

Stock Market and Investment Good Prices: Implications for Macroeconomics*

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Abstract

Stock market prices are procyclical, while investment good prices are countercyclical. A real business cycle model calibrated to these observations implies that 75% of the cyclical variation in aggregate output is due to an investment-specific technology shock, while the rest is due to an aggregate productivity shock. To test this conclusion, we investigate the model's implications for asset prices and business cycles. The model does not do significantly worse than existing models on these dimensions, and on two dimensions it does notably better. It is consistent with the facts: (i) employment and investment across different sectors comove over the business cycle; and (ii) high interest rates lead low aggregate output. Fact (ii) is often interpreted as reflecting the business cycle effects of monetary policy shocks. Our results suggest that (ii) may, at least to some extent, also reflect the effects of real shocks.

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1. Introduction

A fundamental question in macroeconomics is, ‘What are the sources of business cycle shocks?’ We exploit the information in the cyclical behavior of investment good and stock market prices to answer this question. The key feature of the price data which we exploit is that stock prices are procyclical, while investment good prices are countercyclical.¹ The divergent cyclical behavior of these prices is also a source of information about the nature of adjustment costs in the installation of new capital. This is because we interpret stock prices as a measure of the marginal cost of installed capital and investment good prices as a measure of the marginal cost of new capital goods. Under this interpretation, adjustment costs are the wedge between these two prices. Our results are:

- About 75 percent of the variance in output is due to a shock to the supply of investment goods. The rest is due to a technology shock that raises output and the demand for installed capital.
- The countercyclical nature of investment good prices reflects the effects of the supply shock.
- The elasticity of a firm’s marginal cost of installed capital with respect to new capital goods is roughly $3/4$.
- Stock prices respond countercyclically to the investment-specific shock, but are nevertheless procyclical because they are dominated by the demand shock.

Although we reach them by a different route, our first two conclusions are qualitatively similar to those in Greenwood, Hercowitz and Krusell (1998). The estimate of the marginal installation cost elasticity in our third finding is somewhat smaller than the ones reported in the q -theory literature.

The intuition underlying our fourth result is simple. It is based on a basic property of neoclassical models, namely, that the price of a firm’s stock equals the marginal cost of its installed capital.² This relationship is useful for understanding the impact of various shocks on a stock price. Investment-specific shocks play a relatively small role in stock prices because they trigger two offsetting effects on the marginal cost of installed capital. The fall in investment prices associated with a positive investment-specific shock pushes marginal cost down, while the rise in investment works in the other direction by increasing marginal installation costs. Our finding that the latter effect is relatively small

¹The procyclical nature of stock prices is well known. For documentation, see Christiano and Fisher (1995) and Fischer and Merton (1984) and the references they cite. The cyclical behavior of investment good prices is documented in Greenwood, Hercowitz and Krusell (1997, 1998) and Murphy, Shleifer and Vishny (1989). Additional documentation is provided in section 2 below.

²This is a fundamental relationship in the literature on q -theory. See, for example, Hayashi (1982), equation 8.

explains our conclusion that stock prices are countercyclical relative to investment-specific shocks. The marginal cost of installed capital responds strongly and procyclically to a demand shock because the two effects just cited now work in the same direction. Taken together, these considerations explain our finding that stock prices are procyclical because they are dominated by the demand shock.

The identification assumptions used to map from the data to the conclusions just described are the ones contained in an explicitly formulated general equilibrium model. To build confidence in our conclusions, we test the model's implications for other phenomena. Obviously, our choice of model and implications to emphasize in the test play a crucial role in our analysis. We now briefly discuss how we made these choices.

The phenomena that interest us lie at the intersection of macroeconomics and finance. To have confidence in the inferences we draw from the data, we need to be sure that they are not inconsistent with the salient facts in these two areas. For example, since the procyclicality of equity prices is a feature of asset prices, it is particularly important to verify that our explanation does not conflict with existing explanations of other features of asset markets. Similarly, since our focus is on characteristics of the business cycle, we want to verify that our hypothesis is not at odds with basic business cycle facts.

These are the considerations that lead us to work with a version of the model of Boldrin, Christiano and Fisher (1995) (BCF). That model incorporates assumptions designed to help account for basic features of mean asset returns, and does at least as well as standard real business cycle models in accounting for the business cycle. We find that when we introduce the modifications necessary to accommodate the basic characteristics of stock market and investment good prices, the model's asset pricing implications are comparable to those of BCF.

Regarding the business cycle implications of our model, with one exception, it does as well as standard models with respect to conventional business cycle statistics. The exception is that the adjustment costs that our hypothesis invokes hurt the model's ability to account for the volatility of hours worked.

There are two other dimensions, however, on which the model significantly dominates standard models. First, the factors that help it account for prices and rates of return on assets also help it account for the fact that employment and investment across a broad range of sectors move together over the cycle. This is significant, since comovement is perhaps *the* defining characteristic of business cycles (see Burns and Mitchell (1946, p. 3), Lucas (1981, p. 217) and Sargent (1979, p. 215).) Interestingly, standard business cycle models have difficulty accounting for comovement. Second, the model can account for what King and Watson (1996) refer to as the 'inverted leading indicator' property

of interest rates, namely, that high interest rates are correlated with low future output. Standard business cycle models also have difficulty accounting for this phenomenon (King and Watson (1996)). The inverted leading indicator property of interest rates has received a lot of attention, and is often interpreted as reflecting the role in business cycles of monetary policy shocks.³ Our results suggest that the inverted leading indicator phenomenon may, at least to some extent, reflect non-monetary shocks.

The paper is organized as follows. Section 2 elaborates on the relation between stock market prices and the marginal cost of installed capital that we invoked above. In addition, it summarizes the empirical properties of prices and quantities of investment goods. Section 3 presents our model. Section 4 explains why our model can account for comovement, and relates our analysis to that in the existing comovement literature. Section 5 discusses our strategy for assigning values to the model's parameters. Section 6 evaluates our calibrated model's empirical implications. Section 7 concludes. A number of technical details related to our analysis are relegated to an appendix available from the authors on request (see Christiano and Fisher (1998) CF.)

2. Stock Market and Investment Good Prices

Two basic aspects of our analysis are our assumption that stock prices are at least a rough measure of the marginal cost of installed capital, and that investment good prices are countercyclical. We discuss these issues here, and we also summarize the trend properties of investment good prices, since these play an important role in our model calibration. More detailed discussions about the properties of investment good prices appear in Christiano and Fisher (1995), Greenwood, Hercowitz and Krusell (1997), and Murphy, Shleifer, and Vishny (1989).

2.1. Stock Market Prices and Marginal Costs

We take two steps to infer from the observed procyclicality of stock prices that the marginal cost of installed capital must be procyclical too. Both steps are literally correct only under strict neoclassical assumptions: linear homogeneity of production functions and perfect competition. Our first step is to note Hayashi (1982)'s result that under these conditions the value of equity is the product of the marginal value of capital, $P_{k'}$, and the stock of capital. Then, given the slow-moving nature of the stock of capital, we can infer from the procyclicality of stock prices that $P_{k'}$ is itself procyclical.⁴ Our

³See Chari, Christiano and Eichenbaum (1995) and Christiano and Eichenbaum (1995). The vector autoregression-based literature on monetary policy shocks also interprets the low output following interest increases as reflecting the operation of monetary policy shocks (for a review, see Christiano, Eichenbaum and Evans (1998)).

⁴Actually, the stock market does not measure the entire value of productive capital, because there is also debt. However, the data provided in Masulis (1988, Table 1-3) suggests that the debt to equity ratio, γ , is procyclical, which

second step is to observe that an efficiency condition of the firm requires that the level of investment be adjusted until $P_{k'}$ is equated to the marginal cost of installed capital.

We expect that the inferences we draw about marginal costs are robust to plausible deviations from the neoclassical assumptions and from our (implicit) assumption of a homogeneous capital stock. For example, it is safe to infer from procyclical stock prices that the *average* value of capital is procyclical. What we need, though, is that procyclicality of the average implies procyclicality of the marginal value of capital. Hayashi's result says the two objects are identical under neoclassical assumptions, but this is obviously more than what we need. This is fortunate, since the neoclassical assumptions are at best only approximations in the data. Tobin and Brainard (1977, p. 243) conjecture that the average and marginal values respond in similar ways to shocks, and the empirical evidence in Abel and Blanchard (1986) appears to support the conjecture.

Some deviations from perfect competition can also be tolerated, as long as the ratio of $P_{k'}$ to the marginal cost of installed capital does not fluctuate too much. Finally, some degree of heterogeneity in capital can be accommodated too. If multiple capital goods enter in the same weakly separable way in the production and adjustment cost functions, then the stock of capital valued in equity and debt markets could be thought of as an aggregator of multiple capital goods (including such intangible capital as R&D and patents). For further discussion of the implications of capital heterogeneity in this context, see Megna and Klock (1993) and Wildasin (1984).

2.2. Investment Good Prices

We analyze quarterly, constant 1992 dollar data on GDP, investment and the implicit price deflators for aggregate investment (fixed investment plus investment in consumer durables) and its components. These data cover the period 1959I-1995IV.⁵ We emphasize three basic features of these data. First, the price index for aggregate investment shows essentially no trend until 1980, whereupon it takes a sharp turn down, falling at a rate of 2 percent per year. This can be seen in Figure 1A, which displays

only strengthens the point made in the text. Masulis reports estimates of γ for all US corporations covering the years 1947 to 1986. The correlation of the cyclical component of $1 + \gamma$ with the cyclical component of aggregate GDP is 0.15 when debt is measured at market value, and 0.40 when it is measured at book value. By the cyclical component of a variable, we mean that variable after it has been logged and then filtered as in Hodrick and Prescott (1997). We are not aware of data on γ corresponding to sectors of the economy.

⁵The data are taken from the DRI Basic Economics database. Real GDP is mnemonic $GDPQF$ in 1992 dollars. All investment good prices are deflated by the NIPA implicit price deflator for consumption of nondurables and services. The price index for aggregate investment is $(GCD + GIF)/(GCDQF + GIFQF)$. The numerator is composed of current dollar data on household purchases of consumer durables (GCD) and business fixed investment (GIF). The denominator is composed of the corresponding 1992 quantities.

the logged data (solid line) together with the associated HP trend (dashed line).⁶ Most of the evidence for this trend break comes from investment in residential and nonresidential structures (Figures 1D and E), which account for about 35 percent of aggregate investment. There is much less evidence of any trend break in the other components of investment, namely, business and household durable good investment (Figures 1B and 1C).

Second, at cyclical frequencies, the price index for aggregate investment has a correlation of -0.26 (0.12) with output. The standard error (in parentheses) suggests that this negative correlation is statistically significant. The cross-correlation functions with output, together with 95 percent confidence intervals, are reported for aggregate investment (Figure 2A) and the components of investment in Figure 2. If we consider only the business component of investment, namely, investment in equipment and structures, we find that its price is also significantly countercyclical (Figure 2F). This is due in large part to the very significant countercyclical behavior of the price of business equipment investment (Figure 2C).⁷ Business investment is more countercyclical than aggregate investment primarily because residential investment, which is a part of the latter but not the former, is significantly procyclical (Figure 2D). Third, evidence reported in Christiano and Fisher (1995) suggests that, to a first approximation, the share of aggregate investment in GDP is constant, consistent with the balanced growth specification adopted in the model of this paper.

To summarize, the level of aggregation at which our model is specified suggests that the appropriate empirical measure of investment is aggregate investment. The stock price measures on which we base our analysis suggest that the relevant measure of investment is business investment in equipment and structures. Either way, the associated price index is significantly countercyclical. Also, there appears to be a break in the trend in these series. This is something that the model we analyze will not be consistent with.

Our results are robust to using chain-weighted measures of GDP and investment prices.⁸ When we consider the price of equipment investment alone, or the price of equipment and structures, we find that the contemporaneous correlation between the cyclical component of the price of investment goods and output is -0.43 (0.08) and -0.44 (0.10), respectively. Unlike the constant 1992 dollar data, these data are available back to 1947I. When we use data covering the period 1947I-1995IV, the results are

⁶The HP trend is the one suggested in Hodrick and Prescott (1997). We use this throughout our analysis. We follow convention by setting the smoothing parameter to 1600.

⁷This is the investment time series analyzed by Greenwood, Hercowitz and Krusell (1997, 1998).

⁸The data were taken from the DRI Basic Economics database. The mnemonic for chain-weighted GDP is $GDPQ$. We computed the price of investment goods relative to the price index for consumption of nondurables and services. The latter was computed as $p = (GCN + GCS)/(GCNQ + GCSQ)$. The relative price of business equipment is $(GIPNR/GIPNRQ)/p$. The relative price of business equipment plus structures is $(GIN/GINQ)/p$.

-0.27 (0.10) and -0.21 (0.12). Evidently, the correlations are somewhat less negative in the pre-1959 period. This reflects strong comovement in the price of investment goods and in GDP around the time of the Korean war.

3. Model Economy

In the first subsection that follows, we describe the preferences, technology and information assumptions in the model. Since prices and rates of return are a central concern of our analysis, we also discuss the underlying competitive decentralization. We do this in the second subsection below.

3.1. Preferences, Technology and Information

There are two sets of assumptions that differentiate ours from a standard one-sector, representative agent real business cycle model. (i) We adopt the assumptions proposed in BCF to account for mean asset returns: preferences have the habit persistence form proposed in Constantinides (1990) and Sundaresan (1989); investment and consumption goods are produced in spatially isolated locations; and the allocation of capital and labor to sectors must be made prior to the realization of the current period technology shocks. The assumptions on preferences result in a particular cyclical pattern in the demand for productive capital. The assumptions on technology guarantee that the supply of capital is inelastic in the short run.⁹ BCF show that the combined effects of these two features produce a cyclical pattern in capital gains that allows a model to generate a large equity premium. (ii) We adopt modifications designed to help the model account for the dynamic behavior of stock market and investment good prices: a stationary technology shock perturbs both production technologies; the investment good technology is also perturbed by a second shock with positive drift; and there are costs associated with the installation of new investment goods.

A few observations about assumptions (ii) are in order. Following Greenwood, Hercowitz and Krusell (1998), the positive drift in the investment-specific shock is introduced in order to render the model consistent with the post 1980s trend characteristics of the investment data documented in the previous section.¹⁰ The stationary shock that perturbs both production technologies corresponds to the ‘demand’ shock discussed in the introduction. The consumption smoothing motive has the effect

⁹There also exists a one sector specification of technology which can account for inelastic capital supply. In some respects, this alternative is simpler conceptually and computationally. We nevertheless prefer our two-sector specification because we believe it ultimately has greater potential for matching up with sectoral data. Although we do not pursue this further here, an extended discussion and analysis appears in Hornstein and Praschnik (1997) and Huffman and Wynne (1998).

