The Chicago Fed DSGE Model:

Version 2

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FEDERAL RESERVE BANK *of* CHICAGO

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Abstract

The Chicago Fed dynamic stochastic general equilibrium (DSGE) model is used for policy analysis and forecasting at the Federal Reserve Bank of Chicago. This guide describes its specification, estimation, dynamic characteristics, and how it is used to forecast the U.S. economy. In many respects the model resembles other medium-scale New Keynesian frameworks, but there are several features which distinguish it: the monetary policy rule includes anticipated future deviations, productivity is driven by both neutral and investment specific technical change, multiple price and wage indices identify price and wage inflation, the data are measured in a model consistent way, and market-expected interest rates are used to measure the expected path of the federal funds rate that is taken into account by the model's agents when they make their decisions. The model also incorporates a new method introduced by Ferroni, Fisher, and Melosi (2023) to address the unusual Covid pandemic macroeconomic dynamics.

JEL Classification Numbers: E1, E2, E3, E4, E5
Keywords: New Keynesian model, DSGE models, Covid-19, pandemic, Survey of Professional Forecasters, business cycles, forecasting, policy analysis

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This guide describes the construction and estimation of Version 2 of the DSGE model used at the Chicago Fed for policy analysis and forecasting.[1] Originally, it was largely based on Justiniano, Primiceri, and Tambalotti (2010). We published results that are based on the estimated model in Campbell, Evans, Fisher, and Justiniano (2012), Campbell, Fisher, Justiniano, and Melosi (2016), Campbell, Ferroni, Fisher, and Melosi (2019), and Ferroni, Fisher, and Melosi (2023).

The model contains many features familiar from other DSGE analyses of monetary policy and bussiness cycles. External habit in preferences, i-dot costs of adjusting investment, price and wage stickiness based on Calvo's (1983) adjustment probabilities, and partial indexation of unadjusted prices and wages using recently observed price and wage inflation. The features which distinguish our analysis from many otherwise similar undertakings are

- **Forward Guidance Shocks:** An interest-rate rule which depends on recent (and expected future) inflation and output and is subject to stochastic disturbances governs our model economy's monetary policy rate. Standard analysis prior to the great recession restricted the stochastic disturbances to be unforecastable. Our model deviates from this historical standard by including forward guidance shocks, as in Laséen and Svensson (2011). A j-quarter ahead forward guidance shock revealed to the public at time t influences the interest-rate rule's stochastic intercept only at time $t+j$. Each period, the model's monetary authority reveals a vector of these shocks with one element for each quarter from the present until the end of the forward guidance horizon. The vector's elements may be correlated with each other, so the monetary authority could routinely reveal persistent shifts in the interest-rate rule's stochastic intercept. However, the forward guidance shocks are serially uncorrelated *over time*, as is required for them to match the definition of "news."

- **Investment-Specific Technological Change:** As in the Real Business Cycle models from which modern DSGE models decend (King, Plosser, and Rebelo, 1988a), stochastic trend productivity growth both short-run and long-run fluctuations. Our model features two such stochastic trends, one to Hicks-neutral productivity (King, Plosser, and Rebelo, 1988b) and one to the technology for converting consumption goods into investment goods (as in Fisher (2006)). This investment-specific technological change allows our model to reproduce the dynamics of the relative price of investment goods to consumption goods, which is a necessary input into the formula we use to create Fisher-ideal chain-weighted index of real GDP.

- **A Mixed Calibration-Bayesian Estimation Empirical Strategy:** Bayesian estimation of structural business cycle models attempts to match all features of the data's probability distribution using the model's parameters. Since no structural model embodies Platonic "truth," this exercise inevitably requires trading off between the model's ability to replicate first moments with its fidelity to the business cycles in second moments. Since the criteria for this tradeoff are not always clear, we adopt an alternative "first-moments-first" strategy. This selects the values of model parameters which govern the model's steady-state growth path, such as the growth rates of

[1] See Brave, Campbell, Fisher, and Justiniano (2012) for a description of Version 1 of the model.

Hicks-neutral and investment-specific technology, to match estimates of selected first moments. These parameter choices are then fixed for Bayesian estimation, which chooses values for model parameters which only influence second moments, such as technology innovation variances. (Since we employ a log linear solution of our model and all shocks to its primitives have Gaussian distributions, our analysis has no non-trivial implications for third and higher moments of the data.)

- **Pandemic shocks:** We construct a synthetic shock approximating the expected macroeconomic effects of the COVID-19 pandemic. The synthetic shock is a combination of unexpected and anticipated surprises; to economize on the parameters to estimate we assume a correlated (factor) structure across the various horizons of the surprises. Moreover, we assume that the synthetic shock has *hybrid* nature meaning that it could affect contemporaneously different margins of the economy, e.g. demand and supply sides. We estimate the parameters that capture the magnitude and transmission of the COVID-19 shock using 2020Q2 data and Survey of Professional Forecasters expectations about the likely evolution of GDP and inflation over the next four quarters. The propagation of the COVID-19 shock is re-assessed over time, i.e. we re-estimate the propagation parameters sequentially using 2020Q2, 2020Q3 and 2020Q4 data.

The guide proceeds as follows. The next section presents the model economy's primitives, while Section 2 presents the agents' first-order conditions. Section 3 gives the formulas used to remove nominal and technological trends from model variables and thereby induce model stationarity, and Sections 4 and 5 discuss the stationary economy's steady state and the log linearization of its equilibrium necessary conditions around it. Section 6 discusses measurement issues which arise when comparing model-generated data with data measured by the BEA and BLS. Section 7 describes our mixed Calibration-Bayesian Estimation empirical strategy and presents the resulting parameter values we use for model simulations and forecasting. Section 8 describes how we incorporate the pandemic into the model.

1 The Model's Primitives

Eight kinds of agents populate the model economy:

- Households,
- Investment producers,
- Competitive final goods producers,
- Monopolistically-competitive differentiated goods producers,
- Labor Packers,
- Monopolistically-competitive guilds,
- a Fiscal Authority and

- a Monetary Authority.

These agents interact with each other in markets for

- final goods used for consumption
- investment goods used to augment the stock of productive capital
- differentiated intermediate goods
- capital services
- raw labor
- differentiated labor
- composite labor
- government bonds
- privately-issued bonds, and
- state-contingent claims.

The households have preferences over streams of an aggregate consumption good, leisure, and the real value of the fiscal authority's bonds in their portfolios. Our specification for preferences displays balanced growth. They also feature *external* habit in consumption; which creates a channel for the endogenous propagation of shocks. Our bonds-in-the-utility-function preferences follow those of Fisher (2015), and they allow us to incorporate a persistent spread between the monetary policy rate and the return on productive capital. The aggregate consumption good has a single alternative use, as the only input into the linear production function operated by investment producers. These firms sell their output to the households. In turn, households produce capital services from their capital stocks, which they then sell to differentiated goods producers. Producers of final goods operate a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, which are differentiated goods produced by the monopolistically-competitive firms. These firms operate technologies with affine cost curves (a constant fixed cost and linear marginal cost), which employs capital services and composite labor as inputs. The labor packers produce composite labor using a constant-returns-to-scale technology with a constant elasticity of substitution between its inputs, the differentiated labor sold by guilds. Each of these produces differentiated labor from the raw labor provided by the households with a linear technology, and they sell their outputs to the labor packers. There is a nominal unit of account, called the "dollar." The fiscal authority issues one-period nominally risk-free bonds, provides public goods through government spending, and assesses lump-sum taxes on households. The monetary authority sets the interest rate on the fiscal authority's one-period bond according to an interest-rate rule.

All non-financial trade is denominated in dollars, and all private agents take prices as given with two exceptions: the monopolistically-competitive differentiated-goods producers and guilds. These choose output prices to maximize the current value of expected future profits taking as given their demand curves and all relevant input prices. Financial markets are complete, but all securities excepting equities in differentiated-goods producers are in zero net supply. These producers' profits and losses are rebated to the households (who own the firms' equities) lump-sum period-by-period, as are the profits and losses of the guilds. Given both a process for government spending and taxes and a rule for the monetary authority's interest rate choice, a competitive equilibrium consists of allocations and prices that are consistent with households' utility maximization, firms' profit maximization, guilds' profit maximization, and market clearing.

The economy is subject to stochastic disturbances in technology, preferences, and government policy. Without nominal rigidities, the economy's real allocations in competitive equilibrium can be separated from inflation and other dollar-denominated variables. Specifically, monetary policy only influences inflation. To connect real and nominal variables in the model and thereby consider the impact of monetary policy on the business cycle, we introduce Calvo-style wage and price setting. That is, nature endows both differentiated goods producers and guilds with *stochastic* opportunities to incorporate all available information into their nominal price choices. Those producers and guilds without such a opportunity must set their prices according to simple indexing formulas. These two pricing frictions create two forward-looking Phillips curves, one for prices and another for wages, which form the core of the new Keynesian approach to monetary policy analysis.

The model economy is stochastic and features complete markets in state-contingent claims. To place these features on a sound footing, we base all shocks on a general Markovian stochastic process s_t. Denote the history of this vector from an initial period 0 through τ with $s^\tau \equiv (s_0, s_1, \ldots, s_\tau)$. All model shocks are implicit functions of s_t, and all endogenous variables are implicit functions of s^t. We refer to all such implicit functions as "state-contingent sequences." We use braces to denote such a sequence. For example, $\{X_t\}$ represents the state-contingent sequence for a generic variable X_t.

1.1 Households

Our model's households are the ultimate owners of all assets in positive net supply (the capital stock, differentiated goods producers, and guilds). They provide labor and divide their current after-tax income (from wages and assets) between current consumption, investment in productive capital, and purchases of financial assets, both those issued by the government and those issued by other households. The individual household divides its current resources between consumption and the available vehicles for intertemporal substitution (capital and financial assets) to maximize a discounted sum of current and expected future felicity.

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \varepsilon^b_{t+\tau} \left(U_{t+\tau} + \varepsilon^s_{t+\tau} L \left(\frac{B_{t+\tau}}{P_{t+\tau} R_{t+\tau}} \right) \right) \right]$$

with

$$U_t = \frac{1}{1-\gamma_c} \left((C_t - \varrho \bar{C}_{t-1})(1 - H_t^{1+\gamma_h}) \right)^{(1-\gamma_c)} \tag{1}$$

The function $L(\cdot)$ is strictly increasing, concave, and differentiable everywhere on $[0, \infty)$. In particular, $L'(0)$ exists and is finite. Without loss of generality, we set $L'(0)$ to one. The argument of $L(\cdot)$ equals the real value of government bonds in the household's portfolio: their period $t+1$ redemption value B_t divided by their nominal yield R_t expressed in units of the consumption good with the nominal price index P_t. The time-varying coefficient multiplying this felicity from bond holdings, ε_t^s, is the liquidity preference shock introduced by Fisher (2015). A separate shock influences the household's discounting of future utility to the present, ε_t^b. Specifically, the household discounts a certain utility in $t+\tau$ back to t with $\beta^\tau \mathbb{E}_t\left[\varepsilon_{t+\tau}^b / \varepsilon_t^b\right]$. In logarithms, these two preference shocks follow independent autoregressive processes.

$$\ln \varepsilon_t^b = (1-\rho_b) \ln \varepsilon_*^b + \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim \mathbb{N}(0, \sigma_b^2) \tag{2}$$

$$\ln \varepsilon_t^s = (1-\rho_s) \ln \varepsilon_*^s + \rho_s \ln \varepsilon_{t-1}^s + \eta_t^s, \eta_t^s \sim \mathbb{N}(0, \sigma_s^2). \tag{3}$$

A household's wealth at the beginning of period t consists of its nominal government bond holdings, B_t, its net holdings of privately-issued financial assets, and its capital stock K_{t-1}. The household chooses a rate of capital utilization u_t, and the capital services resulting from this choice equal $u_t K_{t-1}$. The cost of increasing utilization is higher depreciation. An increasing, convex and differentiable function $\delta(U)$ gives the capital depreciation rate. We specify this as

$$\delta(u) = \delta_0 + \delta_1 (u - u_\star) + \frac{\delta_2}{2} (u - u_\star)^2 .$$

A household can augment its capital stock with investment, I_t. Investment requires paying adjustment costs of the "i-dot" form introduced by Christiano, Eichenbaum, and Evans (2005). Also, an *investment demand shock* alters the efficiency of investment in augmenting the capital stock. Altogether, if the household's investment in the previous period was I_{t-1}, and it purchases I_t units of the investment good today, then the stock of capital available in the *next* period is

$$K_t = (1 - \delta(u_t)) K_{t-1} + \varepsilon_t^i \left(1 - S\left(\frac{A_{t-1}^K I_t}{A_t^K I_{t-1}}\right)\right) I_t. \tag{4}$$

In (4), A_t^K equals the productivity level of capital goods production, described in more detail below, and ε_t^i is the investment demand shock. In logarithms, this follows a first-order autoregression with a normally-distributed innovation.

$$\ln \varepsilon_t^i = (1-\rho_i) \ln \varepsilon_*^i + \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim \mathbb{N}(0, \sigma_i^2) \tag{5}$$

1.2 Production

The producers of investment goods use a linear technology to transform the final good into investment goods. The technological rate of exchange from the final good to the investment good in period t is A_t^I. We denote $\Delta \ln A_t^I$ with ω_t, which we call the *investment-specific technology shock* and which follows first-order autogregression with normally distributed innovations.

$$\omega_t = (1-\rho_\omega)\omega_\star + \rho_\omega\omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim \mathbb{N}(0,\sigma_\omega^2) \tag{6}$$

Investment goods producers are perfectly competitive.

Final good producers also operate a constant-returns-to-scale technology; which takes as inputs the products of the differentiated goods producers. To specify this, let Y_{it} denote the quantity of good i purchased by the representative final good producer in period t, for $i \in [0,1]$. The representative final good producer's output then equals

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{1}{1+\lambda_t^p}} di\right)^{1+\lambda_t^p}.$$

With this technology, the elasticity of substitution between any two differentiated products equals $1 + 1/\lambda_t^p$ in period t. Although this is constant across products within a time period, it varies stochastically over time according to an ARMA(1,1) in logarithms.

$$\ln\lambda_t^p = (1-\rho_p)\ln\lambda_\star^p + \rho_p \ln\lambda_{t-1}^p - \theta_p\eta_{t-1}^p + \eta_t^p, \eta_t^p \sim \mathbb{N}(0,\sigma_p^2) \tag{7}$$

Given nominal prices for the intermediate goods P_{it}, it is a standard exercise to show that the final goods producers' marginal cost equals

$$P_t = \left(\int_0^1 P_{it}^{-\frac{1}{\lambda_t^p}} di\right)^{-\lambda_t^p} \tag{8}$$

Just like investment goods firms, the final goods' producers are perfectly competitive. Therefore, profit maximization and positive final goods output together require the competitive output price to equal P_t. Therefore, we can define inflation of the nominal final good price as $\pi_t \equiv \ln(P_t/P_{t-1})$.

The intermediate goods producers each use the technology

$$Y_{it} = (K_{it}^e)^\alpha \left(A_t^Y H_{it}^d\right)^{1-\alpha} - A_t\Phi \tag{9}$$

Here, K_{it}^e and H_{it}^d are the capital services and labor services used by firm i, and A_t^Y is the level of neutral technology. Its growth rate, $\nu_t \equiv \ln(A_t^Y/A_{t-1}^Y)$, follows a first-order autogregression.

$$\nu_t = (1-\rho_\nu)\,\nu_* + \rho_v\nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim \mathbb{N}(0,\sigma_\nu^2), \tag{10}$$

The final term in (9) represents the fixed costs of production. These grow with

$$A_t \equiv A_t^Y \left(A_t^I\right)^{\frac{\alpha}{1-\alpha}}. \tag{11}$$

We demonstrate below that A_t is the stochastic trend in equilibrium output and consumption, measured in units of the final good. We denote its growth rate with

$$z_t = \nu_t + \frac{\alpha}{1-\alpha}\omega_t \tag{12}$$

Similarly, define

$$A^K_t \equiv A_t A^I_t \tag{13}$$

In the specification of the capital accumulation technology, we labelled A^K_t the "productivity level of capital goods production." We demonstrate below that this is indeed the case with the definition in (13).

Each intermediate goods producer chooses prices subject to a Calvo (1983) pricing scheme. With probability $\zeta_p \in [0,1]$, producer i has the opportunity to set P_{it} without constraints. With the complementary probability, P_{it} is set with the indexing rule

$$P_{it} = P_{it-1}\pi_{t-1}^{\iota_p}\pi_\star^{1-\iota_p}. \tag{14}$$

In (14), $\pi_\star$ is the gross rate of price growth along the steady-state growth path, and $\iota_p \in [0,1]$.[2]

1.3 Labor Markets

Households' hours worked pass through two intermediaries, guilds and labor packers, in their transformation into labor services used by the intermediate goods producers. The guilds take the households' homogeneous hours as their only input and produce differentiated labor services. These are then sold to the labor packers, who assemble the guilds' services into composite labor services.

The labor packers operate a constant-returns-to-scale technology with a constant elasticity of substitution between the guilds' differentiated labor services. For its specification, let H_{it} denote the hours of differenziated labor purchased from guild i at time t by the representative labor packer. Then that packer's production of composite labor services, H^s_t are given by

$$H^s_t = \left(\int_0^1 (H_{it})^{\frac{1}{1+\lambda^w_t}} di\right)^{1+\lambda^w_t}.$$

As with the final good producer's technology, an ARMA(1,1) in logarithms governs the constant elasticity of substitution between any two guilds' labor services.

$$\ln\lambda^w_t = (1-\rho_w)\ln\lambda^w_\star + \rho_w \ln\lambda^w_{t-1} - \theta_w\eta^w_{t-1} + \eta^w_t, \eta^w_t \sim \mathbb{N}(0,\sigma^2_w) \tag{15}$$

Just as with the final goods producers, we can easily show that the labor packers' marginal

[2] To model firms' price-setting opportunities as functions of s_t, define a random variable u^p_t which is independent over time and uniformly distributed on $[0,1]$. Then, firm i gets a price-setting opportunity if either $u^p_t \geq \zeta_p$ and $i \in [u^p_t - \zeta_p, u^p_t]$ or if $u^p_t < \zeta_p$ and $i \in [0, u^p_t] \cup [1 + u^p_t - \zeta_p, 1]$.

cost equals

$$W_t = \left(\int_0^1 (W_{it})^{-\frac{1}{\lambda_t^w}} di \right)^{-\lambda_t^w}. \tag{16}$$

Here, W_{it} is the nominal price charged by guild i per hour of differentiated labor. Since labor packers are perfectly competitive, their profit maximization and positive output together require that the price of composite labor services equals their marginal cost.

