Learning Monetary Policy Strategies at the Effective Lower Bound with Sudden Surprises

Spencer Krane, Leonardo Melosi, and Matthias Rottner

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Learning Monetary Policy Strategies at the Effective Lower Bound with Sudden Surprises*

Spencer Krane
FRB Chicago

Leonardo Melosi
FRB Chicago, CEPR

Matthias Rottner
Deutsche Bundesbank

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Abstract

Central banks around the world have revised their operating frameworks in an attempt to counter the challenges presented by the effective lower bound (ELB) on policy rates. We examine how private sector agents might learn such a new regime and the effect of future shocks on that process. In our model agents use Bayesian updating to learn the parameters of an asymmetric average inflation targeting rule that is adopted while at the ELB. Little can be discovered until the economy improves enough that rates would be near liftoff under the old policy regime; learning then proceeds until either the new parameters are learned or the average inflation target is reached. Recessionary shocks forcing a return to the ELB would thus delay learning while large inflationary shocks could outright stop it and so inhibit the ability of the new rule to address future ELB episodes. We show the central bank can offset some of the inflation-induced learning loss by deviating from its new rule, but it must weigh the benefits of doing so against the costs of higher near-term inflation and greater uncertainty about the policy function.

Keywords: New framework, central bank's communications, deflationary bias, asymmetric average inflation targeting, imperfect credibility, liftoff, Bayesian learning.

JEL classification: E52, C63, E31

*Emails: spencer.krane@chi.frb.org, leonardo.melosi@chi.frb.org, matthias.rottner@bundesbank.de. We thank seminar participants at the Federal Reserve Bank of Chicago and Deutsche Bundesbank. The views in this paper are solely those of the authors and should not be interpreted as reflecting the views of the Deutsche Bundesbank, the Federal Reserve Bank of Chicago, or any other person associated with the Eurosystem or the Federal Reserve System.

1 Introduction

Over the course of the 2000s, the effective lower bound (ELB) on policy rates went from being a theoretical curiosity to a prominent feature of the monetary policy environment. For many well-known structural reasons, equilibrium policy rates fell below where they were in the 1980s and 1990s. Even average business cycle shocks now have the potential to drive policy rates to the ELB, let alone the large shocks of the Great Financial Crisis or the Covid-19 pandemic. The experience of the past 20 years demonstrated the challenges that the ELB places on central banks' efforts to achieve their mandated policy objectives.

In response, central banks adopted new strategies aimed at offsetting the challenges posed by the ELB. Notably, ELB risks were an important factor behind the Federal Reserve's new statement on Longer-Run Goals and Monetary Policy Strategy it adopted in August 2020 and the ECB's new Monetary Policy Strategy Statement issued in July 2021.[1]

A growing body of research shows that under traditional monetary policy strategies the limits on reducing policy rates presented by the proximity of the ELB imparts a downward bias to inflation and inflation expectations relative to the central bank's target. The expectational bias is a feature of the distribution of inflation over time; it's not just an occurrence associated with a large negative shock to the economy. In turn, this bias impinges on the central bank's ability to stabilize output and inflation.[2]

This literature has proposed a range of alternative monetary policy frameworks to address this inflation bias. These include average inflation targeting, price level targeting, shadow rate makeup polices, running "dovish policies" away from the ELB, and asymmetric reaction functions and target ranges.[3]

The theoretical solutions proposed in this literature generally involve strong assumptions: completely rational and forward-looking agents; complete credibility of the monetary authority; and no adjustment lags or other inertias in the economy. Under such conditions, these models offer clear policy prescriptions and their implementation works

[1]See Federal Reserve Board - Monetary Policy (https://www.federalreserve.gov/monetarypolicy.html) and the ECB's monetary policy strategy statement (https://www.ecb.europa.eu/home/search/review/html/ecb.strategyreview_monpol_strategy_statement.en.html).

[2]See Adam and Billi (2007), Nakov (2008) and Hills et al. (2019).

[3]Discussions of average inflation and price level targeting are found in Evans (2012), Bernanke (2017), Hebden and López-Salido (2018), and Bernanke et al. (2019). Shadow rate make up policies were proposed in Reifschneider and Willams (2000). Dovish policies are discussed in Mertens and Williams (2019). Bianchi et al. (2021) examine asymmetric reaction functions and target ranges. Similar asymmetric strategies are advocated by Caballero and Simsek (2022) during temporary supply contractions when aggregate demand has inertia.

perfectly. Rational agents immediately align their views with the bias-corrected distribution of inflation generated by the new policy rule.

Of course, the real world is not so clean. New monetary policy strategies need to be learned and their credibility established. This process can be complicated even away from the ELB as agents need to estimate the new policy parameters, and it can take time to hone in on precise values given the noisy relationships between policy rates and economic outcomes.[4] But there are added challenges if a new policy rule is introduced while rates are mired at the ELB because there is no co-movement between economic variables and interest rates to inform agents. The central bank's proclamations may lead agents to form priors, but there is little they can do in terms of statistical inference until they actually observe different policy outcomes from what would have occurred under the old strategy.

This paper considers how such a learning process might evolve when a central bank introduces a new operating framework while at the ELB. We start with a standard New Keynesian framework in which the economy is driven to the ELB by large negative demand shocks. The central bank then announces it is adopting a new policy rule to counter the ELB. But we also assume the central bank does not announce all of the elements the new policy function, or cannot do so with full credibility. Instead, agents must learn them.

The new rule we consider is a form of the average inflation targeting, which adds a term in the weighted average of past shortfalls of inflation from target to the central bank's old policy function. The weighted average inflation shortfall is known to agents, but the intensity of the central bank's response to the shortfall is not. The new rule also contains a policy shock that is uncorrelated with the inflation shortfall. Although modeled as a random shock, this term is also meant to capture purposeful deviations from the new policy rule that could be a feature of optimal policy, such as when the central bank adjusts policy due to risk management considerations.[5] Agents know such shocks can occur, but have to learn about their volatility.

Agents form priors over the unknown parameters when the rule is announced. We then roll the problem sequentially forward through time, with agents using Bayesian updating to revise their estimates of the unknown parameters each period. The process is constructed

[4] Erceg and Levin (2003a) for instance study a model in which agents have to learn the inflation target of the central bank.

[5] Any rule is only an approximation to optimal policy, and the shock could stand in for the approximation error. Relevant for our case, even with its asymmetric average inflation target addition, our rule might not completely capture the optimal response to asymmetric risks posed by the ELB. For example, Evans et al. (2015) discuss how optimal policy will prescribe slower increases in rates in response to heightened uncertainty in the neighborhood of the ELB. They also find some statistical evidence that historically the Federal Reserve has deviated from its baseline policy rule for risk management reasons.

so that the updated estimates are consistent with equilibrium output and inflation on a period-by-period basis. We let the negative demand shocks dissipate over time, so that after some time the economy is able to exit the ELB and then eventually converge to its stochastic steady-state.

Little can be learned about the parameters of the new policy function as long as economic conditions are such that the shadow rate under the old monetary policy rule is well below the ELB - for all agents know, the central bank is engaging in cheap talk, and there is no reason to move away from their priors. The information structure evolves as the economy improves enough that rates would be near liftoff under the old policy rule, and then changes radically when the old rule would prescribe liftoff but the central bank keeps rates at the ELB. Agents can now use the difference between the prescriptions of the old rule and the ELB to make more accurate inferences about the new rule's unknown parameters. And once liftoff occurs, agents can hone in more precisely on the new policy function. This means that the key time for policymakers to establish the credibility of the new strategy is around the period when the old rule would have called for liftoff and in the early stages of rate increases under the new regime.

Technically, this evolution of the learning process reflects the fact that the likelihood function component of the posterior distribution of the unknown parameters resembles a Tobit regression in which the modification to the interest rate rule is the dependent variable, the explanatory variable is the average inflation shortfall, the error term is the policy shock, and the censoring point is where the new monetary policy rule prescribes liftoff. Deep in recession, when there is little chance of being near the censoring point, the likelihood is flat and the posterior puts almost all its weight on the prior. As a positive rate becomes increasingly probable, the slope of the likelihood steepens and moves the posterior away from the prior; the information content of the data increases further once liftoff occurs and non-censored observations enter the likelihood function.

In terms of macroeconomic outcomes, the new policy rule is calibrated to offset the disinflationary bias in the ELB and produce long-run inflation expectations that are anchored at target in a full information environment. In the learning problem, and under a baseline scenario with no shocks to the policy function, agents' long-run inflation expectations are stuck below target until just before the old policy function would have called for a lift-off in rates. Inflation expectations subsequently rise towards target, with the pace quickening once lift-off would have occurred under the old rule. This path of expectations convergence contrasts the immediate jump to the inflation target seen in the full-information analysis. But is still a marked improvement relative to the old policy regime, in which long-run inflation expectations would remain stuck below target. Similarly, the paths for output and inflation generated by the new rule with learning are an improvement over outcomes under the old regime, though under our baseline parameter assumptions they fall short of the full information economy.

To see how the new policy performs, we hit the economy with either further recessionary shocks that drive it back to the ELB or positive demand shocks that generate a bout of inflation. By construction, if learning is completed when the shocks hits, the learned-rule performs just like the full information model. In the case of the recessionary shocks, the declines in output and inflation are much smaller than under the old rule. In the case of the inflationary shocks, the new rule performs very much like the old rule, as by the time learning has been completed the past average inflation shortfall has been eliminated and the new and old rules look alike. If a recessionary shocks hit before learning is complete, the new rule performs better than the old rule to the degree that the new parameters have been learned prior to the shock. Learning then progresses as the economy makes its second exit from the ELB. For an inflationary shock, the new model will generate somewhat higher inflation than the old rule until the average inflation shortfall is eliminated. This highlights an important feature of the new rule – because high inflation will drive the average past shortfalls to zero, there is a natural limit to the ”extra” inflation the new rule can generate. However, if the inflation shortfall is driven to zero before the new rule is learned completely, then the economy will still suffer extra losses in output and inflation generated when the next ELB-inducing recessionary shock hits.

The extra risks posed by such inflation shocks suggest that the central bank might be tempted to delay liftoff to extend the learning process. We find this does improve important aspects of the learning process. However, the delay comes at the cost of higher near-term inflation and greater uncertainty over the monetary policy reaction function. Such uncertainty is not very problematic in the simple New Keynesian model we consider, but may be in more complex specifications. The central bank might also be tempted to hike rates faster than the new rule to battle the higher inflation, but this curtails the learning process even more severely, with adverse consequences in the event of future ELB inducing shocks.

