H^{-1} + \Sigma^{-1} \otimes X'_t X_t\right)^{-1}$$

.

2. $G\left(\Sigma|\Psi\right)$.This conditional posterior is inverse Wishart: $IW\left(u'_t u_t + \Sigma_0, T + T_0\right)$.

C IV Identification

For a given draw of B, Σ and u_t, we obtain the first column of $A_0$by using the procedure proposed by Mertens and Ravn (2013). We assume that the instrument is relevant and exogenous:

$$\begin{aligned} cov\left(m_t, \varepsilon_{1t}\right) &= \alpha \\ cov\left(m_t, \varepsilon_t^-\right) &= 0 \end{aligned}$$

where ε_{1t} denotes the structural shock of interest that is ordered first for convenience, while ε_t^-represent all remaining shocks and $\varepsilon_t = \begin{pmatrix} \varepsilon_{1t} & \varepsilon_t^- \end{pmatrix}$. Re-writing the relevance and exogeneity conditions in vector form:

$$E(m_t \varepsilon_t') = [\alpha \quad 0] \tag{C.4}$$

$$E(m_t \varepsilon_t' A_0') = [\alpha \quad 0] A_0' \tag{C.5}$$

$$E(m_t u_t') = \alpha a_0 \tag{C.6}$$

where a_0 is a $(1 \times R)$ vector corresponding to the first row of A_0' (hence first column of A_0). An estimate of $E(m_t u_t') = \begin{pmatrix} E(m_t u_{1t}') \\ E(m_t u_{2t}') \\ . \\ . \\ E(m_t u_{Nt}') \end{pmatrix}'$ can be easily obtained by using a linear regression. However, α on the RHS of equation C.6 is unknown. This parameter can be eliminated by normalising the left and the right hand side by dividing by the first element of $E(m_t u_t')$ and a_0, respectively. Therefore the impulse vector to a unit shock is given by $\tilde{a}_0 = \begin{pmatrix} 1 \\ \frac{E(m_t u_{2t}')}{E(m_t u_{1t}')} \\ . \\ . \\ \frac{E(m_t u_{Nt}')}{E(m_t u_{1t}')} \end{pmatrix}'$.

D FAVAR IRF

D.1 IRF of hours worked and income at different percentile groups

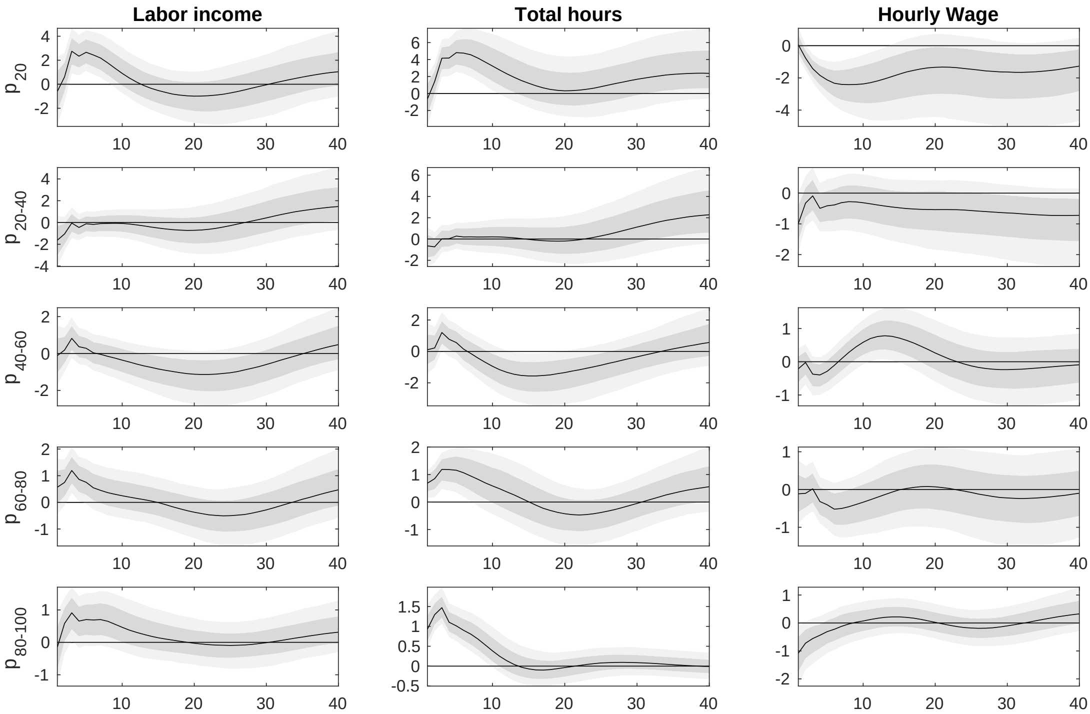


Figure D.1: Impulse responses of hours worked in each percentile group for the US - CEX.

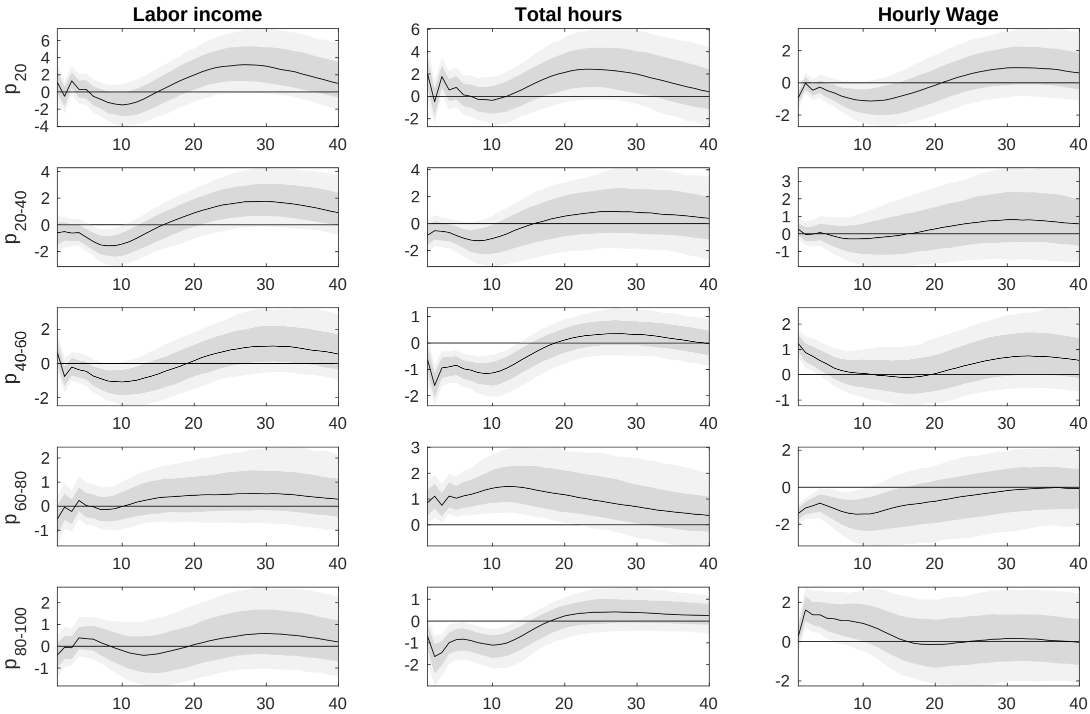


Figure D.2: Impulse responses of hours worked in each percentile group for the UK - FES.

D.2 3D responses

D.2.1 US CEX

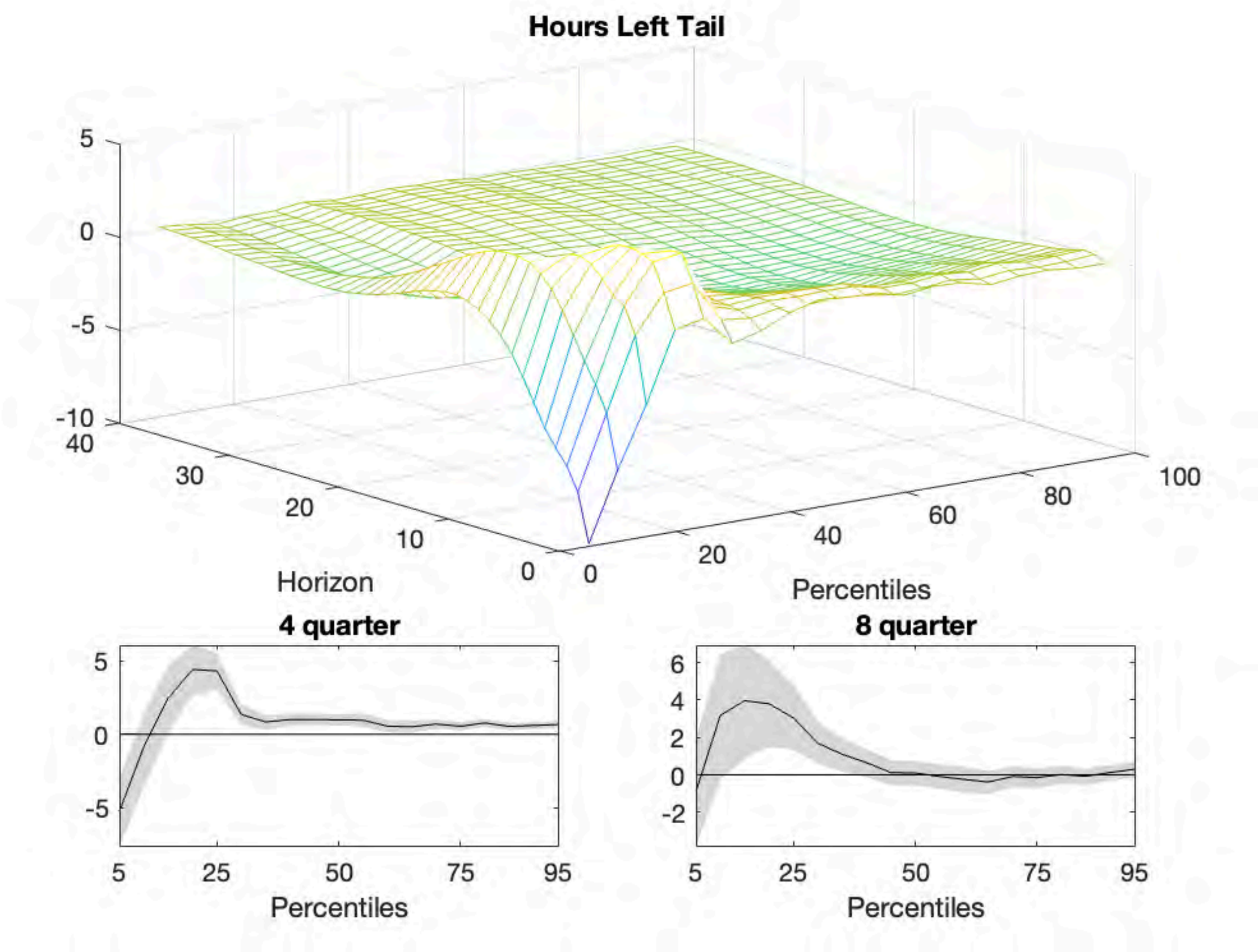


Figure D.3: 3D Impulse response of hours worked of the left tail of the income distribution in the US. (CEX)

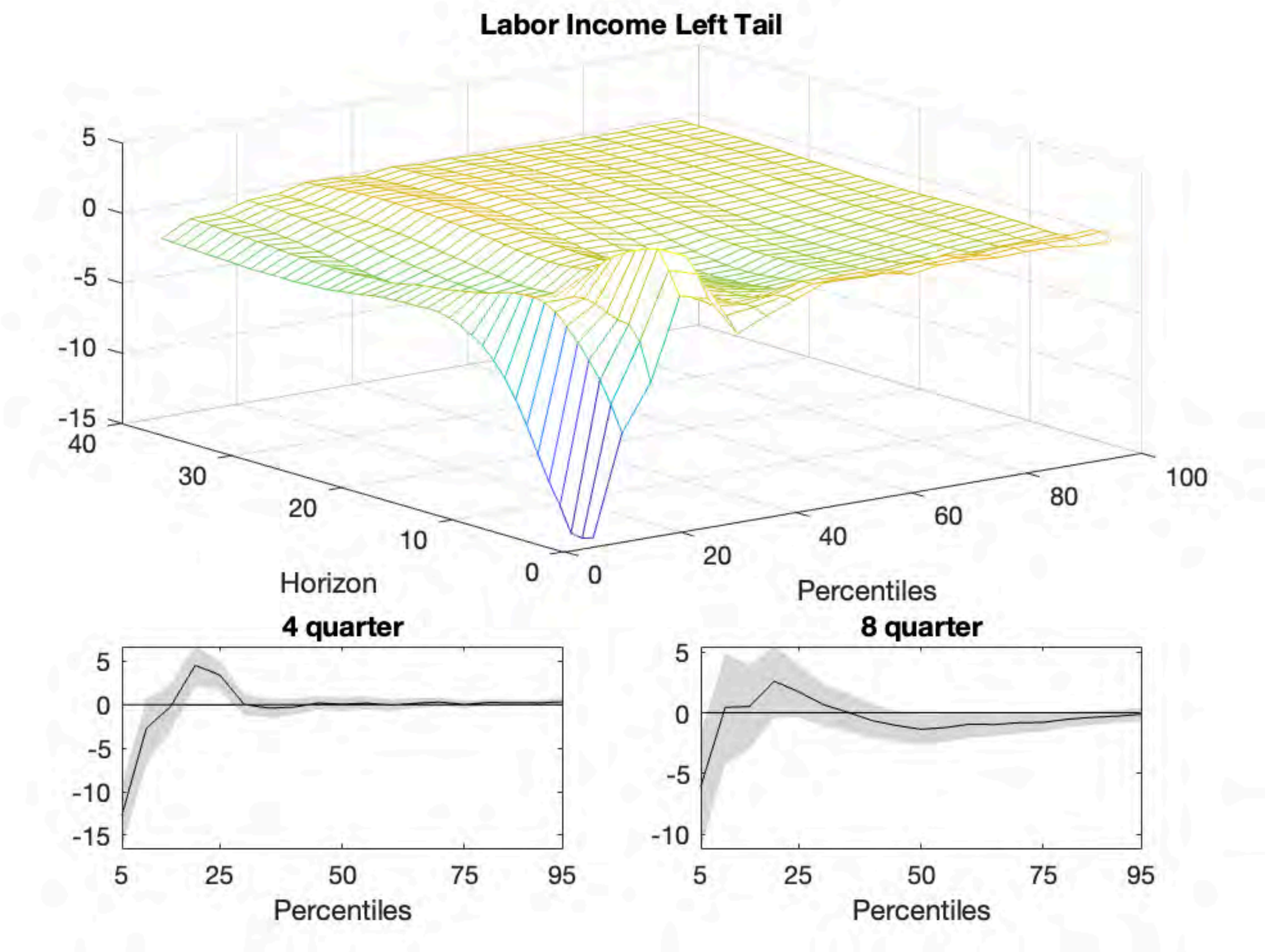


Figure D.4: 3D Impulse response of labor income of the left tail of the income distribution in the US. (CEX)

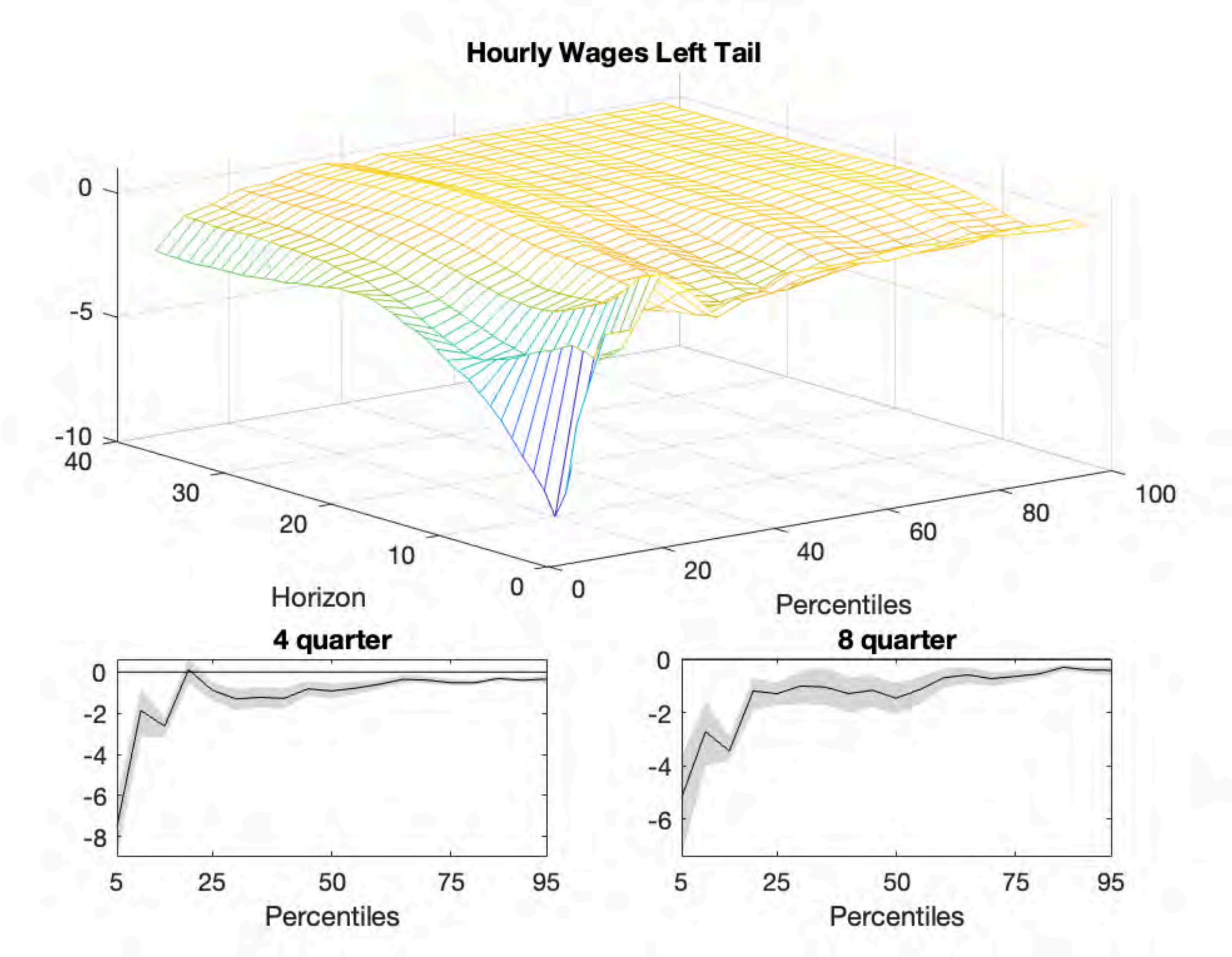


Figure D.5: 3D Impulse response of hourly wages of the left tail of the income distribution in the US. (CEX) - These are constructed by subtracting the IRFs of hours from the ones of labor income.