Each guild produces it's differentiated labor service using a linear technology with the household's hours worked as its only input. A Calvo (1983) pricing scheme similar to that of the differentiated goods producers constrains their nominal prices. Guild i has an unconstrained opportunity to choose its nominal price with probability $\zeta_w \in [0,1]$. With the complementary probability, W_{it} is set with an indexing rule based on π_{t-1} and last period's trend growth rate, z_{t-1}.

$$W_{it} = W_{it-1} \left(\pi_{t-1} e^{z_{t-1}}\right)^{\iota_w} \left(\pi_\star e^{z_\star}\right)^{1-\iota_w}. \tag{17}$$

In (17), $z_\star \equiv \nu_\star + \frac{\alpha}{1-\alpha}\omega_\star$ is the unconditional mean of z_t and $\iota_w \in [0,1]$.

1.4 Fiscal and Monetary Policy

The model economy hosts two policy authorities, each of which follows exogenously-specified rules that receive stochastic disturbances. The fiscal authority issues bonds, B_t, collects lump-sum taxes T_t, and buys "wasteful" public goods G_t. Its period-by-period budget constraint is

$$G_t + B_{t-1} = T_t + \frac{B_t}{R_t}. \tag{18}$$

The left-hand side gives the government's uses of funds, public goods spending and the retirement of existing debt. The left-hand side gives the sources of funds, taxes and the proceeds of new debt issuance at the interest rate R_t. We assume that the fiscal authority keeps its budget balanced period-by-period, so $B_t = 0$. Furthermore, the fiscal authority sets public goods expenditure equal to a stochastic share of output, expressed in consumption units.

$$G_t = (1 - 1/g_t) Y_t, \tag{19}$$

with

$$\ln g_t = (1-\rho_g) \ln s_\star^g + \rho_g \ln g_{t-1} + \eta_t^g, \eta_t^g \sim \mathbb{N}(0, \sigma_g^2). \tag{20}$$

The monetary authority sets the nominal interest rate on government bonds, R_t. For this, it employs a Taylor rule with interest-rate smoothing and forward guidance shocks.

$$\ln R_t = \rho_R \ln R_{t-1} + (1-\rho_R) \ln R_t^n + \sum_{j=0}^{M} \xi_{t-j}^j. \tag{21}$$

The monetary policy disturbances in (21) are $\xi_t^0, \xi_{t-1}^1, \ldots, \xi_{t-M}^M$. The public learns the value of ξ_{t-j}^j in period $t - j$. The conventional unforecastable shock to current monetary policy is ξ_t^0, while for $j \geq 1$, these disturbances are *forward guidance shocks*. We gather all monetary shocks revealed at time t into the vector ε_t^R. This is normally distributed and *i.i.d.* across time. However, *its elements may be correlated with each other*. That is,

$$\varepsilon_t^R \equiv \left(\xi_t^0, \xi_t^1, \ldots, \xi_t^M\right) \sim \mathbb{N}(0, \Sigma_\varepsilon). \tag{22}$$

The off-diagonal elements of Σ^1 are not necessarily zero, so forward-guidance shocks need not randomly impact expected future monetary policy at two adjacent dates independently. Current economic circumstances influence R_t through the notional interest rate, R_t^n.

$$\ln R_t^n = \ln r_\star + \ln \pi_t^\star + \frac{\phi_1}{4}\mathbb{E}_t \sum_{j=-2}^{1} \left(\ln \pi_{t+j} - \ln \pi_t^\star\right) + \frac{\phi_2}{4}\mathbb{E}_t \sum_{j=-2}^{1} \left(\ln Y_{t+j} - \ln y^\star - \ln A_{t+j}\right). \tag{23}$$

The constant $r_\star$ equals the real interest rate along a steady-state growth path, and $\pi_t^\star$ is the central bank's intermediate target for inflation. We call this the *inflation-drift shock.* it follows a first-order autoregression with a normally-distributed innovation. Its unconditional mean equals $\pi_\star$, the inflation rate on a steady-state growth path.

$$\ln \pi_t^\star = (1 - \rho_\pi)\pi_\star + \rho_\pi \ln \pi_{t-1}^\star + \eta_t^\pi, \eta_t^\pi \sim \mathbb{N}(0, \sigma_\pi^2) \tag{24}$$

Allowing $\pi_t^\star$ to change over time enables our model to capture the persistent decline in inflation from the early 1990s through the early 2000s engineered by the Greenspan FOMC.

1.5 Other Financial Markets and Equilibrium Definition

All households participate in the market for nominal risk-free government debt. Additionally, they can buy and sell two classes of privately issued assets without restriction. The first is one-period nominal risk-free *private* debt. We denote the value of household's net holdings of such debt at the beginning of period t with B_{t-1}^P and the interest rate on such debt issued in period t maturing in $t+1$ with R_{t+1}^P. The second asset class consists of a complete set of *real* state-contingent claims. As of the end of period t, the household's ownership of securities that pay off one unit of the aggregate consumption good in period τ if history s^τ occurs is $Q_t(s^\tau)$, and the nominal price of such a security in the same period is $J_t(s^\tau)$.

We define an equilibrium for our economy in the usual way: Households maximize their utility given all prices, taxes, and dividends from both producers and guilds; final goods producers and labor packers maximize profits taking their input and output prices as given; differentiated goods producers and guilds maximize the market value of their dividend streams taking as given all input and financial-market prices; differentiated goods producers and guilds produce to satisfy demand at their posted prices; and otherwise all product, labor, and financial markets clear.

2 First Order Conditions

In this section we present the first-order conditions associated with the optimization problems that the agents in our model solve.

2.1 Households

Given initial financial asset holdings holdings, a stock of productive capital, investment in the previous period (which influences investment adjustment costs), and the external habit stock; households' choices of consumption, capital investment, capital utilization, hours worked, and financial investments maximize utility subject to the constraints of the capital accumulation and utilization technology and a sequence of one-period budget constraints. To specify these budget constraints, denote the nominal wage-per-hour paid by labor guilds to households with W_t^h, the nominal rental rate for capital services with R_t^k, the nominal price of investment goods with P_t^I, and the dividends paid by labor guilds added to those paid by differentiated good producers with D_t. With this notation, writing the period t budget constraint with uses of funds on the left and sources of funds on the right yields

$$C_t + \frac{P_t^I I_t}{P_t} + \frac{B_t}{R_t P_t} + \frac{B_t^P}{R_t^P P_t} + \frac{T_t}{P_t} \leq \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^P}{P_t} + \frac{W_t^h H_t}{P_t} + \frac{R_t^k u_t K_{t-1}}{P_t} + \frac{D_t}{P_t} \tag{25}$$

Denote the Lagrange multiplier on (25) with $\beta^t \Lambda_t^1$, and that on the capital accumulation constraint in (4) with $\beta^t \Lambda_t^2$. With these definitions, the first-order conditions for a household's utility maximization problem are

$$\begin{aligned}
\Lambda_t^1 &= \varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (1 - \varepsilon_t^h H_t^{1+\gamma_h}) \\
\Lambda_t^1 \frac{W_t^h}{P_t} &= (1+\gamma_h)\varepsilon_t^b \left((C_t - \varrho \bar{C}_{t-1})(1 - \varepsilon_t^h H_t^{1+\gamma_h}) \right)^{-\gamma_c} (C_t - \varrho \bar{C}_{t-1}) \varepsilon_t^h H_t^{\gamma_h} \\
\frac{\Lambda_t^1}{R_t P_t} - \varepsilon_{t+q}^b L'\left(\frac{B_t}{R_t P_t}\right) \frac{\varepsilon_t^s}{R_t P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right] \\
\frac{\Lambda_t^1}{R_t^P P_t} &= \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}^1}{P_{t+1}} \right] \\
\Lambda_t^2 &= \beta \mathbb{E} \left[\Lambda_{t+1}^1 \frac{R_{t+1}^k u_{t+1}}{P_{t+1}} + \Lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right] \\
\frac{\Lambda_t^1 R_t^k}{P_t} &= \Lambda_t^2 \delta'(u_t) \\
\Lambda_t^1 &= \varepsilon_t^i \Lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) \\
&\quad + \beta \mathbb{E}_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right]
\end{aligned}$$

In equilibrium, $\bar{C}_t = C_t$ always.

2.2 Goods Sector

2.2.1 Final Goods Producers

The nominal marginal cost of final goods producers equals the right-hand side of (8). A producer of final goods maximizes profit by shutting down if P_t is less than this marginal cost and can make an arbitrarily large profit if P_t exceeds it. When (8) holds, an individual final goods producer's output is indeterminate.

Final goods producers' demand for intermediate goods takes the familiar constant-elasticity form. If we use Y_t to denote total final goods output, then the amount of differentiated good i demanded by final goods producers is

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\frac{1+\lambda_t^p}{\lambda_t^p}}.$$

Given the choice of a reset price, we wish to calculate the overall price level. All intermediate goods producers with a price-setting opportunity choose $\tilde{P}_t$. The remaining producers use the price-indexing rule in (14). The aggregate price level is given by

$$P_t = \left[(1-\zeta_p)\tilde{P}_t^{\frac{1}{\lambda_{p,t-1}}} + \zeta_p \left((\pi_{t-1})^{\iota_p} (\pi_*)^{1-\iota_p} P_{t-1}\right)^{\frac{1}{\lambda_{p,t-1}}}\right]^{\lambda_{p,t-1}}$$

where $\tilde{P}_t$ is the optimal reset price

2.2.2 Intermediate Goods Producers

Intermediate goods producers' cost minimization reads as follows:

$$\max_{H_{t,i},K^e_{i,t}} W_t H^d_{t,i} + R^k_t K^e_{i,t}$$
$$\text{s.t. } Y_{t,i} = \varepsilon^a_t (K^e_{t,i})^\alpha \left(A^y_t H^d_{t,i}\right)^{1-\alpha} - A_t \Phi$$

We get the following optimal capital-labor ratio.

$$\frac{\alpha}{1-\alpha}\frac{W_t}{R^k_t} = \frac{(K^e_{it})^s}{H^d_{t,i}}$$

Notice how for each firm, the idiosyncratic capital to labor ratio is not a function of any firm-specific component. Hence, each firm has the same capital to labor ratio. In equilibrium,

$$K^e_t = u_t K_{t-1}$$

To find the marginal cost, we differentiate the variable part of production with respect to output, and substitute in the capital-labor ratio.

$$MC_{t,i} = (\varepsilon^a_t)^{-1} (A^y_t)^{-(1-\alpha)} W_t^{1-\alpha} R_t^{k\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$$

Again, notice that each firm as the same marginal cost.

Given cost minimization, a differentiated goods producer with an opportunity to adjust its nominal price does so to maximize the present-discounted value of profits earned until the next opportunity to adjust prices arrives. Formally,

$$\max_{\tilde{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \left[\tilde{P}_{t,i} X_{t,s}^y - MC_{t+s}\right] Y_{t+s,i}$$

$$\text{s.t. } Y_t(i) = \left(X_{t,s}^y \frac{\tilde{P}_{t,i}}{P_t} \right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t$$

$$\text{where } X_{t,s}^y = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{l=1}^{s} \pi_{t+l-1}^{\iota_p} \pi_*^{1-\iota_p} & : s = 1, \ldots, \infty \end{array} \right\}$$

This problem leads to the following price-setting equation for firms that are allowed to reoptimize their price:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} Y_{it+s} \left[\lambda_{p,t+s} MC_{t+s} - X_{t,s} \tilde{P}_{it} \right]$$

It can be shown that the producers that are allowed to reoptimize choose the same price. So henceforth, $\tilde{P}_{it} = \tilde{P}_t$.

2.2.3 Investment Goods Producers

Characterizing the profit-maximizing choices of investment goods and final goods producers is straightforward. If $P_t^I > P_t/A_t^I$, then each investment goods producer can make infinite profit by choosing an arbitrarily large output. On the other hand, if $P_t^I < P_t/A_t^I$, then investment goods producers maximize profits with zero production. Finally, their profit-maximizing production is indeterminate when

$$P_t^I = P_t/A_t^I. \tag{26}$$

The relative price of investment to consumption is equal to $(A_t^I)^{-1}$. Making this substitution into the household f.o.c and noting that $P_t Y_t^I$ is an intermediate input that will not be reflected in the aggregate resource constraint, it suffices to substitute the relative price $(A_t^I)^{-1}$ in the constraint for the household.

2.3 Labor Sector

2.3.1 Labor Packers

The labor packers choose the the labor inputs supplied by guilds, pack them into a composite labor service to be sold to the intermediate goods producers. Formally, labor packers' problem reads as follows:

$$\max_{H_t^s, H_{it}} W_t H_t^s - \int_0^1 W_{it} H_{it} di$$
$$s.t. \left[\int_0^1 H_{it}^{\frac{1}{1+\lambda_{w,t}}} di \right]^{1+\lambda_{w,t}} = H_t^s$$

We obtain the following labor demand equation for guild i:

$$H_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} H_t \tag{27}$$

As for the goods sector, we can show that aggregate wage is given by the following equation:

$$W_t = \left[(1-\zeta_w) \tilde{W}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(\left(e^{z_{t-1}} \pi_{t-1} \right)^{\iota_w} \left(\pi_* e^{z_*} \right)^{1-\iota_w} W_{t-1} \right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

where $\tilde{W}$ is the optimal reset wage for guilds.

2.3.2 Guilds

Each guild with an opportunity to set its nominal price does so to maximize the current value of the stream of dividends returned to the household. Formally, their problem reads

$$\max_{\tilde{W}_{it}} E_t \sum_{s=0}^{\infty} \zeta_w^s \left(\frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} \right) \left[X_{t+s}^l \tilde{W}_{it} - W_{t+s}^h \right] H_{it+s}$$
$$\text{s.t. } H_{it+s} = \left(\frac{X_{t,s}^l \tilde{W}_{it}}{W_{t+s}} \right)^{-\frac{1+\lambda_{w,t+s}}{\lambda_{w,t+s}}} H_{t+s}$$
$$\text{where } X_{t,s}^l = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \prod_{j=1}^s \left(\pi_{t+j-1} \frac{A_{t+j-1}}{A_{t+j-2}} \right)^{1-\iota_w} (\pi e^{\gamma})^{\iota_w} & : s = 1, \dots, \infty \end{array} \right\}$$

$\tilde{W}_t$ is the optimal reset wage. This optimal wage is chosen by the guilds who are allowed, with probability ζ_w, to change their prices in a given period. Also, we index the nominal wage inflation rate with ι_w.

This maximization problem gives a wage-setting equation that reads as follows:

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Lambda_{t+s}^1 P_t}{\Lambda_t^1 P_{t+s}} H_{it+s} \frac{1}{\lambda_{w,t+s}} \left((1+\lambda_{w,t+s}) W_{t+s}^h - X_{t,s}^l \tilde{W}_{it} \right)$$

It can be shown that the guilds that are allowed to reoptimize choose the same wage. So henceforth, $\tilde{W}_{it} = \tilde{W}_t$.

3 Detrending

To remove nominal and real trends, we deflate nominal variables by their matching price deflators, and we detrend any resulting real variables influenced permanently by technological change. All scaled versions of variables are the lower-case counterparts.

$$
\begin{aligned}
c_t &= \frac{C_t}{A_t} & i_t &= \frac{I_t}{A_t A_t^I} \\
k_t &= \frac{K_t}{A_t A_t^I} & k_t^e &= \frac{K_t^e}{A_t A_t^I} \\
w_t &= \frac{W_t}{A_t P_t} & \tilde{w}_t &= \frac{\tilde{W}_t}{A_t P_t} \\
\tilde{p}_t &= \frac{\tilde{P}_t}{P_t} & \pi_t &= \frac{P_t}{P_{t-1}} \\
y_t &= \frac{Y_t}{A_t} & mc_t &= \frac{MC_t}{P_t} \\
r_t^k &= \frac{R_t^k A_t^I}{P_t} & w_t^h &= \frac{W_t^h}{A_t P_t} \\
\lambda_t^1 &= \Lambda_t^1 A_t^{\gamma_C} & \lambda_t^2 &= \Lambda_t^2 A_t^{\gamma_C} A_t^I \\
\varepsilon_t^s &= A_t^{\gamma_C} \varepsilon_t^s
\end{aligned}
$$

3.1 Detrended Equations

The detrended equations describing our model are listed in the following sections.