The literature about learning new policy rules in the presence of the ELB is limited. One example is Bodenstein et al. (2022), who consider learning about a price-level targeting strategy. In their setting agents believe the new rule is changing continuously over time, but their beliefs are predetermined each period when the model is solved. This allows the learning problem to be solved analytically using the Kalman filter. Andolfatto and Gomme

(2003) and Erceg and Levin (2003b) develop structural models with regime changes for monetary policy in which private sector agents have imperfect information about the regimes. Schorfheide (2005) and Gust et al. (2018) estimate this type of models using likelihood methods. Bianchi and Melosi (2016, 2018) develop a class of models where agents are uncertain about the persistence of policy regimes.

In all these papers, agents learn about the new policy regime, but there is no intra-period interaction between the learning process and economic outcomes. This is convenient for solving the models, but it also means they cannot account for general-equilibrium effects that could have an important influence on the actions of agents and the central bank. One contribution of our paper is to simultaneously solve the model for agents' updated beliefs about the new strategy and macroeconomic outcomes, and so capture these influences. Another contribution is the censored likelihood function that allows for learning even before rates lift off from the ELB and sharpens intuition about the importance of the periods around liftoff for learning about the new framework.

2 The model

We start with an off-the-shelf New Keynesian model, but work with its nonlinear formulation and not the log-linearization. The model features a standard Euler equation and Phillips curve (using Rotemberg pricing), where π_t is inflation, y_t is output, π and y are their deterministic steady state values, mc_t are marginal costs, and $\Lambda_{t,t+1}$ is the stochastic discount factor:

$$1 = \beta R_t \mathbb{E}_t \left[\left(\frac{\epsilon_{d,t+1}}{\epsilon_{d,t}} \right) \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \frac{1}{\pi_{t+1}} \right] \tag{1}$$

$$\varphi \left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = (1-\epsilon) + \epsilon mc_t + \varphi \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t} \right] \tag{2}$$

The only uncertainty entering this part of the model is the preference shock in the Euler equation, ϵ_t^d, which is assumed to follow an AR(1) process. All of the parameters in these equations and the shock process are known to the agents and the central bank. The new features of the model involve the announcement by the central bank that it will now follow an asymmetric average targeting monetary policy rule given by:

$$R_t = \max \left[1, R \left(\frac{\pi_t}{\pi} \right)^{\phi_\Pi} \left(\frac{y_t}{y} \right)^{\phi_Y} + \eta_t^z \cdot \mathbf{1}_{z_t<0} \right] \tag{3}$$

$$\eta_t^z = \xi_z z_t + \sigma_R \epsilon_{R,t} \tag{4}$$

$$z_t = \rho_z z_{t-1} + \phi_z \left(\pi_{t-1} - \pi\right) \tag{5}$$

R_t is the (gross) nominal interest rate, and it equals the maximum of the ELB, which we assume is the zero lower bound of 1, and the interest rate determined by the new rule.[6] The first term in the new rule is the usual Taylor prescription, where R is equilibrium real rate of interest. This is the policy rule followed prior to the introduction of the new rule. The second term, $\eta_t^z \cdot \mathbf{1}_{z_t<0}$, is the adjustment announced by the central bank, where $\mathbf{1}_{z_t<0}$ is an indicator function for when $z_t < 0$. This adjustment is a function of the weighted average of past inflation shortfalls, z_t, the intensity of the central bank's response to the shortfall, $\xi_z \geq 0$, and a random shock, $\epsilon_{R,t} \sim \mathcal{N}(0,1)$, and is in place as long as z_t is negative. Confining the adjustment to periods when average inflation is below target reflects the new rule's explicit mission to counter the downward inflation bias generated by the ELB. The random shock is meant to capture either purposeful deviations from the new rule, random factors such as measurement error, or technical issues that may prevent the central bank from setting policy exactly where it wants to. Conceptually, such a shock should enter the policy rule even if $z_t > 0$, but we did not do so in order to highlight the implications of such deviations for learning the new policy rule. We calibrate the new rule so that it exactly offsets the ELB inflation bias in a full-information setting.

The functional form of the new rule is known to the public, as are the parameters determining the weighted average of inflation shortfalls, ρ_z and ϕ_z. However, we assume that even though the central bank can announce a new policy, it cannot do so with full credibility. This is captured by assuming agents do not know the intensity of the central bank's response to inflation shortfall, ξ_z, nor the standard deviation, σ_R, of the error term in the policy function adjustment, $\epsilon_{R,t}$. Agents use Bayesian updating to estimate all unknown parameters, and then act as if policy is dictated by their resulting perception of the monetary policy rule, $\hat{R}_t$:

$$\hat{R}_t = \max\left[1, R\left(\frac{\pi_t}{\pi}\right)^{\phi_\Pi}\left(\frac{y_t}{y}\right)^{\phi_Y} + \hat{\eta}_t^z \cdot \mathbf{1}_{z_t<0}\right] \tag{6}$$

$$\hat{\eta}_t^z = \hat{\xi}_{z|t} z_t + \hat{\sigma}_{R|t} \hat{\epsilon}_{R,t|t} \tag{7}$$

$$z_t = \rho_z z_{t-1} + \phi_z \left(\pi_{t-1} - \pi\right) \tag{8}$$

[6] Our framework is general enough to incorporate a negative lower bound, including one that is endogenously determined within a structural model as in Darracq Pariès et al. (2020).

where $\hat{\xi}_{z|t}$, $\hat{\sigma}_{R|t}$, and $\hat{\epsilon}_{R,t|t}$ are the estimates agents make at time t for ξ_z, σ_R, and $\epsilon_{R,t}$.

In the absence of the new policy rule, the model displays the familiar features of the simple three-equation New Keynesian model. Important for our analysis, future events have a powerful influence on the present, so that the chance of reaching the ELB sometime in the future induces a downward bias to inflation expectations that can impede economic performance today.

3 Estimating the new monetary policy rule

Suppose $z_t < 0$ but that the ELB is not binding under the new rule, so $R_t > 1$ is observed. Then agents also can observe η_t^z. Given $\epsilon_{R,t} \sim \mathcal{N}(0,1)$, the likelihood of this observation is:

$$\frac{1}{\sigma_R}\phi\left(\frac{\eta_t^z - \xi_z z_t}{\sigma_R}\right)$$

where ϕ is the standardized normal density.

When the ELB is binding under the new rule, R_t is censored at 1 and η_t^z is unobservable. All agents know is that η_t^z is small enough to keep rates from lifting off, and hence is less than an upper bound, $\bar{\eta}_t^z$. The likelihood of this event occurring is given by:

$$\Phi\left(\frac{\bar{\eta}_t^z - \xi_z z_t}{\sigma_R}\right) \tag{9}$$

where

$$\bar{\eta}_t^z = 1 - R\left(\frac{\pi_t}{\pi}\right)^{\phi_\Pi}\left(\frac{y_t}{y}\right)^{\phi_Y} > \eta_t^z \tag{10}$$

and Φ is the standardized cumulative normal density. Note that $\bar{\eta}_t^z$ is the difference between the ELB and the shadow interest rate under the old monetary policy rule. The bound thus reveals how much work η_t^z has to do to prevent liftoff under the old rule given the observed values for output and inflation.

With both censored and uncensored observations, the likelihood function is a Tobit regression in which η_t^z is the dependent variable, z_t is the explanatory variable, the unknown parameters are ξ_z and σ_R and the censoring point is $\bar{\eta}_z$. However, our problem is different than the standard Tobit, because we assume the central bank announces the new policy framework at time $t = 1$ when $R_t = 1$, so at the start of the learning process agents only have censored observations. Accordingly, maximum likelihood estimation is infeasible during this time.[7]

Instead, we assume at the time of the announcement, agents form the perceived monetary policy rule, equation (6), and a set of priors over the unknown parameters, $\Omega(\xi_{z|P}, \sigma_{R|P})$. The agents then engage in sequential Bayesian updating as they move forward through time. Specifically, in every period when $z_t < 0$, they choose $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ to maximize the posterior distribution:

$$\prod_{\tau \leq t} \left\{ \left[\frac{1}{\hat{\sigma}_{R|t}} \phi \left(\frac{\eta^z_\tau - \hat{\xi}_{z|t} z_\tau}{\hat{\sigma}_{R|t}} \right) \right] \mathbb{I}\left(R_\tau > 1\right) \cdot \left[\Phi \left(\frac{\bar{\eta}^z_\tau - \hat{\xi}_{z|t} z_\tau}{\hat{\sigma}_{R|t}} \right) \right] \mathbb{I}\left(R_\tau = 1, z_\tau < 0\right) \right\} \cdot \Omega\left(\hat{\xi}_{z|P}, \hat{\sigma}_{R|P}\right) \tag{11}$$

Once $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ are in hand, $\hat{\epsilon}_{R|t}$ can be estimated as the residual that sets $\hat{R}_t = R_t$:

$$\hat{\epsilon}_{R,t|t} = \frac{\eta_{z,t} - \hat{\xi}_{z|t} z_t}{\hat{\sigma}_{R|t}} \tag{12}$$

or as the expected value of the standard truncated normal if the ELB is binding:

$$\hat{\epsilon}_{R,t|t} = \mathbb{E}\left(\epsilon_{z,t} | \hat{\xi}_{z|t}, \hat{\sigma}_{R|t}\right) = -\hat{\sigma}_{R|t} \frac{\phi\left(\frac{\bar{\eta}_{z,t} - \hat{\xi}_{z|t} z_t}{\hat{\sigma}_{R|t}}\right)}{\Phi\left(\frac{\bar{\eta}_{z,t} - \hat{\xi}_{z|t} z_t}{\hat{\sigma}_{R|t}}\right)} \tag{13}$$

Agents stop updating their beliefs when inflation has been running above the central bank's target enough that z_t reaches zero. When $z_t \geq 0$ and $\eta^z_t = 0$ there is nothing agents can learn about the unknown parameters, ξ_z and σ_R, because the policy rate R_t is no longer dependent on those parameters.

Learning proceeds as follows. The central bank announces the new policy rule when the economy is deep in recession and $\bar{\eta}^z_1 >> 0$ and $z_1 << 0$. At this time the likelihood function is uninformative. This can be seen in figure 1, which plots the likelihood when the data are censored, with the red dot to the right depicting the $t = 1$ starting point. The likelihood here is essentially 1 one for many values of $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$, and so maximization of the posterior distribution doesn't move the parameters from their priors. As the economy improves, $\bar{\eta}^z$ falls and z_t rises, moving $\left(\bar{\eta}^z_t - \hat{\xi}_{z|t} z_t\right)/\hat{\sigma}_{R|t}$ to left (the blue arrows). The likelihood becomes informative, and maximization of the posterior will begin to change the values of $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ from the priors.