¹⁰Accommodation of the evidence of a possible break in trend is beyond the scope of this paper.

of converting a positive realization of this shock into an increase in demand for installed capital and investment goods.

Turning to the formal description of the model, household preferences have the following form:

$$E_t \sum_{j=0}^{\infty} \beta^j [\log(C_{t+j} - bC_{t+j-1}) + \eta(1 - H_{c,t+j} - H_{i,t+j})], \quad (3.1)$$

where η is a positive scalar, $H_{c,t}, H_{i,t}$ denote employment in the consumption and investment goods-producing industries, and habit persistence corresponds to the case $b > 0$. Here, E_t is the expectation operator conditioned on all variables dated t and earlier. Also, C_t denotes consumption, and we specify that utility is linear in leisure following Rogerson (1988) and Hansen (1985).

The technology for producing consumption goods in t is

$$C_t \leq K_{c,t}^\alpha (\exp(\theta_t) H_{c,t})^{1-\alpha}, \quad (3.2)$$

where θ_t is a covariance stationary shock to technology:

$$\theta_t = \rho\theta_{t-1} + \varepsilon_t, \quad (3.3)$$

$0 < \rho < 1$. The technology for producing new investment goods in t is

$$I_{c,t} + I_{i,t} \leq V_t K_{i,t}^\alpha (\exp(\theta_t) H_{i,t})^{1-\alpha} \quad (3.4)$$

where

$$V_t = \exp(\bar{\mu} + \mu_t) V_{t-1}. \quad (3.5)$$

Here, ε_t and μ_t are zero-mean, normally distributed random variables which are independent of each other and over time and which have standard deviations σ_ε and σ_μ , respectively.

The technology for producing end of period t capital, $K_{x,t}$, for industry x is

$$K_{x,t} \leq Q^x(y, z), \quad (3.6)$$

where

$$Q^x(y, z) = [a_1 y^\psi + a_2 z^\psi]^{1/\psi}, \quad (3.7)$$

for $x = c, i$ and $\psi \leq 1$. In (3.6)–(3.7), y denotes previously installed capital and z denotes new

investment goods.¹¹ When $\psi = 1$, (3.7) corresponds to the conventional linear capital accumulation equation. In the case of adjustment costs, $\psi < 1$, the marginal product of new investment goods is decreasing in the flow of investment. We choose the constants, $a_1 > 0$ and $a_2 > 0$, to guarantee $Q_1^x = Q_2^x = 1$ in nonstochastic steady state. This has the effect of making the nonstochastic steady state properties of the model invariant to the value of ψ .¹²

We require that the date t allocation of capital and labor to sectors be determined prior to the date t realization of the exogenous shocks. We refer to the assumption that labor must be allocated prior to the shocks as the *limited labor mobility* assumption. The planning problem for this economy is as follows. Select contingency plans for investment, capital, consumption and employment which respect the information constraints, and which maximize (3.1) for each t , subject to (3.2)-(3.7). In addition, leisure, employment, capital, and investment must be non-negative. A detailed discussion of the planning problem appears in CF. The contingency plans are found using the linearization method described in Christiano (1998).

3.2. Market Prices and Rates of Return

One market decentralization which allows us to define precisely the prices and rates of return that interest us is discussed in detail in CF. We summarize that discussion here. We assume that households and firms interact anonymously in competitive markets and confront the following prices and rates of return:

- $P_{k_x,t} \sim$ period t price of previously installed, beginning of period t capital,
- $P_{i,t} \sim$ period t price of period t investment goods,
- $P'_{k_x,t} \sim$ period t price of beginning of period $t + 1$ installed capital, $K_{x,t+1}$,
- $1 + r_{x,t}^e \sim$ period t consumption good payoff on one unit of equity acquired in industry x in period $t - 1$,
- $1 + r_t^f \sim$ period $t + 1$ consumption good payoff for each one unit of risk free debt acquired in period t ,

¹¹Our adjustment cost formulation of investment resembles the one originating with Eisner and Strotz (1963), Lucas (1967,a,b), and Lucas and Prescott (1971), among others. For applications in the q -theory literature, see Abel (1980), Abel and Blanchard (1986), Cochrane (1991), Cummins, Hassett and Oliner (1997), Eberly (1997) and Hayashi (1982).

¹²The formulas for a_1 and a_2 are

$$a_1 = [(1 - \delta) \exp(-\bar{\mu}/(1 - \alpha))]^{1-\psi}, \quad a_2 = [1 - ((1 - \delta) \exp(-\bar{\mu}/(1 - \alpha)))^\psi]^{1-\psi}.$$

where $x = c$ corresponds to the consumption good sector and $x = i$ corresponds to the investment good sector. Prices are denominated in units of consumption goods. Rates of return are per unit of consumption goods invested. The fact that there is no subscript x on the price of investment goods reflects the implication of (3.4) that the marginal rate of transformation between $I_{c,t}$ and $I_{i,t}$ is unity. The fact that the rate of return on debt also has no subscript, x , reflects its non-state contingent nature. The rate of return on equity in industry x is:

$$1 + r_{x,t+1}^e = (1 + \gamma_t^x) \left[\frac{mpk_{x,t+1} + (1 - \delta)P_{k_x,t+1}}{P_{k'_x,t}} \right] - \gamma_t^x (1 + r_t^f), \quad (3.8)$$

where γ_t^x denotes the firm's debt to equity ratio. Here, $mpk_{x,t+1}$ denotes the marginal product of capital in industry x , denoted in period $t + 1$ consumption units. As is well known, γ_t^x is not determined in equilibrium, although the equilibrium quantity allocations in the model are unique and coincide with the allocations that emerge from the planning problem.

The overall return on equity, r_{t+1}^e , is:

$$r_{t+1}^e = \frac{P_{k'_{c,t}} K_{c,t+1}}{\tilde{K}_{t+1}} r_{c,t+1}^e + \frac{P_{k'_{i,t}} K_{i,t+1}}{\tilde{K}_{t+1}} r_{i,t+1}^e, \quad (3.9)$$

where $\tilde{K}_{t+1} = P_{k'_{c,t}} K_{c,t+1} + P_{k'_{i,t}} K_{i,t+1}$.

In CF we show that $P_{k'_{x,t}}$, $P_{k_x,t}$ and $P_{i,t}$ satisfy the following efficiency condition for investment:

$$P_{k'_{x,t}} = \frac{P_{i,t}}{Q_{2,t}^x} = \frac{P_{k_x,t}}{Q_{1,t}^x}. \quad (3.10)$$

The first relation in (3.10) is central to q -theory, and so we refer to it as the ' q -theory relation' (see Hayashi (1982, equation 8).) Here, $Q_{1,t}^x$ and $Q_{2,t}^x$ are the partial derivatives of the functions Q^x in (3.7) with respect to its first and second argument, respectively, for $x = c, i$. Our observations in the introduction about the response of the marginal cost of installed capital to shocks to the demand and supply of investment goods are based on this relationship. We can see here that a demand shock, which drives $P_{i,t}$ and $I_{x,t}$ both up leads to mutually reinforcing movements of the numerator and denominator terms in the q -theory relation, while a supply shock leads to offsetting movements.

In addition to satisfying the q -theory relation, the object, $P_{k'_{x,t}}$, is also shown in CF to satisfy two present value relations. In the first, $P_{k'_{x,t}}$ is the present discounted value of the earnings of the

underlying physical capital that equity is used to finance:

$$P_{k'_x,t} = E_t \sum_{j=1}^{\infty} \left(\prod_{i=1}^j p_{c,t+i} \left[Q_{1,t+i}^x (1-\delta) \right] \right) mpk_{x,t+j}, \quad (3.11)$$

where p_c is the usual intertemporal rate of substitution in consumption, defined precisely in CF. The presence of the square-bracketed term in (3.11) reflects the depreciation of capital. In CF we show that (3.11) also holds with $mpk_{x,t+j}$ replaced by the date $t+j$ dividends paid to holders of equity and with the object in square brackets replaced by unity. The measure of dividends in the model includes the direct earnings of capital, mpk_x , but subtracts for the depreciation costs (inclusive of installation costs) of capital, and allows for the possibility, $\gamma_t^x \neq 0$. These considerations lead us to refer to $P_{k'_x}$ as the price of a share of stock, or a share of equity, where a share corresponds to a claim on a unit of the underlying capital being financed.¹³

In our quantitative analysis, we study the dynamics of an aggregate measure of the price of equity. We define this as the ratio of capital goods valued in current consumption units, divided by capital goods valued in base year consumption units. In our model, we identify the base year with nonstochastic steady state, when $P_{k'_c,t} = P_{k'_i,t} = 1$. Thus,

$$P_{k',t} = \frac{K_{c,t+1}}{K_{t+1}} P_{k'_c,t} + \frac{K_{i,t+1}}{K_{t+1}} P_{k'_i,t},$$

where $K_{t+1} = K_{c,t+1} + K_{i,t+1}$.

Finally, recall that Tobin's q is equal to the marginal value to the firm of $K_{x,t+1}$, divided by the marginal cost, $P_{i,t}$, of a new investment good. CF shows that $P_{k'_x,t}$ corresponds to the relevant marginal value term. Thus, using (3.10), Tobin's q in industry x is

$$q_t^x \equiv \frac{P_{k'_x,t}}{P_{i,t}} = \frac{1}{Q_{2,t}^x} = \frac{1}{a_2} \left[a_1 \left(\frac{(1-\delta)K_{x,t}}{I_{x,t}} \right)^\psi + a_2 \right]^{\frac{\psi-1}{\psi}}, \quad (3.12)$$

$x = c, i$, which is unity when $\psi = 1$, since $a_2 = 1$ in that case. Our specification of a_1 and a_2 has the effect of forcing Tobin's q to be unity on a steady state growth path.

¹³There is an ambiguity, in a discrete time setting, in the definition of equity: does the stream of earnings that it generates start in the current period or in the next period? The definition adopted in the paper specifies the latter. Under the former, the price of stock is P_{k_x} , the price of already installed current period capital. According to (3.10), $P_{k_x} = Q_1^x P_{k'_x}$, where Q_1^x is a function of the ratio K'_x/K_x , which cannot fluctuate much over the cycle. This suggests that P_{k_x} and $P_{k'_x}$ exhibit similar cyclical properties. Consistent with this, we found, in results not reported in the paper, that our quantitative results are not sensitive to whether we adopt P_{k_x} or $P_{k'_x}$ as our measure of the price of equity. For this reason, we only discuss results based on $P_{k'_x}$.

4. Implications for Comovement

In the quantitative results reported below, we find that our model accounts well for business cycle comovement: the observation that inputs, outputs and investment across all major economic sectors move up and down together over the business cycle.¹⁴ Traditionally, comovement has been taken to be a defining characteristic of the business cycle. Yet, it has proved difficult to construct business cycle models which are consistent with this phenomenon.¹⁵ For this reason, it is of independent interest to understand what features of our model are crucial for generating comovement, and to relate these to the features explored in the literature. This is what we do in this section.

4.1. Comovement with Habit Persistence and Limited Labor Mobility

We first argue that both the limited labor mobility and habit persistence assumptions are necessary for comovement in our model. To see this, consider the Euler equation for employment in the consumption sector, which says that the conditional mean of the product of the marginal utility of consumption and the marginal product of labor in the consumption sector must equal the marginal utility of leisure:

$$\nu \frac{E_{t-1} \Lambda_{c,t} C_t}{H_{c,t}} = (1 - H_{c,t} - H_{i,t})^{-\xi}. \quad (4.1)$$

Here, $\nu > 0$ and $\Lambda_{c,t}$ is the marginal utility of consumption. In our model $\xi = 0$, but it is useful for pedagogical purposes to allow for $\xi > 0$ too. When the habit persistence assumption is dropped (i.e., $b = 0$), then the value of output in the consumption sector, $\Lambda_{c,t} C_t$, is a constant. Rearranging (4.1):

$$H_{c,t} = \nu(1 - H_{c,t} - H_{i,t})^\xi. \quad (4.2)$$

Evidently, if $\xi > 0$, then $H_{i,t}$ and $H_{c,t}$ *must* move in opposite directions. The most favorable case for comovement corresponds to $\xi = 0$, when $H_{c,t}$ is a constant. Either way, the model does not exhibit comovement in employment.

Now suppose we maintain habit persistence, but drop the limited labor mobility assumption, so that the expectation operator in (4.1) is deleted. The exact expression for $\Lambda_{c,t}$ is complicated, but

¹⁴Statistically, we say two variables comove over the cycle if their correlation with aggregate output and with each other is positive at business cycle frequencies, that is, after the variables have been logged and Hodrick-Prescott filtered. In the statistical definition of comovement, the requirement that two variables be positively correlated with each other, in addition to aggregate output, is important. For example, suppose $y = c + i$. If c and i are uncorrelated with each other they clearly do not ‘move up and down together’. Still, the correlation of each of these variables with y is positive.

¹⁵See Benhabib, Rogerson and Wright (1991) and Murphy, Shleifer and Vishny (1989) for the observation that standard real business cycle models are inconsistent with comovement.

we can approximate it by ignoring the impact of current consumption on next period's utility, in which case $\Lambda_{c,t} = 1/(C_t - bC_{t-1})$. In this case, the value of the output in the consumption sector, $\Lambda_{c,t}C_t = 1/(1 - bC_{t-1}/C_t)$, falls when a shock drives up C_t . It is readily verified that if $\xi = 0$ then $H_{c,t}$ must drop and if $\xi > 0$ then $H_{c,t}$ and $H_{i,t}$ must move in opposite directions. Either way, comovement in employment fails. Although this does not constitute a proof that comovement fails when endogeneity of the habit stock is taken into account, it is suggestive, and the implications of the argument are consistent with the quantitative results reported later. These indicate that, for plausible parameter values, both limited labor mobility and habit persistence are necessary for comovement.

To understand why our two assumptions allow the model to generate comovement, suppose there is a surge in C_t which is transitory, so that it generates an expectation of a rise in C_t/C_{t+1} and, hence, of $1/(1 - bC_t/C_{t+1})$ too. The value of output in the consumption sector drops in period t , but rises in period $t + 1$. These considerations alone hurt comovement in the period of the shock, but help in period $t + 1$. The limited mobility of labor assumption in effect neutralizes the negative contemporaneous impact on comovement of habit persistence by forcing labor to be decided prior to the shock.

