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A Theoretical Model of Leaning Against the Wind

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Abstract

Policymakers have long debated whether central banks should raise interest rates if they are concerned about a potential bubble, a policy known as *leaning against the wind*. In this paper, we develop a framework to study this question. We begin with the seminal work of Galí (2014) and argue that a small change in the model rules out his finding that raising rates amplifies bubbles rather than dampening then. More importantly, we argue that his setup does not provide any reason to intervene against bubbles. This leads us to consider an alternative model in which bubbles are credit-driven. In the simplest case, raising rates dampens bubbles but exacerbates the distortions bubbles introduce. But in this simple case, bubbles only cause harm when they arise, not when they burst. When we add default costs, so that bubbles also cause harm when they burst, raising rates increases welfare for sufficiently large default costs. Yet even when default costs are small, we find that a commitment to raise rates if a bubble persists can raise welfare *ex-ante* even though raising rates itself reduces welfare.

Introduction

Policymakers have long debated how to respond to asset bubbles. One option, advocated by Borio and Lowe (2002) among others, involves raising interest rates to dampen the bubble, a policy that has been dubbed *leaning against the wind*. An alternative approach, most associated with Bernanke and Gertler (1999), argues for waiting to see if asset prices collapse and then cleaning up, or intervening to stimulate the economy, if necessary. As the recent global financial crisis highlighted both the potential of a collapsing bubble to trigger a financial crisis and the limits on central banks in responding to such a crisis, policymakers have became more leery about the wait-and-see approach. This shifted the debate from whether to respond to bubbles to how to best respond to them. The debate between *lean vs clean* essentially evolved into a debate between *lean vs screen*, i.e., a debate between raising rates to dampen bubbles and using macroprudential regulation to curb lending against bubble assets with the aim of depriving bubbles of their fuel.

More recent work has gone beyond the notion that there are preferable alternatives to interest rate hikes and argued that leaning against the wind is in fact counterproductive. One example is the work of Galí (2014). He introduced monetary policy and nominal price rigidity into a model of a dynamically inefficient economy in which it is well known that bubbles can arise, and found that raising rates can amplify bubbles rather than dampen them. As another example, Svensson (2014) argues that raising rates does more harm than good given all the implications of tighter monetary policy, citing the Swedish Riksbank attempt to lean against a housing bubble as an example of where the policy arguably failed.

In this paper, we propose a theoretical framework to analyze whether and when leaning against the wind is productive. Does raising rates dampen bubbles or amplify them? If it dampens them, do higher rates increase or decrease welfare? We begin by arguing that raising rates dampens bubbles. As Galí (2014) observes, his model features multiple equilibria. In some equilibria, higher rates amplify bubbles; in others they dampen them. Galí argues that the equilibria in which higher rates amplify bubbles are more natural. By contrast, Dong, Miao, and Wang (2017) and Ikeda (2017), who study models that similarly feature multiple equilibria, focus on equilibria in which higher rates dampen bubbles and argue that those are more natural. We consider a variation of Galí's framework in which asset prices are uniquely determined, allowing us to sidestep the question of which equilibrium is most natural. Once we eliminate multiple equilibria, we find that raising the real interest rate unambiguously dampens bubbles.

Even after we modify Galí's model so higher rates necessarily dampen bubbles, his setup still poses a problem for the lean-against-the-wind view in that it implies no reason for policymakers to lean against bubbles. This is because the friction that allows bubbles in his model is dynamic inefficiency, and bubbles ameliorate this inefficiency. This observation leads us to modify Galí's environment so that bubbles arise not because of dynamic inefficiency but because of a different friction altogether. Specifically, we modify the economy he considers to be dynamically efficient, but we also introduce credit together with information frictions that interfere with lending. Bubbles arise when agents borrow and, against the interests of their lenders who cannot monitor them, buy risky assets and bid up their price. In this setting, asset bubbles

reduce welfare by crowding out productive activities that could have been funded instead. This feature implies that there may be scope for intervention against bubbles.

We find that in our model, even though raising rates dampens the bubble, it also exacerbates the under-funding of those productive activities crowded out by the bubble. Intuitively, contracting the economy leaves fewer resources either to buy assets or to finance productive activities, so both are crowded out. Hence, there is a sense in which leaning against a bubble indeed does more harm than good. However, the typical argument for intervening against bubbles is not to undo the distortions that arise while bubbles are present but to mitigate the harm bubbles cause when they burst. Our model can speak to this possibility, since in our model a fall in asset prices will drive those who borrowed against the bubble asset to default. The concern is that the losses lenders incur as a result will make them unable or unwilling to extend subsequent credit, leading to a fall in output. Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that financial crises were historically associated with a fall in GDP per capita of between 9 and 16%. Atkinson, Luttrell, and Rosenblum (2013) estimate the cumulative loss in output in the US from the most recent financial crisis was even larger. We can capture these effects in a reduced-form way by allowing for default costs that are proportional to what agents borrow. This specification implies that agents have fewer resources to consume the more that speculators borrow and then default on when asset prices fall, in line with what would happen if the collapse of the bubble resulted in a financial crisis. We confirm that raising rates can indeed make society better off if default costs are sufficiently large: Even if raising rates leaves fewer resources for productive activities, reducing the amount agents borrow against bubble assets and thus the amount they would default on if the bubble burst can make society as a whole better off.

Finally, we find that even when default costs are small, it might still be possible to increase welfare by committing to raise rates in the future if the bubble persists. Intuitively, if the bubble is likely to burst, a threat to act if it persists is desirable: Such a threat both mitigates the distortions due to the bubble and reduces the amount agents would default on if the bubble crashed, yet it is unlikely that the costly intervention will actually be necessary. Our model can therefore reconcile the seemingly conflicting views about the lean-against-the-wind approach. Raising rates to dampen a bubble is indeed costly, as emphasized by Svennson (2014). But dampening the bubble might mitigate the fallout if and when the bubble bursts, and a commitment by a central bank to raise rates in the future should a bubble persist may promote welfare *ex ante* even if raising rates is costly *ex post*.

Beyond these particular results, one of our contributions is to offer a simple framework that can be used to explore a host of issues related to asset bubbles. For example, since in our model credit plays a key role and is essential in allowing bubbles to arise, one can use our setup to compare macroprudential regulations such as leverage restrictions to lean-against-the-wind type policies. We comment on some potential extensions of our model in the Conclusion.

The paper is organized as follows. In Section 1, we revisit Galí's analysis of a dynamically inefficient economy. In Section 2, we describe an environment in which bubbles arise not because of dynamic inefficiency but because of information frictions. We then conclude in Section 3.

1 Dynamic Inefficiency, Bubbles, and Monetary Policy

We begin by reconsidering previous work on monetary policy and bubbles. We focus on a model that draws on the seminal work of Galí (2014). He introduced price rigidity into a dynamically inefficient economy that is well known to allow bubbles, and found somewhat surprisingly that tighter monetary policy could amplify bubbles. While this is also true in our model, we find that once we modify it so that asset prices are uniquely determined, higher rates unambiguously dampen bubbles. We also show how we can capture the effects of monetary policy in a reduced-form endowment economy without explicitly modelling the monetary transmission mechanism. Finally, we argue that if bubbles are due to dynamic inefficiency as in Galí's setup, there will be no reason to intervene against them. This motivates our shift in the second half of the paper to bubbles that are due to different frictions than dynamic inefficiency.

1.1 Dynamic Inefficiency and Bubbles

Consider an overlapping generations economy where agents live for two periods. For convenience, suppose agents only value consumption in their second period of life. That is, agents born at date t value consumption c_t and c_{t+1} at dates t and $t+1$ according to

$$u\left(c_t, c_{t+1}\right) = c_{t+1} \tag{1}$$

Agents are endowed with resources only when young. Let $e_t > 0$ denote the endowment of the agents born at date t. Endowments grow at rate $g > 0$, so agents who are born later are wealthier:

$$e_t = (1 + g)^t e_0 \tag{2}$$

Given their preferences, agents are only concerned with converting the goods they are endowed with when young into goods they can consume when old. We give them two options: They can either store their endowment to consume later, or exchange their endowment for assets. We posit a fixed supply of assets, normalized to 1, where each asset yields a constant dividend flow of $d \geq 0$ consumption goods per period. Galí assumed $d = 0$. We assume this initially as well. All assets are initially endowed to the old at date 0. Let s_t denote the amount of goods agents store at date t and x_t denote the amount they spend on assets. Then $s_t + x_t = e_t$. Old agents will sell any assets they own and consume the goods they trade their assets for together with any they previously stored. They will therefore consume $c_{t+1} = s_t + p_{t+1} \cdot (x_t / p_t)$.

The only market in this economy is the one for assets. An equilibrium is a path of asset prices $\{p_t\}_{t=0}^{\infty}$, denominated in goods, which ensures the asset market clears at each date t. That is, prices must be such that at each date, the old must be willing to sell all their assets and the young must be willing to buy them. Although $\{p_t\}_{t=0}^{\infty}$ can in principle be stochastic, we restrict attention to deterministic price paths.

As is well known, this setup admits multiple equilibria, including equilibria in which $p_t > 0$ for all t. Since the asset yields no dividend, we can think of such equilibria as bubbles in the sense that the price of the

asset exceeds the present discounted value of its dividends. There is also an equilibrium in which $p_t = 0$ for all t and there is no bubble. In this equilibrium, agents must rely on storage to convert their endowment into consumption next period. But note that storage is dynamically inefficient: Since $g > 0$, if all agents agreed to transfer their endowment to the previous generation instead of using storage, everyone would get to consume more. Thus, the equilibrium without a bubble is inefficient.

To characterize the set of deterministic equilibria in our model, let r_t denote the rate of return that agents who buy the asset at date t anticipate to earn from it in equilibrium. Since the asset yields no dividends, this return is just the rate at which the price of the asset grows between dates t and $t + 1$:

$$1 + r_t = \frac{p_{t+1}}{p_t} \tag{3}$$

Suppose the equilibrium return r_t from buying the asset at date t were positive. The cohort born at date t would then strictly prefer the asset to storage, so $s_t = 0$. Since we normalized the supply of the asset to 1, agents will spend all of their endowment on the asset and so

$$p_t = e_t$$

Hence, in any date t in which the equilibrium interest rate $r_t > 0$, the asset price and interest rate are uniquely determined.

Next, suppose the equilibrium return r_t at date t was equal to 0. In this case, the young would be indifferent between storing their endowment and buying assets. While the equilibrium price p_t is no longer uniquely pinned down, we can still say something about the path of prices. Since $r_t = 0$, the price doesn't grow between t and $t + 1$, i.e., $p_{t+1} = p_t$. Given $p_t = e_t - s_t \leq e_t$, then when $r_t = 0$, the price in the next period p_{t+1} will necessarily be less than the endowment of agents, as follows from the fact that

$$p_{t+1} = p_t \leq e_t < e_{t+1}$$

Hence, $s_{t+1} = e_{t+1} - p_{t+1} > 0$, meaning agents at date $t + 1$ store some of their endowment at date. But they will only agree to do so if $r_{t+1} = 0$. Hence, a zero real interest rate is absorbing: Once the real interest rate falls to 0, it will remain there indefinitely. Since the price of the asset grows at the rate of interest, it follows that the price of the asset will remain constant starting from any date t in which $r_t = 0$.

Since a zero interest rate is absorbing while a positive interest rate is associated with a unique p_t, any deterministic equilibrium can be described in terms of a cutoff date $t^* \in \{0, 1, 2, ..., \infty\}$ such that $r_t > 0$ before date t^* and $r_t = 0$ from t^* on. Before t^*, the asset price must equal e_t. From t^* on, the price cannot grow. If we define $e_{-1} \equiv 0$, the price at date t^* can assume any value between e_{t^*-1} and e_{t^*}. Formally,

Proposition 1 *Suppose $d = 0$. A deterministic path $\{p_t\}_{t=0}^{\infty}$ is an equilibrium iff there exists a cutoff date t^* with $0 \leq t^* \leq \infty$ and some value $p_{t^*} \in [e_{t^*-1}, e_{t^*})$ such that*

$$p_t = \begin{cases} e_t & \text{if } t < t^* \\ p_{t^*} & \text{if } t \geq t^* \end{cases} \tag{4}$$

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The proofs of this and other propositions not derived in the text are in Appendix A. Figure 1 illustrates some sample equilibrium price paths. These equilibria can be indexed by the asymptotic price of the asset, $\lim_{t\to\infty} p_t$. For any $p \geq 0$, there exists a unique deterministic equilibrium for which $\lim_{t\to\infty} p_t = p$. When $p \leq e_0$, the threshold t^* is equal to 0. For $p > e_0$, the threshold t^* is the value for which $e_{t^*-1} < p \leq e_{t^*}$.

Finally, Figure 1 suggests that equilibria that feature higher price growth p_{t+1}/p_t also feature higher price levels. To put it another way, equilibria with higher interest rates feature larger bubbles. In the Appendix, we prove that this is indeed true for the set of equilibria in Proposition 1. Intuitively, other things equal, a higher interest rate would induce agents to shift from storage to buying more assets. But since the supply of assets is fixed, this will only bid up the price of the asset.[1]

1.2   Monetary Policy and Nominal Price Rigidity

The previous subsection describes a standard model of bubbles due to dynamic inefficiency. To study how monetary policy affects such bubbles, we need to move beyond an endowment economy so that monetary policy can matter. This was the innovation in Galí (2014), who first incorporated a production economy with rigid prices into a setting with bubbles. In Appendix B, we consider a similar model, but one where preferences and incomes are consistent with the endowment economy from the previous subsection. In contrast to the endowment economy above, young agents in the model described there are endowed with productive inputs rather than goods. Their incomes depend on the output produced in equilibrium, and can in principle vary with monetary policy. Given the incomes of agents, asset prices are determined just as if agents were exogenously endowed with these incomes. In this sense, the endowment economy in the previous subsection can be viewed as the reduced form of the production economy in Appendix B.

The full details of the production economy are in Appendix B. Here, we only sketch the outline of that model. The way we capture monetary policy is by introducing a central bank that can announce a nominal interest rate $1 + i_t$ at which it will borrow and lend money to agents. We only consider equilibria in which agents do not trade with the central bank, meaning the inflation rate in equilibrium must be such that the real return to trading with the central bank is the same as the return on the intrinsically worthless asset. To allow for nominal rigidities, we assume sellers set the price of their goods each period before a sunspot variable ξ_t is realized while the monetary authority sets the nominal interest rate after observing ξ_t. Thus, price-setters cannot perfectly anticipate what nominal interest rate the central bank will set.

An equilibrium is a path of prices (goods prices, input prices, and the real interest rate) for every realization of sunspots $\{\xi_t\}_{t=0}^{\infty}$ and the nominal interest rates given these realizations such that markets clear when agents behave optimally. The formal conditions that define an equilibrium are in Appendix B.

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[1] This result is equivalent to the result in overlapping generations monetary models that, under certain conditions, the real value of money will be higher the lower the inflation rate is. This is because our asset is akin to money in that it pays no dividends, and the return on the asset is akin to the growth in the value of money. See Blanchard and Fischer (1989, pp158-9).

The model admits multiple equilibria. In some of these equilibria, a higher nominal interest rate leads to a higher real interest rate and a smaller bubble. But there are other equilibria in which a higher nominal interest rate leads to a higher real interest rate and a larger bubble.

To elaborate, there exists an equilibrium in which a higher nominal interest rate is associated with lower input prices, lower output, lower asset prices, and a higher real return to buying the intrinsically worthless asset. This equilibrium corresponds to the typical way in which higher nominal interest rates are contractionary in models with nominal price rigidity. In the analogous endowment economy, this would correspond to changing the path of endowments in a way that makes agents poorer. As this induces agents to save less, their demand for assets falls. Given the asset is in fixed supply, its price falls.

But the model admits another equilibrium in which a higher nominal interest rate is associated with the same levels of output, higher asset prices, and a higher real return to buying the intrinsically worthless asset. We can use Figure 1 to understand this equilibrium. Recall that this figure illustrates all deterministic equilibria that are possible for a given path of endowments $\{e_t\}_{t=0}^{\infty}$. If a higher nominal interest rate had no effect on output or what agents earn, the economy could still switch from an equilibrium in Figure 1 with a lower path for real interest rates to one with a weakly higher path for real interest rates. As we observed in the previous subsection, the equilibrium with higher real interest rates are associated with larger bubbles. In this equilibrium, a higher nominal interest rate induces agents to buy assets rather than store their goods. This increases demand for the asset. Given the asset is in fixed supply, its price rises.

