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Signaling Effects of Monetary Policy

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Abstract

We develop a DSGE model in which the policy rate signals the central bank's view about macroeconomic developments to incompletely informed price setters. The model is estimated with likelihood methods on a U.S. data set including the Survey of Professional Forecasters as a measure of price setters' expectations. The signaling effects of monetary policy are found to be empirically important and dampen the effects of monetary disturbances on inflation. While the signaling effects enhance the Federal Reserve's ability to stabilize the economy in the face of demand shocks, they play a small role in stabilizing the economy after technology shocks.

Keywords: Higher-order expectations, imperfect common knowledge, Bayesian econometrics, persistent real effects of nominal shocks.

JEL classification: E52, D82, C11.

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1 Introduction

An important feature of economic systems is that information is dispersed across market participants and policy makers. Dispersed information implies that publicly observable policy actions transfer information to market participants. An important example is the monetary policy rate, which conveys information about the central bank's view on macroeconomic developments. Such an information transfer may strongly influence the transmission of monetary impulses and the central bank's ability to stabilize the economy. Consider the case in which a central bank expects that a disturbance will shrink economic activity in the next few quarters. On the one hand, as predicted by standard macroeconomic models, cutting the policy rate has the effect of countering the contractionary effects of the shock. On the other hand, lowering the policy rate might well accelerate the recession if this action convinces market participants that a contractionary shock will hit the economy. While the first type of effect has been intensively investigated by the theoretical and empirical literature, the signaling effects of monetary policy have received far less attention. The paper develops a dynamic stochastic general equilibrium (DSGE) model to study the empirical relevance of the signaling effects of monetary policy and their implications for the macroeconomic effects of monetary impulses.

The model developed in the paper is characterized by the following two ingredients: (1) monopolistically competitive price setters have limited and dispersed information about aggregate developments and (2) the nominal interest rate set by the central bank can be publicly observed. The model features technology, monetary, and demand shocks. Technology shocks are idiosyncratic but they have a persistent aggregate component that is not observed by the price setters. Price setters observe their idiosyncratic technology, which conveys information about the persistent aggregate component of technology, and an exogenous private signal about the current demand shock. Since firms set their prices in response to changes in their nominal marginal costs, they raise their prices, *ceteris paribus*, when they expect the price level to increase, that is, when the other price setters, on average, are expected to raise their prices. Such a coordination motive in price setting and the availability of private information make it optimal for price setters to forecast the forecast of other price setters (Townsend 1983a and 1983b). Furthermore, price setters observe the interest rate, which is set by the central bank according to a Taylor-type reaction function. This policy signal provides public information about the central bank's view on current inflation and the output gap to price setters.

The model features two channels of monetary transmission. The first channel emerges because the central bank can affect the real interest rate due to both nominal rigidities, as in standard New Keynesian models, and to dispersed information (Woodford, 2002). Changes in the real interest rate induce households to adjust their consumption. The second channel arises because while information is dispersed across price setters, the policy rate is perfectly observable. We call this second channel *the signal channel* of monetary transmission. How the signaling effects of monetary policy influence the propagation of shocks critically relies on how price setters interpret the change in the policy rate. A rise in the policy rate can be interpreted by price-setting firms in two alternative ways. First, a monetary tightening might imply that the central bank is responding to a contractionary monetary shock, leading the central bank to deviate from the Taylor-type rule. Second, an interest rate rise may also be interpreted as the response of the central bank to an inflationary non-policy shock, which, in the model, is either an adverse technology shock or a positive demand shock. If the first interpretation prevails among price setters, stabilization policies in the face of short-run disturbances can be successfully conducted by the central bank, as tightening monetary policy curbs firms' inflation expectations and hence inflation. If the second interpretation prevails, a rise in the policy rate induces firms to expect higher inflation. In this case, monetary policy cannot successfully stabilize inflation. In fact, any attempt by the central bank to counter the inflationary effects of non-policy shocks by raising the policy rate ends up bringing about even higher inflation.

The model is estimated through likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. The data range includes the 1970s, which were characterized by one of the most notorious episodes of a substantial rise in inflation and inflation expectations in recent U.S. economic history. When the model is taken to the data, we find that an interest rate rise signals that either a positive demand shock or a contractionary monetary shock may have hit the economy. Firms, however, do not sensibly change their expectations about the aggregate technology shock when they observe that the policy rate has increased. This result has a number of important implications. First, the signal channel dampens the effects of a monetary disturbance on inflation because price setters interpret a rise in the policy rate as the central bank's response to a positive preference shock, which pushes up inflation expectations. Second, such a mistaken interpretation gives rise to a positive response of inflation expectations to a monetary policy shock. Third, the signal channel does not improve the Federal Reserve's ability to counter the inflationary consequences of technology shocks. The reason is that when the Federal Reserve raises the policy rate to counter a negative technology shock, firms believe that the central bank is reacting to either a positive demand shock or a contractionary monetary shock that have conflicting effects on firms' inflation expectations. These two effects turn out to cancel each other out. Fourth, the signal channel turns out to improve the effectiveness of stabilization policies in the face of demand shocks. The reason is that when the central bank raises the interest rate, firms start believing that a contractionary monetary shock might have occurred. Expecting a contractionary monetary shock tends to lower inflation expectations, curbing the inflationary consequences of the positive demand shock.

Furthermore, the estimated model features a fairly sluggish response of inflation to structural shocks and, at the same time, an average duration of price contracts that is in line with micro studies (Bils and Klenow, 2004, Klenow and Kryvtsov, 2008, Nakamura and Steinsson, 2008, and Klenow and Malin, 2010). As described by Hellwig (2002) and Woodford (2002) and econometrically validated by Melosi (2010), the very slow adjustment of firms' forecasting the forecasts of other price setters accounts for the sluggish responses of inflation to shocks. The paper also shows that the model with the signal channel fits the data better than a canonical New Keynesian DSGE model in which firms have perfect information about the history of shocks and, hence, the signal channel is inactive. In particular, the model with the signal channel does better at explaining the dynamics of the observed inflation expectations (i.e., the SPF). Finally, the paper emphasizes that a strong systematic response to inflation critically raises the central bank's ability to stabilize prices in the model. The reason is that an aggressive policy toward inflation stabilization mitigates the expected inflationary consequences of non-policy shocks that a rise in the policy rate may lead firms to expect.

The paper makes a methodological contribution by providing an algorithm to solve DSGE models in which agents forecast the forecast of other agents. The solution routine proposed in the paper turns out to be sufficiently fast and reliable to allow likelihood-based estimation of a medium-scale model. The proposed algorithm belongs to the general solution methods developed by Nimark (2011). The proposed algorithm improves upon the one used in Nimark (2008) as it does not require solving a system of non-linear equations. This task would be too computationally burdensome to allow likelihood-based estimation.[1]

The model studied in this paper is built on Nimark (2008). A nice feature of Nimark's model is that the supply side of this economy can be analytically worked out and turns out to be characterized by an equation that nests the standard New Keynesian Phillips curve. The model studied in this paper shares this feature. Nonetheless, in Nimark (2008) the signal channel does not arise because assumptions on the Taylor-rule specification imply that the policy rate conveys only redundant information to price setters.