Households' FOC

$$
\begin{aligned}
\lambda_t^1 &= \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \right]^{-\gamma_c} \left(1 - \varepsilon_t^h h_t^{1+\gamma_h} \right) \\
\lambda_t^1 w_t^h &= (1+\gamma_h) \varepsilon_t^b \left[\left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \left(1 - \varepsilon_t^h h_t^{(1+\sigma_h)} \right) \right]^{-\gamma_c} \left(c_t - \varrho \frac{c_{t-1}}{e^{z_t}} \right) \varepsilon_t^h h_t^{\gamma_h} \\
\frac{\lambda_t^1}{R_t^P} &= \beta E_t \left[\frac{\lambda_{t+1}^1 e^{-\gamma_C z_{t+1}}}{\pi_{t+1}} \right] \\
\frac{\lambda_t^1}{R_t} - L'(0) \frac{\varepsilon_t^b \varepsilon_t^s}{R_t} &= \beta E_t \frac{\lambda_{t+1}^1}{\pi_{t+1}} e^{-z_{t+1}\gamma_C} \\
\lambda_t^1 &= \varepsilon_t^i \lambda_t^2 \left((1 - S_t(\cdot)) - S_t'(\cdot) \frac{i_t}{i_{t-1}} \right) + \beta E_t \left[\varepsilon_{t+1}^i e^{(1-\gamma_C) z_{t+1}} \lambda_{t+1}^2 S_{t+1}'(\cdot) \frac{i_{t+1}^2}{i_t^2} \right] \\
\lambda_t^2 &= \beta E_t \left[e^{-\gamma_C z_{t+1} - \omega_{t+1}} \left(\lambda_{t+1}^1 r_{t+1}^k u_{t+1} + \lambda_{t+1}^2 (1 - \delta(u_{t+1})) \right) \right] \\
\lambda_t^1 r_t^k &= \lambda_t^2 \delta'(u_t) \\
k_t &= (1 - \delta(u_t)) k_{t-1} e^{-z_t - \omega_t} + \varepsilon_t^i (1 - S(\cdot)) i_t \\
k_t^e &= u_t k_{t-1} e^{-z_t - \omega_t}
\end{aligned}
$$

Final Goods Price Index

$$1 = \left[(1-\zeta_p)\tilde{p}_t^{\frac{1}{1-\lambda_{p,t}}} + \zeta_p \left(\pi_{t-1}^{\iota_p}\pi^{*(1-\iota_p)}\pi_t^{-1}\right)^{\frac{1}{1-\lambda_{p,t}}} \right]^{1-\lambda_{p,t}}$$

Intermediate Goods Firms: Capital-Labor Ratio

$$\frac{k_t^e}{h_t^d} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

Intermediate Goods Firms: Real Marginal Costs

$$mc_t = \frac{w_t^{1-\alpha}\left(r_t^k\right)^{\alpha}}{\varepsilon_t^a \alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

Intermediate Goods Firms: Price-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \lambda_{t+s}^1 \frac{\tilde{y}_{t,t+s}}{\lambda_{p,t+s}-1}\left(\frac{A_{t+s}}{A_t}\right)^{1-\gamma_C}\left[\lambda_{p,t+s} mc_{t+s} - \tilde{X}_{t,s}^p \tilde{p}_t\right]$$

where

$$\tilde{X}_{t,s}^p = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \pi_{t+j-1}^{1-\iota_p}\pi_*^{\iota_p}}{\prod_{j=1}^s \pi_{t+j}} & : s = 1, \ldots, \infty \end{array} \right\}$$

$\tilde{y}_{t,t+s}$ denotes the time $t+j$ output sold by the producers that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing producers all choose the same price, then we do not have to carry the i-subscript.

Labor Packers: Aggregate Wage Index

$$w_t = \left[(1-\zeta_w)\tilde{w}_t^{-\frac{1}{\lambda_{w,t}}} + \zeta_w \left(e^{\iota_w z_{t-1} - z_t} e^{(1-\iota_w) z_*} \pi_{t-1}^{\iota} \pi_t^{-1} \pi_*^{1-\iota_w} w_{t-1}\right)^{-\frac{1}{\lambda_{w,t}}} \right]^{-\lambda_{w,t}}$$

Guilds: Wage-Setting Equation

$$0 = E_t \sum_{s=0}^{\infty} \zeta_w^s \beta \lambda_{t+s}^1 \left(\frac{A_{t+s}}{A_t}\right)^{1-\gamma_C} \frac{\tilde{h}_{t,t+s}}{\lambda_{w,t+s}} \left((1+\lambda_{w,t+s}) w_{t+s}^h - \tilde{X}_{t,s}^l \tilde{w}_t\right)$$

where

$$\tilde{X}_{t,s}^l = \left\{ \begin{array}{ll} 1 & : s = 0 \\ \frac{\prod_{j=1}^s \left(\pi_{t+j-1} e^{z_{t+j-1}}\right)^{1-\iota_w} (\pi\gamma)^{\iota_w}}{\prod_{j=1}^s \pi_{t+j} e^{z_{t+j}}} & : s = 1, \dots, \infty \end{array} \right\}$$

$\tilde{h}_{t,t+s}$ denotes the time $t+j$ labor supplied by the guild that have optimized at time t the last time they have reoptimized. Since it can be shown that optimizing guilds all choose the same wage, then we do not have to carry the i-subscript.

Monetary Authority

$$R_t = R_{t-1}^{\rho_R} \left[r_* \pi_t^* \left(\prod_{j=-2}^{1} \frac{\pi_{t+j}}{\pi_t^*} \right)^{\frac{\psi_1}{4}} \left(\prod_{j=-2}^{1} \frac{y_{t+j}}{y^*} \right)^{\frac{\psi_2}{4}} \right]^{1-\rho_R} \prod_{j=0}^{M} \xi_{t-j,j}$$

The Aggregate Resource Constraint

$$\frac{y_t}{g_t} = c_t + i_t$$

Production Function

$$y_t = \varepsilon_t^a \left(k_t^e\right)^{\alpha} \left(h_t^d\right)^{1-\alpha} - \Phi$$

Labor Market Clearing Condition

$$h_t = h_t^d$$

4 Steady State

We normalize most shocks and the utilization rate:

$$u_\star = 1 \qquad \varepsilon^i = 1$$

$$\varepsilon^a = 1 \qquad \varepsilon^b = 1$$

Next, we set the following restriction on adjustment costs:

$$\begin{aligned} S(\cdot_*) &\equiv 0 \\ S'(\cdot_*) &\equiv 0 \end{aligned}$$

4.1 Prices and Interest Rates

Given β, z_*, γ_C, and π_*, we can solve for the steady-state nominal interest rate on private bonds R^P_* by using the FOC on private bonds:

$$R^P_* = \frac{\pi_*}{(\beta e^{-\gamma_C z_*})} \tag{28}$$

From the definition of $\delta(u)$, we have

$$\begin{aligned} \delta(1) =& \delta_0 \\ \delta'(1) =& \delta_1. \end{aligned}$$

Next, given ω_*, δ_0, and the above, we can solve for the real return on capital r^k_* using the FOC on capital:

$$r^k_* = \frac{e^{\gamma_C z_* + \omega_*}}{\beta} - (1 - \delta_0) \tag{29}$$

4.2 Ratios

Moving to the production side, we can use the aggregate price equation to solve for $\tilde{p}_*$:

$$\tilde{p}_* = 1$$

Using this result and given $\lambda_{p,*}$, we can use the price Phillips curve to solve for mc_*:

$$mc_* = \frac{1}{1 + \lambda_{p,*}} \tag{30}$$

Given values for α and ε^a_*, we can use the marginal cost equation to solve for w_*:

$$w_* = \left(mc_* \alpha^\alpha (1-\alpha)^{1-\alpha} (r^k_*)^{-\alpha}\right)^{\frac{1}{1-\alpha}} \tag{31}$$

The definition of effective capital gives us a value for k^e_* in terms of k_*:

$$k^e_* = k_* e^{-z_* - \omega_*}$$

Calculating y_* using the labor share of output $1 - \alpha$:

$$y_* = \frac{w_* h_*}{1 - \alpha}$$

Using capital shares based off our value of α, we can calculate the output to capital ratio as follows:

$$\frac{y_*}{k_*^e} = \frac{r_*^k}{\alpha}$$
$$\frac{y_*}{k_*} = e^{-z_*-\omega_*}\frac{r_*^k}{\alpha}$$

Using the capital accumulation equation, we can get a value for $\frac{i_*}{k_*}$

$$\frac{i_*}{k_*} = 1 - (1-\delta_0)e^{-z_*-\omega_*}$$

Using the resource constraint, we can get $\frac{c_*}{k_*}$:

$$\frac{c_*}{k_*} = \frac{y_*}{k_* s_*^g} - \frac{i_*}{k_*}$$

These ratios will give us the remaining steady-state levels and ratios:

$$k_* = y_*\left(\frac{y_*}{k_*}\right)^{-1} \qquad i_* = \frac{i_*}{k_*}k_*$$
$$c_* = \frac{c_*}{k_*}k_* \qquad g_* = g_y y_*$$

4.3 Liquidity Premium

Using the aggregate wage equation, we can get the following for $\tilde{w}_*$:

$$\tilde{w}_* = w_*$$

Combining this result with the wage Phillips curve, we get the following:

$$w_*^h = \frac{w_*}{1+\lambda_{w,*}}$$

We can use the FOC for consumption and the labor supply to pin down ε^h and λ_*^1

$$\varepsilon^b\left[c_*\left(1-\frac{\varrho}{e^z}\right)\right]^{-\gamma_c}\left(1-\varepsilon^h h_*^{(1+\gamma_h)}\right) - \lambda_*^1 = 0$$
$$-(1+\gamma_h)\varepsilon^b c_*^{(1-\gamma_c)}\left(1-\frac{\varrho}{\varepsilon^z}\right)^{(1-\gamma_c)}\left(1-\varepsilon^h h_*^{(1+\gamma_h)}\right)^{-\gamma_c}\varepsilon^h h_*^{\gamma_h} + \lambda_*^1 w_*^h = 0$$

Finally, the government bond rate is calculated from

$$\lambda_*^1 - \varepsilon_*^b\varepsilon_*^s = \beta R_*\frac{\lambda_*^1}{\pi_*}e^{-\gamma_C z}$$

$$\underbrace{\frac{\pi_*}{\beta e^{-\gamma_C z}}}_{R_*^P} - \varepsilon_*^b \varepsilon_*^s \frac{\pi_*}{\beta e^{-\gamma_C z} \lambda_*^1} \quad = \quad R_*$$

Noting that $R_*^P = \frac{\pi_*}{\beta e^{-\gamma_C z}}$ we can write

$$\frac{R_*^P - R_*}{R_*^P} = \frac{\varepsilon_*^b \varepsilon_*^s}{\lambda_*^1}.$$

This is the liquidity premium in steady state.

5 Log Linearization

Hatted variables refer to log deviations from steady-state ($\hat{x} = \ln\left(\frac{x_t}{x_*}\right)$):

$$\ln \varepsilon_t^j = \rho_j \ln \varepsilon_{t-1}^j + \eta_t^j$$

In the cases of z_t, ω_t, and ν_t, we have that $\hat{x} = x_t - x_*$ as these variables are already in logs.

Households' First Order Conditions

$$\hat{\varepsilon}_t^b - \hat{\lambda}_t^1 - \gamma_c \frac{1}{1 - \frac{\varrho}{e^z}} \hat{c}_t + \gamma_c \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{32}$$

$$\hat{\lambda}_t^1 + \hat{w}_t^h - \hat{\varepsilon}_t^b - \hat{\varepsilon}_t^h - \frac{1 - \gamma_c}{1 - \frac{\varrho}{e^z}} \hat{c}_t + (1 - \gamma_c) \frac{\frac{\varrho}{e^z}}{1 - \frac{\varrho}{e^z}} (\hat{c}_{t-1} - \hat{z}_t) \tag{33}$$

$$- \left(\gamma_h + \gamma_c (1 + \gamma_h) \frac{\varepsilon^h h_*^{1+\gamma_h}}{(1 - \varepsilon^h h_*^{1+\gamma_h})^2} \right) \hat{h}_t = 0$$

$$\hat{\lambda}_t^1 = \frac{R_*^P - R_*}{R_*^P} (\hat{\varepsilon}_t^s + \hat{\varepsilon}_t^b) + \frac{R_*}{R_*^P} (\hat{R}_t + E_t[(\hat{\lambda}_{t+1}^1 - \hat{\pi}_{t+1} - \gamma_C \hat{z}_{t+1}]) \tag{34}$$

$$\hat{\lambda}_t^1 = E_t \left[\hat{\lambda}_{t+1}^1 - \gamma_C \hat{z}_{t+1} + \hat{R}_t - \hat{\pi}_{t+1} \right] \tag{35}$$

$$\hat{\lambda}_t^1 = \left(\ln \varepsilon_t^i + \hat{\lambda}_t^2 \right) - S'' (\hat{\imath}_t - \hat{\imath}_{t-1}) + \beta e^{(1-\gamma_C)\gamma} S'' E_t (\hat{\imath}_{t+1} - \hat{\imath}_t) \tag{36}$$

$$\lambda_*^2 \hat{\lambda}_t^2 = \beta e^{-\gamma_C z_* - \omega_*} \left[\lambda_*^1 u_* r_*^k E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}_{t+1}^1 + \hat{r}_{t+1}^k + \hat{u}_{t+1} \right) \right] + \tag{37}$$

$$+ \beta e^{-\gamma_C z_* - \omega_*} \left[(1 - \delta_0) \lambda_*^2 E_t \left(-\gamma_C \hat{z}_{t+1} - \hat{\omega}_{t+1} + \hat{\lambda}_{t+1}^2 \right) - \lambda_*^2 \delta_1 u_* E_t \hat{u}_{t+1} \right]$$

$$\hat{\lambda}_t^1 = \hat{\lambda}_t^2 + \frac{\delta_2}{\delta_1} u_* \hat{u}_t - \hat{r}_t^k \tag{38}$$

$$\hat{k}_t = \left(1 - \frac{\varepsilon_*^i i_*}{k_*} \right) \left(\hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \right) + \frac{\varepsilon_*^i i_*}{k_*} \left(\hat{\varepsilon}_t^i + \hat{\imath}_t \right) - \delta_1 u_* e^{-z_* - \omega_*} \hat{u}_t \tag{39}$$

$$\hat{k}_t^e = \hat{u}_t + \hat{k}_{t-1} - \hat{z}_t - \hat{\omega}_t \tag{40}$$

Capital-Labor Ratio

$$\hat{k}_t^e = \hat{w}_t - \hat{r}_t^k + \hat{h}_t^d \tag{41}$$

Real Marginal Costs

$$\widehat{mc}_t = (1-\alpha)\,\hat{w}_t + \alpha \hat{r}^k_t - \hat{\varepsilon}^a_t \tag{42}$$

The New Keynesian Phillips Curve for Inflation

$$\hat{\pi}_t = \frac{(1-\beta\zeta_p e^{(1-\gamma_C)z_*})(1-\zeta_p)}{(1+\beta\iota_p e^{(1-\gamma_C)z_*})\zeta_p}\left[\frac{\lambda_{p,*}}{1+\lambda_{p,*}}\hat{\lambda}_{p,t} + \widehat{mc}_t\right] + \tag{43}$$
$$+ \frac{\iota_p}{1+\beta\iota_p e^{(1-\gamma_C)z_*}}\hat{\pi}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta\iota_p e^{(1-\gamma_C)z_*}}E_t\hat{\pi}_{t+1}$$

Wage Mark-Up

$$\hat{\mu}^w_t = \hat{w}_t - \hat{w}^h_t \tag{44}$$

The New Keynesian Phillips Curve for Wages

$$\hat{w}_t = \frac{1}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t-1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\hat{w}_{t+1} + \frac{\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(E_t\hat{\pi}_{t+1} + E_t\hat{z}_{t+1}) + \tag{45}$$
$$\frac{\iota_w}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_{t-1} + \hat{z}_{t-1}) - \frac{1+\iota_w\beta e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}(\hat{\pi}_t + \hat{z}_t) +$$
$$\frac{1-\beta\zeta_w e^{(1-\gamma_C)z_*}}{1+\beta e^{(1-\gamma_C)z_*}}\frac{1-\zeta_w}{\zeta_w}\left[\frac{\lambda_{w,*}}{1+\lambda_{w,*}}\hat{\lambda}_{w,t} - \hat{\mu}^w_t\right]$$

The Aggregate Resource Constraint

$$\frac{y_*}{g_*}(\hat{y}_t - \hat{g}_t) = \frac{c_*}{c_*+i_*}\hat{c}_t + \frac{i_*}{c_*+i_*}\hat{\imath}_t \tag{46}$$

The Production Function

$$\hat{y}_t = \frac{1}{mc_*}\left(\ln\varepsilon^a_t + \alpha\hat{k}^e_t + (1-\alpha)\,\hat{h}^d_t\right) \tag{47}$$

Labor Market Clearing Condition

$$\hat{h}_t = \hat{h}^d_t \tag{48}$$

Monetary Authority's Reaction Function

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[(1-\psi_1)\,\hat{\pi}^*_t + \frac{\psi_1}{4}\left(\sum_{j=-2}^{1}\hat{\pi}_{t+j}\right) + \frac{\psi_2}{4}\left(\sum_{j=-2}^{1}\hat{y}_{t+j}\right)\right] + \sum_{j=0}^{M}\hat{\xi}_{t-j,j} \tag{49}$$

6 Measurement

Unless otherwise noted all underlying data are from Haver Analytics.

6.1 National Income Accounts

The model economy's basic structure, with the representative household consuming a single good and accumulating capital using a different good, differs in some important ways from the accounting conventions of the Bureau of Economic Analysis (BEA) underlying the National Income and Product Accounts (NIPA). In particular

1. The BEA treats household purchases of long-lived goods inconsistently. It classifies purchases of residential structures as investment and treats the service flow from their stock as part of Personal Consumption Expenditures (PCE) on services. The BEA classifies households purchases of all other durable goods as consumption expenditures. No service flow from the stock of household durables enters measures of current consumption. In the model, all long-lived investments add to the productive capital stock.

2. The BEA treats all government purchases as government consumption. However, government at all levels makes purchases of investment goods on behalf of the populace. In the model, these should be treated as additions to the single stock of productive capital.

3. The BEA sums PCE and private expenditures on productive capital (Business Fixed Investment and Residential Investment), with government spending, inventory investment, and net exports to create Gross Domestic Product. The model features only the first three of these.

To bridge these differences, we create four *model consistent* NIPA measures from the BEA NIPA data.

1. Model-consistent GDP. Since the model's capital stock includes both the stock of household durable goods and the stock of government-purchased capital, a model-consistent GDP series should include the value of both stocks' service flows. To construct these, we followed a five-step procedure.

   (a) We begin by estimating a constant (by assumption) service-flow rate by dividing the nominal value of housing services from NIPA Table 2.4.5 by the beginning-of-year value of the residential housing stock from the BEA's Fixed Asset Table 1.1. We use annual data and average from 1947 through 2014. The resulting estimate is 0.096. That is, the annual value of housing services equals approximately 10 percent of the housing stock's value each year.

   (b) In the second step, we estimate estimate constant (by assumption) depreciation rates for residential structures, durable goods, and government capital. We constructed these by first dividing observations of value lost to depreciation over a calendar year by the end-of-year stocks. Both variables were taken from the BEA's

Fixed Asset Tables. (Table 1.1 for the stocks and Table 1.3 for the deprecation values.) We then averaged these ratios from 1947 through 2014. The resulting estimates are 0.021, 0.194, and 0.044 for the three durable stocks.

