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Interest Rates or Haircuts? Prices Versus Quantities in the Market for Collateralized Risky Loans

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Markets for risky loans clear on two dimensions - an interest rate (or equivalently a spread above the riskless rate) and a specification of the amount of collateral per dollar of lending. The latter is summarized by the margin or "haircut" associated with the loan. Some key models of endogenous collateral constraints imply that the primary equilibrating force will be in the form of haircuts rather than movements in interest rate spreads. Indeed, an important benchmark model, derived in a two-state world, implies that haircuts will adjust to render all lending riskless, and that a loss of risk capital on the part of borrowers has profound effects on asset prices. Quantitative analysis of a model of collateral equilibrium with a continuum of states turns these results on their heads. The bulk of the response to lenders' perception of increased default risk is in the form of higher default premia. Further, with high initial leverage, reductions in risk capital decrease equilibrium margins almost proportionately, while asset prices barely move. To the extent that one believes that it is a stylized fact that haircuts move more than spreads - as seen, for example, in bilateral repo data from 2007-2008 - this reversal is disturbing.

Keywords: leverage cycle, margins, financial crises, repo, risk, collateral, belief disagreements

JEL Classification: D53, E44, G00, G01

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Markets for risky loans clear on two dimensions - an interest rate (or equivalently a spread above the riskless rate) and a specification of the amount of collateral per dollar of lending. The latter is summarized by the margin or "haircut" associated with the loan. Geanakoplos (2012) stresses the strong association of major booms and busts in financial markets with substantial movements in haircuts, a phenomenon he calls the "leverage cycle". Interest spreads, on the other hand, show far more modest time-variation. Put differently, financial crises seem to be first and foremost periods in which the quantity of leverage falls, and only secondarily periods in which the price of leverage is high.

The primacy of haircuts rather than spreads as the equilibrating mechanism shows up clearly in the Gorton-Metrick (2012) data from bilateral repo markets during the dramatic 2007-2008 episode. Figure I displays eight panels, each of which corresponds to a class of relatively risky collateral assets. For each asset class the figure shows data on average haircuts and repo spreads from three periods: i) the pre-crisis first half of 2007; ii) the second half of 2007, which might be thought of as the period of the relatively contained "subprime crisis"; and iii) 2008 as a whole, the period of the general financial crisis centered on the shadow banking system.

Across all eight asset classes, bilateral repo in the first half of 2007 appears nearly riskless, with both haircuts and spreads close to zero. In the second period, there are modest increases in the repo spread, and larger (in some cases an order of magnitude so) but still not dramatic increases in haircuts. Finally, the third period shows truly striking spikes in harcuts, as high as 60% for some private label assets (see Gorton and Metrick for details). While the interest spread also rises sharply in period 3, this increase is easily an order of magnitude less than the rise in haircuts, with the highest repo spreads on the order of 200 basis points.

Table 1, which shows descriptive statistics, pooling the time series and cross section data, indicates that whether one focuses on means, variances (across time and across asset classes), or higher moments, variation in haircuts trumps variation in spreads in every respect. While Krishnamurthy et al (2014) cast doubt on the notion that the same dramatic increase in haircuts characterized the larger triparty repo market, one might just as readily interpret the complete disappearance of repo for many kinds of collateral in the triparty market as equivalent to a haircut of 100 percent.

At a broad brush level, the tendency for financial market scares to manifest themselves in a sharp drop in the equilibrium *quantity* of lending against risky assets without a commensurately sharp increase in the *price* of loans seems to go far beyond repo. For instance a key feature of the 2007-2008 crisis was the collapse of the large market for asset-backed commercial paper (Krishnamurthy, et al). Why did the quantity of securitized lending fall so dramatically, instead of contracting more modestly with a greater share of the adjustment to increased default risk occurring through spikes in interest rates on the risky loans?

There are a number of reason to regard the "spreads vs. haircuts" (or more generally, the "prices vs. quantities" in financial markets) question as one of first order importance. If financial market scares manifest themselves in increased haircuts without large increases in risky interest rates, the monitoring of spreads alone would provide insufficient warning of financial stress. Likewise, the policy implications for issues such as the lender of last resort function of central banks might well depend on the relative importance of spreads vs. haircuts as equilibrating mechanisms, especially if increased margins are not merely an equilbrium an equilibrium outcome but a reflection of a significant degree of "credit rationing" (Geanakoplos, 2010; Fostel and Geanakoplos, 2013). Second, margins and spreads are key (probably *the* key) statistics on which to evaluate the empirical relevance of models - indeed, entire classes of models - of collateralized risky lending. Finally, the question of how markets clear, particularly when non-price in addition to price mechanisms are at work, is at the very core of economists' underlying intellectual agenda.

Fostel and Geanakoplos (2015) provide an elegant theory of a collaterized loan market with heterogeneous beliefs in which haircuts are always sufficient to preclude equilibrium default and all lending is likely to occur at the riskless rate. Their version of the theory of collateral equilibrium is constructed in the context of "binomonial economies" in which there are only two continuation states. Simsek (2013) studies an otherwise nearly identical model in which there is a continuum of states and finds that the equilibrium features default in some states of the world, that collateralized loans consequently trade at spreads above the riskless interest rate, and that the variation in those spreads depends in interesting ways on both the downside risk and uncertainty perceived by lenders and the upside opportunities perceived by optimistic borrowers.

What Simsek's results do not tell us directly is whether the predictions of the theory of collateral equilibrium in the binomial case might nevertheless be a reasonable approximation to the truth. Should one expect dramatic gyrations in default premia in response to shocks such as increased fear on the part of lenders or a loss of risk capital on the part of borrowers? This paper offers quantitative theory aimed at answering that question. One might thus view our work as an examination of Fostel-Geanakoplos (2015) through the lens of Simsek (2013).

The plan of the paper is as follows. In Section II-a, we present a streamlined derivation of the Simsek model, with special attention to the equilibrium haircuts and interest rates on which Simsek did not explicitly focus, while referring the reader to Simsek (2013) for proofs of existence and uniqueness, etc. In Section II-B, we discuss the somewhat degenerate case of the Simsek model in which there are only two discrete states. We show that in the two-state case the existence of any risky borrowing implies the asset must sell at the pessimist's valuation. We also show that there is another equilibrium at the pessimist's price that has lower leverage and zero default risk, with ex post consumption allocations for both agents that are the same as those that would be achieved with equilibrium risky borrowing. Thus the equilibrium with zero-value-at-risk haircut and no interest rate spread is "essentially unique" in the sense of Fostel and Geanakoplos (2015), and the result is effectively a weak Fostel-Geanakoplos nondefault theorem for the Simsek model (the stronger version, which Fostel-Geanakoplos obtain in the binomial economy when there is a continuum of agents, rules out the equilibrium trading of *any* risky loan contracts.) Though hardly surprising, the FG result for the Simsek model is a critical benchmark because it verifies that we are not dealing with two fundamentally incompatible enviroments - on the contrary, Simsek (2013) is a natural generalization of Fostel-Geanakoplos (2015).