Specifically, consider the first order conditions for the log posterior with respect to $\hat{\xi}_{z|t}$

[7] With only censored values, agents are maximizing the standard normal cdf, which is done with $\hat{\xi}_z|t \to \infty$ or $\hat{\sigma}_{R|t} \to 0$ and $\Phi\left(\frac{\bar{\eta}^z_\tau - \hat{\xi}_{z|t} z_\tau}{\hat{\sigma}_{R|t}}\right) \to 1$

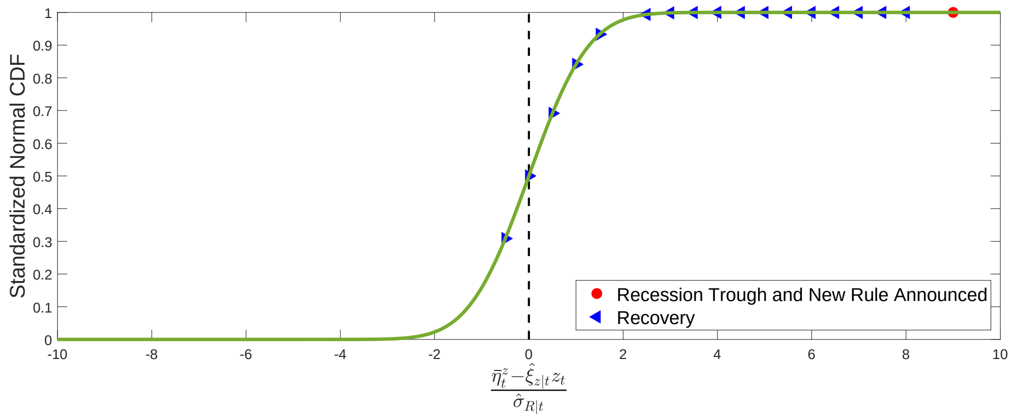

Figure 1: The likelihood function of the censored data. The likelihood (vertical axis) becomes informative as the economy improves, as indicated by movement of $\left(\bar{\eta}^z_t - \hat{\xi}_{z|t} z_t\right)/\hat{\sigma}_{R|t}$ (horizontal axis) to the left (blue arrows). The starting point is the red dot.

when there are only censored data (so $R_t = 1$ for all $\tau < t$):

$$\sum_{\tau \le t} \frac{-\phi\left(\frac{\bar{\eta}^z_\tau - \hat{\xi}_{z|t} z_\tau}{\hat{\sigma}_{R|t}}\right)}{\Phi\left(\frac{\bar{\eta}^z_\tau - \hat{\xi}_{z|t} z_\tau}{\hat{\sigma}_{R|t}}\right)} \frac{z_\tau}{2\hat{\sigma}_{R|t}} - \omega\left(\frac{\hat{\xi}_{z|t} - \xi_{z,p}}{\sigma_{R,p}}\right) = 0$$

where ω is the derivative of $\ln \Omega$. As long as $\left(\bar{\eta}^z_t - \hat{\xi}_{z|t} z_t\right)/\hat{\sigma}_{R|t}$ is very large, $\phi\left[(\bar{\eta}^z_t - \hat{\xi}_{z|t} z_t)/\hat{\sigma}_{R|t}\right]$ is near zero and nearly all of the weight in estimation is on the prior; but as $\bar{\eta}^z_t$ falls and z_t increases, so does $\phi\left[(\bar{\eta}^z_t - \hat{\xi}_{z|t} z_t)/\hat{\sigma}_{R|t}\right]$, the posterior begins putting meaningful weight on the likelihood, and the parameter estimates will change.

Note that the likelihood provides information even before the old rule would have dictated liftoff conditional on $\{y_t, \pi_t\}$. This is because as $\bar{\eta}^z_t$ falls but rates stay at the ELB, agents put lower weight on: 1) values of σ_R that would admit a meaningful probability on large positive realizations of $\epsilon_{R,t}$ that would accelerate lift-off; and 2) small values of ξ_z that would imply little response to the observed z_t. More information is gleaned when liftoff would have occurred under the old rule ($\bar{\eta}^z_t$ turns negative) but rates remain at the ELB. Once liftoff occurs, non-censored observations on η^z_t enter the calculations, and the likelihood portion of the estimation is now a standard Tobit regression. Learning slows as $z_t \to 0$ and $\partial\phi\left[(\eta^z_t - \hat{\xi}_{z|t} z_t)/\hat{\sigma}_{R|t}\right] \;/\; \partial\xi_z \;\to\; 0$ for all ξ_z and stops completely when $z_t \;=\; 0$ and policy reverts back to being determined by the old rule.

Parameters	Sign	Value	Parameters	Sign	Value
		Model Parameters			
Discount factor	β	0.9975	Production function	α	1
Relative risk aversion	σ	1	Inverse Frisch elasticity	η	1
Disutility of labor	χ	0.87	Price elasticity of demand	ϵ	7.67
Rotemberg pricing	φ	79.41	Response Inflation	κ_Π	2
Response Output	κ_Y	0.25	Inflation Target	$4\log(\Pi)$	2%
Persistence AIT	ρ_Z	0.875	Deviations AIT	ϕ_Z	0.1
Response to AIT	ξ_Z	1	Std. Dev. of AIT Shock	σ_R	0
Persistence Pref. Shock	ρ_ζ	0.6	Std. Dev. Pref. Shock	σ_ζ	1.14%
		Priors			
Prior Mean ξ_Z	$\mathbb{E}_{\text{Prior}}(\xi_Z)$	0.1	Prior Std. Dev. ξ_Z	$\text{Std}_{\text{Prior}}(\xi_Z)$	0.25
Prior Mean σ_R	$\mathbb{E}_{\text{Prior}}(\sigma_R)$	0.25%	Prior Stad. Dev. σ_R	$\text{Std}_{\text{Prior}}(\sigma_R)$	0.03%

Table 1: Model calibration.

4 Solving and simulating the model

We solve the model using global methods.[8] We do so to allow the model's nonlinearities to influence economic outcomes. If we solved a linearized version of the model, the only feedback of learning about the monetary policy rule onto current behavior would occur through the rule's effect on expectations of hitting the ELB in the future.[9] But learning about the new rule also reshapes agents' views about the future volatility of inflation and output and hence the welfare risks posed by the economy returning to the ELB. These higher order effects could be important; the multiplicative interactions between variables in the model have the potential to amplify the effects of shocks relative to solutions from linear approximations, and so could provide a richer description of the effects of the ELB than linearized models. Such factors are particularly relevant because the new rule is specifically aimed at eliminating the non-linear downside risks imposed by the ELB constraint. Solving the nonlinear model in which the entire distribution of agents' beliefs influences their current behavior is necessary to capture them.

We simulate the model sequentially in periods t = 1 through T. Most of the parameters are fixed and known to agents. The model is calibrated so that $\xi_Z = 1$ will produce long-run expected inflation at the central bank's target under full rational expectations. The parameters are shown in Table 1:

[8] We use a time iteration algorithm with linear interpolation based on Richter et al. (2014).

[9] In the standard New Keynesian analysis, higher odds that the ELB will bind and reduce consumption tomorrow induce expected utility smoothing agents to lower consumption today.

Note that we picked the prior means $\mathbb{E}_{\text{Prior}}(\xi_Z) = 0.1$ and $\mathbb{E}_{\text{Prior}}(\sigma_R) = 0.25$ to be far away from the true values of 1 and 0. This means agents are taking the central bank's announcement with a substantial grain of salt; accordingly, they will have a good deal of learning to do to discover the new policy framework. Note, too, that $\mathbb{E}_{\text{Prior}}(\sigma_R) = 0.25$ is in terms of the quarterly gross interest factor, and so translates into about a 1 percentage point standard deviation in the annualized policy rate. Shocks of such size can have a meaningful impact on economic outcomes in the model when interest rates are low, as they then have a good chance of generating an episode at the ELB. The effects, however, are small and transitory when interest rates are far from the ELB given the shock is i.i.d. and our model has no added inertia from lagged values of consumption or inflation in the IS or Phillips curves. We explore some alternative priors in the appendix.

We first set a sequence of shocks to the demand curve and the monetary policy function for the entire simulation sample, $\{\epsilon_{d,t}, \epsilon_{R,t}\}_t^T$.[10] The current value of $\epsilon_{d,t}$ is observed by the agents at time t, but its future values are unknown. All the $\epsilon_{R,t}$ are unknown. Each period we initialize $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ at the values estimated in period t-1 and make a guess for $\hat{\epsilon}_{R,t}$. We then calculate outcomes based on the agents' perceived policy rule; that is, we plug these parameter estimates into equation (6) to generate y_t and π_t. We then plug these values for y_t and π_t for into the actual monetary policy rule (3) to obtain the actual interest rate, R_t. If $z_t < 0$, we then re-estimate $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ by maximizing the posterior distribution (11) and recalculate $\{\hat{\epsilon}_{R,\tau|t}\}$ using equations (12) or (13). If the changes in $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ are small enough to meet our convergence criteria, we go on to period t+1. Otherwise, we repeat the process until convergence is achieved.

This general equilibrium approach means that every period economic outcomes are jointly determined with agents' parameter estimates and the central bank's execution of the policy rule. There are no surprises for either agents or the central bank to evaluate at the end of the day.[11]

Figure 2 shows our baseline simulation. Starting in t=1, the economy is hit with a series of negative preference shocks, $\epsilon_{d,t}$ (upper left), which drive interest rates to the ELB (upper right). In period 2, the central bank announces the new policy rule. The depth of the policy shortfall can be seen by the negative shadow rate under the new rule (stars, upper right) as well as the declines in output and inflation well below their steady state

[10] $\{\epsilon_{R,t} = 0\}_t^T$ in most of the simulations we consider.

[11] The general equilibrium approach also saves us the complications of modeling game-theoretic considerations that would arise if we specified a more stylized sequencing of decision-making by agents and the central bank.

