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Realized Volatility

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Summary. Realized volatility is a nonparametric ex-post estimate of the return variation. The most obvious realized volatility measure is the sum of finely-sampled squared return realizations over a fixed time interval. In a frictionless market the estimate achieves consistency for the underlying quadratic return variation when returns are sampled at increasingly higher frequency. We begin with an account of how and why the procedure works in a simplified setting and then extend the discussion to a more general framework. Along the way we clarify how the realized volatility and quadratic return variation relate to the more commonly applied concept of conditional return variance. We then review a set of related and useful notions of return variation along with practical measurement issues (e.g., discretization error and microstructure noise) before briefly touching on the existing empirical applications.

1 Introduction

Given the importance of return volatility on a number of practical financial management decisions, there have been extensive efforts to provide good real-time estimates and forecasts of current and future volatility. One complicating feature is that, contrary to the raw return, actual realizations of return volatility are not directly observable. A common approach to deal with the fundamental latency of return volatility is to conduct inference regarding volatility through strong parametric assumptions, invoking, e.g., an ARCH or

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a stochastic volatility (SV) model estimated with data at daily or lower frequency. An alternative approach is to invoke option pricing models to invert observed derivatives prices into market-based forecasts of “implied volatility” over a fixed future horizon. Such procedures remain model-dependent and further incorporate a potentially time-varying volatility risk premium in the measure so they generally do not provide unbiased forecasts of the volatility of the underlying asset. Finally, some studies rely on “historical” volatility measures that employ a backward looking rolling sample return standard deviation, typically computed using one to six months of daily returns, as a proxy for the current and future volatility level. Since volatility is persistent such measures do provide information but volatility is also clearly mean reverting, implying that such unit root type forecasts of future volatility are far from optimal and, in fact, conditionally biased given the history of the past returns. In sum, while actual returns may be measured with minimal (measurement) error and may be analyzed directly via standard time series methods, volatility modeling has traditionally relied on more complex econometric procedures in order to accommodate the inherent latent character of volatility.

The notion of realized volatility effectively reverses the above characterization. Given continuously observed price or quote data, and absent transaction costs, the realized return variation may be measured without error along with the (realized) return. In addition, the realized variation is conceptually related to the cumulative expected variability of the returns over the given horizon for a wide range of underlying arbitrage-free diffusive data generating processes. In contrast, it is impossible to relate the actual (realized) return to the expected return over shorter sample periods in any formal manner absent very strong auxiliary assumptions. In other words, we learn much about the expected return volatility and almost nothing about the expected mean return from finely-sampled asset prices. This insight has fueled a dramatic increase in research into the measurement and application of realized volatility measures obtained from high frequency, yet noisy, observations on returns. For liquid financial markets with high trade and quote frequency and low transaction costs, it is now prevailing practice to rely on intra-day return data to construct ex-post volatility measures. Given the rapidly increasing availability of high-quality transaction data across many financial assets, it is inevitable that this approach will continue to be developed and applied within ever broader contexts in the future.

This chapter provides a short and largely intuitive overview of the realized volatility concept and the associated applications. We begin with an account of how and why the procedure works in a simplified setting and then discuss more formally how the results apply in general settings. Next, we detail more formally how the realized volatility and quadratic return variation relate to the more common conditional return variance concept. We then review a set of related and useful notions of return variation along with practical measurement issues before briefly touching on the existing empirical applications.

2 Measuring Mean Return versus Return Volatility

The theory of realized volatility is tied closely to the availability of asset price observations at arbitrarily high frequencies. Hence, it is natural to consider the volatility measurement problem in a continuous-time framework, even if we ultimately only allow sampling at discrete intervals. We concentrate on a single risky asset whose price may be observed at equally-spaced discrete points in time over a given interval, $[0, T]$, namely $t = 0, 1/n, 2/n, \ldots, T - (1/n), T$, where n and T are positive integers and the unit interval corresponds to the primary time period over which we desire to measure return volatility, e.g., one trading day. We denote the logarithmic asset price at time t by $s(t)$ and the continuously compounded returns over $[t - k, t]$ is then given by $r(t, k) = s(t) - s(t - k)$ where $0 \leq t - k < t \leq T$ and $k = j/n$ for some positive integer j. When $k = 1$ it is convenient to use the shorthand notation $r(t) = r(t, 1)$, where t is an integer $1 \leq t \leq T$, for the unit period, or "daily," return.