Section III, which contains the main results of the paper, consists of quantitative theory, studying the behavior of the Simsek model in a laboratory-type setting with specific belief distributions and endowments carefully chosen to lay bare the fundamental mechanisms and to stress-test the model's predictions for margins and spreads. Representing beliefs by means of the simple triangular family of distributions, we compute equilibrium loan size, interest rates, margins, and the price of the risky asset, and present diagramatic representations of the underlying workings of the loan market equilibria that determine them. Not surprisingly, lenders' fear of increased defaults raises both interest rates and haircuts, and (as Simsek's logic already lead us to expect) the extent of the rise in the interest rate is greater, and the fall in leverage (rise in the endogenous haircut) smaller when the optimist's belief distribution is skewed towards positive events. However, nothing prepared us for the sharp variation in interest rates, and the extremely limited variation in haircuts, that the model generates in response to belief shocks. The quantitative predictions of Simsek in response to a "fear shock" are far from those of Fostel-Geanakoplos and far from what is seen in the data.

Surely, however, there are disturbances other than shocks to beliefs. Might these not help us better match the data? Most observers of the 2007-2008 crisis, not least Geanakoplos (2010), consider the loss of much of the wealth that constituted the optimistic borrowers' risk capital as the other most salient characteristic of the downward "loss-haircut" spiral. The stylized analogue of the loss of risk capital in the model is an adverse endowment shock. In the model, such a shock is anything but helpful. The wealth loss increases the interest rate and *reduces* the haircut more or less proportionately - i.e. a halving in risk capital leads to an approximate halving of the haircut (i.e. a doubling of leverage at the height of financial crisis!) The underlying reason for this is the fundamentally inessential nature of risk capital in the Simsek model in the first place. The optimist's endowment serves to reduce his desired borrowing, thereby reducing the likelihood of default and taking some pressure off of the interest rate. However, because the default premium does the heavy lifting, there is no need for loans to be overcollateralized in this model. Ineed, there is no analytical reason that loan to value ratios cannot exceed 100 percent (as they in fact sometimes did in nonconfirming mortgage markets). Simsek, the natural extension of Fostel-Geanakoplos beyond the binomial case, in a sense turns the FG results on their head. Instead of the simple but powerful expression anchoring leverage to the borrower's risk capital and the lender's worst conceivable realization of the continuation value of the collateral, we have an "endogeneous haircut" in which risk capital plays primarily a mechanical role. The fundamental economics is about the optimist's trading off his perceived wedge between the market price and the true value of the collateral, on the one hand, and the pessimist's demand for an "excessive" default premium on the other.

Section IV discusses the implications of the counterfactual predictions of the Simsek model for the leverage cycle and for financial theory in general. We suspect that many of the fascinating theoretical results about the leverage cycle that Geanakoplos and his coauthors have generated are largely independent of their foundations in collateral equilibrium, but should perhaps be constructed on alternative principles, probably incorporating counterparty risk on top of uncertainty about the collateral alone.

II. Interest Rates and Haircuts in the Simsek Model

a) The Simsek Model With General Continuous Belief Distributions

Except where noted explicitly we follow closely the derivations in Simsek (2013). The model has two dates {0,1} and two types of risk neutral agents {o, p}, denoting optimists and pessimists, respectively. There is a continuum of each type of agent. There are two assets - a risky asset that we will call a tree, and a consumption good that we will call fruit. Fruit can be stored at a constant real return of 0, so that it functions as a riskless asset; we will sometimes call it "cash". Agents receive their endowments of the assets at date 0 but consume at date 1 only. Unlike Simsek, who has a third set of agents that are endowed with the risky asset and the sole sellers of it, we assign the initial endowments of trees (normalized to unity in the aggregate) to the pessimists, who are also endowed with cash. Without loss of generality, we endow the optimists with cash only. The important thing is that they use that fruit (which we intepret as their risk capital), along with fruit borrowed from pessimists, to buy trees. Pessimists are both lenders of fruit and sellers of trees. They cannot, however, bundle these activities - i.e. provide the buyers of their trees with cash financing. The two activities occur in separate competive securities markets.[1]

[1] Simsek makes two assumptions concerning the size of the cash endowments of the two groups that guarantee that the optimists will hold all the trees in equilibrium *and* that this can be accomplished only with at least some risky borrowing. These together ensure that the set of possible equilibrium prices for the risky trees will lie strictly

Next we turn to the characterization of the beliefs of the two kinds of agents. The optimist first and foremest believes that the expected payoff from the trees is higher than does the pessimist. Sufficient conditions for the existence and uniqueness of the solution to the priincipal agent problem described immediately below also require the relative optimism of the optimist to be increasing in the state s. This can be expressed either in terms of increasing vertical distance between the two agents' inverse CDFs as s gets large, or equivalently in the non-crossing of the hazard conditions. The intuition can be understand either through the proofs contained in Simsek's appendix or via our quantitative examples in which the hazard function and the inverse CDF are exhibited graphically in each case.

The work of Geanakoplos and his coauthors on "collateral equilibrium" is based on the application of competitive general equilibrium theory to commodities consisting of contracts - in the case of the "simple debt contracts" which are the sole contracts in the current paper, representable by ordered pairs consisting of an interest rate and an amount borrowed per unit collateral. Except in the limiting case where the price of trees falls to the pessimists' level , pessimists confine their period zero activities to storage and lending. If the contractual interest rate on a loan of size b collateralized by one tree is r, the payment received by the lender in state s is $\min[s,\ \phi]$, where $\phi = (1 + r)b$. In state s, the lender either receives his full repayment ϕ or the salvage value s, whichever is less. Thus under risk neutrality, arbitrage between storing and lending establishes the size of the loan collateralizable with one tree as $\mathbb{E}_p[\min(s, \phi)]$.