In sum, monetary policy has an indeterminate effect on bubbles in our model. Galí (2014) focuses on equilibria that resemble the second equilibrium we describe, in which higher nominal interest rates raise demand for assets.[2] But in the model he studies, there similarly exist equilibria in which higher nominal interest rates impoverish agents and decrease demand for assets. Dong, Miao, and Wang (2017) and Ikeda (2017), who study models of bubbles that feature a similar indeterminacy, instead focus on equilibria that resemble the first equilibrium we describe, in which higher nominal interest rates reduce demand for assets.[3] But there exist equilibria in their models in which higher nominal interest rates induce agents to spend more on assets. How monetary policy operates in these models depends on the equilibrium we choose. Our discussion suggests that the equilibria in which a higher nominal rate leads to a larger bubble rely on the indeterminacy of asset prices, which allows agents to coordinate on an equilibrium in which they all

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[2] Galí focuses on an equilibrium in which the economy tends to a stable steady state in the long run and in which the sunspot that leads the central bank to set a higher nominal interest rate also results in a higher asset price one period later. This is because he assumes the sunspot is associated with the creation of new bubble assets with no offsetting decline in the value of existing assets. That requires that agents next period agree to spend more on assets in total.

[3] These papers focus on equilibria in which the economy tends to an unstable or saddle-path stable steady state. This restriction implies asset prices cannot change in the long run in response to a shock. But if asset prices are unchanged in the long run and the price of the asset grows at the rate of interest, higher interest rates require lower initial prices for asset prices to remain at the same level in the long run. Note that in these two papers bubbles are due to borrowing constraints rather dynamic inefficiency, but the indeterminacy in these models is qualitatively similar.

spend more on assets when the nominal interest rate is high. If this were indeed the case, then eliminating the indeterminacy of asset prices should eliminate the equilibrium in which a higher nominal interest rate leads agents to coordinate to buy more assets. In the next subsection, we show that changing one feature of the model yields a unique equilibrium in the asset market, and that this equilibrium still corresponds to a bubble. We then confirm that in this case, if a higher nominal interest rate leads to a higher real interest rate, it must impoverish agents and dampen the bubble.

1.3 Eliminating Indeterminacy

While bubbles may seem to be inextricably linked with indeterminacy, Tirole (1985) already showed that in some circumstances, a dynamically inefficient economy could feature a bubble as the unique equilibrium outcome. This occurs if (1) there is an asset that pays out positive dividends, and (2) without a bubble, the real interest rate agents earn would tend to zero or to a negative value. Intuitively, in these circumstances the present discounted value of dividends would tend to infinity if there were no bubble, and agents would not be able to afford the asset. The only possible equilibrium, then, is one that features a bubble.

We can use this insight to modify our model to eliminate its indeterminacy. For simplicity, let us return to our original endowment economy. Suppose we replace the intrinsically worthless asset in which $d = 0$ with an asset that pays a fixed positive dividend $d > 0$ at all dates. Note that over time, this dividend becomes small relative to the endowment agents would like to save. This implies that without a bubble, when returns can only be due to dividends, the return on investment that agents can expect would tend to zero, the exact condition Tirole identified with uniqueness.[4]

To confirm that $d > 0$ implies a unique equilibrium, we first show that the equilibrium return to holding the asset r_t can never equal 0. For suppose r_t did equal 0 at some date t. Then (3) would imply

$$p_{t+1} = p_t - d < p_t$$

Since $p_t \leq e_t$, it again follows that $p_{t+1} < p_t \leq e_t < e_{t+1}$. But if agents store goods at date $t + 1$, then $r_{t+1} = 0$. By this logic, the price would continue to decline in increments of d from date $t + 1$ on, until it would eventually turn negative. But a negative price cannot be an equilibrium, since the cohort that owns the assets would refuse to sell them. The only candidate equilibrium price path is one with strictly positive interest rates at all dates. But in that case, storage is dominated and $p_t = e_t$ for all t. Formally, we have

Proposition 2 *Suppose $d > 0$. Then the unique equilibrium path $\{p_t\}_{t=0}^{\infty}$ is given by*

$$p_t = e_t \tag{5}$$

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[4]More generally, we can allow for an asset whose dividends d_t vary over time. Our proof of Proposition 2 shows the equilibrium is unique iff $\sum_{t=0}^{\infty} d_t = \infty$ with $d_t > 0$ for all t. The fact that this equilibrium can only correspond to a bubble if $\lim_{t\to\infty} d_t/e_t = 0$, or if we have an asymptotically vanishing dividend yield, was shown by Tirole (1985) and Rhee (1991).

and the unique equilibrium interest rate is given by

$$r_t = \frac{p_{t+1} + d}{p_t} - 1 = g + \frac{d}{e_t} \tag{6}$$

To confirm that the equilibrium in Proposition 2 still represents a bubble, we first need to define the fundamental value of the asset. In particular, we need to take a stand on the rate at which to discount dividends. If we took resources from a young agent at date t, she would demand $1 + r_t$ units at date $t + 1$ to remain equally well off. Hence, the market interest rate r_t captures the way society trades off resources between adjacent dates. This leads us to define the fundamental value of the asset as

$$f_t = \sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + r_{t+i}} \right) d \tag{7}$$

From Proposition 2, $r_t > g > 0$ for all t. Hence, (7) is bounded above for all t, since

$$f_t \leq \sum_{j=1}^{\infty} \left(\frac{1}{1 + g} \right)^j d = \frac{d}{g}$$

Furthermore, $\lim_{t \to \infty} f_t = d/g < \infty$. The fundamental value of the asset is thus bounded. At the same time, the asset price grows without bound, since $\lim_{t \to \infty} p_t = \lim_{t \to \infty} (1 + g)^t e_0 = \infty$. The asset price will eventually exceed its fundamental value, at which point it will be a bubble. But we can show that the equilibrium price of the asset exceeds the fundamental value at all dates rather than just asymptotically. To see this, note that the equilibrium interest rate r_t in Proposition 2 implies that

$$p_t = \frac{d + p_{t+1}}{1 + r_t} \tag{8}$$

At the same time, the fundamental value f_t in (7) satisfies

$$f_t = \frac{d + f_{t+1}}{1 + r_t} \tag{9}$$

Subtracting the latter expression from the former reveals that the difference $b_t \equiv p_t - f_t$, which is just the size of the bubble at date t, must satisfy

$$b_t = \frac{b_{t+1}}{1 + r_t} \tag{10}$$

Since $b_T > 0$ as $T \to \infty$, it follows that $b_0 > 0$. Hence, the price of the asset exceeds its fundamental value at all dates, and the asset is necessarily a bubble.

1.4 Monetary Policy Revisited

The fact that $d > 0$ yields a unique equilibrium extends to the production economy we lay out in Appendix B. We can therefore use that framework to analyze the effects of monetary policy in a production economy where asset prices are uniquely determined. For simplicity, suppose that instead of a sunspot at each date t, there is a single sunspot at date 0. In Appendix B, we show that in this case monetary policy has no

effect on real variables beyond $t = 0$. Intuitively, since producers can set the price of goods each period, they can perfectly anticipate monetary policy from date $t = 1$ on.

While the formal analysis of the production economy is contained in Appendix B, we can use our endowment economy to illustrate the key insights. The path of incomes in equilibrium when the nominal interest rate is low at date 0 can be represented by a path of endowments $e_t = (1 + g)^t e_0$. Since monetary policy has no real effects beyond date 0, the incomes agents earn at dates $t \geq 1$ must be the same regardless of what nominal interest rate the central bank chose at date 0, and hence e_t must be the same from date $t = 1$. Let τ denote the difference in the equilibrium income of young agents at date 0 when nominal interest rates are high and when they are low, meaning young agents earn $e_0 - \tau$ if the monetary authority sets a high nominal interest rate at date 0. Proposition 2 tells us that when the asset pays a fixed dividend, the unique equilibrium is one in which agents spend all of their income to buy assets. Hence, the equilibrium asset price when nominal interest rates are high at $t = 0$ is given by

$$p_t = \begin{cases} e_0 - \tau & \text{if } t = 0 \\ e_t & \text{if } t = 1, 2, 3, ... \end{cases} \tag{11}$$

and the equilibrium real return on buying the asset is given by

$$r_t = \begin{cases} \dfrac{e_1 + d}{e_0 - \tau} & \text{if } t = 0 \\ g + \dfrac{d}{e_t} & \text{if } t = 1, 2, 3, ... \end{cases} \tag{12}$$

Equation (12) shows that the real interest rate maps into a unique τ and equation (11) shows that the asset price p_0 maps into a unique τ. The only way monetary policy can increase the real interest rate r_t is if $\tau > 0$, i.e., if it depresses output and the price of the asset. To confirm that it also dampens the bubble $b_t = p_t - f_t$, observe that destroying τ goods at date 0 has no effect on either p_t or r_t for $t \geq 1$, since these depend on endowments from date $t = 1$ on. Hence, neither p_t nor f_t change beyond date 1, and so b_t is unchanged for $t \geq 1$. But we know from (10) that $b_0 = \frac{b_1}{1+r_0}$. Since the bubble b_1 is unchanged and r_0 is higher, b_0 must fall. Thus, once we eliminate the indeterminacy in asset prices, a higher nominal interest rate that increases the real interest rate must also dampen the bubble, and equilibria like the ones we discussed earlier in which increasing the real interest rate amplifies bubbles will no longer arise.

For the remainder of the paper, we will use an endowment economy to capture the effect of tighter monetary policy by looking at the effects of destroying part of the endowment of some cohort in lieu of explicitly modeling production and monetary policy. We should note that other policy interventions also imply the same reduced form effect of destroying part of the endowment. For example, a lump-sum tax on the young at date 0 that is used to finance government consumption would be similarly captured by destroying part of the endowment of young agents. In addition, when we introduce credit in the next section, a restriction on the amount agents can lend along the lines of how Allen and Gale (2000) model credit tightening can also be represented as a destruction of endowments. While we interpret this thought experiment as contractionary monetary policy, it captures other interventions as well.

1.5 Welfare

To recap, our analysis so far has dealt with the problem of indeterminacy in models of bubbles due to dynamic inefficiency. In the standard model, the effects of monetary policy are ambiguous. Although Galí (2014) emphasized the surprising fact that a contractionary monetary policy can amplify bubbles, it can also dampen bubbles. We showed that if we change some of the assumptions of the model to ensure a unique equilibrium in asset markets, a higher real rate is necessarily associated with a smaller bubble. This suggests policymakers can dampen bubbles by increasing rates. But in the model of bubbles we have explored thus far, there is in fact no reason for a policymaker to intervene against bubbles. This is because bubbles only serve to alleviate the underlying dynamic inefficiency that allows for bubbles in the first place.

To see this, suppose we destroy τ units of the endowment of those born at date 0. The policy is clearly costly, with the cost fully borne by the old at date 0 who would otherwise have consumed those τ units when they exchanged them for their assets. But there are no countervailing benefits to reducing the bubble: All other agents are unaffected, and each cohort born at date $t \geq 0$ will continue to consume $e_{t+1} + d$ for all $t \geq 0$, since our preferences imply that each period all goods will go to that period's old. More generally, the cost of destroying τ goods at date 0 could be borne by young agents at date 0 as well as by later cohorts. But such a policy could never be welfare improving. As Tirole (1985) and others have argued, asymptotic bubbles in which $\lim_{t \to \infty} b_t/e_t > 0$ are dynamically efficient in these economies. Any intervention which changes what agents consume must make at least some agents worse off.

While our analysis assumed the supply of assets of assets is fixed, this insight extends to the case where new assets can be created. Even though the price of the asset exceeds the present discounted value of dividends it generates, there is no benefit to raising rates in order to discourage the creation of assets. We show this formally in Appendix C, where we allow for an endogenous supply of assets.

In sum, the problem with models of bubbles due to dynamic inefficiency for the lean-against-the-wind view is not that they imply raising rates amplifies bubbles, but that they imply there is no reason to intervene against such bubbles. To make a case for leaning against the wind, we need a model in which bubbles do not alleviate the friction that allows them to arise in the first place. In the next section, we construct such a model in which the friction that gives rise to bubbles is different than dynamic inefficiency.

2 Credit-Driven Bubbles

In the previous section, we argued that models like Galí (2014) cannot be used to justify leaning-against-the-wind policies, not because higher interest rates in these models amplify bubbles but because there is no reason to dampen bubbles in these models. To the contrary, it is the equilibrium without a bubble in these models that is generally inefficient. These models also fail to capture the main concern of policymakers, who worry that bubbles financed by bank lending may collapse and threaten the financial system. Mishkin

(2011) nicely summarizes this concern:

> [N]ot all asset price bubbles are alike. Financial history and the financial crisis of 2007-2009 indicates that one type of bubble, which is best referred to as a credit-driven bubble, can be highly dangerous. With this type of bubble, there is the following typical chain of events: Because of either exuberant expectations about economic prospects or structural changes in financial markets, a credit boom begins, increasing the demand for some assets and thereby raising their prices... At some point, however, the bubble bursts. The collapse in asset prices then leads to a reversal of the feedback loop in which loans go sour, lenders cut back on credit supply, the demand for the assets declines further, and prices drop even more. The resulting loan losses and declines in asset prices erode the balance sheets at financial institutions, further diminishing credit and investment across a broad range of assets. The decline in lending depresses business and household spending, which weakens economic activity and increases macroeconomic risk in credit markets. In the extreme, the interaction between asset prices and the health of financial institutions following the collapse of an asset price bubble can endanger the operation of the financial system as a whole

In this section, we show how, starting with the framework developed in the previous section, we can generate credit-driven bubbles along the lines in Mishkin's quote. In particular, we eliminate dynamic inefficiency and introduce a different friction that allows bubbles to arise. To do this, we modify our model in several ways. First, we assume the endowment no longer grows over time. This ensures the economy is dynamically efficient. Second, we add a credit market and introduce information frictions that prevent lenders from monitoring their borrowers. For the latter friction to matter, we also assume dividends are stochastic, although we continue to assume they are always positive. The fact that assets are risky allows agents to gamble at the expense of creditors, which gives rise to a bubble.

2.1   An Economy with Credit

Our starting point is the endowment economy in the previous section in which agents can invest in an asset that yields a constant dividend $d > 0$ each period. Our first modification is to assume $g = 0$ so that all cohorts receive the same endowment $e_t = e$. Since the proof of Proposition 2 does not rely on the value of g, we can still use the proposition to determine the equilibrium. Hence, the unique equilibrium is given by $p_t = e$ for all t and $r_t = d/e \equiv r$ for all t. However, setting $g = 0$ leads to an important difference. The value of all dividends discounted by the return on the asset $1 + r$ is given by

$$f_0 = \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t d = d/r = e$$

The fundamental value of the asset is now equal to the price, in contrast to the case where $g > 0$. Intuitively, without growth there is no role for intergenerational transfers that allow old agents to benefit from the larger

endowment of the young generation. The economy is thus dynamically efficient, and a bubble no longer emerges. We will set $g = 0$ from now on, so any bubble that arises cannot be due to dynamic inefficiency.

We next introduce a credit market. For agents to trade in this market requires some heterogeneity among young agents, since they are the only ones willing to trade intertemporally. We continue to assume each cohort includes a unit mass of savers who are born with an endowment of goods e and whose preferences are given by (1) and whose only concern is to convert their endowment when young into consumption when old. But we now assume that each cohort also includes an infinitely large mass of potential entrepreneurs. The latter are endowed with no resources, but they have access to a technology that converts goods at date t into goods at date $t + 1$.

To be more precise, we assume each entrepreneur can transform up to one unit of the good at date t into $y > 1$ units at date $t + 1$. Potential entrepreneurs differ in their productivity y. Let $N(y)$ denote the mass of entrepreneurs whose productivity is at least y. We assume that $N(y)$ is differentiable and satisfies

$$N'(y) < 0, \ \lim_{y \downarrow 1} N(y) = \infty, \ \lim_{y \to \infty} N(y) = 0.$$

The assumption that $N(y)$ is strictly decreasing implies $N(y) > 0$ for all $y \geq 1$. Thus, for any finite interest rate that savers might charge, there would always be a positive mass of entrepreneurs who are sufficiently productive that they would find it profitable to borrow resources and produce.

Savers and entrepreneurs can trade in a centralized credit market with a single interest rate R_t. We assume that trade takes the form of loans. Entrepreneur must repay a fixed amount $1 + R_t$ units of the good at date $t + 1$ for each unit they borrowed at date t. An entrepreneur who cannot meet the required payment, for whatever reason, is said to be in default and we assume a court can compel the transfer of all of his or her resources to the creditor. Entrepreneurs who default incur a small utility cost of ϕ per unit borrowed. In practice, we will take the limit as $\phi \to 0$. As long as $\phi > 0$, borrowers will never take out a loan they expect to default on with certainty. This helps us avoid equilibria in which agents are willing to take out loans they will certainly default on. For now, we assume there are no other costs of default. We will consider the consequences of allowing for costs of default to lenders later on.