To convey the basic rationale behind the realized volatility approach, we initially consider a simplified setting with the continuously compounded returns driven by a simple time-invariant Brownian motion, so that

$$ds(t) = \alpha dt + \sigma dW(t), \; 0 \leq t \leq T \,, \tag{1}$$

where α and σ ($\sigma > 0$) denote the constant drift and diffusion coefficients, respectively, scaled to correspond to the unit time interval.

For a given measurement period, say $[0, K]$, where K is a positive integer, we have $n \cdot K$ intraday return observations $r(t, 1/n) = s(t) - s(t - 1/n)$ for $t = 1/n, \ldots, (n-1) \cdot K/n, K$, that are i.i.d. normally distributed with mean α/n and variance σ^2/n. It follows that the maximum likelihood estimator for the drift coefficient is given by

$$\hat{\alpha}_n = \frac{1}{K} \sum_{j=1}^{n \cdot K} r(j/n, 1/n) = \frac{r(K, K)}{K} = \frac{s(K) - s(0)}{K} . \tag{2}$$

Hence, for a fixed interval the in-fill asymptotics, obtained by continually increasing the number of intraday observations, are irrelevant for estimating the expected return. The estimator of the drift is independent of the sampling frequency, given by n, and depends only on the span of the data, K. For example, one may readily deduce that

$$\mathrm{Var}(\hat{\alpha}_n) = \frac{\sigma^2}{K} \, . \tag{3}$$

In other words, although the estimator is unbiased, the mean drift cannot be estimated consistently over any fixed interval. Even for the simplest case of a constant mean, long samples (large K) are necessary for precise inference. Thus, in a setting where the expected returns are stipulated to vary conditionally on features of the underlying economic environment, auxiliary identifying assumptions are required for sensible inference about α. This is the reason why critical empirical questions such as the size of the equity premium and the pattern of the expected returns in the cross-section of individual stocks remain contentious and unsettled issues within financial economics.

The situation is radically different for estimation of return volatility. Even if the expected return cannot be inferred with precision, nonparametric measurement of volatility may be based on un-adjusted or un-centered squared returns. This is feasible as the second return moment dominates the first moment in terms of influencing the high-frequency squared returns. Specifically, we have,

$$\mathrm{E}\left[\, r(j/n, 1/n)^2 \,\right] = \frac{\alpha^2}{n^2} + \frac{\sigma^2}{n}\,, \tag{4}$$

and

$$\mathrm{E}\left[\, r(j/n, 1/n)^4 \,\right] = \frac{\alpha^4}{n^4} + 6\frac{\alpha^2\sigma^2}{n^3} + 3\frac{\sigma^4}{n^2}\,. \tag{5}$$

It is evident that the terms involving the drift coefficient are an order of magnitude smaller, for n large, than those that pertain only to the diffusion coefficient. This feature allows us to estimate the return variation with a high degree of precision even without specifying the underlying mean drift, e.g.,[3]

$$\hat{\sigma}_n^2 = \frac{1}{K}\sum_{j=1}^{n\cdot K} r^2(j/n, 1/n). \tag{6}$$

It is straightforward to establish that

$$\mathrm{E}\left[\, \hat{\sigma}_n^2 \,\right] = \frac{\alpha^2}{n} + \sigma^2\,, \tag{7}$$

while some additional calculations yield

$$\mathrm{Var}\left[\, \hat{\sigma}_n^2 \,\right] = 4\frac{\alpha^2\sigma^2}{n^2 K} + 2\frac{\sigma^4}{nK}\,. \tag{8}$$

It follows by a standard L^2 argument that, in probability, $\hat{\sigma}_n^2 \to \sigma^2$ for $n \to \infty$. Hence, the realized variation measure is a biased but consistent estimator of the underlying (squared) volatility coefficient. Moreover, it is evident that, for n large, the bias is close to negligible. In fact, as $n \to \infty$ we have the distributional convergence,

$$\sqrt{n \cdot K}\,(\hat{\sigma}_n^2 - \sigma^2) \;\to\; N(0, 2\sigma^4)\,. \tag{9}$$

These insights are not new. For example, within a similar context, they were stressed by Merton [97]. However, the lack of quality intraday price data and the highly restrictive setting have long led scholars to view them as bereft

[3] The quantity $(K \cdot \hat{\sigma}_n^2)$ is a "realized volatility" estimator of the return variation over $[0, K]$ and it moves to the forefront of our discussion in the following section.

of practical import. This situation has changed fundamentally over the last decade, as it has been shown that the basic results apply very generally, high-frequency data have become commonplace, and the measurement procedures, through suitable strategies, can be adapted to deal with intraday observations for which the relative impact of microstructure noise may be substantial.