An implication of this argument is that comovement in the next period is more likely, the more transitory is the shock to C_t . It is interesting that transitoryness not only enhances the likelihood of comovement, but it also enhances the ability of the model to account for the equity premium (see Boldrin, Christiano and Fisher (1997).)¹⁶

4.2. Comovement in the Literature

Condition (4.2) also characterizes the equilibrium¹⁷ in standard real business cycle models. For the most part, the literature attempts to understand comovement by identifying factors which raise the value of the output of the sector which produces consumption goods in a boom.¹⁸ We briefly discuss

¹⁶The underlying intuition is based on simple permanent income-type reasoning. Thus, consider the case in which consumption growth is positively autocorrelated. Then, a positive innovation in consumption next period is expected to be associated with further increases in subsequent periods. Permanent income considerations lead households to sell assets in such states of nature. Because everyone tries to do this, the equilibrium must have the property that the price of equity is low in such a state of nature. With the price of equity low in high consumption states (and, hence, high in low consumption states), equity is a good hedge against risk and the equilibrium equity premium must be small. Similar reasoning leads to the conclusion that small or negative autocorrelation in consumption results in a high equity premium.

¹⁷For example, it holds for the model studied by Hansen (1985), in which $b = 0$, $\xi = 1$, $\Lambda_{c,t} = 1/C_t$ and the Euler equation for labor is given by (4.1) without the expectation operator. It also holds in the version of Hansen's model in which the elasticity of substitution in utility between consumption and leisure is unity.

¹⁸Equation (4.1) suggests two alternative strategies. One, pursued by Einarsson and Marquis (1997), focuses on the marginal utility of leisure over the cycle. They introduce an additional use of time, H_m , say. It is evident that, by replacing the expression on the right of (4.1) with $(1 - H_{c,t} - H_{i,t} - H_{m,t})^{-\xi}$, it is possible to avoid the implication that H_c and H_i must move in opposite directions. However, this requires that $H_{m,t}$ be sufficiently countercyclical. Einarsson and Marquis model H_m as time spent out of the home and the market in accumulating human capital. Their explanation for comovement is suggestive. Still, it must remain tentative until one (i) identifies an empirical measure for H_m , and (ii) verifies that the cyclical behavior of H_m in their model matches roughly the cyclical behavior of its empirical counterpart.

two of these approaches here. One focuses on the role of durable goods in utility, and the other focuses on the role of intersectoral linkages in production.¹⁹

Baxter (1996) shows that allowing for some substitutability in utility between durables and market consumption can lead to comovement by enhancing the value of market consumption in a boom. For example, in the extreme case when C_t and the service flow from durables, D_t , are perfect substitutes, and utility from these is given by $\log(C_t + D_t)$, the value of consumption is $1/(1 + D_t/C_t)$. This is increasing in C_t in the plausible case that D_t is slow to respond to a shock.²⁰ While Baxter’s model appears to make substantial headway towards understanding comovement, it still falls a little short on comovement in sectoral investment. Although capital investment in the nondurable and durable good sectors covary positively with aggregate output, they are essentially uncorrelated with each other.²¹

In her model, Baxter assumes that the output of the nondurable good sector is used exclusively for consumption. However, Hornstein and Praschnik (1997) argue that a significant fraction of the output of the nondurable good sector is actually used as an intermediate input, m_t , in the production of investment goods. They adopt an extended version of Baxter’s framework and calibrate it to accommodate this feature of the data.²² In their model, procyclical investment translates into procyclical m_t . This helps comovement because, with m_t sufficiently procyclical, the value of the output of the nondurable goods sector becomes procyclical.²³ Interestingly, even though they introduce an additional source of comovement relative to Baxter, there is a sense in which Hornstein and Praschnik get less comovement than she does. Their model implies counterfactually that investment in the nondurable and durable goods sectors is negatively correlated, while, as noted above, Baxter’s model predicts an essentially zero correlation. The difference in results presumably has to do with differences in the details of model specification and in calibration.

One interpretation of Hornstein and Praschnik’s results is that intersectoral linkages are not strong

Another strategy for escaping the straightjacket of equation (4.2) is to allow the elasticity of substitution between capital and labor in production to differ substantially from unity. To our knowledge, the potential for this to help account for comovement has not been explored.

¹⁹Another approach to comovement was pursued by Huffman and Wynne (1998). However, their focus is primarily on comovement in investment and output. They largely abstract from comovement in the labor input by making assumptions which make labor in the consumption sector essentially constant. They assume that the elasticity of substitution between labor and capital in the consumption sector is nearly unity, and that $\xi = 0$. As a result, their model has an equation that is virtually identical to (4.2), with the implication that H_c is essentially constant.

²⁰Baxter actually uses an elasticity of substitution equal to only 1.5, and reports she can get ‘realistic labor comovement’ for elasticities as low as 0.5 (p.149).

²¹Footnote 14 explains why it is important that comovement between two variables involves a positive correlation between the two.

²²See also Horvath (1997) and Long and Plosser (1983) for models in which production occurs in multiple sectors, each linked by the exchange of intermediate goods.

²³The introduction of an intermediate good link between durables and nondurables amounts to replacing $E_{t-1}\Lambda_{c,t}C_t$ in (4.1) with $(C_t + m_t)/C_t$. With m_t/C_t sufficiently procyclical, the value of the output of the nondurable good sector is procyclical, enhancing the likelihood of comovement in employment.

enough to produce full comovement. An analysis of subsectors of the nondurable goods sector gives more reason to suspect that intermediate good linkages between sectors are not a primary reason for comovement. Christiano and Kouparitsas (1998) show that there is substantial variation across subsectors in terms of how much intermediate goods they send to the investment sector. But, almost all of these sectors nevertheless exhibit a substantial degree of procyclicality. For example, less than one percent of the gross output of the retail trade sector is used as intermediate goods in the production of investment goods. Still, employment in that sector is strongly procyclical. Something other than an intermediate good link to the investment sector must be behind this procyclicality. Perhaps it is habit persistence and limited labor mobility.

5. Assigning Values to the Parameters

In this section, we discuss the values we assign to our model’s parameters. The basic approach is as follows. We choose the capital adjustment cost parameter, ψ , and the variance of the investment-specific shock, σ_μ , to match the cyclical properties of stock market and investment good prices. We estimate the innovation standard deviation of our aggregate shock, σ_ε , from the innovation standard deviation of measured consumption. The remaining parameters are selected using the methodology in BCF, to replicate as closely as possible the mean properties of asset returns, and to reproduce various first moments like average growth rates and the average share of income paid to labor. A key aspect of our method is that we do not use business cycle statistics or second moment properties of asset returns to estimate the model. These are reserved for testing purposes.

There are 10 model parameters:

$$\beta, \alpha, \delta, \gamma, \bar{\mu}, b, \rho, \sigma_\varepsilon, \psi, \sigma_\mu.$$

The first set of five parameters control the steady state properties of the model.²⁴ The second set controls the business cycle and asset pricing properties. We now consider these in sequence.

5.1. Parameters Controlling Steady State

We set

$$\beta = 0.99999, \alpha = 0.36, \delta = 0.021, \gamma = 1, \bar{\mu}\alpha/(1 - \alpha) = 0.004. \tag{5.1}$$

²⁴The parameter η , in (3.1), just controls scale, and we normalize it to 1.

The indicated value of β was selected to maximize the model's ability to account for the observed low risk free rate. The value of α was chosen based on evidence on the share of GDP earned by capital reported in Christiano (1988, ftn.3). Also, the value of δ was chosen based on Christiano and Eichenbaum (1992, p.441)'s finding that the sample average of $[1 - (K_{t+1} - I_t)/K_t]$ is 0.021. We chose a conservative value for γ . It matches the lower bound of the range of empirical estimates of the debt to equity ratio reported in Benninga and Protopapadakis (1990).²⁵ It lies below the debt to equity ratios considered in Abel and Blanchard (1986, p.258). They consider $\gamma = 2, 3$. Finally $\bar{\mu}\alpha/(1 - \alpha)$ is the steady state growth rate of consumption in the model, and $\bar{\mu}$ was selected so that it coincides with the corresponding sample average reported in Christiano and Eichenbaum (1992, p.441).

The model's steady state properties are inconsistent with the evidence reported in Section 2 that suggests there has been a break in the trend of investment good prices (see Figure 1). We expect that this shortcoming of our model does not significantly distort our results. Our value of $\bar{\mu}$ has the implication that the rate at which the price of investment goods falls in steady state is 2.84 percent per year. Although this misses the pre-1980s trend, it matches the 1980s and 1990s price trend reasonably well. The implicit price deflator for fixed investment plus consumer durables falls at the rate of 2.04 percent per year during this period.

5.2. Parameters Controlling Business Cycles and Asset Pricing

The remaining parameters are $b, \psi, \rho, \sigma_\varepsilon, \sigma_\mu$. In our model, the standard deviation in the innovation in equilibrium, log consumption is $(1 - \alpha)\sigma_\varepsilon$. We set $\sigma_\varepsilon = 0.0069$, the empirical standard deviation of the innovation in consumption, divided by $(1 - \alpha)$.²⁶ Values for the remaining four model parameters were chosen using the following four moments of the data:

$$\rho(Y, P_i) = -0.26, \quad \rho(Y, P_{k'}) = 0.28, \quad (5.2)$$

and

$$E(r^e - r^f) = 6.63, \quad Er^f = 1.19. \quad (5.3)$$

²⁵They argue (footnote 6, p. 54) that the annual average debt to total asset ratio of all U.S. corporations between 1937 and 1984 ranged between 53% and 75%. Our assumption, $\gamma = 1$, translates into a debt to total asset value of 50%.

²⁶To obtain this estimate, we regressed the first difference of the log of consumption on one lag of itself, with estimation period 1959I-1995IV. The resulting fitted residual has standard error 0.0044. The data used for these regressions were taken from DRI Basic Economics database. Consumption is measured as consumption of nondurables and services, (GCNQF+GCSQF).

Here, $\rho(x, y)$ denotes the correlation between the logged, HP filtered variable x and the similarly filtered variable, y . Also, Y denotes aggregate GDP, measured in base year prices.²⁷ Finally, E denotes the unconditional expectation operator. The first two statistics in (5.2) are the ones that motivate this paper. The two statistics in (5.3) are reproduced from Cecchetti, Lam, and Mark (1993) (CLM).

We could not find values of the parameters to reproduce (5.2) and (5.3) exactly. Instead, we choose values for b , ψ , ρ , σ_μ which make the model reproduce (5.2) exactly, and simply come as close as possible to (5.3). Our algorithm proceeds as follows. For given values of b and ρ , we select values for ψ and σ_μ so that the model replicates (5.2).²⁸ This implicitly defines ψ and σ_μ as functions of b and ρ . We then vary b and ρ over a grid of 50 admissible points to minimize the following criterion:²⁹

$$\mathcal{L}(b, \rho) = [\hat{\nu}_T - f(b, \rho)]' \hat{V}_T^{-1} [\hat{\nu}_T - f(b, \rho)], \quad (5.4)$$

where $f(b, \rho)$ is the 2×1 vector composed of the model's implication for the risk free rate and the equity premium, and $\hat{\nu}_T$ is the vector composed of the corresponding empirical estimates reported in (5.3). In addition, the 2×2 matrix \hat{V}_T is the CLM estimate of the underlying sampling variance in $\hat{\nu}_T$. Let

$$J = \mathcal{L}(\hat{b}_T, \hat{\rho}_T), \quad (5.5)$$

where \hat{b}_T , $\hat{\rho}_T$ solve the minimization problem. A value for ρ was admissible if it lies inside the unit circle. In addition, only those b, ρ combinations were considered admissible if they had the property that $C_t \leq bC_{t-1}$ and $\Lambda_{c,t} \leq 0$ are never observed in the Monte Carlo simulations used to evaluate f .

We obtained the following results:

$$\hat{b} = 0.60, \hat{\rho} = 0.625, J = 3.30. \quad (5.6)$$

The corresponding estimates of ψ , σ_μ are:³⁰

$$\hat{\psi} = 0.23, \hat{\sigma}_\mu = 0.023. \quad (5.7)$$

²⁷The data on P_i and Y are taken from DRI Basic Economics database, and are discussed in section 2. The variable, $P_{k'}$, also from DRI, is the Dow Jones Industrial average, deflated by the NIPA implicit price deflator for consumption of nondurables and services. These data cover the period 1959I-1995IV

²⁸For a given parameterization of the model, we computed its implication for a statistic, say s , by the following Monte Carlo method. We used the model to simulate 500 artificial datasets, each of length 120, giving 500 simulated values of s . The model's implication for s is identified with the means of these s 's.

²⁹We examined 50 grid points, corresponding to the five values of b , 0.450, 0.500, 0.550, 0.600, 0.650, and the 10 values of ρ , 0.450, 0.500, 0.525, 0.550, 0.600, 0.625, 0.650, 0.675, 0.700. Evaluating \mathcal{L} once requires about 10 minutes on a 200 Mhz Pentium Pro computer.

³⁰The implied values of a_1 and a_2 are 0.98 and 0.07, respectively.

We can get some idea about the shape of $\mathcal{L}(b, \rho)$ from Figure 3. It shows the 50 values of \mathcal{L} corresponding to our grid of b, ρ 's. The b, ρ combination that optimizes the criterion is indicated by a small circle, and the associated value of J is indicated in parentheses. The values of \mathcal{L} appear in 5 clusters of 10, with each cluster associated with one of the 5 values of b . The underlying value of ρ declines as we move from southeast to northwest in a cluster. Evidently, reducing ρ increases the equity premium and the risk free rate at the same time. Although our model cannot exactly replicate the empirical equity premium and risk free rate, it can get well inside a 5 percent confidence sphere.

We can at this point say something about the rationale for the particular specification of habit persistence preferences adopted in (3.1). In a more general specification, utility would have been a function of $C_t - X_t$, where the habit stock, X_t , obeys the law of motion $X_t = hX_{t-1} + bC_{t-1}$. We set $h = 0$ because this is the value which optimizes the metric in (5.5).³¹ Although this metric has nothing directly to do with business cycles, $h = 0$ is an important factor behind the model's ability to account for comovement. With a large positive value of h , a surge in consumption leaves the habit stock relatively unaffected in the following period and so there is not much of an incentive to target resources to the consumption sector then. For example, in BCF, $h > 0$ and their model does not display comovement. Christiano and Fisher (1995) discuss the reasons BCF estimate $h > 0$ while we find $h = 0$. One has to do with the specification of the driving shocks adopted here to render the model consistent with the business cycle and post-1980s trend properties of investment good prices. Another has to do with the different way leisure enters the utility function (see Section 4). We assume it enters linearly, while BCF assume it enters log-linearly.