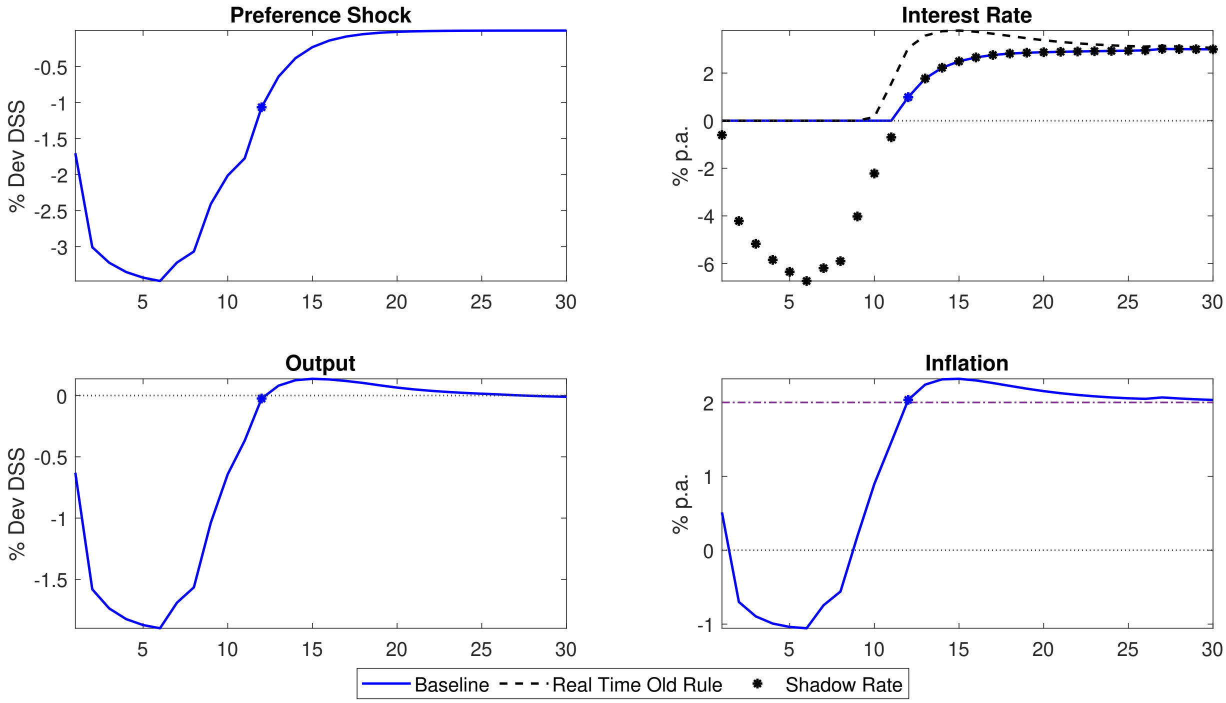

Figure 2: Baseline simulation for the new monetary policy strategy introduced during a recession (blue lines). The economy is hit by a series of negative preference shocks starting in period 1 (upper left); the new rule is introduced in period 2. The dynamics of interest rates (upper right) are compared with the shadow rate in the absence of the ELB (black dots) and the real-time old rule without η_t^z (black dashed line). Output and inflation are shown in the bottom panels; the blue dots identify the period of liftoff from the ELB.

values. The upper right-hand panel also shows the interest rate path under the old policy rule given the output and inflation rates generated by the learning model (red line).

As the demand shocks dissipate, the economic situation improves. By period 10, the old monetary policy rule would dictate liftoff, with rates rising quickly to above their long-run level of 2-1/2 percent before gradually returning to steady state. In contrast, policy under the new rule delays liftoff until period 12, and rates then slowly asymptote to the steady state. The new policy is consistent with some overshooting of both output and inflation above their long-run targets; notably inflation reaches 2-1/2 percent and then gradually returns to 2 percent over time.

Figure 3 describes the learning process. The blue solid lines show agents' parameter estimates, while the stars show the true values set by the central bank. The upper panels show the components of η_t^z, while the lower panels show the parameters that enter its calculation. A small amount of learning occurs before liftoff would have taken place under the old rule (period 10) as agents are able to eliminate values of ξ_z and σ_R that would generate early lift-off under the new rule. Learning about ξ_z picks up notably in period 10 as agents observe rates remaining at the ELB instead of lifting off as would have been occurred under the old rule. Once lift off occurs under the new rule in period 12, agents can observe η_t^z, but they still do not know its individual components, $\xi_z z_t$ and $\sigma_R \epsilon_{R,t}$, and must continue to estimate them. Indeed, $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ remain far from their true values for some time. Learning continues at a moderate pace until z_t gets very close to zero and the interest rate prescriptions under the new and old rule are essentially the same.

Note that when we solve the model, there is a discrete jump in the estimated parameters to their true values in period 26. This is largely technical in nature. By that time the estimated parameters have moved so far from the priors and $\left(\bar{\eta}_t^z - \hat{\xi}_{z|t} z_t\right)/\hat{\sigma}_{R|t}$ is so negative that the combined weights in the posterior of the prior distribution and the censored component of the likelihood effectively collapse to zero. This means only the non-truncated portion of the likelihood covering the post liftoff period influences estimation, and because $\{\epsilon_{R,t}\}_t^T = 0$ it perfectly fits the data with $\{\hat{\xi}_{z,t}, \hat{\sigma}_{R,t}\} = \{1, 0\}$. The jump would be minimal if we added a small amount of random noise, $\epsilon_{R,t}$, to the true policy rule, equation (3), but we would then have to average over a large number of draws for $\{\epsilon_{R,t}\}_t^T$ to derive the parameter estimates and this would greatly increase the computational time for each simulation. Alternatively, the jump would be smaller if we had more diffuse priors; for example, as seen in the appendix, larger values of $\text{Std}_{\text{Prior}}(\sigma_R)$ would admit a longer overlap between the tails of the prior for ξ_z and the non-truncated portion of the likelihood and results in a smoother transition of $\hat{\xi}_{z,t}$ to its true value of 1.

The new monetary policy rule is designed to eliminate the downward biases presented by the ELB under conventional policy rules. Figure 4 examines how well the learning model performs this job. To do so, we compare it to two alternatives. The first is the old monetary policy regime. In this alternative the central bank never announces a new monetary policy; there is no learning problem, and the path for $\{y_t, \pi_t\}$ is the rational expectations equilibrium generated by equations (1), (2), and (3) with $\xi_z = \sigma_R = 0$.[12,13] The second alternative is the new policy rule in the full-information rational expectations

[12]The old regime differs from our earlier calculations of the shadow rate under the old monetary policy rule. Those shadow rates, which are key for the agents' inference problem under learning, were calculated by plugging the model's output for $\{y_t, \pi_t\}$ into equation (3) with $\eta_t^z = 0$; those $\{y_t, \pi_t\}$, however, depend on $\{\hat{\xi}_{z|t}, \hat{\sigma}_{R|t}\}$ and all the previous periods' data. In contrast, under the old policy regime, ξ_z, σ_R, $\hat{\xi}_{z|t}$ and $\hat{\sigma}_{R|t}$ do not enter the solutions for $\{y_t, \pi_t, R_t\}$ at all; that is, the outcomes are what would happen if the central bank never announced a new policy rule.

[13]Note that agents cannot learn anything extra from comparing the output and inflation paths between these two scenarios. This is because the difference between the paths solely reflects $\{\hat{\xi}_{z|t}, \hat{\sigma}_{R|t}\}$, and agents already know those values when observing $\{y_t, \pi_t, R_t\}$ under the new regime.

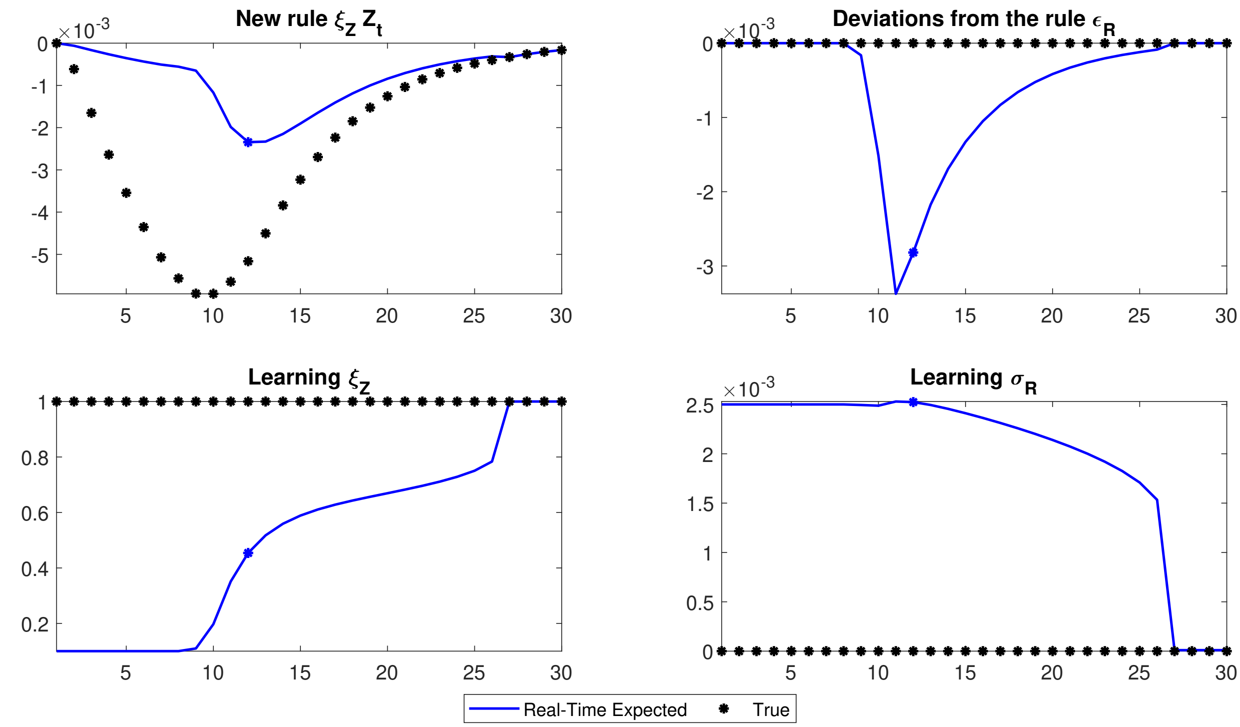

Figure 3: Learning the new rule. The blue lines are the real-time expected values of the agents; the black dashed lines are the true values. Upper panels: Components of η_t^z: structural effects of the average inflation shortfall (left) and the policy shock (right graph). The lower panels show the parameters agents estimate: intensity of central bank's response to shortfall (left) and the standard deviation of the policy shock (right). The blue dots identify the period of liftoff from the ELB

environment. This is the case in which agents immediately know all the elements of the new policy rule; again there is no learning, and the path for $\{y_t, \pi_t\}$ is the rational expectations equilibrium generated by equations (1) - (3) with ξ_z and σ_R fixed at their true values.