The idea that the monetary authority sends public signals in an economy in which agents

[1] An alternative solution algorithm based on re-writing the equilibrium dynamics partly as an MA process and setting the lag with which the state is revealed to be a very large number is analyzed by Hellwig (2002) and Hellwig and Vankateswaran (2009).

have dispersed information has been pioneered by Morris and Shin (2003a). Morris and Shin (2003b), Angeletos and Pavan (2004), Hellwig (2005), and Angeletos and Pavan (2007) focus on the welfare effects of disclosing public information in models with dispersed information and complementarities. Angeletos et al. (2006) study the signaling effects of policy decisions in a coordination game. Nonetheless, this theoretical literature is based on models that are too stylized to be taken to the data. Dispersed information models have also been used for studying economic fluctuations (Townsend, 1983a, 1983b; Adam, 2009; Angeletos and La'O, 2009; Nimark, 2012, and Rondina, 2008) and the propagation of monetary disturbances to real variables and prices (Phelps, 1970; Lucas, 1972; Woodford, 2002; Adam, 2007; Gorodnichenko, 2008; Mackowiak and Wiederholt, 2009 and 2010; and Paciello, forthcoming).[2] Lorenzoni (2009 and 2010) studies a model in which aggregate fluctuations are driven by the private sector's uncertainty about the economy's fundamentals. In Lorenzoni's papers, while monetary policy affects agents' incentives to respond to private and public signals, the signaling effect of monetary policy is not investigated. The paper is also related to Walsh (2010), who shows that the (perceived or actual) signaling effects of monetary policy alter the central bank's decisions, resulting in a bias (i.e., an opacity bias) that distorts the central bank's optimal response to shocks. Unlike this paper, Walsh's study is based on a model that does not feature dispersed information. Cogley et al. (2011) address the problem of a newly-appointed central bank governor who inherits a high average inflation rate from the past and wants to disinflate. In the model, agents conduct Bayesian learning over the coefficients that characterize the conduct of monetary policy. The paper characterizes an optimal Taylor-type rule and study how learning affects the choice of policy.

The paper is also related to a quickly growing empirical literature that uses the SPF to study the response of public expectations to monetary policy decisions. Del Negro and Eusepi (2010) perform an econometric evaluation of the extent to which the inflation expectations generated by DSGE models are in line with the observed inflation expectations. The main differences with this paper are as follows. First, in our settings, price setters have heterogeneous and dispersed higher-order expectations as they observe private signals. Second, this paper fits the model to a data set that includes the 1970s, whereas Del Negro and Eusepi (2010) use a data set starting from the early 1980s. Coibion and Gorodnichenko (2011) find that the Federal Reserve raises the policy rate more gradually if the private sector's inflation expectations are lower than the Federal Reserve's forecasts of inflation. This empirical evidence can be rationalized in a model in which monetary policy has signaling effects and the central bank acts strategically to stabilize public inflation expectations. Other

[2] See Mankiw and Reis (2006), who develop models with information frictions that do not feature dispersed information but can also generate sizeable persistence.

empirical studies related to this paper include Melosi (2010), who conducts an econometric analysis of a stylized DSGE model with dispersed information. Bianchi (2010) studies how agents' beliefs react to shifts in the monetary policy regime and the associated implications for the transmission mechanism of monetary policy. Bianchi and Melosi (2010) develop a DSGE model that features waves of agents' pessimism about how aggressively the central bank will react to future changes in inflation to study the welfare implications of monetary policy communication.

The paper is organized as follows. Section 2 describes the incomplete information model. In this section, we also describe a model in which firms have complete information. The latter model will be used as a benchmark to evaluate the empirical performance of the model with incomplete information. Section 3 deals with the empirical analysis of the paper. Section 4 concludes.

2 Models

Section 2.1 introduces the model with dispersed information. In section 2.2, I present the time protocol of the model. Section 2.3 presents the problem of households. Section 2.4 presents the price-setting problem of firms, which have incomplete information. In Section 2.5, the central bank's and government's behavior is modeled. Section 2.6 deals with the log-linearization of the model equations. Section 2.7 presents the perfect information model, which will turn out to be useful in evaluating the empirical significance of the signal channel. Finally, Section 2.8 analyzes how the signal channel works.

2.1 The Incomplete Information Model (IIM)

The economy is populated by a continuum $(0, 1)$ of households, a continuum $(0, 1)$ of monopolistically competitive firms, a central bank (or monetary authority), and a government (or fiscal authority). A Calvo lottery establishes which firms are allowed to re-optimize their prices in any given period t (Calvo, 1983). Those firms that are not allowed to re-optimize index their prices to the steady-state inflation. Households consume the goods produced by firms, demand government bonds, pay taxes to or receive transfers from the fiscal authority, and supply labor to the firms in a perfectly competitive labor market. Firms sell differentiated goods to households. The fiscal authority has to finance maturing government bonds. The fiscal authority can issue new government bonds and can either collect lump-sum taxes from households or pays transfers to households. The central bank sets the nominal interest rate at which the government's bonds pay out their return.

Aggregate and idiosyncratic shocks hit the model economy. The aggregate shocks are a technology shock, a monetary policy shock, and a demand shock. All of these shocks are orthogonal to each other at all leads and lags. Idiosyncratic shocks include a firm-specific technology shock and the outcome of the Calvo lottery for price optimization.

2.2 The Time Protocol

Any period t is divided into three stages. All actions that are taken in any given stage are simultaneous. At stage 0, shocks are realized and the central bank observes the realization of the aggregate shocks and sets the interest rate. At stage 1, firms observe (i) their idiosyncratic technology $A_{j,t}$, (ii) their signal about preference shocks $g_{j,t}$, and (iii) the interest rate set by the central bank. Given these observations, firms set their price. At stage 2, households learn the realization of all the shocks in the economy and decide their consumption, C_t, their demand for (one-period) nominal government bonds, B_t, and their labor supply, N_t. At this stage, firms hire labor and produce so as to deliver the demanded quantity $C_{j,t}$ at the price they have set at stage 1. The fiscal authority issues bonds and collects taxes from households or pay transfers to households. The markets for goods, labor, and bonds clear.

2.3 Households

Households have perfect information and, hence, we can use the representative household to solve their problem:

$$\max_{C_{t+s},B_{t+s},N_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} g_{t+s} \left[\ln C_{t+s} - \chi_n N_{t+s}\right]$$

where β is the deterministic discount factor, g_t denotes a preference shifter that scales up or down the overall period utility. The logarithm of the preference shifter follows an AR process: $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$ with Gaussian shocks $\varepsilon_{g,t} \backsim \mathcal{N}(0,1)$. These preference shocks play the role of demand shocks in the economy. Disutility from labor linearly enters the period utility function. χ_n is a parameter that affects the marginal disutility of labor.

The flow budget constraint of the representative household in period t reads

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t + T_t \tag{1}$$

where P_t is the price level of the composite good consumed by households and W_t is the (competitive) nominal wage rate, R_t stands for the nominal (gross) interest rate, Π_t are the (equally shared) dividends paid out by the firms, and T_t stands for government lump-sum transfers/taxes. Composite consumption in period t is given by the Dixit-Stiglitz aggregator

$$C_t = \left(\int_0^1 C_{j,t}^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}}, \tag{2}$$

where $C_{j,t}$ is consumption of a differentiated good j in period t.