To help evaluate our parameter estimates, we computed the implied elasticity of investment with respect to Tobin's q .³² That quantity is 1.31 in our model. For comparison, Abel (1980, p. 74)'s estimates of this quantity range from 0.50 to 1.14. Eberly (1997)'s estimates for US data, based on ordinary least squares and instrumental variables, are 0.56 and 1.06, respectively.³³ Finally, Cummins, Hassett, and Oliner (1997)'s estimates are 0.42 and 0.55 based on ordinary least squares and instru-

³¹When we minimized (5.4) setting $h = 0.1$ we found $\hat{b} = \hat{\rho} = 0.55$. The measure of fit associated with these parameter values, $J = 4.9$, is substantially worse than what we found for $h = 0$ (see (5.6).) At these parameter values, the risk free rate is 2.81, and the equity premium is 5.40.

³²The elasticity of investment in industry x with respect to Tobin's q is, using (3.12)

$$\frac{d \log I_{x,t}}{d \log q_t^x} = \left[1 + \frac{a_2}{a_1} \left(\frac{I_{x,t}}{(1-\delta)K_{x,t}} \right)^\psi \right] / (1-\psi) = \left[1 + \frac{a_2}{a_1} \left(\frac{\exp(\bar{\mu}) - 1 + \delta}{1-\delta} \right)^\psi \right] / (1-\psi)$$

in steady state.

³³To compute the implications of Eberly (1997)'s estimates for the elasticity discussed in the text, we used the average values of q , 1.56, and I/K , 0.25, reported for the US in her Table 1. We then combined these with the estimates of $d(I/K)/dq$ reported for OLS in the first panel of her Table 3 and for instrumental variables in the third panel.

mental variables, respectively.³⁴ The inverse of the above elasticity is the elasticity of the marginal cost of installed capital with respect to investment, holding prices fixed. This is the object referred to in the introduction. Our estimate of this quantity, 0.76, is evidently smaller than what is reported in the literature.

6. Quantitative Results

This section has two purposes. First, we evaluate the interpretation summarized in the introduction regarding the divergent cyclical behavior of stock market and investment good prices. We document how the various features of our calibrated model contribute to accounting for this phenomenon. We then build confidence in our interpretation by evaluating the model’s business cycle and asset pricing implications. Overall, the model does well on this test. The modifications we introduce into BCF to accommodate our hypothesis do not lead to a significant deterioration of the empirical performance of that model. Moreover, the modifications lead to two substantial improvements, by enabling the model to account for comovement and for the inverted leading indicator role of interest rates.

Second, we evaluate an alternative hypothesis about the observed cyclical behavior of stock market and investment good prices. A key fact motivating our analysis is the procyclical nature of stock market prices. Our calibrated model interprets this as reflecting their response to the demand shock (i.e., θ_t). But, with a high enough marginal installation cost elasticity, stock prices could even be procyclical with respect to the investment-specific shock (the intuition for this is implicit in the discussion in the introduction). In this case, only the investment-specific shock would be needed to explain the divergent cyclical behavior of stock market and investment good prices. This leads us to also consider a one-shock hypothesis, that business cycle fluctuations are driven only by the investment-specific shock. Interestingly, we find that stock prices are predicted to be procyclical with an installation elasticity not much larger than the ones reported in the literature. We nevertheless reject the one shock hypothesis because, as documented below, it destroys the ability of the model to account for several salient features of asset returns and business cycles.³⁵

³⁴To compute the implications of Cummins, Hassett, and Oliner (1997)’s estimates for the elasticity discussed in the text, we used the average values of q , 1.376, and I/K , 0.29, reported for the US in their Table 1. We then combined these with the estimates of $d(I/K)/dq$ reported for OLS in their Table 2, column 2, row 3 and for instrumental variables in Table 3, column 2, row 3.

³⁵There are at least five ways in which it does this. Under the one-shock hypothesis: (i) the short run predictability of consumption is counterfactually great and the model’s ability to replicate the equity premium is lost; (ii) the volatility of consumption and its correlation with output is reduced to almost zero; (iii) the higher marginal cost elasticity reduces the volatility of hours worked to essentially zero, makes hours worked countercyclical, and reduces the model’s internal sources of propagation; (iv) the model loses its ability to account for the inverted leading indicator property of interest rates; and (v) with the investment-specific shock only, we are not able to account for all the volatility of output.

The following discussion is organized into two parts. In the first, we study the model’s implications for financial market prices and rates of return, and for the price of investment goods. In the second, we study the model’s implications for business cycles.

6.1. Financial Markets

Table 1 presents various statistics which capture the implications of our model for financial variables. The first column presents results based on the data (‘Data’). The column marked ‘Calibrated’ reports results using the parameter values from the calibration described in the previous section. The columns to the right of that report results based on perturbing the value, as indicated in the column header, of one (or, in two cases, two) of the model parameters. In each case, the other parameter values are kept at their calibrated values.

6.1.1. Stock Market and Investment Good Prices

We begin by documenting the contributions of the two-shock and adjustment cost assumptions to explaining the sign switch in the correlation with output of stock market and investment good prices. When adjustment costs are eliminated (‘ $\psi = 0.9$ ’), or there is only an aggregate shock (‘ $\sigma_\mu = 0$ ’), then the model loses its ability to account for the sign switch. This is to be expected. Adjustment costs are the wedge between stock market and investment good prices (recall (3.10)). Eliminate the wedge, and the difference in the dynamic behavior between the two prices disappears too. Also, with only aggregate shocks there are only sources of procyclicality in the two prices.

The one-shock hypothesis mentioned above *is* capable of accounting for the sign switch, if the marginal installation cost elasticity is sufficiently large. Simply setting the variance of the aggregate shock to zero without also raising the cost elasticity does not work, since both prices are countercyclical with respect to the investment-specific shock in the calibrated model (see the column marked ‘ $\sigma_\varepsilon = 0$, $\psi = 0.23$ ’). With ψ reduced to -0.145 , stock prices are procyclical with respect to the investment-specific shock and, moreover, the model replicates the empirical correlation between the stock price and output (‘ $\sigma_\varepsilon = 0$, $\psi = -0.145$ ’). Interestingly, the new value of ψ does not imply an inordinately high marginal cost elasticity. The implied elasticity of investment to Tobin’s q is 0.87, and this lies well within the range of the empirical estimates discussed above. Thus, the one-shock hypothesis is capable of accounting for the sign switch observation. However, we show below that the one-shock hypothesis has other bad empirical implications. A relatively small problem that is already evident in Table 1 is that the correlation of P_i with output is predicted to be close to minus one.

Further insight into our calibrated model’s account for the sign switch may be obtained from

Figure 4. That figure displays the response of prices and rates of return to a one-standard deviation innovation in the aggregate technology shock (solid line) and in the investment-specific shock (dashed line). Consider the response to the aggregate shock. There is a sharp rise in both the investment good price, P_i , and the price of equity, $P_{k'}$ (Figures 4A and 4B). Consistent with the intuition in the introduction and the discussion in Section 3, the jump in the equity price exceeds that in the price of investment goods. Consider the response to an investment-specific technology shock. As anticipated in the introduction, Figure 4 shows that $P_{k'}$ falls relatively little, by comparison with P_i .

Although the model accounts well for the cyclical comovement with output of investment and equity prices, it does not account well for the magnitude of their cyclical volatility. In the data, the standard deviation of equity prices is a little below 10 percent, while the standard deviation of investment good prices is a little above 1 percent. In the model, these two prices have roughly the same standard deviation, equal to the midpoint between the two empirical standard deviations. Interestingly, the Shiller (1981) “excess volatility” puzzle stands here. Despite its (counterfactually, as we will see) high volatility in interest rates, the model still cannot account for the observed high volatility of stock prices.

6.1.2. The Equity Premium and Risk Free Rate

The asset pricing performance of our model is comparable to that of BCF. As noted above, the equity premium and risk free rate implied by our calibrated model lie well inside a 5 percent confidence ellipse of the corresponding empirical estimates (Figure 3). Table 1 shows that the equity premium is reduced by a factor of 10 by eliminating habit persistence in preferences (see the column, ‘ $b = 0$ ’), or by dropping the limited labor mobility assumption (see the column, “Full Labor”).³⁶ This is consistent with BCF’s conclusion that limitations on factor mobility and habit persistence are important for producing an equity premium in a business cycle model. Our model has essentially the same implications as BCF’s for risk aversion, and so we do not discuss this here.

As can be anticipated from earlier discussion, the stochastic properties of the exogenous shocks are important for the model’s ability to replicate the equity premium. Increased autocorrelation in the aggregate technology shock reduces the equity premium (‘ $\rho = 0.99$, $\psi = 0.23$ ’).³⁷ Also, the equity premium disappears when the innovation of the aggregate technology shock is set to zero (‘ $\sigma_\varepsilon = 0$, $\psi = 0.23$ ’ or ‘ $\sigma_\varepsilon = 0$, $\psi = -0.145$ ’), and is virtually unaffected by setting the innovation in the investment-specific shock to zero (‘ $\sigma_\mu = 0$ ’). These latter findings reflect that the equity premium

³⁶Recall, the limited labor mobility assumption specifies that households’ sectoral labor supply decision is made before the current-period realization of the technology shocks.

³⁷See footnote 16 for the underlying intuition.

depends on the one-step-ahead uncertainty in aggregate consumption.³⁸ Consistent with this, under the one-shock hypothesis discussed above, the model loses its ability to account for the equity premium.

To gain further insight into the dynamic behavior of rates of return in the model, consider Figures 4C and 4D. The risk free rate hardly responds to the investment specific shock (see also the ‘ $\sigma_\varepsilon = 0$, $\psi = 0.23$ ’ column in Table 1), but responds massively to the aggregate shock. This reflects (i) that the investment-specific shock has no current period impact on consumption, while the aggregate shock does, (ii) the primary avenue by which a shock affects the risk free rate in the model is via the current period marginal utility of consumption and (iii) the marginal utility of consumption is sensitive to current consumption under habit persistence.

The strong response in the risk free rate to the aggregate shock is the reason our model overstates by a factor of three the volatility of the risk free rate (see Table 1).³⁹ The response of the rate of return on equity to the aggregate shock is also very strong, and explains why the model overstates by a factor of two the volatility of the equity premium and the return on equity (see Table 1). The poor performance of the model on these dimensions is comparable to that of BCF.

There is an aspect of the second moment properties of asset returns on which the model does reasonably well. In particular, like BCF, our model does reasonably well on the Sharpe ratio, Er^e/σ_{r^e} .⁴⁰ According to Table 1, the model’s Sharpe ratio of 0.14 lies just barely outside a two-standard deviation bound of the corresponding empirical estimate reported in CLM. According to the results in BCF (their Table 1a), the Sharpe ratio for a standard real business cycle model is 0.0001. The results in Table 1 indicate that the key features of our model that account for its relatively good performance on the Sharpe ratio are habit persistence in preferences and limited labor mobility.

6.2. Business Cycles

We now turn to the business cycle implications of our model. We first discuss the implications for the measures of volatility and correlation that are typically considered in the business cycle literature. We then turn to features of business cycles which have attracted relatively less attention: business cycle comovement, and the inverted leading indicator property of interest rates.

³⁸See equation (B.18) in CF.

³⁹Our findings on the volatility of asset returns are consistent with those in of Heaton (1995), who estimates and tests an endowment model with habit persistence preferences. One of Heaton’s findings is that the ability to account for first moments of asset returns comes at a cost of poor performance on second moments. Although this appears to be a feature of our model too, the results of Campbell and Cochrane (1995) suggest that this need not be a feature of models with habit persistence generally.

⁴⁰See, for example, Hansen and Jagannathan (1991) for a discussion of why this statistic is of interest in the analysis of asset returns.

6.2.1. Standard Business Cycle Statistics

Table 2 presents the implications of our model for standard business cycle statistics. The format of that table corresponds to that of Table 1. The results in the ‘Data’ column are based on data from the NIPA, which are computed using base year prices. To put our simulated output and investment data on a comparable basis, we measure them in base year prices too. The base year in our simulations is the initial observation, when the state of investment-specific technology, V_t , is set to unity, and the model is assumed to be on a steady state growth path, so that the relative price of investment goods is one. Thus, base year output in the model is the simple sum of the physical quantity of consumption and investment goods produced. Similarly, base year investment is the quantity of investment goods.⁴¹

6.2.2. Standard Business Cycle Statistics

Aggregate Output

Interestingly, the model’s overall volatility implication is quite close to what we observe in the data (compare σ_Y in the ‘Data’ and ‘Calibrated’ columns). This represents a notable confirmation for the model, since the overall volatility of output plays no role in the model calibration. The results suggest that the investment-specific shock is the most important source of impulses to the business cycle fluctuations in output. When the aggregate shock variance is shut down (see the ‘ $\sigma_\varepsilon = 0, \psi = 0.23$ ’ column) the variance of business cycle fluctuations falls by only 25 percent (i.e., $100 \times [1 - 1.40^2/1.61^2]$). The remaining variance is due to the investment-specific shock. Figure 5A displays the dynamic response function of aggregate output to the two shocks.

Consumption

A key success of standard real business cycle models, shared by BCF, lies in their ability to account for the observed smoothness of consumption and relative volatility of investment. Our model does well on this dimension too. Both shocks play an important role in this. For example, if the investment-specific shock is too small (‘ $\sigma_\mu = 0$ ’), then the relative volatility of consumption is too high. If the aggregate shock is too small, then the relative volatility of consumption is too low (see ‘ $\sigma_\varepsilon = 0, \psi = 0.23$ ’). Figure 5B displays the response of consumption to the two shocks, respectively. Note that,

⁴¹An alternative strategy would have been to compare quantities measured in consumption units in the model with quantities measured in consumption units in the data. There are two reasons that we do not adopt this alternative. First, the approach we take, of focussing on quantities measured in base year prices, is standard practice. Second, the fact, documented in the previous subsection, that the volatility of P_i in our model is counterfactually high, implies that output measured in consumption units, $C + P_i I$, is dominated by P_i .

because of the limitations on the intersectoral mobility of resources, consumption responds immediately to the aggregate shock and slowly to the investment-specific shock. The response to the first shock generates negative autocorrelation in consumption growth ($\sigma_\mu = 0$), while the response to the second generates positive autocorrelation ($\sigma_\varepsilon = 0, \psi = 0.23$). The net outcome with both shocks is negative, and reflects mostly the aggregate shock. This is because around 98 percent of the business cycle variation in consumption reflects the aggregate shock.