Simsek reformulates the determination of collateral equilbrium as a principal-agent problem, albeit with one key additional step. The optimization problem faced by optimists can be written as:

between the optimist's full price (the maximum he would pay for the asset, which is the integral of the possible payoffs weighted by his perceived probabilities and discounted at the riskless rate) and the price at which the pessimist's short sale constraint ceases to bind and he is marginally willing to hold the trees. Although this will hold in almost all of our examples, we make neither assumption a priorri, as we find the limiting cases in which a) the optimist can purchases the entire supply of trees with riskless borrowing only and, more importantly b) the opposite case in which the price must fall to the level at which the pessimist is willing to go long in trees to be of some interest (see in particular Section II. b, where we discuss the version of the Fostel-Geanakoplos nondefault result that holds in our model).

$$\max_{(a_1,\phi)\in\mathbb{R}^2_+} a_o\mathbb{E}_o[s] - a_o\mathbb{E}_o[\min(s,\phi)] \tag{1}$$

$$\text{s.t. } a_o p = n_o + a_o\mathbb{E}_p[\min(s,\phi)] \tag{2}$$

In words, the optimist chooses an amount a_o of risky trees to purchase with collaterized loans (in addition to his endowment of fruit) by maximizing his expected payoff from the trees net of his expected debt repayment, subject to a budget constraint that takes the price of trees as given and the interest rate as increasing in borrowing per tree in order to satisfy the lender's "participation constraint" $b = \mathbb{E}_p[\min(s,\phi)]$. How do we know that the participation constraint holds with equality? Perhaps the pessimist instead receives some surplus $\mathbb{E}_p[\min(s,\phi)] - b$. The answer (which happens to represent that key step mentioned above) is that the expected return on collateralized lending must be precisely zero (or more generally, the return on storage).

Importantly, because he perceives less lower tail risk, the optimist believes that his expected loan repayment is greater than the pessimist's expectation; $\mathbb{E}_o[\min(s,\phi)] > \mathbb{E}_p[\min(s,\phi)]$. As we will soon see, the optimist's belief that the system is rigged against him and that he must pay an excessive default premium will play a crucial role in discouraging the optimistic from borrowing as much as he otherwise might, and in depressing the price of the risky asset.

Simsek proves that, under the above assumptions, the solution to the principal-agent problem and a characterization of the associated collateral equilibrium is characterized by the following equation, which implicitly determines a bankruptcy threshhold or "loan riskiness" $\bar{s}$:

$$p = p^{opt}(\bar{s}) \equiv \int_{s^{min}}^{\bar{s}} s dF_p + (1 - F_p(\bar{s})) \int_{\bar{s}}^{s^{max}} s \frac{dF_O}{1 - F_O(\bar{s})}$$

Following Simsek we will call the first equation, the *optimality condition. There is also an equilibrium condition:*

$$p = n_1 + \mathbb{E}_p[min(s,\bar{s})] \tag{3}$$

which we will refer to *as the market-clearing condition.* This says that the price of a tree is just covered by the risk capital of the optimist plus the maximal loan per tree that the pessimist is willing to provide. Note that the pessimist's participation constraint appears both in the decision of the optimist represented by the optimality curve, and in the market clearing condition, which combines the otpimist's budget constraint and the pessimists' participation constraint. In the next section of the paper we will plot the two curves and find the equilibrium at their intersection, solved numerically for exogenously determined values of the endowments and subjective probability distributions.

A new way of looking at this price equation emerges from the following thought experiment. The first term represents the value of a security entitling its owner to the salvage value of a tree in bankruptcy. Suppose the lender cedes this security to the borrower in exchange for a higher "zero recovery" interest rate. Then the relevant disagreement is fully characterized by the pessimist's higher default probability. The second additive piece, $(1 - F_p(\bar{s})) \int_{\bar{s}}^{s^{max}} s \frac{dF_O}{1-F_O(\bar{s})}$, represents optimists' valuation of the fruit he keeps in nonbankruptcy states, taking into account that it is on the margin purchased with money borrowed from someone with whom there is a fundamental disagreement about bankruptcy risk. If there were no default disagreement, given the risk neutrality of both agents the optimist would discount at the riskless rate of zero, and that upside piece would be simply $\int_{\bar{s}}^{s^{max}} sdF_{O.}$Instead the optimist sees the appropriate breakeven interest rate as $(1 + r^{opt}) = [1 - F_o(\bar{s})]^{-1}$, while the pessimist requires the higher rate $(1 + r) = [1 - F_p(\bar{s})]^{-1}$; thus the optimist discounts his winnings in the nondefault state by $\frac{1-F_0(\bar{s})]}{1-F_p(\bar{s})} > 1$, in Simsek's terminology the optimist's *perceived* [gross] *interest rate*. From the point of view of the optimist it might be thought of as an "unfairness" measure. In all of our numerical exercises we will present the statistic $0 \leq \frac{1-F_p(\bar{s})]}{1-F_o(\bar{s})} \leq 1$,which we refer to as the "agreement ratio". When this ratio is unity, both parties are confident that loans will be paid in full, while a ratio of zero implies that the pessimist (but not the optimist) is certain that default will occur.

Finally, Simsek rewrites the optimality condition in terms of conditional expectations and provides a nice compact "price equation":

$$p = F_p(\bar{s})\mathbb{E}_p[s|s < \bar{s}] + (1 - F_p(\bar{s}))\mathbb{E}_o[s|s > \bar{s}].$$

This "price equation" is however not a reduced form solution for the asset price. Although our focus is shifted towards the equilibrium price of the tree rather than the choice of optimal loan size $\bar{s}$ for a parametrically given asset price, it is mathematically equivalent to the optimality equation as $\bar{s}$ is still an endogenous variable, pinned down only by coupling the optimality condition with the equilibrium condition. This has an important implication. A key theme in Simsek is the distinction between upside and downside disagreement. Simsek is normally quite clear that whether or not the disagreement is "downside" depends on whether or not it concerns default states, - i.e. states for which $s < \bar{s}$. However, it can be easy to forget that upside versus downside, though it has much to do with the shapes of the belief distributions, cannot be inferred from looking at the agents' pdfs alone. The crucial dividing line is the equilibrium loan riskiness $\bar{s}$, indicated in the quantitative exercises of the next section by the dotted vertical line, which depends on endowments in addition to beliefs.

b) Two Discrete States: A Fostel-Geanakoplos Result for the Simsek Model

Suppose, as do Fostel and Geanakoplos, that there are just two discrete states, H and L. The optimist believes state H will occur with probability $\pi_{H,o}$and the pessimist believes that it occurs with probability $\pi_{H,p}$. Substituting into the Simsek pricing formula eq. 3, we have

$$(1-\pi_{H,p})L + \frac{\pi_{H,p}}{\pi_{H,o}}(\pi_{H,o}H) = (1-\pi_{H,p})L + \pi_{H,p}H \tag{4}$$

The r.h.s is precisely the formula for pessimistic valuation. Thus, *if there is any risky borrowing, the asset must sell at the pessimist's price.*

The pessimist's price can be supported without any risky borrowing because pessimists are now indifferent between purchasing the asset from other pessimists and storing fruit. Thus in the two state model there is always an equilibrium that does not feature default, just as in Fostel-Geanakoplos. In the Simsek two agent model, there is non-uniqueness in the sense that an equilibrium at the pessimist's price can also be achieved with risky borrowing – e.g. optimists can hold all of the asset by borrowing at a risky rate just high enough to discount their expected payoff to the pessimistic valuation. However, just as in F-G, the non-uniqueness is "inessential" in the sense that not only the asset price, but the consumption allocations are invariant to whether or not the risky debt contract is actively in use. This is well illustrated by the following example.