Early during the ELB period, the learning model and the old policy regime produce similar paths for output, but the learning model generates modestly higher inflation, reflecting the non-zero prior on ξ_z. Bigger differences emerge as agents begin to gain knowledge of ξ_z and σ_R . The differences are largest for inflation. Reflecting the powerful biases generated by the ELB, inflation under the old rule asymptotes to a level below target, while soon after liftoff inflation under the new rule is close to the full-information path.[14]

The lower-right panel shows the evolution of long-term inflation expectations, which

[14]Recall in the New Kenyesian model, events in the future have a powerful influence on outcomes today. Importantly, the output and inflation losses under future expected ELB episodes cause lower output and inflation today through the forward expectations terms in the Euler equation and Philips curve equations.

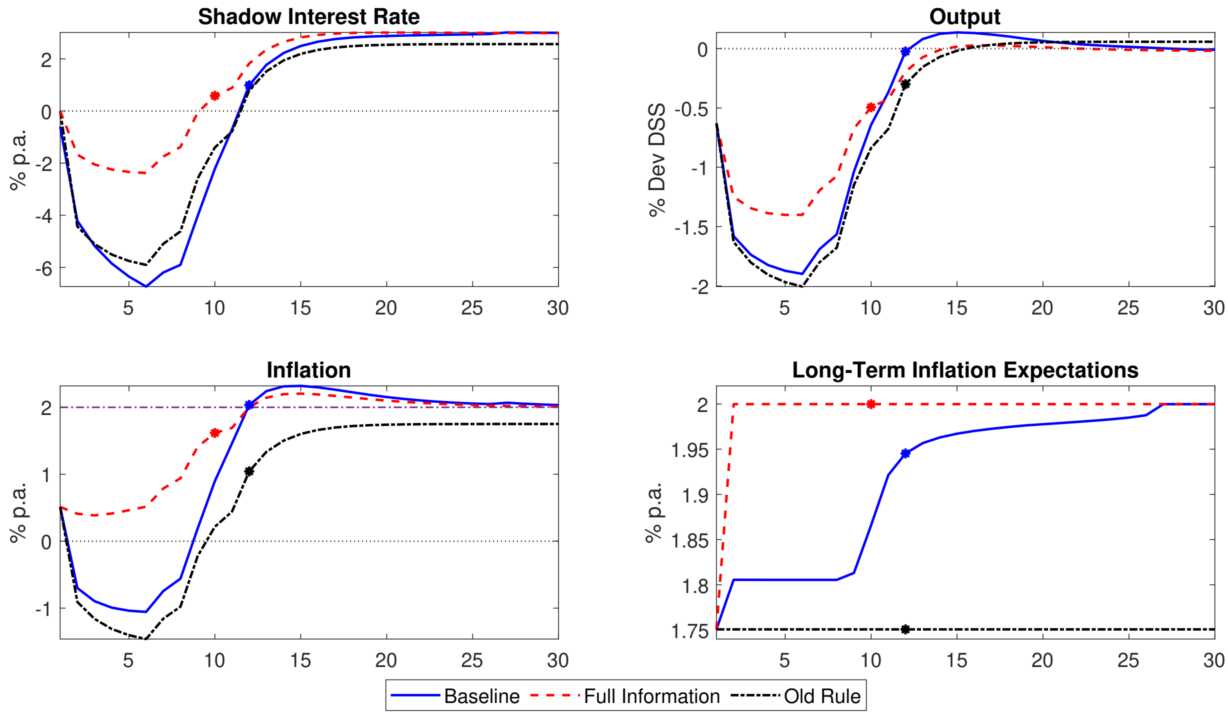

Figure 4: Comparison of outcomes under the baseline simulation of the learning model (blue) with those for the old monetary regime (black) and the new policy rule under full-information, in which agents learn the rule immediately (red). The stars identify the period of liftoff from the ELB in each regime.

we define as the model's stochastic steady state conditioned on $\{\hat{\xi}_{z|t}, \hat{\sigma}_{R|t}\}$.[15] Expectations under the old rule are stuck below the central bank's 2 percent target - this is the well-known downward bias to inflation under the ELB. In contrast, with the new rule, long-run inflation expectations begin to move up a bit before lift-off would have occurred under the old rule, reflecting the small degree of learning that takes place then. Expectations rise quickly after agents see liftoff is delayed, quickly moving to near the 2 percent target. Complete convergence to target is slow, however, reflecting the sluggish final stages of the learning process. By design, the full-information model exhibits an immediate jump to target upon adoption of the new rule; furthermore, output and inflation do not fall nearly as much during the ELB episode as in the old regime or the model with learning.

[15]Specifically, for the learning rule this inflation rate is calculated on a period-by-period basis as what the model converges to when it is run forward from $\{y_t, \pi_t, R_t\}$ conditional on $\{\hat{\xi}_{z|t}, \hat{\sigma}_{R|t}\}$ and with no future shocks to demand or monetary policy. For the old regime, we run only equations (1)-(3) forward with $\eta_t^z = 0$; the full information case runs these equations forward using the true values for $\xi_z = 1$ and $\sigma_R = 0$.

5 Performance over time: Future recessions and inflations

Our analysis so far has focused on the performance of the new rule when the only disturbance to the economy is the ELB episode that motivated its adoption. How well will it perform in a less tranquil environment? To address this question, we examine the new rule's performance in the face of subsequent shocks to the economy. We consider two types of demand shocks; ones that generate another episode at the ELB and ones that cause a surge in inflation well beyond the central bank's target.

We first examine shocks that occur in period 31, after the learning process has been completed and z_t has returned to zero. Starting with the recessionary scenario, a large negative shock to aggregate demand drives the economy to the zero lower bound. As seen in figure 5, since agents have honed in on the parameters of the new rule before the shock hit, the performance of the economy under the new rule with learning is identical to the full information case, delivering a substantially shorter stay at the ELB and smaller deviations of output and inflation from target than the old policy rule. The change in policy has been a success.

In the case of the inflationary shock, we impose demand shocks that push inflation above 3 percent. The results are shown in figure 6. Because of the downward inflation bias in the old policy rule, inflation rises somewhat slower under it than under the new rule, but the overall inflation experience is not much different between the two. This is because by the time the inflation shocks hit, $z_t = 0$, $\eta_t^z \cdot \mathbf{1}_{z_t<0}$ is no longer operative, and the old and new policy rules look alike; the only difference between the two is that agents' long-run inflation expectations are somewhat lower under the old rule. Once the shocks have dissipated, inflation and output quickly return to their pre-shock steady states.

We next consider the case when shocks occur in period 14, after liftoff has occurred, but before agents have completed learning the new policy rule and while $z_t < 0$. The consequences of the recessionary shock depend on the degree to which learning has progressed. In the simulation, the shock hits when $\hat{\xi}_{z|t} = 0.6$, and as seen in figure 8, this value is large enough that the performance of the economy is only modestly worse than the full information case and substantially better than under the old rule. Note, though, that in contrast to the first recession, learning does not re-commence until liftoff under the under new rule. This is because due to its inferior economic performance, in the second recession liftoff under the old rule would occur well after liftoff under the new rule. So

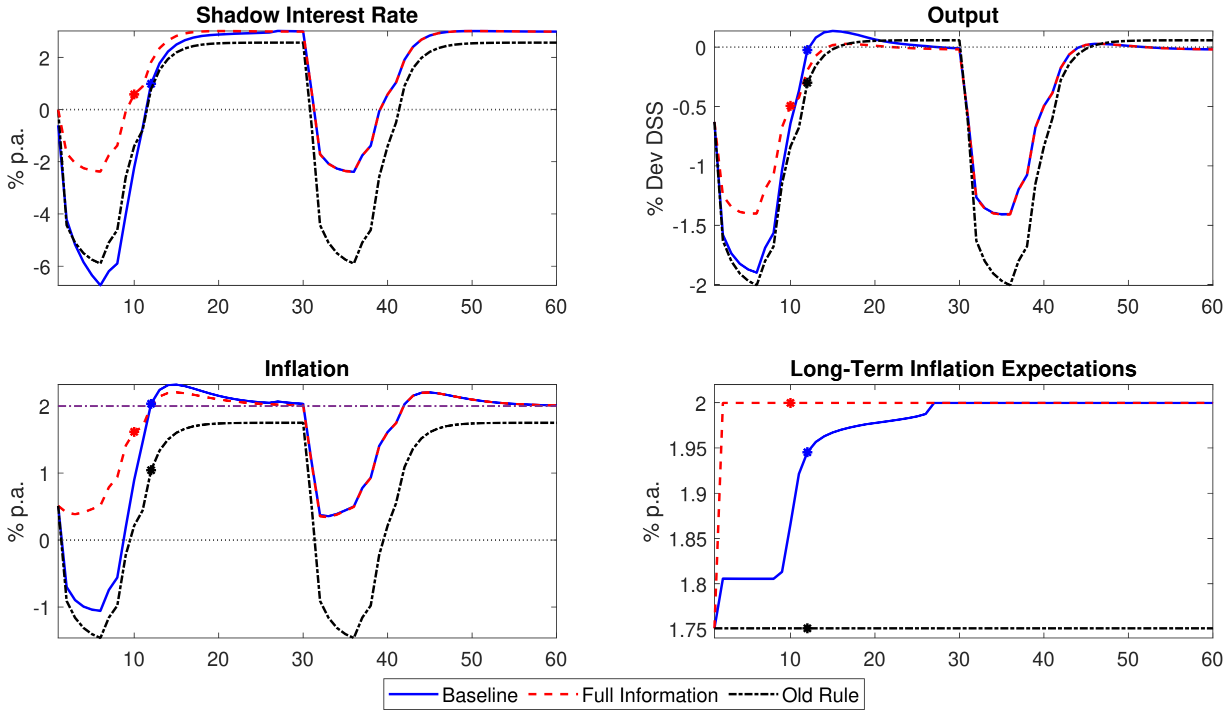

Figure 5: Recession shock that occurs after learning is complete. Comparison of outcomes under baseline simulation (blue) with those for the old monetary regime (black) and the new policy rule under full-information (red).

learning resumes at only a modest pace after liftoff under the new rule, and then accelerates once liftoff would have occurred under the old rule and agents can compare the two paths. In sum, the second recession has significantly slowed the learning process, but has not derailed it.