The model's negative autocorrelation in consumption growth misses the corresponding empirical estimate, which is positive and statistically significant.⁴² We were concerned that the model's ability to account for the equity premium with low risk aversion might be an artifact of this empirical shortcoming. To investigate this, we carried out two more experiments. We increased the autocorrelation of equilibrium consumption growth to zero by raising ρ to 0.99 (see the column marked ' $\rho = 0.99, \psi = 0.23$ '). Table 1 shows that this reduces the equity premium and the risk free rate. However, these numbers are still well within the 10% confidence ellipse in Figure 3.⁴³ At the same time, the parameter change causes employment in the consumption sector to become countercyclical. Table 2 indicates that this is corrected, without significantly hurting other implications, by increasing ψ ($\rho = 0.99, \psi = 0.40$). Not surprisingly, this change cuts a little into the model's ability to account for the cyclical correlation of stock market and investment good prices. However, there is still a sign switch in these statistics. We conclude from these two experiments that the model's ability to account for the equity premium with low risk aversion in consumption is not an artifact of counterfactual implications for consumption growth.

Employment

The model does poorly on the relative volatility of employment. In the data, hours worked fluctuate about as much, in percent terms, as output. However, the calibrated model implies that hours worked fluctuate a little over 10 percent as much as output.

The low volatility of labor reflects in part the high degree of persistence in the shocks. This works by reducing the motive to intertemporally substitute leisure in response to shocks. For example, when the persistence of the aggregate shock is increased ($\rho = 0.99, \psi = 0.23$), the relative volatility of hours worked drops to nearly zero. When the highly persistent investment-specific shock is eliminated, relative hours volatility jumps by 50 percent ($\sigma_\mu = 0$).

⁴²One interpretation, pursued in, for example, Christiano, Eichenbaum and Marshall (1991), is that the positive autocorrelation in consumption growth reflects temporal aggregation. We do not pursue this interpretation here.

⁴³To be precise, the p-value associated with this risk free rate, equity premium combination is 0.086, or 8.6 percent.

The low degree of volatility also reflects, in part, the various types of adjustment costs in the model. When the adjustment costs in installing new capital are reduced ($\psi = 0.9$), relative hours volatility jumps by a factor of four. When the limited labor mobility assumption is removed (*‘Full Labor’*), relative volatility doubles.

One interpretation of these findings is that they warrant rejecting a basic thesis of this paper: that adjustment costs play a role in accounting for the different cyclical behavior in investment good and stock market prices. An interpretation we find more appealing is that they reflect misspecification of some other aspect of the model. For example, it would be of interest to explore specifications with less persistence in the exogenous shocks. Another promising avenue would be to explore alternative specifications of aspects of the labor market. For example, Mortensen (1992) conjectures that an important reason for the high observed volatility of employment is that search activities are very sensitive to changing labor market conditions.⁴⁴ Alexopoulos (1998) shows that incentives to shirk when there is asymmetric information in the labor market can give rise to substantial cyclical labor volatility. Finally, Rupert, Rogerson and Wright (1997) argue that home production considerations can go a long way to accounting for observed labor volatility. Exploring these possibilities would be of interest, but is beyond the scope of this paper.

Persistence

Macroeconomists have for a long time sought to identify factors which propagate the effects of economic shocks over time. Standard real business cycle models do poorly on this dimension. For example, it is known that the persistence of equilibrium output growth in these models closely matches the persistence properties of the underlying economic shocks (see Christiano (1988), Cogley and Nason (1995) and Watson (1993)). One indicator that our calibrated model introduces persistence is that its implied growth rate of the Solow residual has autocorrelation -0.06 , while equilibrium output growth has autocorrelation 0.02 (see Table 2).⁴⁵

It is interesting to note how curvature in adjustment costs dampens the model’s internal propagation mechanism. Thus, when curvature is essentially eliminated ($\psi = 0.9$), then output persistence jumps noticeably, to 0.32 . Another bad empirical implication of the one-shock hypothesis ($\sigma_\varepsilon = 0$, $\psi = -0.145$) is that the increased curvature cuts significantly into the model’s ability to generate persistence.

⁴⁴See also Andolfatto (1996), Cole and Rogerson (1996), den Haan, Ramey and Watson, (1997) and Merz (1995).

⁴⁵We measure the Solow residual in the same way that it is measured in the data, as $Y_t / [K_t^\alpha H_t^{1-\alpha}]$, where Y_t is aggregate output (in base year prices) and K_t and H_t denote the aggregate stock of capital and employment, respectively.

Campbell and Mankiw (1989,1991) and Christiano (1989) argue that standard real business cycle models also fail to account for another form of persistence, namely, that the forecastable component in consumption growth is related to the forecastable component in income growth. BCF show that their model is able to account for this observation. The model in this paper performs about as well as BCF on this dimension. To save space, we do not present these results here.

6.2.3. Comovement

It has been extensively documented that employment and investment move up and down together across economic sectors, so there is no need to repeat this here.⁴⁶ Section 4 explains how limited labor mobility and habit persistence help our model produce comovement. Table 3 documents that the model produces very strong comovement over the cycle in sectoral employment and investment. In addition, it shows that habit persistence and limited labor mobility are necessary for our model to produce comovement. Without either of these (see the ‘Full Labor’ and ‘ $b = 0$ ’ columns) employment in the consumption sector is countercyclical. Figures 5E-5H document the dynamic response of sectoral investment and hours worked to the two shocks. With one exception, employment in both sectors respond positively to each shock. The exception is hours worked in the consumption sector, which does not respond to an investment-specific shock. Note that the investment response in the two sectors is very similar, with the response in the investment good sector being slightly stronger.

6.2.4. Inverted Leading Indicators

Here, we consider the implications of our model for the inverted leading indicator role of real and nominal rates of interest. We begin with the real rate of interest, which we approximate in the data by the *ex post* real return.

Real Interest Rates

The data column in Table 4 displays the results for the US.⁴⁷ It shows the dynamic cross correlation between the HP-filtered *ex post* real return on the Federal Funds rate and logged, HP-filtered aggregate output. The correlation between the interest rate and future output is negative, while the correlation between the interest rate and past output is positive. Our calibrated model reproduces this pattern,

⁴⁶See, for example, Christiano and Fisher (1995) for evidence on comovement of labor inputs. They note that the degree of business cycle comovement is especially striking in view of the fact that the trend behavior of labor inputs in different sectors are so different. For other recent discussions of the evidence on comovement, see Baxter (1996), Hornstein and Praschnik (1998), Huffman and Wynne (1988), and Murphy, Shleifer and Vishny (1989).

⁴⁷See Fiorito and Kollintzas (1994), who show that the inverted leading indicator property of *ex post* real returns also holds for other countries too.

and the results in the table shed light on why this is so. Note first that the dynamic correlation primarily reflects the effects of the aggregate shock. When the investment-specific shock is shut down ($\sigma_\mu = 0$), the correlations are qualitatively unaffected. When the aggregate shock is shut down ($\sigma_\varepsilon = 0, \psi = 0.23$), the result is gone. Note, too, that habit persistence plays only a small role in this result. When it is eliminated, the inverted leading indicator property of interest rates continues to hold ($b = 0$), although it is less pronounced.

An important factor underlying the model result is its implication that a positive realization of the aggregate shock causes consumption to ‘overshoot’: it shifts up quickly and then declines back to trend over time (Figure 5B). The reduction in expected consumption growth triggered by a positive realization of the aggregate shock is the reason why the rate of interest jumps. These considerations alone imply a strongly counterfactual negative autocorrelation in consumption growth (compare the ‘ $\sigma_\mu = 0$ ’ and the ‘Data’ columns of Table 2). However, equilibrium consumption growth in our calibrated model is not as negative as the aggregate shock alone suggests, because of the effects of the investment specific shock (see Figure 5B). Still, as noted above, the calibrated model does imply a large negative autocorrelation in equilibrium consumption growth (see ‘Calibrated’ column in Table 2). To verify that this counterfactual implication is not at the heart of the model’s ability to account for the inverted leading indicator phenomenon, consider the results in the Table 2 column marked ‘ $\rho = 0.99, \psi = 0.40$ ’. Under this parameterization of the model, equilibrium consumption growth is approximately a random walk, and yet the model’s inverted leading indicator property is preserved.

Interestingly, consumption overshooting is not a feature of standard business cycle models. For example, we simulated a version of the model in which $b = 0, \psi = 1$, the investment-specific shock, V_t , is fixed at unity, and capital and labor are freely transferrable across sectors. This is essentially the model studied in Hansen (1985). This model implies a hump-shaped consumption response to an aggregate shock: the initial rise in consumption is followed by further increases before consumption converges back down to steady state. As a consequence, this version of the model is not capable of reproducing the inverted leading indicator property of real interest rates.⁴⁸ King and Watson (1996) report a similar finding for a real business cycle model in which the first difference of θ_t has the first

⁴⁸In the model, $Er_{t+k}^f Y_t$, $k = -2, -1, 0, 1, 2$ is 0.18, 0.39, 0.75, 0.29, 0.03, where r_t^f is the risk free rate expressed at an annual rate and then HP-filtered, and Y_t is the HP-filter of the logged output series. These numbers were computed as the others in the tables, by averaging over the results based on 500 artificial data sets of length 120 each. The implications for consumption overshooting and for inverted leading indicators are essentially the same when we replace the period utility function with $\log(C_t) + \eta \log(1 - H_t)$, where η was chosen so that the steady state value of H_t is 0.3. When this example is modified, by requiring that investment be determined prior to the realization of the technology shock, then consumption overshoots and the model is consistent with the inverted leading indicator result. However, in this case, consumption growth is negatively autocorrelated.

order autoregressive representation in (3.3).⁴⁹

A conventional interpretation of the leading indicator role of interest rates is that it reflects the dynamic economic effects of disturbances to monetary policy. The popularity of this interpretation is in part due to the fact that standard real business cycle models cannot account for it. In addition, several authors have argued that a monetary model with a strong liquidity effect could in principle account for the inverted leading indicator phenomenon.⁵⁰ The monetary interpretation of the inverted leading indicator result seems to be reinforced by the fact that it is also a characteristic of the nominal rate of interest, a rate of return that is widely believed to be determined by the actions of the Federal Reserve.

Nominal Interest Rates

To investigate the compatibility of our model with the inverted leading indicator role of *nominal* interest rates, we have to monetize it. We do this in a way that preserves the spirit of the model, namely, by imposing an extreme form of monetary neutrality. Following the strategy in King and Watson (1996), we assume that real variables are the outcome of the planning problem of this paper, while nominal variables are determined separately by a standard money demand equation, a Fisher relation, and a specification for the money supply. To accommodate the observed procyclical nature of money, we specify that the money supply responds positively to the current and past values of the two shocks.⁵¹

We found that the model replicates key features of the dynamic cross correlation between the nominal rate of interest rate, money, and prices on one hand, and output on the other hand. To see this, consider the results in Table 5, which exhibits the cyclical correlation between these variables. Note first how the nominal Federal Funds rate displays the inverted leading indicator property. Although it is positively correlated contemporaneously with output, it is negatively correlated with output two quarters later. While the model misses on the first of these observations, it is consistent with the fact

⁴⁹The model of Christiano and Todd (1996) does generate consumption overshooting. It would be interesting to explore the inverted leading indicator implications of that model.

⁵⁰See, for example, Chari, Christiano and Eichenbaum (1995). However, these authors and King and Watson (1996) show that early versions of the liquidity effects monetary model fail to reproduce the inverted leading indicator phenomenon. More recently, Christiano and Gust (1998) display a version of such a model that is consistent with this phenomenon.

⁵¹We specify the following money demand equation: $M_t = P_t + \log(C_t) - m_R R_t$, where M_t is the log of the money stock, P_t is the log of the price level, R_t is the nominal rate of interest (quarterly rate), and m_R is the money demand elasticity. Following King and Watson (1996), we set $m_R = 0.01$. In addition, we specify that the nominal interest rate is determined according to the Fisher relation, $1 + R_t = r_t^f + E_t \log(P_{t+1}) - \log(P_t)$, where E_t is the conditional expectation operator. Finally, we specify that the money supply has the form, $M_t = \alpha(L) [\varepsilon_t + \mu_t]$, with $\alpha(L) = 0.01/(1 - 0.8L)$. We solved for the price level, the rate of inflation and the nominal rate of interest using the approach used by Sargent (1979) to solve the Cagan model of money.

that the nominal rate of interest is negatively correlated with future output.

Turning to the money supply, note that in both the model and the data, it covaries positively with output. The model is also consistent with the negative cyclical correlation between output and the price level. It is even, to some extent, consistent with the positive correlation that exists between output and inflation. This sign switch between the price-output and inflation-output correlation has been interpreted by some as support for a sticky price model (see, e.g., Ball and Mankiw (1994).) Interestingly, here we account for this in a flexible price model.

These results suggest an intriguing possibility. Evidence cited in support of the notion that monetary policy shocks play a significant role in business cycles may, at least to some extent, reflect the effects of real economic shocks originating in the private economy.⁵² For example, estimated orthogonalized interest rate shocks are typically interpreted as isolating shocks to monetary policy. They may in fact also include a shock like our aggregate technology shock. That is because what happens after a bad realization of the aggregate technology shock in our model is similar to what happens after a positive orthogonalized interest rate shock in the vector autoregression (VAR) literature (for a survey, see Christiano, Eichenbaum and Evans (1998).) The bad technology shock signals low output in the future, a high current interest rate (whether nominal or real), and a temporarily high price level.

The notion that estimated orthogonalized interest rate shocks in the VAR literature are distorted by a real shock has some appeal because it could help resolve two puzzles. First, the fact that there is a transient rise in the price level after a bad aggregate shock suggests that our model may help account for the so-called ‘price puzzle’ analyzed in the VAR literature. This is the tendency for prices to rise for a while after a positive interest rate shock, something that is hard to reconcile with the notion that an orthogonalized jump in the interest rate reflects a monetary policy tightening. Second, Christiano, Eichenbaum and Evans (1998) document that even though different empirical measures of monetary policy shocks tend to have a very low correlation, they nevertheless tend to have similar implications for what happens after a ‘monetary’ policy shock. They conjecture that this reflects that the different measures confound, in various degrees, some other shock which has a similar dynamic impact on the economy. The aggregate technology shock in this paper could be that shock.