Suppose the two states are $H = 1$ and $L = .5$. The optimist attaches probability .75 to H, while the pessimist believes there is a 75% chance of state L. Let pessimists be endowed (in the aggregate) with one tree and no fruits, and optimists be endowed with no trees but with risk capital in the form of .1 fruit. We verify that there is an equilibrium in which all of the risky tree is held by the optimist at the pessimist's fundamental valuation of .625 ($.25 \times 1 + .75 \times .5$). In this equilibrium the optimist borrows .525 at a risky interest rate of 14.29% - which gives the pessimist his required expected return of zero, since $.25 \times (1.1429 \times .525) + .75 \times .5 = .525$. In this equilibrium the optimist consumes $(1 - 1.143 \times .525) \cong .4$ in state H and zero in the default state L, while the pessimist consumes $1.143 \times .525 \cong .6$ in state H and .5 in state L.

Alternatively, suppose the optimists borrow only .4, which they will be able to pay back with certainty, at an interest rate of zero. Adding this borrowing to their risk capital of .1, optimists will now hold only 80% (.5/.6265) of the trees, but at the rock-bottom price of .6265 the pessimist is willing to take up the slack and hold the remaining 20%. The optimist consumes .4 (proceeds of .8 from his tree minus his debt of .4) in state H and 0 in state L, while the pessimist consumes .6 (proceeds of .2 from equity in the tree plus debt repayment of .4) in state H and .5 (.1 from equity in the tree plus the debt repayment of .4) in state L.

Thus we have established and illustrated a weak Fostel-Geanakoplos result for the two state case of the Simsek model. It is not impossible that risky debt is traded in equilibrium, but if it is, the equilbrium is "esssentially equivalent" to one in which only riskless loans are traded. Either way, the price is at the pessimist's fundamental valuation, and the consumption allocations for each of the two agents in each of the two states are the same whether or not risky borrowing is observed.

Fostel and Geanakoplos derive a stronger result that flatly rules out the equilbrium trading of risky debt contracts in a two-state model in which agents lie along a continuum from most optimistic to least optimistic. But even a case like ours when the equilibrium is only "essentially unique" there are at least two reasons to focus, in the two state case, on the equilibrium without risky debt. First, as F-G note, the use of collateral is costly; collateral must be evaluated, contractual arrangements clearly specified, etc. Second, if the required risk spread drives the asset price down to the pessimist's valuation, the reason d'etre for repo and other collaterized lending disappears. In a sense, this situation represents a sort of joint collapse of the market for the risky asset and the loans collateralized by it.

III. Quantitative Theory

We represent the beliefs of the two kinds of agents by means of the 3-parameter triangular family of distributions. We take the view that the agents have little precise knowledge, but have a sense of the worst and best case scenarios (a and b, respectively, with b strictly greater than a), as well as a "most likely" scenario c. Our benchmark case is a "right triangular distribution" with $b = c = 1$, so that the mostly likely outcome coincides with the best case scenario - which we take to be "business as usual" with minimal defaults on the underlying mortgages or accounts receivable which constitute the cash flows beyond the security. With a satisfying $0 \leq a < b, 1 - a$ has an interpretation as the agent's maximum conceivable loss. We will be particualrly interested in sharp reductions in the pessimist's a, what we call a "scare" or a "fear shock". When we depart from the right triangular case and allow (for the optimist in particular) $b > c$, the interpretation is that the optimist may have picked up assets at fire sale prices during a previous scare and is hoping to experience a capital gain as the scare dies down.

The density is given by: $$f(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x < c \\ \frac{2}{b-a} & \text{for } x = c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \\ 0 & \text{for } b < x \end{cases}$$

Figure II shows the belief distributions of the two agents, and the determination of equilibrium loan size, interest rate, and asset price. The optimist is confident that the asset will not fall in value by more than a couple of percent. The pessimist, on the other hand, believes the collateral could lose as much as a third of its value. Neither agent sees any upside potential at this point. Note the key role of $\bar{s}$. One this equilibrium value is determined in the diagram at the top right of Figure II (which was first constructed in Simsek's paper to determine the equilibrium asset price), we can easily find the equilibrium loan size, interest rate, and the "endogenous haircut".

Figure III contains our first comparative statics exercise, a "fear shock". The pessimist, who initially believes the collateral can lose at most a third of its value, comes to believe that the potential loss may be as large as fifty percent. Note the key result that the interest rate more than doubles, from 2.2% to 5.7% (more than twice the maximum spread seen in the Gorton-Metrick data for 2008), while the haircut barely budges. Figure IV repeats the exercise, with one crucial difference. The optimist is now depicted (throughout) as perceiving a substantial upside - he believes he may have purchased the collateral at a fire sale price and that it might rise in value by as much as 20%. The initial interest rate is higher, and the post-scare interest rate has risen more (in terms of basis points, though not in percentage terms) than in the absence of optimists' perceived upside potential. A comparison of Figures III and IV (panel by panel, horizontally) can also be interpreted as revealing the effect of an "excitement shock" - the optimist perceives an increase in upside potential holding constant his down side and that of the pessimist as well.

Figure V illustrates the effect of a loss of the optimistic borrower's risk capital. The optimist's endowment falls from 0.1 to 0.05, while beliefs are unchanged. When initial risk capital is small relative to borrowing, the level of borrowing is nearly unchanged by this wealth shock, and thus a halving of risk capital also nearly halves the haircut. As stressed in our introduction, this fact strikes us as quite telling, both about the workings of the model and about its essential unrealism.