An equilibrium in this economy is a path of asset prices $\{p_t\}_{t=0}^{\infty}$ and a path of interest rates on loans $\{R_t\}_{t=0}^{\infty}$ such that markets clear when agents optimize. To solve for an equilibrium, we need to describe supply and demand for both assets and credit. The optimal behavior of agents in their second period of life is straightforward: They collect repayment on loans they made in the past and pay back the loans they have outstanding. If they own assets, they sell them if the price is nonnegative. As for young agents, savers must choose between buying assets and lending. They allocate their wealth to the investments that offer the highest returns. Entrepreneurs choose whether to borrow the resources they need to produce and whether to borrow to buy assets. Given the interest rate on loans, R_t, entrepreneurs with productivity $y > 1 + R_t$ will borrow to produce, regardless of whether they also borrow to buy assets. Whether entrepreneurs borrow to buy assets depends on whether that activity is profitable after netting out interest costs.

For any interest rate R_t, the marginal entrepreneur has productivity $y_m = 1 + R_t$ and the demand for loans from entrepreneurs who want to produce will be $N(y_m) = N(1 + R_t) > 0$. As a result, the amount of the endowment available to purchase the asset, either directly or with borrowed funds, is less than e. Market clearing requires that

$$N(1 + R_t) + p_t = e \qquad (13)$$

Our assumptions on $N(y)$ imply there is a unique market-clearing interest rate $R_t = \rho(p_t)$ for any price of the asset. Since $N'(y) < 0$, the function $\rho(p_t)$ is increasing: A higher p_t reduces the amount of goods available for productive investment, so the interest rate on loans R_t must rise to lower demand from entrepreneurs who want to borrow in order to produce.

Since entrepreneurs would borrow unbounded amounts to buy the asset if they could earn positive profits, the interest rate on loans must be at least as high as what agents can earn from buying the asset. At the same time, the interest rate on loans cannot exceed the return to buying the asset, or else no one would buy the asset: Young savers would strictly prefer to make loans, and young entrepreneurs would refuse to borrow to buy the asset knowing they will default. Thus, in equilibrium the interest rate on loans must equal the return to buying the asset:

$$p_{t+1} + d = (1 + R_t) p_t = (1 + \rho(p_t)) p_t \qquad (14)$$

Define $\varphi(p) \equiv (1 + \rho(p)) p - d$. We can immediately see that $\varphi(p)$ has the following properties:

$$\varphi'(p) \quad > \quad 1, \text{ for any } p;$$
$$\varphi(p) \quad < \quad 0, \text{ for } p > 0 \text{ sufficiently small};$$
$$\varphi(p) \quad > \quad e, \text{ for } p > 0 \text{ sufficiently large}.$$

The first property follows from the fact that $\rho'(p) > 0$; the second follows from the definition of $\varphi(p)$ and the fact that

$$\lim_{p \to 0} (1 + \rho(p)) p = 0;$$

and the third follows from the fact that $\rho'(p) \geq 0$.

The graph of φ is illustrated in Figure 2. The red curve is the graph of $p_{t+1} = \varphi(p_t)$ and the black curve is the 45° line. The two lines intersect at a unique point where $p^* = \varphi(p^*)$. For any initial condition, the law of motion $p_{t+1} = \varphi(p_t)$ defines a unique path of asset prices, but for any initial condition other than $p_0 = p^*$, the path will drift away from p^* until either $p_t < 0$ or $p_t > e$ in finite time, neither of which can be an equilibrium. Thus, the unique equilibrium path is a steady state in which $p_t = p^*$ and $R_t = \rho(p^*)$ for all t. A fact that will be useful later is that the steady state price p^* is increasing in the dividend d. Setting $p_t = p_{t+1} = p^*$ in the zero-profit condition (14), we see that

$$d = \rho(p^*) p^*.$$

The right hand side is increasing because $\rho(p^*)$ is increasing, so an increase in d must increase the equilibrium asset price p^*. Graphically, a larger d will lead the red curve in Figure 2 to shift down, and so the steady state p^* must rise.

Finally, observe that the equilibrium return on the asset $r_t = d/p^*$ is the same as the equilibrium interest rate on loans, since $R_t = \rho(p^*) = d/p^*$. The value of dividends discounted at this rate of return is given by

$$f_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+R^*}\right)^j d = d/R^* = p^*$$

Hence, the introduction of credit does not on its own introduce a wedge between the price of the asset and its fundamental value. We collect these results in a summary proposition.

Proposition 3 *The unique equilibrium for the economy with credit and a constant dividend $d > 0$ features a constant asset price $p_t = p^*$, a constant interest rate on loans $R_t = \rho(p^*)$, and a constant fundamental $f_t = p^*$ for all t, where p^* is the fixed point of the equation $p = \varphi(p)$.*

The introduction of credit *per se* does not lead to bubbles. For that, we require some additional frictions. In the next subsection, we assume asymmetric information in the market for credit. We also assume assets are risky, which means that our assumption that agents must use non-contingent debt contracts represents an additional market imperfection.

2.2 Risky Assets and Information Frictions

To allow for risky assets, we introduce a regime switching process inspired by Zeira (1999). The asset initially pays a dividend $d_t = D > 0$, but there is a probability $\pi > 0$ in each period that the dividend falls to a lower level $0 < d < D$. Once the dividend d_t falls to d, it remains at the lower level forever.

The combination of risky assets and credit contracts introduce the possibility of default. Because agents can borrow unlimited amounts to buy assets, market clearing still requires that agents cannot expect to earn positive profits from borrowing to buy assets. That is, the interest rate must be high enough to ensure zero profits to buying the asset even when the return on the asset is highest. But in that case, entrepreneurs will be forced to default whenever the return on the asset is below the maximum possible. Below we show that the maximal return to buying the asset when its future dividend is uncertain occurs if the dividend remains high, so entrepreneurs who borrow to buy the asset default if and when the dividend falls.

The second friction is that savers cannot observe an entrepreneur's productivity and cannot monitor what the borrower does with the funds he obtains. This information friction is the key feature that allows bubbles to emerge, because it implies that less productive entrepreneurs will be able to borrow and speculate on assets instead of producing. Allen and Gorton (1993), Allen and Gale (2000), and Barlevy (2014) previously showed that such information frictions allow bubbles to arise. But these papers do not explore interest rate policy and its consequences for welfare, as we do here.

Equilibrium is defined in two steps, corresponding to the two regimes in equilibrium: either the dividend has not yet fallen and the asset is risky, or the dividend has fallen and the asset is safe. Let p_t^D (respectively,

p_t^d) denote the price of the asset at date t if $d_t = D$ (respectively, $d_t = d$), and similarly let R_t^D (respectively, R_t^d) denote the interest rate on loans at date t if $d_t = D$ (respectively, $d_t = d$). Once the asset's dividend has fallen to d, the economy is identical to the one analyzed in the previous subsection. As summarized in Proposition 3, there is a unique equilibrium with a constant asset price $p_t^d = p^d$, which we previously denoted by p^*, a constant interest rate $R_t^d = R^d$ which we previously denoted by $\rho(p^*)$, and no bubble.

The equilibrium in the regime before the dividend falls is similar in some respects. Forget for a moment that the dividend on the asset may fall, and consider what would happen if the dividend were D forever. The interest rate on loans would have to equal the return to buying the asset:

$$(1 + R_t)\, p_t = p_{t+1} + D \tag{15}$$

By the same argument as before, there is a unique solution to equations (13) and (15), which takes the form of a constant asset price $p_t = p^D$ and a constant interest rate $R_t = R^D$. As we noted above, an increase in the dividend increases the steady state asset price, so $p^D > p^d$.

We now argue that setting $\left(p_t^D, R_t^D\right) = \left(p^D, R^D\right)$ as long as the dividend is high constitutes an equilibrium path for the economy where dividends are stochastic. We know by construction that this path satisfies the equilibrium condition (13), so we only need to show that at these prices agents cannot profit from borrowing to buy the asset. Since $\left(1 + R^D\right) p^D = p^D + D$, we need to verify that agents cannot expect to earn positive profits from borrowing to buy the asset and hoping for the low dividend regime, i.e., that $\left(1 + R^D\right) p^D \geq p^d + d$. Recall that the equilibrium interest rate $R_t = \rho(p_t)$ where $\rho(p_t)$ is an increasing function that does not depend on the dividend. This implies

$$R^D = \rho\left(p^D\right) > \rho\left(p^d\right) = R^d$$

Since $p^D > p^d$ and $R^D > R^d$, it follows that $\left(1 + R^D\right) p^D > \left(1 + R^d\right) p^d = p^d + d$, where the last equality follows from the definition of p^d. So an agent who borrows to buy the asset will indeed earn zero profits given these prices.

Thus, in the economy where the dividend is stochastic, there exists an equilibrium in which during the high dividend regime the price of the asset p_t^D and the interest rate on loans R_t^D are the same as they would be if the dividend remained high forever. But is this the only equilibrium? Let $\left\{\left(p_t^D, R_t^D\right)\right\}$ denote any alternative equilibrium path for the high-dividend regime. If we can show that $p_t^D + D > p^d + d$ for every t, then the zero profit condition that defines the equilibrium would correspond to (15). But under (15), if $\left(p_t^D, R_t^D\right) \neq \left(p^D, R^D\right)$ for some t, the price of the asset in the high dividend regime would eventually either turn negative or exceed e, neither of which can be an equilibrium. Suppose that in fact $p_t^D + D \leq p^d + d$ for some t. This would require that $p_t^D < p^d$. Since $\rho(p_t)$ is increasing and the equilibrium interest rate on loans R_t is always equal to $\rho(p_t)$, this would imply

$$R_t^D = \rho\left(p_t^D\right) < \rho\left(p^d\right) = R^d$$

But then we would have

$$\left(1 + R_t^D\right) p_t^D < \left(1 + R^d\right) p^d = p^d + d.$$

This means that an entrepreneur can make positive profits at date t if the dividend falls next period, which cannot be true in equilibrium. This contradiction implies $p_t^D \geq p^d$, and so $p_t^D + D > p^d + d$ in any equilibrium. The equilibrium we constructed, in which $\left(p_t^D, R_t^D\right) = \left(p^D, R^D\right)$ for all t, is unique.

Finally, we observe that when the dividend is d, savers are indifferent between buying the asset and lending, while entrepreneurs are indifferent about borrowing to buy the asset. Hence, while the interest rate on loans is uniquely determined, the amount of lending is not. By contrast, when the dividend is D, savers will not be willing to invest in the asset. Let $1 + \bar{r}^D$ denote the expected return to buying the asset. Then

$$1 + \bar{r}^D = \frac{(1 - \pi)\left(D + p^D\right) + \pi\left(d + p^d\right)}{p^D}$$

If a saver were to lend in the credit market, she wouldn't know whether she was lending to an entrepreneur who will produce or an entrepreneur who plans to speculate on assets. In the first case, she would earn the interest rate on loans $1 + R^D$, which from (15) we know is equal to $\left(D + p^D\right)/p^D$, and in the second she would earn the expected return on the asset $1 + \bar{r}^D$. Since we argued above that $d + p^d < D + p^D$, the interest rate on loans exceeds the expected return to buying the asset, i.e., $\overline{R}^D > \bar{r}^D$. Hence, the expected return from lending is greater than the expected return from buying the asset. Only low productivity entrepreneurs hold the asset. We summarize the equilibrium with the following proposition:

Proposition 4 *The equilibrium for the economy with credit and stochastic dividends is unique. As $\phi \to 0$, the prices and interest rates in this equilibrium are given by*

$$(p_t, R_t) = \begin{cases} \left(p^D, R^D\right) & \text{if } d_t = D \\ \left(p^d, R^d\right) & \text{if } d_t = d \end{cases}$$

While $d_t = D$, savers strictly prefer to lend out their savings, and low productivity entrepreneurs with $y < 1 + R^D$ are the only ones who invest in the asset.

The fact that the price of the asset when the dividend is stochastic is identical to the price of the asset when the dividend remains high forever suggests the asset is overpriced. This interpretation is consistent with the fact that savers avoid buying the asset in the high dividend regime. This would suggest the asset is a bubble during the high dividend regime. We confirm this in the next subsection.

2.3   Credit-Driven Bubbles

To determine whether the asset is a bubble, we need to calculate the fundamental value of the asset in each dividend regime. Once the dividend has fallen to $d_t = d$, the economy is just the one with a safe asset. As we have already seen, the equilibrium of this economy is stationary, and the fundamental value of the asset is constant and equal to the asset price:

$$f^d = p^d = d/R^d$$

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When the dividend is still high, so $d_t = D$, the equilibrium is stationary conditional on the dividend being high. In particular, the returns agents earn on their investments are constant. The fundamental value must therefore also be constant. The question is at what rate we should discount dividends.

Up to now, the discount rate we used to define the fundamental value of the asset was the compensation we need to provide an agent at date $t+1$ if we took a unit of resources from her at date t. In the dynamically inefficient economy in the previous section, the necessary compensation was the return to buying the asset $1+r_t$ or $(p_{t+1} + d)/p_t$, which was effectively the only option available for agents. In the economy with credit and a safe asset, the return to lending $1 + R_t$ is the same as the return to the asset $1 + r_t$ in equilibrium, so there is again no ambiguity about the relevant discount rate. But in the economy with credit and a risky asset, the return agents can earn varies in equilibrium both across investments (the expected return on loans exceeds the expected return to buying the asset) and across individuals (more productive entrepreneurs can earn higher returns). So the question of what is the appropriate discount rate becomes relevant.

It will be useful to single out three particular rates of return in our credit economy during the high dividend regime, when the return on the asset is risky. One is the interest rate on loans, $1 + R^D$. Since (15) holds in equilibrium, this interest rate is given by

$$1 + R^D = \frac{D + p^D}{p^D} \tag{16}$$

Another rate of return in this economy is the expected return to buying the asset. We previously denoted this rate by $1 + \bar{r}^D$. This expected return is given by

$$1 + \bar{r}^D = \frac{\pi \left(d + p^d\right) + (1 - \pi) \left(D + p^D\right)}{p^D} \tag{17}$$

Finally, let us define the expected return to lending $1 + \overline{R}^D$. Since some agents will default on their loans, this expression will differ from the interest rate on loans $1 + R^D$. In particular, lenders collect $1 + R^D$ from each unit they lend out that is used in production, and expect to earn $1 + \bar{r}^D$ from each unit they lend out that is used to buy assets. Let α denote the fraction of the endowment e that is invested in the asset, so $\alpha = p^D/e$. Then the expected return to lending is just a weighted average of $1 + R^D$ and $1 + \bar{r}^D$, specifically

$$1 + \bar{R}^D = 1 + (1 - \alpha) R^D + \alpha \bar{r}^D \tag{18}$$

Since the price of the asset is positive, $\alpha > 0$. At the same time, because the mass of entrepreneurs who borrow in equilibrium is positive given our assumptions on $N(y)$, some of the endowment will be used for production, so $\alpha < 1$. We can therefore rank the three returns, with $\bar{r}^D < \overline{R}^D < R^D$.

So, which of these rates, if any, should we use to discount dividends? Effectively, this amounts to asking how much society would value an additional unit of resources during the high dividend regime. Under full information, that extra unit would be allocated to the most productive entrepreneur, whose productivity is $1 + R^D$. But since information frictions make it impossible to direct resources to the most productive entrepreneur, the best we can do is lend it out to any willing borrower, and the expected return on a loan

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is equal to $1 + \bar{R}^D$. Following this logic, we will define the fundamentals using $1 + \bar{R}^D$ as our discount rate. But regardless of whether we discount using the return on a loan to a random borrower or a loan that is directed towards more productive entrepreneurs, this return would exceed the expected return to buying the asset $1 + \bar{r}^D$. We now argue that as long as we discount dividends at a rate above $1 + \bar{r}^D$, the price of the asset will exceed the fundamental value.

Since the equilibrium during the high dividend regime is stationary, we can define the fundamental value of the asset f^D recursively as

$$\left(1 + \bar{R}^D\right) f^D = \pi \left(d + p^d\right) + (1 - \pi) \left(D + f^D\right) \tag{19}$$

Equation (19) incorporates $1 + \bar{R}^D$ as the relevant discount rate. It also uses our result in Proposition 3 that there is no bubble in the low dividend regime, implying $p^d = f^d$. If we compare (19) to (17), we see that since $\bar{r}^D < \bar{R}^D$, then

$$\frac{\pi \left(d + p^d\right) + (1 - \pi) \left(D + p^D\right)}{p^D} < \frac{\pi \left(d + p^d\right) + (1 - \pi) \left(D + f^D\right)}{f^D}$$

But this inequality can only be true if $p^D > f^D$. As long as we discount dividends at a rate that exceeds the average return to buying the asset $1 + \bar{r}^D$, the fundamental will be lower than the price. We summarize this with a proposition.

Proposition 5 *In the credit economy with stochastic dividends, if the fundamental value f^D discounts dividends at the expected return on loans $\bar{R}_t$, then $f^D < p^D$ and $f^d = p^d$. The bubble in the initial high dividend regime, $b^D = p^D - f^D$, is given by*

$$b^D = \left(\pi \left(d + p^d\right) + (1 - \pi) D\right) \left[\frac{1}{\pi + \bar{r}^D} - \frac{1}{\pi + \bar{R}^D}\right] > 0 \tag{20}$$

Proposition 5 formalizes the notion that, in this economy, the asset is overpriced during the high dividend regime. Demand for the asset by less productive entrepreneurs pushes the price of the asset up. At this high price, the return on the asset is low relative to alternative investments, which is why savers avoid buying it.