7. Concluding Remarks

We used the business cycle properties of stock market prices and investment good prices to draw inferences about the source of business cycle shocks and the nature of capital installation costs. We

⁵²See Sims (1980, 1997) for a similar conjecture.

achieved identification using an explicitly formulated model.

As in Greenwood, Hercowitz and Krusell (1997, 1998), we infer that the countercyclical nature of investment good prices reflects the effects of supply shocks in the investment sector.⁵³ Our analysis suggests that a second shock also plays a role in the business cycle. In addition, we interpret the fact that investment good and stock market prices behave differently as reflecting the presence of costs for installing new capital.

To test these inferences, we go beyond simply constructing a model that can account quantitatively for the observations that interest us. We also investigate the model's implications for business cycle and asset pricing phenomena. We find that the model does not do significantly worse than other models on these dimensions. In some respects, the model actually represents a step forward. For example, on the business cycle dimension, the model contains an endogenous source of propagation. In addition, the model is consistent with observations on the sectoral comovement of investment, output and employment.

But, how do we reconcile our real business cycle model with the growing literature that suggests that at least a small part of business cycle fluctuations may be due to monetary policy shocks? Our model suggests the possibility that some of the evidence of a business cycle role for monetary policy shocks may actually reflect the effects of private sector technology shocks. Working this possibility out in detail is well beyond the scope of this paper, but is clearly an important topic for further research.

As in any empirical research, our model has shortcomings. We did our best to draw these out as clearly as possible. On the asset pricing side, our model inherits the difficulties that BCF has in accounting for the second moment of asset returns. On the business cycle side, our model has difficulties with the labor market. Resolving these shortcomings constitutes a task for future research.

⁵³Alternative interpretations of the negative correlation between investment prices and output are possible. For example, Murphy, Shleifer and Vishny (1989) infer that the business cycle is driven by a shock to the demand for investment goods, and that there are increasing returns in the production of investment goods. The empirical evidence seems to go against this interpretation, however. For example, although Harrison (1998) provides some empirical support for the notion that there are increasing returns associated with capital and labor in the production of investment goods, the degree of increasing returns she finds is not sufficient to generate increasing returns in the factor of production, labor, that is variable in the short run. This is consistent with the findings of Goolsby (1998) and Shea (1993), who find that supply curves slope up. Goolsby's analysis focuses specifically on the supply of capital goods. Yet another possible interpretation of countercyclical investment good prices is that markups in the investment good industry are countercyclical. It would be of interest to investigate this hypothesis further.

References

- Abel, Andrew, 1980, "Empirical investment equations: An integrative framework," *Carnegie-Rochester Conference Series on Public Policy*, vol. 12, pp. 39–91.
- Abel, Andrew, and Olivier Blanchard, 1986, 'The Present Value of Profits and Cyclical Movements in Investment,' *Econometrica*, vol. 54, no. 2 March, pp. 249-73.
- Alexopoulos, Michelle, 1998, 'Monetary Shocks and Shirking,' unpublished manuscript, department of economics, Northwestern University.
- Andolfatto, David, 1996, 'Business Cycles and Labor-Market Search,' *American Economic Review*, 86, no. 1 March, pp. 112-32.
- Ball, Laurence and N. Gregory Mankiw, 1994, 'A Sticky-Price Manifesto,' *Carnegie-Rochester Conference Series on Public Policy*, vol. 41, December, pp. 127-51.
- Baxter, Marianne, 1996, 'Are Consumer Durables Important for Business Cycles?', *Review of Economics and Statistics*, vol.78, no. 1, February.
- Benhabib, Jess, Richard Rogerson, and Randall Wright, 1991, 'Homework in macroeconomics: Household production and aggregate fluctuations,' *Journal of Political Economy*, vol. 99, pp. 1166–1187.
- Benninga, Simon and Aris Protopapadakis, 1990, 'Leverage, time preference and the equity premium puzzle,' *Journal of Monetary Economics*, vol. 25, pp. 49–58.
- Boldrin, Michele, Lawrence J. Christiano, and Jonas D.M. Fisher, 1995, 'Asset pricing lessons for modeling business cycles,' *National Bureau of Economic Research working paper 5262*.
- Boldrin, Michele, Lawrence J. Christiano, and Jonas D.M. Fisher, 1997, 'Habit Persistence and Asset Returns in an Exchange Economy,' *Macroeconomic Dynamics*, vol. 1, no. 2.
- Burns, Arthur, and W. C. Mitchell, 1946, 'Measuring Business Cycles, *Studies in Business Cycles*,' No. 2, National Bureau of Economic Research.
- Campbell, John Y., and John H. Cochrane, 1995, 'By force of habit: A consumption-based explanation of aggregate stock market behavior,' *National Bureau of Economic Research working paper 4995*.
- Campbell, John, and N. Gregory Mankiw, 1989, 'Consumption, income, and interest rates: reinterpreting the time series evidence,' in Blanchard and Fischer, editors, *NBER Macroeconomics Annual 1989*.
- Campbell, John, and N. Gregory Mankiw, 1991, 'The response of consumption to income: a cross-country investigation,' *European Economic Review*, vol. 35, pp. 723-767.
- Cecchetti, Stephen G., Pok-sang Lam, and Nelson C. Mark, 1993, 'The equity premium and the risk-free rate: Matching the moments,' *Journal of Monetary Economics*, vol. 31, pp. 21–45.
- Chari, V.V., Lawrence J. Christiano and Martin Eichenbaum, 1995, 'Inside Money, Outside Money, and Short-Term Interest Rates,' *Journal of Money, Credit and Banking*, vol. 27, no. 4, part 2, November, pp. 1354-1401.
- Christiano, Lawrence J., 1988, 'Why does inventory investment fluctuate so much?' *Journal of Monetary Economics*, vol. 21, pp. 247–280.

- Christiano, Lawrence J., 1989, "Comment on 'consumption, income, and interest rates: Reinterpreting the time series evidence,'" in Olivier Jean Blanchard and Stanley Fischer, editors, NBER Macroeconomics Annual 1989, pp. 216–233.
- Christiano, Lawrence J., 1998, 'Solving Dynamic Equilibrium Models by a Method of Undetermined Coefficients', National Bureau of Economic Research Technical Working Paper, January.
- Christiano, Lawrence J., and Martin Eichenbaum, 1992, 'Current real-business-cycle theories and aggregate labor-market fluctuations,' American Economic Review, vol. 82, pp. 430–450.
- Christiano, Lawrence J., Martin Eichenbaum and Charles Evans, 1998, 'Monetary Policy Shocks: What Have We Learned and to What End?', National Bureau of Economic Research Working Paper number 6400.
- Christiano, Lawrence J., Martin Eichenbaum and David Marshall, 1991, 'The Permanent Income Hypothesis Revisited,' Econometrica 59, no. 2, March.
- Christiano, Lawrence J. and Jonas D. M. Fisher, 1995, 'Tobin's q and Asset Returns: Implications for Business Cycle Analysis,' NBER working paper 5292.
- Christiano, Lawrence J. and Jonas D. M. Fisher, 1998, 'Technical Appendix to Stock Market and Investment Good Prices: Implications for Macroeconomics', manuscript.
- Christiano, Lawrence J. and Christopher Gust, 1998, 'Interest Rate Targeting Rules,' manuscript in preparation.
- Christiano, Lawrence J. and Michael Kouparitsas, 1998, 'Comovement and Sectoral Linkages: Some Evidence,' manuscript, Federal Reserve Bank of Chicago.
- Christiano, Lawrence J. and Richard Todd, 1996, 'Time to plan and aggregate fluctuations,' Federal Reserve Bank of Minneapolis Quarterly Review 20, 14–27.
- Cochrane, John, 1991, 'Production-Based Asset Pricing and the Link between Stock Returns and Economic Fluctuations,' Journal of Finance, vol. 46, no. 1, March, pp. 209–37.
- Cogley, Timothy, and James M. Nason, 1995, 'Output Dynamics in Real-Business-Cycle Models,' American Economic Review, 85(3), June, pages 492–511.
- Cole, Hal, and Richard Rogerson, 1996, 'Can the Mortensen-Pissarides Matching Model Match the Business Cycle Facts?', Research Department Staff Report 224, Federal Reserve Bank of Minneapolis, December.
- Constantinides, George, 1990, 'Habit formation: A resolution of the equity premium puzzle,' Journal of Political Economy, vol. 98, pp. 519–542.
- Cummins, Jason, K. A. Hassett and S. D. Oliner, 1997, 'Investment Behavior, Observable Expectations and Internal Funds,' manuscript, New York University.
- den Haan, Wouter J., Garey Ramey, and Joel Watson, 1997, 'Job Destruction and Propagation of Shocks,' NBER working paper #6275.
- Eberly, Janice C., 1997, 'International Evidence on Investment and Fundamentals,' European Economic Review, vol. 41, no. 6, June, pp. 1055–78.
- Einarsson, Tor, and Milton M. Marquis, 1997, 'Home production with endogenous growth,' Journal of Monetary Economics, 39, pp. 551–569.

- Eisner, Robert and Robert H. Strotz, 1963, 'Determinants of Business Investment,' in Commission on Money and Credit Impacts of Monetary Policy, Englewood Cliffs, New Jersey: Prentice-Hall, pp. 59-337.
- Fiorito, Riccardo and Tryphon Kollintzas, 1994, 'Stylized Facts of Business Cycles in the G7 from a Real Business Cycles Perspective,' *European Economic Review*, vol. 38, no. 2, pp. 235-69.
- Fischer, Stanley, and Robert C. Merton, 1984, 'Macroeconomics and Finance: The Role of the Stock Market,' in Karl Brunner and Allan H. Meltzer, eds., *Essays On Macroeconomic Implications of Financial and Labor Markets and Political Processes*, Carnegie-Rochester Conference Series on Public Policy, 21, pp. 57-108.
- Goolsbee, Austan, 1998, 'Investment Tax Incentives, Prices, and the Supply of Capital Goods,' *Quarterly Journal of Economics*, vol. 113, issue 1, pp. 121-148, February.
- Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, 1997, 'Long-Run Implications of Investment-Specific Technological Change,' *American Economic Review*, vol. 87, no. 3, June, pp. 342-362.
- Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell, 1998, 'The Role of Investment-Specific Technological Change in the Business Cycle,' manuscript, University of Rochester, January.
- Hansen, Gary, 1985, 'Indivisible labor and the business cycle,' *Journal of Monetary Economics*, vol. 16, pp. 309-327.
- Hansen, Lars Peter, and Ravi Jagannathan, 1991, 'Implications of Security Market Data for Models of Dynamic Economics,' *Journal of Political Economy*, pp. 225-262.
- Harrison, Sharon G., 1998, Evidence on the empirical plausibility of externalities and indeterminacy in a two sector model, unpublished manuscript, Barnard College, Columbia University.
- Hayashi, Fumio, 1982, 'Tobin's Marginal q and Average q: A Neoclassical Interpretation,' *Econometrica*, vol. 50, no. 1, January, pp. 213-24.
- Heaton, John, 1995, 'An empirical investigation of asset pricing with temporally dependent preference specifications,' *Econometrica* 63, 681-717.
- Hodrick, R. J. and E. C. Prescott, 1997, 'Postwar US business cycles: an empirical investigation', *Journal of Money, Credit, and Banking* 29, 1-16.
- Hornstein, Andreas, and J. Praschnik, 1997, 'Intermediate Inputs and Sectoral Comovement in the Business Cycle,' *Journal of Monetary Economics*, pp. 573-595.
- Horvath, Michael, 1997, 'Sectoral Shocks and Aggregate Fluctuations', manuscript.
- Huffman, Gregory, and Mark Wynne, 1998, 'The Role of Intratemporal Adjustment Costs in a Multisector Economy,' forthcoming, *Journal of Monetary Economics*.
- King, Robert and Mark Watson, 1996, 'Money, Prices, Interest Rates and the Business Cycle,' *The Review of Economics and Statistics*, vol. LXXVIII, no. 1, pp. 35-53.
- Long, John, and Charles Plosser, 1993, 'Real Business Cycles,' *Journal of Political Economy*, pp. 39-69.
- Lucas, Robert E., Jr., 1967a, 'Adjustment Costs and the Theory of Supply,' *Journal of Political Economy*, 75, pp. 321-334.

- Lucas, Robert E., Jr., 1967b, 'Optimal Investment Policy and the Flexible Accelerator,' *International Economic Review*, 8, pp. 78-85.
- Lucas, Robert E., Jr., 1978, 'Asset Prices in an Exchange Economy,' *Econometrica*, 46: 1429-1445.
- Lucas, Robert E., Jr., 1981, 'Understanding Business Cycles,' in R.E. Lucas, Jr., *Studies in Business-Cycle Theory*, The MIT Press, reprinted from *Stabilization of the Domestic and International Economy*, vol. 5 of *Carnegie-Rochester Series on Public Policy*, eds., Karl Brunner and Allan H. Meltzer, Amsterdam: North-Holland Publishing Company, 1977.
- Lucas, Robert E., Jr., and Edward Prescott, 1971, 'Investment under uncertainty,' *Econometrica*, vol. 39, pp. 659-681.
- Masulis, Ronald W, 1988, *The debt/equity choice*. Cambridge, MA: Ballinger.
- Megna, Pamela, and Mark Klock, 1993, 'The Impact on Intangible Capital on Tobin's q in the Semiconductor Industry,' *American Economic Review*, vol. 83, no. 2, May, pp. 265-69.
- Merz, Monika, 1995, 'Search in the Labor Market and the Real Business Cycle,' *Journal of Monetary Economics*, 36 no. 2, pp. 269-300.
- Mortensen, Dale, 1992, 'Equilibrio de Búsqueda y Ciclos Economicos Reales,' *Cuadernos Economicos*, Numero 51.
- Murphy, Kevin M., Andrei Shleifer, and Robert W. Vishny, 1989, 'Building blocks of market clearing business cycle models,' *NBER Macroeconomics Annual 1989*, edited by Olivier J. Blanchard and Stanley Fischer, MIT Press.
- Rogerson, Richard, 1988, 'Indivisible labor, lotteries and equilibrium,' *Journal of Monetary Economics*, vol. 21, pp. 3-16.
- Rupert, Peter, Richard Rogerson and Randall Wright, 1997, 'On the Macroeconomic Implications of Models with Household Production,' manuscript, University of Pennsylvania.
- Sargent, Thomas J., 1979, *Macroeconomic theory*, Academic Press.
- Shea, John, 1993, 'Do Supply Curves Slope Up?', *Quarterly Journal of Economics*, vol. 108, February, pp. 1-32.
- Shiller, Robert, 1981, 'Do stock prices move too much to be justified by subsequent changes in dividends?' *American Economic Review*, vol. 71, pp. 421-436.
- Sims, Christopher A., 1980, 'International Evidence on Monetary Factors in Macroeconomic Fluctuations,' *Center for Economic Research Discussion Paper no. 80-137*, Department of Economics, University of Minnesota, September.
- Sims, Christopher A., 1997, Comments on 'Systematic Monetary Policy and the Effects of Oil Price Shocks', by Ben S. Bernanke, Mark Gertler and Mark Watson', *Brookings Papers on Economic Activity*, pages 143 - 148.
- Sundaresan, Suresh M., 1989, 'Intertemporally dependent preferences and the volatility of consumption and wealth,' *Review of Financial Studies*, vol. 2, pp. 73-89.
- Tobin and Brainard (1977, p.243)

Watson, Mark W., 1993, 'Measures of Fit for Calibrated Models,' *Journal of Political Economy*, 101(6), December, pages 1011-41.