IV. Concluding Remarks

We found that the quantitative predictions of Simsek's constinuous state model of collateral equilibrium, which constitutes a natural generalization of the Fostel-Geanakoplos two-state model, are wide of the mark relative to the Gorton-Metrick data as well more generally recognized stylized facts, according to which spreads move considerably less than margins. In particular, the predictions for a fear shock, and even more strikingly a loss of risk capital are highly counterfactual. The present paper should be interpreted neither as an assault on the empirical facts underlying the leverage cycle notion nor on the rich theoretical implications derived in the fascinating series of papers by Geanakoplos and his coauthors, most of which we suspect are not dependent on the precise way in which the determination of leverage is modeled. Similarly, our results do not challenge the analytical value of the results in Simsek (2013), which we continue to regard as the appropriate way to further study the prices vs. quantitites properties of collateral equilibrium problems. Rather, our paper might be characterized in part as an indication for resting the leverage cycle on a somewhat different foundation than collateral equilibrium, in which uncertainty and disagreement focus exclusively o the properties of hte colateral itself. Perhaps collateral equilibrium should be replaced or modified to account for the importance ascribed to counterparty risk in institutionally motivated studies of markets for collateralized loans.

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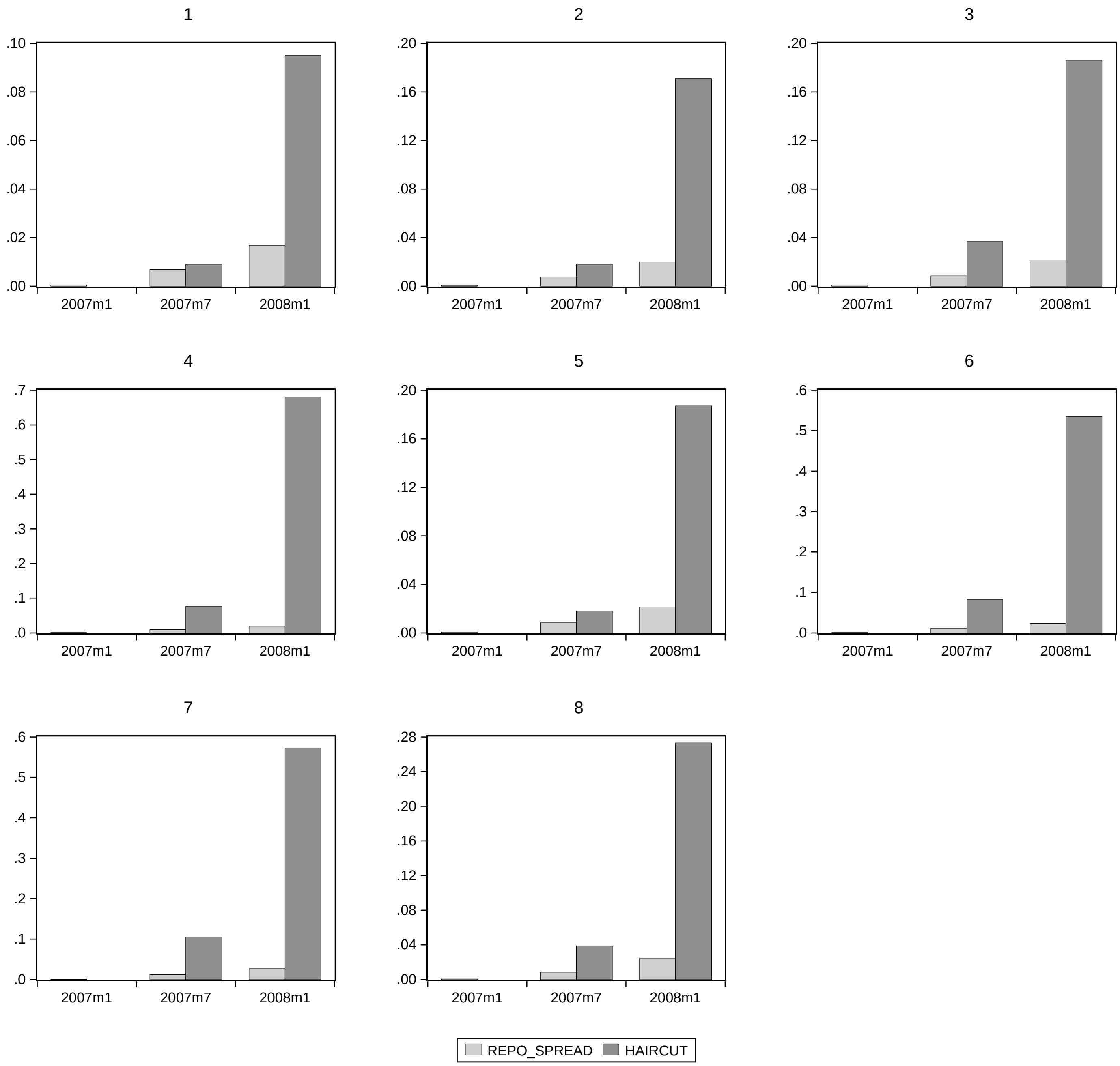
Table I: Descriptive Statistics for Haircut and Spread, Pooled

	HAIRCUT	REPO_SPREAD
Mean	0.128583	0.010502
Median	0.038000	0.008524
Maximum	0.680000	0.026839
Minimum	0.000000	0.000444
Std. Dev.	0.196709	0.009027
Skewness	1.767162	0.373831
Kurtosis	4.910127	1.751665

Source: Gorton and Metrick (2012)

Figure I: Mean Repo Spread and Haircut for Eight Asset Classes in Bilateral Repo Market: First Half of 2007, Second Half of 2007, All of 2008



Source: Gorton and Metrick (2012)