3 Quadratic Return Variation and Realized Volatility

This section outlines the main steps in generalizing the above findings to an empirically relevant setting with stochastic volatility. We still operate within the continuous-time diffusive setting, for simplicity ruling out price jumps, and assume a *frictionless market*. In this setting the asset's logarithmic price process s must be a semimartingale to rule out arbitrage opportunities (e.g., Back [29]). We then have,

$$ds(t) = \mu(t)dt + \sigma(t)\, dW(t)\,, \qquad 0 \leq t \leq T\,, \tag{10}$$

where W is a standard Brownian motion process, $\mu(t)$ and $\sigma(t)$ are predictable processes, $\mu(t)$ is of finite variation, while $\sigma(t)$ is strictly positive and square integrable, i.e., $\mathrm{E}\left(\int_0^t \sigma_s^2 \mathrm{d}s\right) < \infty$. Hence, the processes $\mu(t)$ and $\sigma(t)$ signify the instantaneous conditional mean and volatility of the return. The continuously compounded return over the time interval from $t-k$ to t, $0 < k \leq t$, is therefore

$$r(t,k) = s(t) - s(t-k) = \int_{t-k}^{t} \mu(\tau)d\tau + \int_{t-k}^{t} \sigma(\tau)dW(\tau)\,, \tag{11}$$

and its *quadratic variation* $QV(t,k)$ is

$$QV(t,k) = \int_{t-k}^{t} \sigma^2(\tau)d\tau\,. \tag{12}$$

Equation (12) shows that innovations to the mean component $\mu(t)$ do not affect the sample path variation of the return. Intuitively, this is because the mean term, $\mu(t)dt$, is of lower order in terms of second order properties than the diffusive innovations, $\sigma(t)dW(t)$. Thus, when cumulated across many high-frequency returns over a short time interval of length k they can effectively be neglected. The *diffusive* sample path variation over $[t-k,t]$ is also known as the *integrated variance* $IV(t,k)$,

$$IV(t,k) = \int_{t-k}^{t} \sigma^2(\tau)d\tau\,. \tag{13}$$

Equations (12) and (13) show that, in this setting, the quadratic and integrated variation coincide. This is however no longer true for more general return process like, e.g., the stochastic volatility jump-diffusion model discussed in Section 5 below.

Absent microstructure noise and measurement error, the return quadratic variation can be approximated arbitrarily well by the corresponding cumulative squared return process. Consider a partition $\{t-k+\frac{j}{n}, j=1,\dots n\cdot k\}$ of the $[\,t-k,\,t\,]$ interval. Then the realized volatility (RV) of the logarithmic price process is

$$RV(t,k;n) = \sum_{j=1}^{n\cdot k} r\left(t-k+\frac{j}{n},\frac{1}{n}\right)^2 . \tag{14}$$

Semimartingale theory ensures that the realized volatility measure converges in probability to the return quadratic variation QV, previously defined in equation (12), when the sampling frequency n increases:

$$RV(t,k;n) \longrightarrow QV(t,k) \qquad \text{as } n \to \infty\,. \tag{15}$$

This finding extends the consistency result for the (constant) volatility coefficient discussed below equation (8) to a full-fledged stochastic volatility setting. This formal link between realized volatility measures based on high-frequency returns and the quadratic variation of the underlying (no arbitrage) price process follows immediately from the theory of semimartingales (e.g., Protter [102]) and was first applied in the context of empirical return volatility measurement by Andersen and Bollerslev [9]. The distributional result in equation (9) also generalizes directly, as we have, for $n \to \infty$,

$$\sqrt{n\cdot k}\left(\frac{RV(t,k;n)-QV(t,k)}{\sqrt{2\,IQ(t,k)}}\right) \to N(0,1)\,, \tag{16}$$

where $IQ(t,k) \equiv \int_{t-k}^{t}\sigma^4(\tau)d\tau$ is the integrated quarticity, with $IQ(t,k)$ independent from the limiting Gaussian distribution on the right hand side. This result was developed and brought into the realized volatility literature by Barndorff-Nielsen and Shephard [37].[4]

Equation (16) sets the stage for formal ex-post inference regarding the actual realized return variation over a given period. However, the result is not directly applicable as the so-called integrated quarticity, $IQ(t,k)$, is unobserved and is likely to display large period-to-period variation. Hence, a consistent estimator for the integrated quarticity must be used in lieu of the true realization to enable feasible inference. Such estimators, applicable for any integrated power of the diffusive coefficient, have been proposed by Barndorff-Nielsen and Shephard [37]. The realized power variation of order p, $V(p;t,k;n)$ is the (scaled) cumulative sum of the absolute p-th power of the high-frequency returns and it converges, as $n \to \infty$, to the corresponding power variation of order p, $V(p;t,k)$. That is, defining the p-th realized power variation as,