D.2.2 US CPS

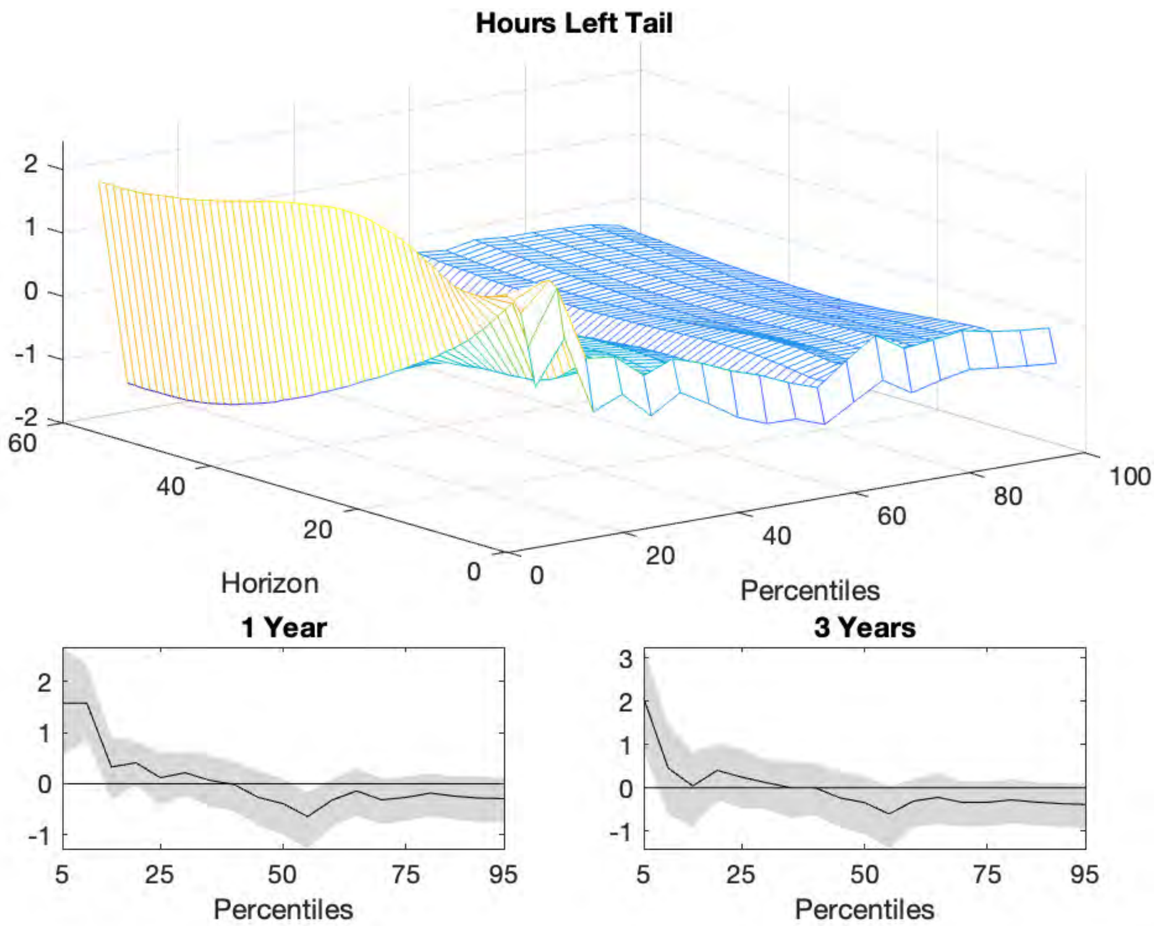


Figure D.6: 3D Impulse response of hours worked of the left tail of the income distribution in the US. (CPS)

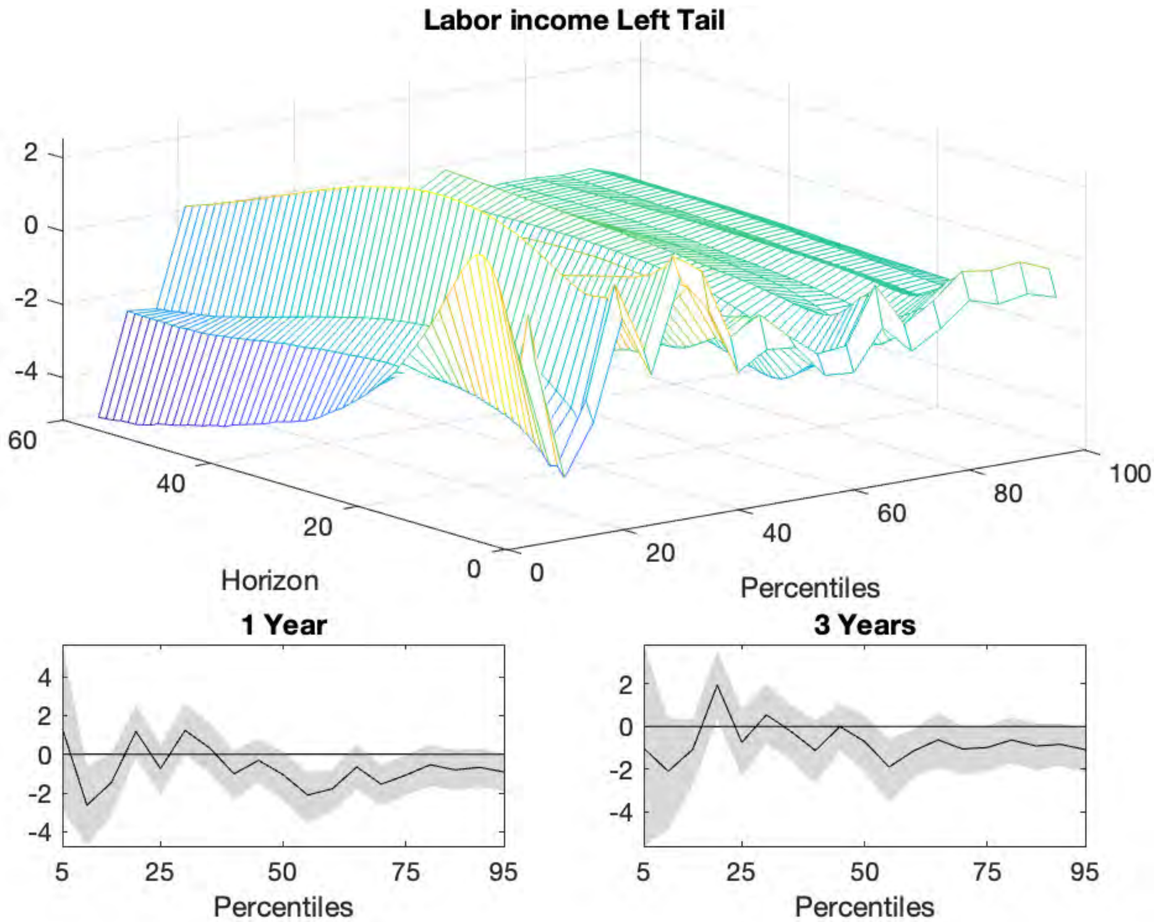


Figure D.7: 3D Impulse response of labor income of the left tail of the income distribution in the US. (CPS) - These are constructed by adding up the IRFs of hours and wages.

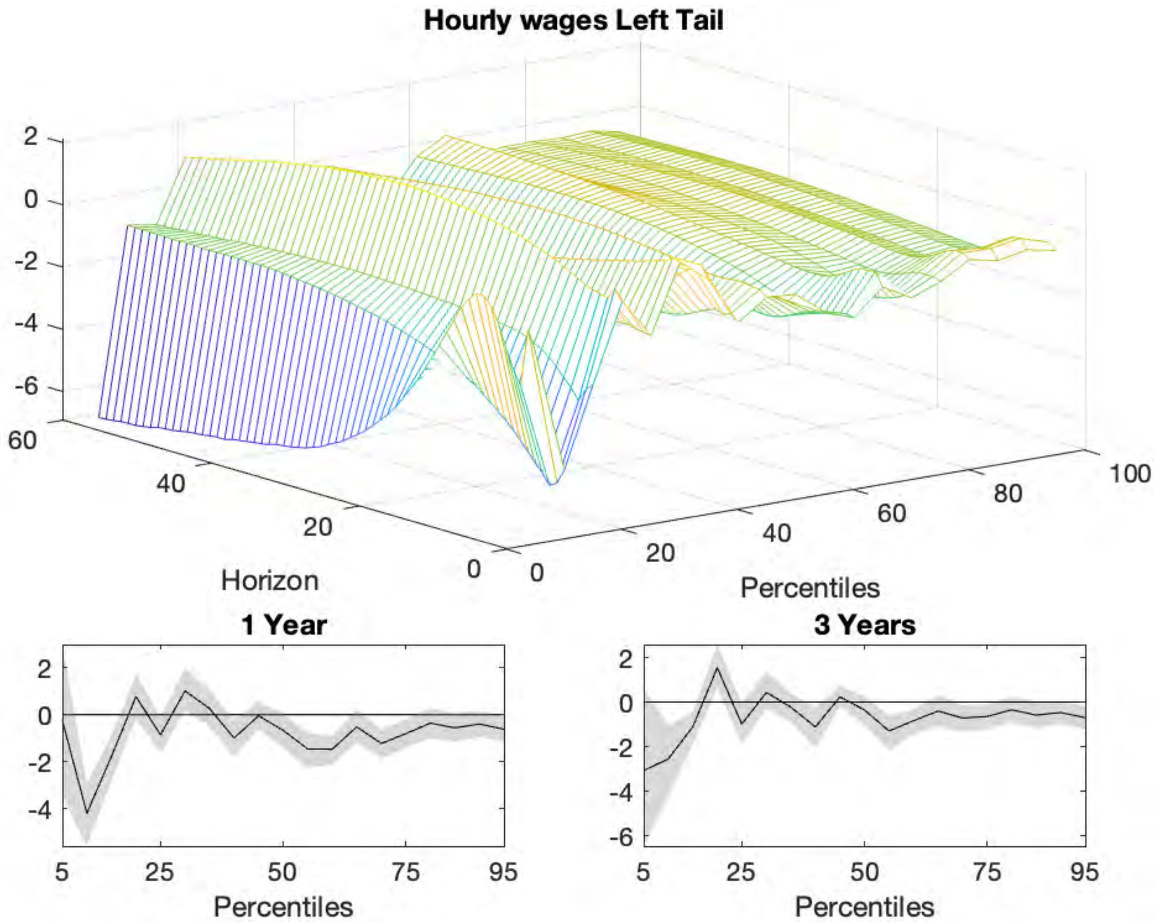


Figure D.8: 3D Impulse response of hourly wages of the left tail of the income distribution in the US. (CPS)

D.2.3 UK FES

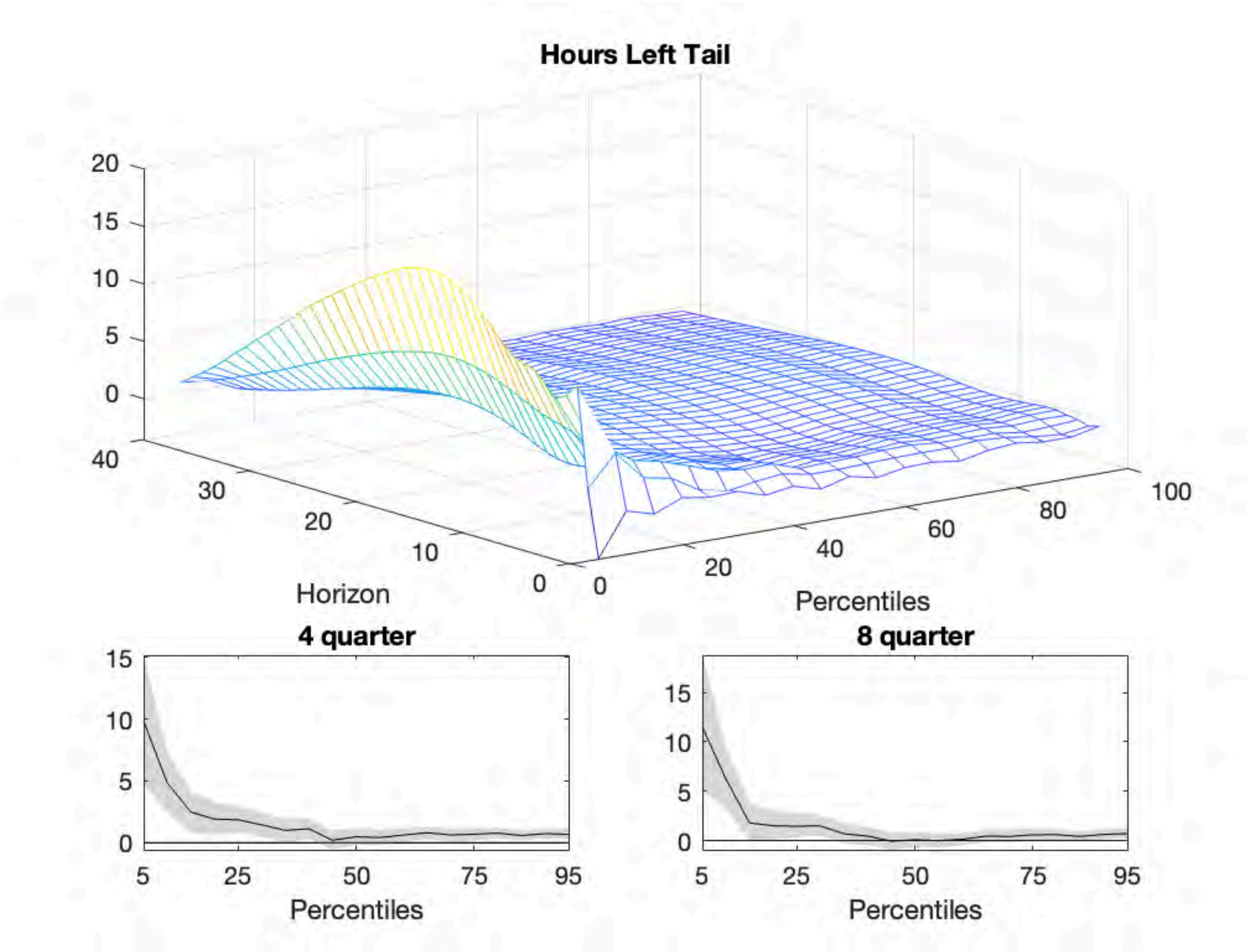


Figure D.9: 3D Impulse response of hours worked of the left tail of the income distribution in the UK. (FES)

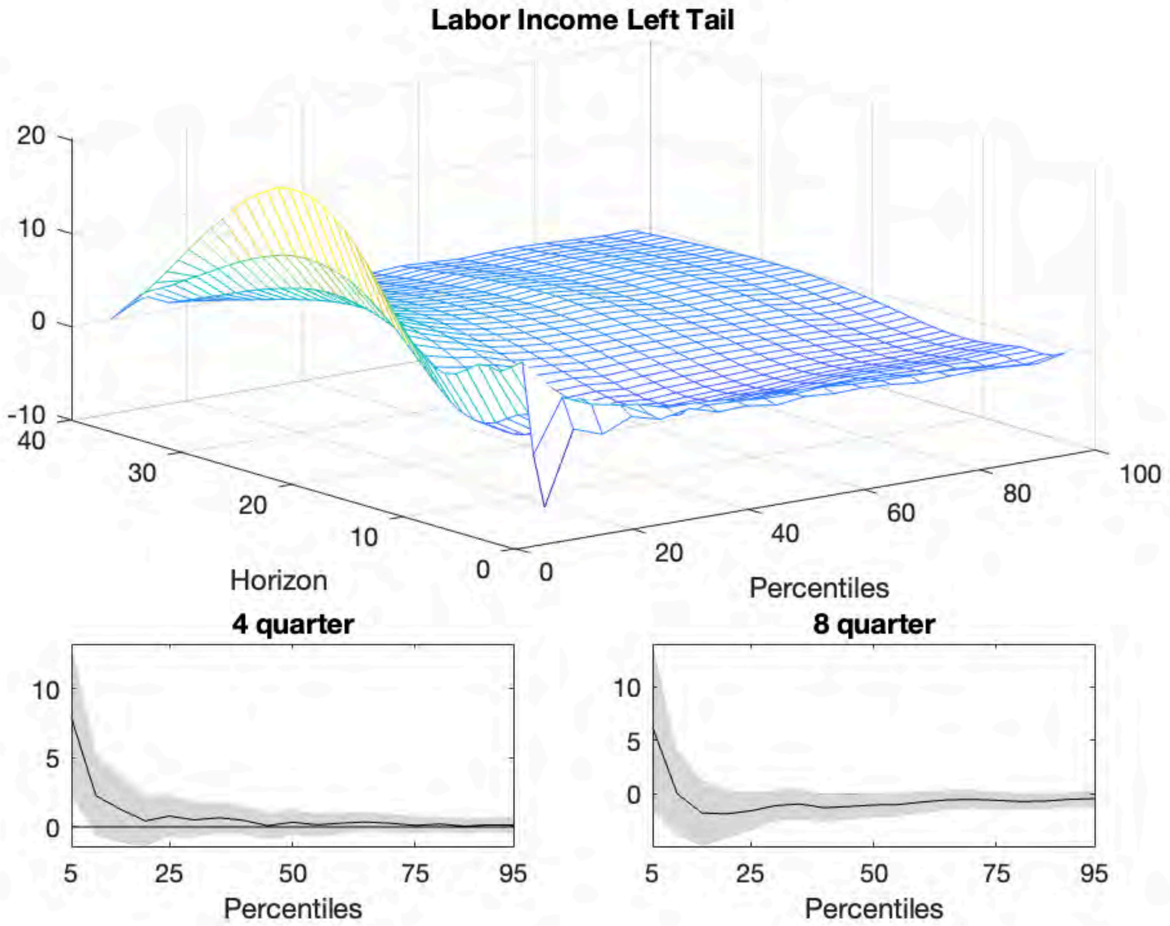


Figure D.10: 3D Impulse response of labor income of the left tail of the income distribution in the UK. (FES)

Figure D.11: 3D Impulse response of hourly wages of the left tail of the income distribution in the UK. (FES) - These are constructed by subtracting the IRFs of hours from the ones of labor income.

D.2.4 UK LFS

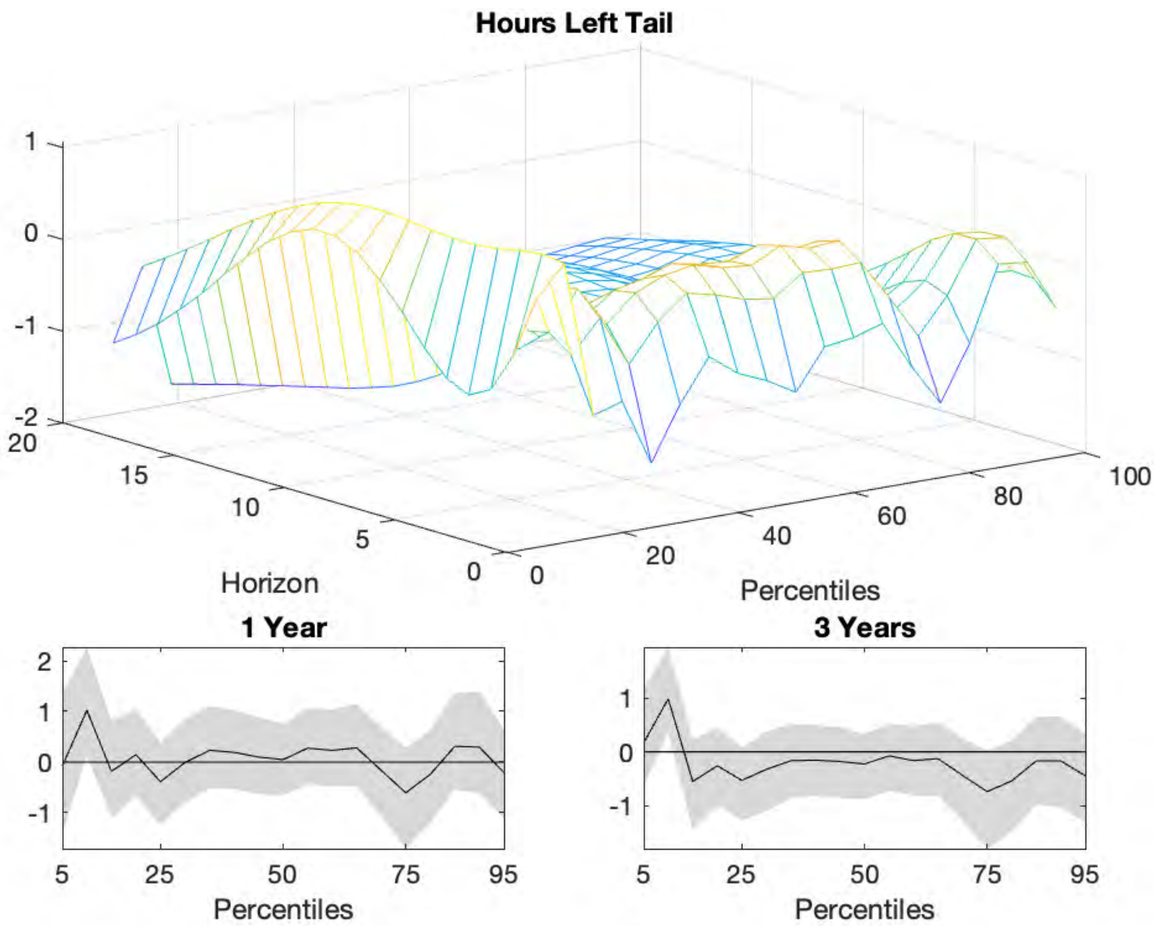


Figure D.12: 3D Impulse response of hours worked of the left tail of the income distribution in the UK. (LFS)