Figure II: Belief Distributions and Determination of Equilibrium

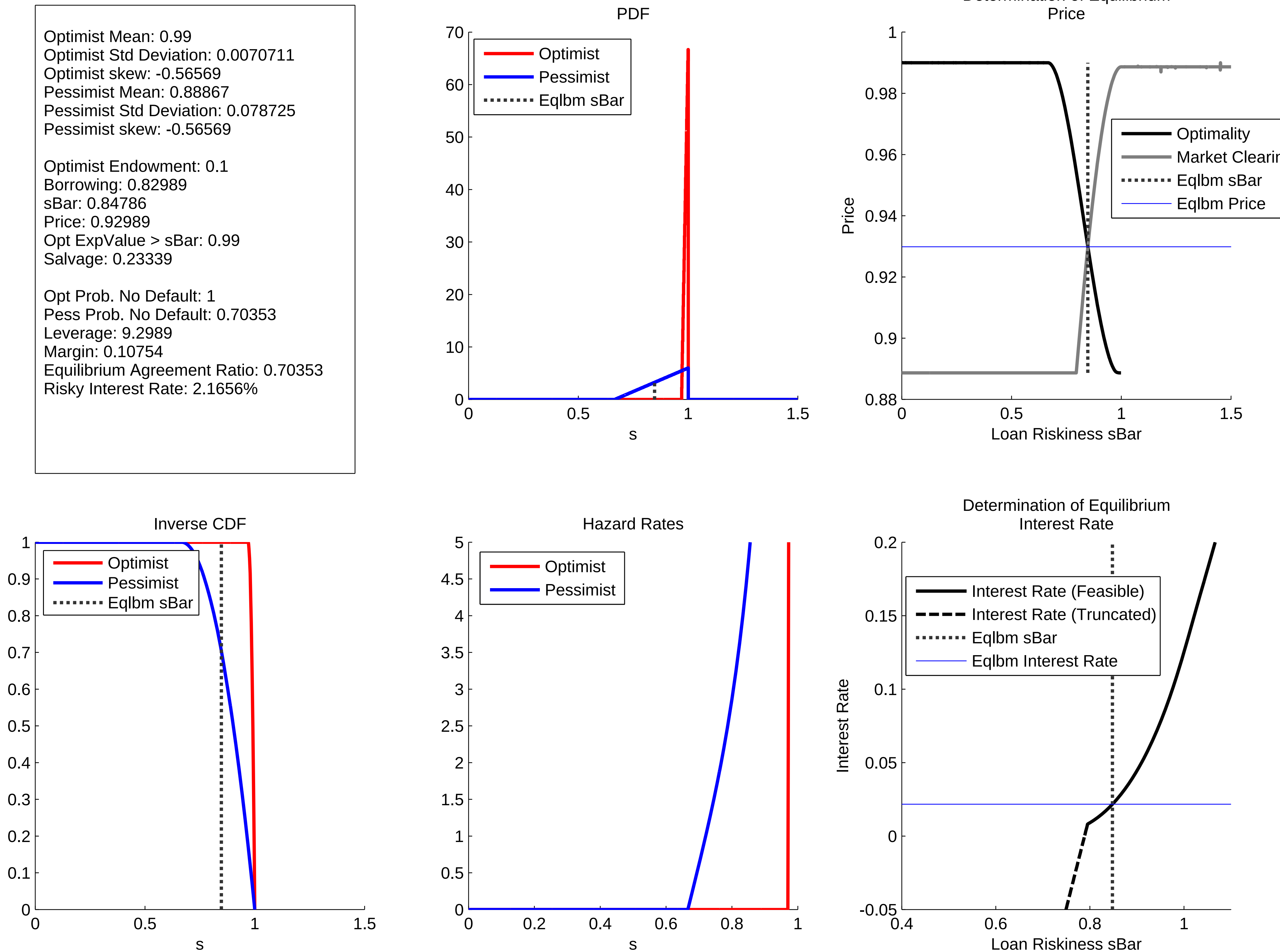


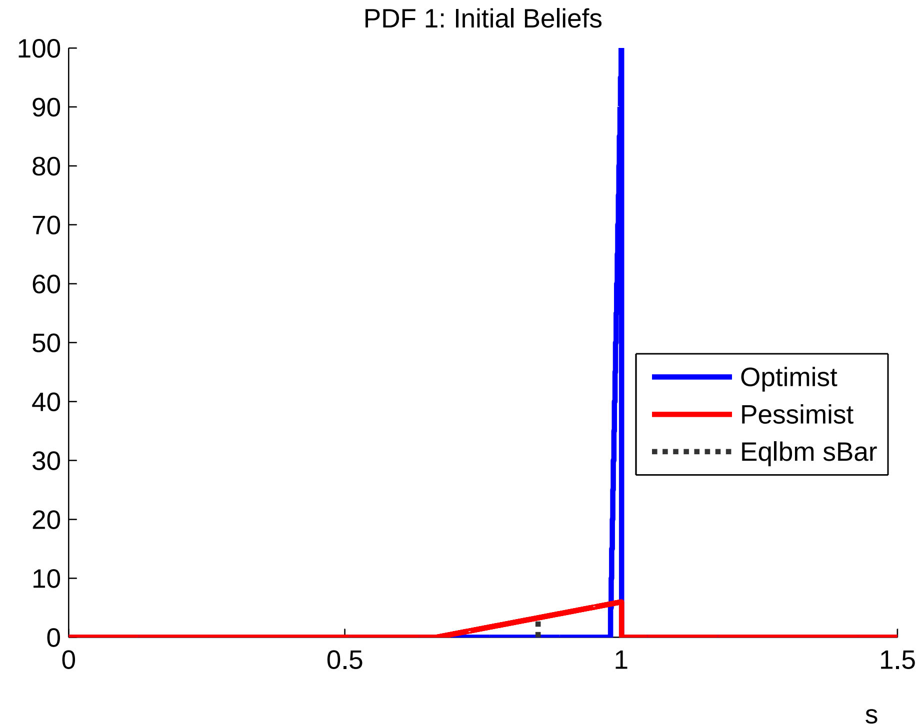
Figure III: A Fear Shock

Pre-Shock Beliefs

Optimist Mean: 0.99333
Optimist Std Deviation: 0.004714
Pessimist Mean: 0.88867
Pessimist Std Deviation: 0.078725

Optimist Endowment: 0.1
Borrowing: 0.83129
sBar: 0.84986
Price: 0.93129
Opt ExpValue > sBar: 0.99333
Salvage: 0.23895

Opt Prob. No Default: 1
Pess Prob. No Default: 0.69698
Leverage: 9.3129
Margin: 0.10738
Equilibrium Agreement Ratio: 0.69698
Risky Interest Rate: 2.234%



Post-Shock Beliefs

Optimist Mean: 0.99333
Optimist Std Deviation: 0.004714
Pessimist Mean: 0.83333
Pessimist Std Deviation: 0.11785

Optimist Endowment: 0.1
Borrowing: 0.77764
sBar: 0.82227
Price: 0.87764
Opt ExpValue > sBar: 0.99333
Salvage: 0.29696

Opt Prob. No Default: 1
Pess Prob. No Default: 0.58458
Leverage: 8.7764
Margin: 0.11394
Equilibrium Agreement Ratio: 0.58458
Risky Interest Rate: 5.738%