[4] The unpublished note by Jacod [88] implies the identical result but this note was not known to the literature at the time.

$$V(p;t,k;n) \equiv n^{p/2-1}\,\mu_p^{-1}\sum_{j=1}^{n\cdot k}\left|r\left(t-k+\frac{j}{n},\frac{1}{n}\right)\right|^p, \tag{17}$$

where μ_p denotes the p-th absolute moment of a standard normal variable, we have, in probability,

$$V(p;t,k;n) \to \int_{t-k}^{t}\sigma^p(\tau)d\tau \equiv V(p;t,k). \tag{18}$$

In other words, $V(4;t,k;n)$ is a natural choice as a consistent estimator for the integrated quarticity $IQ(t,k)$. It should be noted that this conclusion is heavily dependent on the absence of jumps in the price process which is an issue we address in more detail later. Moreover, the notion of realized power variation is a direct extension of realized volatility as $RV(t,k;n) = V(2;t,k;n)$ so equation (18) reduces to equation (15) for $p = 2$.

More details regarding the asymptotic results and multivariate generalizations of realized volatility may be found in, e.g., Andersen et al. [16, 17], Barndorff-Nielsen and Shephard [36, 37, 38], Meddahi [95], and Mykland [98].

4 Conditional Return Variance and Realized Volatility

This section discusses the relationship between quadratic variation or integrated variance along with its associated empirical measure, realized volatility, and the conditional return variance. In the case of constant drift and volatility coefficients, the conditional (and unconditional) return variance equals the quadratic variation of the log price process. In contrast, when volatility is stochastic we must distinguish clearly between the conditional variance, representing the (ex-ante) expected size of future squared return innovations over a certain period, and the quadratic variation, reflecting the actual (ex-post) realization of return variation, over the corresponding horizon. Hence, the distinction is one of a priori expectations versus subsequent actual realizations of return volatility. Under ideal conditions, the realized volatility captures the latter, but not the former. Nonetheless, realized volatility measures are useful in gauging the conditional return variance as one may construct well calibrated forecasts (conditional expectations) of return volatility from a time series of past realized volatilities. In fact, within a slightly simplified setting, we can formally strengthen these statements. If the instantaneous return is the continuous-time process (10) *and* the return, mean, and volatility processes are uncorrelated (i.e., $dW(t)$ and innovations to $\mu(t)$ and $\sigma(t)$ are mutually independent), then $r(t,k)$ is normally distributed conditional on the cumulative drift $\mu(t,k) \equiv \int_{t-k}^{t}\mu(\tau)d\tau$ and the quadratic variation $QV(t,k)$ (which in this setting equals the integrated variance $IV(t,k)$ as noted in equations (12) and (13)):

$$\big(r(t,k)\,|\,\mu(t,k), IV(t,k)\big) \sim \mathrm{N}\big(\mu(t,k), IV(t,k)\big). \tag{19}$$

Consequently, the return distribution is mixed Gaussian with the mixture governed by the realizations of the integrated variance (and integrated mean) process. Extreme realizations (draws) from the integrated variance process render return outliers likely while persistence in the integrated variance process induces volatility clustering. Moreover, for short horizons, where the conditional mean is negligible relative to the cumulative absolute return innovations, the integrated variance may be directly related to the conditional variance as,

$$\mathrm{Var}[\, r(t,k) \,|\, \mathcal{F}_{t-k} \,] \approx \mathrm{E}[\, RV(t,k;n) \,|\, \mathcal{F}_{t-k} \,] \approx \mathrm{E}[\, QV(t,k) \,|\, \mathcal{F}_{t-k} \,] \,. \qquad (20)$$

A volatility forecast is an estimate of the conditional return variance on the far left-hand side of equation (20), which in turn approximates the expected quadratic variation. Since RV is approximately unbiased for the corresponding unobserved quadratic variation, the realized volatility measure is the natural benchmark against which to gauge the performance of volatility forecasts. Goodness-of-fit tests may be conducted on the residuals given by the difference between the ex-post realized volatility measure and the ex-ante forecast. We review some of the evidence obtained via applications inspired by these relations in Section 7. In summary, the quadratic variation is directly related to the *actual* return variance as demonstrated by equation (19) and to the *expected* return variance, as follows from equation (20).