It is worth commenting on the differences between the bubble in the dynamically inefficient economy from the previous section and the credit-driven bubble that emerges here. In the former, the price of the asset exceeded the value of dividends discounted according to the return on the asset, $1 + r_t$ or $(p_{t+1} + d)/p_t$. This is because in the dynamically inefficient economy, the return to buying the asset consists of not only a dividend yield but also a transfer from the subsequent generation. The fact that the return on the asset is high reduces the fundamental value. By contrast, in the dynamically efficient economy with credit, discounting dividends according to the expected return on the asset yields a value equal to its price. The reason the asset can still be viewed as a bubble is that the return on the asset is too low compared with the return on other investment opportunities available to agents.

Another difference between the two types of bubbles is that the bubble in our credit economy can grow more slowly than the rate of interest. In particular, the bubble doesn't grow even though agents can earn a positive return on their savings. By contrast, in the dynamically inefficient economy, (10) implies the bubble grows at the rate of interest, since $b_{t+1} = (1 + r_t) b_t$. Intuitively, in the credit-driven bubble in our model, the agents who effectively own the bubble, namely savers, do so reluctantly. Thus, they do not require it to offer returns that compete with other assets they might hold, as is the case in models without information frictions where agents know which assets they hold.

That said, credit-driven bubbles are not inherently fixed in size. The only reason the bubble b^D remains constant is that we assumed a stationary economy to simplify the analysis. If we had instead allowed the endowment to grow at a rate $g > 0$ in the high dividend regime and then remain constant thereafter, the economy would continue to be dynamically efficient and a bubble would only arise because of information frictions. But in that case, the price p_t and the bubble b_t would both grow over time, as would total credit. This is consistent with Mishkin's observation on the connection between bubbles and credit booms.

Finally, the conditions that allow a bubble to arise differ in the two cases. In the dynamically inefficient economy, a necessary condition for a bubble to occur is that the asymptotic growth rate of the economy is at least as high as the interest rate, in which case intergenerational transfers can make all agents better off. Credit-driven bubbles instead arise because information frictions create a situation in which entrepreneurs who borrow to produce cross-subsidize those who borrow to buy risky assets. Less productive entrepreneurs keep borrowing until they drive up both the asset price p_t^D and the interest rate on loans R_t^D enough to push the return to speculation to zero. A bubble thus requires both that assets are risky, so there is a risk to shift onto creditors, and someone to cross-subsidize speculation. This explains why the bubble term in (20) tends to zero if either $\pi \to 0$ or $d/D \to 1$, meaning the asset isn't risky, or the share of resources that go to production α tends to zero, meaning there are no agents to cross-subsidize speculation. All three cases imply $\overline{R}^D \to \overline{r}^D$, in which case b^D would tend to 0.

Several features of our model line up nicely with historical episodes believed to be bubbles. Our model is certainly consistent with Mishkin's description that credit fuels demand for the asset, pushing its price up until eventually the bubble bursts, which in our model occurs when the dividend falls. The asset price falls when the bubble bursts, from p^D to p^d. The collapse of the bubble triggers default on the loans used to finance asset purchases. If we had assumed loans were intermediated, then the default by speculators could have triggered additional defaults by financial intermediaries. Thus, our model can be extended to explain why the collapse of asset bubbles can coincide with financial crises. The collapse of the bubble in our model is also associated with a period of low interest rates, consistent with the recent experience of low real rates in the wake of the collapse of house prices. At the same time, the cross-subsidization that is central to our model implies that even during the bubble phase agents who borrow to buy assets are paying too low of an interest rate. Moreover, the expected return on the asset during the bubble phase, $1 + \overline{r}^D$, is low relative to the return on other investments. However, even if the *expected* return to buying the asset is low during the bubble phase, the *realized* return on the asset is high during the bubble phase. And while lenders suffer

losses when the bubble bursts, they make profits on loans during the bubble phase.

2.4 Monetary Policy and Welfare

Now that we have established that our model admits a bubble, we can turn to the question of whether a policy intervention to raise the real interest rate in order to dampen the bubble is desirable. Recall from the previous section that a reduced-form way to capture monetary policy tightening in our endowment economy is to destroy some of the endowment. We will use the same approach in our economy with credit. We confirm that destroying part of the endowment at date 0 is still associated with higher interest rates, increasing both the interest rate on loans and the expected return on the asset. It will also dampen the bubble, at least under certain conditions. However, we find that, as our model is currently specified, a tighter monetary policy cannot make society better off even though the bubble is distortionary. We then argue that this is because the bubble only causes harm while it persists, not when it bursts. When we allow for costly default in the next subsection, the size of the bubble will matter for the collapse of the bubble affects agents. In that case, tighter monetary policy could be welfare improving.

As in the previous section, we capture the contractionary effects of monetary policy in an economy with price rigidity by looking at the effect of destroying the endowment at date 0. That is, we consider an alternative path for endowments in which

$$
e_t = \begin{cases} e - \tau & \text{if } t = 0 \\ e & \text{if } t = 1, 2, 3, ... \end{cases}
$$

Since the economy at dates $t \geq 1$ is identical to our original economy, the intervention has no effect at these dates. We can therefore focus on what happens at date 0. The equilibrium conditions at date 0 are the same as before, except that the endowment has been reduced. Market clearing requires that the amount borrowed by entrepreneurs for production and speculation must equal the reduced endowment, $e - \tau$, so

$$
N\left(1 + R_0^D\right) + p_0^D = e - \tau \tag{21}
$$

The zero profit condition for speculators is the same as before, where now we use the fact that $p_t^D = p^D$ for all $t \geq 1$:

$$
\left(1 + R_0^D\right) p_0^D = p^D + D \tag{22}
$$

Using the second equation to express the interest rate on loans $1 + R_0^D$ at date 0 in terms of the asset price p_0, we obtain a single equation in p_0,

$$
N\left(\frac{p^D + D}{p_0}\right) + p_0 = e - \tau
$$

Differentiating this with respect to τ, we get

$$
\left(-\frac{p^D + D}{\left(p_0^D\right)^2} N'\left(\frac{p^D + D}{p_0^D}\right) + 1\right) \frac{dp_0^D}{d\tau} = -1 \tag{23}
$$

20

Since $N'(y) < 0$ is negative for all $y \geq 1$, it follows that $-1 < \frac{dp_0^D}{d\tau} < 0$. Destroying the endowment will lower the price of the asset at date 0.

The fact that a higher τ lowers p_0^D implies that both the expected return on the asset $1 + \bar{r}_0^D$ at date 0 and the interest rate on loans $1 + R_0^D$ at date 0 increase with τ, since

$$1 + \bar{r}_0^D = \frac{\pi \left(p^D + D\right) + (1 - \pi) \left(p^d + d\right)}{p_0^D}$$

and

$$1 + R_0^D = \frac{D + p^D}{p_0^D}$$

In both cases, the numerator is a constant while the denominator is decreasing in τ. By contrast, the effect of increasing τ on the expected return on loans $1 + \overline{R}_0^D$ at date 0 is ambiguous. Recall that the expected return on loans $\overline{R}_0^D$ is equal to the weighted average $(1 - \alpha_0) R_0^D + \alpha_0 \bar{r}_0^D$ with $\alpha_0 = p_0^D / (e - \tau)$. The effect of increasing τ on α_0 is ambiguous, since both the numerator and denominator are decreasing in τ. For α_0 to increase with τ requires that $\frac{dp_0^D}{d\tau}$ be close to -1. From (23), this requires that $N'\left(1 + \frac{D}{p^D}\right)$ be close to 0. Hence, if $N'\left(1 + R^D\right)$ is sufficiently close to 0, increasing τ will raise the expected return on loans because it increases both R_0^D and r_0^D as well as the share of loans that earn R_0^D. Intuitively, if there are only a few marginal entrepreneurs who would be affected by the crowding out of resources due to contractionary monetary policy, tighter monetary policy will primarily drive out speculation and shift the composition of borrowers towards those who wish to produce.

If increasing τ raises the expected return on loans $\overline{R}_0^D$, it will also decrease the fundamental value of the asset as we have defined it. To see this, note the fundamental value of the asset f_0^D at date 0 can still be defined recursively as

$$f_0^D = \frac{\pi \left(d + f^d\right) + (1 - \pi) \left(D + f^D\right)}{1 + \overline{R}_0^D} \tag{24}$$

An intervention that raises $\overline{R}_0^D$ will thus reduce the fundamental value f_0^D at date 0. Hence, both the price of the asset p_0^D and the fundamental value f_0^D at date 0 decline with τ. To determine the effect of τ on the bubble term $b_0^D = p_0^D - f_0^D$ involves some tedious algebra we delegate to Appendix A. There, we show that increasing τ dampens the bubble, as summarized in the next proposition.

Proposition 6 *For $N'\left(1 + \frac{D}{p^D}\right)$ sufficiently small, the expected return on loans $\overline{R}_0^D$ at date 0 is increasing in τ and the bubble b_0^D at date 0 is decreasing in τ at $\tau = 0$.*

To recap, under some additional restrictions on the distribution of productivity across entrepreneurs, contractionary monetary policy will dampen the bubble in our economy just as in the dynamically inefficient economy. The key difference is that now there may be a reason for a policymaker to intervene. This is because the marginal entrepreneur in our economy can earn a return equal to the interest rate on loans, R^D, which exceeds the expected return on the asset, $\bar{r}^D$. Society would thus be better served by diverting

some of the resources used to buy the asset into funding entrepreneurial activity. Thus, there may be scope for government intervention. But it turns out that even though tighter monetary policy can dampen the bubble, it does not help divert resources towards entrepreneurial activity. To the contrary, it only further crowds out such activity. Recall that we argued above that an increase in τ will raise the interest rate on loans R_0^D at date 0. This means the marginal entrepreneur who produces must be even more productive, meaning there must be fewer entrepreneurs producing than without the intervention.

The crowding out of entrepreneurial activity means that a monetary contraction cannot be welfare improving, even though it dampens the bubble. To see why, let us consider how each cohort fares under the intervention. The cohort that is old at date 0 gets to consume the proceeds p_0^D from selling their asset, which is decreasing in τ. The cohort born at date 0 consume the return to their investments from date 0, namely the proceeds from entrepreneurial activity initiated at date 0 and the returns on the assets they bought. In expectation, these are given by

$$\int_{1+R_0^D}^{\infty} \left[-N'(y)\right] y \, dy + \pi \left(p^d + d\right) + (1-\pi)\left(p^D + D\right) \tag{25}$$

Since R_0^D is increasing in τ, this expression too is decreasing in τ. Finally, since the equilibrium is unchanged from date $t=1$ on, the consumption of all other cohorts is unchanged. The fact that both of the first two cohorts consume less as τ increases means it will be impossible to make all cohorts better off: The only way to make the old at date 0 better off is if the young at date 0 give them some of their resources, but this would leave the young with even less to consume at date 1, unless some other cohort got to consume less. Intuitively, the presence of asymmetric information in this economy allows less productive entrepreneurs to speculate, which raises the interest rate on loans and crowds out entrepreneurship. Increasing rates further thus fails to correct the distortion in this economy, and only exacerbates the misallocation of resources. Dampening the bubble by contracting economic activity does not undo the distortion associated with bubbles. In Appendix C, we confirm that leaning against the wind cannot make society better in the case where the quantity of assets is endogenous as well, even though destroying the endowment reduces the quantity of bubble assets. Increasing rates may dampen bubbles, and bubbles in our setting are distortionary, but such interventions do more harm than good.

2.5 Costly Default and Welfare-Increasing Interventions

An important caveat to our result from the previous subsection on the counterproductive nature of raising rates is that our setup abstracts from one of the main reasons policymakers cite for intervening against bubbles, namely that it can help temper the negative consequences if and when a bubble collapses. For example, one concern we already discussed is that if loans to those who speculate on the asset are intermediated, a collapse in asset prices may force financial intermediaries to default, which would limit their ability to provide credit to future entrepreneurs. Yet in our model, the bubble only causes harm while it is present, when it distorts the allocation of resources, not when it collapses.

In principle, we should explicitly add intermediation to our model. We follow a simpler approach by assuming that when borrowers default, the lenders who financed them incur default costs that are proportional to the amount they lend. That is, in addition to the vanishing utility cost ϕ that borrowers suffer if they default, we now assume savers incur a cost of $\Phi > 0$ per unit lent to those agents who default. While we model this as a cost of verifying and recovering payment from borrowers who are in default, this assumption implies that once the bubble burst, available resources will fall in proportion to the amount of loans taken out against assets. As we point out below, this is equivalent to allowing a recession when the bubble bursts. In line with the evidence on the costs of financial crises, we will think of Φ as large.

Before turning to the effects of any policy interventions, we first need to discuss how adding a cost of default would affect the equilibrium in our economy absent any intervention. The equilibrium conditions (13) and (15) remain unchanged: The resources of the young in each period would still either finance entrepreneurs or be used to buy the asset, and the interest rate on loans must still equal the maximal return an agent could earn from buying the asset, which is unaffected by Φ. Equilibrium prices are therefore the same as before. However, savers might no longer strictly prefer lending to buying the asset themselves. The expected return from buying an asset, $1 + \bar{r}^D$, is unchanged, but the expected return on a loan will now reflect the cost Φ incurred when their borrower defaults. Once again, let α denote the share of loans issued that are invested in the asset. Then the expected return on loans is given by

$$1 + \overline{R}^D = (1 - \alpha)\left(1 + R^D\right) + \alpha\left(1 + \bar{r}^D - \pi\Phi\right)$$

If only entrepreneurs bought the asset, as was the case when $\Phi = 0$, then α would equal p^D/e. But holding α fixed, if we increase default costs Φ by enough, $1 + \overline{R}^D$ will fall by enough to make savers indifferent between buying the asset and making loans. Since equilibrium prices are determined independently of Φ, the only variable that can adjust to sustain an equilibrium is α. Low productivity entrepreneurs are indifferent about buying the asset in equilibrium, so they would always be willing to buy fewer assets and let savers buy them instead. Specifically, the share of loans used to buy the asset α must be such that savers are indifferent between buying the asset directly and making loans, meaning $\overline{R}^D = \bar{r}^D$. The value of α that equates these two returns is given by

$$\alpha^* = \frac{R^D - \bar{r}^D}{\pi\Phi + R^D - \bar{r}^D}$$

Recall from our previous discussion that the asset can be viewed as a bubble only if the rate $1 + \overline{R}^D$ that we use to discount dividends in defining the fundamental value exceeds $\bar{r}^D$. Hence, for sufficiently large Φ, the asset would no longer be a bubble. We therefore restrict attention to the case where, given a value for Φ, the ratio $p^D/e < \alpha^*$. That is, we assume the share of the asset in the total endowment of savers is small, ensuring that expected default costs are small enough that $\overline{R}^D$ exceeds $\bar{r}^D$ and the asset is a bubble.

Under the additional restriction that $p^D/e < a^*$, destroying τ units of the endowment at date 0 will now, under certain conditions, yield a Pareto improvement. Once again, consider the amount the different cohorts can consume. The old at date 0 consume p_0^D, which is decreasing in τ. Those who are born at date 0 and consume at date 1 again consume the returns on their investment, although now we have to net out

default costs. Their expected consumption is thus equal to

$$\int_{1+R_0^D}^{\infty} [-N'(y)] \, y \, dy + \pi \left(p^d + d \right) + (1-\pi) \left(p^D + D \right) - \pi \Phi p_0^D \tag{26}$$

All remaining cohorts born after date 0 are unaffected. Note that we can reinterpret (26) to mean that when dividends fall, the asset price is not p^d but $p^d - \Phi p_0^D$, as would be the case if young savers at date 1 spent Φp_0^D less on the asset they buy from the old because their endowment was smaller. In this sense, our model is equivalent to one in which default causes a recession that impoverishes the young.

The two middle terms of the expression in (26) are independent of τ, so we are left with two terms:

$$\int_{1+R_0^D}^{\infty} [-N'(y)] \, y \, dy - \pi \Phi p_0^D$$

Differentiating and substituting in for $1 + R_0^D$ reveals the effects of increasing τ:

$$\left(-\frac{p^D + D}{p_0^2} N' \left(\frac{p^D + D}{p_0} \right) - \pi \Phi \right) \frac{dp_0^D}{d\tau}$$

This expression will be positive if and only if

$$-\frac{p^D + D}{p_0^2} N' \left(\frac{p^D + D}{p_0} \right) < \pi \Phi$$

Thus, if both π and Φ are sufficiently large, and the mass of entrepreneurs who are marginal is sufficiently small, increasing τ would allow the cohort of agents born at date 0 to consume more. In contrast to the case where $\Phi = 0$, we now have one cohort that is better off and another that is worse off. This allows for the possibility that all agents can be made better off if we allowed for lump-sum transfers.