At stage 2 of every period t, the representative household chooses a consumption vector, labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum transfers/taxes, and the prices of all consumption goods. It can be shown that the demand for the good produced by firm j is:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\nu} C_t \tag{3}$$

where the price level of the composite good is defined as

$$P_t = \left(\int (P_{j,t})^{1-\nu} di \right)^{\frac{1}{1-\nu}} \tag{4}$$

2.4 Firms' Price-Setting Problem

Firms are endowed with a linear technology:

$$Y_{j,t} = A_{j,t} N_{j,t} \tag{5}$$

where $Y_{j,t}$ is the output produced by the firm j at time t, $N_{j,t}$ is the amount of labor employed by firm j at time t, and $A_{j,t}$ is the firm-specific level of technology that can be decomposed into a persistent aggregate component, A_t, and a white-noise firm-specific component, $\varepsilon_{j,t}^a$. More specifically, we have:

$$\ln A_{j,t} = \ln A_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a \tag{6}$$

with $\varepsilon_{j,t}^a \overset{iid}{\backsim} \mathcal{N}(0,1)$ and $A_t = \gamma^t a_t$ with $\gamma > 1$ and a_t is the de-trended level of aggregate technology that evolves according to the process: $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_{a,t}$ with Gaussian shocks $\varepsilon_{a,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$.

Following Calvo (1983), we assume that a fraction θ of firms are not allowed to re-optimize their prices in any given period. Those firms that are not allowed to re-optimize are assumed to index their prices to the steady-state inflation rate. We assume that the firms that are allowed to re-optimize take their price-setting decisions based on incomplete knowledge about the history of shocks that have hit the economy. More specifically, it is assumed that firms' information set at stage 1 of time t (i.e., when prices are set) includes the history of firm-specific technology, the history of a private signal on the preference shifter, the history of the nominal interest rate set by the central bank, and the history of the price set by the firm. To put it in symbols, the information set $\mathcal{I}_{j,t}$ of firm j at stage 1 of time t is given by

$$\mathcal{I}_{j,t} \equiv \{\ln A_{j,\tau}, \ln g_{j,\tau}, R_\tau, P_{j,\tau} : \tau \leq t\} \tag{7}$$

where $\ln g_{j,t}$ denotes the exogenous private signal concerning the preference shifter g_t. This signal is defined as follows:

$$\ln g_{j,t} = \ln g_t + \widetilde{\sigma}_g \varepsilon^g_{j,t} \tag{8}$$

where $\varepsilon^g_{j,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. This signal is meant to capture the fact that arguably firms are used to carry out market analyses to gather information about demand conditions before setting their price.[3] Furthermore, firms are assumed to know the model transition equations and their structural parameters.

Let us denote the steady-state (gross) inflation rate as π_*, the nominal marginal costs for firm j as $MC_{j,t} = W_t/A_{j,t}$, the time t value of one unit of the composite consumption good in period $t+s$ to the representative household as $\Xi_{t|t+s}$, and the expectation operator conditional on firm j's information set $\mathcal{I}_{j,t}$ as $\mathbb{E}_{j,t}$. At stage 1, an arbitrary firm j that is allowed to re-optimize its price solves

$$\max_{P_{j,t}} \quad \mathbb{E}_{j,t}\left[\sum_{s=0}^{\infty} (\beta\theta)^s \, \Xi_{t|t+s} \left(\pi^s_* P_{j,t} - MC_{j,t+s}\right) Y_{j,t+s}\right]$$

subject to $Y_{j,t} = C_{j,t}$ (i.e., firms commit themselves to satisfying any demanded quantity that will arise at stage 2) and the firm's specific demand in equation (3). When solving their price-setting problem, firms have to form expectations about the development of their nominal marginal costs $MC_{j,t}$, the price for the composite good P_t, and the output level of the composite good Y_t, conditional on their information set $\mathcal{I}_{j,t}$. Firms take these three unknown variables as given.

[3]Note that observing the history of their price $P_{j,t}$ does not convey any information about the state of the economy to firms because their price is either adjusted to the steady-state inflation rate, which is known by firms, or a function of the history of the signals that have been observed.

2.5 The Monetary and Fiscal Authority

There is a monetary authority and a fiscal authority. The monetary authority sets the nominal interest rate according to the reaction function

$$R_t = (r_* \pi_*) \left(\frac{\pi_t}{\pi_*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*} \right)^{\phi_y} \eta_{r,t} \tag{9}$$

where r_* is the steady-state real interest rate, π_t is the (gross) inflation rate, and Y_t^* is the potential output, that is, the output level that would be realized if prices were perfectly flexible (i.e., $\theta = 0$). $\eta_{r,t}$ is a random variable that affects the nominal interest rate in period t and is driven by the process: $\ln \eta_{r,t} = \rho_r \ln \eta_{r,t-1} + \sigma_r \varepsilon_{r,t}$, with Gaussian shocks $\varepsilon_{r,t} \overset{iid}{\backsim} \mathcal{N}(0,1)$. We refer to the innovation $\varepsilon_{r,t}$ as a monetary policy shock. We choose to model the log of $\eta_{r,t}$ as a first-order autoregressive process. The alternative would have been to include a lagged nominal interest rate on the right-hand side of equation (9) and specify the $\ln \eta_{r,t}$ as a white noise process. The reason for our modeling choice is that we want to treat symmetrically the three exogenous state variables of the model (i.e., the state of technology a_t, the state of monetary policy $\eta_{r,t}$, and the preference shifter g_t), and therefore, we model each exogenous stochastic process as a first-order autoregression. Furthermore, including lagged endogenous variables turns out to raise the computational burden of solving the model, preventing the likelihood-based estimation conducted in this paper.[4]

The central bank observes the contemporaneous realization of aggregate shocks (i.e., $\varepsilon_{a,t}$, $\varepsilon_{r,t}$, and $\varepsilon_{g,t}$) at stage 0 of every period and sets the interest rate R_t according to equation (9). Note that at stage 0 the central bank knows the output gap (Y_t/Y_t^*) and inflation, albeit not yet realized, because it is assumed to know the model and to observe the history of aggregate shocks. Furthermore, note that the central bank cannot simply tell firms the history of shocks since there is an incentive for the central bank to lie to firms to generate surprise inflation with the aim of raising output growth.[5] Unexpected inflation raises output due to nominal rigidities and dispersed information. This rise in output has benefits because producers have monopoly power and the unexpected inflation reduces the monopoly distortion. Since there is no commitment device that would back up the central bank's words, any central bank statements about real output, inflation, and shocks are not deemed as credible by price setters.

[4] Nimark (2009) introduces a method to improve the efficiency of solution methods for models with dispersed information and lagged endogenous variables.

[5] The fact that the central bank sets the interest rate before firms set their prices cannot be considered as a viable commitment device to communicate current inflation and output to firms. The reason is that the central bank's reaction function (13) makes the interest rate depend on the output gap and inflation only up to the shock $\eta_{r,t}$, which is not observed by firms.

The flow budget constraint of the fiscal authority in period t reads

$$R_{t-1}B_{t-1} - B_t = T_t$$

The fiscal authority has to finance maturing government bonds. The fiscal authority can collect lump-sum taxes or issue new government bonds. Since there is neither capital accumulation nor government consumption, the resource constraint implies $Y_t = C_t$.