Finally, note that the realized volatility concept is associated with the return variation measured over a discrete time interval rather than with the so-called spot or instantaneous volatility. This distinction separates the realized volatility approach from a voluminous literature in statistics seeking to estimate spot volatility from discrete observations, predominantly in a setting with a constant diffusion coefficient. It also renders it distinct from the early contributions in financial econometrics allowing explicitly for time-varying volatilities, e.g., Foster and Nelson [73]. In principle, the realized volatility measurement can be adapted to spot volatility estimation: as k goes to zero, $QV(t,k)$ converges to the instantaneous volatility $\sigma^2(t)$, i.e., in principle RV converges to instantaneous volatility when both k and k/n shrink. For this to happen, however, k/n must converge at a rate higher than k, so as the interval shrinks we must sample returns at an ever increasing frequency. In practice, this is infeasible, because intensive sampling over tiny intervals magnifies the effects of microstructure noise. We return to this point in Section 6 where we discuss the bias in RV measures when returns are sampled with error.

5 Jumps and Bipower Variation

The return process in equation (10) is continuous under the stated regularity conditions, even if σ may display jumps. This is quite restrictive as asset prices often appear to exhibit sudden discrete movements when unexpected news hits the market. A broad class of SV models that allow for the presence of jumps in returns is defined by

$$ds(t) = \mu(t)dt + \sigma(t)dW(t) + \xi(t)\, dq_t\,, \tag{21}$$

where q is a Poisson process uncorrelated with W and governed by the jump intensity λ_t, i.e., Prob$(dq_t = 1) = \lambda_t\, dt$, with λ_t positive and finite. This assumption implies that there can only be a finite number of jumps in the price path per time period. This is a common restriction in the finance literature, though it rules out infinite activity Lévy processes. The scaling factor $\xi(t)$ denotes the magnitude of the jump in the return process if a jump occurs at time t. While explicit distributional assumptions often are invoked for parametric estimation, such restrictions are not required as the realized volatility approach is fully nonparametric in this dimension as well.

In this case, the quadratic return variation process over the interval from $t-k$ to t, $0 \leq k \leq t \leq T$, is the sum of the diffusive integrated variance and the cumulative squared jumps:

$$QV(t,k) = \int_{t-k}^{t} \sigma^2(s)ds + \sum_{t-k \leq s \leq t} J^2(s) \equiv IV(t,k) + \sum_{t-k \leq s \leq t} J^2(s)\,, \tag{22}$$

where $J(t) \equiv \xi(t)dq(t)$ is non-zero only if there is a jump at time t.

The RV estimator (14) remains a consistent measure of the total QV in the presence of jumps, i.e., result (15) still holds; see, e.g., Protter [102] and the discussion in Andersen, Bollerslev, and Diebold [11]. However, since the diffusive and jump volatility components appear to have distinctly different persistence properties it is useful both for analytic and predictive purposes to obtain separate estimates of these two factors in the decomposition of the quadratic variation implied by equation (22).

To this end, the h-skip bipower variation, BV, introduced by Barndorff-Nielsen and Shephard [39] provides a consistent estimate of the IV component,

$$BV(t,k;h,n) = \frac{\pi}{2} \sum_{i=h+1}^{n \cdot k} \left| r\left(t-k+\frac{ik}{n}, \frac{1}{n}\right) \right| \left| r\left(t-k+\frac{(i-h)k}{n}, \frac{1}{n}\right) \right|. \tag{23}$$

Setting $h = 1$ in definition (23) yields the 'realized bipower variation' $BV(t,k;n) \equiv BV(t,k;1,n)$. The bipower variation is robust to the presence of jumps and therefore, in combination with RV, it yields a consistent estimate of the cumulative squared jump component:

$$RV(t,k;n) - BV(t,k;n) \underset{n\to\infty}{\longrightarrow} QV(t,k) - IV(t,k) = \sum_{t-k \leq s \leq t} J^2(s)\,. \tag{24}$$

The results in equations (22)-(24) along with the associated asymptotic distributions have been exploited to improve the accuracy of volatility forecasts and to design tests for the presence of jumps in volatility. We discuss these applications in Section 7 below.