(c) In the third step, we calculated the average rates of real price depreciation for the three stocks. For this, we began with the nominal values and implicit deflators for PCE Nondurable Goods and PCE Services from NIPA Table 1.2. We used these series and the Fisher-ideal formula to produce a chain-weighted implicit deflator for PCE Nondurable Goods and Services. Then, we calculated the price for each of the three durable good's stocks in consumption units as the ratio of the implicit deflator taken from Fixed Asset Table 1.2 to this deflator. Finally, we calculated average growth rates for these series from 1947 through 2014. The resulting estimates equal 0.0029, −0.0223, and 0.0146 for residential housing, household durable goods, and government-purchased capital.

(d) The fourth combines the previous steps' calculations to estimate constant (by assumption) service-flow rates for household durable goods and government-purchased capital. To implement this, we assumed that all *three* stocks yield the same financial return along a steady-state growth path. These returns sum the per-unit service flow with the appropriately depreciated value of the initial investment. This delivers two equations in two unknowns, the two unknown service-flow rates. The resulting estimates are 0.29 and 0.12 for household durable goods and government-purchased capital.

(e) The fifth and final step uses the annual service-flow rates to calculate real and nominal service flows from the real and nominal stocks of durable goods and government-purchased capital reported in Fixed Asset Table 1.1. This delivers an annual series. Since the stocks are measured as of the end of the calendar year, we interpret these as the service flow values in the *next* year's first quarter. We create quarterly data by linearly interpolating between these values.

With these real and nominal service flow series in hand, we create nominal model-consistent GDP by summing the BEA's definition of nominal GDP with the nominal values of the two service flows. We create the analogous series for model-consistent real GDP by applying the Fisher ideal formula to the nominal values and price indices for these three components.

2. Model-consistent Investment. The nominal version of this series sums nominal Business Fixed Investment, Residential Investment, PCE Durable Goods, and government investment expenditures. The first three of these come from NIPA Table 1.1.5, while government investment expenditures sums Federal Defense, Federal Nondefense, and State and Local expenditures from NIPA Table 1.5.5. We construct the analogous series for real Model-consistent Investment by combining these series with their real chain-weighted counterparts found in NIPA Tables 1.1.3 and 1.5.3 using the Fisher ideal formula. By construction, this produces an implicit deflator for Model-consistent investment as well.

3. Model-consistent Consumption. The nominal version of this series sums nominal PCE

Nondurable Goods, PCE Services, and the series for nominal services from the durable goods stock. The first two of these come from NIPA Table 1.1.5. We construct the analogous series for real Model-consistent consumption by combining these series with their real chain-weighted counterparts using the Fisher ideal formula. The two real PCE series come from NIPA Table 1.1.3. Again, this produces an implicit deflator for Model-consistent consumption as a by-product.

4. Model-consistent Government Purchases. Conceptually, the model's measure of Government Purchases includes all expenditures not otherwise classified as Investment or Consumption: Inventory Investment, Net Exports, and actual Government Purchases. We construct the nominal version of this series simply by subtracting nominal Model-consistent Investment and Consumption from nominal Model-consistent GDP. We calculate the analogous real series using "chain subtraction." This applies the Fisher ideal formula to Model-consistent GDP and the *negatives* of Model-consistent Consumption and Investment.

Our empirical analysis requires us to compare model-consistent series measured from the NIPA data with their counterparts from the model's solution. To do this, we begin by solving the log-linearized system above, and then we feed the model specific paths for all exogenous shocks starting from a particular initial condition. for a given such simulation, the growth rates of Model-consistent Consumption and Investment equal

$$\begin{aligned} \Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + z_t \text{ and} \\ \Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{i}_t + z_t + \omega_t \end{aligned}$$

The measurement of GDP growth in the model is substantially more complicated, because the variables Y_t and y_t denote model output *in consumption units.* In contrast, we mimic the BEA by using a chain-weighted Fisher ideal index to measure model-consistent GDP. Therefore, we construct an analogous chain-weighted GDP index from model data. Since such an ideal index is invariant to the units with which nominal prices are measured, we can normalize the price of consumption to equal one and employ the prices of investment goods and government purchases relative to current consumption. Our model identifies the first of these relative prices as with investment-specific technology. However, the model characterizes only government purchases *in consumption units*, because private agents do not care about their division into "real" purchases and their relative price. For this reason, we use a simple autoregression to characterize the evolution of the price of government services in consumption units. Denote this price in quarter t with P_t^g. We construct this for the US economy by dividing the Fisher-ideal price index for model-consistent government purchases by that for model-consistent consumption. Then, our model for its evolution is

$$\pi_t^{g,obs} = \ln(P_t^g / P_{t-1}^t) = (1 - \beta_{2,1} - \beta_{2,2})\pi_g^* + \beta_{2,1} \ln(P_{t-1}^g / P_{t-2}^g) + \beta_{2,2} \ln(P_{t-2}^g / P_{t-3}^g) + u_t^g. \quad (50)$$

Here, $u_t^g \sim \mathbb{N}(0, \sigma_g^2)$. Given an arbitrary normalization of P_t^g to one for some time period, simulations from (50) can be used to construct simulated values of P_t^g for all other time periods. With these and a simulation from the model of all other variables in hand, we can calculate the simulation's values for Fisher ideal GDP growth using

$$\frac{Q_t}{Q_{t-1}} \equiv \sqrt{\dot{Q}_t^P \dot{Q}_t^L}, \tag{51}$$

where the Paasche and Laspeyres indices of quantity growth are

$$\dot{Q}_t^P \equiv \frac{C_t + P_t^I I_t + P_t^G (G_t/P_t^G)}{C_{t-1} + P_t^I I_{t-1} + P_t^G (G_{t-1}/P_{t-1}^g)} \text{ and} \tag{52}$$

$$\dot{Q}_t^L \equiv \frac{C_t + P_{t-1}^I I_t + P_{t-1}^G (G_t/P_t^G)}{C_{t-1} + P_{t-1}^I I_{t-1} + P_{t-1}^G (G_{t-1}/P_{t-1}^G)}. \tag{53}$$

In both (52) and (53), P_t^I is the relative price of investment to consumption. In equilibrium, this always equals A_t^I.

The above gives a complete recipe for *simulating* the growth of model-consistent real GDP growth. However, we also embody its insights into our estimation with a log-linear approximation. For this, we start by removing stochastic trends from all variables in (52) and (53), and we proceed by taking a log-linear approximation of the resulting expression. Details are available from the authors upon request.

6.1.1 Output Growth Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one- to four-quarter ahead real GDP growth from the Survey of Professional Forecasters with the analogous expectations from our model. The Survey of Professional Forecasters did not report these expectations prior to 2007, so we use them only in the second sample. As discussed in previous section, the quarterly per-capita model-consistent real GDP growth ($\Delta \ln Q_t$) does not map one-to-one with the SPF forecast of the BEA annual real GDP growth ($\Delta \ln Y_t^{BEA}$). So we transform the former into the latter by adding back population growth to the per-capita model-consistent real GDP growth and by fitting a linear regression model of BEA real GDP growth on model-consistent real GDP growth over the sample 1993:Q1-2016Q4. In particular, we estimate the following model

$$\Delta \ln Y_t^{BEA} = \underbrace{a}_{-0.14} + \underbrace{b}_{1.06} [4 \times (\Delta \ln Q_t^{obs} + pop_t)] \quad R^2 = 0.996$$

When we bridge model and SPF forecasts, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\Delta \ln Y_t^{l,obs} = a + 4b(\Delta \ln Q_t^{l,obs} + pop_t^l), \quad l = 1, 2, 3, 4;$$

and we assume that population forecast is at 1 percent at annual rate throughout. The two measurement errors follow mutually-independent first-order autoregressive processes.

6.2 Hours Worked Measurement

Empirical work using DSGE models like our own typically measure labor input with hours worked per capita, constructed directly from BLS measures of hours worked and the civilian non-institutional population over age 16. However, this measure corresponds poorly with business cycle models because it contains underlying low frequency variation. This fact led us to construct a new measure of hours for the model using labor market trends produced for the FRB/US model and for the Chicago Fed's in-house labor market analysis.

We begin with a multiplicative decomposition of hours worked per capita into hours per worker, the employment rate of those in the labor force, and the labor-force participation rate. The BLS provides CPS-based measures of the last two rates for the US as a whole. However, its measure of hours per worker comes from the Establishment Survey and covers only the private business sector. If we use hours per worker in the business sector to approximate hours per worker in the economy as a whole, then we can measure hours per capita as

$$\frac{H_t}{P_t} = \frac{H_t^E}{E_t^E}\frac{E_t^C}{L_t^C}\frac{L_t^C}{P_t^C}.$$

Here, H_t and P_t equal total hours worked and the total population, H_t^E/E_t^E equals hours per worker measured with the Establishment survey, E_t^C/L_t^C equals one minus the CPS based unemployment rate, and L_t^C/P_t^C equals the CPS based labor-force participation rate. Our measure of *model-relevant* hours worked deflates each component on the right-hand side by an exogenously measured trend. The trend for the unemployment rate comes from the Chicago Fed's Microeconomics team, while those for hours per worker and labor-force participation come from the FRB/US model files.

6.3 Inflation

Our empirical analysis compares model predictions of price inflation, wage inflation, inflation in the price of investment goods relative to consumption goods, and inflation expectations with their observed values from the U.S. economy. We describe our implementations of these comparisons sequentially below.

6.3.1 Price Inflation

Our model directly characterizes the inflation rate for Model-consistent Consumption. In principle, this is close to the FOMC's preferred inflation rate, that for the implicit deflator of PCE. However, in practice the match between the two inflation rates is poor. In the data, short-run movements in food and energy prices substantially influences the short-run evolution of PCE inflation. Our model lacks such a volatile sector, so if we ask it to match observed short-run inflation dynamics, it will attribute those to transitory shocks to intermediate goods' producers' desired markups driven by λ_t^p.

To avoid this outcome, we adopt a different strategy for matching model and data inflation rates, which follows that of Justiniano, Primiceri, and Tambalotti (2013). This relates three observable inflation rates – core CPI inflation, core PCE inflation, and market-based PCE inflation – to Model-consistent consumption inflation using auxiliary observation equations.

For core PCE inflation, this equation is

$$\pi_t^{1,obs} = \pi_* + \pi_*^1 + \beta^{\pi,1}\hat{\pi}_t + \gamma^{\pi,j}\pi_t^{d,obs} + u_t^{\pi,1}, \tag{54}$$

In (54) as elsewhere, π_* equals the long-run inflation rate. The constant π_*^1 is an adjustment to this long-run inflation rate which accounts for possible long-run differences between realized inflation and the FOMC's goal of $\pi_\star$ (for PCE inflation π_*^1 is set to zero). The right-hand side's inflation rates, $\hat{\pi}_t$ and $\pi_t^{d,obs}$ equal Model-consistent consumption inflation and PCE Durables inflation. We refer to the coefficients multiplying them, $\beta^{\pi,1}$ and $\gamma^{\pi,1}$, as the *inflation loadings*. We include PCE Durables inflation on the right-hand side of (54) because the principle adjustment required to transform Model-consistent inflation into core PCE inflation is the replacement of the price index for durable goods services with that for durable goods purchases. The disturbance term $u_t^{\pi,1}$ follows a zero-mean first-order autoregressive process.

The other two observed inflation measures, market-based PCE inflation and core CPI inflation, have identically specified observation equations. We use 2 and 3 in superscripts to denote these equations parameters and error terms, and we use the same expressions as subscripts to denote the parameters governing the evolution of their error terms. We assume that the error terms $u_t^{\pi,1}$, $u_t^{\pi,2}$, and $u_t^{\pi,3}$ are independent of each other at all leads and lags.

To produce forecasts of inflation with these these three observation equations, we must forecast their right-hand side variables. The model itself gives forecasts of $\hat{\pi}_t$. The forecasts of durable goods inflation come from a second-order autoregression.

$$\pi_t^{d,obs} = (1 - \beta_{1,1} - \beta_{1,2})\pi_*^d + \beta_{1,1}\pi_{t-1}^{d,obs} + \beta_{1,2}\pi_{t-2}^{d,obs} + u_t^d \tag{55}$$

Its innovation is normally distributed and serially uncorrelated.

6.3.2 Wage Inflation

Although observed wage inflation does not feature the same short-run variability as does price inflation, it does include the influences of persistent demographic labor-market trends which we removed ex ante from our measure of hours worked. Therefore, we follow the same general strategy of relating observed measures of wage inflation to the model's predicted wage inflation with a error-augmented observation equation. For this, we employ two measures of compensation per hour, Earnings per Hour and Total Compensation per Hour. In parallel with our notation for inflation measures, we use 1 and 2 to denote these two wage measures of wage inflation. The observation equation for Earnings per Hour is

$$\Delta \ln w_t^{1,obs} = z_* + w_*^j + \beta^{w,1}\left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{w,1}, \tag{56}$$

where "Δ" is the first difference operator. Just as with the price inflation measurement errors, $u_t^{w,1}$ follows a zero-mean first-order autoregressive process. The observation equation for Total Compensation per Hour is analogous to (56).

6.3.3 Relative Price Inflation

To empirically ground investment-specific technological change in the model, we use an error-augmented observation equation to relate the relative price of investment to consumption, both model-consistent measures constructed from NIPA and Fixed Asset tables as described above, with the model's growth rate of the rate of technological transformation between these two goods, ω_t.

$$\pi_t^{i,obs} = \omega_* + \hat{\omega}_t + u_t^{c/i};$$

Here, $\pi_t^{i,obs}$ denotes the price of $c$onsumption relative to $i$nvestment. The measurement error $u_t^{c/i}$ follows a i.i.d. zero-mean normally-distributed innovation.

6.3.4 Inflation Expectations

We also discipline our model's inferences about the state of the economy by comparing expectations of one- to four-quarter ahead and 10-year inflation from the Survey of Professional Forecasters with the analogous expectations from our model. Just as with all of the other inflation measures, we allow these two sets of expectations to differ from each other by including serially correlated measurement errors. The observation equations are

$$\begin{aligned}
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \beta^{l,j} E_t \hat{\pi}_{t+l} + u_t^{l,j,\pi}, \; j = 1, 2, \; l = 1, \ldots 4; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l} \sum_{i=1}^{l} E_t \hat{\pi}_{t+i} + u_t^{l,j,\pi}, \; j = 1, 2, \; l = 40;
\end{aligned}$$

The measurement errors follow mutually-independent first-order autoregressive processes.

6.4 Interest Rates and Monetary Policy Shocks

Since our model features forward guidance shocks, it has non-trivial implications for the current policy rate as well as for expected future policy rates. To discipline the parameters governing their realizations, the elements of Σ_ε, using data, we compare the model's monetary policy shocks to high-frequency interest-rate innovations informed by event studies, such as that of Gürkaynak, Sack, and Swanson (2005). Those authors applied a factor structure to innovations in implied expected interest rates from futures prices around FOMC policy announcement dates. Specifically, they show that the vector of M implied interest rate changes following an FOMC policy announcement, Δr_t, can be written as

$$\Delta r_t = \Lambda f_t + \eta_t$$

Where f is a 2×1 vector of factors, Λ is a $H \times 2$ matrix of factor loadings, and η is an $H \times 1$ vector of mutually independent shocks. Denoting the 2×2 diagonal variance covariance matrix of f with Σ_f and the $H \times H$ diagonal variance-covariance matrix of η with Ψ, we can express the observed variance-covariance matrix of Δr as $\Lambda \Sigma_f \Lambda' + \Psi$.

Our model has implications for this same variance covariance matrix. For this, use the model's solution to express the changes in current and future expected interest rates following monetary policy shocks as $\Delta r = \Gamma_1 \varepsilon^R$. Here, ε_t^R is the vector which collects the current monetary policy shock with $M-1$ forward guidance shocks, and Γ_1 is an $H \times H$ matrix. In general, Γ_1 does *not* simply equal the identity matrix, because current and future inflation and output gaps respond to the monetary policy shocks and thereby influence future monetary policy "indirectly" through the interest rate rule.