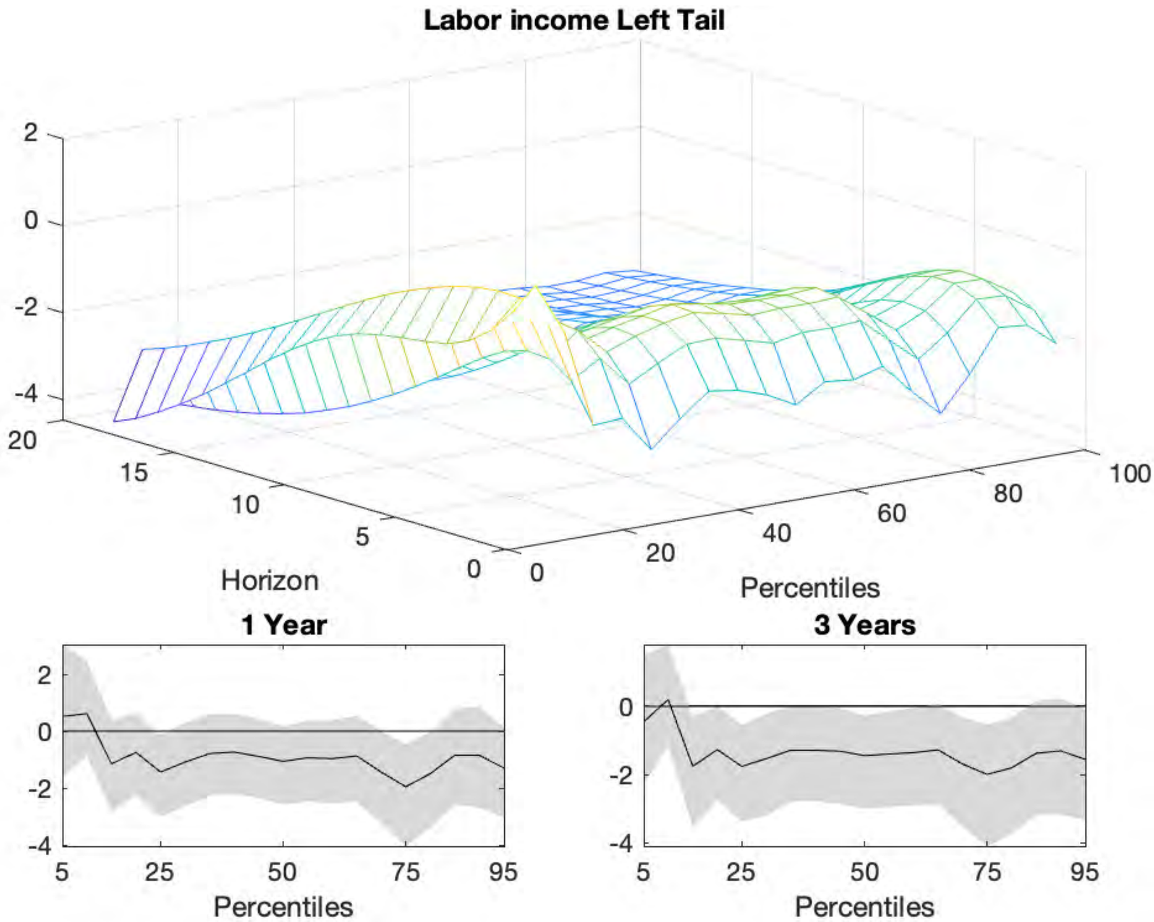


Figure D.13: 3D Impulse response of labor income of the left tail of the income distribution in the UK. (LFS) - These are constructed by adding up the IRFs of hours and wages.

Figure D.14: 3D Impulse response of hourly wages of the left tail of the income distribution in the UK. (LFS)

E Robustness

E.1 US

We employ two alternative instruments. First, we use the measure built by Miranda-Agrippino and Ricco (2021). Miranda-Agrippino and Ricco (2021) argue that high frequency instruments such as those used in Gertler and Karadi (2015) contain information about the policy shock and a signal regarding central bank's information about the state of the economy. Miranda-Agrippino and Ricco (2021) construct their proxy as the high frequency changes in federal funds futures that are orthogonal to Greenbook forecasts and data revisions. Second, we use the well known measure constructed by Romer and Romer (2004). The Romer and Romer (2004) is constructed by purging movements in the intended federal funds rate of information regarding future developments in the economy.[64] To check robustness of this IV approach, we use sign restrictions as an additional identification scheme. Under this approach, a monetary contraction is restricted to increase the 1 year rate and reduce CPI, the PCE deflator, the GDP deflator, real M2 and real GDP. With the exception of real GDP, the restrictions are imposed for the contemporaneous and subsequent three quarters. The restrictions on GDP are imposed at horizon 1 to 3, with no restriction placed on the contemporaneous response. The top panel of Figure E.1 presents the impulse responses of CEX hours in each percentile group to a contractionary policy shock. As in the benchmark case, hours rise for households below the 20th percentile.

The bottom panel of the Figure presents the response CPS hours form FAVARs that use the instrument of Miranda-Agrippino and Ricco (2021) and sign restrictions. As our sample from the CPS begins in 1994, we do not use the Romer and Romer (2004) sample.[65] The results are qualitatively similar to those reported in the benchmark.

In figure E.2 we present the impulse responses of hours in each percentile to a supply shock. As before the top panel corresponds to hours constructed in the CEX while the bottom to the CPS. We estimated supply shocks following two identification methods: a) defining the shock as the business cycle one of Angeletos, Collard and Dellas (2020); b) using sign restrictions. In both cases we see how, differently from Monetary policy shocks, hours in the left tail are procyclical. Similar to monetary policy shocks the magnitude of the response at the bottom of the distribution is the largest.

[64] As these instruments are monthly, we aggregate them by summing across the quarter.

[65] Because of the short sample the relevance of the instrument declines substantially.

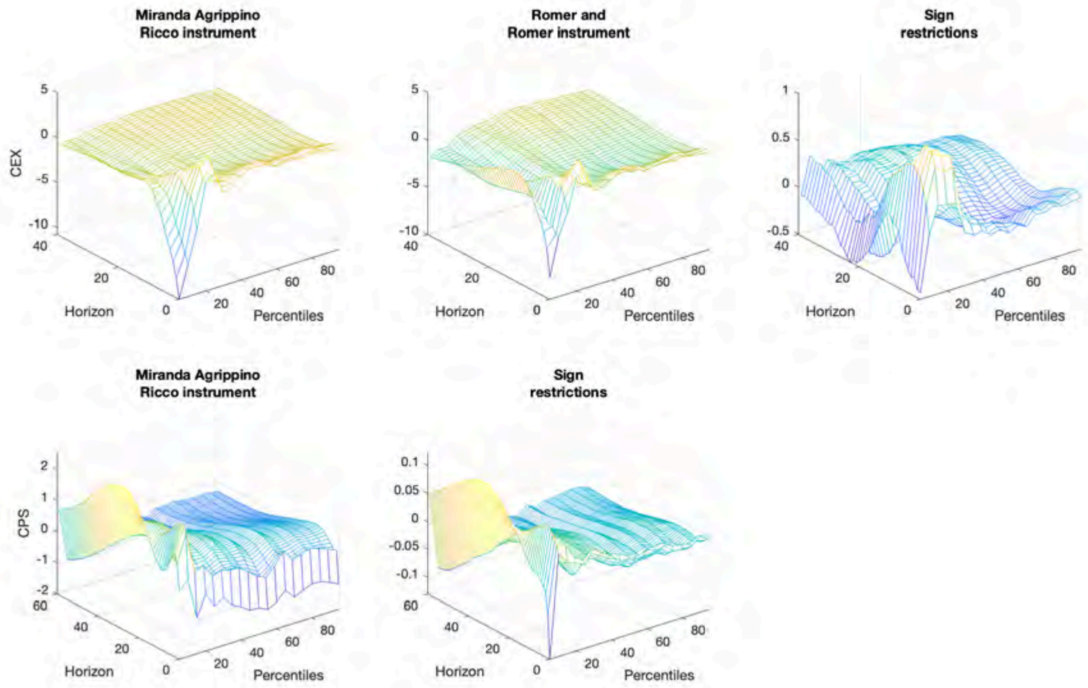


Figure E.1: Impulse responses of hours of the left tail in each percentile group for the US using alternative identifications.

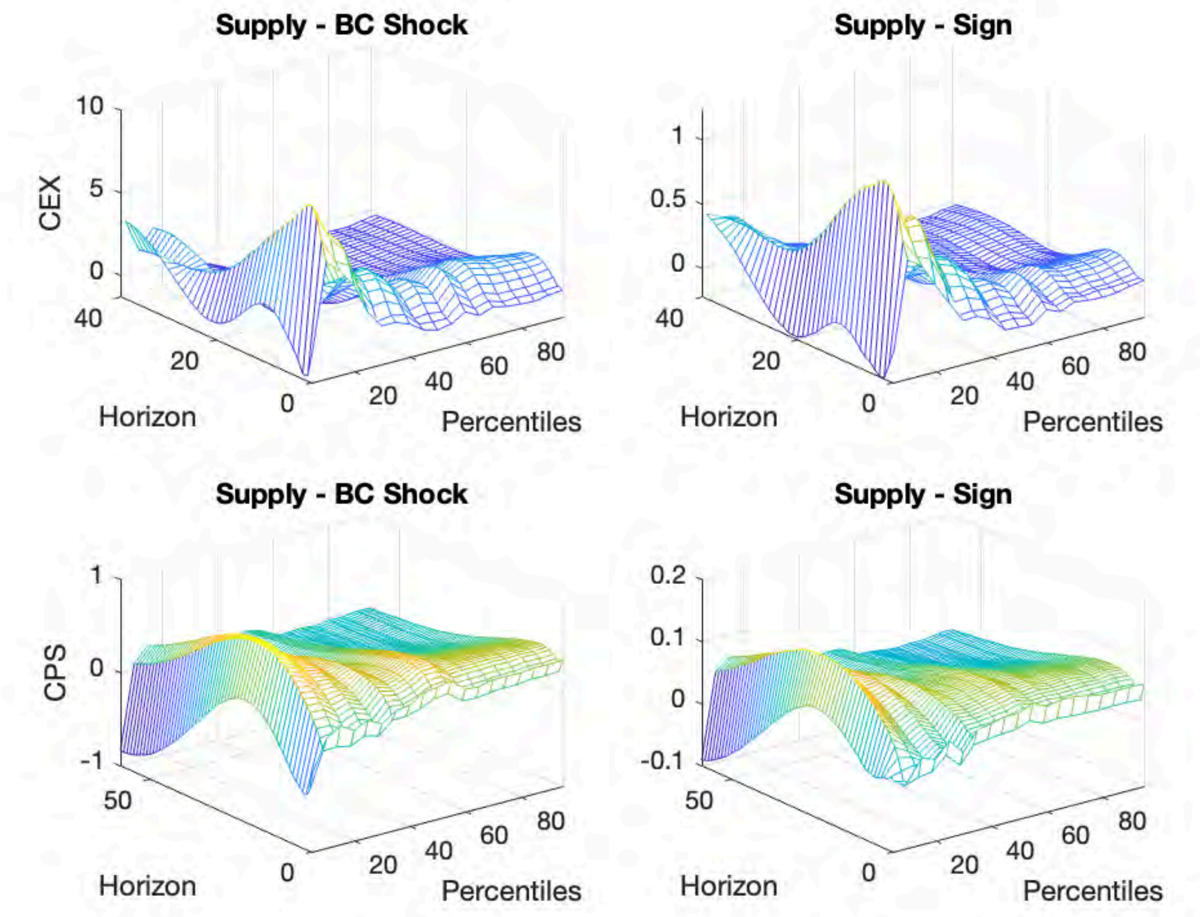


Figure E.2: Impulse responses to a supply shock of hours of the left tail in each percentile group for the US.

E.2 UK

We use the proxy for monetary policy shock estimated by Cloyne and Hurtgen (2016) by applying the procedure of Romer and Romer (2004) to the UK. This is used as an alternative instrument when we use the FES data over the longer sample period[66] As in the case of the US, a sign restriction scheme is used to check the robustness of the IV estimates. A monetary contraction is restricted to increase the 1 year rate and reduce CPI, RPI, the GDP deflator, M4 and real GDP. With the exception of real GDP, the restrictions are imposed for the contemporaneous and subsequent three quarters. The restrictions on GDP are imposed at horizon 1 to 3, with no restriction placed on the contemporaneous response.

The results are presented in Figure E.3. The top panel of the Figure shows the response of the distribution of hours from the FES. The bottom panel shows the results based on LFS data for hours. In all cases, the response of hours is similar to benchmark with hours rising for households (individuals) on the left tail of the income (wage) distribution.

In figure E.4 we present the impulse responses of hours in each percentile to a supply shock. As before the top panel corresponds to hours constructed in the FES while the bottom to the LFS. We estimated supply shocks following two identification methods: a) defining the shock as the business cycle one of Angeletos et al. (2020); b) using sign restrictions. Results are in line with what we find for the US. Hours at the bottom exhibit stronger and procyclical responses to supply shocks.

[66] This proxy is only available in the pre-2009 period. Therefore the relevance of this instrument drops when the sample is restricted to start from 1994 for the estimation with LFS survey data.

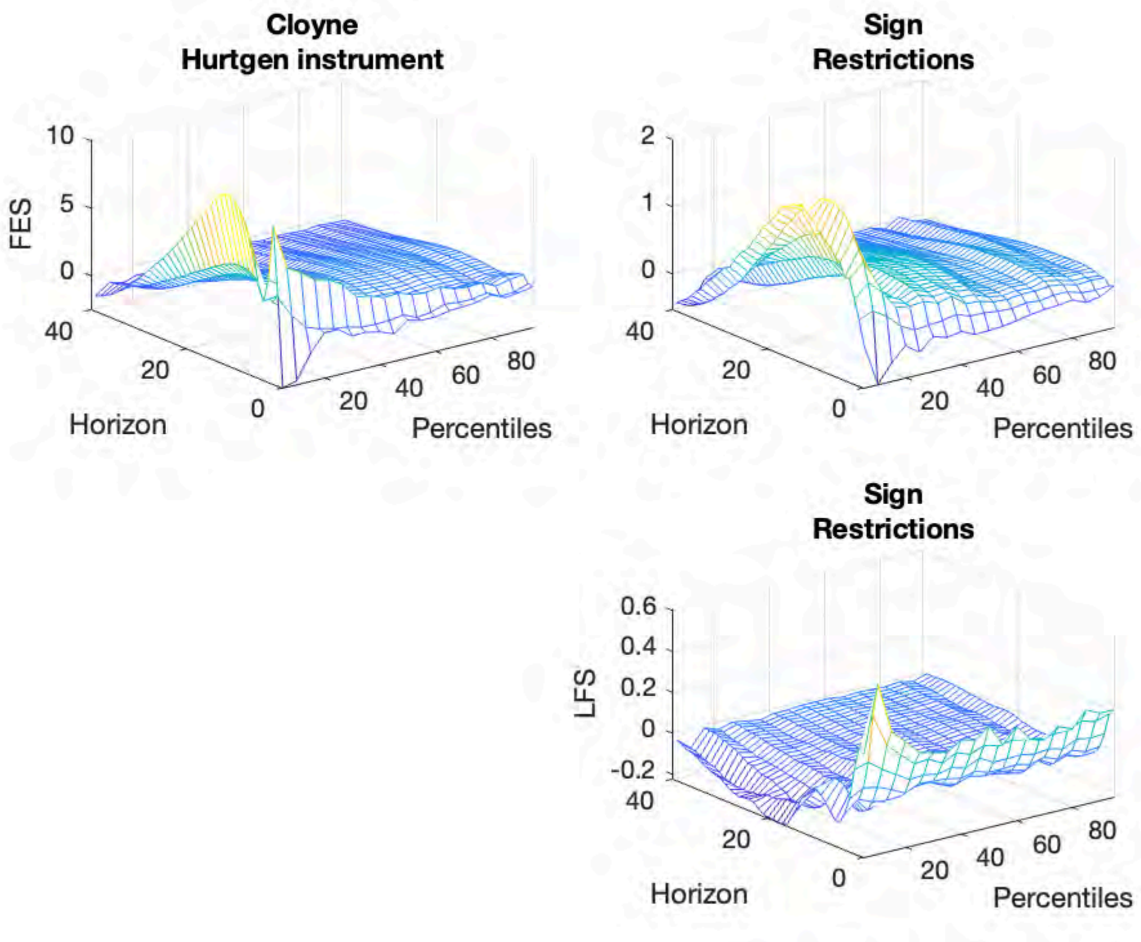


Figure E.3: Impulse responses of hours of the left tail in each percentile group for the UK using alternative identifications.

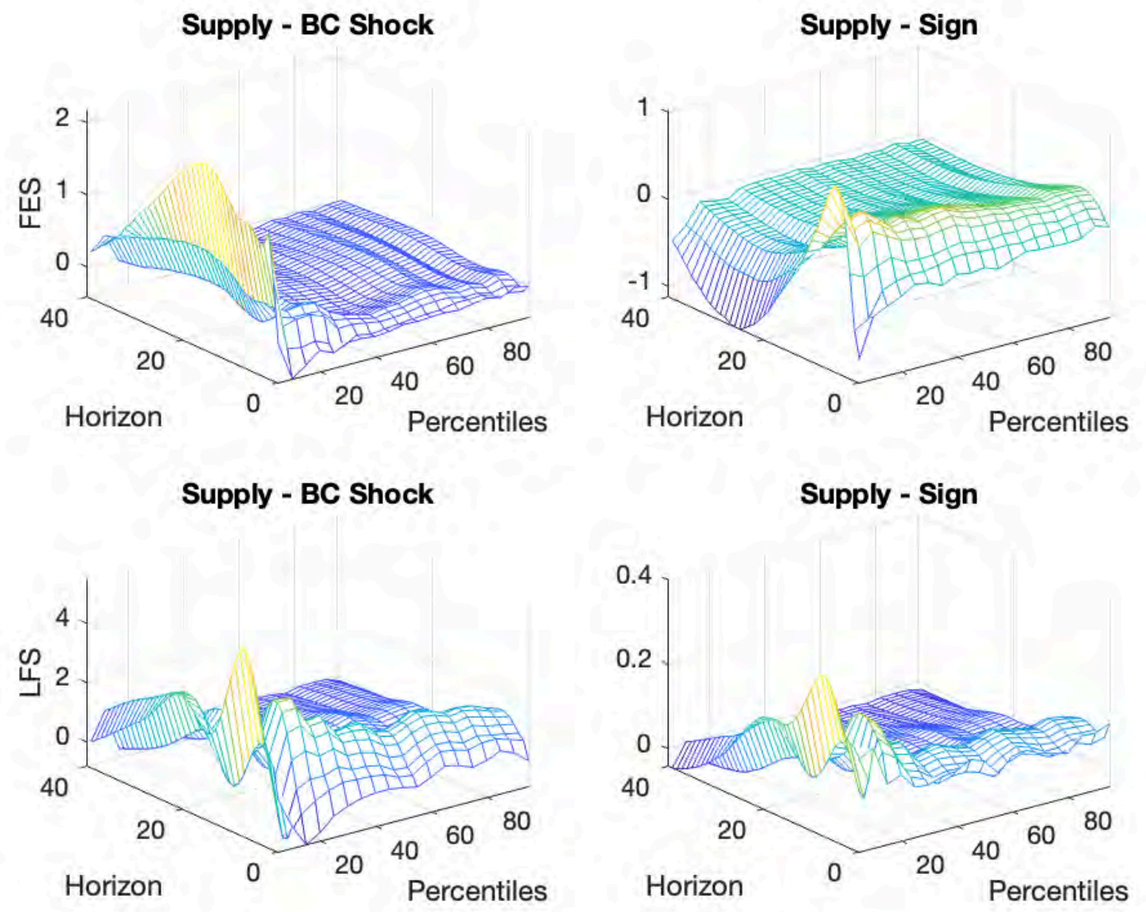


Figure E.4: Impulse responses to a supply shock of hours of the left tail in each percentile group for the UK.