With the inflation shock, the economy generates much more inflation under the new rule until the past inflation shortfall has been completely offset, z_t returns to 0 (which occurs in period 22), the extra term $\eta^z_t \cdot \mathbf{1}_{z_t<0}$ in the monetary policy function is no longer operative and policy is determined by the same function as in the old regime.[16] This result highlights an important feature of the new rule: Because high inflation will eventually drive z_t to zero, there is a natural limit to the "extra" inflation the new rule can generate.

However, there are longer-term costs. Because z_t reaches zero before the learning process is complete, so agents carry forward incorrect perceptions of ξ_z and σ_r. Accordingly the economy still displays some long-run disinflationary bias and will suffer larger shortfalls in output and inflation in the event of a future ELB episode. So not only has the surge in

[16]Note the very similar interest rate paths under the two rules, as the higher expected inflation under the new rule offsets offsets the dampening effect of the ξ_z term.

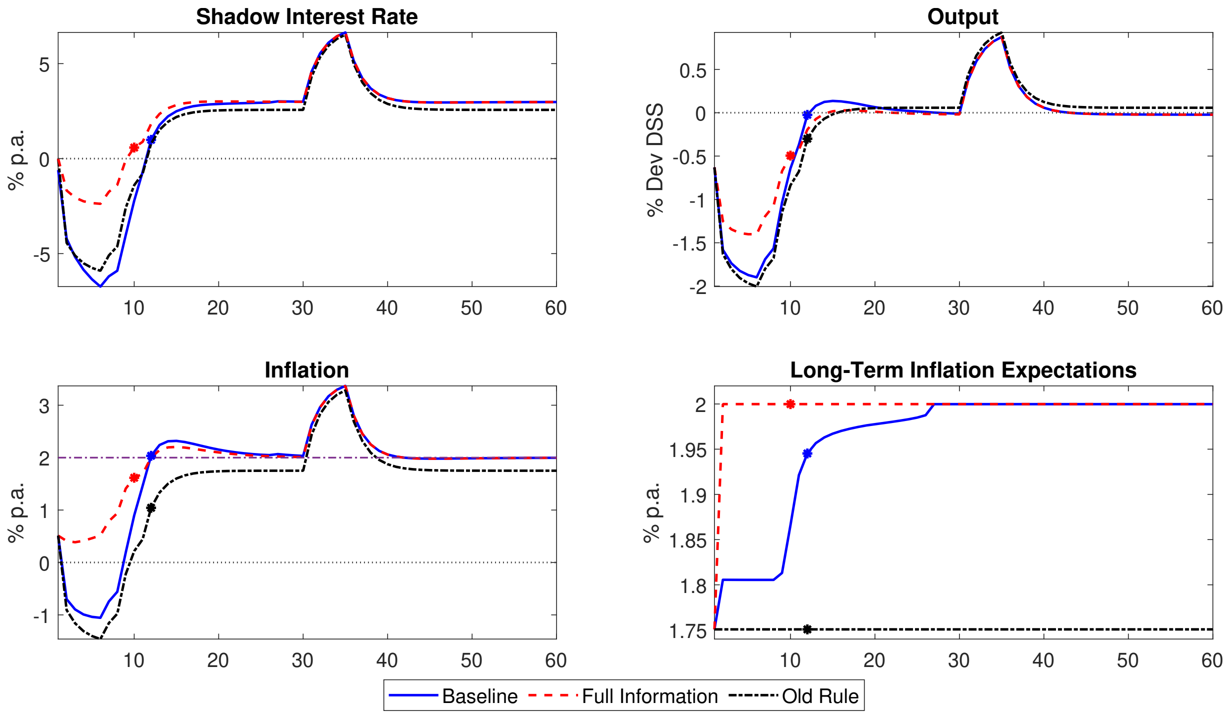

Figure 6: Inflation shock that occurs after learning is complete. Comparison of outcomes under the baseline simulation (blue) with those for the old monetary regime (black) and the new policy rule under full-information (red).

inflation been costly in direct terms of lost current utility, it also has prevented the new rule from being completely learned. Ironically, this means that high inflation might be successful in achieving an average inflation target in the short run, but could also generate longer-term costs.

6 A policy dilemma

The results in the previous section suggest that the central bank may tempted to take some kind of action to influence the learning process in the event of a large inflation shock. In our model, this means deviating from the new rule. For example, if interest rates have not yet lifted off from the ELB, the central bank could delay liftoff, allowing some higher inflation today to emphasize its commitment to eliminating past inflation shortfalls. Agents would attribute the delay in part to a larger reaction to z_t than they had been estimating, therefore moving $\hat{\xi}_{z,t}$ closer to its true value of 1, and in part as a policy shock, $\epsilon_{R,t}$. Accordingly, the delayed liftoff and associated stronger expected response to inflation shortfalls would have the benefit of reducing the losses incurred with

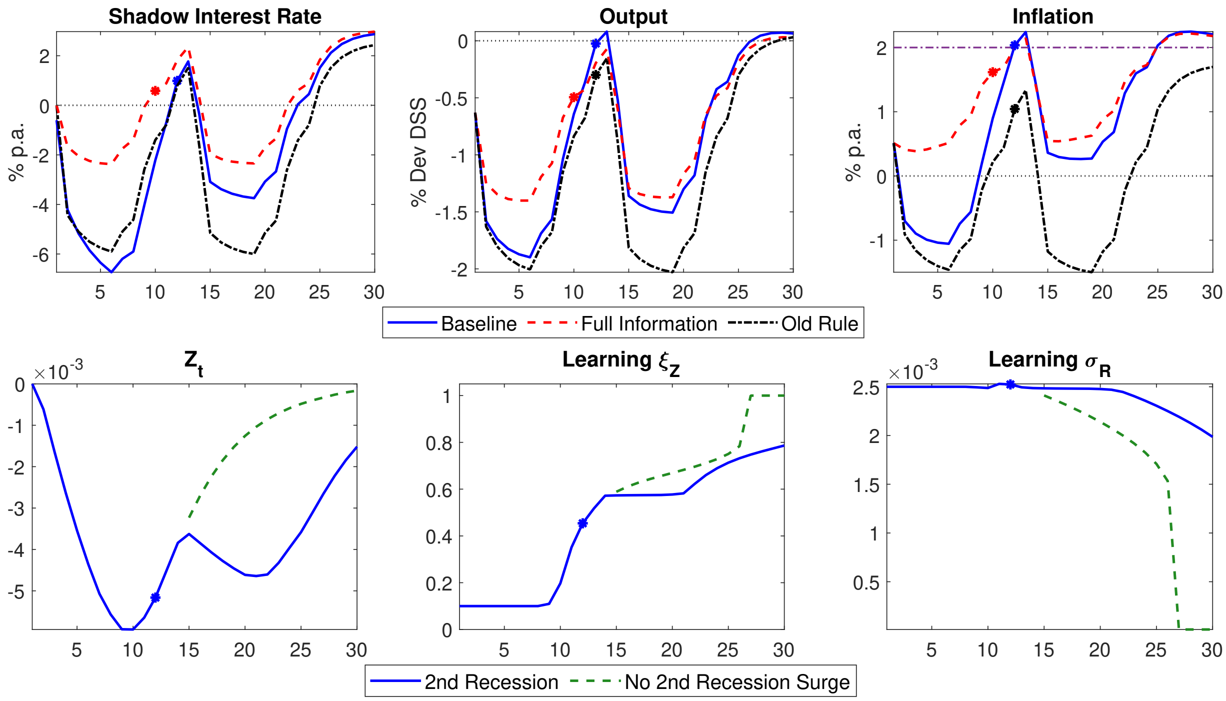

Figure 7: Recession shock before learning is completed. Top panel: Comparison of outcomes under baseline simulation (blue) with those for the old monetary regime (black) and the new policy rule under full-information (red). Bottom panel: Comparison of learning in the baseline model with (blue) and without (green) the second recession.

future recessionary shocks that threaten to send the economy back to the ELB, but come with the cost of higher inflation today and increased uncertainty about the monetary policy reaction function. So the central bank is faced with a policy dilemma – should it incur the costs of deviating from the new rule for better insurance against future ELB risks, or should it strictly adhere to the new rule?

To examine these trade-offs, we consider a stark example in which an unexpected surge in inflation occurs while the economy is still in the neighborhood of the ELB. We assume large inflationary demand shocks occur in periods 12 and 13 – so just when liftoff would have taken place in the absence of these shocks. The outcomes of this exercise under the baseline new policy rule without any deviations are shown in blue lines in Figure 9. Immediately after the shock, inflation is not yet high for long enough to push $z_t > 0$ and rates continue to be held down by η_t^Z. Inflation is strong enough to generate a sharp liftoff in the policy rate, but not one as large as would have occurred under the old rule. By period 17, the persistent high inflation brings $z_t = 0$ and rates look like they would under the old rule. Learning stops and $\{\hat{\xi}_{z,t}, \hat{\sigma}_{R,t}\}$ remain at the values that had been estimated up to this point, 0.5 and 0.24, below the true $\{\xi_z, \sigma_R\}$. Output and inflation return to