2.6 Log-linearization and Model Solution

First, I solve firms' and households' problems, described in Sections 2.3 and 2.4, and obtain the consumption Euler equation and the price-setting equation. Second, I detrend the non-stationary variables before log-linearizing the model equations around their value at the non-stochastic steady-state equilibrium. Let us define the de-trended real output as $y_t \equiv Y_t/\gamma^t$. We denote the log-deviation of an arbitrary (stationary) variable x_t from their steady-state value as $\widehat{x}_t$. From the linearized price-setting equation, one can obtain the imperfect-common-knowledge Phillips curve (Nimark, 2008):

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^k \widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^k \widehat{\pi}_{t+1|t}^{(k)} \tag{10}$$

where $\widehat{\pi}_{t+1|t}^{(k)}$ denotes the average k-th order expectations about the next period's inflation rate, $\widehat{\pi}_{t+1}$, that is, $\widehat{\pi}_{t+1|t}^{(k)} \equiv \underbrace{\int \mathbb{E}_{j,t} \ldots \int \mathbb{E}_{j,t}}_{k} \widehat{\pi}_{t+1} dj...dj$, any integer $k > 1$. $\widehat{mc}_{t|t}^{(k)}$ denotes the average k-th order expectations about the real aggregate marginal costs $\widehat{mc}_t \equiv \int \widehat{mc}_{j,t} dj$, which evolve according to the equation:

$$\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)} \tag{11}$$

any integer $k > 1$. The log-linearized IS equation is standard and reads

$$\widehat{g}_t - \widehat{y}_t = \mathbb{E}_t\widehat{g}_{t+1} - \mathbb{E}_t\widehat{y}_{t+1} - \mathbb{E}_t\hat{\pi}_{t+1} + \hat{R}_t \tag{12}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the complete information set, which includes the history of the three aggregate shocks. The central bank's reaction function (9) boils down to

$$\hat{R}_t = \phi_\pi\hat{\pi}_t + \phi_y(\hat{y}_t - \widehat{y}_t^*) + \widehat{\eta}_{r,t} \tag{13}$$

The preference shifter evolves according to $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \varepsilon_{g,t}$. The process for technology becomes $\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{a,t}$. The process leading the state of monetary policy becomes $\widehat{\eta}_{r,t} = \rho_r \widehat{\eta}_{r,t-1} + \sigma_r \varepsilon_{r,t}$. The signal equation concerning the preference shifter (8) is written as:

$$\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon^g_{j,t} \tag{14}$$

We de-trend and then log-linearize the signal equation concerning the aggregate level of technology (6) and obtain

$$\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon^a_{j,t} \tag{15}$$

The signal about monetary policy is given by equation (13). When solving their price-setting problem, firms have to form expectations about the dynamics of their nominal marginal costs $MC_{j,t}$, the price for the composite good P_t, and the output level of the composite good Y_t. To this end, they solve a signal extraction problem using the log-linearized model equations, which are listed above, and the signal equations (14), (15), and (13).[6] Note that the policy signal is endogenous. A detailed description of how we solve the model is provided in the Appendix. When the model is solved, the law of motion of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]'$ reads:

$$\mathbf{s}_t = \mathbf{v}_0 X^{(0:k)}_{t|t} \tag{16}$$

where $X^{(0:k)}_{t|t} \equiv \left[\widehat{a}^{(s)}_{t|t}, \widehat{\eta}^{(s)}_{r,t|t}, \widehat{g}^{(s)}_{t|t} : 0 \leq s \leq k\right]'$ is the vector of the higher-order expectations (HOEs) about the exogenous state variables (i.e., $\widehat{a}_t, \widehat{\eta}$, and $\widehat{g}_t$) that follows a VAR(1)[7]

$$X^{(0:k)}_{t|t} = \mathbf{M} X^{(0:k)}_{t-1|t-1} + \mathbf{N} \boldsymbol{\varepsilon}_t \tag{17}$$

The parameter set of the log-linearized incomplete information model is given by the vector

$$\Theta_{IIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_g, \rho_a, \rho_r, \widetilde{\sigma}_g, \sigma_g, \sigma_a, \widetilde{\sigma}_a, \sigma_r, \gamma\right]'$$

2.7 The Perfect Information Model (PIM)

If the noise variance of the private exogenous signals, $\widetilde{\sigma}_a$ and $\widetilde{\sigma}_g$, is equal to zero, higher-order uncertainty would fade away (i.e., $X^{(k)}_{t|t} = X_t$, any integer k) and the model would

[6] Observing R_t, as indicated in the information set (7), or $\widehat{R}_t$, as specified in the signal equation (13), does not change the price-setting decisions in that firms are assumed to know the model and, hence, the non-stochastic steady-state, including the nominal interest rate $r_* \pi_*$.

[7] As is standard in the literature (e.g., Woodford, 2002 and Nimark, 2011), we focus on equilibria where the higher-order expectations about the exogenous state variables follow a VAR(1) process. To solve the model we also assume common knowledge of rationality. See Nimark (2008), Assumption 1, p. 373 for a formal formulation of the assumption of common knowledge of rationality in this context.

boil down to a canonical three-equation New Keynesian model with Calvo sticky prices (e.g., Rotemberg and Woodford (1997) and Rabanal and Rubio-Ramirez, 2005). More specifically, the imperfect-common-knowledge Phillips curve (10) would become $\hat{\pi}_t = \kappa_{pc}\widehat{mc}_t + \beta\mathbb{E}_t\widehat{\pi}_{t+1}$, where $\kappa_{pc} \equiv (1-\theta)(1-\theta\beta)/\theta$ and the real marginal costs $\widehat{mc}_t = \hat{y}_t - \widehat{a}_t$. The IS equation and the Taylor rule would be the same as in the incomplete information model. See equations (12) and (13). In this perfect information model, the monetary shock propagates by affecting the intertemporal allocation of consumption. The real effects of money solely emerge as a result of price-stickiness as opposed to the sluggish adjustments of firms' expectations in the incomplete information model. We call this canonical New Keynesian model the perfect information model. The parameter set of the log-linearized perfect information model is given by the vector

$$\Theta_{PIM} = \left[\theta, \phi_\pi, \phi_y, \beta, \rho_g, \rho_a, \rho_r, \sigma_g, \sigma_a, \sigma_r, \gamma\right]'$$

2.8 The Signal Channel of Monetary Transmission

A salient feature of the incomplete information model is that the central bank can transfer information about the output gap and inflation to price setters by setting its policy rate. We call this the signal channel of monetary transmission. Price setters use the policy rate as a signal that helps them to track non-policy shocks (namely, preference and technology shocks) and, at the same time, to infer potential exogenous deviations from the monetary rule (i.e., $\eta_{r,t}$).

In this section, we analyze the signaling effects of monetary policy on the response of inflation to the structural shocks of the model (i.e., demand, technology, and monetary policy shocks). It is illustrative to use equation (16) to decompose the effects of shocks on inflation as follows:

$$\frac{\partial\pi_{t+h}}{\partial\varepsilon_{i,t}} = \mathbf{v}_a \cdot \frac{\partial X^a_{t+h}}{\partial\varepsilon_{i,t}} + \mathbf{v}_m \cdot \frac{\partial X^m_{t+h}}{\partial\varepsilon_{i,t}} + \mathbf{v}_g \cdot \frac{\partial X^g_{t+h}}{\partial\varepsilon_{i,t}} \tag{18}$$

where $i \in \{a, r, g\}$ is the subscript that determines the shock of interest (i.e., supply, monetary, or demand shock) and the row vectors $\mathbf{v}_a, \mathbf{v}_m$, and $\mathbf{v}_g$ are subvectors of the second row of the matrix $\mathbf{v}_0$ in equation (16). X^a_{t+h} is the column vector of h-step-ahead higher-order expectations (HOEs) about the aggregate technology $\widehat{a}_t$, that is, $X^a_{t+h} \equiv \left[\widehat{a}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$.[8] Analogously, $X^m_{t+h} \equiv \left[\widehat{\eta}^{(s)}_{r,t+h|t} : 0 \leq s \leq k\right]$ and $X^g_{t+h} \equiv \left[\widehat{g}^{(s)}_{t+h|t} : 0 \leq s \leq k\right]$ are the column vectors of h-step-ahead higher-order expectations about the state of monetary policy and the preference shifter, respectively.