E.3 Housing Tenure

Households are grouped according to tenure status. Renters are generally lower income. Mortgagors higher income. Proportion of outright owners is fairly constant across income groups.

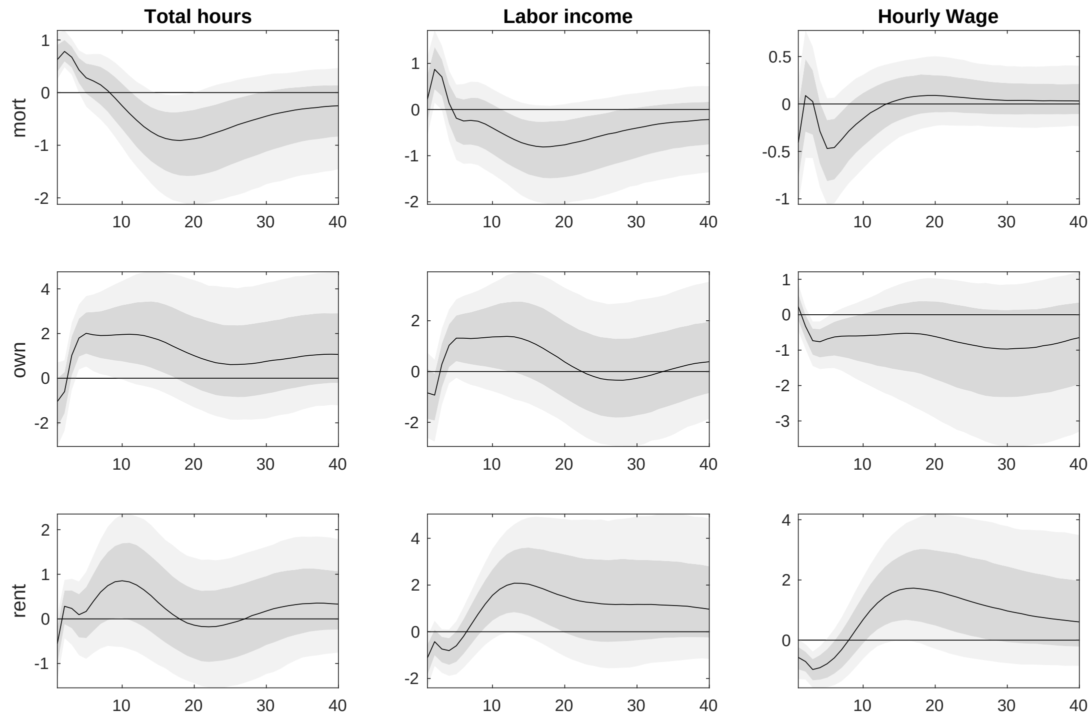


Figure E.5: Impulse responses of labor income, hours and wages for morgators, outright owners and renters after a monetary policy tightening in the US.

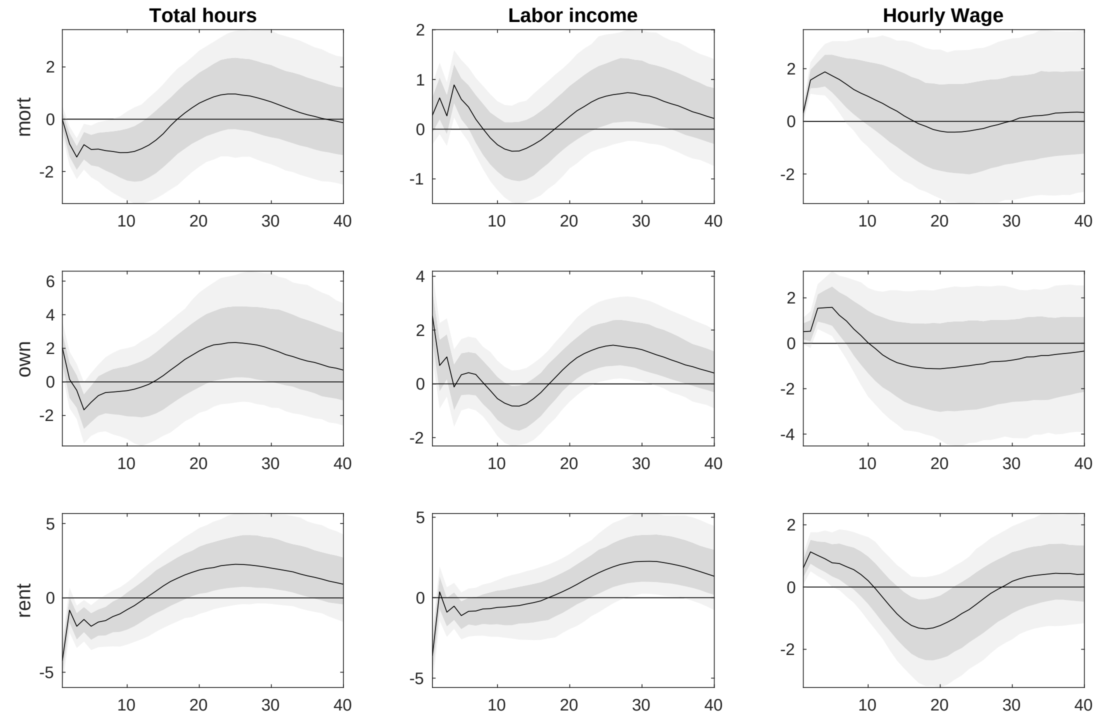


Figure E.6: Impulse responses of labor income, hours and wages for morgators, outright owners and renters after a monetary policy tightening in the UK.

E.4 Industry

E.4.1 US: CPS

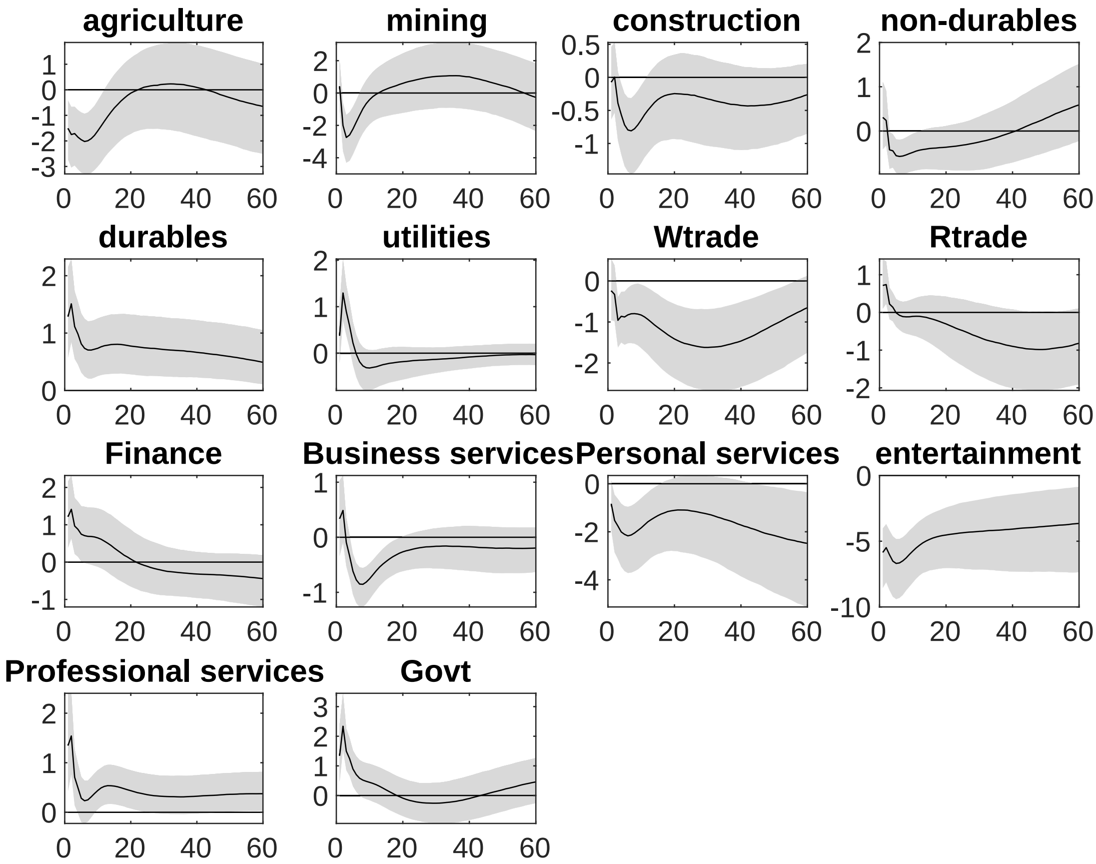


Figure E.7: Impulse responses of hours by industry of employment in the US.

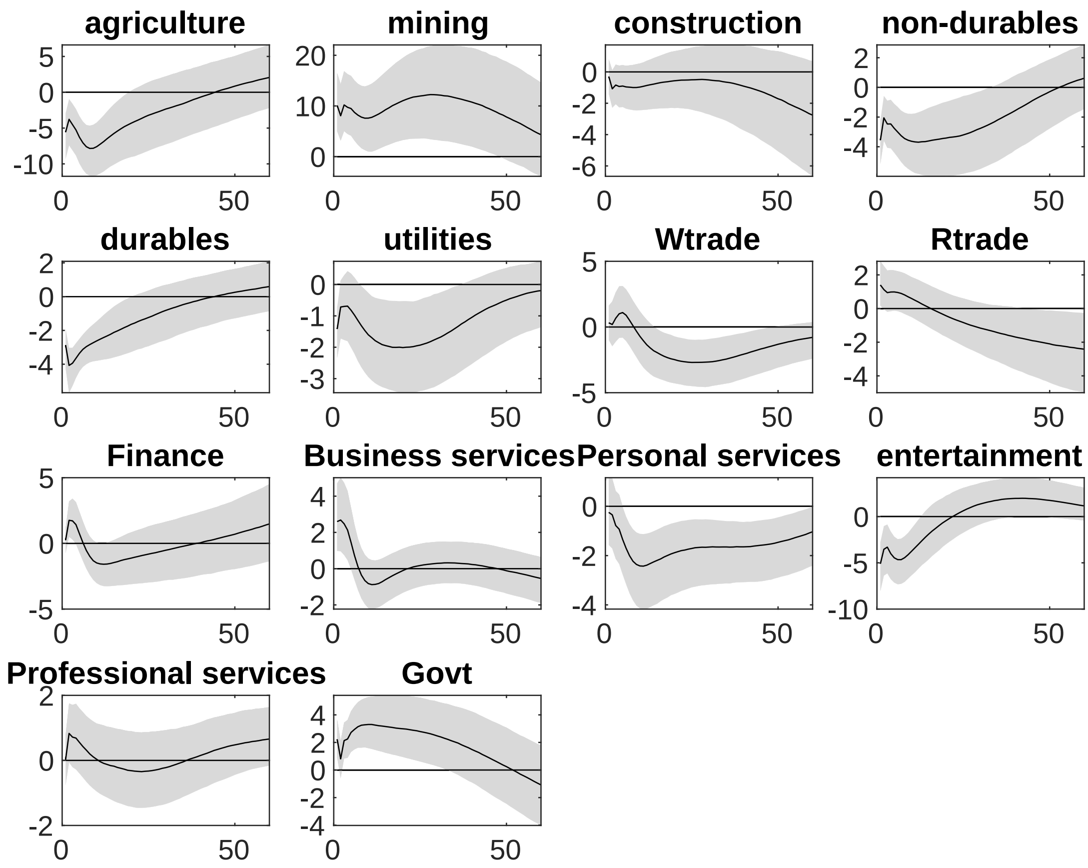


Figure E.8: Impulse responses of hourly wages by industry of employment in the US.

E.4.2 UK: LFS

The names of the industries are abbreviated in the Figures below. The full names are reported her:

- A - Agriculture, forestry and fishing
- B,D,E - Energy and water
- C -Manufacturing
- F - Construction
- G,I -Distribution, hotels and restaurants
- H,J -Transport and communication
- K,L,M,N - Banking and finance
- O,P,Q - Public admin, education and health

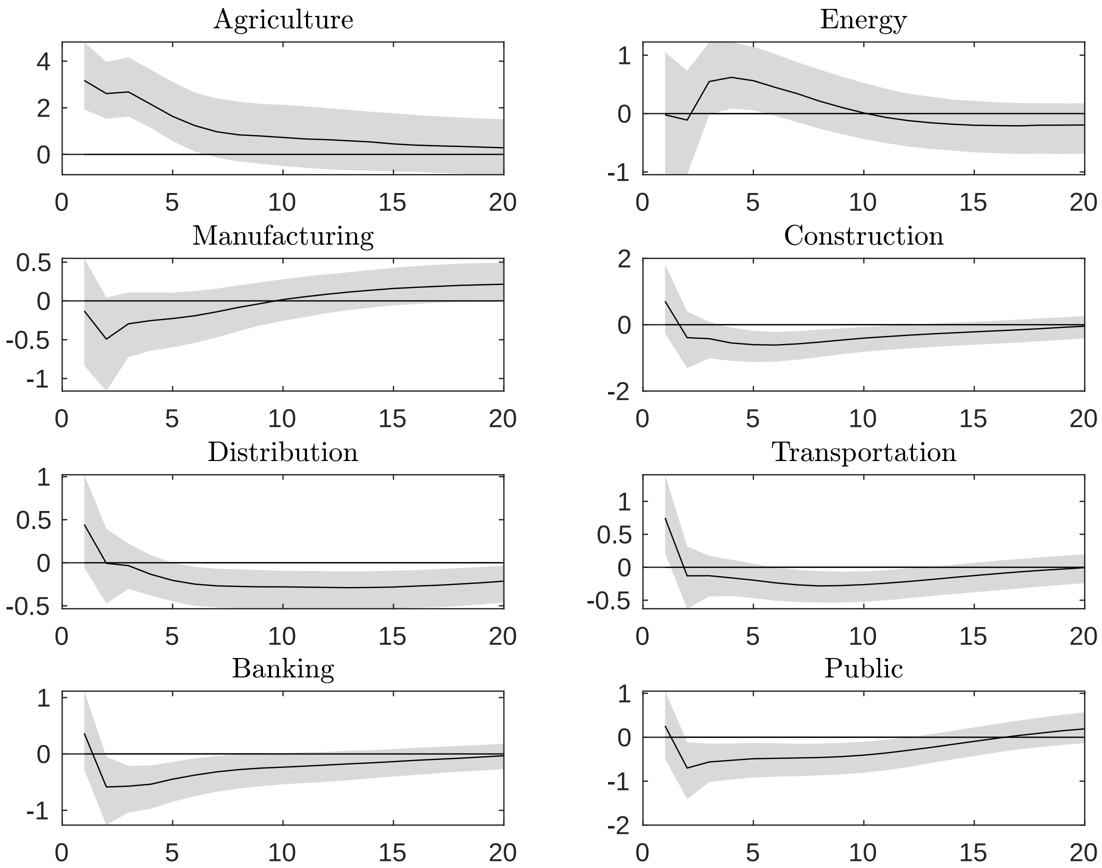


Figure E.9: Impulse responses of hours by industry of employment in the UK.

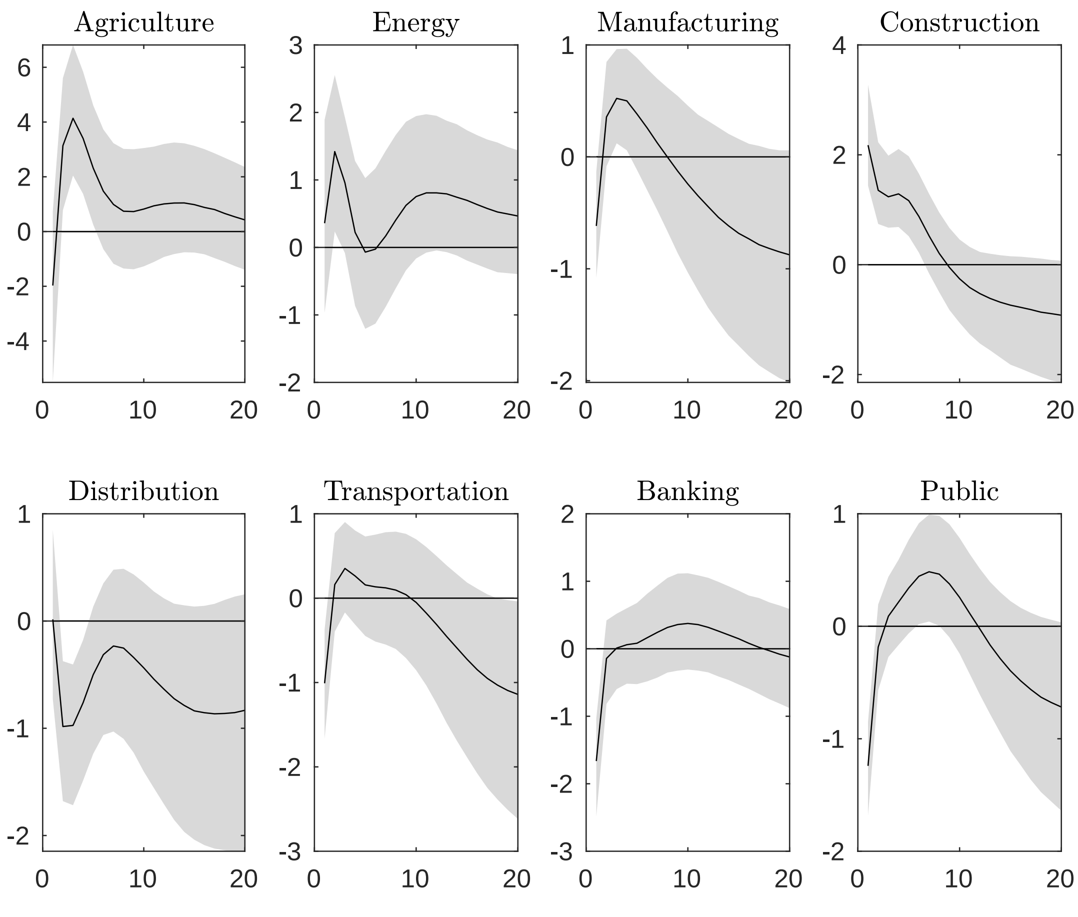


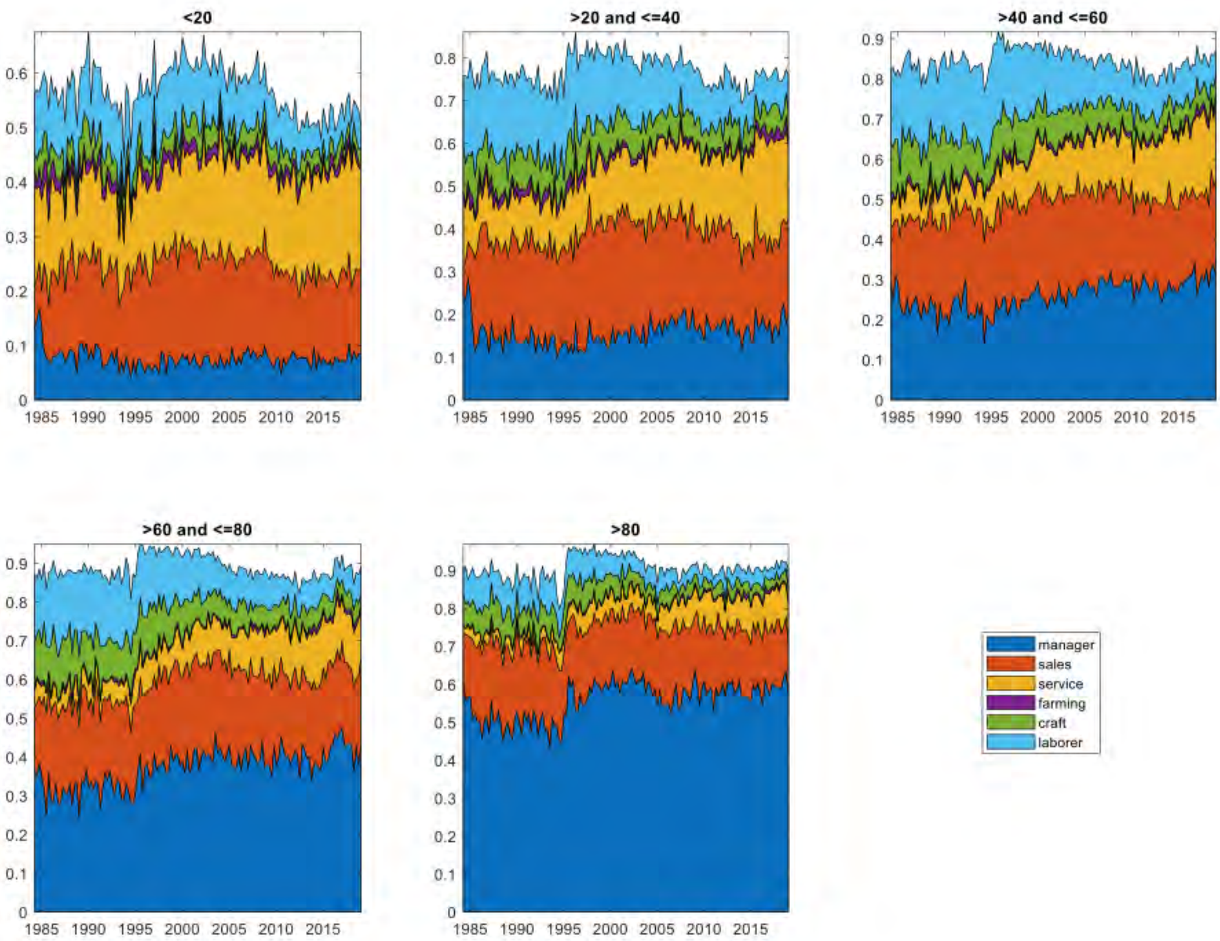
Figure E.10: Impulse responses of hourly wages by industry of employment in the UK.