We assume that a factor structure determines the cross-correlations among monetary policy shocks. Specifically, we assume

$$\varepsilon_{R,t}^j = \alpha_j f_t^\alpha + \beta_j f_t^\beta + \eta_t^j,$$

where the factors f_t^α and f_t^β and factor loadings α_i and β_i are scalars, η_t^j is a measurement error. The factors and shocks have zero means and are independent and normally distributed. In matrix notation, we have

$$\varepsilon_t^R = \boldsymbol{\alpha} f_t^\alpha + \boldsymbol{\beta} f_t^\beta + \eta_t,$$

where $\boldsymbol{\alpha} = [\alpha_0, \ldots, \alpha_H]'$, $\boldsymbol{\beta} = [\beta_0, \ldots, \beta_H]'$. Let $\Sigma_\eta = E(\eta_t \eta_t')$ denote the variance-covariance matrix of the idiosyncratic shocks, and σ_α^2 (σ_β^2) denote the variance of f_t^α (f_t^β). Therefore we have that

$$\Lambda \Sigma_f \Lambda' + \Psi = \Gamma_1 (\boldsymbol{\alpha}\boldsymbol{\alpha}' \sigma_\alpha^2 + \boldsymbol{\beta}\boldsymbol{\beta}' \sigma_\beta^2) \Gamma_1' + \Gamma_1 \Sigma_\eta \Gamma_1'$$

6.5 Measurement Equations Synthesis

To summarize the measurement equations are as follows:

$$\begin{aligned}
\Delta \ln Q_t^{obs} &= f\left(\hat{c}_t, \hat{c}_{t-1}, \hat{i}_t, \hat{i}_{t-1}, \hat{g}_t, \hat{\omega}_t, \hat{\pi}_t^{g,obs}\right) \equiv \Delta \ln Q_t^j; \\
\Delta \ln Y_t^{l,obs} &= a + 4b(\Delta \ln Q_t^l + pop_t^l), \quad l = 1, 2, 3, 4; \\
\Delta \ln C_t^{obs} &= z_* + \Delta \hat{c}_t + \hat{z}_t; \\
\Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta \hat{\imath}_t + \hat{z}_t + \hat{\omega}_t; \\
\log H_t^{obs} &= \hat{H}_t; \\
\pi_t^{i,obs} &= \omega_* + \hat{\omega}_t + u_t^i; \\
R_t^{obs} &= R_* + \hat{R}_t; \\
R_t^{j,obs} &= R_* + E_t \hat{R}_{t+j}, \; j = 1, 2, \ldots, H; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \beta^{l,j} E_t \hat{\pi}_{t+l} + u_t^{l,j,\pi}, \; j = 1, 2, \; l = 1, \ldots 4; \\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l} \sum_{i=1}^{l} E_t \hat{\pi}_{t+i} + u_t^{l,j,\pi}, \; j = 1, 2, \; l = 40; \\
\pi_t^{j,obs} &= \pi_* + \pi_*^j + \beta^{\pi,j} \hat{\pi}_t + \gamma^{\pi,j} \pi_t^{d,obs} + u_t^{j,p}, \; \text{ with } \beta^{\pi,1} = 1, j = 1, 2, 3; \\
\Delta \ln w_t^{j,obs} &= z_* + w_*^j + \beta^{w,j} \left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{j,w}, \; \text{ with } \beta^{w,1} = 1, j = 1, 2; \\
\pi_t^{d,obs} &= (1 - \beta_{1,1} - \beta_{1,2}) \pi_*^d + \beta_{1,1} \pi_{t-1}^{d,obs} + \beta_{1,2} \pi_{t-2}^{d,obs} + u_t^d; \\
\pi_t^{g,obs} &= (1 - \beta_{2,1} - \beta_{2,2}) \pi_*^g + \beta_{2,1} \pi_{t-1}^{g,obs} + \beta_{2,2} \pi_{t-2}^{g,obs} + u_t^g.
\end{aligned}$$

The left hand side variables represent data (Q denotes chain-weighted GDP). The function f in the first equation represents the linear approximation to the chain-weighted GDP formula. As previously discussed, two variables are included to complete the mapping from model to data but are not endogenous to the model. Specifically, the consumption price of government consumption plus net exports, $\pi_t^{g,obs}$, helps map model GDP to our model-consistent measure of chain-weighted GDP, and inflation in the consumption price of consumer durable goods, $\pi_t^{d,obs}$, is used to complete the mapping from model inflation to measured inflation.

The measurement equations indicate we use 21 time series to estimate the model in the first sample. In addition to the real quantities and federal funds rate that are standard in the literature our estimation includes multiple measures of wage and consumer price inflation, two measures each of average inflation expected over the next ten years and over one quarter, and $H = 4$ quarters of interest rate futures. Our second sample estimation is restricted to estimating the parameters of the stochastic process for forward guidance news with $H = 10$ plus the processes driving $\pi_t^{g,obs}$ and $\pi_t^{d,obs}$ (only the constant and the standard deviation). This estimation uses the measurement equations involving the current federal funds rate and 10 quarters of expected future policy rates plus the last two equations. We take into account the change in steady state but keep the remaining structural parameters at their first sample values. Because our estimation forces data on real activity, wages and prices to coexist with the interest rate futures data, we expect the estimation to mitigate the forward guidance puzzle. Finally, it is worth stressing that our estimation respects the ELB in the second sample. This is because we measure expected future rates in the model, the $E_t\hat{R}_{t+j}$, using the corresponding empirical futures rates, $R_t^{j,obs}$, and we use futures rates extending out 10 quarters. Finally, in the second sample we extend the use the Survey of Professional Forecasts about near term inflation expectations using the 1Q-4Q ahead CPI and PCE inflation expectations, and introduce the SPF expectations about near term real GDP growth expectations, i.e. 1Q ahead until 4Q ahead.

6.6 Data Synopsis

Model-Consistent Output: gdp_pcLD100

- The DSGE model output is the chained sum of conventional GDP with government capital services and durable goods services. This series is de-trended by population growth.

Model-Consistent Consumption: cons_pcLD100

- DSGE consumption is defined as the chained sum of conventional PCE nondurable goods with PCE services and durable goods services. This series is de-trended by population growth.

Model-Consistent Investment: inv_pcLD100

- Model-consistent Investment is the chained sum of durable goods purchases, fixed investment, and government investment. This series is de-trended by population growth.

Model-Consistent Residual Output Inflation: gnx_CONSINF

- The residual output is the chained difference of model consumption and investment from model GDP. Residual output reflects government spending and net exports.

Relative Price of Consumption to Investment: RPCtoI_LD100

- The relative price is constructed by dividing the consumption price series and investment price series.

Deflators for Consumer Durables: JCD_LD100

- We take the log difference[3] of the PCE Durable Goods Chain Price Index for the deflators for consumer durables.

Inflation Expectations: inf_10YQ_PCE, ASAF1CPX, inf_10YQ_CPI, ASAF1CX

- Our inflation expectations series are quarterly inflation expectations data from the Survey of Professional Forecasters at the Philadelphia Fed. They report inflation expectations at various horizons for both PCE and CPI measures. We use measures of 1Q ahead and 40Q ahead CPI and core PCE inflation expectations. The 40Q ahead series are the ten-year ahead expectations, not the annual average over the next ten years. The SPF did not report expectations for core PCE prior to 2007, so we do not have many observations for the first sample of our data. However, we continue to include these few observations in order to initialize the kalman filter for second sample estimation. We have the full data for CPI expectations.

Real GDP Growth Expectations: GDP_1Q_SPF-GDP_4Q_SPF

- Our real GDP growth expectations series are annualized expectations data from the Survey of Professional Forecasters at the Philadelphia Fed. They report BEA real GDP growth expectations at various horizons, from 1Q to 4Q ahead. The SPF did not report these expectations prior to 2007, so we use them only in the second sample.

Real Wages: lepriva_CORE, ls_CORE

- We have two different measures of wages in the model - average hourly earnings and employment compensation. We take the average hourly earnings and divide by the chain price index of core PCE, then take the log difference.
- We repeat the same steps to calculate employment compensation but use the employment cost index for the compensation of civilian workers.

Price Inflation: JCXFE_LD100, JCMXFE_LD100, PCUSLFE_LD100

- We use three different measures of price inflation: Core PCE, Market-Based Core PCE, and Core CPI.

Hours: hours_L

[3] All log differenced series are multipled by 100.

- We construct our hours series with the methodology as described in *Forward Guidance and Macroeconomic Outcomes Since the Financial Crisis* (Campbell et al., 2016).

Effective Federal Funds Rate: ffed_q

- For the first sample (1993q1-2008q3), we use the federal funds target rate observed as the average over the last month of the quarter.
- For the second sample (2008q4-2018q4), we use the federal funds target rate observed at the end of the quarter.
- We divide the series by 4 to convert to quarterly rates.

Expected Federal Funds Rate (FFR): 1-10QAhead

- From 1993Q1 to 2005Q4, our 4-quarter ahead path comes from Eurodollar futures. Eurodollar futures have expiration dates that lie about two weeks before the end of each quarter. Eurodollar rate is closely tied to expectations for the Federal Funds rates over the same period, so the Eurodollar futures rate corresponds with the Fed Funds rate at the middle of the last month of each quarter.
- Beginning with 2006Q1, our 4-quarter ahead, and later, 10-quarter ahead path comes from the Overnight Index Swaps (OIS). The OIS data are converted into a point estimate of the Fed Funds for a particular date using a Svensson term structure model. The dates of the OIS data reflect the middle of the quarter values, and we interpolate to obtain the end of quarter values.
- From 2014Q1, we began to use the expected Fed Funds from the Survey of Market Participants (SMP). The SMP correspond to the survey participants' expected Fed Funds at the end of the quarter.
- The path for the current forecasting quarter is the most recently released SMP path adjusted with the difference between the SMP date OIS and the forecasting date OIS.
- All expected FFR series are in quarterly rates.

7 Calibration and Bayesian Estimation

As we discussed, we follow a two-stage approach to the estimation of our model's parameters. In a calibration stage, we set the values of selected parameters so that the model has empirically-sensible implications for long-run averages from the U.S. economy. In this stage, we also enforce several normalizations and a judgemental restriction on one of the measurement error variances. In the second stage, we estimate the model's remaining parameters using standard Bayesian methods.

7.1 Calibration

Our calibration strategy is the same as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we address the well-known evidence of secular declines in economic growth and rates of return on nominally risk free assets. We address these developments by imposing a change in steady state in 2008q4 (the choice of this date is motivated in the next subsection). Steady state GDP growth is governed by the mean growth rates of the neutral and investment-specific technologies, ν_* and ω_*. We adjust ω_* down to account for the slower decline in the relative price of investment since 2008q4. Given this change we then lower ν_* so that steady state GDP growth is reduced to 2%. To match a lower real risk-free rate of 1% we increase the steady state marginal utility of government bonds using ε^s_*.[4] These adjustments leave the other calibrated parameters unchanged but do change the steady state values of the endogenous variables and therefore the point at which the economy is log-linearized.[5]

We observe the long-run average of the following aggregates: nominal federal funds rate, labor share, government spending share, investment spending share, the capital-output ratio, real per-capita GDP growth (g_y), inflation in price of government, net exports and inventory investment relative to non-durables and services consumption, and the growth rate of the consumption-investment relative price.

- The labor share can be used to calibrate the parameter α.
- The government spending share determines s^g_*.
- The government price growth rate pins down π^g_*.
- The growth rate of the consumption-investment relative price pins down ω_*.
- The investment share pins down i_*/y_*.
- The capital output ratio pins down k_*/y_*.
- Calculate the consumption-output share

$$\frac{c_*}{y_*} = \left(1 - \frac{i_*}{y_*} - \frac{g_*}{y_*}\right). \tag{57}$$

- The growth rate of real chain-weighted GDP is used to pin down the growth rate of the common trend z_*. First

$$g_y = e^{z_*}\sqrt{\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi^g_*\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi^g_*\frac{g_*}{y_*}}}$$

[4]The targets for steady state GDP growth and risk-free rate reflect a variety of evidence including the Fed's Summary of Economic Projections.

[5]Our re-calibration changes the return on private assets by a little. This small change is consistent with Yi and Zhang (2017) who show that rates of return on private capital have stayed roughly constant in the face of declines in risk free rates.

All the variables in this equation are known except for z_*. So we can solve for z_*:

$$z_* \quad = \quad g_y - \frac{1}{2} \ln \left(\frac{\frac{c_*}{y_*} + e^{\omega} \frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1} \frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega} \frac{i_*}{y_*} + \pi_*^g \frac{g_*}{y_*}} \right) \tag{58}$$

- The growth rate of the labor-augmenting technology ν_* can be easily obtained by exploiting the following equation:

$$z_* = \upsilon_* + \frac{\alpha}{1-\alpha} \omega_*. \tag{59}$$

- We are now in a position to identify the depreciation rate δ_0 using the steady-state equation pinning down the investment capital ratio:

$$\begin{aligned} \frac{i_*}{k_*} &= 1 - (1-\delta_0) e^{-z_* - \omega_*} \\ &\Rightarrow \quad \delta_0 = 1 + \left(\frac{i_*}{k_*} - 1 \right) e^{z_* + \omega_*} \end{aligned}$$

where the investment capital ratio is obtained combining the investment share and the capital output ratio:

$$\frac{i_*}{k_*} = \frac{i_*/y_*}{k_*/y_*}. \tag{60}$$

- From the steady-state equilibrium we have that

$$\frac{y_*}{k_*} = e^{-z_* - \omega_*} \frac{\delta_1}{\alpha}. \tag{61}$$

Therefore

$$\delta_1 \quad = \quad \alpha \left(\frac{k_*}{y_*} \right)^{-1} e^{z_* + \omega_*} \tag{62}$$

where the capital output ratio is given above.

- In steady state, the real rate of return on private bonds is derived from the first order condition for private bonds:

$$r_*^p \equiv \frac{R_*^P}{\pi_*} = \frac{e^{\gamma_c z_*}}{\beta}. \tag{63}$$

In steady state the real rental rate of capital is derived from the first order condition for capital:

$$r_*^k = \left[\frac{e^{\gamma_c z_*}}{\beta}\right] e^{\omega_*} - (1 - \delta_0) \tag{64}$$

Combining these last two equations yields

$$r_*^k = r_*^p e^{\omega_*} - (1 - \delta_0)$$

and hence

$$r_*^p = \left[r_*^k + 1 - \delta_0\right] e^{-\omega_*}.$$

Note that $r_*^k = \delta_1$ from the first order condition for capacity utilization. It follows that

$$r_*^p \quad = \quad (1 - \delta_0 + \delta_1)\, e^{-\omega_*}$$

- The liquidity premium in steady state (i.e., $\frac{R_*/\pi_*}{r_*^p}$) can be computed now by assuming a *nominal* average federal funds rate, R_*, and an annualized average inflation rate.
- Using equation (64) and the fact that $r_*^k = \delta_1$, we can calibrate the discount factor β :

  $$\beta = (1 - \delta_0 + \delta_1)^{-1} e^{\omega_*} e^{\gamma_c z_*}$$

  where γ_c is a parameter of the utility function to be estimated.

7.2 Bayesian Estimation

Our Bayesian estimation uses the same split-sample strategy as in Campbell, Fisher, Justiniano, and Melosi (2016) except that we incorporate the change in steady state described above and one other change noted below. As in Campbell, Fisher, Justiniano, and Melosi (2016) our sample begins in 1993q1. This date is based on the availability and reliability of the overnight interest rate futures data. The sample period ends in 2016q4 but we impose a sample break in 2008q4. Our choice of this latter date is motivated by three main considerations. First, there is the evidence that points to lower interest rates and economic growth later in the sample. Second, it seems clear that the horizon over which forward guidance was communicated by the Fed lengthened substantially during the ELB period. Finally, the downward trends in inflation and inflation expectations from the early 1990s appear to come to an end in the mid-2000s. Splitting the sample in 2008q4 and assuming some parameters change at that date is our way of striking a balance between parsimony and addressing the multiple structural changes that seem to occur around the same time.

We estimate the full suite of non-calibrated structural parameters in the first sample under the assumption that forward guidance extends for $H = 4$ quarters. Starting in 2008q4 we assume the model environment changes in three ways. First we assume the change in the steady state described above. Second, forward guidance lengthens to $H = 10$ quarters. Third, the time-varying inflation target from the first sample becomes a constant equal to the steady state rate of inflation, 2% at an annual rate. All three changes are assumed to be unanticipated and permanent.

We employ standard prior distributions, but those governing monetary policy shocks deserve further elaboration. Our estimation requires the variance-covariance matrix of monetary policy shocks to be consistent with the factor-structure of interest rate innovations used by Gürkaynak, Sack, and Swanson (2005), as described above. Therefore, we parameterize Σ_ε in terms of factors STD (σ_α and σ_β), factor loadings (α and β) and STD of the idiosyncratic errors ($\sigma_{\eta,j}$). We then center our priors for these parameters at their estimates from event-studies. However, we do not require our estimates to equal their prior values. Our Bayesian estimation procedure employs quarterly data on expected future interest rates, the posterior likelihood function includes them as free parameters. It is well known that factors STD and loadings are not separately identified, so we impose two scale normalizations and one rotation normalization on α and β. The rotation normalization requires that the first factor, which we label "Factor A", is the only factor influence the current policy rate. That is, the second factor, "Factor B" influences only future policy rates. Gürkaynak, Sack, and Swanson (2005) call Factors A and B the "target" and "path" factors.

Finally, in the second sample we also use the Survey of Professional Forecasts about near term (1Q- until 4Q-head) inflation expectations and real GDP growth expectations and estiamte the parameters of the respective measurement equations.

7.3 Posterior Estimates

We report the results of our two-stage two-sample estimation in a series of tables. Table 1 reports our most notable calibration targets. The long-run policy rate equals 1.1 percent on a quarterly basis. We target a two percent growth rate of per capita GDP. Given an average population growth rate of one percent per year, this implies that our potential GDP growth rate equals three percent. The other empirical moments we target are a nominal investment to output ratio of 26 percent and nominal government purchases to output ratio of 15 percent. Finally, we target a capital to output ratio of approximately 10 on a quarterly basis.

Table 2 lists the parameters which we calibrate along with their given values. The table includes many more parameters than there are targets in Table 1. This is because Table 1 omitted calibration targets which map one-to-one with particular parameter values. For example, we calibrate the steady-state capital depreciation rate (δ_0) using standard methods applied to data from the Fixed Asset tables. It is also because Table 2 lists several parameters which are normalized prior to estimation. Most notable among these are the three factor loadings listed at the table's bottom. Tables 3 and 6 report prior distributions and posterior modes for the model's remaining parameters, for the first and second samples respectively.

8 Incorporating the pandemic into the model

We use the framework introduced by Ferroni, Fisher, and Melosi (2023) to identify the propagation of the COVID-19 shock in the Chicago Fed DSGE model. The initial outbreak is represented as the onset of a new shock process where the shock is defined as a combination of the model's wedges that comprise the other structural shocks. Realizations of the pandemic shock come with news about its propagation. We identify pandemic shocks and their propagation with revisions to private sector forecasts of GDP and inflation. The event-study assumptions comprise priors on the pandemic shock's contribution to aggregate dynamics. We discuss the use of this framework next.

8.1 Methodology and Assumptions

We start by introducing some modeling assumptions about the new COVID-19 shock. These assumptions introduce enough structure to be able to separately parameterize the nature of COVID shock and its expected persistence.

Definition 1 *The propagation of COVID–19 is modeled as a combination of anticipated iid shocks ψ_t^j that are governed by the factor model*

$$\psi_t^j = \lambda(j) f_t, \quad j \in \{0, ..., n\} \tag{65}$$

where $\lambda = \{\lambda(j)\}_{j=0}^n$ denotes the loadings for the n anticipated shocks. $f_t \overset{iid}{\backsim} \mathcal{N}(0,1)$ is the COVID shock that is assumed to be $f_t = 0$ in the pre-COVID period that is set to zero in the periods preceding the onset of the COVID-19 pandemic. Notationally, ψ_t^j denotes shocks that are known at time t and will hit the economy in period $t + j$.