Wildasin, David E., 1984, 'The q Theory of Investment with Many Capital Goods,' *American Economic Review*, vol. 74, no. 1, March, pp. 203-10.

Technical Appendix to:
 Stock Market and Investment Good Prices: Implications for Macroeconomics

by

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The equation numbers in this document refer to the equation numbers in the Christiano and Fisher paper. References may be found there as well.

A. The Planner's Problem

The quantities in a competitive equilibrium of our model can be computed by solving the following planner problem: maximize

$$\begin{aligned}
 E_0 \sum_{t=0}^{\infty} \beta^t \{ & \ln(C_t - X_t) + \eta(1 - H_{c,t} - H_{i,t}) \\
 & + \Lambda_{x,t} [X_{t+1} - bC_t] \\
 & + \Lambda_{c,t} [K_{c,t}^{\alpha} (\exp(\theta_t) H_{c,t})^{1-\alpha} - C_t] \\
 & + \Lambda_{i,t} [V_t K_{i,t}^{\alpha} (\exp(\theta_t) H_{i,t})^{1-\alpha} - I_{c,t} - I_{i,t}] \\
 & + \Omega_{c,t} [Q^c ((1 - \delta)K_{c,t}, I_{c,t}) - K_{c,t+1}] \\
 & + \Omega_{i,t} [Q^i ((1 - \delta)K_{i,t}, I_{i,t}) - K_{i,t+1}] \}
 \end{aligned}$$

subject to

$$\theta_t = \rho\theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2)$$

$$V_t = \exp(\bar{\mu} + \mu_t)V_{t-1}, \quad \mu_t \sim N(0, \sigma_{\mu}^2)$$

Here, $\Lambda_{y,t}$, $y = x, c, i$ and $\Omega_{y,t}$, $y = c, i$ are Lagrange multipliers. The planner is assumed to choose $H_{c,t}$ and $H_{i,t}$ prior to the realization of the date t state of nature, while the remaining choice variables (including the multipliers) are selected afterward.

To solve the planner problem we first have to transform it into its stationary form. The variables in the transformed problem are c_t , $k_{c,t+1}$, $k_{i,t+1}$, x_{t+1} , $i_{c,t}$, $i_{i,t}$, $\lambda_{c,t}$, $\lambda_{i,t}$, $\lambda_{x,t}$, $\omega_{c,t}$ and $\omega_{i,t}$. These are

defined as follows (not all variables are transformed in the same way):

$$\begin{aligned}
c_t &= \frac{C_t}{V_t^{\alpha/(1-\alpha)}}, & k_{c,t+1} &= \frac{K_{c,t+1}}{V_t^{1/(1-\alpha)}}, & k_{i,t+1} &= \frac{K_{i,t+1}}{V_t^{1/(1-\alpha)}}, & x_{t+1} &= \frac{X_{t+1}}{V_t^{\alpha/(1-\alpha)}} \\
i_{c,t} &= \frac{I_{c,t}}{V_t^{1/(1-\alpha)}}, & i_{i,t} &= \frac{I_{i,t}}{V_t^{1/(1-\alpha)}}, \\
\lambda_{c,t} &= V_t^{\alpha/(1-\alpha)} \Lambda_{c,t}, & \lambda_{i,t} &= V_t^{1/(1-\alpha)} \Lambda_{i,t}, & \lambda_{x,t} &= V_t^{\alpha/(1-\alpha)} \Lambda_{x,t} \\
\omega_{c,t} &= V_t^{1/(1-\alpha)} \Omega_{c,t}, & \omega_{i,t} &= V_t^{1/(1-\alpha)} \Omega_{i,t}
\end{aligned}$$

In what follows it will also be helpful to use the following:

$$y_{c,t} = \frac{Y_{c,t}}{V_t^{\alpha/(1-\alpha)}}, \quad y_{i,t} = \frac{Y_{i,t}}{V_t^{1/(1-\alpha)}},$$

where $Y_{c,t} = K_{c,t}^\alpha (\exp(\theta_t) H_{c,t})^{1-\alpha}$ and $Y_{i,t} = V_t K_{i,t}^\alpha (\exp(\theta_t) H_{i,t})^{1-\alpha}$.

The first order conditions for an interior solution to the stationary version of the planner problem can be rearranged to form the following system of equations:

$$\lambda_{x,t} - \beta E \left[\exp(-\hat{\mu}_{t+1}) \frac{1}{c_{t+1} - \exp(-\hat{\mu}_{t+1}) x_{t+1}} \middle| \Omega_t \right] = 0 \quad (\text{A.1})$$

$$\begin{aligned}
\frac{\lambda_{i,t}}{Q_{2,t}^c} - \beta E \left[\left(\frac{1}{c_{t+1} - \exp(-\hat{\mu}_{t+1}) x_{t+1}} - b \lambda_{x,t+1} \right) \alpha \frac{y_{c,t+1}}{k_{c,t+1}} \right. \\
\left. + \exp(-\tilde{\mu}_{t+1}) (1 - \delta) \frac{\lambda_{i,t+1}}{Q_{2,t+1}^c} Q_{1,t+1}^c \middle| \Omega_t \right] = 0 \quad (\text{A.2})
\end{aligned}$$

$$\frac{\lambda_{i,t}}{Q_{2,t}^i} - \beta E \left[\lambda_{i,t+1} \left(\alpha \frac{y_{i,t+1}}{k_{i,t+1}} + \exp(-\tilde{\mu}_{t+1}) (1 - \delta) \frac{\lambda_{i,t+1}}{Q_{2,t+1}^i} Q_{1,t+1}^i \right) \middle| \Omega_t \right] = 0 \quad (\text{A.3})$$

$$E \left[\left(\frac{1}{c_t - \exp(-\hat{\mu}_t) x_t} - b \lambda_{x,t} \right) (1 - \alpha) \frac{y_{c,t}}{H_{c,t}} - \eta \middle| \Omega_t^* \right] = 0 \quad (\text{A.4})$$

$$E \left[\lambda_{i,t} (1 - \alpha) \frac{y_{i,t}}{H_{i,t}} - \eta \middle| \Omega_t^* \right] = 0 \quad (\text{A.5})$$

$$\frac{x_{t+1}}{b} - y_{c,t} = 0 \quad (\text{A.6})$$

$$i_{c,t} + i_{i,t} - y_{i,t} = 0 \quad (\text{A.7})$$

In these expressions Ω_t denotes the information set that includes realizations of the technology shocks up to and including time t , and Ω_t^* denotes the information set identical to Ω_t except that the time

t realizations of the technology shocks are excluded. Also, $Q_{j,t}^x$, $j = 1, 2$ denote partial derivatives of Q_t^x with respect to its first and second arguments, respectively, evaluated at the time t values of the arguments, for $x = c, i$. Finally, $\hat{\mu}_t = \alpha\mu_t/(1 - \alpha)$ and $\tilde{\mu}_{t+1} = \hat{\mu}_t/\alpha$.

Making use of the resource constraints implicit in the formulation of the planner problem, we can collapse the model to the following seven equations

$$E[v_j(k_{c,t}, k_{c,t+1}, k_{c,t+2}, k_{i,t}, k_{i,t+1}, k_{i,t+2}, x_t, x_{t+1}, x_{t+2}, H_{c,t}, H_{c,t+1}, H_{i,t}, H_{i,t+1},$$

$$\lambda_{x,t}, \lambda_{x,t+1}, \lambda_{i,t}, \lambda_{i,t+1}, \theta_t, \theta_{t+1}, \mu_t, \mu_{t+1}) | \Omega_t] = 0, \quad j = 1, 2, 3, 6, 7;$$

$$E[v_j(k_{c,t}, k_{c,t+1}, k_{c,t+2}, k_{i,t}, k_{i,t+1}, k_{i,t+2}, x_t, x_{t+1}, x_{t+2}, H_{c,t}, H_{c,t+1}, H_{i,t}, H_{i,t+1},$$

$$\lambda_{x,t}, \lambda_{x,t+1}, \lambda_{i,t}, \lambda_{i,t+1}, \theta_t, \theta_{t+1}, \mu_t, \mu_{t+1}) | \Omega_t^*] = 0, \quad j = 4, 5.$$

Here $v_j(\cdot)$, $j = 1, 2, 3, 6, 7$ are equations (A.1), (A.2), (A.3), (A.6), and (A.7) respectively, and $v_j(\cdot)$, $j = 4, 5$ are equations (A.4) and (A.5), respectively. These equations were solved using the methods described in Christiano (1998).

We can use the multipliers from a solution to the planner problem to compute the relative prices studied in the main text. First, the relative price of the new investment good is given by

$$P_{i,t} \equiv \frac{\Lambda_{i,t}}{\Lambda_{c,t}} = \frac{\lambda_{i,t}}{\lambda_{c,t}} \frac{1}{V_t}.$$

Second, the prices for $K_{c,t+1}$ and $K_{i,t+1}$ are

$$P_{k'_{c,t}} \equiv \frac{\Omega_{c,t}}{\Lambda_{c,t}} = \frac{\omega_{c,t}}{\lambda_{c,t}} \frac{1}{V_t}, \quad P_{k'_{i,t}} \equiv \frac{\Omega_{i,t}}{\Lambda_{c,t}} = \frac{\omega_{i,t}}{\lambda_{c,t}} \frac{1}{V_t},$$

respectively. Third, the prices for installed capital are

$$P_{k_{c,t}} \equiv \frac{Q_{1,t}^c \Omega_{c,t}}{\Lambda_{c,t}} = \frac{\omega_{c,t}}{\lambda_{c,t}} \frac{Q_{1,t}^c}{V_t}, \quad P_{k_{i,t}} \equiv \frac{Q_{1,t}^i \Omega_{i,t}}{\Lambda_{c,t}} = \frac{\omega_{i,t}}{\lambda_{c,t}} \frac{Q_{1,t}^i}{V_t}.$$

Notice that each of these prices will trend downward if V_t has a positive trend. To use these formulas we require expressions for $\lambda_{c,t}$, $\omega_{c,t}$ and $\omega_{i,t}$. These can be computed from the first order conditions from the planner problem. The expressions derived in this way are given by

$$\lambda_{c,t} = \frac{1}{y_{c,t} - \exp(\hat{\mu}_t)x_t} - b\lambda_{x,t},$$

$$\omega_{c,t} = \frac{\lambda_{i,t}}{Q_{2,t}^c}, \text{ and}$$

$$\omega_{i,t} = \frac{\lambda_{i,t}}{Q_{2,t}^i}.$$

B. A Market Decentralization

Following is a description of the household and firm problems.

B.1. Households

The representative household's budget constraint is:

$$\begin{aligned} C_t + S_t^c + S_t^i + B_t \\ \leq (1 + r_{ct}^e)S_{t-1}^c + (1 + r_{it}^e)S_{t-1}^i + (1 + r_{t-1}^f)B_{t-1} + w_t^c H_{c,t} + w_t^i H_{i,t}, \end{aligned} \quad (\text{B.1})$$

for $t = 0, 1, \dots$. Here, S_t^x denotes date t purchases of equity in industry x , for $x = c, i$, and B_t denotes purchases of risk-free debt. The rate of return on equity purchased in period t , $(1 + r_{x,t+1}^e)$, depends upon the period $t + 1$ state of nature, while the rate of return on debt, $(1 + r_t^f)$, depends on the date t state of nature. Also, w_t^x denotes the wage rate in industry x , which is a function of the date t state of nature.

At the beginning of date t , the household knows $S_{t-1}^c, S_{t-1}^i, B_{t-1}, X_t$ and all date and state contingent prices and rates of return henceforth. In the limited labor mobility version of the model, households must choose $H_{c,t}$ and $H_{i,t}$ prior to the realization of the current state of nature. It makes its remaining decisions, C_t, S_t^c, S_t^i, B_t , after the date t realization of uncertainty. The household's objective is to maximize (3.1) subject to (3.2), (B.1) and the condition that the future choice variables satisfy the same information constraints. The household's intratemporal first order necessary conditions for labor are

$$E_{t-1} w_t^x \Lambda_{c,t} = \eta, \quad x = c, i. \quad (\text{B.2})$$

Its intertemporal first order conditions are

$$E_t p_{c,t+1} (1 + r_{x,t+1}^e) = 1 = E_t p_{c,t+1} (1 + r_t^f), \quad (\text{B.3})$$

for $x = c, i$, where

$$p_{c,t+1} = \frac{\beta \Lambda_{c,t+1}}{\Lambda_{c,t}}, \quad (\text{B.4})$$

and $\Lambda_{c,t}$ is the derivative of (3.1) with respect to C_t , when E_{t-1} is replaced by E_t .

To keep from cluttering the notation, we do not explicitly include competitive markets for state contingent, one period-ahead consumption goods. If we did include such markets, then $p_{c,t+1}$ would be the market price, denominated in terms of date t consumption units, of a state-contingent unit of date $t + 1$ consumption, scaled by the conditional probability of that state of nature.¹

B.2. Firms

We suppose there are four representative, competitive firms producing each of the four types of period t good: consumption goods, investment goods, and $K_{x,t+1}$ for $x = c, i$. These firms interact anonymously in markets for investment goods, previously installed capital, new capital, equity and labor. Before discussing the individual firm problems, we list the prices and rates of return in these markets. Firms take these as given and beyond their control.