6 Efficient Sampling versus Microstructure Noise

The convergence relation in equation (15) states that RV approximates QV arbitrarily well as the sampling frequency n increases. Two issues, however, complicate the application of this result. First, even for the most liquid assets a continuous price record is unavailable. This limitation introduces an inevitable discretization error in the RV measures which forces us to recognize the presence of a measurement error. Although we may gauge the magnitude of such errors via the continuous record asymptotic theory outlined in equations (16)-(18), such inference is always subject to some finite sample distortions and it is only strictly valid in the absence of price jumps. Second, a wide array of microstructure effects induces spurious autocorrelations in the ultra-high frequency return series. The list includes price discreteness and rounding, bid-ask bounces, trades taking places on different markets and networks, gradual response of prices to a block trade, difference in information contained in order of different size, strategic order flows, spread positioning due to dealer inventory control, and, finally, data recording mistakes. Such "spurious" autocorrelations can inflate the RV measures and thus generate a traditional type of bias-variance trade off. The highest possible sampling frequency should be used for efficiency. However, sampling at ultra-high frequency tends to bias the RV estimate.

A useful tool to assess this trade-off is the *volatility signature plot*, which depicts the sample average of the RV estimator over a long time span as a function of the sampling frequency. The long time span mitigates the impact of sampling variability so, absent microstructure noise, the plot should be close to a horizontal line. In practice, however, for transaction data obtained from liquid stocks the plot spikes at high sampling frequencies and decays rather smoothly to stabilize at frequencies in the 5- to 40-minute range. In contrast, the opposite often occurs for returns constructed from bid-ask quote midpoints as asymmetric adjustment of the spread induces positive serial correlation and biases the signature plot downward at the very highest sampling frequencies. Likewise, for illiquid stocks the inactive trading induces positive return serial autocorrelation which renders the signature plot increasing at lower sampling frequencies, see, e.g., Andersen, Bollerslev, Diebold, and Labys [14]. Aït-Sahalia, Mykland, and Zhang [4] and Bandi and Russell [33] extend this approach by explicitly trading off efficient sampling versus bias-inducing noise to derive optimal sampling schemes.

Other researchers have suggested dealing with the problem by using alternative QV estimators that are less sensitive to microstructure noise. For instance, Huang and Tauchen [87] and Andersen, Bollerslev, and Diebold [12] note that using staggered returns and BV helps reduce the effect of noise, while Andersen, Bollerslev, Frederiksen, and Nielsen [20] extend volatility signature plots to include power and h-skip bipower variation. Other studies have instead relied on the high-low price range measure (e.g., Alizadeh, Brandt, and Diebold [5], Brandt and Diebold [48], Brandt and Jones [49], Gallant et al.

[74], Garman and Klass [76], Parkinson [99], Schwert [104], and Yang and Zhang [114]) to deal with situations in which the noise to signal ratio is high. Christensen and Podolskij [55] and Dobrev [62] generalize the range estimator to high-frequency data in distinct ways and discuss the link to RV.

A different solution to the problem is considered in the original contribution of Zhou [119] who seeks to correct the bias of RV style estimators by explicitly accounting for the covariance in lagged squared return observations. Hansen and Lunde [82] extend Zhou's approach to the case of non-i.i.d. noise. In contrast, Aït-Sahalia et al. [4] explicitly determine the requisite bias correction when the noise term is i.i.d. normally distributed, while Zhang et al. [116] propose a consistent volatility estimator that uses the entire price record by averaging RVs computed from different sparse sub-samples and correcting for the remaining bias. Aït-Sahalia et al. [3] extend the sub-sampling approach to account for certain types of serially correlated errors. Another prominent and general approach is the recently proposed kernel-based technique of Barndorff-Nielsen et al. [40, 41].

7 Empirical Applications

Since the early 1990s transaction data have become increasingly available to academic research. This development has opened the way for a wide array of empirical applications exploiting the realized return variation approach. Below we briefly review the progress in different areas of research.

7.1 Early Work

Hsieh [86] provides one of the first estimates of the daily return variation constructed from intra-daily S&P500 returns sampled at the 15-minute frequency. The investigation is informal in the sense that there is no direct association with the concept of quadratic variation. More in-depth applications were pursued in publications by the Olsen & Associates group and later surveyed in Dacorogna et al. [60] as they explore both intraday periodicity and longer run persistence issues for volatility related measures. Another significant early contribution is a largely unnoticed working paper by Dybvig [65] who explores interest rate volatility through the cumulative sum of squared daily yield changes for the three-month Treasury bill and explicitly refers to it as an empirical version of the quadratic variation process used in analysis of semimartingales. More recently, Zhou [119] provides an initial study of RV style estimators. He notes that the linkage between sampling frequency and autocorrelation in the high-frequency data series may be induced by sampling noise and he proposes a method to correct for this bias. Andersen and Bollerslev [8, 10] document the simultaneous impact of intraday volatility patterns, the volatility shocks due to macroeconomic news announcements, and the long-run dependence in realized volatility series through an analysis of the cumulative absolute and squared five-minute returns for the Deutsche Mark-Dollar exchange rate. The pronounced intraday features motivate the focus on (multiples of) one trading as the basic aggregation unit for realized volatility measures since this approach largely annihilates repetitive high frequency fluctuations and brings the systematic medium and low frequency volatility variation into focus. Comte and Renault [58] point to the potential association between RV measures and instantaneous volatility. Finally, early empirical analyses of daily realized volatility measures are provided in, e.g., Andersen et al. [15] and Barndorff-Nielsen and Shephard [36].