Definition 2 *We introduce a (new) set of current and anticipated shocks in our DSGE model $\{\varepsilon_t(i)\}_{i=0}^m$, with anticipation horizons $j \in \{0, ..., n\}$ and $i \in \mathcal{S}^C$, where $\mathcal{S}^C$ denotes the subset of DSGE shocks chosen to approximate the propagation of COVID-19.*

While the shocks in the set $\mathcal{S}^C$ are shocks that have never realized before the start of the COVID-19 pandemic, their nature is identical to that of the original set of shocks in our DSGE models (e.g., liquidity preference shocks or temporary technology shocks). Furthermore, the set $\mathcal{S}^C$ includes anticipated version of those shocks.

Definition 3 *We assume that the current and anticipated iid shocks ψ_t^j, capturing the propagation of COVID-19, are linear combinations of the current and anticipated shocks in our DSGE model $\{\varepsilon_t(i)\}_{i=0}^m$. Formally,*

$$\varepsilon_t^j(i) = \phi_i \psi_t^j, \quad j \in \{0, ..., n\} \tag{66}$$

where $\varepsilon_t^j(i)$ denotes the $i-th$ shock in the set $\mathcal{S}^C$ with anticipation horizon $j \in \{0, ..., n\}$ and ϕ_i is a scalar that controls the weight of the $i-th$ shock in affecting the shocks ψ_t^j, which approximate the propagation of COVID-19.

Note that these weights $\{\phi_i\}_{i=0}^m$ are shock-specific and do not depend on the anticipation horizon of the shocks. We make this assumption in order to economize on the number of parameters needed to be estimated. This assumption implies that the composition of the DSGE shocks, which is used to approximate the propagation of the COVID-19, does not vary across anticipation horizons. This seems a very natural assumption to make.

To sum up, the loadings λ identify the *persistence* of the COVID-19 shock. The vector $\phi = \{\phi_i\}_{i=1}^m$ identifies the *nature* of the COVID shocks - defined as a particular combination of shocks in the subset $\mathcal{S}^C$. The exogenous iid variable f_t should be interpreted as the forecasters' *revision* of their expectations regarding the magnitude of the COVID-19 shock.

8.2 Estimation and Identification

This structure leaves us with $m+n$ new parameters to be estimated. One way to go is to denote the parameters that we need to estimate as $\Xi = [\lambda,\phi]$ and use the Bayes theorem to obtain a distribution of these parameters conditional on the expectation data X. [6] In symbols,

$$p\left(\Xi|X,\Theta,s_{t-1};\mathcal{M}\right) \propto \mathcal{L}\left(X|\Xi,\Theta,s_{t-1};\mathcal{M}\right) p\left(\Xi\right), \tag{67}$$

where Θ denotes the parameters of the DSGE model, s_{t-1} denotes our model's state vector estimated one quarter earlier (initial conditions), and $\mathcal{M}$ denotes our DSGE model. The density $p(\Xi)$ is a prior on the new parameters capturing the propagation of COVID-19 in our DSGE model. The density $\mathcal{L}\left(X|\Xi,\Theta;\mathcal{M}\right)$ denotes the likelihood function associated with the data set X.

We assume that $p(\Xi)$ is a diffuse prior and we maximize the log of the posterior kernel $\ln\mathcal{L}\left(X|\Xi,\Theta;\mathcal{M}\right)p\left(\Xi\right)$. The intuition here is to find the combination of DSGE shocks that can best explain the propagation of COVID-19 expected by a large set of leading forecasters and our judgement reflecting our best knowledge about the effects of the pandemic on macro variables.

To sum up, at this stage we use the data X and our model $(s_{t-1},\mathcal{M})$ to pin down both the nature of the COVID shock, ϕ, and its expected persistence, λ, as well as forecasters' *revisions* of their expectations regarding the magnitude of the COVID-19 shock, f_t. In every quarters for which macro forecasts are made available (i.e., the sample size of X), we can use the smoother to evaluate forecasters' revision of their expectations regarding the magnitude of the COVID-19 shock.

The full list of assumptions of our estimation exercises is reported below

- We estimate the parameters of the COVID-19 shock using 2020Q2, 2020Q3 and 2020Q4 data. The loading on structural shocks ($\phi = \{\phi_i\}_{i=1}^m$) are estimated using 2020Q2 data only. The propagation of the COVID-19 shock (λ's) is re-estimated over time, i.e. using sequentially 2020Q2, 2020Q3 and 2020Q4.

- We reduce the standard deviation of the measurement error shocks on the 1Q-4Q ahead SPF expectations of inflation and real GDP growth in order to induce the model to take seriously the professionals forecasts.

[6] In a later stage, it would be nice to choose the $\mathcal{S}^C$ so as to maximize the model's fit of the expectation data/judgement used to estimate Ξ.

- The COVID-19 shock loads on the following structural shocks: Permanent Neutral, Marginal Efficiency of Investment, Discount Factor, Price Markup and Liquidity Preference.

The parameters estimates are reported in table 7.

Table 1: First Sample Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.2597
Capital to Output Ratio	K_t/Y_t	10.7629
Fraction of Final Good Output Spent on Public Goods	G_t/Y_t	0.1532
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.371

Table 2: First Sample Calibrated Parameters

Parameter	Symbol	Value
Discount Factor	β	0.9857
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ_*^w	1.500
Steady-State Price Markup	λ_*^p	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.0162
Steady-State Marginal Depreciation Cost	δ_1	0.0385
Core PCE, 1Q Ahead and 10Y Ahead Expected PCE		
Constant	$\pi_*^1, \pi_*^{l,1}$	0.000
Loading 1	$\beta^{\pi,1}, \beta^{l,1}$	1.000
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.010
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.418
2nd Lag Coefficient	$\beta_{1,2}$	0.379
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.311
2nd Lag Coefficient	$\beta_{2,2}$	0.0057
Compensation		
Constant	w_*^1	-0.202
Loading	$\beta^{w,1}$	1.000
Earnings Constant	w_*^2	-0.237
Loading 0 Factor A	α_0	0.981
Loading 0 Factor B	β_0	0.000
Loading 4 Factor B	β_4	0.951

Table 3: First Sample Estimated Parameters

Parameter	Symbol	Density	Prior Mean	Prior Std.Dev	Posterior Mode
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.474
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.409
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.077
External Habit Weight	λ	B	0.7500	0.025	0.780
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.589
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.831
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.914
Adjustment Cost of Investment	φ	G	3.0000	0.750	5.354
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.319
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.791
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.398
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.801
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.813
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.887
Investment-Demand	ρ_i	B	0.5000	0.100	0.791
Liquidity Preference	ρ_s	B	0.6000	0.200	0.887
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.136
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.469
Neutral Technology	ρ_ν	B	0.3000	0.150	0.492
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.303
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.307
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.391
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.768
Inflation Drift	σ_π	I	0.0150	0.0075	0.077
Exogenous Spending	σ_g	U	1.0000	2.000	4.139
Investment-Demand	σ_i	I	0.2000	0.200	0.549
Liquidity Preference	σ_s	U	0.5000	2.000	0.341
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.101
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.035
Neutral Technology	σ_ν	U	0.5000	0.250	0.530
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.259
Relative Price of Cons to Inv	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.675
Monetary Policy					
Unanticipated	σ_{η_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{η_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{η_2}	N	0.0050	0.0025	0.008

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	Symbol	Density	Prior Mean	Prior Std.Dev	Posterior Mode
3Q Ahead	σ_{η_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{η_4}	N	0.0050	0.0025	0.012
Compensation					
Standard Deviation of $u_t^{1,w}$		I	0.0500	0.100	0.194
AR(1) Coefficient of $u_t^{1,w}$		B	0.4000	0.100	0.458
Earnings					
Loading 1	$\beta^{w,2}$	N	0.8000	0.100	0.904
Standard Deviation of $u_t^{2,w}$		I	0.0500	0.100	0.143
AR(1) Coefficient of $u_t^{2,w}$		B	0.4000	0.100	0.674
Core PCE					
Loading 2	$\gamma^{\pi,1}$	N	0.0000	1.000	0.045
Standard Deviation of $u_t^{1,p}$		I	0.0500	0.100	0.046
AR(1) Coefficient of $u_t^{1,p}$		B	0.2000	0.100	0.108
Core CPI					
Loading 1	$\beta^{\pi,2}$	N	1.0000	0.100	0.808
Loading 2	$\gamma^{\pi,2}$	N	0.0000	1.000	0.087
Standard Deviation of $u_t^{2,p}$		I	0.1000	0.100	0.077
AR(1) Coefficient of $u_t^{2,p}$		B	0.4000	0.200	0.586
Market-Based Core PCE					
Constant	π_*^3	N	-0.1000	0.100	-0.037
Loading 1	$\beta^{\pi,3}$	N	1.0000	0.100	1.121
Loading 2	$\gamma^{\pi,3}$	N	0.0000	1.000	0.015
Standard Deviation of $u_t^{3,p}$		I	0.0500	0.100	0.035
AR(1) Coefficient of $u_t^{3,p}$		B	0.2000	0.100	0.144
1Q Ahead Expected PCE					
Standard Deviation of $u_t^{1,1,\pi}$		I	0.0500	0.100	0.026
AR(1) Coefficient of $u_t^{1,1,\pi}$		B	0.2000	0.100	0.196
1Q Ahead Expected CPI					
Loading	$\beta^{1,2}$	N	1.0000	0.100	0.980
Standard Deviation of $u_t^{1,2,\pi}$		I	0.0500	0.100	0.062
AR(1) Coefficient of $u_t^{1,2,\pi}$		B	0.2000	0.100	0.198
10Y Ahead Expected PCE					
AR(1) Coefficient of $u_t^{40,1,\pi}$		B	0.2000	0.100	0.271
10Y Ahead Expected CPI					
Loading	$\beta^{40,2}$	N	1.0000	0.100	1.021
AR(1) Coefficient of $u_t^{40,2,\pi}$		B	0.2000	0.100	0.213
PCE Durable Goods Inflation					
Constant	π_*^d	N	-0.3500	0.100	-0.360
Standard Deviation of u_t^d		I	0.2000	2.000	0.286

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Prior Std.Dev**	**Posterior Mode**
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption					
Constant	π_*^g	N	0.1980	1.000	-0.666
Standard Deviation of u_t^g		I	0.5000	2.000	1.861
Factor A					
Loading 1	α_1	N	0.6839	0.200	1.305
Loading 2	α_2	N	0.5224	0.200	0.877
Loading 3	α_3	N	0.4314	0.200	0.306
Loading 4	α_4	N	0.3243	0.200	-0.012
Standard Deviation	σ_α	N	0.1000	0.0750	0.040
Factor B					
Loading 1	β_1	N	0.3310	0.200	0.656
Loading 2	β_2	N	0.6525	0.200	1.104
Loading 3	β_3	N	0.8059	0.200	1.162
Standard Deviation	σ_β	N	0.1000	0.0750	0.078

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 4: Second Sample Calibration Targets (Different from First Sample)

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.007
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.003
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.171

Table 5: Second Sample Calibrated Parameters (Different from First Sample)

Parameter	Symbol	Value
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.171
Steady-State Marginal Depreciation Cost	δ_1	0.038
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.020
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.000
2nd Lag Coefficient	$\beta_{1,2}$	0.000
Inflation in Relative Price of Government,		
Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.320
2nd Lag Coefficient	$\beta_{2,2}$	-0.240
Compensation Loading	$\beta^{w,1}$	1.000
Loading 5 Factor A	α_5	0.932
Loading 8 Factor B	β_8	0.210
Loading 10 Factor B	β_{10}	0.000

Table 6: Second Sample Estimated Parameters

Parameter	Symbol	Prior Mean	Prior Std.Dev	Posterior Mode
Compensation				
Constant	w_*^1	-0.2023	0.000	-0.2023
Standard Deviation of $u_t^{1,w}$		0.1941	0.100	0.284
AR(1) Coefficient of $u_t^{1,w}$		0.4579	0.000	0.4579
Earnings				
Constant	w_*^2	-0.2370	0.000	-0.237
Loading 1	$\beta^{w,2}$	0.9039	0.000	0.9039
Standard Deviation of $u_t^{2,w}$		0.1434	0.100	0.304
AR(1) Coefficient of $u_t^{2,w}$		0.6741	0.000	0.6741
Core PCE				
Loading 2	$\gamma^{\pi,1}$	0.0449	0.000	0.0449
Standard Deviation of $u_t^{1,p}$		0.0457	0.100	0.274
AR(1) Coefficient of $u_t^{1,p}$		0.1081	0.000	0.1801
Core CPI				
Loading 1	$\beta^{\pi,2}$	0.8083	0.00	0.8083
Loading 2	$\gamma^{\pi,2}$	0.0868	0.000	0.0868
Standard Deviation of $u_t^{2,p}$		0.0770	0.100	0.2517
AR(1) Coefficient of $u_t^{2,p}$		0.5856	0.000	0.5856
Market PCE				
Constant	π_*^3	-0.0367	0.000	-0.0367
Loading 1	$\beta^{\pi,3}$	1.1213	0.000	1.1213
Loading 2	$\gamma^{\pi,3}$	0.0153	0.000	0.0153
Standard Deviation of $u_t^{3,p}$		0.0349	0.100	0.2553
AR(1) Coefficient of $u_t^{3,p}$		0.1436	0.000	0.1436
1Q Ahead Expected PCE				
Standard Deviation of $u_t^{1,1,\pi}$		0.0259	0.020	0.0412
AR(1) Coefficient of $u_t^{1,1,\pi}$		0.1960	0.050	0.1832
2Q Ahead Expected PCE				
Standard Deviation of $u_t^{2,1,\pi}$		0.0259	0.020	0.0175
AR(1) Coefficient of $u_t^{2,1,\pi}$		0.1960	0.050	0.2140
3Q Ahead Expected PCE				
Standard Deviation of $u_t^{3,1,\pi}$		0.0259	0.020	0.0193
AR(1) Coefficient of $u_t^{3,1,\pi}$		0.1960	0.050	0.2202
4Q Ahead Expected PCE				
Standard Deviation of $u_t^{4,1,\pi}$		0.0259	0.020	0.0156
AR(1) Coefficient of $u_t^{4,1,\pi}$		0.1960	0.050	0.2075
1Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.0022
Standard Deviation of $u_t^{1,2,\pi}$		0.0622	0.020	0.095

Second Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
AR(1) Coefficient of $u_t^{1,2,\pi}$		0.1982	0.050	0.206
2Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2433
Standard Deviation of $u_t^{2,2,\pi}$		0.0622	0.020	0.0411
AR(1) Coefficient of $u_t^{2,2,\pi}$		0.1982	0.050	0.2532
3Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2662
Standard Deviation of $u_t^{3,2,\pi}$		0.0622	0.020	0.0399
AR(1) Coefficient of $u_t^{3,2,\pi}$		0.1982	0.050	0.2607
4Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	1.2354
Standard Deviation of $u_t^{4,2,\pi}$		0.0622	0.020	0.0406
AR(1) Coefficient of $u_t^{4,2,\pi}$		0.1982	0.050	0.2782
10Y Ahead Expected PCE				
AR(1) Coefficient of $u_t^{40,1,\pi}$		0.2711	0.000	0.2711
10Y Ahead Expected CPI				
Loading	$\beta^{40,2}$	1.0207	0.000	1.0207
AR(1) Coefficient of $u_t^{40,2,\pi}$		0.2133	0.000	0.2133
1Q Ahead Expected GDP				
Standard Deviation of $u_t^{1,1,Y}$		0.10	0.100	0.9827
AR(1) Coefficient of $u_t^{1,1,Y}$		0.20	0.100	0.1300
2Q Ahead Expected GDP				
Standard Deviation of $u_t^{2,1,Y}$		0.10	0.100	0.6263
AR(1) Coefficient of $u_t^{2,1,Y}$		0.20	0.100	0.1825
3Q Ahead Expected GDP				
Standard Deviation of $u_t^{3,1,Y}$		0.10	0.100	0.9779
AR(1) Coefficient of $u_t^{3,1,Y}$		0.20	0.100	0.1767
4Q Ahead Expected GDP				
Standard Deviation of $u_t^{4,1,Y}$		0.10	0.100	0.3664
AR(1) Coefficient of $u_t^{4,1,Y}$		0.20	0.100	0.2747
PCE Durable Goods Inflation				
Constant	π_*^d	-0.4500	0.200	-0.4858
Standard Deviation of u_t^d		0.5000	0.150	0.325
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption				
Constant	π_*^g	0.8900	0.400	-0.1177
Standard Deviation of u_t^g		0.8143	0.150	1.267
Factor A				
Loading 0	α_0	0.0180	0.250	0.099
Loading 1	α_1	0.0574	0.250	0.202

Second Sample Estimated Parameters (Continued)

Parameter	Symbol	Prior Mean	Std.Dev	Posterior Mode
Loading 2	α_2	0.1941	0.250	0.397
Loading 3	α_3	0.3996	0.250	0.591
Loading 4	α_4	0.6520	0.250	0.792
Loading 6	α_6	1.2266	0.250	1.116
Loading 7	α_7	1.5237	0.250	1.281
Loading 8	α_8	1.8139	0.250	1.406
Loading 9	α_9	2.0914	0.250	1.517
Loading 10	α_{10}	2.3523	0.250	2.851
Standard Deviation	σ_α	0.0442	0.100	0.056
Factor B				
Loading 0	β_0	-0.0181	0.300	0.051
Loading 1	β_1	0.2211	0.300	0.083
Loading 2	β_2	0.3679	0.300	0.125
Loading 3	β_3	0.4424	0.300	0.152
Loading 4	β_4	0.4612	0.300	0.167
Loading 5	β_5	0.4370	0.300	0.181
Loading 6	β_6	0.3817	0.300	0.192
Loading 7	β_7	0.3032	0.300	0.203
Loading 9	β_9	0.1074	0.300	0.210
Standard Deviation	σ_β	0.0334	0.100	0.449
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{η_0}	0.0061	0.005	0.011
1Q Ahead	σ_{η_1}	0.0021	0.005	0.010
2Q Ahead	σ_{η_2}	0.0004	0.005	0.010
3Q Ahead	σ_{η_3}	0.0019	0.005	0.010
4Q Ahead	σ_{η_4}	0.0001	0.005	0.010
5Q Ahead	σ_{η_5}	0.0025	0.005	0.009
6Q Ahead	σ_{η_6}	0.0019	0.005	0.010
7Q Ahead	σ_{η_7}	0.0011	0.005	0.009
8Q Ahead	σ_{η_8}	0.0001	0.005	0.009
9Q Ahead	σ_{η_9}	0.0014	0.005	0.010
10Q Ahead	$\sigma_{\eta_{10}}$	0.0028	0.005	0.0001