To compensate the old at date 0 requires a transfer of $\left(p_0^D - p^D \right)$ resources from the young to the old at date 0. If this is small, those resources would have earned an expected return of approximately $1 + \overline{R}_0^D$. Thus, to ensure both the old and young cohorts at date 0 are no worse off requires that the consumption of the latter cohort increase even after netting out the $\left(1 + \overline{R}_0^D \right) \left(p_0^D - p^D \right)$ required to keep the old at date 0 no worse off. This will be true if and only if

$$1 + \overline{R}_0^D - \frac{p^D + D}{p_0^2} N' \left(\frac{p^D + D}{p_0} \right) < \pi \Phi$$

The threshold Φ that ensures all agents can be made better off is higher than the one that ensures the young are better off. Still, there exists a value of Φ large enough that increasing τ will be Pareto-improving.

To better appreciate these results, it is helpful to contrast them with those in previous work on the welfare implications of intervening against bubbles by Grossman and Yanagawa (1993). They introduce production externalities into an overlapping generations economy which imply that eliminating bubbles can make future generations better off, just as in our analysis. But in their setup, the only way to make all those generations better off is by taking resources from the old at date 0 and deploying them to more productive

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uses. This means that the only way to make the young better off is to take away resources from the old and leave them worse off. The reason the intervention we consider can make all agents better off is that in our economy we can make the young better off without taking resources away from the old and deploying them to another use. Instead, the problem is that transferring resources to the old through debt financing is costly in a way that transferring them directly is not. The welfare gains we find come not from deploying resources more efficiently, but from discouraging borrowing and reducing the amount agents would default on if their speculation failed.[5]

To recap, the argument for leaning against the wind in our model is based on the benefits of mitigating the harm due to the collapse of a bubble rather than the benefits to undoing any distortions that bubbles introduce when they are present. This is similar in spirit to Svensson (2014). He also argued that leaning against the wind can be counterproductive, although he focused not on whether contractionary monetary policy exacerbates distortions but on whether it increases agents' debt burden. At the same time, he recognized there may be potential benefits from mitigating the fallout from the collapse of a bubble. He focused on whether tighter monetary policy would increase the likelihood of a financial crisis and argued the effect was small. By contrast, in our model the probability that the bubble will burst in any given period is π, which is exogenous and unaffected by monetary policy. But even if raising rates has no effect on the probability of a crisis, it will have an effect on the severity of the financial crisis, and that effect might in principle be quite large.[6]

2.6 Threats of Future Intervention

Our analysis so far suggests that leaning against the wind can be useful, but only if the costs associated with defaults triggered by the collapse of a bubble are sufficiently large. In this last subsection, we argue that even when the cost of default is small, there may still be scope for using monetary policy to make society better off. In particular, suppose that rather than raising rates in the face of a bubble, the monetary authority promises to raise rates in the future if a bubble persists. The reason this intervention can be useful is that it can serve to dampen the bubble without immediately inflicting the pain associated with contractionary monetary policy. In particular, the agents in our model who borrow to buy risky assets are motivated by the maximal profits they can earn. These accrue if the bubble persists. By threatening to intervene and drive down asset prices in that state of the world, the monetary authority can make buying the asset today less profitable, which would lower the price of the asset without any actual tightening. That

---

[5] Another way around Grossman and Yanagawa's impossibility result is if the old at date 0 are not endowed with assets but must produce them. As we discuss in Appendix C, in that case we can make the young better off by redirecting resources to produce fewer assets and deploying them to alternative uses even as we maintain the same consumption for the old.

[6] Biswas, Hanson, and Phan (2017) offer a different reason for why the collapse of a bubble can be socially costly. In their model, a bubble improves the allocation of resources and leads to higher wages. At the same time, they assume wages are downwardly rigid. When the bubble collapses, wages are stuck at too high of a level and economic activity contracts. This suggests an additional benefit to dampening the bubble above and beyond costs associated with default.

can free up resources for entrepreneurial activity as well as reduce the expected default costs if the bubble should collapse. If the bubble is very likely to burst, the monetary authority can achieve these benefits and will likely not have to intervene.

Consider the economy from the previous subsection in which default is costly. Let us examine an intervention that involves a commitment to destroy τ units of the endowment at date 1 if $d_1 = D$ and the bubble persists. Once again, the effect of policy can be seen by analyzing the equilibrium for an economy with a particular path for the endowment. The endowment path is now given by

$$
e_t = \begin{cases} e - \tau & \text{if } t = 1 \text{ and } d_1 = D \\ e & \text{otherwise} \end{cases}
$$

As before, the equilibrium from date $t = 2$ on will be unaffected by policy. Moreover, the equilibrium at $t = 1$ corresponds to the equilibrium we solved for at date 0 in the previous subsection. The only case that still needs to be analyzed is date 0, the period before the endowment is destroyed.

Market clearing at date 0 is the same as in (13), i.e.,

$$
N \left(1 + R_0^D \right) + p_0^D = e
$$

The interest rate on loans $1 + R_0^D$ at date 0 must ensure that buying the risky asset is unprofitable even in the most favorable state. A similar logic as before implies this will be if the dividend remains high at date 1. This implies

$$
1 + R_0^D = \frac{D + p_1^D}{p_0^D}
$$

As before, we can substitute in $R_0^D = \rho \left(p_0^D \right)$ to rewrite the latter equation as

$$
\left(1 + \rho \left(p_0^D \right) \right) p_0^D = D + p_1^D
$$

Differentiating with respect to τ yields

$$
\left[\left(1 + \rho' \left(p_0^D \right) \right) p_0^D + 1 + \rho \left(p_0^D \right) \right] \frac{dp_0^D}{d\tau} = \frac{dp_1^D}{d\tau}
$$

We already showed that $\frac{dp_1^D}{d\tau} < 0$, since above we argued that in the period where τ units are destroyed, the price of the asset falls. This result is unaffected by the presence of default costs, since those only affect p_1^d and not p_1^D. It follows that $\frac{dp_0^D}{d\tau} < 0$ as well, i.e., a threat of tighter monetary policy at date 1 if the bubble persists will reduce the price of the asset at date 0. Intuitively, if speculators know the monetary authority will intervene and drive down the price of the asset if the bubble persists, they will find buying the asset less attractive. This would cause the price of the asset to fall even before any intervention.

From the market-clearing condition (13), a lower p_0^D requires a higher value for $N \left(1 + R_0^D \right)$. Since $N'(y) < 0$ for all $y \geq 0$, this implies that $\frac{dR_0^D}{d\tau} < 0$. Hence, a threat to increase τ if the bubble persists will divert resources at date 0 from those who wish to buy assets to those who intend to produce.

We now argue that this intervention can increase welfare. Again, consider the effect on each cohort. Agents who are old at date 0 still consume the proceeds they earn from selling the asset, p_0^D, which is lower. Agents born at date 0 expect to consume

$$\int_{1+R_0^D}^{\infty} \left[-N'(y) \right] y dy + \pi \left(p^d + d \right) + (1 - \pi) \left(p_1^D + D \right) - \pi \Phi p_0^D$$

Agents born at date 1 will consume the same amount if $d_1 = d$. If $d_1 = D$, they will expect to consume

$$\int_{1+R_1^D}^{\infty} \left[-N'(y) \right] y dy + \pi \left(p^d + d \right) + (1 - \pi) \left(p^D + D \right) - \pi \Phi p_1^D$$

All cohorts born at date 2 on will be unaffected.

We want to argue that when π is large, the above intervention can be Pareto improving. For π close to 1, the cohort born at date 0 will be better off: Since $\frac{dR_0^D}{d\tau} < 0$, they will enjoy the results of greater entrepreneurial activity at date 0 and they will suffer smaller losses in case of default. The latter benefit applies even if we took the resources that went to fund entrepreneurial activity and gave them back to the cohort that was old at date 0 directly to make sure they are no worse off. Since transferring resources directly to the old does not involve debt financing, it still achieves a benefit of reducing default costs. However, we now need to compensate the cohort born at date 1, since although they will be unaffected if the bubble bursts at date 1, will be made worse if the bubble persists and the monetary authority intervenes. But as $\pi \to 1$, this becomes an unlikely scenario. As long as the cohort born at date 0 shares any of their benefits from smaller default costs with the cohort born at date 1 if the bubble collapses, the cohort born at date 1 will also be better off ex-ante. Thus, as long as the bubble is very likely to collapse, a threat to lean against the wind should a bubble persist can be used to make all agents better off *ex ante* even though the intervention itself makes agents worse off *ex post*. Of course, this requires that the central bank can credibly commit to intervene if the bubble persists. Whether such promises are credible is an interesting question, but beyond the scope of this paper.

3   Conclusion

In this paper, we developed a framework to explore the merits of the lean-against-the-wind approach towards bubbles. We argued that tighter monetary policy helps dampen bubbles, even though when Galí (2014) first introduced nominal rigidities into models that allowed bubbles, he found that raising rates can amplify bubbles in some circumstances. The problem with his setting for the lean-against-the-wind view is that the bubble in his setup alleviates the friction that allows a bubble to arise in the first place, so there is no reason to act against the bubble. We therefore considered a different set of frictions that give rise to bubbles that serve no useful social role and looked at the effects of monetary policy in that environment. Our model speaks to many of the issues that have come up in debate over how policymakers ought to respond to bubbles. For example, we find that raising rates is a blunt tool that, even if it dampens the bubble, exacerbates the distortions a bubble causes. However, if the collapse of a bubble leads those who

borrowed against the bubble to default, which in turn incurs costs that are increasing in the amount of borrowing against the bubble, raising rates to dampen bubbles can increase welfare. Finally, even when raising rates is bad for welfare, a commitment in advance to raise rates if a bubble persists may be welfare improving since it can dampen bubbles when they first arise without immediately inflicting the pain that goes with monetary tightening.

One of the themes that emerges from our analysis is the importance of the friction that allows bubbles to emerge. The information frictions we consider imply bubbles serve no useful role, which allows scope for intervention. Other models, such as Allen, Morris, and Postlewaite (1993), Conlon (2004), and Doblas-Madrid (2012), have also generated bubbles in models with information frictions but which do not rely on credit. In those models, bubbles do not serve a useful role either. By contrast, in models where bubbles arise because of dynamic inefficiency and serve to ameliorate that inefficiency, eliminating a bubble would be inefficient, at least in the simplest versions of these models. The same is true in models where bubbles arise because of binding borrowing constraints. Various papers have now demonstrated that when firms cannot borrow all the resources they could use productively, bubbles can arise as a way to transfer resources to firms who cannot borrow these resources. Examples include Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2012), Hirano and Yanagawa (2017), Miao and Wang (2015), and Martin and Ventura (2016).[7] Recent work by Dong, Miao, and Wang (2017) and Ikeda (2017) have looked at the effect of monetary policy in environments where bubbles arise because of borrowing constraints. And indeed, neither paper concludes that monetary policy should aggressively fight bubbles but manage them.

Finally, given our model is both simple and seems to connect to various issues the policymakers cite in regard to bubbles, we view our model as a potential workhorse for exploring various policy questions. For example, as we noted in the Introduction, the policy debate in the wake of the recent financial crisis has focused on whether interest rate policy or macroprudential policy offer a better response to a potential bubble. If we extended our model to allow for entrepreneurs who differ both in productivity and in initial endowments, we could explore the effect of policies like leverage restrictions. Here, we should emphasize that while our analysis reveals when leaning against the wind can improve welfare, we are not arguing that such a policy is optimal. Understanding the merits of different interventions remains an area for future work. Another way in which we could extend our model is to more explicitly model the role of financial intermediaries to have a better understanding of how to model (and calibrate) the cost of default. Still another extension would be to incorporate our analysis into a small monetary open economy model as in Galí and Tomacelli (2005) to explore how policy implications differ with the possibility of international capital flows that might flow into a country that successfully increases its real interest rate. And given the potential benefits we document from a commitment to raise rates if a bubble persists, understanding the problem of time inconsistency and how to deal with it stands as another direction for future work.

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[7]Earlier work by Kocherlakota (1992), Santos and Woodford (1997) and Kocherlakota (2008) argued that the existence of borrowing constraints can give rise to bubbles in endowment economies where consumers are borrowing constrained. Rocheteau and Wright (2013) argue that, at least in some cases, constraints on consumers and firms are isomorphic.

$$e_t = (1+g)^t e_0$$
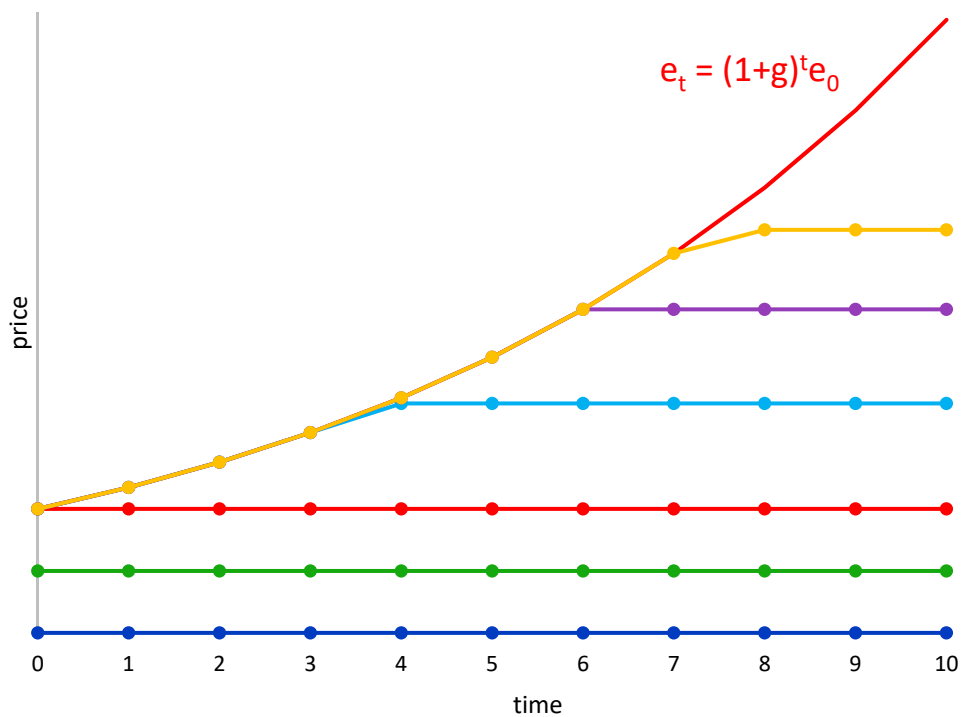
Figure 1: Sample equilibrium price paths for the case where d = 0
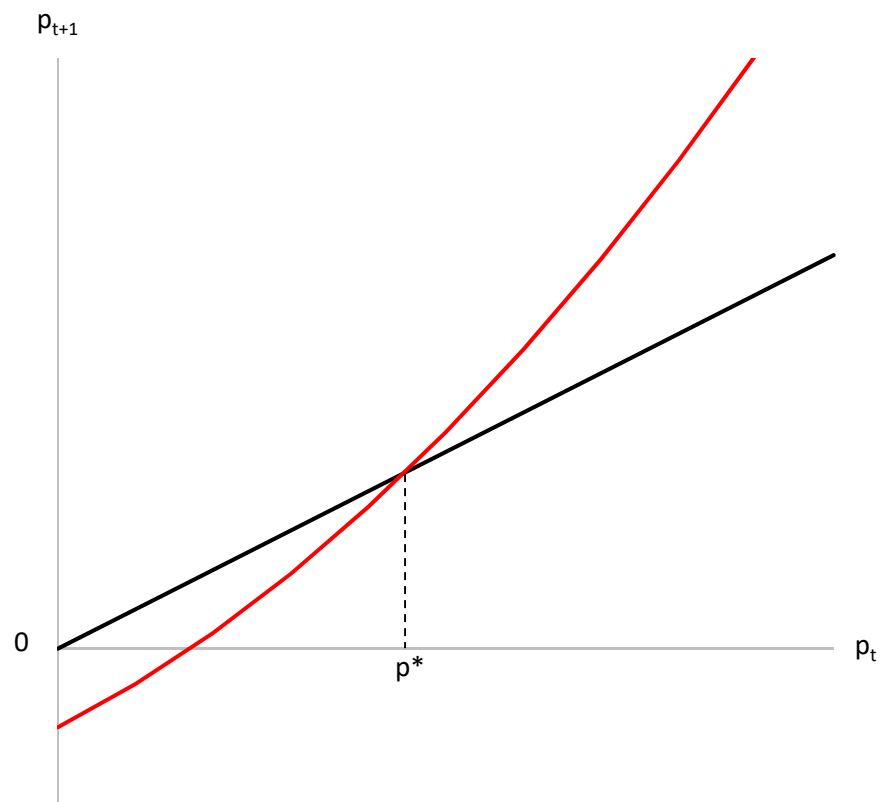
Figure 2: Equilibrium relationship between p_t and p_{t+1}

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Appendix A: Proofs of Propositions

Proof of Proposition 1: In the text, we argued that $r_t = 0$ is an absorbing state. That is, there exists a t^* where $0 \leq t^* \leq \infty$ such that $r_t > 0$ for $t < t^*$ and $r_t = 0$ for $t \geq t^*$. For $t < t^*$, we know that $r_t > 0$ implies storage is dominated, and so $p_t = e_t$ for $t < t^*$. Since $r_t = 0$ for $t \geq t^*$ we can use (3) to conclude that $p_{t+1} = p_t$ for $t \geq t^*$, and by induction we can infer $p_t = p_{t^*}$ for all $t \geq t^*$.