7.2 Volatility Forecasting

As noted in Section 3, RV is the natural benchmark against which to gauge volatility forecasts. Andersen and Bollerslev [9] stress this point which is further developed by Andersen et al. [17, 22, 23] and Patton [100] through different analytic means.

Several studies pursue alternative approaches in order to improve predictive performance. Ghysels et al. [77] consider Mixed Data Sampling (MIDAS) regressions that use a combination of volatility measures estimated at different frequencies and horizons. Related, Engle and Gallo [67] exploit the information in different volatility measures, modelled with a multivariate extension of the multiplicative error model suggested by Engle [66], to predict multistep volatility. A rapidly growing literature studies jump detection (e.g., Aït-Sahalia and Jacod [1], Andersen et al. [21, 19], Fleming and Paye [71], Huang and Tauchen [87], Jiang and Oomen [89], Lee and Mykland [90], Tauchen and Zhou [108], and Zhang [117]). Andersen et al. [12] show that separating the jump and diffusive components in QV estimates enhances the model forecasting performance. Related, Liu and Maheu [91] and Forsberg and Ghysels [72] show that realized power variation, which is more robust to the presence of jumps than RV, can improve volatility forecasts.

Other researchers have been investigating the role of microstructure noise on forecasting performance (e.g., Aït-Sahalia and Mancini [2], Andersen et al. [24, 25], and Ghysels and Sinko [78]) and the issue of how to use noisy overnight return information to enhance volatility forecasts (e.g., Hansen and Lunde [81] and Fleming et al. [70]).

A critical feature of volatility is the degree of its temporal dependence. Correlogram plots for the (logarithmic) RV series show a distinct hyperbolic decay that is described well by a fractionally-integrated process. Andersen and Bollerslev [8] document this feature using the RV series for the Deutsche Mark-Dollar exchange rate. Subsequent studies have documented similar properties across financial markets for the RV on equities (e.g., Andersen et al. [13], Areal and Taylor [28], Deo et al. [61], Martens [93]), currencies (e.g., Andersen and Bollerslev [10], Andersen et al. [16, 17], and Zumbach [120]), and bond yields (e.g., Andersen and Benzoni [6]). This literature concurs on the value of the fractional integration coefficient, which is estimated in the 0.30–0.48 range, i.e., the stationarity condition is satisfied. Accounting for long memory in volatility can prove useful in forecasting applications (e.g., Deo et al. [61]). A particularly convenient approach to accommodate the persistent behavior of the RV series is to use a component-based regression to forecast the k-step-ahead quadratic variation (e.g., Andersen et al. [12], Barndorff-Nielsen and Shephard [36], and Corsi [59]):

$$RV(t+k,k) = \beta_0 + \beta_D RV(t,1) + \beta_W RV(t,5) + \beta_M RV(t,21) + \varepsilon(t+k)\,. \quad (25)$$

Simple OLS estimation yields consistent estimates for the coefficients in the regression (25), which can be used to forecast volatility out of sample.

7.3 The Distributional Implications of the No-Arbitrage Condition

Equation (19) implies that, approximately, the daily return $r(t)$ follows a Gaussian mixture directed by the IV process. This is reminiscent of the mixture-of-distributions hypothesis analyzed by, e.g., Clark [57] and Tauchen and Pitts [106]. However, in the case of equation (19) the mixing variable is directly measurable by the RV estimator which facilitates testing the distributional restrictions implied by the no-arbitrage condition embedded in the return dynamics (10). Andersen et al. [15] and Thomakos and Wang [110] find that returns standardized by RV are closer to normal than the standardized residuals from parametric SV models estimated at the daily frequency. Any remaining deviation from normality may be due to a bias in RV stemming from microstructure noise or model misspecification. In particular, when returns jump as in equation (21), or if volatility and return innovations correlate, condition (19) no longer holds. Peters and de Vilder [101] deal with the volatility-return dependence by sampling returns in 'financial time,' i.e., they identify calendar periods that correspond to equal increments to IV, while Andersen et al. [19] extend their approach for the presence of jumps. Andersen et al. [21] apply these insights, in combination with alternative jump-identification techniques, to different data sets and find evidence consistent with the mixing condition. Along the way they document the importance of jumps and the asymmetric return-volatility relation. Similar issues are also studied in Fleming and Paye [71] and Maheu and McCurdy [92].