Table 7: Covid Params

Parameter	2020Q2	2020Q3	2020Q4
ϕ_b	0.004	0.004	0.004
ϕ_s	1.000	1.000	1.000
ϕ_i	0.170	0.170	0.170
ϕ_ν	-0.440	-0.440	-0.440
ϕ_{pm}	0.010	0.010	0.010
λ_0	1.000	1.000	1.000
λ_1	-0.130	-0.392	-0.011
λ_2	-0.104	-0.354	-0.105
λ_3	-0.141	-0.225	-0.295
λ_4	-0.107	0.172	0.279
σ_{c19}	11.686	11.673	9.780

Figure 1: DiscountFactor

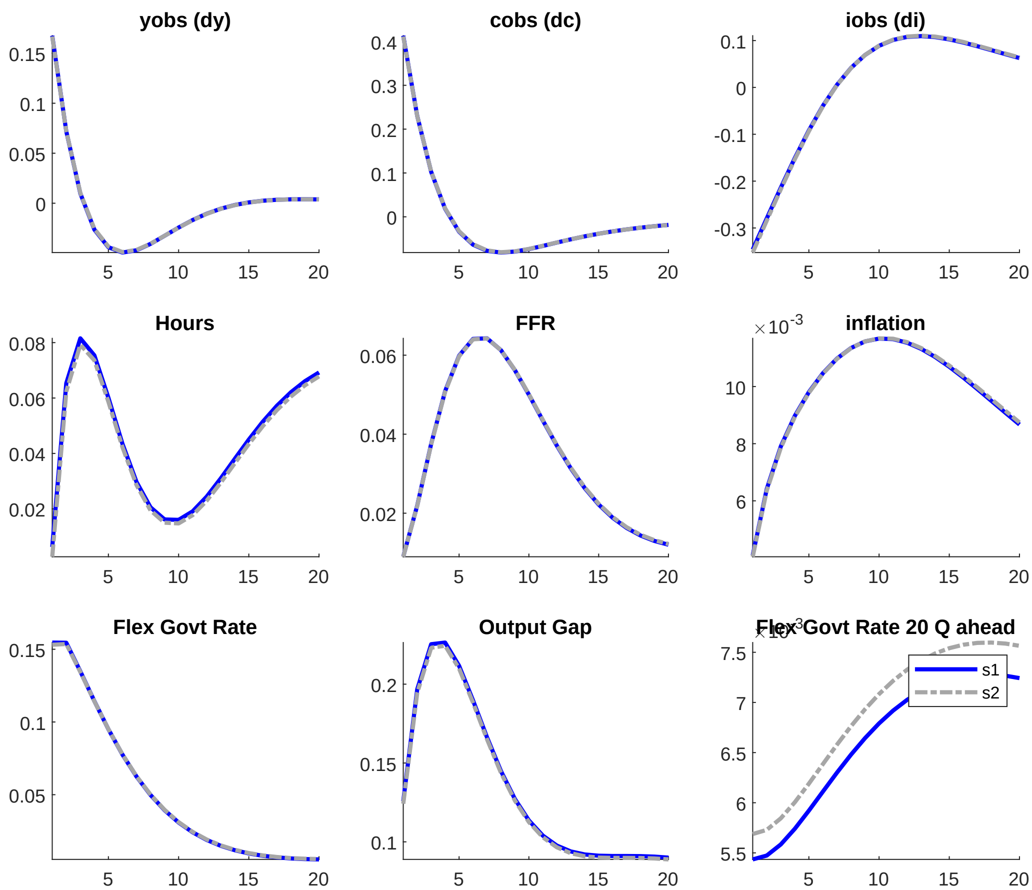


Figure 2: InflationDrift

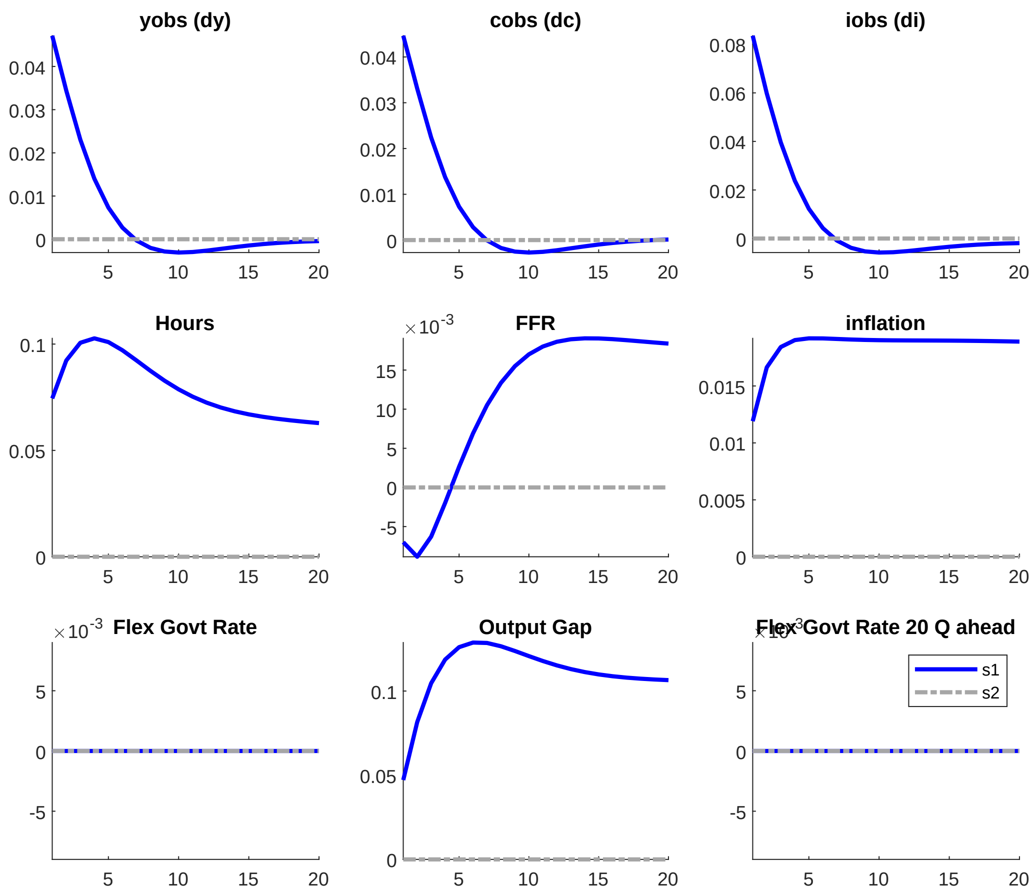


Figure 3: FactorA

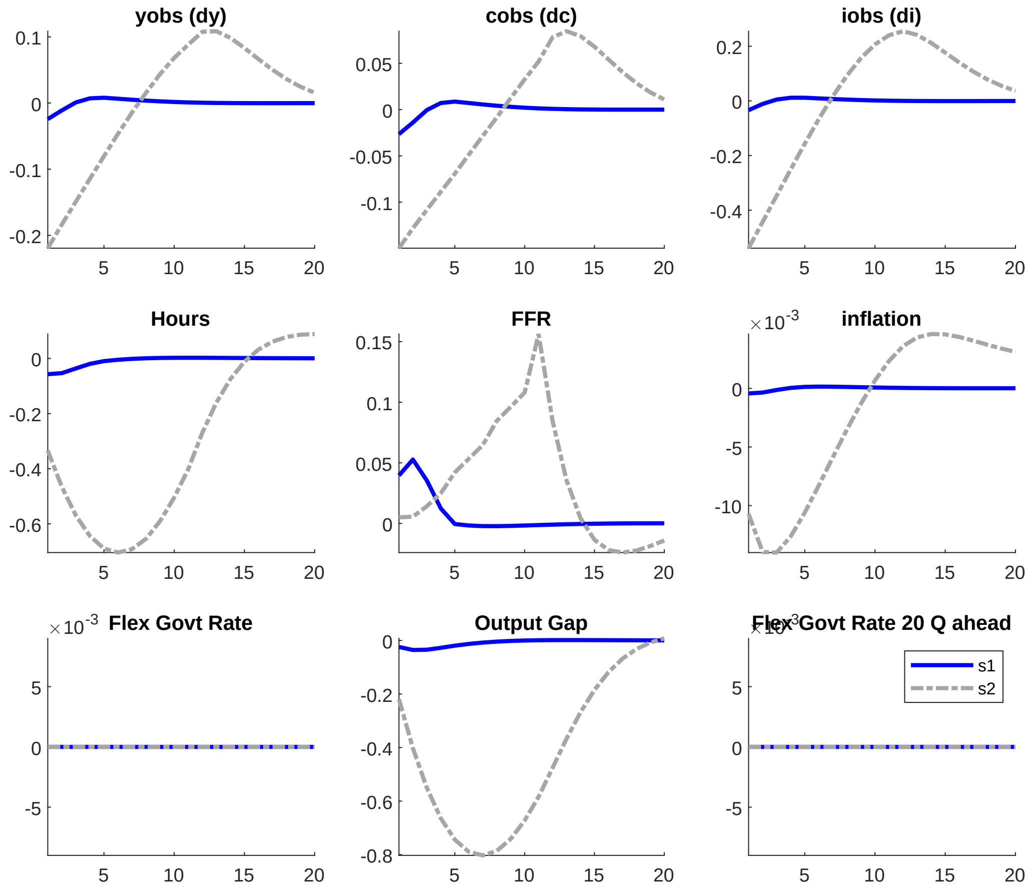


Figure 4: FactorB

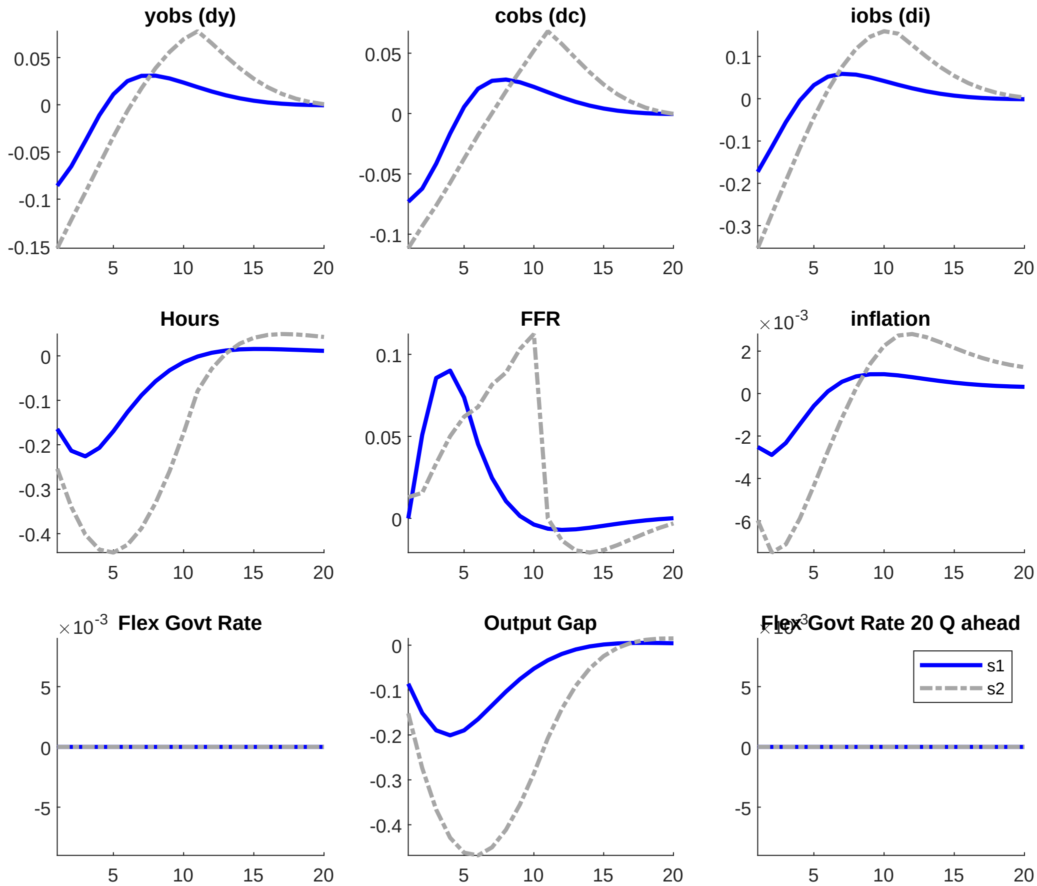


Figure 5: InvestmentShock

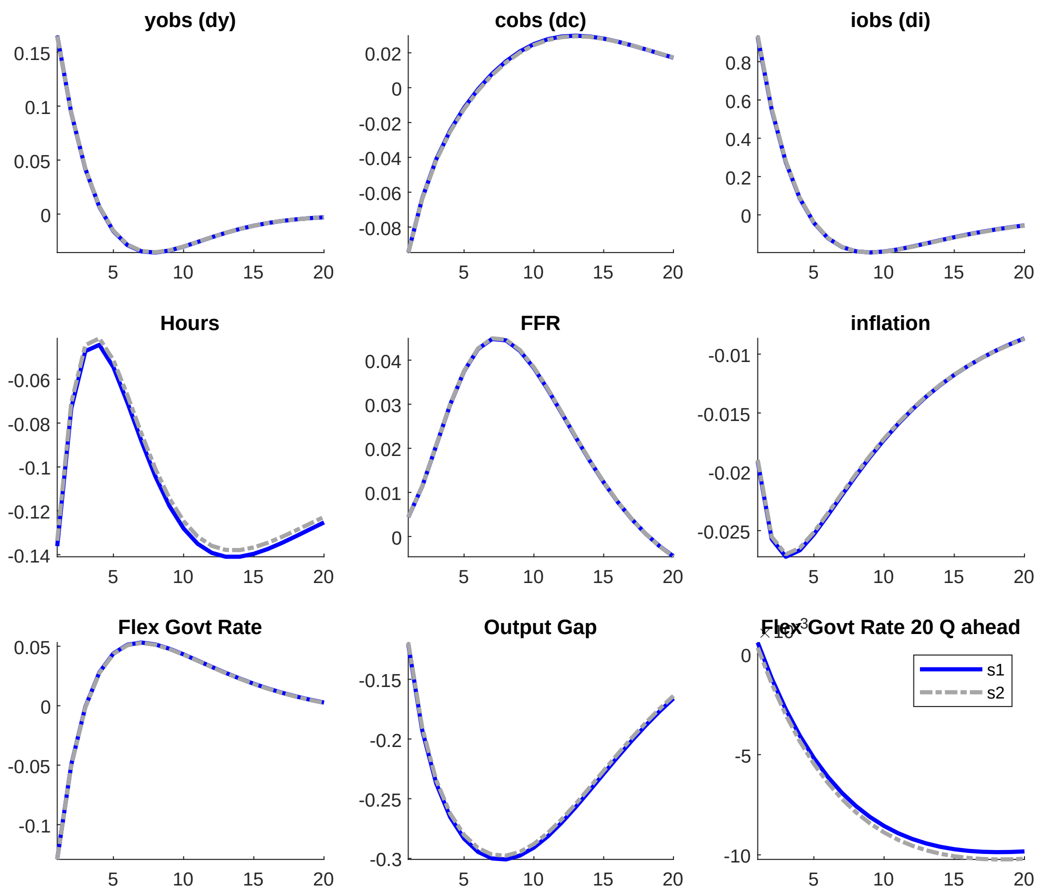


Figure 6: PermanentNeutral

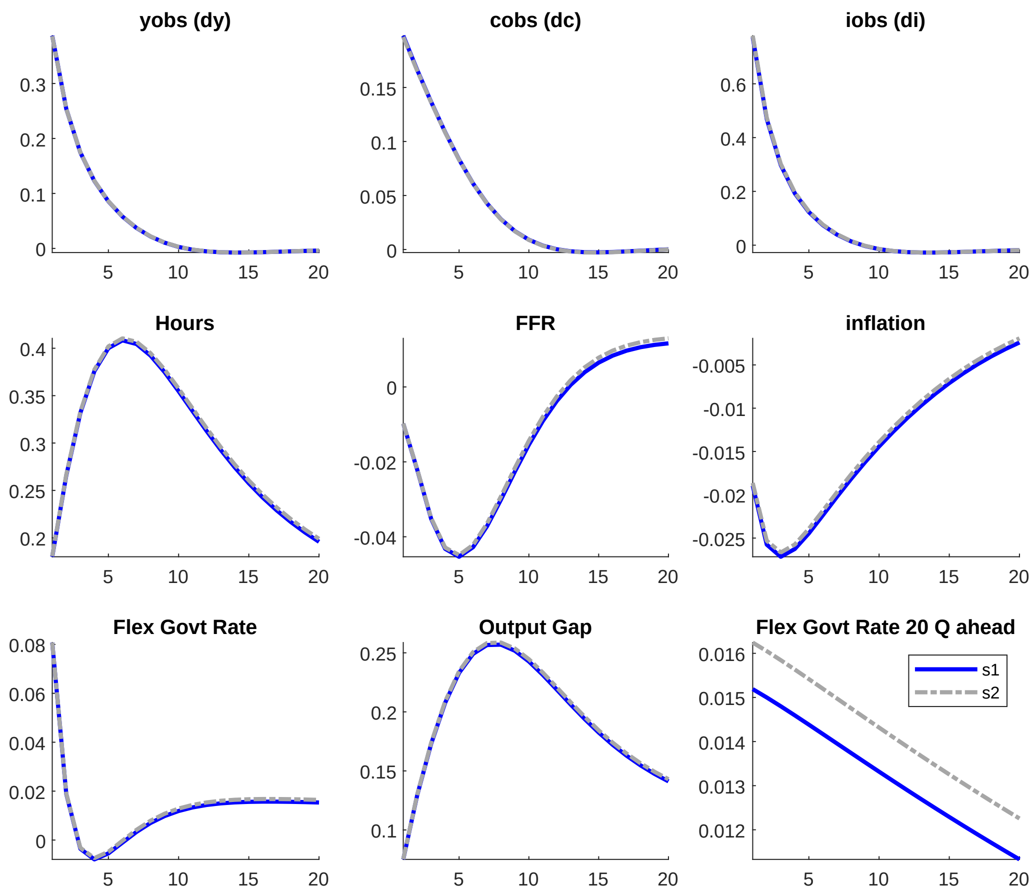


Figure 7: PriceMarkup

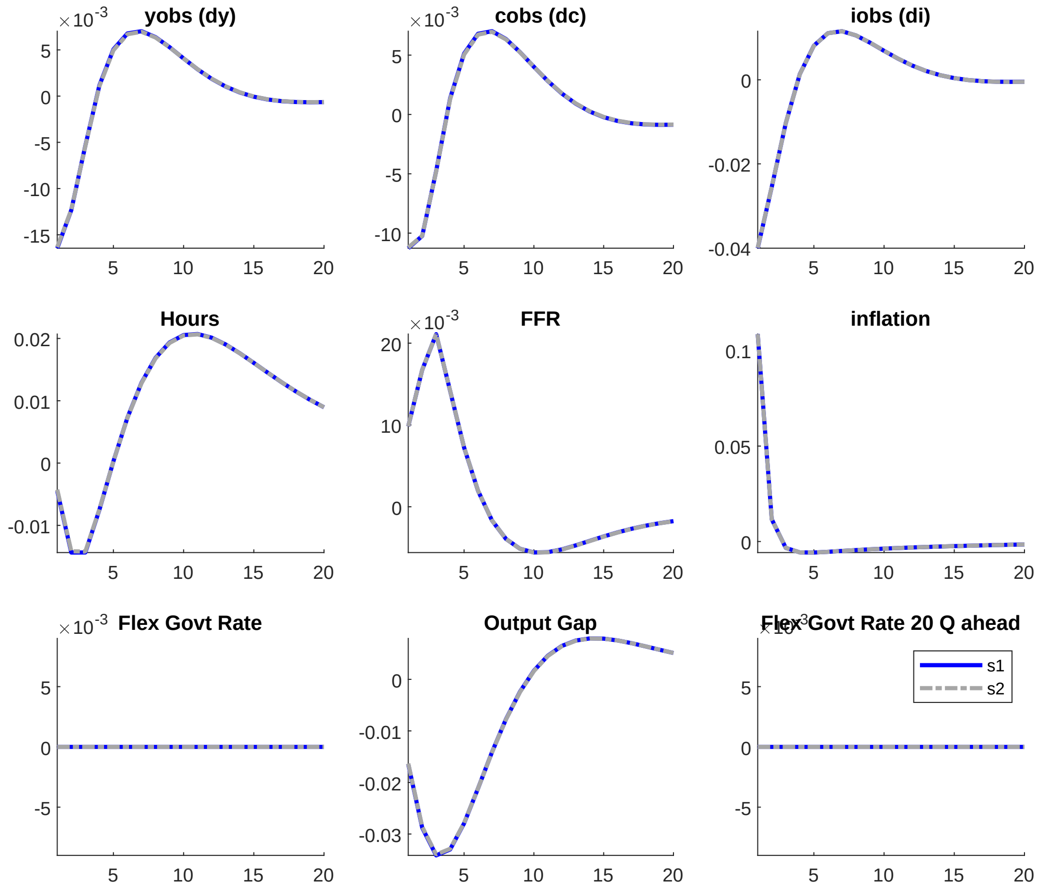


Figure 8: WageMarkup

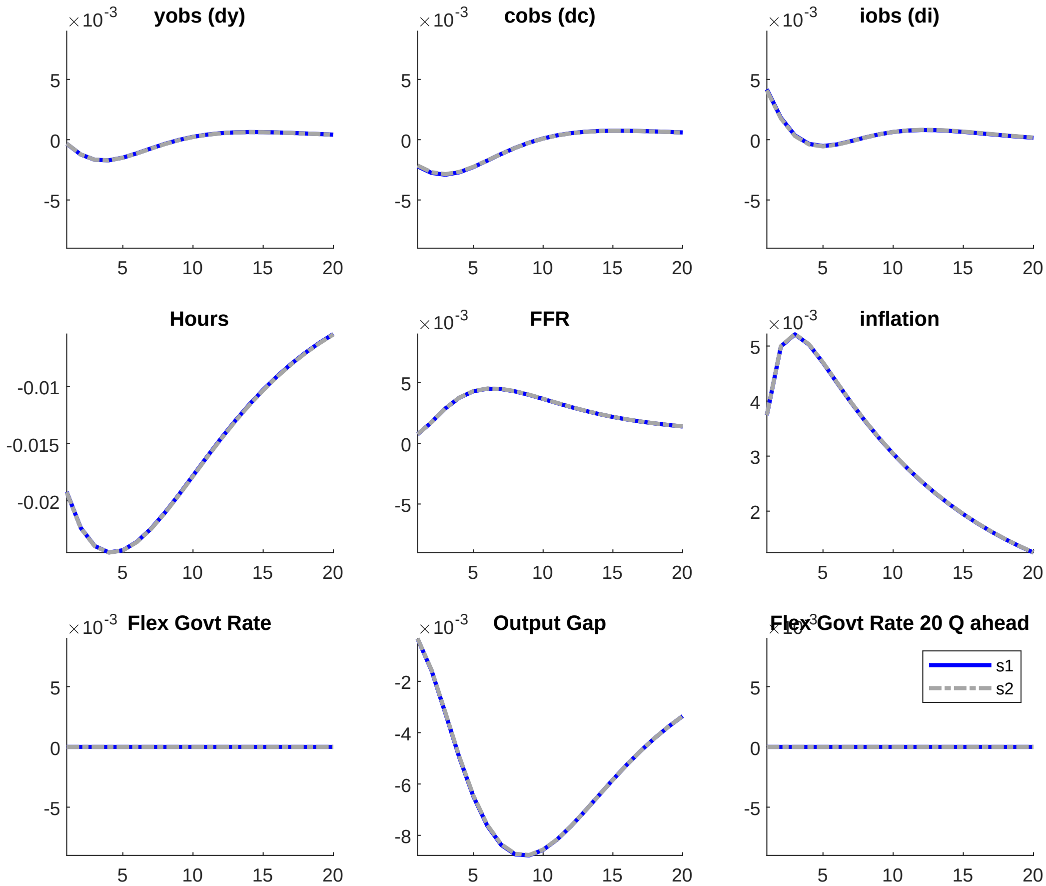


Figure 9: LiquidityPreference

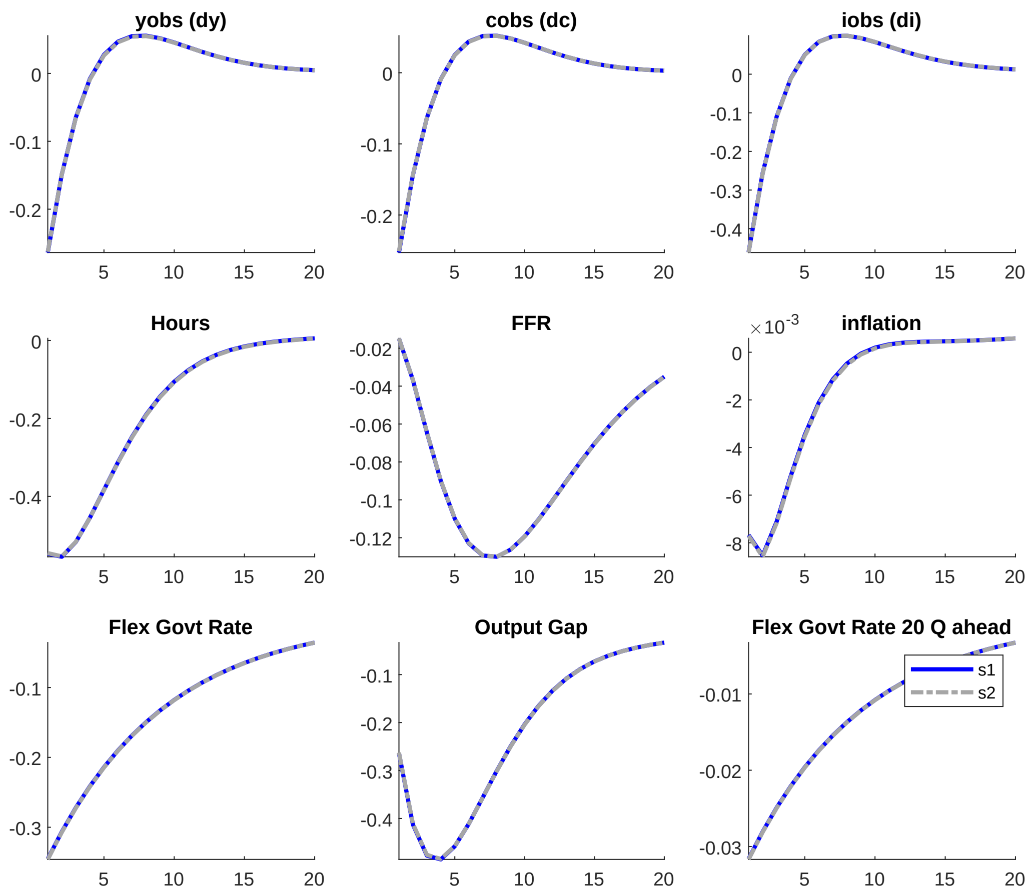


Figure 10: ISTS

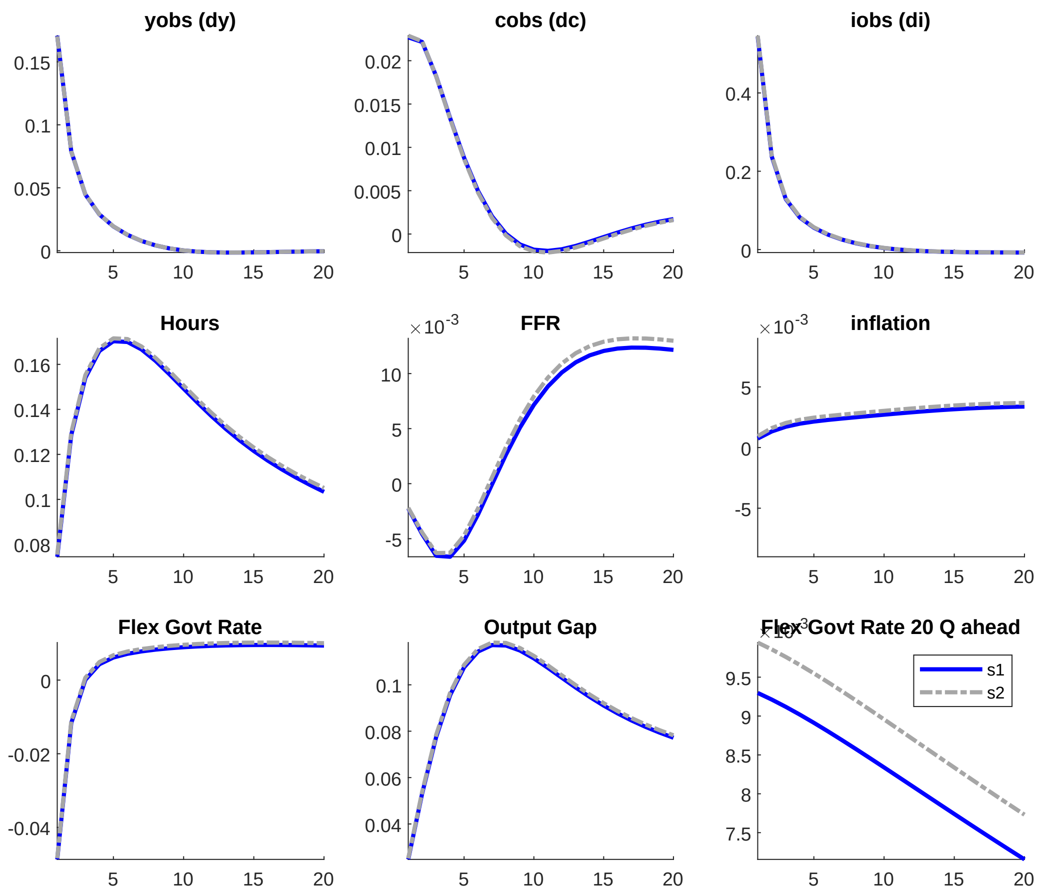
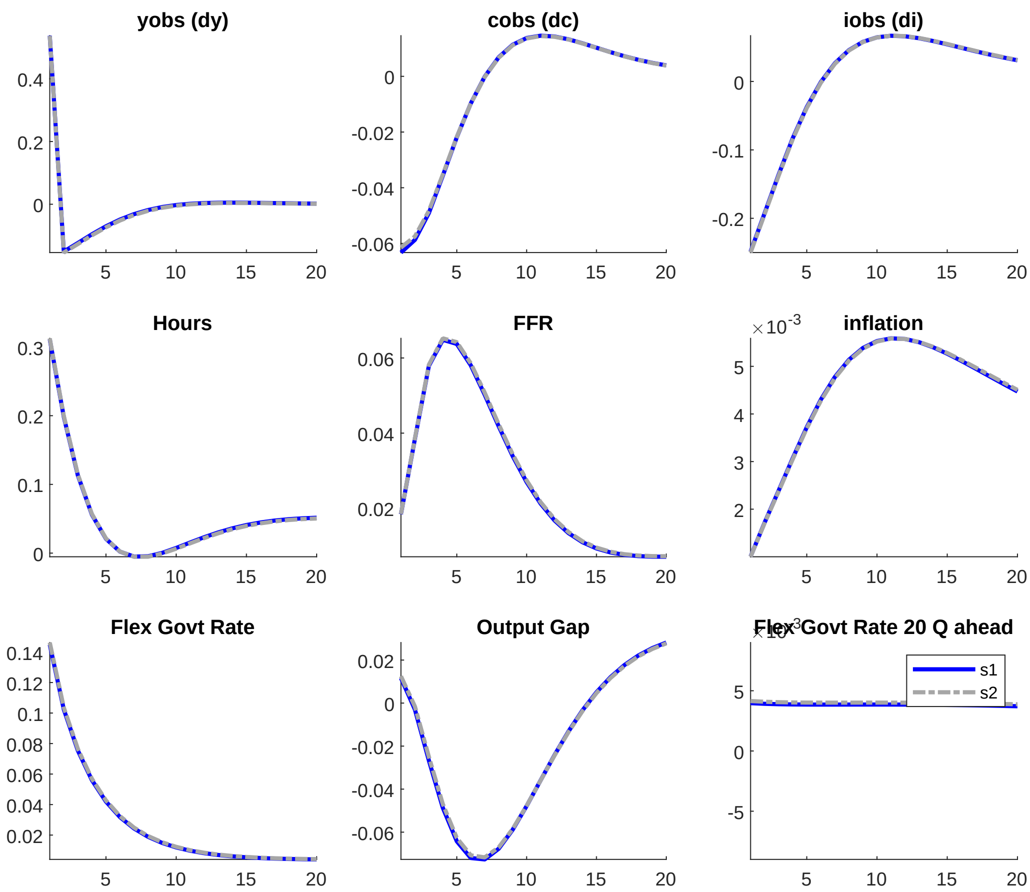


Figure 11: GovernmentSpending



References

Brave, S., J. R. Campbell, J. D. Fisher, and A. Justiniano (2012). The Chicago Fed DSGE Model. Chicago Fed working paper no. 2012-02. 1

Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics 12*, 383–398. 1, 7, 8

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012). Macroeconomic effects of Federal Reserve forward guidance. *Brookings Papers on Economic Activity Spring*, 1–54. 1

Campbell, J. R., F. Ferroni, J. D. Fisher, and L. Melosi (2019, December). The limits of forward guidance. *Journal of Monetary Economics 108*, 118–134. 1

Campbell, J. R., J. D. M. Fisher, A. Justiniano, and L. Melosi (2016). Forward guidance and macroeconomic outcomes since the financial crisis. In M. Eichenbaum and J. Parker (Eds.), *NBER Macroeconomics Annual 2016*, Volume 31. University of Chicago Press. 1, 32, 34