[8] Conventionally, the average zero-order expectation about a random variable (say, the level of aggregate technology, $\widehat{a}_t$) is equal to the variable itself, that is, $\widehat{a}^{(0)}_{t+h|t} = \widehat{a}_{t+h}$, for any h.

Note that under perfect information, following, say, a monetary shock $\varepsilon_{r,t}$, the vectors $\frac{\partial X^g_{t+h}}{\partial \varepsilon_{r,t}}$ and $\frac{\partial X^a_{t+h}}{\partial \varepsilon_{r,t}}$ will be equal to a vector of zeros. Since firms can observe the nature of shocks, average higher-order expectations about preferences and aggregate technology do not respond to a monetary shock. Furthermore, under perfect information $\frac{\partial X^m_{t+h}}{\partial \varepsilon_{r,t}} = \left(\rho^h_r \sigma_r \varepsilon_{r,t}\right) \mathbf{1}_{(k+1)\times 1}$, where $\mathbf{1}_{(k+1)\times 1}$ is a $(k+1)\times 1$ vector of ones, since it is common knowledge that every firm observes the nature of shocks that have hit the economy. More generally, under perfect information: $\frac{\partial X^i_{t+h}}{\partial \varepsilon_{i,t}} = \left(\rho^h_i \sigma_i \varepsilon_{i,t}\right) \mathbf{1}_{(k+1)\times 1}$ and $\frac{\partial X^j_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{0}_{(k+1)\times 1}$ for all $i, j \in \{a, r, g\}^2$ and $i \neq j$. These restrictions on these vectors of partial derivatives do not hold when firms observe the policy signal, which has the effect of confusing price setters about the nature of disturbances that have hit the economy. For instance, a rise in the policy rate can be interpreted as the central bank's response to a contractionary monetary shock, or to an adverse technology shock, or to a positive preference shock. This happens because the policy signal is the policy rate that responds to endogenous variables (i.e., inflation and the output gap), which are functions of the history of all three structural shocks (i.e., demand, technology, and monetary policy shock). Thus, the signal channel (i.e., the presence of the policy signal, $R_t \in \mathcal{I}_{j,t}$) relaxes the restrictions $\frac{\partial X^j_{t+h}}{\partial \varepsilon_{i,t}} = \mathbf{0}_{(k+1)\times 1}$ for all $i, j \in \{a, r, g\}^2$ and $i \neq j$. In contrast, since private signals are orthogonal, private information does not give rise to any confusion about the nature of the shocks that have hit the economy.

The decomposition in equation (18) allows us to shed light on the signaling effects of monetary policy on the response of inflation to structural shocks. To this end, we conduct a simple numerical experiment. For simplicity, we shut down the preference shock, that is, $\sigma_g = 0$, and set the value of the other parameters as indicated in Table 1.[9] Let us first consider a propagation of a contractionary monetary policy shock when the signal noise $\widetilde{\sigma}_a$ is set to be equal to the standard deviation of the aggregate technology shock, σ_a. The bottom plots of Figure 1 report the response of the average higher-order expectations (HOEs) about aggregate technology $\partial X^a_{t+h}/\partial \varepsilon_{r,t}$ (bottom left plot) and about the state of monetary policy $\partial X^m_{t+h}/\partial \varepsilon_{r,t}$ (bottom right plot) from order 0[10] up to the third order, h periods after the monetary shock.[11] The bottom left plot shows that average expectations about aggregate technology promptly fall into negative territory after a contractionary monetary shock. This means that the rise in the policy rate is mostly interpreted by firms as the central bank responding to a negative technology shock. The bottom right plot shows that average expectations about a monetary shock respond only a little to a monetary shock. As a result, firms expect inflation to go up in the next quarter. See the top right plot of Figure 1.

[9] These values are the ones we will use to center the prior distribution. See Section 3.2.

[10] Conventionally, the average zero-order expectations about a variable are the realization of the variable itself.

[11] Note that since we shut down the preference shock $\partial X^g_{t+h}/\partial \varepsilon_{r,t} = \mathbf{0}_{(k+1)\times 1}$.

The top left plot of Figure 1 shows that the response of inflation to the contractionary monetary shock is negative. Note that the response of inflation to monetary shocks combines both the deflationary effects of the contractionary monetary shock (the solid line in the bottom right plot),[12] and the inflationary effects from the signal channel. The former effects work through the fall in the real interest rate associated with the contractionary monetary shock, as is common to virtually every New Keynesian model with perfect information. The latter effects work through the strongly negative response of the average higher-order expectations about technology, as reported in the bottom left plot. The vertical bars in the top left plot show the effects of the change in the higher-order expectations (HOEs) about technology $\mathbf{v}_a' \partial X_{t+h}^a / \partial \varepsilon_{r,t}$, and those about the state of monetary policy $\mathbf{v}_m' \cdot \partial X_{t+h}^m / \partial \varepsilon_{r,t}$ on inflation, $\widehat{\pi}_t$, h periods after the monetary shock. The top left plot illustrates that the effects of HOEs about aggregate technology on inflation (see the white vertical bars) are sizeable. This happens because the observed rise in the policy rate misleads firms, inducing them to believe that the central bank is reacting to a negative technology shock. Such signaling effect from monetary policy has quite distortive effects on inflation, dampening the fall in inflation owing to a monetary tightening. Furthermore, the signal channel has even more distortive effects on firms' inflation expectations, which turn out to respond positively to monetary shocks. See the top right plot of Figure 1.

Let us turn our attention to the case in which firms have less precise information about the dynamics of aggregate technology. To this end, we set the noise variance so that the signal-to-noise ratio $\sigma_a / \widetilde{\sigma}_a$ is equal to 0.2. As firms become less informed about aggregate technology, the distortive effects of the signal channel on inflation are larger because firms use the policy signal to extract more information about technology shocks. As a result, as illustrated by the solid line in the top left plot of Figure 2, inflation responds positively to a monetary tightening. Two effects lead the signal channel to be so distortive as to bring about a positive response of inflation in the aftermath of a monetary contraction. One effect is related to the fact that firms are poorly informed about technology shocks. When the signal-to-noise ratio $\sigma_a / \widetilde{\sigma}_a$ is small, the policy signal is the only reliable source of information for firms to learn about aggregate technology. The other effect has to do with the informative content of the policy rate, which can be evaluated by looking at the variance decomposition of the Taylor rule. If the variability of the current interest rate (conditional on the past interest rate R_{t-1}) is mostly explained by the technology shock, then firms will mostly rely on the central bank's actions to learn about the state of technology. In the numerical examples we are studying about 91% of the variability of the policy rate stems from the aggregate

[12] In the bottom left plot the average zero-order expectations about the technology shock do not respond to a monetary shock as the two shocks are orthogonal.