7.4 Multivariate Quadratic Variation Measures

A growing number of studies uses multivariate versions of realized volatility estimators, i.e., realized covariance matrix measures, in portfolio choice (e.g., Bandi et al. [34] and Fleming et al. [70]) and risk measurement problems (e.g., Andersen et al. [13, 18] and Bollerslev and Zhang [44]). Multivariate applications, however, are complicated by delays in the security price reactions to price changes in related assets as well as by non-synchronous trading effects. Sheppard [105] discusses this problem but how to best deal with it remains largely an open issue. Similar to Scholes and Williams [103], some researchers include temporal cross-correlation terms estimated with lead and lag return data in covariance measures (e.g., Hayashi and Yoshida [83, 84] and Griffin and Oomen [79]). Other studies explicitly trade off efficiency and noise-induced bias in realized covariance estimates (e.g., Bandi and Russell [32] and Zhang [115]), while Bauer and Vorkink [42] propose a latent-factor model of the realized covariance matrix.

7.5 Realized Volatility, Model Specification and Estimation

RV gives empirical content to the latent variance variable and is therefore useful for specification testing of the restrictions imposed on volatility by parametric models previously estimated with low-frequency data. For instance, Andersen and Benzoni [6] examine the linkage between the quadratic variation and level of bond yields embedded in some affine term structure models and reject the condition that volatility is spanned by bond yields in the U.S. Treasury market. Christoffersen et al. [56] reject the Heston [85] model implication that the standard deviation dynamics are conditionally Gaussian by examining the distribution of the changes in the square-root RV measure for S&P 500 returns.

Further, RV measures facilitate direct estimation of parametric models. Barndorff-Nielsen and Shephard [37] decompose RV into actual volatility and realized volatility error. They consider a state-space representation for this decomposition and apply the Kalman filter to estimate different flavors of the SV model. Bollerslev and Zhou [45] and Garcia et al. [75] build on the results of Meddahi [96] to obtain efficient moment conditions which they use in the estimation of continuous-time stochastic volatility processes. Todorov [112] extends the analysis for the presence of jumps.

8 Possible Directions for Future Research

In recent years the market for derivative securities offering a pure play on volatility has grown rapidly in size and complexity. Well-known examples are the over-the-counter markets for variance swaps, which at maturity pay the difference between realized variance and a fixed strike price, and volatility swaps with payoffs linked to the square root of realized variance. These financial innovations have opened the way for new research on the pricing and hedging of these contracts. For instance, while variance swaps admit a simple replication strategy through static positions in call and put options combined with dynamic trading in the underlying asset (e.g., Britten-Jones and Neuberger [50] and Carr and Madan [53]), it is still an open issue to determine the appropriate replication strategy for volatility swaps and other derivatives that are non-linear functions of realized variance (e.g., call and put options). Carr and Lee [52] make an interesting contribution in this direction.

Realized volatility is also a useful source of information to learn more about the volatility risk premium. Recent contributions have explored the issue by combining RV measures with model-free option-implied volatility gauges like the VIX (e.g., Bollerslev et al. [43], Carr and Wu [54], and Todorov [112]). Other studies are examining the linkage between volatility risk and equity premia (Bollerslev and Zhou [46]), bond premia (Wright and Zhou [113]), credit spreads (Tauchen and Zhou [109] and Zhang et al. [118]), and hedge-fund performance (Bondarenko [47]). In addition, new research is studying the pricing of volatility risk in individual stock options (e.g., Bakshi and Kapadia [30], Carr and Wu [54], Driessen et al. [63], and Duarte and Jones [64]) and in the cross section of stock returns (e.g., Ang et al. [26, 27], Bandi et al. [31], and Guo et al. [80]).

Finally, more work is needed to better understand the linkage between asset return volatility and fluctuations in underlying fundamentals. Several studies have proposed general equilibrium models that generate low-frequency conditional heteroskedasticity (e.g., Bansal and Yaron [35], Campbell and Cochrane [51], McQueen and Vorkink [94], and Tauchen [107]). Related, Engle and Rangel [69] and Engle et al. [68] link macroeconomic variables and long-run volatility movements. An attempt to link medium and higher frequency realized volatility fluctuations in the bond market to both business cycle variation and macroeconomic news releases is initiated in Andersen and Benzoni [7], but clearly much more work on this front is warranted.

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