The last step is to show that at date t^*, any $\rho \in [e_{t^*-1}, e_{t^*}]$ can be an equilibrium, and only these values can be an equilibrium. Since $r_t \geq 0$, $p_{t^*} \geq p_{t^*-1} = e_{t^*-1}$. Since $p_t \leq e_t$ at all dates t, this is also true for date t^*. For any $\rho \in (e_{t^*-1}, e_{t^*}]$, the rate of return on the asset to those purchasing the asset at date $t^* - 1$ will be positive, so they would buy the asset at this price at date $t^* - 1$ and sell all their holdings at date t^*. Since $r_{t^*} = 0$, the young at date t^* are indifferent between storage and buying the asset, so they would be willing to buy any amount the old sell. Hence, we can construct an equilibrium where $p_{t^*} = \rho$ for any $\rho \in (e_{t^*-1}, e_{t^*}]$. ∎

Claim: Given any two equilibrium paths $\{p_t\}_{t=0}^{\infty}$ and $\{p_t'\}_{t=0}^{\infty}$, define $r_t = p_{t+1}/p_t$ and $r_t' = p_{t+1}'/p_t'$. If $r_t \geq r_t'$ for all t and $r_0 > r_0'$, then $p_0 \geq p_0'$ and $p_t > p_t'$ for all $t \geq 1$.

Proof: Since $r'_0 \geq 0$, then $r_0 > 0$. From Proposition 1, this implies $p_0 = e_0$. At the same time, since in equilibrium the price of the asset cannot exceed the endowment of young agents, we know $p'_0 \leq e_0$. Hence, $p_0 \geq p'_0$. Next, since $p_{t+1} = (1 + r_t) p_t$ and $p'_{t+1} = (1 + r'_t) p'_t$, we can use the fact that $r_0 > r'_0$ to show by induction that $p_t > p'_t$ for all $t \geq 1$. In particular, for $t = 1$, we have

$$p_1 = (1 + r_0) p_0 > (1 + r'_0) p'_0 = p'_1$$

Next, if $p_t > p'_t$, then since $r_t \geq r'_t$, it follows that

$$p_{t+1} = (1 + r_t) p_t > (1 + r'_t) p'_t = p'_{t+1}$$

The claim then follows. ∎

Proof of Proposition 2: We prove a more general result which implies the Proposition. In particular, we show that for a sequence of nonnegative dividends $d_t \geq 0$, there is a unique equilibrium iff $\sum_{t=0}^{\infty} d_t = \infty$. Since the case where $d_t = d > 0$ satisfies this condition, the claim will follow.

Suppose $d_t \geq 0$ for all t and $\sum_{t=0}^{\infty} d_t = \infty$. We argue that the interest rate r_t must be positive at all dates. For suppose $r_t = 0$ for some date t. Then $p_{t+1} = p_t - d_t$ and since $p_t \leq e_t$, it follows that $p_{t+1} < e_t < e_{t+1}$, which implies that $r_{t+1} = 0$. By this logic, the price declines by d_{t+i} at each date $t + i$, for $i = 1, 2, \dots$. Since $\sum_{t=0}^{\infty} d_t = \infty$ implies $\lim_{k \to \infty} \sum_{i=1}^{k} d_{t+i} = \infty$, there must be a date $t + k$ such that $p_{t+k} = p_t - \sum_{i=1}^{k} d_{t+i} < 0$, i.e., there must be some date $t + k$ at which the price of the asset turns negative. But this is incompatible with equilibrium. Hence, $r_t > 0$ at all dates. This implies storage is dominated, and so the unique equilibrium price is $p_t = e_t$ for all t. The corresponding return on holding the asset is then

$$1 + r_t = \frac{d_t + p_{t+1}}{p_t} = (1 + g) + \frac{d_t}{e_t}$$

We can further show that the condition $\sum_{t=0}^{\infty} d_t = \infty$ is a necessary condition for uniqueness. The proof is constructive. Suppose $\sum_{t=0}^{\infty} d_t < \infty$. Then there must be some date t^* such that $e_{t^*} - \sum_{t=t^*}^{\infty} d_t > 0$. We now propose an equilibrium price sequence as follows. For $t < t^*$, set $p_t = e_t$; for $t = t^*$, set p_t to be any value such that $\max \left\{ e_{t^*-1} - d_{t^*}, \sum_{s \geq t^*} d \right\} < p_t \leq e_{t^*}$; and for $t > t^*$, let $p_t = p_{t*} - d_{t^*+1} - \dots - d_t$. The price sequence is positive for every date t because $p_t = e_t > 0$ for $t < t^*$ and $p_t \geq p_{t*} - \sum_{s \geq t^*} d_s > 0$ for $t > t^*$. The price sequence p_t corresponds to an equilibrium because, for $t < t^*$, the interest rate is positive and the entire endowment is invested in the asset and, for $t \geq t^*$, the interest rate is zero. Thus, there are multiple equilibria in this case. ∎

Proof of Proposition 6: To compute the effect of τ on b_0^D, we first rearrange the fundamental value f_0^D at date 0. Recursively, we know

$$f_0^D = \frac{\pi \left(d + f^d \right) + (1 - \pi) \left(D + f^D \right)}{1 + \overline{R}_0^D}$$

Next, recall that

$$1 + \overline{R}_0^D = (1 - \alpha) \left(1 + R_0^D \right) + \alpha \left(1 + \overline{r}_0^D \right)$$

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where

$$\alpha = p_0^D / (e - \tau)$$

Substituting in, we get

$$f_0^D = \frac{\pi \left(d + f^d\right) + (1 - \pi) \left(D + f^D\right)}{(1 - \alpha) \left(1 + R_0^D\right) + \alpha \left(1 + \bar{r}_0^D\right)}$$

Next, we can write the interest rate on loans and the expected return on the asset as

$$
\begin{aligned}
1 + R_0^D &= \left(D + p^D\right)/p_0^D \\
1 + \bar{r}_0^D &= \pi \left(d + p^d\right) + (1 - \pi)\left(D + p^D\right)
\end{aligned}
$$

Substituting in these two expressions, we get

$$f_0^D = \frac{\pi \left(d + f^d\right) + (1 - \pi)\left(D + f^D\right)}{(1 - \alpha)\left(D + p^D\right) + \alpha\left(\pi\left(d + p^d\right) + (1 - \pi)\left(D + p^D\right)\right)} p_0^D$$

and so

$$b_0^D = \left[1 - \frac{\pi \left(d + f^d\right) + (1 - \pi)\left(D + f^D\right)}{(1 - \alpha)\left(D + p^D\right) + \alpha\left(\pi\left(d + p^d\right) + (1 - \pi)\left(D + p^D\right)\right)}\right] p_0^D$$

We already established in the text that $\frac{dp_0^D}{d\tau} < 0$. To show that $\frac{db_0^D}{d\tau} < 0$, it will suffice to show that the expression in the brackets is decreasing in τ. The only place where τ appears in this expression is in $\alpha = \frac{p_0^D}{e - \tau} = 1 - \frac{N\left(\frac{D + p^D}{p_0^D}\right)}{e - \tau}$. In the limit as $N'\left(1 + \frac{D}{p^D}\right)$ tends to zero, α must decrease with τ. It follows that the expression in the brackets is decreasing in τ, and hence so does b_0^D.

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Appendix B: Monetary Policy in a Production Economy

In this Appendix we consider a production economy that is analogous to the economy we examine in the text, and then use it to study monetary policy and nominal price rigidity as in Galí (2014). As a brief preview, young agents engage in production which nets them all of the income they earn over their lifetime. Since they only value consumption when old, they will need to convert their income into consumption while old. The can store the goods they earn as income, exchange these goods for the one asset in the economy, or exchange for money issued by the central bank.

We study equilibria in which there is no trade in money. The central bank announces a nominal interest rate, and the real return on the asset together with the inflation in goods prices adjust to make sure agents do not wish to hold money. In the bulk of what follows, we will assume the yields a positive dividend $d > 0$, so that agents spend all of their income on buying assets in equilibrium. The case where $d = 0$ introduces additional equilibria in which agents are indifferent between buying the asset and storing goods. Although in the text we begin with the case where $d = 0$, we defer the discussion of this case towards the end.

We begin by describing the economic environment in detail, and then define and discuss equilibria for this setup.

B.1 Endowments and Preferences

Consider an economy in which a new cohort of mass 2 is born each period and lives for two periods. Half of each cohort is endowed with a unit of labor in their first period of life. We will refer to these agents as workers. The other half is endowed in their first period of life with the knowledge of how to use labor to produce. We will refer to these agents as producers. Note that this structure is more similar to the Adam (2003) monetary OLG model, in which agents are heterogeneous and only earn income in their first period of life, than Galí's model in which agents and homogeneous and work as workers when young and producers when old. We prefer the Adam formulation to ensure agents only earn income when young as in our endowment economy.

Agents have the same preferences as in (1), i.e., the only value consumption when old. However, those agents who are workers also incur a disutility from labor. In particular, a worker born at date t who supplies n_t units of effort and who consumes c_t^w and c_{t+1}^w when young and old, respectively, has utility given by

$$u\left(c_t^w, c_{t+1}^w, n_t\right) = c_{t+1}^w - v_t\left(n_t\right) \tag{27}$$

where

$$
\begin{aligned}
v_t\left(n_t\right) &= (1+g)^t \, v\left(n_t\right) \\
&\equiv A_t v\left(n_t\right)
\end{aligned}
$$

for some convex differentiable function $v(\cdot)$. We assume $\lim_{n\to 0} v'(n) = 0$ and $\lim_{n\to 1} v'(n) = \infty$ to ensure an interior solution. Later cohorts value leisure more. Below we assume these cohorts are also more productive, and the two will cancel out.

For those agents who are producers, utility is only defined only over consumption. A producer born at date t who consumes c_t^p and c_{t+1}^p when young and old, respectively, has utility given by

$$u\left(c_t^p, c_{t+1}^p\right) = c_{t+1}^p \tag{28}$$

B.2 Production and Pricing

Workers and producers combine efforts to produce a variety of intermediate goods which are then combined to form final goods. We index the unit mass of producers by $i \in [0, 1]$. Each producer knows how to produce a different intermediate good, indexed by the same i as the producer. If producer i hires n_{it} units of labor at date t he can produce y_{it} units of good i, where

$$y_{it} = (1+g)^t n_{it} \equiv A_t n_{it}$$

Thus, as we anticipated above, later cohorts are more productive. The different intermediate goods can be combined to produce final goods according to a Dixit-Stiglitz production function, i.e., y_{it} of each good $i \in [0, 1]$ can be combine to yield Y_t of final goods according to

$$Y_t = \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{29}$$

Any agent can produce final goods this way, so the production of these is goods is perfectly competitive. Any agent who produces final goods producers will choose the amount of intermediate goods y_{it} to maximize their profits. If we let P_t denote the price of final goods, any such producer will solve

$$\max_{y_{it}} P_t \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} y_{it} di$$

The first-order condition with respect to y_{it} implies

$$y_{it} = Y_t \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}} \tag{30}$$

To obtain the cost of producing one unit of the final good, we set $Y_t = 1$ and compute the price of the optimal bundle of intermediate goods given intermediate goods prices, $y_{it} = \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}}$:

$$\int_0^1 P_{it} y_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

We assume the market for final goods is competitive. In this case, the price of final goods P_t must equal the per unit cost of producing a good. The price of final goods P_t will thus equal the Dixit-Stiglitz aggregator of intermediate good prices:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{31}$$

Each intermediate goods producer chooses the price P_{it} to maximize expected profits given demand (30) and the wage, which we denote by W_t. We will consider cases both where price setters observe the interest rate $1 + i_t$ that the central bank sets before choosing their prices, which we will call the flexible price case, and where price setters set prices before observing the nominal interest rate, which we will call the sticky price case. Let Ω_{it} denote producer i's information set at date t when they choose their price. Then each producer i will set P_{it} to solve

$$\max_{P_{it}} E\left[\left(P_{it} - \frac{W_t}{A_t}\right) Y_t \left(\frac{P_t}{P_{it}}\right)^{\frac{1}{\sigma}} \middle| \Omega_{it}\right]$$

The optimal price for each intermediate goods producer is then just

$$P_{it} = \frac{E\left[W_t Y_t | \Omega_t\right]}{(1 - \sigma) A_t E\left[Y_t | \Omega_t\right]} \tag{32}$$

Given the symmetry in production technology and information sets, all producers will charge the same price. Hence, the amount of intermediate goods produced in equilibrium will be the same, i.e., y_{it} is the same for all $i \in [0, 1]$. All producers will therefore hire the same amount of labor, so $n_{it} = n_t = \int_0^1 n_{it} di$. Substituting in, we can express aggregate output Y_t as a function of aggregate labor:

$$Y_t = A_t n_t$$

This output is divided between producers and workers, with workers receiving $(W_t/P_t)\, n_t$ and producers receiving $(A_t - W_t/P)\, n_t$. Since both types only wish to consume when old, they will want to save all of their income to consume next period. We now turn to the options young agents have to save their income.

B.3 Asset and Money Markets

Agents who wish to convert the income they earn from production when young into consumption when old have several options. The first two are the same as in our endowment economy: Agents can store their goods at a zero rate of return, or they can trade their goods for an asset which pays a fixed dividend of $d > 0$ consumption goods each period and which available in a fixed supply we normalize to 1. In this case, storage will be dominated, for the same arguments we used for the endowment economy. This will not be true when $d = 0$. Thus, we abstract from any additional equilibria that may arise when $d = 0$ in which agents are willing to store goods. We will discuss the case where $d = 0$ briefly at the end. Let p_t denote the price of the asset in terms of goods, and denote the (real) return on the asset by

$$1 + r_t \equiv \frac{d + p_{t+1}}{p_t}$$

In contrast to the endowment economy, we now allow agents to save using a third option: They can exchange their endowment for cash, deposit it with the central bank, and receive cash from the central bank next period at a nominal interest rate $1 + i_t$ announced by the central bank. We also allow people to borrow from the central bank at this rate. Following Galí, we restrict attention to equilibria in which agents do not hold

cash or trade with the central bank. Hence, the real return from lending or borrowing cash to the central bank must be the same as the return agents earn elsewhere on their savings. Thus, if we let $\Pi_t = P_{t+1}/P_t$ denote the gross inflation rate between dates t and $t+1$, then in equilibrium

$$\frac{1+i_t}{\Pi_t} = \frac{d+p_{t+1}}{p_t} \tag{33}$$

When the central bank sets a different nominal interest rate i_t, either inflation Π_t, the return to the asset $1+r_t$, or both must adjust to ensure agents do not want to either borrow or lend from the central bank.

With storage dominated and agents holding no cash in equilibrium, young agents will exchange all of their income to buy the asset from the old who wish to sell it. It follows that

$$p_t = A_t n_t \tag{34}$$

and the real return to buying the asset in equilibrium will equal

$$1+r_t = \frac{d+A_{t+1}n_{t+1}}{A_t n_t} \tag{35}$$

B.4 Labor Market

Finally, we consider the labor market. Demand for labor by intermediate goods producers is indirect; producers set a price P_{it} for the goods they sell, which determines how much of each goods final goods producers will demand. Producers then simply hire the workers they need to meet this demand. On the supply side, workers know that if they put in n_t units of labor, they will earn $(W_t/P_t)\,n_t$ goods. Above we argued they will use these goods to buy assets and earn a return of $1+r_t$, where $1+r_t$ is given by (35). Substituting this into the utility function for workers implies they will choose n_t to solve

$$\max_{n_t} (1+r_t)\frac{W_t}{P_t}n_t - A_t v\,(n_t)$$

The first order condition for labor is then given by

$$A_t v'\,(n_t) = (1+r_t)\frac{W_t}{P_t} \tag{36}$$

B.5 Equilibrium

To recap, in the production economy in this Appendix, young workers and producers join efforts to produce goods which they exchange with the old for assets. When workers and producers turn old, they consume the dividend d on the asset they own and the goods the exchange for the assets when they trade with the next generation of young agents. Productivity growth implies later cohorts can produce more than their predecessors. Hence, this economy will be dynamically inefficient and admit bubbles. The key difference is that the income of young agents is now endogenous and can potentially be affected by monetary policy.