- $P_{k_x,t} \sim$ period t price of previously installed, beginning of period t capital,
- $P_{i,t} \sim$ period t price of period t investment goods,
- $P_{k'_x,t} \sim$ period t price of beginning of period $t + 1$ installed capital, $K_{x,t+1}$,
- $1 + r_{x,t}^e \sim$ period t payoff on one unit of equity acquired in period $t - 1$,
- $1 + r_t^f \sim$ period $t + 1$ payoff on one unit of debt acquired in period t ,
- $p_{c,t+1} \sim$ period t price of a state contingent period $t + 1$ consumption good, scaled by the conditional probability of that state of nature,
- $w_{t+1}^x \sim$ period t wage rate,

for sectors $x = c, i$. The price of investment goods has no subscript, x . This is because the linear rate of transformation between $I_{c,t+1}$ and $I_{i,t+1}$ implicit in (3.4) guarantees that, in equilibrium, the prices of new investment goods for the consumption and investment goods sectors are equalized. The fact that the rate of return on debt also has no subscript, x , reflects its non-state contingent nature.

Capital Producing Firms

¹In particular, let $g_t(s')$ denote the date t conditional probability that state of nature s' will be realized in period $t + 1$. Then $g_t(s')p_{c,t+1}(s')$ is the value—denominated in date t consumption units—of a unit of consumption in date $t + 1$, state of nature s' . Here, $p_{c,t+1}(s')$ is the value of $p_{c,t+1}$ in state of nature s' .

Firms that produce capital operate the ‘adjustment cost’ technology, (3.6)-(3.7). They purchase $K_{x,t}$ and I_t at prices $P_{x,t}$ and $P_{i,t}$ and sell $K_{x,t+1}$ at the price $P_{k'_x,t}$ for $x = c, i$. Their first order conditions are given by the q -theory relation, (3.10). Competition and linear homogeneity of their production functions guarantee that in equilibrium their profits must be zero.

Consumption and Investment Good Producing Firms

Our specification that it takes one period to build capital before it becomes productive generates a financing requirement. We assume that this financing requirement falls entirely on the firms that produce consumption and investment goods. Whatever physical capital such a firm uses to operate the production technology in period $t + 1$ must be acquired from the producers of new capital by the end of period t . To finance the purchase of this capital, the firm issues debt and equity in period t . There are separate equity markets for the two types of firms, and the two types of equity command different, competitively determined, state-contingent rates of return. Since the competitive rate of return on debt is known at the time it is traded, in equilibrium there can be only one rate of return for that financial instrument. When period $t + 1$ occurs, the consumption and investment good firm observes the state of nature and, hence, the prevailing wage rate. It then enters competitive labor markets to hire that amount of labor which maximizes cash flow in that state of nature. Labor does not give rise to a need for finance because it is assumed to be paid out of current receipts from production. The firm’s cash flow is the value of its production, plus the value of its undepreciated stock of capital, net of expenses. The firm’s objective at date t is to maximize the date t value of cash flow at $t + 1$, summed across all possible states of nature.

In our decentralization, adjustment costs for installing productive capital are external to the consumption and investment good firms. This assumption was made for convenience only, in an effort to make precise the notion that $P_{k'_x}$ is the marginal value of new capital. Our results would be the same if we had instead consolidated the capital producing firms and the goods producing firms within each industry. In that case, $P_{k'_x}$ would have been a shadow cost. Perhaps this latter type of decentralization matches up better with the way actual markets are organized.

We now describe the problem of the firm in industry x . Its financing constraint is:

$$P_{k'_x,t}K_{x,t+1} \leq S_{x,t} + B_{x,t}, \tag{B.5}$$

and its period $t + 1$ cash flow constraint is

$$\pi_{t+1}^x = Y_{x,t+1} + (1 - \delta)K_{x,t+1}P_{k_x,t+1} - w_{t+1}^x H_{x,t+1} - (1 + r_{x,t+1}^e)S_t^x - (1 + r_t^f)B_t \geq 0. \tag{B.6}$$

Here, $Y_{x,t+1}$ is the firm's gross output, given by (3.2) or (3.4), measured in date $t + 1$ consumption units.² Also, we assume that if $K_{x,t+1}$ is the amount of capital used by the firm during period $t + 1$, then $(1 - \delta)K_{x,t+1}$ remains at the end of the period, when it is made available for sale. The nonnegativity constraint in (B.6) is essentially a “no blood from stone” constraint. It says firms cannot pay out what they do not have.

The firm's profit function is the value of π_{t+1}^x denominated in units of the date t consumption good, summed across all possible date $t + 1$ states of nature: $E_t p_{c,t+1} \pi_{t+1}^x$. Its objective is to select values for $S_{x,t}$, $B_{x,t}$, $K_{x,t+1}$, and a contingency plan for $H_{x,t+1}$ that solve

$$\max_{S_{x,t}, B_{x,t}, K_{x,t+1}} E_t p_{c,t+1} \max_{H_{x,t+1}} \pi_{t+1}^x, \quad (\text{B.7})$$

subject to the relevant production technology and (B.5), (B.6). The firm's first order condition for hours worked is

$$mpl_{x,t+1} = w_t^x, \quad (\text{B.8})$$

where $mpl_{x,t+1}$ denotes the marginal product of labor, denoted in period $t + 1$ consumption units. The firm's first order conditions for $K_{x,t+1}$, $S_{x,t}$, and $B_{x,t}$ are:

$$E_t p_{c,t+1} [mpk_{x,t+1} + (1 - \delta)P_{k_x,t+1}] = \lambda P_{k'_x,t}, \quad (\text{B.9})$$

$$E_t p_{c,t+1} (1 + r_{x,t+1}^e) = \lambda, \quad (\text{B.10})$$

$$E_t p_{c,t+1} (1 + r_{x,t}^f) = \lambda. \quad (\text{B.11})$$

Here, $mpk_{x,t+1}$ denotes the marginal product of capital, denoted in period $t + 1$ consumption units, and λ is the multiplier on the financing constraint, (B.5). In equilibrium, $\lambda = 1$ by (B.3).

B.3. General Equilibrium

We adopt the usual sequence-of-markets equilibrium concept. Market clearing implies that, in equilibrium, the period t demand for previously installed industry x capital, denoted above by z , equals the supply, $(1 - \delta)K_{x,t}$. Similarly, the demand for period t new investment goods by industry x , denoted

²Thus, for the consumption industry,

$$Y_{c,t+1} = Q^c(y, z)^\alpha (\exp(\theta_{t+1})H_{c,t+1})^{1-\alpha},$$

and for the investment goods industry,

$$Y_{i,t+1} = P_{i,t+1}V_{i+1}Q^c(y, z)^\alpha (\exp(\theta_{t+1})H_{i,t+1})^{1-\alpha}.$$

by y , is $I_{x,t}$.

B.4. Prices and Rates of Return

In this section we derive and interpret an expression for the equilibrium rate of return on equity. In addition, we provide a further discussion of $P_{k'}$. The main thrust of our analysis exploits the fact that this price satisfies the q -theory relationship summarized in the first equality in (3.10). To better understand $P_{k'}$, however, it is useful to know that it must also satisfy two other relationships in equilibrium: $P_{k'}$ must be the present discounted value of the extra output generated by a marginal increase in productive capital, and it must be the present discounted value of dividends. We derive these implications in this subsection. The fact that $P_{k'}$ satisfies the latter means that it is natural to refer to it as the ‘price of a share of equity’. Here, we implicitly adopt the normalization that a share of equity is a unit of the underlying capital financed by the equity.

We now turn to the rate of return on equity. Note that linear homogeneity of profit functions guarantees that, in equilibrium, maximized profits (B.7) are zero. The cash flow constraint, (B.6), then guarantees $\pi_{t+1}^x = 0$ in each date and state of nature. Using this, linear homogeneity in production, and (B.8), one gets the following equilibrium condition after some algebra:

$$1 + r_{x,t+1}^e \tag{B.12}$$

$$= \left[\frac{mpk_{x,t+1} + (1 - \delta)P_{k_x,t+1}}{P_{k'_x,t}} \right] + \gamma_t^x \left\{ \frac{mpk_{x,t+1} + (1 - \delta)P_{k_x,t+1}}{P_{k'_x,t}} - (1 + r_t^f) \right\}.$$

Here, $\gamma_t^x = B_t^x/S_t^x$ denotes the firm’s debt to equity ratio. As is well known, γ_t^x is not determined in equilibrium, even though the equilibrium quantity allocations in the model are unique and coincide with the allocations chosen by a central planner. The overall rate of return on equity, in (3.9) is just

$$\frac{[(1 + r_{c,t+1}^e)S_{c,t} + (1 + r_{i,t+1}^e)S_{i,t}]}{S_{c,t} + S_{i,t}},$$

after making use of the firms’ first order conditions and our restrictions on the debt to equity ratio.

The rate of return formula in (B.12) has the following interpretation. Suppose someone purchases a unit of equity, which finances the acquisition of $1/P_{k'_x,t}$ units of extra time $t + 1$ productive capital. Corresponding to this purchase of equity, γ_t^x units of risk free debt are issued too. Thus one can imagine that each holder of a unit of equity is ‘paired’ up with a holder of γ_t^x units of risk-free debt. The underlying productive capital corresponding to a unit of equity generates income equal to the term in square brackets in (B.12). This income is not necessarily the payoff actually received by the equity

holder, however. For example, the owner of a unit of equity receives more than this if the state non-contingent payment promised to the debt holder with which he/she is paired, $\gamma_t^x(1+r_t^f)$, is less than the income generated by the assets this debt holder finances, namely, $\gamma_t^x(mpk_{x,t+1} + (1-\delta)P_{k_x,t+1})/P_{k'_x,t}$. Evidently, in high income states, there is a transfer from the debt to the equity holder and in low income states, the transfer goes in the other direction. In this way, our model articulates the well-known observation that, in effect, equity holders provide insurance to debt holders.

We now use (B.3) and (B.12) to express the price of equity as a function of future income. Multiply (B.12) by $P_{k'_x,t}p_{c,t+1}$, apply the conditional expectation operator, and make use of (B.3) to obtain:

$$\begin{aligned} P_{k'_x,t} &= E_t p_{c,t+1} [mpk_{x,t+1} + (1-\delta)P_{k_x,t+1}] \\ &\quad + \gamma_t^x \{E_t p_{c,t+1} [mpk_{x,t+1} + (1-\delta)P_{k_x,t+1}] - P_{k'_x,t}\}, \end{aligned} \quad (\text{B.13})$$

or,

$$\begin{aligned} P_{k'_x,t} &= E_t p_{c,t+1} [mpk_{x,t+1} + (1-\delta)P_{k_x,t+1}] \\ &= E_t p_{c,t+1} [mpk_{x,t+1} + (1-\delta)Q_{1,t+1}^x P_{k'_x,t+1}], \end{aligned} \quad (\text{B.14})$$

where the second equality in (B.14) makes use of (3.10). From the first equality in (B.14) we see that the expected transfer payment between debt and equity holders is zero, when the transfers are weighted across states of nature by $p_{c,t+1}$. By iterating forward on this expression and imposing a terminal condition, we obtain (3.11).

It is convenient to relate (3.11) to the present discounted value of dividends relationship that appears in the literature on the stock market (see, for example, Shiller (1981) and the papers cited there). Thus, we can rewrite (3.11) as follows

$$1 + r_{x,t+1}^e = \frac{D_{t+1}^x + P_{k'_x,t+1}}{P_{k'_x,t}}, \quad (\text{B.15})$$

where D_{t+1}^x denotes 'dividend' payments:

$$D_{t+1}^x = mpk_{x,t+1} - \left[1 - (1-\delta)Q_{1,t+1}^x\right] P_{k'_x,t+1} \quad (\text{B.16})$$

$$+ \gamma_t^x \left[mpk_{x,t+1} + (1-\delta)P_{k_x,t+1} - (1+r_t^f)P_{k'_x,t}\right]. \quad (\text{B.17})$$

Here, the q -theory relation, (3.10) has been used to replace $P_{k_x,t+1}$ by $Q_{1,t+1}^x P_{k'_x,t+1}$. This dividend term is a measure of income to equity holders, per unit of the underlying capital financed by equity. It is

the sum of the direct earnings, $mpk_{x,t+1}$, from a unit of capital, minus depreciation expenses (inclusive of installation costs), plus any transfer (positive or negative) from debt holders due to leverage.

Three observations are worth making about $P_{k'_x,t}$. First, it is trivial to verify that a version of (3.11) in which $mpk_{x,t+j}$ is replaced by D_{t+j}^x and the object in square brackets is replaced by unity holds too. This version of (3.11) coincides with the conventional definition of the price of a share of equity. Implicitly, the literature adopts the convention that a share of equity corresponds to a unit of the underlying capital financed by the equity. Second, when it is observed that $P_{k'_x,t}$ is also the present discounted value of dividends, it may seem surprising that γ_t^x is absent from (3.11), in view of the transfer payments passing between holders of equity and debt. This reflects simply that these transfers are zero in the relevant expected value sense, so that

$$E_t p_{c,t+1} D_{t+1}^x = E_t p_{c,t+1} \left\{ mpk_{x,t+1} - \left[1 - (1 - \delta) Q_{1,t+1}^x \right] P_{k'_x,t+1} \right\}.$$

Third, since γ_t^x also does not enter (3.11) indirectly, via p_c or mpk_x (recall the previous observations about the invariance of quantities to γ_t^x), it follows that $P_{k'_x,t}$ is invariant to γ_t^x . This implies that the value of the entire productive stock of capital in the economy is invariant to γ_t^c and γ_t^i . Thus, our environment is consistent with the celebrated Modigliani-Miller theorem of corporate finance, according to which the value of the firm is independent of its debt to equity structure.

Finally, it is useful to have the standard covariance formula expression for the equity premium. To obtain this, note that (B.3) implies $E_t p_{c,t+1} E_t (1 + r_{t+1}^e) = 1 - Cov_t(p_{c,t+1}, 1 + r_{t+1}^e)$ or, using (B.12) and (B.3):

$$\frac{E_t (1 + r_{x,t+1}^e)}{1 + r_t^f} - 1 = -Cov_t \left(p_{c,t+1}, \frac{mpk_{x,t+1} + (1 - \delta) P_{k_x,t+1}}{P_{k'_x,t}} \right) (1 + \gamma_t^x). \quad (\text{B.18})$$

The object on the left of the equality is the date t equity premium in industry x since it is, approximately, $E_t r_{x,t+1}^e - r_t^f$. An implication of this formula is that the Sharpe ratio, the ratio of the equity premium to its standard deviation, is invariant to leverage.