E.5 Occupation

E.5.1 US: CEX

Households are grouped according to the occupation of the household reference person.

- **Manager** = Managerial & professional specialty
- **Sales** = Technical, sales, and administrative support
- **Service** = Services
- **Farming** = Farming, forestry, and fishing
- **Craft** = Precision production, craft, and repair, mechanics, mining
- **Laborer** = Machine operator, fabricators, transport, and laborers



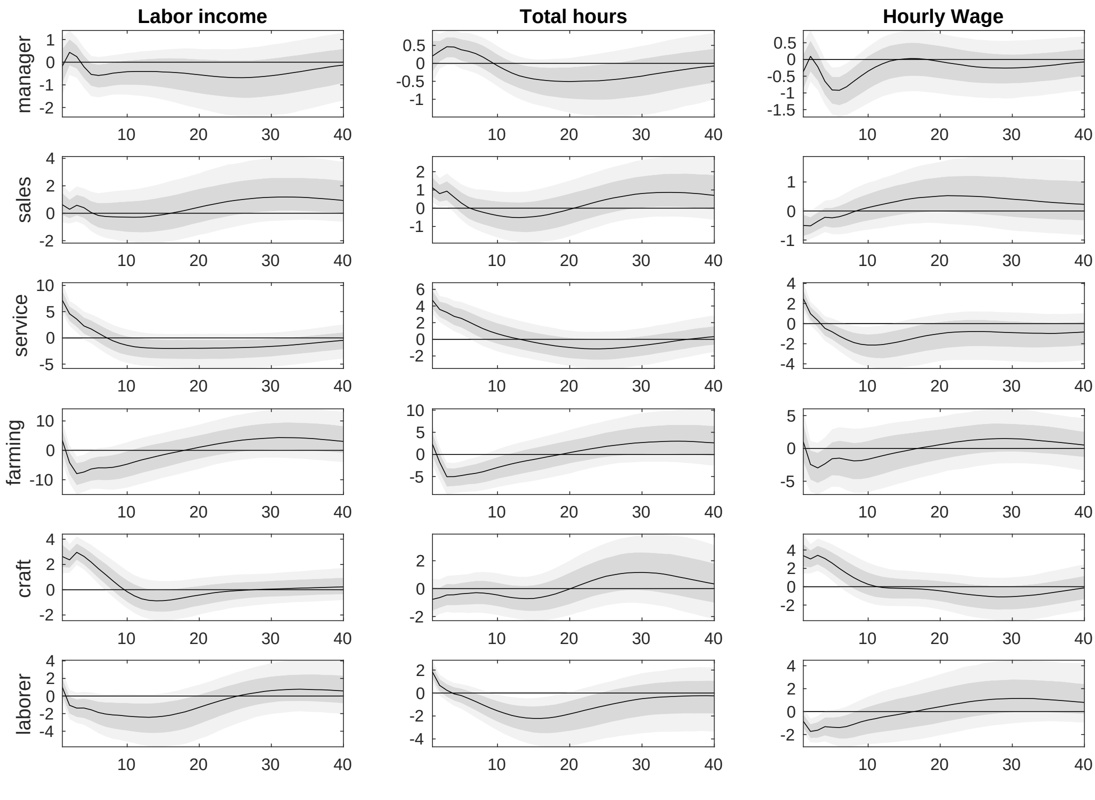
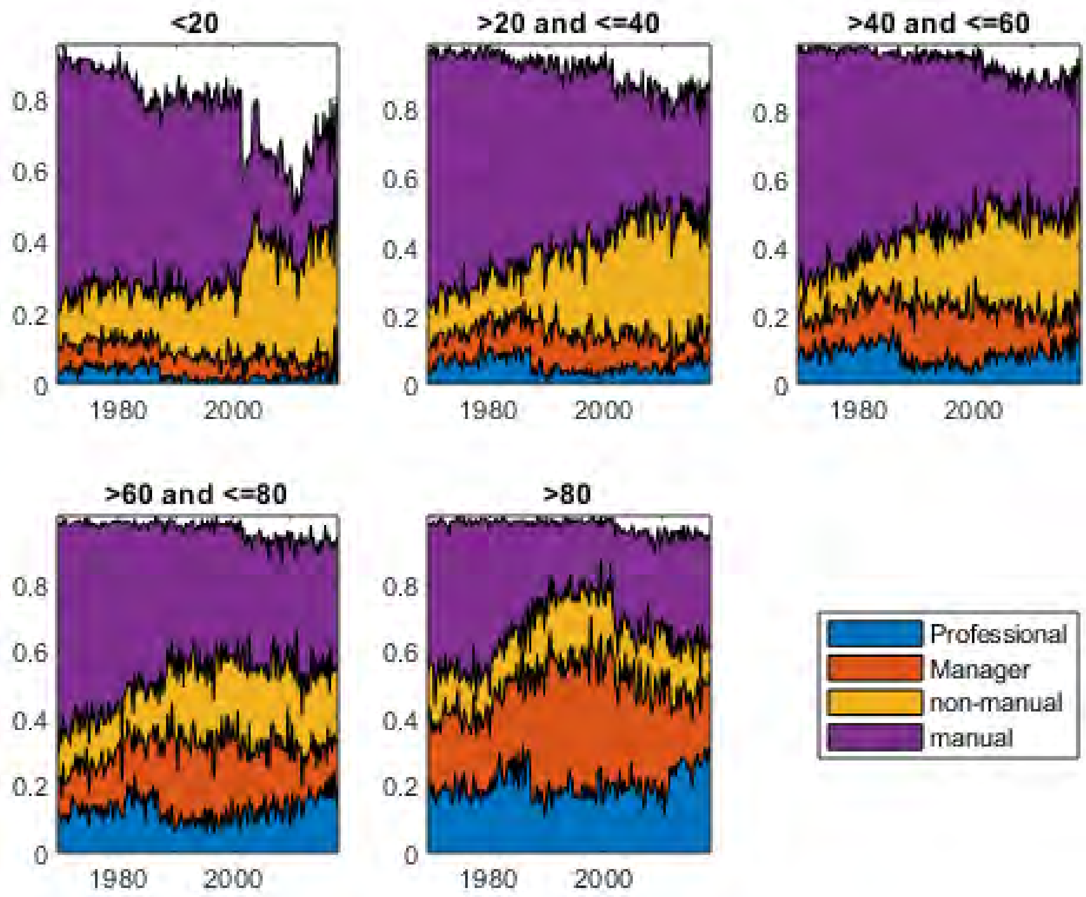


Figure E.11: Impulse responses of labor income, hours and wages by occupation in the US.

E.5.2 UK: FES

Households are grouped according to the occupation of the household reference person.

- **Professional**
- **Manager**
- **Non-manual**
- **Manual**



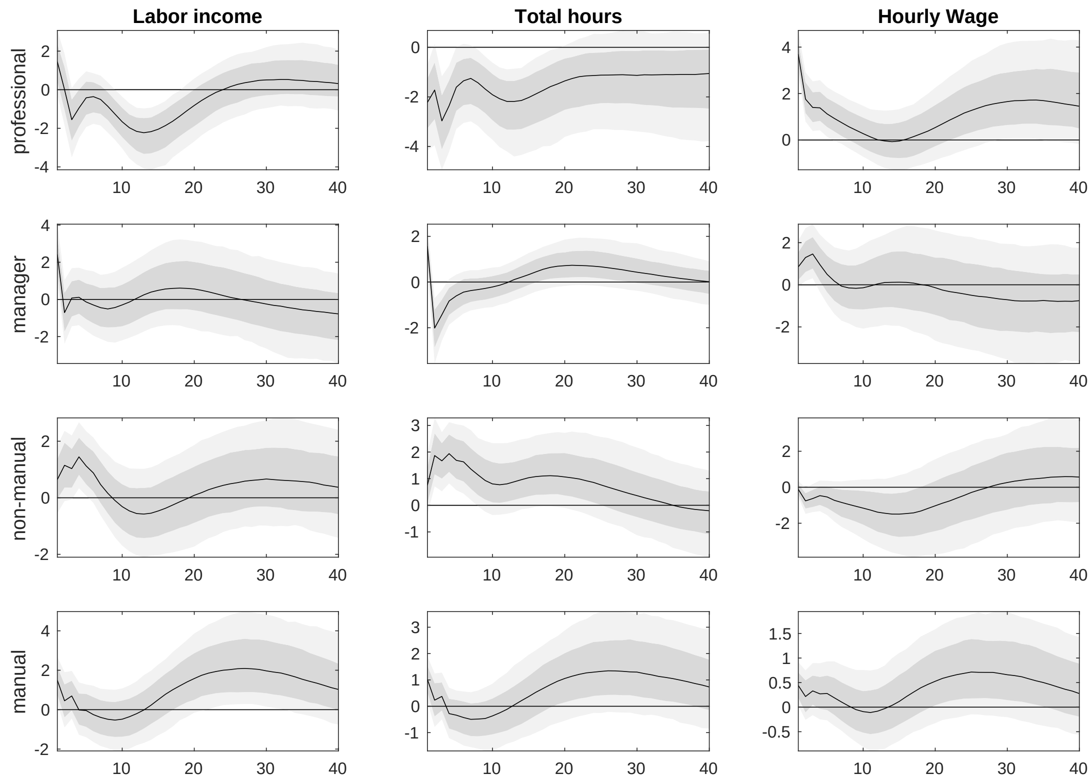


Figure E.12: Impulse responses of labor income, hours and wages by occupation in the UK.

E.6 Sample restricted to Employed

E.7 US

Observations restricted to include only the ones where the reference person reports being in one of the occupations asked by the survey. These corresponds to the 5 categories listed in previous subsection.

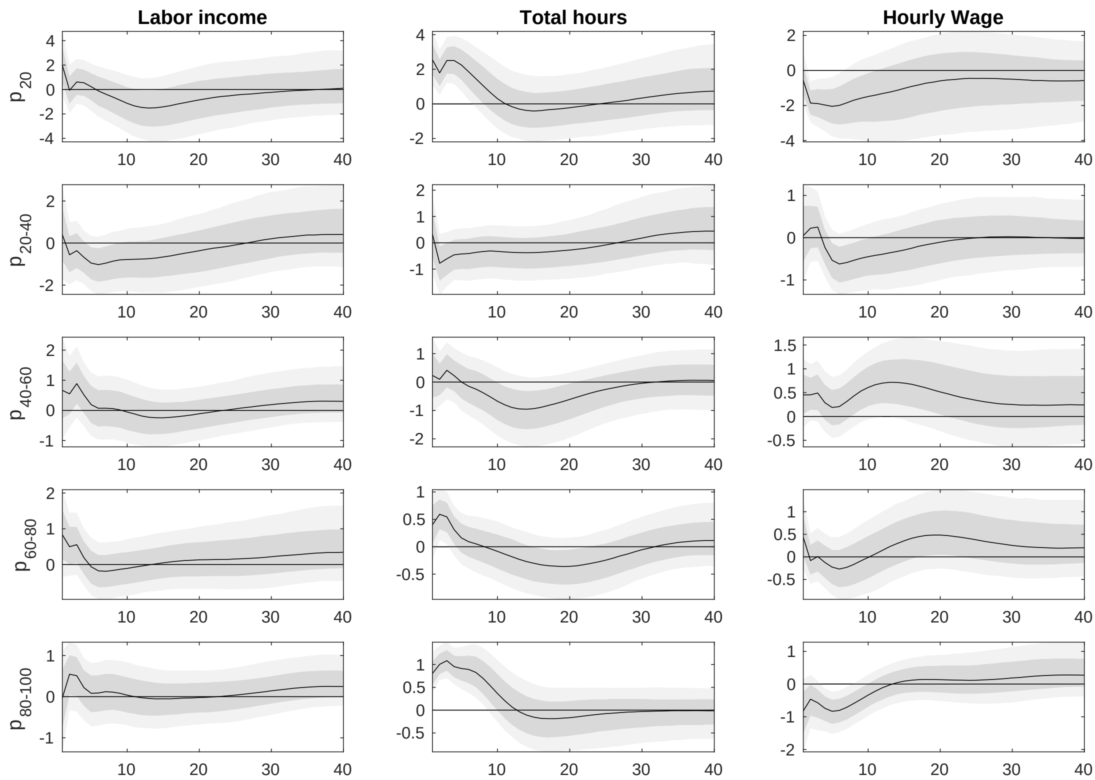


Figure E.13: Impulse responses of labor income, hours and wages in the US with sample restricted to those employed.

F Simple TANK with *poor* Hand to Mouth

Throughout, time is discrete and denoted t = 0, 1, 2,... . Real quantities are in terms of the consumption good and - unless otherwise stated - denoted by lower case letters while nominal variables are denoted by capital letters. Steady-state variables are without time subscript and log-linear variables in deviation from their steady state will be denoted by a ˆ.

This model follows the TANK model in Bilbiie (2008, 2020) allowing for heterogeneity in the intertemporal elasticities (IES) of substitution across agents. As in Bilbiie (2020), there is a production subsidy that induces marginal cost pricing which implies that the steady state of marginal costs is 1 and simplifies substantially the steady state and the log-linearized conditions. Differently from Bilbiie (2008, 2020), and for simplicity, we abstract from fiscal redistribution of profit income.

The economy consists of three sectors: households, firms, and a central bank. The household sector is populated by two different types: **savers** S and **hand-to-mouth** H. The firm sector is the standard one in New Keynesian models. Nominal rigidities are introduced by assuming that intermediate goods producers face quadratic adjustment costs a la Rotemberg (1982) in adjusting their prices. The central bank follows a Taylor type rule to choose the real interest rate. This assumption is made to obtain simpler analytical expressions.

F.1 Model

There is a continuum of households $[0, 1]$. There are two types of households: A share λ of households are HtM (indexed by H) who work and consume all of their labor income, having no access to bonds nor to firm ownership and dividends. The remaining $1 - \lambda$ are savers (indexed by S) who work, hold bonds and shares in monopolistic firms and get firm profits. Savers are standard PIH households.

Savers Savers maximize their lifetime utility subject to their budget constraint, taking prices and wages as given:

$$\max_{c_t^S, b_t^S, H_t^S} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^t \left(\frac{(c_{t+s}^S)^{1-\frac{1}{\sigma_S}}}{1-\frac{1}{\sigma_S}} - \nu^S \frac{(H_{t+s}^S)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^S + b_t^S = \frac{d_t}{1-\lambda} + H_t^S w_t + \frac{R_{t-1}}{\Pi_t} b_{t-1}^S,$$

where c_t^S is consumption, b_t^S bonds, H_t^S hours, Π_t is inflation, w_t are real wages, R_t is the gross nominal interest rate on assets and d_t are firm profits. σ_S is the inter-temporal elasticity of substitution, $\frac{1}{\varphi}$ is the Frish elasticity of labor supply and ν^S indicates how leisure is valued relative to consumption.

The solution to the savers problem yields the standard Euler and labor supply equations:

$$1 = \beta E_t \left[\left(\frac{c^S{}_{t+1}}{c^S{}_t} \right)^{-\frac{1}{\sigma}} \frac{R_t}{\Pi_{t+1}} \right] \tag{F.7}$$

$$w_t = \nu^S \left(H_t^S\right)^{\varphi} \left(c_t^S\right)^{\frac{1}{\sigma}}. \tag{F.8}$$

HtM HtM households have no access to bonds nor firms' shares and therefore rely solely on labor earnings. Using the same CRRA form of utility as for the savers, and allowing for IES heterogeneity, their static problem reads:

$$\max_{c_t^H, H_t^H} \left(\frac{\left(c_t^H\right)^{1-\frac{1}{\sigma_H}}}{1-\frac{1}{\sigma_H}} - \nu^H \frac{\left(H_t^H\right)^{1+\varphi}}{1+\varphi} \right) \quad \text{subject to}$$

$$c_t^H = H_t^H w_t$$

where parameters and variables with H superscript have the same interpretations as the ones for savers.

Since preferences are monotonic, HtM will just consume all of their labor income plus transfers:

$$c_t^H = H_t^H w_t \tag{F.9}$$

$$w_t = \nu^H \left(H_t^H\right)^{\varphi} \left(c_t^H\right)^{\frac{1}{\sigma_H}}. \tag{F.10}$$

Firms The firm sector is split into two. A representative competitive final goods firm aggregates intermediate goods according to a CES technology and a continuum of intermediate goods producers that produce different varieties using labor as an input. To the extent to which the intermediate goods are imperfect substitutes, with ϵ denoting the elasticity of substitution, there is a downward-sloping demand for each intermediate variety, giving the intermediate producers some pricing power. However, importantly, intermediate goods producers are also subject to some costs (ϕ^p) in adjusting prices (Rotemberg (1982)). This generates sticky prices. The production function is CRS and linear in labor. This supply-side setup is the standard one in NK models and follows directly from Bilbiie (2020).[67] We assume that the government implements the standard NK optimal subsidy inducing marginal cost pricing: $\tau^S = (\epsilon - 1)^{-1}$. The optimal firm behavior after having imposed the symmetric equilibrium is characterized by

$$y_t = H_t$$

$$w_t = mc_t \frac{y_t}{H_t}$$

$$\left(1+\tau^S\right)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p \left(\Pi_t - 1\right) + \beta^S E_t \left[\left(\frac{c_{t+1}^S}{c_t^S} \right)^{-\frac{1}{\sigma}} \Pi_{t+1} \phi_p \left(\Pi_{t+1} - 1\right) \frac{y_{t+1}}{y_t} \right] = 0.$$

[67] And we refer to it for details.