technology shock. Hence, firms receive a lot of information about technology shocks from observing the interest rate. When firms have poor private information about the technology shock and the policy rate is very informative about this shock, the distortive effects from the signal channel are very strong, leading inflation to respond positively to a contractionary monetary shock.[13]

The more information about monetary shocks that firms are able to collect from observing the policy rate, the weaker the distortive effects of the signal channel on inflation. The top plots of Figure 3 show what happens when firms are poorly informed about the process of aggregate technology a_t (i.e., $\sigma_a/\widetilde{\sigma}_a = 0.2$) and the state of monetary policy $\eta_{r,t}$ is $\sigma_r = 0.5$ (i.e., five times bigger than that in Table 1). The informative content of the policy rate about aggregate technology shocks is now 45%, as opposed to 91% when σ_r is equal to 0.1. The fact that the policy rate is much less informative about aggregate technology weakens the inflationary distortions from the signal channel and makes a monetary tightening more effective in reducing the inflation rate. The top left plot of Figure 3 illustrates that the inflation rate goes down after a monetary contraction. The signal channel would have even weaker distortive effects and monetary policy would have been even more effective in reducing inflation if firms were more precisely informed about the process of aggregate technology (i.e., $\sigma_a/\widetilde{\sigma}_a$ is large).

The inflationary consequences associated with non-policy shocks also influence the strength of the distortive effects from the signal channel. One parameter that clearly affects the inflationary consequences associated with technology shocks is the policy parameter ϕ_π. If the central bank is known to not effectively fight the inflationary consequences of technology shocks (i.e., small ϕ_π), then optimizing firms will strongly raise their prices whenever they expect a negative technology shock. Thus, the signal channel dampens the fall in inflation or might even cause inflation to rise in the aftermath of a contractionary monetary shock. The bottom plots of Figure 3 show that a more accommodative monetary policy (i.e., $\phi_\pi = 1$) substantially strengthens the inflationary distortions exerted by the signal channel upon the transmission of monetary impulses. This figure shows that inflation responds positively to a monetary tightening when $\phi_\pi = 1$.[14]

[13] The positive response of inflation to a contractionary monetary policy has occasionally been found in empirical work (see Sims, 1992 and, more recently, Hanson, 2004, and Castelnuovo and Surico, 2010) and was dubbed by Eichenbaum (1992) the price puzzle.

[14] Changing the inflation coefficient also affects the informative content of the policy rate. The variance decomposition of the Taylor rule reveals that a fall in the inflation coefficient ϕ_π from two to one raises the information content of the policy rate about technology by about 4%. The reason is that weak responses to inflation tend to raise the variability of inflation in the aftermath of technology shocks. Nonetheless, the impact of more accommodative policy upon the informative content of the policy rate seems to be second order in this numerical example. The direction of the response of inflation is mainly driven by the higher perceived inflationary consequences associated with non-policy shocks, as described in the main text.

To sum up, what we have learned from this numerical exercise is that the functioning of the signal channel is influenced by three factors: (*i*) the quality of private information about non-policy shocks (i.e., technology and demand shocks), (*ii*) the informative content of the policy rate R_t, and (*iii*) the inflation consequences associated with the occurrence of non-policy shocks.

3 Empirical Analysis

This section contains the quantitative analysis of the model. I combine a prior distribution for the parameter set of the three models with their likelihood function and conduct Bayesian inference and evaluation.

Section 3.1 presents the data set and the state-space model for the econometrician. In Section 3.2, we discuss the prior distribution for the model parameters. Section 3.3 presents the posterior distribution. In Section 3.4, we conduct an econometric evaluation of the incomplete information model and the signal channel for monetary transmission. Section 3.5 studies the impulse response functions of the observables (i.e., GDP growth rate, inflation, federal funds rate, and inflation expectations) to an unanticipated monetary shock. Section 3.6 deals with how the signal channel affects the propagation of non-policy shocks, such as the technology shock and the demand shock.

3.1 Econometrician's State-Space model

The data set includes five observable variables: U.S. GDP growth rate, U.S. inflation rate (from the GDP deflator), the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations. The data set ranges from 1970:3 to 2007:4. A detailed description of the data set is provided in Table 2. Data on inflation expectations are obtained from the *Survey of Professional Forecasters* (SPF). The measurement equations are:

$$\left[\ln\left(\frac{GDP_t}{POP_t^{\geq 16}}\right) - \ln\left(\frac{GDP_{t-1}}{POP_{t-1}^{\geq 16}}\right)\right] \cdot 100 = 100 \ln \gamma + \hat{y}_t - \hat{y}_{t-1}$$

$$100 \ln \frac{PGDP_t}{PGDP_{t-1}} = 100 \ln \pi_* + \hat{\pi}_t$$

$$100 \cdot FEDRATE_t = \hat{R}_t + 100 \ln R_*$$

$$\ln\left(\frac{PGDP3_t}{PGDP2_t}\right) 100 = \hat{\pi}^{(1)}_{t+1|t} + 100 \ln \pi_* + \sigma_{m_1} \varepsilon_t^{m_1}$$

$$\ln\left(\frac{PGDP6_t}{PGDP2_t}\right)25 = \hat{\pi}^{(1)}_{t+4|t} + 100\ln\pi_* + \sigma_{m_2}\varepsilon^{m_2}_t$$

where $PGDP2_t$, $PGDP3_t$, $PGDP6_t$ are the SPF's mnemonics for the forecasts about the current, one-quarter-ahead, and four-quarters-ahead GDP price index. We relate these statistics with the first moment of the distribution of firms' expectations implied by the model. To avoid stochastic singularity, we introduce two i.i.d. Gaussian measurement errors $\varepsilon^{m_1}_t$ and $\varepsilon^{m_2}_t$. Furthermore, these errors are meant to capture the difference between the measured expectations from the SPF and their model concepts, $\hat{\pi}^{(1)}_{t+1|t}$ and $\hat{\pi}^{(1)}_{t+4|t}$.

3.2 Priors

The prior medians and the 95% credible intervals are reported in Table 3. At the steady state the discount factor β depends on the linear trend of real output γ and the steady-state real interest rate R_*/π_*. Hence, I fix the value for this parameter so that the steady-state nominal interest rate R_* matches the sample mean of the $FEDRATE_t$ in the sample. Note that the prior medians for the variance of the idiosyncratic technology $a_{j,t}$ and that of the private signal concerning the preference shifter are set so that the model can match the cross-sectional variance of the expectations about current inflation and output as reported in the *Survey of Professional Forecasters.* The prior for the standard deviation of technology shock, σ_a, is centered at 0.70, which is consistent with the real business cycle literature. The prior distribution puts a probability mass to values for the Calvo parameter θ, implying that firms adjust their prices about every three quarters. This belief is derived from micro studies on price setting (Bils and Klenow, 2004, Klenow and Kryvtsov, 2008, Nakamura and Steinsson, 2008, and Klenow and Malin, 2010). The priors for the autoregressive parameters ρ_a, ρ_r, and ρ_g reflect the belief that the corresponding exogenous processes may exhibit sizeable persistence as is usually observed in the macroeconomic data. Nonetheless, these priors are broad enough to accommodate a wide range of persistence degrees for these exogenous processes. Priors for the parameters of the central bank's reaction function (i.e., response to inflation, ϕ_π, response to economic activity, ϕ_y, autoregressive parameter, ρ_r, and the standard deviation of the i.i.d. monetary shock, σ_r) are chosen as follows. The priors for ϕ_π and ϕ_y are centered at 2.00 and 0.25, respectively, and imply a fairly strong response to inflation and the output gap. The volatility of the monetary policy shock, σ_r, and the demand shock, σ_g is informally taken according to the rule proposed by Del Negro and Schorfheide (2008) that the overall variance of endogenous variables is roughly close to that observed in the pre-sample, ranging from 1960:1 to 1970:2. The prior median for the measurement errors (i.e., σ_{m1}, σ_{m2}) is set so as to match the variance of inflation expectations reported in the *Livingston Survey.*