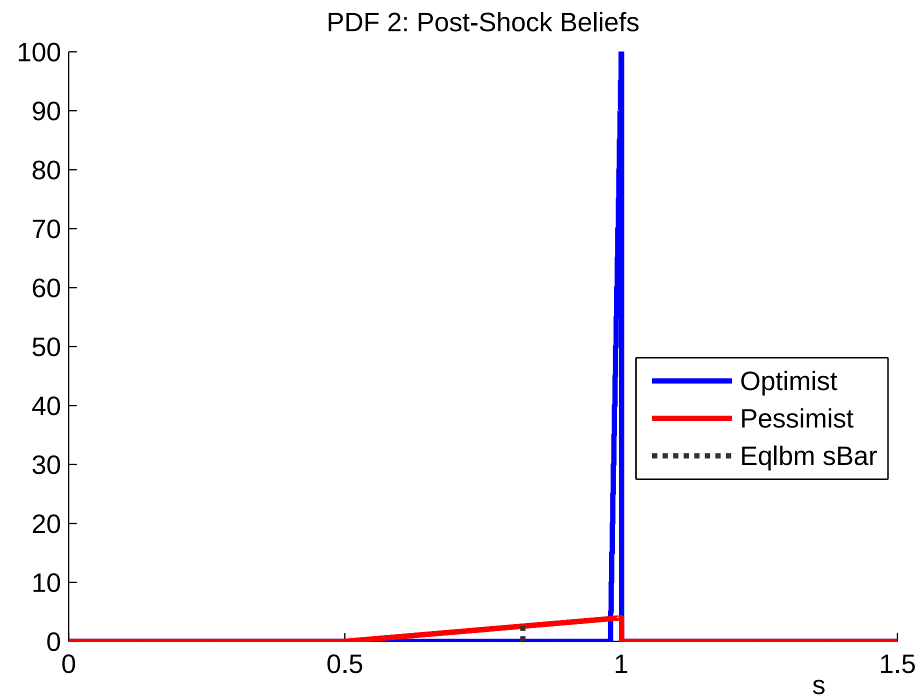


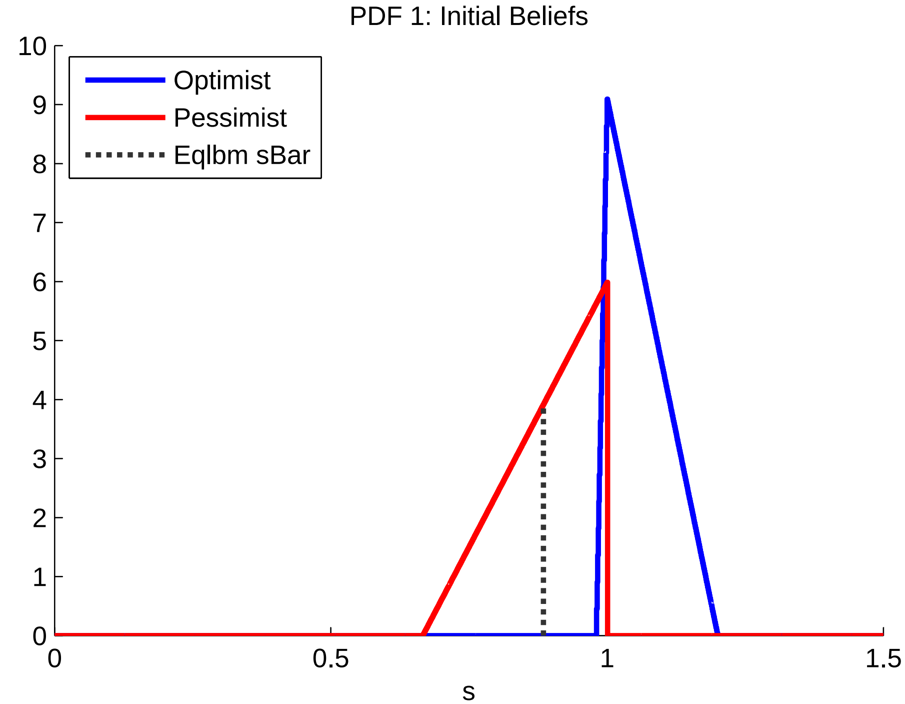
Figure IV A Fear Shock in Presence of High Upside for Optimist

Pre-Shock Beliefs

Optimist Mean: 1.06
Optimist Std Deviation: 0.049666
Pessimist Mean: 0.88867
Pessimist Std Deviation: 0.078725

Optimist Endowment: 0.1
Borrowing: 0.85351
sBar: 0.88481
Price: 0.95351
Opt ExpValue > sBar: 1.06
Salvage: 0.34844

Opt Prob. No Default: 1
Pess Prob. No Default: 0.57082
Leverage: 9.5351
Margin: 0.10488
Equilibrium Agreement Ratio: 0.57082
Risky Interest Rate: 3.668%



Post-Shock Beliefs

Optimist Mean: 1.0767
Optimist Std Deviation: 0.061418
Pessimist Mean: 0.83333
Pessimist Std Deviation: 0.11785

Optimist Endowment: 0.1
Borrowing: 0.79968
sBar: 0.86396
Price: 0.89968
Opt ExpValue > sBar: 1.0767
Salvage: 0.3935

Opt Prob. No Default: 1
Pess Prob. No Default: 0.47013
Leverage: 8.9968
Margin: 0.11115
Equilibrium Agreement Ratio: 0.47013
Risky Interest Rate: 8.039%