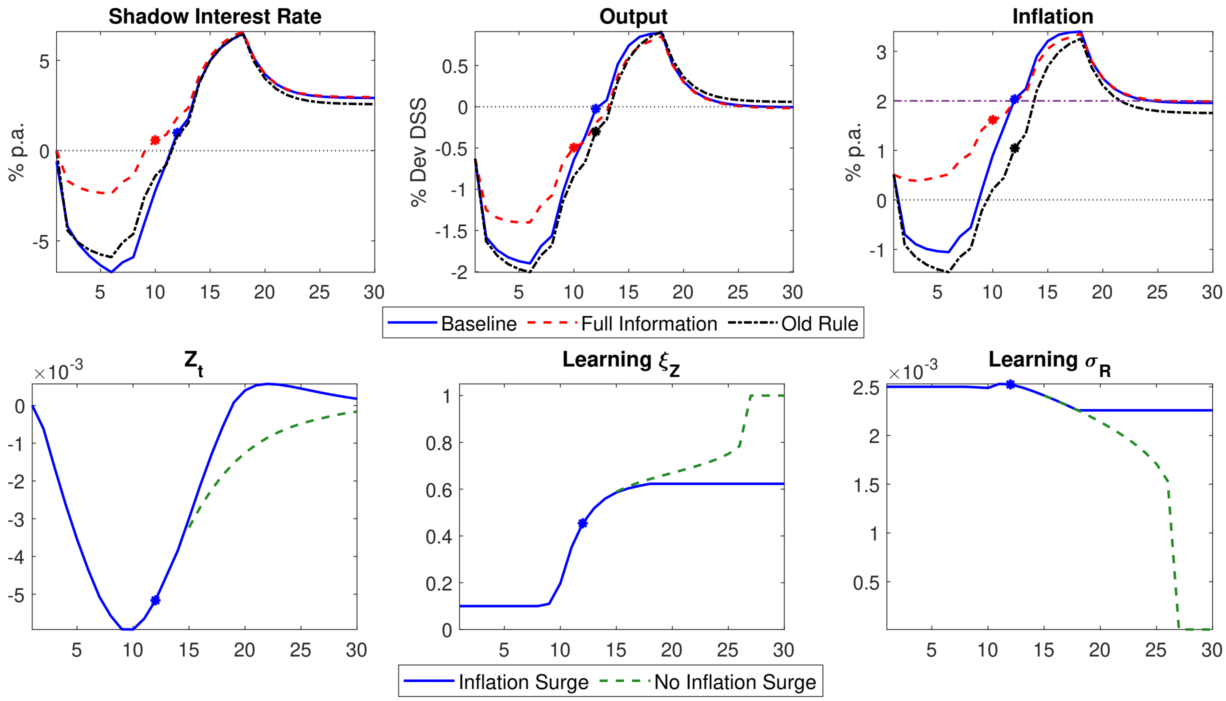

Figure 8: Inflation shock before learning is completed. Top panel: Comparison of outcomes under baseline simulation (blue) with those for the old monetary regime (black) and the new policy rule under full-information (red). Bottom panel: Comparison of learning in the baseline model with (blue) and without (green) the second inflation shock.

target around period 22.

We first compare this simulation to one in which the central bank purposefully does not move rates as aggressively as the new rule would dictate in order to keep the learning process going. This is achieved by introducing negative shocks to the monetary policy rule, $\epsilon_{R,t}$, beginning at the time when rates lift off from the ELB. The red dashed lines in the figure show the associated results. With the weaker response, inflation and output rise faster than in the baseline. By construction, once the shocks return to zero the rate paths and economic outcomes are essentially the same in the two scenarios. However, before the scenarios converge, agents see the delay in liftoff as signaling a stronger response to z_t than they had assumed, and $\hat{\xi}_{z,t}$ is much higher than in the baseline. Note, though, that it still does not converge to one by the time z_t reaches zero. Furthermore, agents do not attribute all of the delay to a larger ξ_z; they put some odds on the delay in rate increases reflecting a policy shock, and so boost their estimate of σ_R, pushing it even further off the mark than in the baseline scenario.

Alternatively, the large inflationary shock could raise central bank concern over its ability to reign in inflation expeditiously enough to prevent inflation expectations from

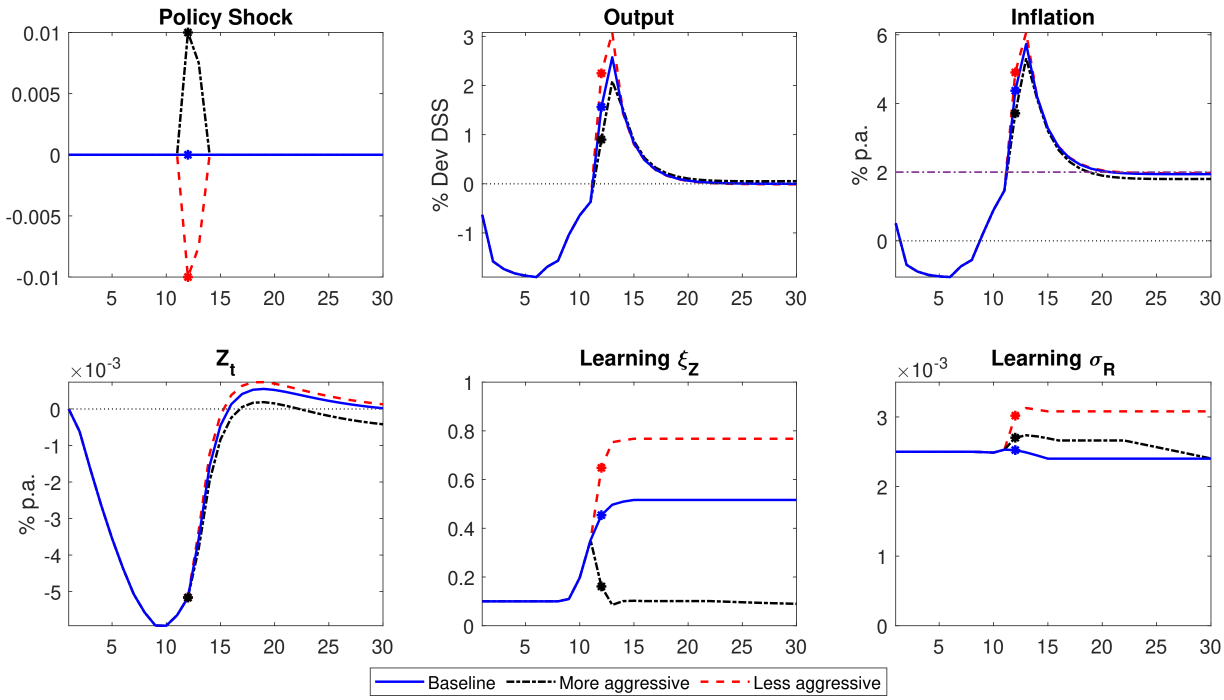

Figure 9: Inflation shock occurring around the time of liftoff. Comparison of outcomes under baseline simulation (blue) with monetary policy shocks that result in a less aggressive (red) or more aggressive (black) response to inflation.

rising above target. If so, it might consider a more aggressive response to the inflation shock in order to re-assure the private sector of its commitment to both sides of the inflation target. We model this by introducing a positive shock to the policy function that results in the central bank lifting off from the ELB more aggressively than dictated by new rule. The black line shows the results of this exercise. The moves results in output and inflation overshooting target by a much smaller degree than in the baseline over the few periods before z_t returns to zero in the baseline simulation. But this comes with a large cost, as $\hat{\xi}_{z,t}$ falls all the way back to the prior and $\hat{\sigma}_R$ increases somewhat. Accordingly, very little would have been achieved in terms of countering the negative effects of future ELB episodes.

These exercises highlight the policy dilemma imposed by large inflationary shocks. The central bank knows that the shock may curtail learning the new policy rule, and that it can successfully offset some of that effect and boost $\hat{\xi}_{z,t}$ by deviating from the rule and providing even more policy accommodation. But it also knows the such deviations are not costless. First, they result in temporarily higher inflation.[17] Second, by delaying

[17]While the less aggressive response boosts $\hat{\xi}_{R,t}$, it also results in higher inflation and thus pushes z_t to zero faster and shortens the learning period (though the effect is not large in the scenarios we consider).

raising rates, the central bank generates additional uncertainty about the new rule. This could result in precautionary behavior that would weigh on output. Alternatively, if the central bank increases rates more aggressively because it is worried about its credibility as an inflation fighter, it must realize that this could amount to agents believing they have abandoned the new framework. Real time assessment of these trade offs could be quite difficult when in the midst of large shocks of unknown duration.

7 The value of the new rule and costs of incomplete learning

This section provides some illustrative calculations of the value of the new rule over the old regime, the costs associated with incomplete learning of the new rule, and trade-offs that might arise from the central bank taking actions to boost $\hat{\xi}_{z,t}$ by generating policy shocks, $\epsilon_{R,t}$. One way to summarize the relative performance of the different policy functions is to consider a loss function that adds up squared deviations of output and inflation from target. Such a function is often used to assess the setting of optimal monetary policy, see, for example, Woodford (2003), Svensson (2010) and the Federal Reserve Board's Tealbook analysis.

We consider the stochastic loss, SL. This is a forward looking calculation that takes into account potential future shocks. Starting at steady state values for output and inflation, we simulate the model T periods ahead conditioned on fixed values for $\hat{\xi}_z$ and $\hat{\sigma}_R$ and N future draws of $\{\epsilon_{d,t}, \epsilon_{R,t}\}_1^T$, and take the average discounted sum of future squared deviations of output and inflation from target:

$$SL \;=\; \frac{1}{N}\sum_{n=1}^{N}\sum_{\tau=0}^{T}\beta^{-\tau}\;\left[(\pi_{t+\tau,n} - \pi)^2 \;+\; (y_{t+\tau,n} - y)^2\right]$$

where $\pi_{t+\tau,n}$ and $y_{t+\tau,n}$ are the values $\pi_{t+\tau}$ and $y_{t+\tau}$ from draw n. Accordingly, the stochastic loss tells us about the average performance the policy regime can be expected to deliver given the entire distribution of future shocks to the economy conditioned on agents' perception of the new rule's parameters.[18] We start with the economy at steady state,

The central bank must also take this trade off into account when calculating the appropriate liftoff delay.

[18]Note that SL does not include a term in the squared change in the policy rate, which enters some commonly used loss functions, such as in the Tealbook. This term is meant to capture financial stability concerns associated with large movements in interest rates as well as the empirical fact that central banks

		Policy shock $\hat{\sigma}_R$ (annualized)				
		1.4	1.0	0.6	0.4	0.0
AIT rule $\hat{\xi}_z$	0.0	2.67	1.92	1.78	1.72	1.69
	0.25	1.40	1.32	1.27	1.26	1.24
	0.50	1.19	1.15	1.12	1.12	1.11
	0.75	1.10	1.07	1.05	1.05	1.04
	1.0	1.04	1.02	1.01	1.00	1.00

Table 2: Stochastic loss of incomplete learning relative to $\{\hat{\xi}_z = 1, \hat{\sigma}_R = 0\}$. The values for $\hat{\sigma}_R$ have been converted to annualized percentage points for the net interest rate.

and then calculate SL for a range of values for $\{\hat{\xi}_z, \hat{\sigma}_R\}$. Table 2 shows the results of these experiments, with the loss relative to that if the new rule was known, $\{\hat{\xi}_z = 1, \hat{\sigma}_R = 0\}$.