3.3 Posteriors

As explained in Fernández-Villaverde and Rubio-Ramírez (2004), a closed-form expression for the posterior distribution is not available, but we can approximate the moments of the posterior distributions via the Metropolis-Hastings algorithm. We obtain $100,000$ posterior draws. The posterior moments for the parameters of the incomplete information model (IIM) and the perfect information model (PIM) are reported in Table 4. As far as the IIM is concerned, the posterior median for the Calvo parameter θ implies very flexible price contracts, which is in line with what is found in micro studies (Bils and Klenow, 2004, Klenow and Kryvtsov, 2008, Nakamura and Steinsson, 2008, and Klenow and Malin, 2010). The posterior median for the autoregressive parameters ρ_a and ρ_g is larger than what is conjectured in the prior. In particular, the autoregressive parameter of technology is close to unity, suggesting that the process of technology is almost a unit root. The posterior median for the variance of the firm-specific technology shock $\widetilde{\sigma}_a$ implies that the signal-to-noise ratio $\sigma_a/\widetilde{\sigma}_a$ is very close to unity. The posterior median for the signal-to-noise ratio $\sigma_g/\widetilde{\sigma}_g$ is smaller than unity, suggesting that firms are less informed about the preference shocks than about the aggregate technology shocks. These estimates imply that, *ceteris paribus*, firms will rely more on the policy signal to learn preference shocks than aggregate technology shocks, since their private signals on the former is relatively less precise. The posterior median for the inflation coefficient of the Taylor rule, ϕ_π, is substantially smaller than its prior median. As discussed in Section 2.8, an accommodative monetary policy raises the inflationary consequences of shocks and, hence, strengthens the distortive effects from the signal channel on inflation. The posterior median for the variance of the monetary shock σ_r is bigger than that conjectured in the prior by a factor of six. As observed in Section 2.8, the larger variance of monetary shocks makes the policy signal $\widehat{R}_t$ less informative about non-policy shocks (i.e., preference and aggregate technology shocks) and, hence, tends to weaken the distortive effects of the signal channel on inflation.

3.4 Model Evaluation

To shed light on the empirical relevance of the signal channel, in Section 3.4.1, we investigate the incomplete information model's ability to fit the data relative to the perfect information model, in which monetary policy does not have signaling effects. Furthermore, in Section 3.4.2, we assess how well the incomplete information model fares at fitting the observed inflation expectations (i.e., the SPF) relative to the perfect information model. Since the signal channel imposes tight restrictions on how the policy rate influences average inflation expectations (see Figures 1-3), this exercise is very informative about whether the channel is empirically relevant.

3.4.1 Marginal Data Density Comparison

Bayesian tests for non-nested models rely on computing the marginal data density (MDD). The marginal data density is needed for updating prior probabilities over a given set of models. Denote the data set, presented in Section 3.1, as Y. The MDD associated with the incomplete information model is defined as $P(Y|\mathcal{M}_{IIM}) = \int \mathcal{L}(Y|\Theta_{IIM})\, p(\Theta_{IIM})\, d\Theta_{IIM}$, where $\mathcal{L}(Y|\Theta_{IIM})$ denotes the likelihood function derived from the model and $p(\Theta_{IIM})$ is the prior density, whose moments are described in Section 3.2.

A Bayesian test of the null hypothesis that the incomplete information model is at odds with the data can be performed by comparing the MDDs associated with this model ($\mathcal{M}_{IIM}$) and the perfect information model ($\mathcal{M}_{PIM}$). Under a $0-1$ loss function the test rejects the null that the incomplete information model is at odds with the data, if the incomplete information model has a larger posterior probability than the alternative model, namely, the perfect information model (Schorfheide, 2000). The posterior probability of the model $\mathcal{M}_s$, where $s \in \{IIM, PIM\}$, is given by:

$$\pi_{T,\mathcal{M}_s} = \frac{\pi_{0,\mathcal{M}_s} \cdot P(Y|\mathcal{M}_s)}{\sum_{s \in \{IIM,PIM\}} \pi_{0,\mathcal{M}_s} \cdot P(Y|\mathcal{M}_s)} \tag{19}$$

where $\pi_{0,\mathcal{M}_s}$ stands for the prior probability of the model $\mathcal{M}_s$. $P(Y|\mathcal{M}_s)$ is the MDD associated with the model $\mathcal{M}_s$. We use Geweke's harmonic mean estimator (Geweke, 1999) to estimate the marginal data density. Table 5 shows that the incomplete information model attains a larger posterior probability and hence the null can be rejected. The null hypothesis can be rejected unless the prior probability in favor of the incomplete information model (i.e., $\pi_{0,\mathcal{M}_{IIM}}$) is as small as $8.5E-7$. Such a low prior probability suggests that only if one has extremely strong *a priori* information against the incomplete information model, one can conclude that the null cannot be rejected.

3.4.2 Predicting the Observed Inflation Expectations

The top plot in Figure 4 reports the one-quarter-ahead inflation expectations (left plots) and the four-quarters-ahead inflation expectations (right plots) predicted by the IIM and the PIM estimated without including the observed inflation expectations (i.e., the data set for estimation includes only the U.S. GDP growth rate, the U.S. inflation rate, and the federal funds rate).[15] This predicted path is compared with the observed inflation expectations obtained from the SPF. Analogously, the bottom plot reports the predictive path from the perfect information model and compares it with the SPF. These plots shed light on how well the incomplete information model fits the observed inflation expectations relatively to the perfect information model. Since the signal channel relies on affecting inflation expectations, it is very important to assess whether the incomplete information model delivers empirically consistent predictions for the inflation expectations. It can be observed that the incomplete information model produces much smoother inflation expectations than the perfect information model. Data on inflation expectations are quite smooth, hence favoring the incomplete information model.

A synthetic measure of the models' ability to fit the observed inflation expectations is the RMSEs associated with their models' predictions. This statistic is reported in Table 6 for the incomplete information model (IIM) and the perfect information model (PIM). The table considers both the full sample and the first part of the sample, which has been characterized by the larger volatility of the observed inflation expectations. For both samples and for both observables, the incomplete information model does better than the perfect information model at fitting the observed inflation expectations.

3.5 Propagation of Monetary Shocks

Figure 5 shows the impulse response functions (and their 95% posterior credible sets in gray) of GDP, the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a 25-basis-point rise in the interest rate. The responses are reported as deviations from the balanced-growth path in units of percentage points of annualized rates. Two features of these impulse response functions have to be emphasized. First, four-quarters-ahead inflation expectations respond positively to a monetary policy shock. Second, inflation and especially inflation expectations seem to react very sluggishly to shocks, although the estimated average duration of the price contract is very short (the posterior median for the Calvo parameter θ is 0.46). The latter result is in line with findings in Woodford (2002), Nimark (2008), and Melosi (2010).