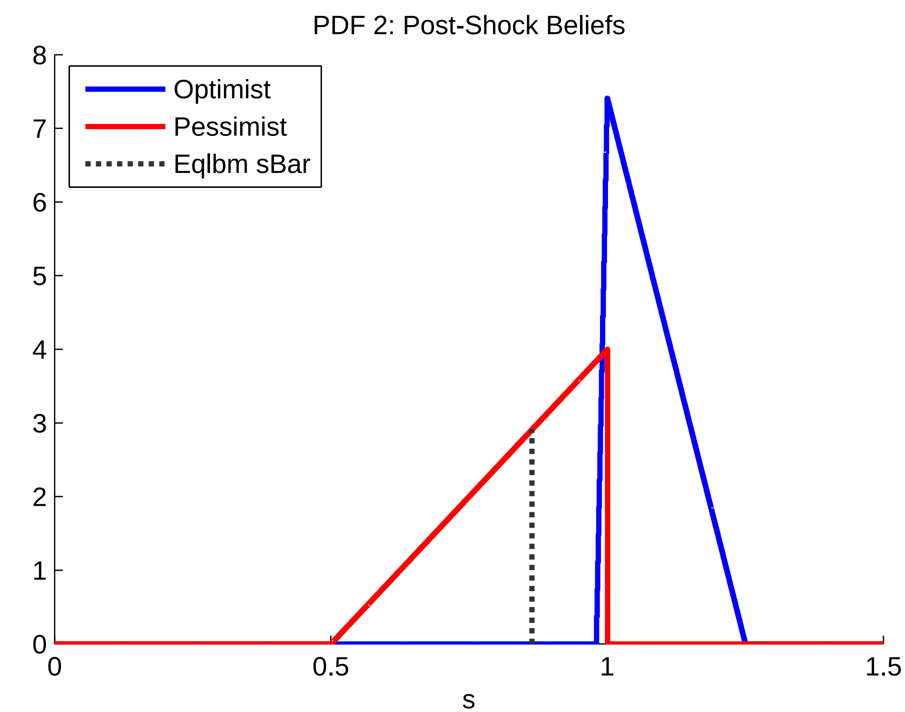


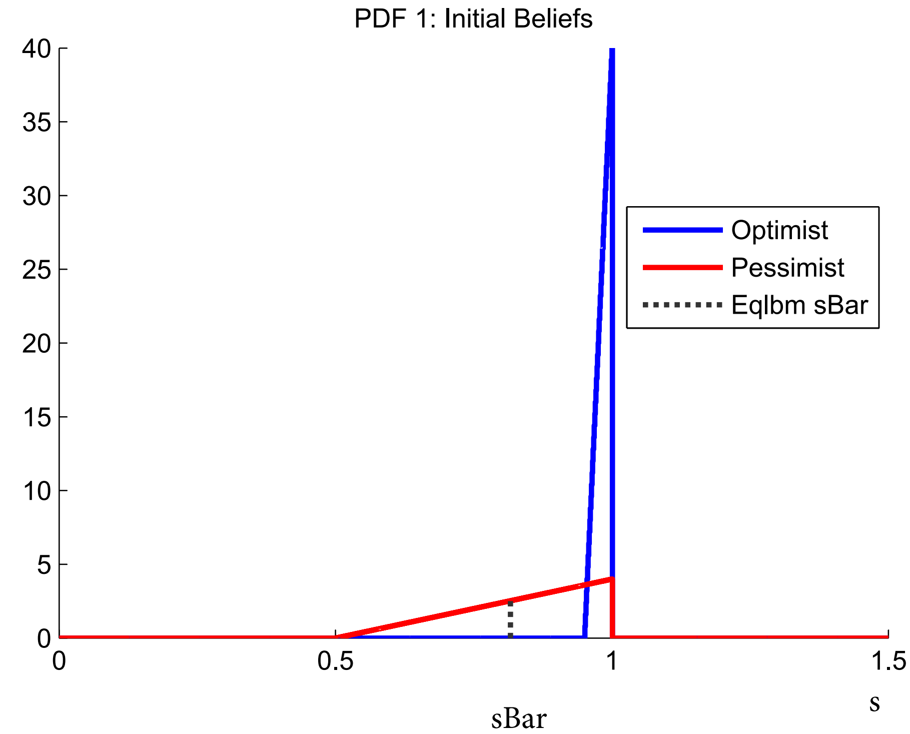
Figure V: Loss of Risk Capital

Pre-Shock: Risk Capital = 0.1

Optimist Mean: 0.98333
Optimist Std Deviation: 0.011785
Pessimist Mean: 0.83333
Pessimist Std Deviation: 0.11785

Optimist Endowment: 0.1
Borrowing: 0.77422
sBar: 0.81649
Price: 0.87422
Opt ExpValue > sBar: 0.9897
Salvage: 0.28486

Opt Prob. No Default: 1
Pess Prob. No Default: 0.59935
Leverage: 8.7422
Margin: 0.11439
Equilibrium Agreement Ratio: 0.59935
Risky Interest Rate: 5.459%

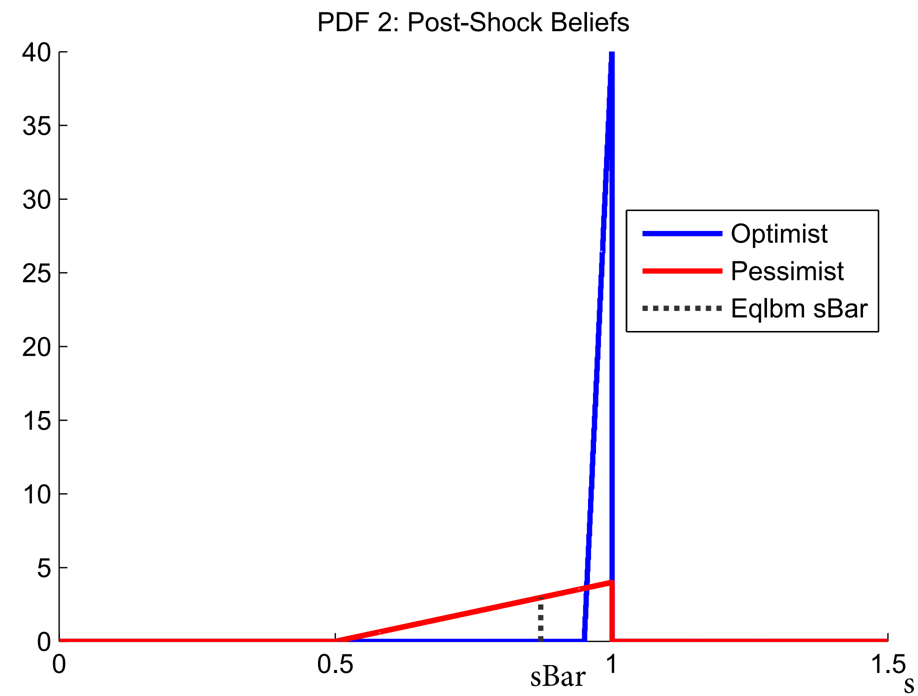


Post-Shock: Risk Capital = 0.05

Optimist Mean: 0.98333
Optimist Std Deviation: 0.011785
Pessimist Mean: 0.83333
Pessimist Std Deviation: 0.11785

Optimist Endowment: 0.05
Borrowing: 0.80319
sBar: 0.87162
Price: 0.85319
Opt ExpValue > sBar: 0.98333
Salvage: 0.41307

Opt Prob. No Default: 0.99988
Pess Prob. No Default: 0.44757
Leverage: 17.0639
Margin: 0.058603
Equilibrium Agreement Ratio: 0.44759
Risky Interest Rate: 8.520%



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