Formally, for any path of nominal interest rates $\{1 + i_t\}_{t=0}^{\infty}$ set by the monetary authority, an equilibrium in this economy is a path of prices $\{P_t, W_t, p_t, r_t\}_{t=0}^{\infty}$ and a path of employment $\{n_t\}_{t=0}^{\infty}$ such that agents behave optimally and all markets clear at each date t. Collecting the relevant conditions from above yields five equations that govern these five variables:

(1) Optimal pricing: $\qquad P_t = \dfrac{E\left[W_t Y_t | \Omega_t\right]}{(1-\sigma) A_t E\left[Y_t | \Omega_t\right]}$

(2) Asset market clearing: $\qquad 1 + r_t = \dfrac{d + p_{t+1}}{p_t}$

(3) Labor market clearing: $\qquad v_t'(n_t) = (1 + r_t) \dfrac{W_t}{A_t P_t}$

(4) Goods market clearing: $\qquad p_t = A_t n_t$

(5) Money market clearing: $\qquad \Pi_t = \dfrac{1 + i_t}{1 + r_t}$

Conditions (1), (3), and (4) are all static and only involve date t prices. Conditions (2) and (5) are dynamic and relate prices at date t to prices at date $t + 1$, since condition (5) involves $\Pi_t = P_{t+1}/P_t$.

B.6 Equilibrium with Flexible Prices

We begin with the flexible price case in which producers can set their price for date t after observing the nominal wage W_t. Since producers can deduce what other producers will do and how much effort workers provide, they can perfectly anticipate total output Y_t. Hence, their information set will be given by $\Omega_t = \{W_t, Y_t\}$. In this case, $E\left[W_t Y_t | \Omega_t\right] = W_t Y_t$ and $E\left[Y_t | \Omega_t\right] = Y_t$, and so from the optimal pricing rule, we have

$$P_t = \frac{W_t}{(1-\sigma) A_t}$$

Substituting this and the asset market clearing condition into the labor market clearing condition yields

$$v_t'(n_t) = \frac{d + A_{t+1} n_{t+1}}{A_t n_t} (1 - \sigma)$$

Hence, n_t is governed by one-dimensional difference equation that is independent of $\{1 + i_t\}_{t=0}^{\infty}$. This equation features a unique unstable steady state for each t. It follows that the equilibrium path for $\{n_t\}_{t=0}^{\infty}$ is unique. Thus, in a flexible price environment, monetary policy has no effect on employment n_t. For reference, we denote the path of employment in the flexible price case by n_t^*. In this case, monetary policy also has no effect on output Y_t, the real interest rate r_t, and the real price of assets p_t, all of which are functions of n_t and exogenous variables. For reference, we will refer to these as Y_t^*, r_t^*, and p_t^*.

The only objects that remain to be solved are the path of prices P_t and W_t. The money market clearing condition yields a law of motion for P_t given the path of monetary policy, since $1 + i_t$ is chosen by the central bank and $1 + r_t^*$ is determined independently of prices. With this law of motion, we can determine P_t for all $t \geq 1$ given an initial value for P_0. Since the real wage W_t/P_t is pinned down in equilibrium by the optimal pricing rule, this means that the equilibrium is uniquely determined up to the initial nominal

wage W_0. Given an initial nominal wage W_0, we can determine P_0, then determine P_t for all $t \geq 1$, and finally, since we know $W_t/P_t = A_t\,(1-\sigma)$, we can determine W_t for $t \geq 1$. But the initial nominal wage W_0 is indeterminate. This is just an illustration of the price level indeterminacy of pure interest rate rules established by Sargent and Wallace (1975).

B.7 Equilibria with Rigid Prices

We now consider the sticky price case. We assume producers must set the price of their intermediate good P_{it} at the beginning of each period t, before the monetary authority moves or nominal wages W_t are set. The monetary authority then sets $1 + i_t$. After this, the timing is the same as before. Goods producers hire workers in the labor market at a nominal wage W_t. Final goods producers buy intermediate goods at the price P_{it} specified in the beginning of the period. Workers and producers use the goods they earn as income they earn to buy assets from the old who sell off their assets.

If monetary policy is deterministic, the change in timing is inconsequential. Producers can perfectly anticipate the nominal interest rate and deduce what the equilibrium nominal wage W_t will be. In that case, $\Omega_t = \{W_t, Y_t\}$ as before.

The same is not true if monetary policy is contingent on some random variable that is realized after producers set their prices. That is, suppose $i_t = i\,(\xi_t)$ where $\{\xi_t\}_{t=0}^{\infty}$ is some sequence of random variables. For simplicity, suppose ξ_t is only random at $t = 0$. That is, suppose

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1 - \chi \end{cases}$$

where $\chi \in (0,1)$, and

$$\xi_t = \emptyset \text{ for } t = 1, 2, 3, \ldots$$

Since we assume ξ_t is realized after producers set their prices, the information available to price setters is $\Omega_0 = \{i : \xi_0 \to \mathbb{R}\}$, i.e., producers know the monetary policy rule from but not the realization of ξ_0. At dates $t \geq 1$, monetary policy is deterministic and so $\Omega_t = \{W_t, Y_t\}$ as we argued before. Thus, monetary policy is irrelevant from date $t = 1$ on, as anything the monetary authority does is anticipated in advance.

For notational ease, let us use a superscript $\xi \in \{H, L\}$ to denote the value of a variable as a function of the realization of the sunspot variable at date 0. Thus, W_t^{ξ} denotes the nominal wage at date t if back at date 0 the realization of the sunspot was given by $\xi_0 = \xi$. We now argue that the equilibrium is indeterminate in up to two values: the nominal wage W_0^H the inflation rate Π_0^H at date 0 when $\xi_0 = H$. Intuitively, monetary policy determines the average inflation rate $E\,[\Pi_0]$, but it does not pin down what inflation must be for each realization of ξ_0. Since the sunspot can assume two values at date 0, the equilibrium features one additional free parameter.

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Formally, we know that $n_t = n_t^*$ for $t \geq 1$. We now show that given W_0^H and Π_0^H, we can use the equilibrium conditions above to pin down any remaining equilibrium objects. First, from the money market clearing condition, given $1 + i_0^H$ and Π_0^H we can deduce the real interest rate $1 + r_0^H$ when $\xi_0 = H$. From the goods market clearing and the asset market clearing conditions, we can deduce n_0^H; this is because we know $n_1 = n_1^*$, and so

$$1 + r_0^H = \frac{d + A_1 n_1^*}{A_0 n_0^H}$$

Next, we can use the labor market clearing condition to pin down the real wage W_0^H / P_0 when $\xi = H$, since

$$\frac{W_0^H}{P_0} = \frac{v'\left(n_0^H\right)}{1 + r_0^H}$$

Note that P_0 does not depend on ξ_0 since producers set their prices before ξ_0 is realized. Once we know the real wage at date 0 if $\xi_0 = H$, we can use the optimal pricing rule together with the various market clearing conditions to solve for the real wage W_0^L / P_0 and n_0^L. In particular, these two variables are determined by the pair of conditions

$$P_0 = \frac{1}{(1-\sigma) A_0} \frac{\chi n_0^H W_0^H + (1-\chi) n_0^L W_0^L}{\chi n_0^H + (1-\chi) n_0^L}$$

$$\frac{W_0^L}{P_0} = \frac{v'\left(n_0^L\right) A_0 n_0^L}{d + A_1 n_1^*}$$

Once we know n_0^L, we also have the return $1 + r_0^L = (d + A_1 n_1^*) / \left(A_0 n_0^L\right)$. Combining this with the other conditions yields the inflation rate $\Pi_0^L = \left(1 + i_0^L\right) / \left(1 + r_0^L\right)$ if $\xi_0 = L$.

Finally, once we have W_0^H and the real price W_0^H / P_0, we can deduce the initial price P_0. We can then use the money market clearing condition to deduce P_t^H and P_t^L for all $t \geq 1$, i.e., the price level at each date as a function of the realization of ξ_0. We can then deduce W_t^H and W_t^L for all $t \geq 1$ from the fact that $W_t^\xi = (1 - \sigma) A_t P_t^\xi$. Hence, given W_0^H and Π_0^H, we can pin down all remaining equilibrium prices and employment levels.

The fact that we now have two equilibrium objects that are not pinned down suggests a greater degree of indeterminacy than under flexible prices. In fact, the set of equilibria in the sticky price economy includes the flexible price equilibrium in which $n_t = n_t^*$ for all dates for both realizes of ξ_0 at date 0 as a special case. In other words, the presence of price rigidity does not change the set of equilibrium outcomes on the real side. Rather, it adds additional equilibria. To see this, suppose we set the inflation rate if $\xi_0 = H$ to equal

$$\Pi_0^H = \frac{1 + i_0^H}{1 + r_0^*}$$

where

$$1 + r_0^* = \frac{d + A_1 n_1^*}{A_0 n_0^*}$$

is the real interest rate that would prevail at date 0 in the flexible price world. Since $n_1^H = n_1^L = n_1^*$, it follows that $n_0^H = n_0^*$. With a little bit of algebra, we can confirm that $n_0^L = n_0^*$ as well. Thus, even when

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prices are rigid, there exists an equilibrium in which employment, output, and the real interest rate are all unaffected by monetary policy, even though producers set their prices before observing ξ_0 and thus what the monetary authority will do. But price rigidity introduces other equilibria in which the actions of the monetary authority at date 0 affect real variables at date 0.[8] We now turn to this possibility.

B.8 Equilibria with Monetary Policy Tightening

The equilibria that we are particularly interested in are those where setting a higher nominal interest rate $1 + i_0$ at date 0 leads to a higher real interest rate $1 + r_0$ at date 0. We will refer to this case as tighter monetary policy. Without loss of generality, suppose the monetary authority sets $i_0^H > i_0^L$. Consider an equilibrium in which we set the free parameter Π_0^H to some value where

$$\Pi_0^H < \frac{1 + i_0^H}{1 + r_0^*}$$

In this case, the only way to satisfy the money market clearing condition is if $r_0^H > r_0^*$, i.e., if the real interest rate if $\xi_0 = H$ exceeds the interest rate that would prevail at date 0 in the flexible price economy. Since $1 + r_0^H = (d + A_1 n_1^*) / (A_0 n_0^H)$, it follows that $n_0^H < n_0^*$. That is, the only way the monetary authority can achieve an equilibrium real interest rate that exceeds r_0^* is if it reduces employment n_0^H below n_0^*. In other words, tighter monetary policy is contractionary. From the labor market clearing condition, we know that for n_0^H to fall below n_0^* requires that the real wage W_0^H/P_0 fall below the its level in the flexible price equilibrium, ensuring workers put in less effort even though they face a higher real interest rate.

We can rearrange the optimal pricing condition to emphasize that the weighted average wage across states must be constant:

$$\frac{\chi n_0^H}{\chi n_0^H + (1 - \chi) n_0^L} \frac{W_0^H}{P_0} + \frac{(1 - \chi) n_0^L}{\chi n_0^H + (1 - \chi) n_0^L} \frac{W_0^L}{P_0} = (1 - \sigma) A_0$$

Since n_0^H and W_0^H/P_0 are both lower than the flexible price levels, we can use this condition together with the labor market clearing condition to confirm that W_0^L/P_0 and n_0^L must exceed the analogous flexible price equilibrium values. We can then deduce that $1 + r_0^L < 1 + r_0^*$, and so

$$\Pi_0^L > \frac{1 + i_0^L}{1 + r_0^*}$$

Thus, with only two states, if the monetary authority manages to raise the real interest rate in some state of the world at date 0 relative to the flexible price level, it must also lower the real interest rate relative to the flexible price level in another state of the world.

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[8] The fact that nominal price rigidity includes the original flexible price equilibrium is due to the way this rigidity is modelled: all producers can set their price unconstrained after one period. If producers could only adjust their prices in a staggered manner or were constrained in terms of how much they adjust their price, the flexible price outcome would no longer necessarily constitute an equilibrium with rigid pricing.

To the extent that a higher nominal interest rate achieves a higher real interest rate, then, it is by inducing agents to work fewer hours when nominal interest rates are high, reducing the income they generate and subsequently spend to buy assets. Since equilibrium output and real prices are unaffected from date 1 on, the agents who save at date 0 buy buying assets receive the same amount at date 1 in exchange for the assets they bought, even though they spent less to buy these assets. Hence, when a higher nominal interest rate contracts economic activity, it implies agents who buy assets earn a higher real return.

Note that we can capture the equilibrium in which a higher nominal interest rate corresponds to a higher real interest rate using our endowment economy. The contractionary effect of a higher nominal interest rate can be captured in an endowment economy by having the endowment of the initial young depend on the realization of a random variable ξ_0, with the endowment being lower in the state that corresponds to a higher nominal interest rate. Endowments from date $t = 1$ on do not depend on the realization of ξ_0. This specification will correctly capture what happens to output, consumption, and asset prices in the equilibrium with tighter monetary policy in the production economy we've described in this Appendix. Thus, we refer to thought experiments in which we destroy part of the endowment of the cohort born at date 0 as a reduced-form representation of the effects of monetary policy in a production economy with nominal price rigidity.

B.9 Intrinsically Worthless Assets

Up to now, we have assumed the asset yields a positive dividend $d > 0$ at all dates. This implies that agents will spend all of their income on assets in equilibrium. If $d = 0$ so the asset is intrinsically worthless, the equilibria we have analyzed in which agents spend all of their income on assets remain. However, additional equilibria arise in which agents are indifferent between buying the assets and storing goods and spend only some of their income to buy assets. We will not attempt to characterize all such equilibria here. To give a flavor of equilibria that can arise when the asset is intrinsically worthless, we consider equilibria in which the path of prices for each realization of ξ_0 are deterministic. Once again, we will use a superscript ξ to denote the value of a variable at date t if ξ_0 back at date 0 was equal to ξ.

From date $t = 1$ on, we know the intermediate goods producers will set their prices so that regardless of ξ,

$$P_t^\xi = \frac{W_t^\xi}{(1 - \sigma) A_t}$$

Hence, the real wage from date $t = 1$ on is uniquely pinned down and will be independent of the realization of ξ_0 back at date 0. Substituting the real wage and the asset market clearing condition into the labor market clearing condition yields

$$v'\left(n_t^\xi\right) = \frac{p_{t+1}^\xi}{p_t^\xi}(1 - \sigma)$$

Suppose $p_t^\xi < A_t n_t^\xi$ for some date t, so at some date the agent is willing to store goods. This requires

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$1 + r_t = p_{t+1}^{\xi}/p_t^{\xi} = 1$. The implication is that equilibrium employment at date t satisfies

$$v'\left(n_t^{\xi}\right) = 1 - \sigma$$

Just as in Proposition 1, we argue that if $r_t^{\xi} = 0$, then $r_{t+1}^{\xi} = 0$ as well. For suppose $1 + r_{t+1}^{\xi} > 1$. In this case, young agents born at date $t + 1$ would strictly prefer the asset to storage, and so $p_{t+1}^{\xi} = A_{t+1}n_{t+1}^{\xi}$. From the labor market clearing condition, we have

$$v'\left(n_{t+1}^{\xi}\right) = \left(1 + r_{t+1}^{\xi}\right)(1 - \sigma) > 1 - \sigma$$

Since $v''\left(\cdot\right) > 0$ and $v'\left(n_t^{\xi}\right) = (1 - \sigma)$, it follows that $n_{t+1}^{\xi} > n_t^{\xi}$. But this implies that $p_{t+1}^{\xi} = A_{t+1}n_{t+1}^{\xi} > A_t n_t^{\xi} \geq p_t^{\xi}$, which contradicts the fact that $r_t^{\xi} = p_{t+1}^{\xi}/p_t^{\xi} - 1 = 0$. Once again, then, a zero interest rate is absorbing. These deterministic equilibria thus have the same structure as the equilibria in Proposition 1: There exists a cutoff date t^* such that before date t^*, the real interest rate $1 + r_t^{\xi} > 0$ and agents spend all of their income on the asset and $p_t^{\xi} = A_t n_t^{\xi}$. From date t^* on, the real interest rate is 0, employment is constant and solves $v'\left(n_t^{\xi}\right) = 1 - \sigma$, and the real price of the asset is constant. We can index the equilibrium by its asymptotic limit, $\lim_{t \to \infty} p_t^{\xi} = p^{\xi}$. Note that the limiting value can vary by ξ. Thus, an equilibrium is determined up to two parameters, p^H and p^L. We now argue that these two parameters pin n_t^{ξ} and r_t^{ξ} for all $t \geq 1$.

In the case where $p^{\xi} = \infty$, we have $p_t = A_t n_t$ for all t and the analysis is the same as in the case where $d > 0$. In that case, the same argument as in the case where $d > 0$ implies $1 + r_t = 1 + g$ for $t \geq 1$, and so n_t^{ξ} solves $v'\left(n_t^{\xi}\right) = (1 + g)(1 - \sigma)$.