The upper right hand corner with $\{\hat{\xi}_z = 0, \hat{\sigma}_R = 0\}$ corresponds to agents believing the central bank has made no change to its policy rule; the associated losses are 69 percent larger than if the rule had been enacted with full credibility. However, just a small degree of learning can result in significantly improved macro outcomes, with SL for $\{\hat{\xi}_z = 0.25, \hat{\sigma}_R = 0\}$ just 24 percent larger than at $\{\hat{\xi}_z = 1, \hat{\sigma}_R = 0\}$. The relative loss shrinks to 11 percent for $\hat{\xi}_z = 0.5$ and 4 percent for $\hat{\xi}_z = 0.75$. Uncertainty over the reaction function is costly, but less so than missing on ξ_z. In the old regime, even a 100 basis point standard deviation in uncertainty over the policy rate increases the normalized loss by only about 23 basis points. With $\hat{\xi}_z = 1$ the difference in loss is just 2 basis points. Indeed, within the range of parameters in the table, the gains from a marginal increase in $\hat{\xi}_z$ always outweigh the costs of a higher $\hat{\sigma}_R$. This suggests that inducing a little uncertainty into agents' perception of the interest rate rule may be a cost worth taking if it improves learning ξ_t, thus reducing the cost of future episodes at the ELB.

We don't want to push these result too far. As is well known, policy rules using common parameterizations such as the ones in this paper will not replicate optimal policy that minimizes a loss function such as SL. Rules with much stronger reactions to inflation and output gaps generaly do a better job at approaching optimal policy, and hence larger values of $\hat{\xi}_z$ can be expected to result in lower losses. Furthermore, as we noted earlier, monetary policy shocks of the size we consider can have a meaningful impact on economic outcomes in the model when interest rates are low, as they then have a good chance of generating an episode at the ELB. The effects, however, are small and transitory when

usually move policy rates in a smooth manner. Such a loss function is consistent with a monetary policy rule that includes inertia in the policy rate, which is not a feature of the rule in our model.

interest rates are far from the ELB (as is the case for most of the time periods in our simulations) as there is nothing in the model other than the ELB to induce persistent effects of a policy shock. Indeed, alternative parameterizations would influence the trade-offs. Greater risk aversion would boost precautionary saving motives, making increased uncertainty over the policy function more costly. And backward inertia in the price setting process would prolong the high inflation period. In addition to direct costs, this could cause agents to doubt that the central bank will revert to its old rule once the average inflation shortfall is eliminated and push long-run inflation expectations above target.

That said, the fundamental results are still strong. Including a make-up for past inflation shortfalls in the policy function can significantly enhance economic outcomes by reducing the underlying disinflationary bias generated by the ELB. And the costs of any additional above-target inflation generated by the new rule are limited as the higher inflation drives z_t to zero and results in policy reverting back to the same inflation-fighting prescriptions the central bank has always followed.

8 Conclusions

Under traditional monetary policy strategies, the limits on reducing policy rates presented by the proximity of the ELB impart a downward bias to inflation and inflation expectations relative to the central bank's target. This bias impinges on the central bank's ability to stabilize output and inflation. Economic theory has provided a number of alternative policy strategies to combat these limitations. In this paper we consider a form of asymmetric average inflation averaging, which adds a term in the weighted average of past shortfalls of inflation from target to the central bank's old policy function. This policy successfully offsets the disinflationary bias.

For this or any other alternative strategy to be effective, agents need to understand a regime change has taken place and adjust their behavior accordingly. This happens automatically in an abstract economy in which agents have complete information about the parameters of the new policy and the central bank has full credibility. However, in the real world, agents need to be convinced that the new policy process is indeed in place and then must learn its parameters. We show how this occurs through observation of the central bank's policy settings, with the period around the time that liftoff from the ELB would have occurred under the old policy regime being particularly crucial for learning process. The interactions between policy decisions, economic outcomes, and agents' learning process are important determinants of the performance of the new policy rule.

Shocks inevitably will hit the economy during this time, and they can present important impediments to the learning process. Recessionary shocks may delay learning, while inflationary shocks that cause the inflation averaging goal to be achieved before agents have fully learned the new policy rule can derail it entirely. This means the central bank could consider not reacting to certain inflation developments in order to better establish its new policy, which would have value the next time a recessionary shock hits the economy. But when doing so it must recognize that holding back on rate increases comes with costs of higher near term inflation and induces additional uncertainty into agents' attempts to learn the new rule. The need to learn a new rule can add complexity to an already challenging policy environment.

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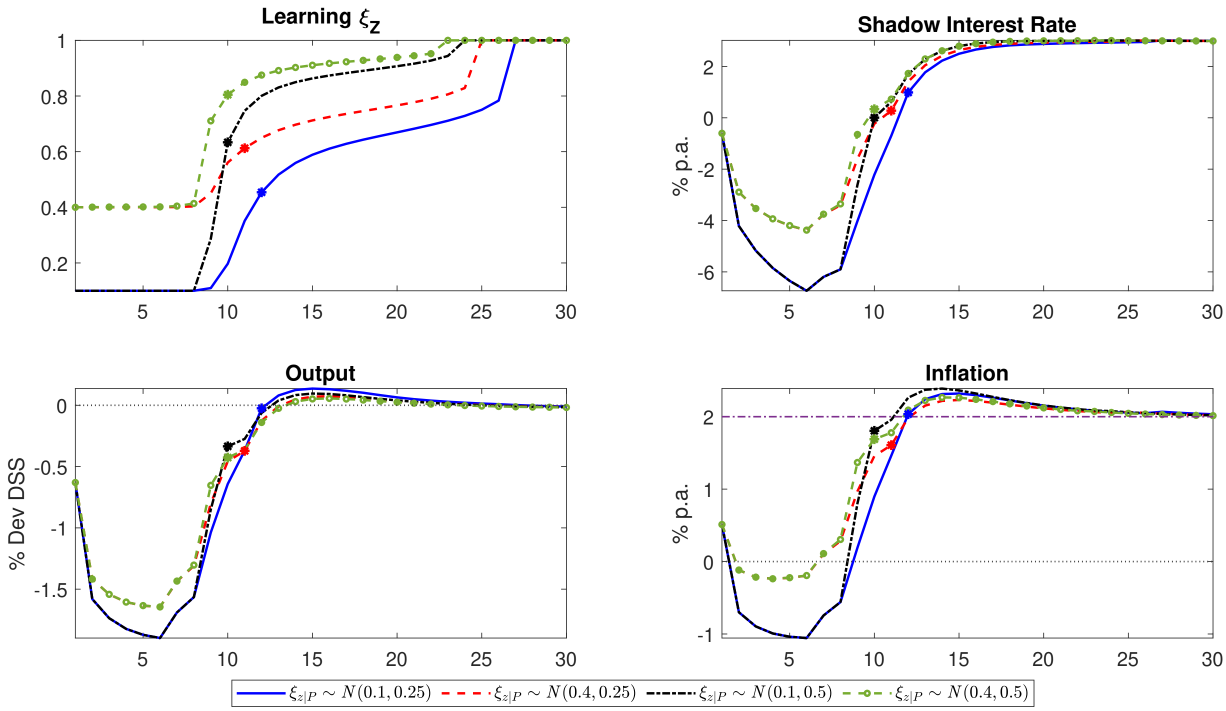

Figure 10: Impact of alternative priors for the structural components η_t^z of the new rule in the baseline simulation. The prior of the baseline case (blue) is compared to a scenario with a higher prior mean (red), a larger prior standard deviation (black) and a combination of higher mean and larger standard deviation (green).

A The role of prior information

The results of our learning model are sensitive to the prior distribution of the unknown parameters. As noted earlier, our baseline case uses prior means that are far from the actual values for ξ_z and σ_R. Figure 10 shows some alternatives for the prior mean and standard deviation of ξ_z, $\mathbb{E}_{\text{Prior}}(\xi_Z)$ and $\text{Std}_{\text{Prior}}(\sigma_R)$, respectively. As would be expected, increasing the prior mean by itself (the red dashed lines) results in a higher shadow rate and earlier liftoff than in the baseline (blue line). This is because agents assume the central bank will do more to offset the ELB bias in the future, which boosts output and inflation today.

A larger prior standard deviation on its own (black line) can also speed learning relative to the baseline, because agents are more receptive to incoming information moving their views away from the prior. However, this can work in two directions–information that is not supportive of the new policy would more readily push perceptions away from the correct ξ_z as well. In particular, if an $\varepsilon_{R,t}$ shocks occur that hastens liftoff relative to the baseline, agents would more readily lower their views of ξ_z if they had a larger prior standard deviation.

As discussed earlier and seen in the upper left panel, when we solve the model there is a discrete jump in the estimated parameters to their true values in period 26. This is because $\{\hat{\xi}_{z,t}, \hat{\sigma}_{R,t}\}$ has moved so far from the priors and $\left(\bar{\eta}_t^z - \hat{\xi}_{z|t} z_t\right)/\hat{\sigma}_{R|t}$ is so negative that the combined weights in the posterior of the prior and the censored component of the likelihood collapse to zero and the non-truncated portion of the likelihood covering the post liftoff period perfectly fits the data with $\{\hat{\xi}_{z,t}, \hat{\sigma}_{R,t}\} = \{1, 0\}$. Note that this jump is much less pronounced the larger the standard deviation in the prior distribution (the black and green lines). This is because with the bigger tails, the prior carries palpable weight in the posterior for a longer period of time as $\hat{\xi}_{t,z}$ moves away from $\mathbb{E}_{\text{Prior}}\left(\xi_Z\right)$.

In terms of the performance of output and inflation, the effects of starting the learning process closer to the truth – a larger $\mathbb{E}_{\text{Prior}}\left(\xi_Z\right)$ – is more important than having a looser prior – a larger $\text{Std}_{\text{Prior}}\left(\sigma_R\right)$ – that admits faster learning. This is seen by the better performance of output and inflation in the green and red lines with the higher prior mean than black dashed line with higher prior standard deviation.

These results point to the benefits of actions the central bank might take to push agents' priors closer to the true policy parameters. Clear communications about the new policy rule at the announcement and while shadow rates are still well below the ELB would be important. So would be concrete actions during this time. For example, as discussed in Engen et al. (2015), large scale asset purchases may have provided monetary accommodation in the U.S. not just through portfolio balance effects, but also by the signal they sent about the Federal Reserve's commitment to operate a lower-for-longer interest rate policy. Through the lens of our model, this role for of asset purchases could be interpreted as one way to influence agents' priors about interest rate policy well before the liftoff phase of the rate cycle.