Central Bank To allow for an analytical solution we assume that the central bank responds one-to-one to inflation expectations as in Bilbiie (2008) and McKay et al. (2016):

$$\frac{R_t}{R} = \left(\frac{\Pi_{t+1}}{\Pi}\right) e^{\epsilon_t^m}.$$

Aggregation and market clearing Aggregate bonds are in 0 net supply and aggregate consumption and hours are:

$$0 = (1-\lambda) b_t^S \tag{F.11}$$

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S \tag{F.12}$$

$$H_t = \lambda H_t^H + (1-\lambda) H_t^S. \tag{F.13}$$

Using firm's profits definition and the two agent's budget constraints we obtain the resource constraint:

$$c_t = y_t - \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2 y_t.$$

Equilibrium Conditions		
1:	Labor Supply S	$w_t = \nu^S \left(H_t^S\right)^{\varphi} \left(c_t^S\right)^{\frac{1}{\sigma_S}}$
2:	Labor Supply H	$w_t = \nu^H \left(H_t^H\right)^{\varphi} \left(c_t^S\right)^{\frac{1}{\sigma_H}}$
3:	Euler S	$1 = \beta E_t \left[\left(\frac{{c^S}_{t+1}}{{c^S}_t}\right)^{-\frac{1}{\sigma_S}} \frac{R_t}{\Pi_{t+1}} \right]$
4:	Budget constraint H	$c_t^H = H_t^H w_t$
5:	Marginal prod. of labor	$w_t = mc_t \frac{y_t}{H_t}$
6:	Phillips Curve	$\left(1+\tau^S\right)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p \left(\Pi_t - 1\right) + \beta E_t \left[\left(\frac{c_{t+1}^S}{c_t^S}\right)^{-\sigma_S} \Pi_{t+1} \phi_p \left(\Pi_{t+1} - 1\right) \frac{y_{t+1}}{y_t} \right] = 0$
7:	Production Function	$y_t = H_t$
8:	Profits	$d_t = \left(1 - mc_t - \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2\right) y_t$
9:	Aggregate C	$c_t = \lambda c_t^H + (1-\lambda) c_t^S$
10:	Aggregate H	$H_t = \lambda H_t^H + (1-\lambda) H_t^S$
11:	Resource constraint	$c_t = y_t - \frac{\phi_p}{2} \left(\Pi_t - 1\right)^2 y_t$
12:	Taylor Rule	$\frac{R_t}{R} = \left(\frac{\Pi_{t+1}}{\Pi}\right) e^{\epsilon_t^m}$

Table F.1: Non linear TANK.

Note: S budget constraint is not included because it is implied by Walras law.

F.2 Steady State

We assume that inflation is zero in steady state, $\Pi = 1$. From the Euler equation of savers we have $R = \frac{1}{\beta}$. From the optimal pricing equation, we have $mc = \frac{\left(1+\tau^S\right)(\epsilon-1)}{\epsilon} = \mathcal{M}^{-1}$. From the production function, and the resource constraint, we have that $c = y = H$. Imposing the optimal subsidy $\tau^s = \frac{1}{\epsilon-1}$ we have $\mathcal{M}^{-1} = mc = w = 1$ which implies, combining both agents budget constraints, $c = c^S = c^H$. Finally we can normalize total hours $H = 1$ which implies also $y = c = 1$.

F.3 Log-linear Model

We log-linearize the model around the steady state. We assume that inflation is zero in steady state. Variables with a ˆ denote log-deviations from steady state. We log-linearize all variables ($\hat{x}_t = \frac{x_t - x}{x}$) except total profits, which we linearize and denote as a share of total output, i.e. $\tilde{d}_t = \frac{d_t - d}{y}$.

The model in Table F.1 can be simplified further removing output and total hours ($y_t = c_t = H_t$) and marginal costs and profits ($w_t = mc_t = -d_t$).

The log-linearization of the equations is straightforward and standard. We denote the slope of the Phillips curve is by $\kappa = \frac{\epsilon}{\phi^p}$. The full list of the log-linear equilibrium conditions is in Table 1 of the main text.

F.4 Derivation of Proposition 1

Solving the Euler equation forward we have

$$\hat{c}_t^S = -\sigma_S \epsilon_t^m$$

Combining the HtM BC and labor supply decision we have

$$\hat{H}_t^H = \frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H$$

Taking the difference between the S and H labor supplies we have

$$\begin{aligned}
\varphi \hat{H}_t^S - \varphi \hat{H}_t^H &= -\frac{1}{\sigma_S} \hat{c}_t^S + \frac{1}{\sigma_H} \hat{c}_t^H \\
\varphi \hat{H}_t^S - \varphi \frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H &= \epsilon_t^m + \frac{1}{\sigma_H} \hat{c}_t^H \\
\varphi \hat{H}_t^S = \epsilon_t^m + \frac{1}{\sigma_H} \hat{c}_t^H + \varphi &\frac{\sigma_H - 1}{\sigma_H(\varphi + 1)} \hat{c}_t^H \\
\hat{H}_t^S = \frac{1}{\varphi} \epsilon_t^m + &\frac{1 + \varphi \sigma_H}{\sigma_H(\varphi + 1)\varphi} \hat{c}_t^H
\end{aligned}$$

Combining aggregate C and H, we have

$$\lambda \hat{H}_t^H - \lambda \hat{c}_t^H = (1 - \lambda) \hat{c}_t^S - (1 - \lambda) \hat{H}_t^S$$

Combining the last equations we get

$$\begin{aligned}
\lambda\frac{\sigma_H-1}{\sigma_H(\varphi+1)}\hat{c}_t^H-\lambda\hat{c}_t^H &= -(1-\lambda)\sigma_S\epsilon_t^m-(1-\lambda)\hat{H}_t^S\\
\lambda\frac{\sigma_H-1-\sigma_H(\varphi+1)}{\sigma_H(\varphi+1)}\hat{c}_t^H &= -(1-\lambda)\sigma_S\epsilon_t^m-(1-\lambda)\hat{H}_t^S\\
\lambda\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m+(1-\lambda)\left[\frac{1}{\varphi}\epsilon_t^m+\frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H\right]\\
\lambda\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\hat{c}_t^H-(1-\lambda)\frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m+\frac{1-\lambda}{\varphi}\epsilon_t^m\\
\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\left[\lambda-\frac{1-\lambda}{\varphi}\right]\hat{c}_t^H &= (1-\lambda)\sigma_S\epsilon_t^m+\frac{1-\lambda}{\varphi}\epsilon_t^m\\
\frac{1+\sigma_H\varphi}{\sigma_H(\varphi+1)}\left[\frac{\lambda\varphi-1+\lambda}{\varphi}\right]\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\varphi}\epsilon_t^m\\
\frac{\lambda\varphi-1+\lambda}{\sigma_H(\varphi+1)}\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\epsilon_t^m\\
\hat{c}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\,\frac{\sigma_H(\varphi+1)}{\lambda(\varphi+1)-1}\epsilon_t^m
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\hat{H}_t^H &= (1-\lambda)\frac{\sigma_S\varphi+1}{\sigma_H\varphi+1}\,\frac{\sigma_H-1}{\lambda(\varphi+1)-1}\epsilon_t^m\\
\hat{H}_t^S &= \frac{1}{\varphi}\epsilon_t^m+\frac{1+\varphi\sigma_H}{\sigma_H(\varphi+1)\varphi}\hat{c}_t^H\\
&= \frac{1}{\varphi}\epsilon_t^m\left[1+\frac{(1+\varphi\sigma_S)(1-\lambda)}{\lambda(\varphi+1)-1}\right]\\
&= \frac{\lambda+(1-\lambda)\sigma_S}{\lambda(\varphi+1)-1}\epsilon_t^m
\end{aligned}$$

Aggregate hours become

$$\begin{aligned}
\hat{H}_t &= \lambda\hat{H}_t^H+(1-\lambda)\hat{H}_t^S\\
&= \frac{\sigma_S(1-\lambda)}{\lambda(\varphi+1)-1}\,\frac{\lambda(1+\varphi)(\sigma_H/\sigma_S-1)+(1+\varphi\sigma_H)}{\sigma_H\varphi+1}\epsilon_t^m\\
&= \frac{\sigma_S(1-\lambda)}{\lambda(\varphi+1)-1}\left[1-\frac{\lambda(1+\varphi)(1-\sigma_H/\sigma_S)}{\sigma_H\varphi+1}\right]\epsilon_t^m
\end{aligned}$$

The numerator in the first term is always positive; the denominator is negative when $\lambda < \frac{1}{1+\varphi}$. When $\lambda < \frac{1}{1+\varphi}$, the terms in square brackets is always positive since $\sigma_H > 0 > \frac{\lambda(\varphi+1)-1}{\sigma_S\varphi+(1+\varphi)\lambda}\ \sigma_S$. This makes the all term multiplying the monetary positive shock negative. Hence aggregate hours decline while hours worked by HtM household increase.

F.5 Derivation of Proposition 2

Substituting the Taylor rule in (7) and solving it forward we get:

$$\hat{c}_t = \sigma_S \frac{1-\lambda}{1-(1+\varphi)\lambda} \frac{\lambda\left(\frac{\sigma_H}{\sigma_S}-1\right)(\varphi+1)+\sigma_H\varphi+1}{\sigma_H\varphi+1}\epsilon_t^m. \tag{F.14}$$

$\frac{\partial \hat{c}_t}{\partial \lambda}$ is a second order equation in λ:

$$\frac{\partial \hat{c}_t}{\partial \lambda} = \frac{1}{\sigma_S\,(\sigma_H\,\varphi+1)\,(\lambda+\lambda\,\varphi-1)^2}(\sigma_H-\sigma_S-2\,\lambda\,\sigma_H+2\,\lambda\,\sigma_S+\sigma_H\,\varphi+\lambda^2\,\sigma_H-...$$
$$-\lambda^2\,\sigma_S+2\,\lambda^2\,\sigma_H\,\varphi-2\,\lambda^2\,\sigma_S\,\varphi+\sigma_H\,\sigma_S\,\varphi^2+\lambda^2\,\sigma_H\,\varphi^2-\lambda^2\,\sigma_S\,\varphi^2-2\,\lambda\,\sigma_H\,\varphi+2\,\lambda\,\sigma_S\,\varphi). \tag{F.15}$$

Solving $\frac{\partial \hat{c}_t}{\partial \lambda}=0$ for λ we obtain:

$$\lambda^1 = \frac{\sigma_H-\sigma_S+\sqrt{-\sigma_H\,\varphi\,(\sigma_H-\sigma_S)\,(\sigma_S\,\varphi+1)}}{\sigma_H-\sigma_S+\sigma_H\,\varphi-\sigma_S\,\varphi}$$
$$\lambda^2 = -\frac{\sigma_S-\sigma_H+\sqrt{-\sigma_H\,\varphi\,(\sigma_H-\sigma_S)\,(\sigma_S\,\varphi+1)}}{\sigma_H-\sigma_S+\sigma_H\,\varphi-\sigma_S\,\varphi}$$

which can be rewritten as

$$\lambda^1 = \frac{1}{1+\varphi} - \frac{\sqrt{\sigma_H\,\varphi\,(\sigma_S-\sigma_H)\,(\sigma_S\,\varphi+1)}}{(\sigma_S-\sigma_H)(1+\varphi)}$$
$$\lambda^2 = \frac{1}{1+\varphi} + \frac{\sqrt{\sigma_H\,\varphi\,(\sigma_S-\sigma_H)\,(\sigma_S\,\varphi+1)}}{(\sigma_S-\sigma_H)(1+\varphi)}$$

Since λ_2 is outside the SADL region, $\lambda^\star=\lambda^1$ as in proposition 2. Finally, $\sigma^\star$ can be found by solving $\lambda^2=0$ for σ_H.

G Medium scale TANK model

The model presented here builds on Bilbiie et al. (2022). The differences from their setup are the following: i) we allow for labor supply heterogeneity; ii) we use Stone-Geary preferences to generate IES increasing in the steady-state consumption level; iii) we abstract from idiosyncratic risk and fiscal redistribution of profits and/or capital income. The economy comprises of households, firms, and a monetary authority. We discuss each sector in turn. As before lowercase letters define real variables while capitals define nominal variables.

Households The Households side consists of a continuum of households on the unit interval of two types: a share $\lambda \in [0,\, 1)$ are hand-to-mouth (H) and the rest 1 - λ are savers (S). Throughout we assume symmetry within types. Households have the same Stone-Geary preferences. Household j (with $j = H, S$) has per period utility $u(c^j, H^j) = \frac{(c^j - \bar{c})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{(H^j)^{1+\varphi}}{1+\varphi}$, with subsistence level of consumption $\bar{c}$ and discount the future at the rate β.

We introduce sticky wages using Rotemberg adjustment costs. As such, hours worked are largely demand-determined, and agents are not always on their labor supply curve. To allow for labor supply heterogeneity we follow Ascari et al. (2017)[68] and assume that wage-setting decisions depend uniquely on the savers. Hand to mouth instead supply their labor taking as given the wage fixed by savers. We assume a continuum of differentiated labor inputs indexed by $i \in [0, 1]$. Each savers household provides the differentiated labor type i, and acts as a monopolist on labor market i. On the contrary, HtM households do not perceive their power in the labor market and supply each labor type taking as given the wage fixed by savers households. Savers households stand ready to supply at the fixed wage $W_t(i)$ as many hours to the labor market i, $H_t^S(i)$, as required by firms, taking into account that part of the demand is satisfied by the supply of HtM agents. The variable $H_t = \left[\int_0^1 \left(H_t^j(i)\right)^{\frac{\eta^w-1}{\eta^w}} di\right]^{\frac{\eta^w}{\eta^w-1}}$ represents the labor input used in the production process by intermediate goods producers and $\eta^w > 1$ is the elasticity of substitution between labor inputs.

Under Rotemberg pricing savers households $j = S$ in the interval $(\lambda, 1]$ face the problem of choosing nominal wage $W_t(i)$ to maximize:

$$E_t \sum_{k=0}^{\infty} \beta^k u\left(c_{t+k}^S, H_{t+k}^S(i)\right), \tag{G.16}$$

taking into account the demand for its labor variety

$$H_{t+k}^S(i) = \left(\frac{W_{t+k}(i)}{W_{t+k}}\right)^{-\eta^w} H_{t+k}^d - \lambda H_{t+k}^H, \tag{G.17}$$

[68] They use Calvo wage settings while we use Rotemberg but the intuition is the same as well as the implied log-linear wage Phillips curve up to first order. See their online appendix for details.

and subject to the budget constraint and the capital accumulation equation:

$$c_t^S + i_t^S + b_t^S = \frac{d_t}{1-\lambda} + H_t^S(i)w_t(i) + r_t^K k_{t-1}^S + \frac{R_{t-1}}{\Pi_t} b_{t-1}^S - \frac{\phi^w}{2}\left(\frac{W_{t+k}(i)}{W_{t+k-1}(i)} - 1\right) y_t \tag{G.18}$$

$$k_t^S = \left(1 - \frac{\iota}{2}\log\left(\frac{i_t^S}{i_{t-1}^S}\right)^2\right) i_t^S + (1-\delta)\, k_{t-1}^S. \tag{G.19}$$

As before we call c_t^S the consumption of Savers, while $H_{t+k}^S(i)$ are hours worked on market i by Savers defined in equation (G.17). The variable H_t^d represents the total demand of the labor bundle by firms, while λH_t^H represents the total supply of the bundle by HtM agents. In each period t Savers can invest (i_t^S) in capital (k_t^S), subject to a quadratic investment adjustment costs, and/or bonds (b_t^S). d_t are firms' profits, R_t is the nominal risk-free interest rate, r_t^K is the rental rate of capital and the last term in the constraint represents the quadratic Rotemberg costs of adjusting the wage, with ϕ^w being the Rotemberg wage adjustment cost parameter. These adjustment costs are measured in units of aggregate output (y_t), and are given by a quadratic function of the change in wages above and beyond steady-state wage inflation assumed to be equal to 1. Defining $u_t^{H^S}$ the marginal disutility of labor and $u_t^{c^S}$ the marginal utility of consumption of savers and making use of the definition of the marginal rate of substitution of the savers $mrs_t^S = -\frac{u_t^{H^S}}{u_t^{c^S}}$ the resulting FOC can be written as:

$$(1-\eta^w) + \frac{mrs_t^S\,\eta^w}{w_t} - (\Pi_t^w - 1)\,\phi^w\,\Pi_t^w + \beta\mathbb{E}_t\left(\phi^w\,\frac{u_{t+1}^{c^S}}{u_t^{c^S}}\,\Pi_{t+1}^w\,\left(\Pi_{t+1}^w - 1\right)\, w_{t+1}\,\frac{H_{t+1}^S}{H_t^S\, w_t}\right) = 0. \tag{G.20}$$

Linearizing around the $\Pi^w = 1$ steady state yields:

$$\hat{\Pi}_t^w = \beta\,\mathbb{E}_t\hat{\Pi}_{t+1}^w + \kappa_w\left(\hat{mrs}_t^S - \hat{w}_t\right)\,, \tag{G.21}$$

where Π_t^w is wage inflation and $\kappa_w = \frac{\eta^w - 1}{\phi_w}$.

The other FOCs that characterise the savers behavior, together with (G.18) and (G.19), are standard and summarised below:

$$mrs_t^S = -\frac{u_t^{H^S}}{u_t^{c^S}} \tag{G.22}$$

$$u_t^{c^S} = \beta\, r_t\, \mathbb{E}_t u_{t+1}^{c^S} \tag{G.23}$$

$$u_t^{c^S} = u_{t+1}^{c^S} \frac{\beta\, \mathbb{E}_t \left(r_{t+1}^K + (1-\delta)\, q_{t+1}\right)}{q_t} \tag{G.24}$$

$$q_t \left(1 - \frac{\iota}{2} \log\left(\frac{i_t^S}{i_{t-1}^S}\right)^2 - \frac{i_t^S}{i_{t-1}^S} \frac{\iota \log\left(\frac{i_t^S}{i_{t-1}^S}\right)}{\frac{i_t^S}{i_{t-1}^S}}\right) + \mathbb{E}_t \left(q_{t+1} \frac{\beta\, u_{t+1}^{c^S}}{u_t^{c^S}} \frac{\iota \log\left(\frac{i_{t+1}^S}{i_t^S}\right)}{\frac{i_{t+1}^S}{i_t^S}} \left(\frac{i_{t+1}^S}{i_t^S}\right)^2\right) = 1. \tag{G.25}$$

The first equation above is the labor supply of savers, the second is the Euler equation for riskless bonds, the third is the Euler for capital, and the last is the investment equation.

Hand-to-mouth household $j = H$ in the interval $[0, \lambda)$ face the following problem:

$$E_t \sum_{k=0}^{\infty} \beta^k u\left(c_{t+k}^H, H_{t+k}^H\right), \tag{G.26}$$

subject to the budget constraint:

$$c_t^H = \int_0^1 H_t^H(i) w_t(i) di. \tag{G.27}$$

$$\tag{G.28}$$

Here $H_t^H = \left(\int_0^1 \left(H_t^H(i)\right)^{\frac{\eta^w-1}{\eta^w}} di\right)^{\frac{\eta^w}{\eta^w-1}}$ and given that the cost of one unit of the bundle is $W_t = \left(\int_0^1 (W_t(i))^{1-\eta^w} di\right)^{1/(1-\eta^w)}$ the budget constraint can be written as

$$c_t^{j,H} = H_t^{j,H} w_t. \tag{G.29}$$

This generates the following labor supply equation for HtM:

$$w_t = -\frac{u_t^{H^H}}{u_t^{c^H}}. \tag{G.30}$$

Firms The firm sector is the same as in section F with the addition that intermediate goods producers now produce output using a combination of capital and labor following a standard Cobb-Douglas production function ($y_t = H_t^{1-\alpha} k_t^{\alpha}$). Firms still face Rotember price adjustment costs (ϕ^p).[69] Cost minimization delivers the following FOCs:

$$w_t = (1-\alpha) \left(\frac{H_t}{k_{t-1}}\right)^{(-\alpha)} mc_t \tag{G.31}$$

$$r_t^K = mc_t\, \alpha \left(\frac{k_{t-1}}{H_t}\right)^{\alpha-1}. \tag{G.32}$$

[69] Here we relax the assumption of the optimal subsidy (so $\tau^S = 0$ now).