In the case where $p^{\xi} < \infty$, there exists a cutoff $t^*\left(\xi\right) < \infty$ such that $p_{t^*(\xi)-1} = A_t n_{t-1}$ and $p_{t^*(\xi)} = p^{\xi}$. For any date $t \geq t^*\left(\xi\right)$, the real interest rate $r_t^{\xi} = 0$, employment n_t^{ξ} solves $v'\left(n_t^{\xi}\right) = 1 - \sigma$, the real price $p_t^{\xi} = p^{\xi}$, and the real wage $W_t^{\xi}/P_t^{\xi} = (1 - \sigma)A_t$, so everything is determined up to the wage level W_t^{ξ}. For any dates $0 < t \leq t^*\left(\xi\right) - 1$, we can solve for these same values recursively. In particular, we have

$$v'\left(n_t^{\xi}\right) = \frac{A_{t+1}n_{t+1}^{\xi}}{A_t n_t^{\xi}}(1 - \sigma)$$

Using the boundary condition for n_t at $t = t^*\left(\xi\right)$, we can solve for n_t for all t between 1 and $t^*\left(\xi\right) - 1$, which would yield the unique interest rate at each such date t and the unique asset price.

As in the case where $d > 0$, the equilibrium at date 0 is determined up to two parameters, Π_0^H and W_0^H. These determine the degree to which changes in the nominal interest rate affect the real wage at date 0, and the level of wages and prices given price-level indeterminacy in the model.

In Section 1 of the paper, we discuss a particular equilibrium in which $n_t^H = n_t^L$ for all t but $p_t^H \neq p_t^L$. Consider two equilibria in which $r_t^H = r_t^L = 0$ from date $t = 1$ on. This ensures employment is the same regardless of ξ from date $t = 1$ on. The initial interest rate $r_0^H > r_0^L$, but the initial real wage W_0^H/P_0^H is lower by enough to ensure $\left(1 + r_0^H\right)W_0^H/P_0^H = \left(1 + r_0^L\right)W_0^L/P_0^L$. That ensures $n_0^H = n_0^L$.

Appendix C: Endogenous Asset Creation

In this Appendix, we consider the case where the supply of assets is endogenous rather than fixed exogenously. Suppose that in any period, old agents can convert consumption goods into assets. At date 0, old agents determine the supply of assets they sell. Beyond date 0, old agents can sell assets they previously purchased as well as create new ones. All other aspects are unchanged: At each date t a new cohort arrives that is endowed with $e_t = (1 + g)^t e_0$ consumption good who wishes to consume at date $t + 1$.

We look at the effect of endogenous asset creation in both the dynamically inefficient economy and the economy with credit. For the dynamically inefficient economy, we show that even when the supply of assets is endogenous, there is still no reason to intervene against bubbles: The high price of the assets does not lead to excessive asset creation. For the economy with information frictions, we show that intervening to raise rates cannot improve welfare even when it frees up resources that would have been used to create assets. We then show that a threat to raise rates if the bubble persists can generate a Pareto improvement that makes all cohorts, including the initial old at date 0, better off. Allowing the supply of assets to be endogenous thus allows us to get around the Grossman and Yanagawa (1993) result on the impossibility of Pareto improvements in overlapping generation models of bubbles.

C.1 Endogenous Asset Creation in the Dynamically Inefficient Economy

We begin with the dynamically inefficient economy in Section 1. Instead of assuming that the supply of assets is fixed and endowed to the old at date 0, suppose no assets exist at the beginning of date 0, but that old agents can convert goods into assets. The technology for converting goods into assets features increasing marginal costs. Formally, the cost of producing an additional asset given a mass q assets has already been produced is $c(q)$ units of the consumption good, where $c(\cdot)$ is a differentiable and increasing function. The total cost for the old to produce the initial q_0 assets that trade at date 0 is thus

$$C(q_0) \equiv \int_0^{q_0} c(j)\, dj$$

Since all assets trade in a decentralized market at a common price p_0, the amount of goods the old can consume at date 0 is equal to $p_0 q_0 - C(q_0)$.

At dates $t = 1, 2, 3, \ldots$ the economy inherits the q_{t-1} assets that the young purchased in the previous period and are now held by the old. Old agents at date t can produce new assets in addition to the ones they previously bought. Let x_t denote the quantity of new assets produced at date t. Assets cannot be destroyed, so $x_t \geq 0$. We assume the cost of producing new assets depends on the number of existing assets q_{t-1}. That is, we assume marginal costs are increasing not in the amount produced in a given period but in the total amount produced. In addition, since the economy is growing over time, we assume the cost of

producing also rises at the same rate. Formally, producing x_t assets requires $C(x_t)$ goods

$$C_t(x_t) = (1+g)^t \int_{q_{t-1}}^{q_{t-1}+x_t} c(j)\, dj$$

As we will see, this specification implies there is no reason to delay creating assets. All assets will be created at date 0.

All $q_{t-1} + x_t$ assets available at date t are traded in a decentralized market at a single price p_t. In equilibrium, old agents will keep creating assets until the marginal cost of the last asset created is equal to the price at which assets trade. Thus, at date 0, we have

$$c(q_0) = p_0 \tag{37}$$

We restrict attention to the case where the dividend $d > 0$, so prices are uniquely pinned down. Since young agents at date 0 will spend all of their endowment to buy the asset, we have

$$p_0 = e_0/q_0 \tag{38}$$

These two equations together determine the price p_0 and quantity q_0 of assets at date 0.

At dates $t = 1, 2, 3, ...$ it will still be the case that young agents born at date t will want to trade all of their endowment for the asset. Hence, the price will equal $p_t = e_t/q_t$. Since assets cannot be destroyed, $q_t \geq q_0$. From this, it follows that

$$p_t = \frac{e_t}{q_t} \leq \frac{e_t}{q_0} = (1+g)^t \frac{e_0}{q_0} = (1+g)^t p_0$$

Since $c(q_0) = p_0$, then the cost of producing new assets at date t

$$(1+g)^t c(q_t) \geq (1+g)^t c(q_0) = (1+g)^t p_0 \geq p_t$$

Thus, no new assets will be created after date 0, so $x_t = 0$ for all $t = 1, 2, 3, ...$

Given that all assets will be created at date 0, beyond date 0 the economy will be the same as in the case where the stock of assets is exogenously fixed. The new issue that arises when the supply of assets is endogenous is whether the quantity of assets created in equilibrium is optimal or whether resources used to create assets at date 0 are wasted.

Since the overlapping generations economy features multiple agents – infinitely many, in fact – it features a set of allocations that are Pareto optimal rather than a single allocation. We now argue that the equilibrium with endogenous creation is Pareto optimal. Thus, it will not be possible to make agents better off by intervening to reduce the quantity of assets created at date 0.

Consider the equilibrium outcome in which the quantity of assets q_0 solves $p_0 = e_0/q_0 = c(q_0)$. Suppose we intervened and produced Δ fewer assets at date 0 than in the equilibrium. To ensure the old at date 0

are no worse off, we would have to give them

$$\int_{q_0-\Delta}^{q_0} [p_0 - c(j)] \, dj$$

This leaves us with $\int_{q_0-\Delta}^{q_0} c(j) \, dj$ resources. Could we use them to make young agents at date 0 better off? Since young agents only value consumption at date 1, they would have to either store these goods or use them to buy the remaining $q_0 - \Delta$ assets. But at date 1, the most old agents can consume is the endowment e_1 of the young at date 1 and the dividends from the $q_0 - \Delta$ assets that were produced at date 0. In the original equilibrium, they already consumed $e_1 + dq_0$. This means that the young now consume $d\Delta$ less than that under the original equilibrium. Thus, there is no way to make all agents who are around at date 0 weakly better off by producing fewer assets. A similar type of argument can be used to show that producing Δ more assets cannot make agents better off.

Intuitively, the best we could do with the resources used to create the marginal asset in equilibrium is to invest them in the highest return option available. But in our economy, this would involve investing it in an equivalent asset. Thus, there is nothing to gain on the margin from producing one less asset. Even though the asset is a bubble, there is no sense in which the equilibrium quantity of assets is excessive. Asset prices exceed the value of dividends these assets generate, but they also reflect the value assets serve in facilitating the intergenerational transfers needed to overcome dynamic inefficiency.

C.2 Endogenous Asset Creation in the Economy with Information Frictions

We now turn to endogenous asset creation in the economy with information frictions. We continue to assume that at date 0, old agents can convert consumption goods into assets according to a features increasing marginal costs such that the total cost to produce q_0 assets is given by

$$C(q_0) \equiv \int_0^{q_0} c(j) \, dj$$

As in the dynamically inefficient economy, old agents will keep creating assets until the marginal cost of the last asset created is equal to the price at which assets trade. Since at date 0 the dividend is equal to D, let us first derive the equilibrium conditional on the dividend remaining high. We begin with the equilibrium when $d_t = D$. The market clearing condition is now given by

$$p_t^D q_t^D + N \left(1 + R_t^D \right) = e \tag{39}$$

and the zero-profit condition for borrowers is given by

$$\left(1 + R_t^D \right) p_t^D = p_{t+1}^D + D \tag{40}$$

Finally, the quantity of assets q_t^D satisfies

$$q_t^D = \begin{cases} q_t^D = c^{-1} \left(p_t^D \right) & \text{if } p_t^D \geq \max \left\{ p_0^D, p_1^D, ..., p_{t-1}^D \right\} \\ q_{t-1}^D & \text{if } p_t^D < \max \left\{ p_0^D, p_1^D, ..., p_{t-1}^D \right\} \end{cases} \tag{41}$$

where $c^{-1}(\cdot)$ is an increasing function. Suppose for a moment that $p_t^D \geq \max\{p_0^D, p_1^D, ..., p_{t-1}^D\}$ for all t. Then $q_t^D = c^{-1}(p_t^D)$, and so we can replace (39) with

$$p_t^D c^{-1}(p_t^D) + N(1 + R_t^D) = e$$

This yields R_t^D as an increasing function of p_t^D, i.e., $R_t^D = \bar{\rho}(p_t^D)$, which we can substitute into (40). Similarly to the case with a fixed supply of assets, this yields an equilibrium law of motion $p_{t+1}^D = \bar{\varphi}(p_t^D)$ which has a unique fixed point p^D. If we consider an equilibrium in which $p_0^D > p^D$, then $p_{t+1}^D = \varphi(p_t^D)$ would yield an increasing sequence of prices. In this case, $q_t^D = c^{-1}(p_t^D)$ for all dates, so the path would satisfy all three equilibrium conditions. However, as before, the total expenditures on the asset, $p_t^D c^{-1}(p_t^D)$, would eventually exceed e, which cannot be an equilibrium. What about paths in which $p_0^D < p^D$? In that case, $p_1 < p_0$, and so $q_1 < c^{-1}(p_1^D)$. However, define $q^D = c^{-1}(p^D)$, and let us replace (39) with

$$p_t^D q^D + N(1 + R_t^D) = e$$

This again yields R_t^D as an increasing function of p_t^D, i.e., $R_t^D = \underline{\rho}(p_t^D)$, which we can substitute into (40). This yields a different law of motion $p_{t+1}^D = \underline{\varphi}(p_t^D)$ that has a unique fixed point, and by construction the fixed point is also p^D. If we consider an equilibrium in which $p_0^D < p^D$, then $p_{t+1}^D = \varphi(p_t^D)$ would yield a decreasing sequence of prices. In this case, $q_t^D = q_{t-1}^D = q^D$ for all dates, so the path would satisfy all three equilibrium conditions. However, in this case the total expenditures on the asset would eventually turn negative, which means the price of the asset eventually turns negative. But this cannot be an equilibrium. Hence, $p_t^D = p^D$ for all t. As long as dividends are high, no new assets will be produced.

We can apply a similar analysis to characterize the price of the asset in the low-dividend regime p_t^d. The main difference is that the quantity q^D which is equal to $c^{-1}(p^D)$ now strictly exceeds $c^{-1}(p^d)$. The case where $p_0^d > p^d$ is now a bit more involved, since the argument that it will eventually exceed e is a bit more involved, but the same argument applies. Once again, no new assets will be produced in the low dividend regime. All assets in the economy will be produced at date 0.

Since the price of the asset in the high dividend regime p^D is the same as the price of the asset paid D forever, there is a sense in which there is excessive creation of assets in this economy. Essentially, the expected return on the last asset produced is $1 + \bar{r}^D$ which is less than the expected return to lending out resources to all willing borrowers, which is $1 + \overline{R}^D$. However, we will now show that raising rates at date 0 continues to crowd out entrepreneurial activity even though this intervention discourages the creation of bubble assets and frees up resources to be used for entrepreneurial activity.

Consider the effect of reducing the initial endowment e_0 from e to $e - \tau$. We begin by solving for the equilibrium at date 0:

$$p_0^D c^{-1}(p_0^D) + N(1 + R_0^D) = e - \tau \tag{42}$$

$$(1 + R_0^D) p_0^D = p_1^D + D \tag{43}$$

From (42), we can express $R_0^D = \rho_\tau(p_0^D)$, where $\rho_\tau(p_0^D)$ is increasing in both τ and p_0^D. If we substitute this into (43), we get $p_1^D = \varphi_\tau(p_0^D)$. Since $\rho_\tau(\rho_0^D)$ is increasing in τ, $\varphi(p^D) > p^D$. This implies that

$p_0^D < p^D$, since otherwise the amount spent on the asset along the equilibrium path in the high dividend regime would eventually exceed e. It follows that in equilibrium, $p_1^D = p^D$. That is, the equilibrium at date 1 is unaffected by τ. If we substitute (43) into (42) and differentiate, we get

$$\left(c^{-1}\left(p_0^D\right) + p_0^D c^{-1\prime}\left(p_0^D\right) - \frac{p^D + D}{\left(p_0^D\right)^2} N'\left(\frac{p^D + D}{p_0^D}\right) \right) \frac{dp_0^D}{d\tau} = -1$$

$$\frac{dp_0^D}{d\tau} = \frac{-1}{c^{-1}\left(p_0^D\right) + p_0^D c^{-1\prime}\left(p_0^D\right) - \frac{p^D+D}{\left(p_0^D\right)^2} N'\left(\frac{p^D+D}{p_0^D}\right)} < 0 \tag{44}$$

Thus, intervening to raise rates at date 0 will depress the price of the asset. This implies $q_0^D < q_1^D$ and $x_1 = q_1^D - q_0^D > 0$. Thus, the intervention implies fewer assets will be created at date 0. However, since $R_0^D = \frac{p^D+D}{p_0^D}$ is increasing in τ, then $N\left(1 + R_0^D\right)$ is decreasing. Raising rates dampens the bubble, but it still crowds out rather than stimulates more entrepreneurial activity.

Finally, we consider the effects of a threat to destroy the endowment at date 1. The equilibrium price at date 0 is now given by

$$p_0^D c^{-1}\left(p_0^D\right) + N\left(1 + R_0^D\right) = e \tag{45}$$
$$\left(1 + R_0^D\right) p_0^D = p_1^D + D \tag{46}$$

The market clearing condition at date 0 is the same as before, and so we can still write $R_0^D = \rho\left(p_0^D\right)$. Substituting in implies $p_1^D = \varphi\left(p_0^D\right)$ where $\varphi\left(p^D\right) = p^D$. But we know from our analysis of what happens in the period when we destroy τ units of the endowment that $p_1^D < p^D$. Hence, $p_0^D < p^D$. Thus, a threat to destroy output at date 1 if the dividend remains high will dampen the price at date 0, and since $R_0^D = \rho\left(p_0^D\right)$ is an increasing function of p_0^D, it will also lower the interest rate on loans R_0^D. Hence, a threat to destroy some of the endowment at date 1 if the bubble persists dampens the bubble and leads to more entrepreneurial activity at date 0. Since the quantity of assets is increasing in the price, this means such a threat would result in fewer assets being produced at date 0. On the margin, the last asset produced yields an expected return of

$$1 + \bar{r}_0^D = \frac{\pi\left(d + p^d\right) + (1 - \pi)\left(D + p_1^D\right)}{p_0^D}$$
$$< \frac{D + p_1^D}{p_0^D} = 1 + R_0^D$$

Hence, the effect of shifting resources to entrepreneurship increases the expected resources available at date 1. Since producers require fewer resources to compensate them than the young spend on assets, i.e., since entrepreneurs only require $p^D - c\left(q\right)$ to leave them no worse off, then $c\left(q\right)$ worth of resources can be freed up for entrepreneurial activity. Thus, even without default costs, a threat to destroy output in the future can be used to generate a Pareto improvement in which both the old and the young at date 0 are at least as well off. If we transfer some of the gains to the young at date 1 if the bubble collapses, as described in Section 2.6, we can ensure all cohorts are at least as well off under the intervention.