Christiano, L., M. Eichenbaum, and C. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45. 5

Ferroni, F., J. D. Fisher, and L. Melosi (2023). Unusual shocks in our usual models. Revision to Chicago Fed working paper no. 2022-39. 0, 1, 35

Fisher, J. (2015, March-April). On the structural interpretation of the smets-wouters "risk premium" shock. *Journal of Money, Credit and Banking 47*(2-3), 511–516. 3, 5

Fisher, J. D. M. (2006). The dynamic effects of neutral and investment-specific technology shocks. *Journal of Political Economy 114*(3), 413–451. 1

Gürkaynak, R. S., B. Sack, and E. T. Swanson (2005, May). Do actions speak louder than words? The response of asset prices to monetary policy actions and statements. *International Journal of Central Banking 1*(1), 55–93. 27, 35

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132 – 145. 1

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013, April). Is there a trade-off between inflation and output stabilization? *American Economic Journal: Macroeconomics 5*(2), 1–31. 25

King, R. G., C. I. Plosser, and S. T. Rebelo (1988a). Production, growth and business cycles I. the basic neoclassical model. *Journal of Monetary Economics 21*, 195–232. 1

King, R. G., C. I. Plosser, and S. T. Rebelo (1988b). Production, growth and business cycles II. new directions. *Journal of Monetary Economics 21*, 309–341. 1

Laséen, S. and L. E. Svensson (2011). Anticipated alternative policy-rate paths in policy simulations. *International Journal of Central Banking Fall*, 215–265. 1

Yi, K. and J. Zhang (2017, March). Understanding global trends in long-run real interest rates. *Federal Reserve Bank of Chicago Economic Perspectives 4*(2), 22–37. 32