The pricing problem delivers the standard Phillips curve under Rotemberg price adjustment costs:

$$(1-\eta) + mc_t\,\eta - (\Pi_t - 1)\;\phi^p\,\Pi_t + \mathbb{E}_t\left(\beta\,\phi^p\,\Pi_{t+1}\,\frac{u_{t+1}^{c^S}}{u_t^{c^S}}\;(\Pi_{t+1}-1)\;\frac{y_{t+1}}{y_t}\right) = 0, \tag{G.33}$$

where η is the elasticity of substitution across goods varieties. Log-linearizing this expression around $\Pi = 1$ steady-state delivers the standard NK Phillips curve:

$$\hat{\Pi}_t = \beta\,\hat{\Pi}_{t+1} + \kappa\hat{mc}_t\,. \tag{G.34}$$

Central Bank Monetary policy follows a standard Taylor rule responding to inflation and output deviations from the steady-state and with interest rate smoothing:

$$\hat{R}_t = \phi^r\,\hat{R}_{t-1} + (1-\phi^r)\,\left(\phi^\pi\hat{\Pi}_t\,+\phi^y\hat{y}_t\right) + \epsilon_t^m. \tag{G.35}$$

Aggregation and market clearing Aggregate bonds are in 0 net supply and aggregate consumption, hours, capital and investment are:

$$0 = (1-\lambda)b_t^S \tag{G.36}$$

$$c_t = \lambda c_t^H + (1-\lambda)c_t^S \tag{G.37}$$

$$H_t = \lambda H_t^H + (1-\lambda)H_t^S \tag{G.38}$$

$$k_t = (1-\lambda)\,k_t^S \tag{G.39}$$

$$i_t = (1-\lambda)\,i_t^S\,. \tag{G.40}$$

Using firm's profits definition and the two agent's budget constraints we obtain the resource constraint:

$$c_t + i_t = y_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 y_t - \frac{\phi_w}{2}\left(\Pi_t^w - 1\right)^2 y_t.$$

G.1 Steady State

We assume a zero net inflation steady state therefore gross inflation is $\Pi = 1$. The same for wage inflation, $\Pi^w = 1$. Steady state marginal cost is equal to the inverse of the steady state price mark-up $mc = \frac{\eta-1}{\eta}$. The steady state wage markup is $\frac{w}{mrs^S} = \frac{\eta^w}{\eta^w-1}$. Tobin's q is 1. The steady-state nominal interest rate is given by the Euler equations for bonds of the Savers, $R = \beta^{-1} - 1$, while the rental rate of capital from the Euler equation for capital, $r^K = R + \delta$. Hours worked of the two types of agents are calibrated as discussed in the main text. This together with the calibration of λ defines the steady-state of aggregate hours. To simplify the notation, in line with equation (9) we define $\sigma_H = \sigma\left(1 - \frac{\bar{c}}{c^H}\right)$ and $\sigma_S = \sigma\left(1 - \frac{\bar{c}}{c^S}\right)$.[70] This implies $\sigma = \frac{c^H\sigma_H - c^S\sigma_S}{c^H - c^S}$. Labor demand implies $w = mc(1-\alpha)\left(\frac{H}{k}\right)^{-\alpha}$. Therefore we can write down the capital labor ratio $\frac{K}{H} = \left(\frac{r^K}{mc\alpha}\right)^{\frac{1}{\alpha-1}}$ and use the steady state of hours

[70] Note that these two equations, together with the calibration of σ_H and σ_S also pin down $\bar{c}$.

to compute the steady state of capital. Using the production function we find the steady state of output $y = H^{1-\alpha}k^{\alpha}$. The capital accumulation equation gives the steady state investment $i = \delta k$. The resource constraint can be used to get aggregate consumption $c = y - i$. Consumption of HtM comes from their budget constraint $c^H = w + H^H$ and it follows from aggregation that consumption of the Savers in steady state is $c^S = \frac{c-\lambda c^H}{1-\lambda}$. Our calibration of consumption inequality assumes positive profits in steady state. These are given by $d = y - wH - r^K k$ and we calibrate the steady state labor share ($\bar{ls} = \frac{wH}{y} = 0.68$) which implies $\alpha = 1 - \frac{\bar{ls}}{mc}$.

G.2 Model summary

Table G.2 summarized the log-linear equilibrium conditions of the model.

Log-linearized Conditions		
1:	Aggregate Hours	$\hat{H}_t = \lambda \hat{H}_t^H \frac{H^H}{H} + (1-\lambda)\hat{H}_t^S \frac{H^S}{H}$
2:	Aggregate Consumption	$\hat{c}_t = \lambda \hat{c}_t^H \frac{c^H}{c} + (1-\lambda)\hat{c}_t^S \frac{c^S}{c}$
3:	Euler Bonds Savers	$\hat{c}_t^S = \mathbb{E}_t \hat{c}_{t+1}^S - \sigma_S \left(\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1}\right)$
4:	Euler Capital Savers	$\hat{q}_t = \beta\,(1-\delta)\,\mathbb{E}_t \hat{q}_{t+1} + (1-\beta\,(1-\delta))\,\mathbb{E}_t \hat{r}_{t+1}^K - \frac{1}{\sigma_S}\left(\mathbb{E}_t \hat{c}_{t+1}^S - \hat{c}_t^S\right)$
5:	Labor Supply Savers	$\varphi \hat{H}_t^S = \hat{mrs}_t^S - \frac{1}{\sigma_S}\hat{c}_t^S$
6:	Labor Supply HtM	$\varphi \hat{H}_t^H = \hat{w}_t - \frac{1}{\sigma_H}\hat{c}_t^H$
7:	Budget constraint HtM	$\hat{c}_t^H = \frac{wH^H}{c^H}(\hat{w}_t + \hat{H}_t^H)$
8:	Production Function	$\hat{y}_t = (1-\alpha)\,\hat{H}_t + \alpha \hat{k}_{t-1}$
9:	Labor Demand	$\hat{w}_t = \hat{y}_t - \hat{h}_t + \hat{mc}_t$
10:	Capital Demand	$\hat{r}_t^K = \hat{mc}_t + \hat{y}_t - \hat{k}_{t-1}$
11:	Profits	$\hat{d}_t = \hat{y}_t \frac{y}{d} - (\hat{w}_t + \hat{H}_t)\frac{wH}{d} - (\hat{r}^K + \hat{k}_{t-1})\frac{r^K k}{d}$
12:	Capital Accumulation	$\hat{k}_t = \delta\,\hat{i}_t + (1-\delta)\,\hat{k}_{t-1}$
13:	Tobin's q Savers	$\hat{i}_t\,(1+\beta) = \beta\,\mathbb{E}_t \hat{i}_{t+1} + \hat{i}_{t-1} + \hat{q}_t\,\frac{1}{\iota}$
14:	Resource Constraint	$\hat{y}_t = \hat{c}_t \frac{c}{y} + \hat{i}_t \frac{i}{y}$
15:	Phillips Curve	$\hat{\Pi}_t = \beta\,\mathbb{E}_t \hat{\Pi}_{t+1} + \kappa \hat{mc}_t$
16:	Wage Phillips Curve	$\hat{\Pi}_t^w = \beta\,\mathbb{E}_t \hat{\Pi}_{t+1}^w + \kappa_w \left(\hat{mrs}_t^S - \hat{w}_t\right)$
17:	Wage Inflation	$\hat{\Pi}_t^w = \hat{w}_t - \hat{w}_{t-1}$
18:	Taylor Rule	$\hat{R}_t = \phi^r\,\hat{R}_{t-1} + (1-\phi^r)\,\left(\hat{\Pi}_t\,\phi^\pi + \hat{y}_t\,\phi^y\right) + \epsilon_t^m$

Table G.2: Log linearized conditions of the model with capital and sticky wages.

G.3 Calibration

The calibration of the parameters used to obtain the results in table 2 of the paper is summarized in table G.3 below. We use standard values in the DSGE literature like assuming an average price/wage rigidity of 3.5 quarters, 10% annual depreciation of capital, investment adjustment costs as estimated by Smets and Wouters (2007), an average price/wage markup of 20% and a central Bank that follows a strict inflation targeting regime without smoothing in the Taylor rule. Here we briefly discuss the difference in the calibration across the different model variants reported in table 2. The model with Homogeneous hours assumes that the hours of both agents are the same (as in Bilbiie et al. (2022)) and therefore, in the presence of sticky wages, the labor union pulls labor variety from both agents.[71] Given this, in that case, we assume a homogeneous IES across the two agents equal to the value we use to calibrate σ_S in the heterogenous IES case ($\sigma_H = \sigma_S = 0.8$). This also applies to the model with heterogeneous hours but not IES. The model with flexible wages implies $\eta^w \to \infty$ and $\phi^w = 0$. The model without capital is approximated by setting $\alpha \to 0$ and $\iota \to \infty$.

Table G.3: Parameters calibration used for results in table 2

Parameter	**Value**	**Description**
β	0.99	Discount Factor
σ^H	0.80/0.1	IES, H
σ^S	0.80	IES, S
φ^{-1}	1	Frish elasticity of Labor Supply
δ	0.025	Capital depreciation
ι	5.5	Investment adjustment costs
η	6	Elasticity of substitution goods
η^w	6	Elasticity of substitution labor
ϕ^r	0	Interest rate smoothing
ϕ^π	1.5	Taylor rule coeff of inflation
ϕ^y	0	Taylor rule coeff of output
λ	0.155	Share of HtM Agents
ϕ^p	42.7	Rotemberg prices
ϕ^w	42.7	Rotemberg wages
$\bar{H}^H$	0.275	Steady State Hours, HtM
$\bar{H}^S$	0.33	Steady State Hours, Savers
$\bar{\Pi}$	1	Steady State Inflation Convention
$\bar{ls}$	0.68	Steady State Labor Share

[71] The model with homogeneous hours also assumes equal steady state hours between the two agents.

G.4 Robustness

In this section we report robustness exercises that show how alternative calibrations affect the value of χ within and between model variants. Figure G.1 plots the value of χ in the flexible wages + capital accumulation version of the heterogeneous Hours and IES model for different values of σ_H and λ. It resembles our analytical results in section 3.1 showing how crucial is the strength of the income effect (σ_H) in affecting the value of χ. The other important parameter affecting χ is the Frisch elasticity. As discussed in the main text, for the case of homogenous IES $\chi = 1 + \varphi$ (Bilbiie (2020)). With heterogeneous IES φ is still an important driver of χ (see equation (5) in the main text). In table G.4 we repeat the same exercise of table G.3 in the main text but this time calibrating $\varphi^{-1} = 0.5$ in line with the evidence of Chetty, Guren, Manoli and Weber (2011). With this calibration, we still see a substantial decline in amplification when allowing for heterogeneous hours and IES but we obtain dampening ($\chi < 1$) only when combined with sticky wages. This result is not sensitive to the strength of the Rotemberg wages adjustment costs. As long as we deviate from flexible wages the value of χ goes below 1.[72]

	Flexible wages	**Sticky wages**	**Flexible wages**	**Sticky wages**
Homogeneous Hours	4.25	1.25	5.06	1.52
Heterogeneous Hours	3.06	0.26	3.64	0.31
Heterogeneous Hours & IES	1.25	0.07	1.35	0.09

Table G.4: Value of χ across model's specifications. $\varphi^{-1} = 0.5$

[72] Results available upon request.

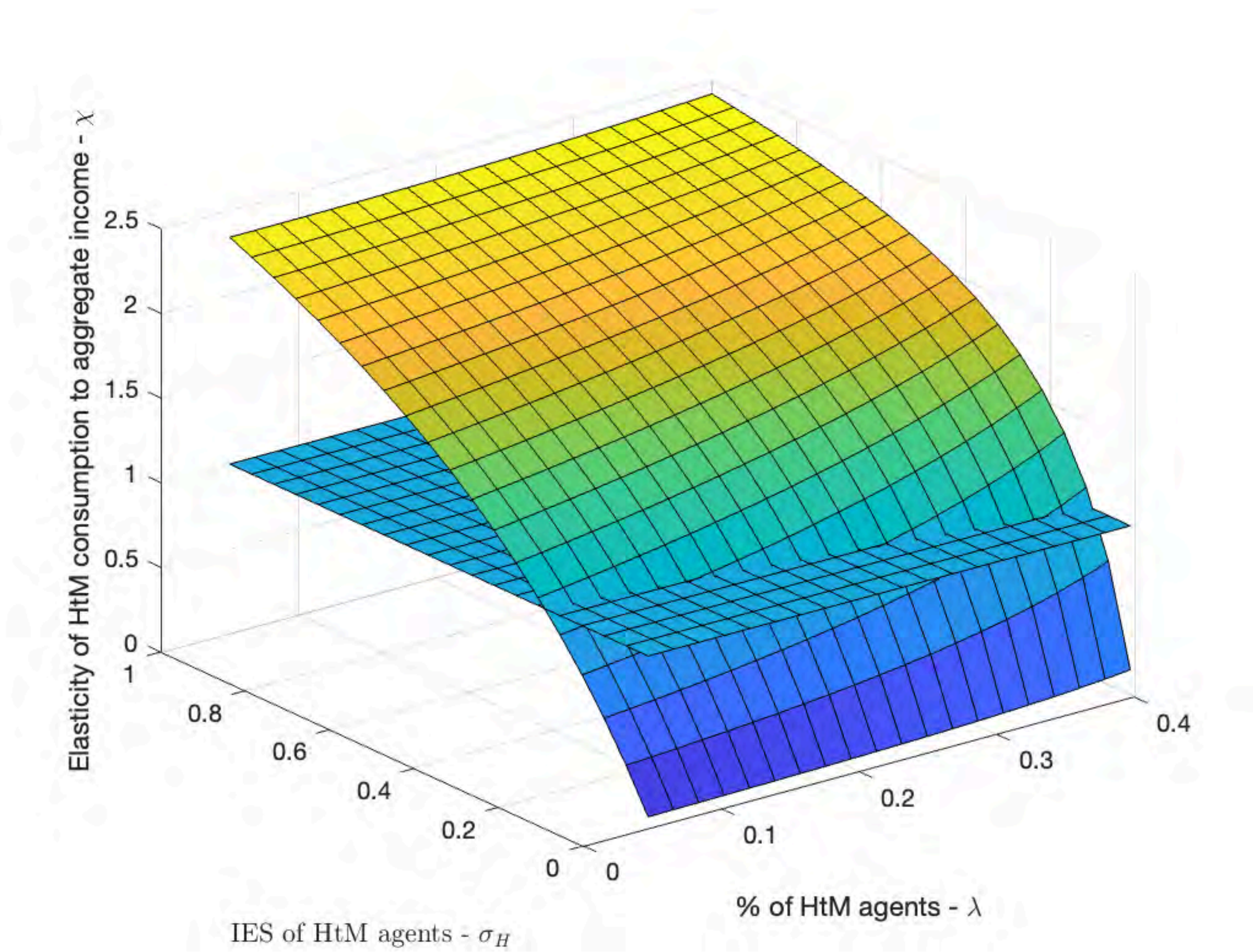


Figure G.1: Value of χ in the calibrated medium scale version of the model with different values of λ and σ_H.

H TANK model with Borrowers and Savers

An alternative way to characterize the household heterogeneity is to assume that some agents are net *borrower* (indexed with B) and some other net *saver*, as in Bilbiie et al. (2013).[73] The key features of this class of models are that borrowers are more impatient than savers, have no access to government bonds, and can borrow up to a limit. Importantly, here we abstract from IES heterogeneity and assume that both agents have the same IES, σ.

The key equations of the log-linearized model are reported in table H.5. Equilibrium

Log-linearized Conditions		
1:	Labor Supply S	$\varphi \hat{H}_t^S = \hat{w}_t - \sigma^{-1}\hat{c}_t^S$
2:	Euler S	$\hat{c}_t^S = \hat{c}_{t+1\|t}^S - \sigma\left(\hat{R}_t - \hat{\Pi}_{t+1\|t}\right)$
3:	Labor Supply B	$\varphi \hat{H}_t^B = \hat{w}_t - \sigma^{-1}\hat{c}_t^B$
4:	Budget constraint B	$\hat{c}_t^B\gamma + \bar{D}(\hat{R}_{t-1} - \hat{\Pi}_t) = \left(\hat{w}_t + \hat{H}_t^B\right)$
5:	Phillips Curve	$\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \kappa \hat{w}_t$
6:	Aggregate C	$\hat{c}_t = \lambda\gamma\hat{c}_t^B + (1-\lambda\gamma)\hat{c}_t^S$
7:	Aggregate B	$\hat{c}_t = \hat{H}_t = \lambda \hat{H}_t^B + (1-\lambda)\hat{H}_t^S$
8:	Taylor Rule	$\hat{R}_t = \hat{\Pi}_{t+1\|t} + \epsilon_t^m$

Table H.5: Log-linearized Conditions of Savers/Borrowers model

conditions in table H.5 and Table 1 are similar (leaving aside IES heterogeneity) with the only difference being the borrowers budget constraint and aggregate consumption. The assumption that borrowers discount more future consumption, $\beta^B < \beta^S = \beta$, implies that they become net borrower in equilibrium with the borrowing limit ($\bar{D}$) always binding.[74] γ is a steady state parameter which captures the consumption inequality between borrowers and savers, i.e. $\gamma = c^B/c = 1 + \bar{D}(\beta - 1) < 1$. Notice that when $\bar{D} = 0 \rightarrow \gamma = 1$ and the model is identical to the one with HtM consumers, and homogenous IES. In this model, as well, a fraction of agents are not on the Euler equation and cannot optimize intertemporally. The key difference with the previous set up is that a change in the nominal rate will have an impact not only on the time t consumption and labor supply decision but also on the $t+1$ decisions because the debt repayments at $t+1$ depend on the time t interest rates.

Under mild conditions, borrowers have an incentive to increase their labor supply after an interest rate hike; this is formalized in the following proposition.

Proposition 3 *Under SADL ($\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)}$) and $\sigma < \frac{1+\bar{D}\kappa}{\gamma}$, a rate hike at time t induces an increases in the borrowers labor supply both at time t and $t+1$.*

[73] See their paper for details on the model derivation. Relative to them we simplify it further by abstracting from government debt, expenditure, and redistribution concerns. As in the previous model, we follow Bilbiie (2020) and assume that there is a production subsidy that induces marginal cost pricing which implies that the steady state of marginal costs is 1 which simplifies substantially the steady state and the log-linearized conditions.

[74] Note that in equation (4) in table H.5 $\bar{D}$ is effectively divided by the steady state of total income/consumption. But this is =1 one in this simple set up.

The proof is in appendix H.2. At the core of this result, we have that consumption and the labor supply of the borrowers move in opposite directions. This can be appreciated when combining the time $t+1$ optimal response of borrowers in terms of consumption and labor supply after a monetary policy shock which are[75]

$$\hat{H}^B_{t+1} = \frac{\bar{D}(\varphi\lambda\gamma\sigma - 1 + \lambda)}{(\varphi\lambda - 1 + \lambda)(1 + \gamma\varphi\sigma)}\epsilon^m_t$$
$$\hat{c}^B_{t+1} = \frac{\varphi\sigma\bar{D}}{(\varphi\lambda - 1 + \lambda)(1 + \gamma\varphi\sigma)}\epsilon^m_t$$

Combining the latter two equations we have that

$$\hat{c}^B_{t+1} = \frac{\varphi\sigma}{\varphi\lambda\gamma\sigma - 1 + \lambda}\hat{H}^B_{t+1}$$

The numerator is positive. Notice that if the conditions of Proposition 3 hold, we have $\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)} < \frac{1}{1+\varphi\sigma\gamma}$, which implies that $(1 - \lambda - \varphi\sigma\lambda\gamma) > 0$; hence the denominator is negative. As mentioned this model nests the HtM model when $\bar{D} = 0$. This holds true also for the sufficient conditions of Proposition 3; notice that when $\bar{D} = 0 \to \gamma = 1 \to \sigma < 1$ and $\lambda < \frac{1}{1+\varphi}$.

H.1 Hours ratio in Borrowers/Savers model

Here we check how the Borrowers/Savers model performs in capturing the different sensitivity of the household's labor supply decision to a monetary policy shock. As the tightness of the borrowing constraint increases (larger $\bar{D}$), the borrower's labor supply becomes more sensitive to changes in the nominal rate; for large enough values of $\bar{D}$ and low enough value of σ, the income effect eventually gets so strong that generates an absolute increase in hours worked larger than one of the unconstrained agents. Since aggregate hours worked decline in the SADL equilibrium, the fraction of HtM households ought not to be too large. These arguments are depicted in Figures H.2-H.3 where we display the absolute ratio between the borrower's and savers' hours worked after a monetary policy shock for different values of $\bar{D}$ and λ. Figures H.2 is obtained by calibrating $\sigma = 1$ and $\varphi = 1$. Cool (warm) colors indicate a smaller (larger) value of the hour's ratio. As we move to the northeast corner (larger $\bar{D}$ and low λ, we generate a relatively more sensitive response of borrower households. However, the hour's ratio remains well below unity and far from our empirical evidence. This is because with $\sigma = 1$ the income effect on labor supply is still not enough to generate a larger sensitivity of the borrower's labor supply. Figure H.3 shows that to move closer to our empirical evidence we need to reduce the economy-wide IES to 0.5.

[75] The time t optimal responses are analogous but more involved. To ease the notation we only discuss the time $t+1$ decisions.

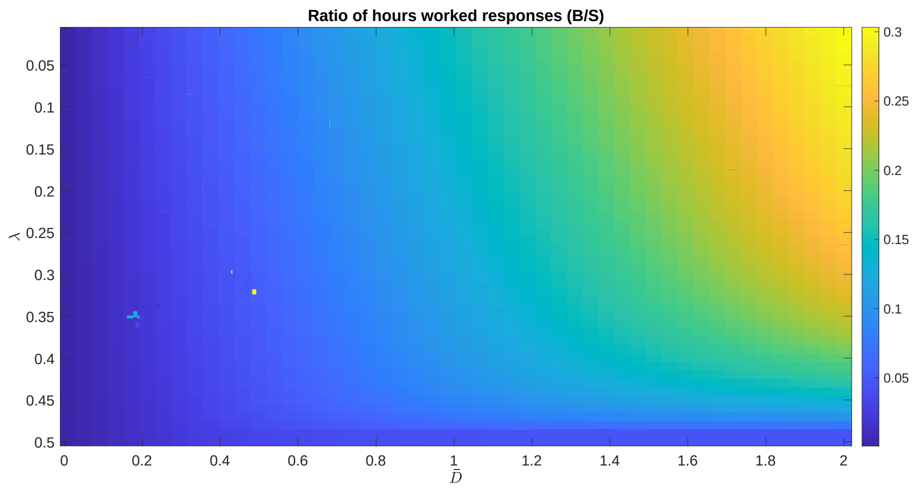


Figure H.2: Relative (absolute) magnitude of the response of borrowers and savers hours worked to a monetary policy shock for different values of $\bar{D}$ and λ and with $\sigma = 1$. Values larger than one indicate larger volatility of borrowers labor supply relative to Savers.

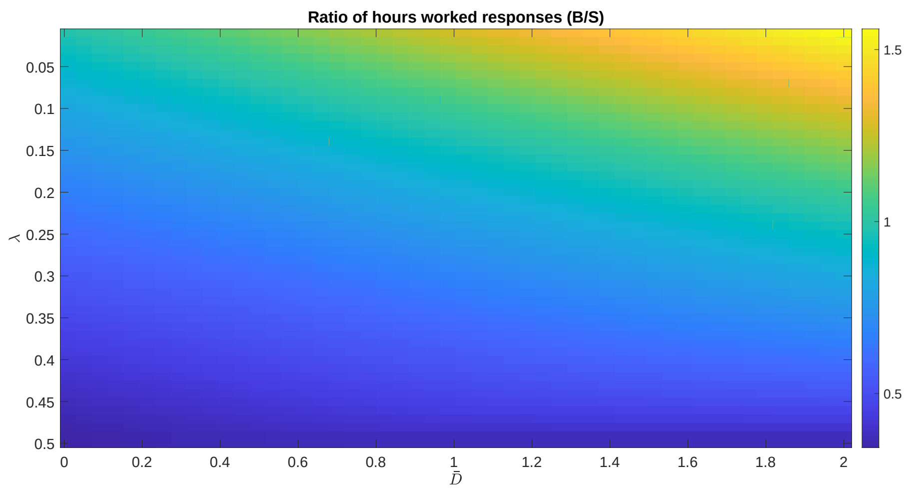


Figure H.3: Relative (absolute) magnitude of the response of borrowers and savers hours worked to a monetary policy shock for different values of $\bar{D}$ and λ and with $\sigma = 0.5$. Values larger than one indicate larger volatility of borrowers labor supply relative to Savers.

H.2 Derivation of Proposition 3

The derivation of the proposition 3 is more involved since R_t enters in the budget constrain of the Borrowers at $t+1$. This requires a backward induction solution approach, i.e. we first solve $t+1$, then given $t+1$ we solve for t. For convenience we report below the time $t+1$ equilibrium conditions.

	Log-linearized Conditions	
1:	Labor Supply S	$\varphi \hat{H}^S_{t+1} = \hat{w}_{t+1} - \sigma^{-1}\hat{c}^S_{t+1}$
2:	Euler S	$\hat{c}^S_{t+1} = \hat{c}^S_{t+2\|t+1} - \sigma\left(\hat{R}_{t+1} - \hat{\Pi}_{t+2\|t+1}\right)$
3:	Labor Supply B	$\varphi \hat{H}^B_{t+1} = \hat{w}_{t+1} - \sigma^{-1}\hat{c}^B_{t+1}$
4:	Budget constraint B	$\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{R}_t - \hat{\Pi}_{t+1}) = \left(\hat{w}_{t+1} + \hat{H}^B_{t+1}\right)$
5:	Phillips Curve	$\hat{\Pi}_{t+1} = \beta\hat{\Pi}_{t+2\|t+1} + \kappa\hat{w}_{t+1}$
6:	Aggregate C	$\hat{c}_{t+1} = \lambda\gamma\hat{c}^B_{t+1} + (1-\lambda\gamma)\hat{c}^S_{t+1}$
7:	Aggregate B	$\hat{c}_{t+1} = \lambda\hat{H}^B_{t+1} + (1-\lambda)\hat{H}^S_{t+1}$
8:	Taylor Rule	$\hat{R}_{t+1} = \hat{\Pi}_{t+2\|t+1} + \epsilon^m_{t+1}$

Table H.6: Log-linearized Conditions of Savers/Borrowers model

Recall that $\epsilon^m_{t+j} = 0$ for $j > 0$ and $\epsilon^m_t \neq 0$. This implies that from $t+2$ onward the economy is back to steady states and all quantities are zero. This means also that $\hat{R}_{t+j} = \hat{\Pi}_{t+j+1|t+j}$ for $j > 0$, which implies that

$$\hat{c}^S_{t+1} = 0$$

The saver labor supply becomes

$$\varphi\hat{H}^S_{t+1} = \hat{w}_{t+1}$$

Using the borrowers BC we have

$$\begin{aligned}
\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{R}_t - \hat{\Pi}_{t+1}) &= \left(\hat{w}_{t+1} + \hat{H}^B_{t+1}\right) \\
\hat{c}^B_{t+1}\gamma + \bar{D}(\hat{\Pi}_{t+1|t} + \epsilon^m_t - \hat{\Pi}_{t+1}) &= (1+\varphi)\hat{H}^B_{t+1} + 1/\sigma\hat{c}^B_{t+1} \\
1/\sigma\hat{c}^B_{t+1} &= \frac{1+\varphi}{\gamma\sigma - 1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma - 1}\epsilon^m_t
\end{aligned}$$

notice that in absence of shocks in $t+1$ $\hat{\Pi}_{t+1} = \hat{\Pi}_{t+1|t}$. Combining the to labor supply conditions we have

$$\begin{aligned}
\varphi\hat{H}^S_{t+1} + \sigma^{-1}\hat{c}^S_{t+1} &= \varphi\hat{H}^B_{t+1} + \sigma^{-1}\hat{c}^B_{t+1} \\
\varphi\hat{H}^S_{t+1} &= \varphi\hat{H}^B_{t+1} + \frac{1+\varphi}{\gamma\sigma - 1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma - 1}\epsilon^m_t \\
\varphi\hat{H}^S_{t+1} &= \frac{1+\gamma\varphi\sigma}{\gamma\sigma - 1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma - 1}\epsilon^m_t
\end{aligned}$$

Combining the aggregate conditions we have

$$
\begin{aligned}
\lambda\gamma\hat{c}^B_{t+1} + (1-\lambda\gamma)\hat{c}^S_{t+1} =& \lambda\hat{H}^B_{t+1} + (1-\lambda)\hat{H}^S_{t+1} \\
\lambda\gamma\sigma\left[\frac{1+\varphi}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\bar{D}}{\gamma\sigma-1}\epsilon^m_t\right] =& \lambda\hat{H}^B_{t+1} + (1-\lambda)\left[\frac{1+\gamma\varphi\sigma}{\varphi(\gamma\sigma-1)}\hat{H}^B_{t+1} - \frac{\bar{D}}{\varphi(\gamma\sigma-1)}\epsilon^m_t\right] \\
\hat{H}^B_{t+1}\left[\frac{(1+\varphi)\lambda\gamma\sigma}{\gamma\sigma-1} - \lambda - (1-\lambda)\frac{1+\gamma\varphi\sigma}{\varphi(\gamma\sigma-1)}\right] =& \left[\frac{\bar{D}\lambda\gamma\sigma}{\gamma\sigma-1} - (1-\lambda)\frac{\bar{D}}{\varphi(\gamma\sigma-1)}\right]\epsilon^m_t \\
\hat{H}^B_{t+1}\left[\frac{(1+\varphi)\varphi\lambda\gamma\sigma - \lambda\varphi(\gamma\sigma-1) - (1-\lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)}\right] =& \frac{\varphi\lambda\gamma\sigma - (1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t \\
\hat{H}^B_{t+1}\left[\frac{\varphi\lambda(\gamma\sigma + \varphi\gamma\sigma - \gamma\sigma + 1) - (1-\lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)}\right] =& \frac{\varphi\lambda\gamma\sigma - (1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t \\
\hat{H}^B_{t+1}\frac{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}{\varphi(\gamma\sigma-1)} =& \frac{\varphi\lambda\gamma\sigma - (1-\lambda)}{\varphi(\gamma\sigma-1)}\bar{D}\epsilon^m_t
\end{aligned}
$$

which yield to

$$
\hat{H}^B_{t+1} = \frac{\varphi\lambda\gamma\sigma - 1 + \lambda}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t \tag{H.41}
$$

This implies that borrower consumption at time $t+1$ is

$$
\begin{aligned}
\hat{c}^B_{t+1} =& \frac{\sigma(1+\varphi)}{\gamma\sigma-1}\hat{H}^B_{t+1} - \frac{\sigma\bar{D}}{\gamma\sigma-1}\epsilon^m_t \\
=& \frac{\sigma(1+\varphi)}{\gamma\sigma-1}\frac{\varphi\lambda\gamma\sigma - 1 + \lambda}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t - \frac{\sigma\bar{D}}{\gamma\sigma-1}\epsilon^m_t \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{(1+\varphi)(-1+\lambda(1+\varphi\gamma\sigma)) - (\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{-(1+\varphi) + (1+\varphi)\lambda(1+\varphi\gamma\sigma) - \varphi\lambda(1+\gamma\varphi\sigma) + (1-\lambda)(1+\gamma\varphi\sigma)}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\sigma\bar{D}}{\gamma\sigma-1}\left[\frac{-1-\varphi+1+\gamma\varphi\sigma}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\right] \\
=& \epsilon^m_t\frac{\varphi\sigma\bar{D}}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}
\end{aligned}
$$

and wages

$$
\begin{aligned}
\hat{w}_{t+1} =& \varphi\hat{H}^B_{t+1} + 1/\sigma\hat{c}^B_{t+1} \\
=& \varphi\frac{\varphi\lambda\gamma\sigma - 1 + \lambda}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)}\bar{D}\ \epsilon^m_t + \epsilon^m_t\frac{\varphi\bar{D}}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)} \\
=& \epsilon^m_t\bar{D}\varphi\frac{\varphi\lambda\gamma\sigma - 1 + \lambda + 1}{(\varphi\lambda - 1 + \lambda)(1+\gamma\varphi\sigma)} \\
=& \epsilon^m_t\frac{\bar{D}\varphi\lambda}{\varphi\lambda - 1 + \lambda}
\end{aligned}
$$

and inflation

$$
\hat{\Pi}_{t+1} = \beta\hat{\Pi}_{t+2|t+1} + \kappa\hat{w}_{t+1} = \epsilon^m_t\frac{\bar{D}\varphi\lambda\kappa}{\varphi\lambda - 1 + \lambda}
$$

Now, we are in a position to solve for time t. Solving the Euler equation forward we have

$$
\hat{c}^S_t = -\sigma\epsilon^m_t
$$

From the NKP we have an expression for today inflation

$$\hat{\Pi}_t = \beta \hat{\Pi}_{t+1|t} + \kappa \hat{w}_t = \epsilon_t^m \frac{\bar{D}\varphi\lambda\kappa\beta}{\varphi\lambda - 1 + \lambda} + \kappa \hat{w}_t$$

Using the borrowers BC we have

$$\gamma \hat{c}_t^B + \bar{D}(\hat{R}_{t-1} - \hat{\Pi}_t) = \hat{w}_t + \hat{H}_t^B$$

$$\gamma \hat{c}_t^B - \epsilon_t^m \bar{D} \frac{\bar{D}\varphi\lambda\kappa\beta}{\varphi\lambda - 1 + \lambda} - \bar{D}\kappa\varphi \hat{H}_t^B - \bar{D}\kappa/\sigma \hat{c}_t^B = \varphi \hat{H}_t^B + 1/\sigma \hat{c}_t^B + \hat{H}_t^B$$

which yields to

$$\hat{c}_t^B = \frac{\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\gamma\sigma - 1 - \bar{D}\kappa} \hat{H}_t^B + \frac{\bar{D}^2 \sigma\varphi\lambda\kappa\beta}{e_1 e_0} \epsilon_t^m$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. Combining the labor supply decision we have

$$\varphi \hat{H}_t^S + \sigma^{-1} \hat{c}_t^S = \varphi \hat{H}_t^B + \sigma^{-1} \hat{c}_t^B$$

$$\varphi \hat{H}_t^S - \epsilon_t^m = \varphi \hat{H}_t^B + \frac{1 + \varphi + \varphi\bar{D}\kappa}{\gamma\sigma - 1 - \bar{D}\kappa} \hat{H}_t^B + \frac{\bar{D}^2 \varphi\lambda\kappa\beta}{(\gamma\sigma - 1 - \bar{D}\kappa)(\varphi\lambda - 1 + \lambda)} \epsilon_t^m$$

which yields to

$$\hat{H}_t^S = \frac{1 + \varphi\gamma\sigma}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} \hat{H}_t^B + \frac{\bar{D}^2 \varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1} \epsilon_t^m$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. Combining the aggregate conditions we have

$$\lambda\gamma \hat{c}_{t+1}^B + (1 - \lambda\gamma)\hat{c}_{t+1}^S = \lambda \hat{H}_{t+1}^B + (1 - \lambda)\hat{H}_{t+1}^S$$

$$\begin{aligned}
&\lambda\gamma \left[\frac{\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\gamma\sigma - 1 - \bar{D}\kappa} \hat{H}_t^B + \frac{\bar{D}^2 \sigma\varphi\lambda\kappa\beta}{e_1 e_0} \epsilon_t^m \right] + (1 - \lambda\gamma)[-\sigma\epsilon_t^m] \\
&\quad = \lambda \hat{H}_{t+1}^B + (1 - \lambda) \left[\frac{1 + \varphi\gamma\sigma}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} \hat{H}_t^B + \frac{\bar{D}^2 \varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1} \epsilon_t^m \right]
\end{aligned}$$

$$\begin{aligned}
&\hat{H}_t^B \left[\frac{\lambda\gamma\varphi\sigma(1 + \varphi(1 + \bar{D}\kappa))}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} - \frac{\lambda\varphi(\gamma\sigma - 1 - \bar{D}\kappa)}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} - \frac{(1 - \lambda)(1 + \varphi\gamma\sigma)}{\varphi(\gamma\sigma - 1 - \bar{D}\kappa)} \right] \\
&\quad = \epsilon_t^m \left[\sigma(1 - \lambda\gamma) - \gamma\lambda \frac{\bar{D}^2 \sigma\varphi\lambda\kappa\beta}{e_1 e_0} + (1 - \lambda) \frac{\bar{D}^2 \varphi\lambda\kappa\beta + e_0 e_1}{\varphi e_0 e_1} \right]
\end{aligned}$$

Focusing on terms inside the left hand side square bracket we have

$$\begin{aligned}
&\varphi^{-1} e_1^{-1} \left[\varphi\lambda(\sigma\gamma + \sigma\gamma\varphi(1 + \bar{D}\kappa)) - \lambda\varphi(\gamma\sigma - 1 - \bar{D}\kappa) - (1 - \lambda)(1 + \varphi\gamma\sigma) \right] \\
&= \varphi^{-1} e_1^{-1} \left[\varphi\lambda(1 + \sigma\gamma\varphi)(1 + \bar{D}\kappa) - (1 - \lambda)(1 + \varphi\gamma\sigma) \right] \\
&= \varphi^{-1} e_1^{-1} (1 + \sigma\gamma\varphi)(\varphi\lambda(1 + \bar{D}\kappa) - 1 + \lambda)
\end{aligned}$$

Focusing on terms inside the right hand side square bracket we have

$$\begin{aligned}
&(\varphi e_0 e_1)^{-1} \left[\varphi\sigma(1 - \lambda\gamma) e_0 e_1 - \varphi\gamma\lambda(\bar{D}^2 \sigma\varphi\lambda\kappa\beta) + (1 - \lambda)(\bar{D}^2 \varphi\lambda\kappa\beta + e_0 e_1) \right] \\
&= (\varphi e_0 e_1)^{-1} \left[\varphi\sigma(1 - \lambda\gamma) e_0 e_1 - \varphi\gamma\lambda(\bar{D}^2 \sigma\varphi\lambda\kappa\beta) + (1 - \lambda)\bar{D}^2 \varphi\lambda\kappa\beta + (1 - \lambda) e_0 e_1 \right] \\
&= (\varphi e_0 e_1)^{-1} \left[e_0 e_1(\varphi\sigma(1 - \lambda\gamma) + 1 - \lambda) - \varphi\gamma\lambda\sigma\bar{D}^2 \varphi\lambda\kappa\beta + (1 - \lambda)\bar{D}^2 \varphi\lambda\kappa\beta \right] \\
&= (\varphi e_0 e_1)^{-1} \left[\varphi\sigma e_0 e_1 + e_0 e_1(1 - \lambda - \varphi\sigma\lambda\gamma) + \bar{D}^2 \varphi\lambda\kappa\beta(1 - \lambda - \varphi\sigma\lambda\gamma) \right] \\
&= (\varphi e_0 e_1)^{-1} \left[\varphi\sigma e_0 e_1 + (e_0 e_1 + \bar{D}^2 \varphi\lambda\kappa\beta)(1 - \lambda - \varphi\sigma\lambda\gamma) \right]
\end{aligned}$$

Rearranging terms we have

$$\hat{H}_t^B = \frac{\varphi\sigma e_0 e_1 + (e_0 e_1 + \bar{D}^2\varphi\lambda\kappa\beta)(1-\lambda-\varphi\sigma\lambda\gamma)}{(1+\sigma\gamma\varphi)(\varphi\lambda(1+\bar{D}\kappa)-1+\lambda)(\varphi\lambda-1+\lambda)}\ \epsilon_t^m \tag{H.42}$$

where $e_1 = \gamma\sigma - 1 - \bar{D}\kappa$ and $e_0 = \varphi\lambda - 1 + \lambda$. While still not very tractable we can derive a set of sufficient conditions such that the latter expression becomes positive. These conditions are

1. $\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)}$
2. $\sigma < \frac{1+\bar{D}\kappa}{\gamma}$

Condition 1 implies that $e_1 = [\varphi\lambda(1+\bar{D}\kappa)-1+\lambda] < 0$; this implies also that $(\varphi\lambda-1+\lambda) < 0$. Thus, if condition 1 holds, the denominator is positive. Condition 2 implies that $e_0 = (\sigma\gamma - 1 - \bar{D}\kappa) < 0$; this implies also that $e_0 e_1 > 0$. Moreover, if condition 2 hold, it is the case that

$$\lambda < \frac{1}{1+\varphi(1+\bar{D}\kappa)} < \frac{1}{1+\varphi\sigma\gamma}$$

The latter implies that $(1-\lambda-\varphi\sigma\lambda\gamma) > 0$. Therefore also the numerator is positive. These conditions imply also that the coefficient in (H.41) is positive.