The vertical bars in the top left plot of Figure 6 dissect the response of inflation to a monetary shock into the effect of average higher-order expectations about monetary policy, the aggregate technology, and the preference shifter. We observe a large effect of the higher-order expectations about the preference shifter, which can be interpreted as a situation in which price setters mistakenly believe that the interest rate has been raised in response to a

[15] For the sake of brevity, the parameter estimates of this exercise are not reported but are available upon request.

positive preference shock. Such a mistaken interpretation gives rise to inflationary pressures (captured by the light gray bars in the graph), which dampens the deflationary consequences (i.e., the dark blue bars in the graph) that are usually associated with contractionary monetary shocks in perfect information models.

We report the response of the higher-order expectations about the three exogenous state variables (i.e., the aggregate technology $\widehat{a}_t$, the state of monetary policy $\widehat{\eta}_{r,t}$, and the preference shifter $\widehat{g}_t$) in the other three plots of Figure 6. Average first-order expectations about aggregate technology go down only moderately (by around 27% of the posterior median of σ_a). Average first-order expectations about the preference shifter rise by 50% of the posterior median of σ_g. The latter is quite a substantial deviation from the zero level, which can be explained by the following two facts. First, firms have relatively less precise private information about the preference shock: The posterior medians for the signal-to-noise ratios $\sigma_a/\widetilde{\sigma}_a$ and $\sigma_g/\widetilde{\sigma}_g$ are 0.95 and 0.62, respectively. This implies that firms have to rely more on the policy signal to learn about preference shocks. Second, the posterior variance decomposition of the Taylor rule (conditional on $\widehat{R}_{t-1}$), which is reported in Table 7, shows that the policy signal is relatively more informative about preference shocks than about aggregate technology shocks. Therefore, price setters interpret a rise in the policy rate as the central bank's response to a positive preference shock. This implies that the signal channel dampens the effects of a monetary disturbance on inflation.

Note that expecting an adverse technology shock, which is not realized, gives rise to disinflationary consequences (see the white bars lying in negative territory in Figure 6). This result emerges because of both the high persistence of aggregate technology shocks and the short average duration of price contracts. Both features prompt firms to anticipate a sharp fall in demand that is expected to depress the developments of their future real marginal costs.

To sum up, the data suggest that the signal channel has the effect of mitigating the fall in the inflation rate and pushing inflation expectations up in the aftermath of a monetary tightening. The reason is that firms tend to attach a non-negligible probability that the central bank has adjusted the policy rate to react to a demand shock.

3.6 Propagation of Non-Policy Shocks

Figure 7 shows the response of GDP, the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a one-standard-deviation negative technology shock. Given that the technology shock is almost unit root, the response of variables exhibits high persistence. Figure 8 plots the decomposition of the response of inflation to a negative technology shock (top left plot) and the response of average higher-order expectations about the three exogenous state variables (i.e., $\widehat{a}_t$, $\widehat{\eta}_{r,t}$, and $\widehat{g}_t$) to the shock. Let us focus, first, on the response of the average higher-order expectations (i.e., the top right plot and the bottom plots). As discussed in Section 2.8, the signaling effects from monetary policy confuse firms about shocks that have not occurred. A rise in the policy rate owing to an adverse technology shock may induce firms to believe that the central bank is responding to a contractionary monetary shock or to a positive demand shock. See the bottom plots of Figure 8 showing that average expectations about the state of monetary policy and the preference shifter respond to technology shocks. If firms are mostly persuaded that a contractionary monetary shock has occurred, then the confusion generated by the signal channel would be a good thing from the perspective of a central bank that wants to limit the response of inflation to technology shocks. Firms' inflation expectations would go down and, hence, the technology shock would have a smaller impact on inflation. However, if the monetary tightening mostly led firms to believe that a positive demand shock has hit the economy, the opposite de-stabilizing effect would prevail. Firms' inflation expectations would increase and inflation would go up.

The top left plot in Figure 8 shows that the response of average expectations about the state of monetary policy (i.e., the dark blue bars) and that about the preference shifter (i.e., the light gray bars) contribute to the adjustment of inflation by similar amounts. Thus, the two effects of the confusion caused by the signal channel on inflation turn out to virtually cancel each other out. This implies that the signal channel has basically very limited effects on the Federal Reserve's ability to stabilize inflation in the aftermath of a technology shock.

The propagation of a preference shock is described in Figure 9. Note that the response of inflation and that of inflation expectations are hump-shaped. Figure 10 sheds light on how the signal channel affects the propagation of preference shocks. There are two effects to be emphasized. First, the signal channel confuses firms, inducing them to believe that a contractionary monetary shock has prompted the central bank to raise the policy rate (see the dark blue bars). Second, the signal channel also leads firms to believe that a negative technology shock might be the reason behind the observed rise in the interest rate (see the white bars). Both effects push inflation expectations down and, hence, limit the adjustment of inflation after a positive preference shock. Note also that while the second effect (captured by the white bars in the top left plot of Figure 10) has a very limited impact on inflation, the first effect (captured by the dark blue bars) seems to substantially contribute to pushing inflation down. Therefore, the signal channel enhances the Federal Reserve's ability to stabilize inflation in the aftermath of a preference shock because it partially induces firms to believe that a contractionary monetary shock has occurred.

4 Concluding Remarks

The paper studies a DSGE model in which incompletely informed price setters use the interest rate set by the central bank to infer the nature of shocks that have hit the economy. Since there is a coordination motive in price setting and price setters observe private signals, the model features dispersed information and higher-order uncertainty. In this model, monetary impulses propagate through two channels: the traditional New Keynesian channel based on price stickiness and the signal channel. The latter arises because changing the policy rate conveys information about inflation and the output gap to price setters.

The paper fits the model to a data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. The paper performs a formal econometric evaluation of the model and finds empirical support for the signal channel of monetary transmission. After having established the empirical importance of the new channel, the paper turns to studying how the signal channel affects the propagation of demand, supply, and monetary shocks. We find that firms interpret an observed interest rate rise as the central bank's response to either a positive demand shock or a contractionary monetary shock. Firms, however, do not sensibly change their expectations about aggregate technology shocks after observing a monetary tightening as they hold fairly accurate private information about aggregate technology shocks and the policy rate turns out not to be particularly informative about this type of shock. The paper shows that this finding implies that the Federal Reserve has limited ability to counter the inflationary consequences of technology shocks. In contrast, the signal channel turns out to improve the effectiveness of monetary policy stabilization in the face of demand shocks. Furthermore, the model with incomplete information features fairly sluggish price adjustments (i.e., a quite flat Phillips curve), while the average duration of price contracts is in line with the micro-evidence on price changes.

In the model, the central bank communicates with price setters only by setting the policy rate. The central bank cannot vocally communicate the state of the economy to price setters because any announcement is not regarded as truthful by price setters. However, it seems that market participants react to the central bank's announcements in practice. Empirically assessing the macroeconomic effects of monetary policy communication in a DSGE model with dispersed information is beyond the scope of this paper but is an interesting venue for future research. Furthermore, estimating a DSGE model where both households and firms have incomplete information is a challenging but fascinating topic for future research. Expanding the analysis to a monetary DSGE model of larger scale (e.g., Christiano, Eichenbaum, and Evans, 2005 and Smets and Wouters, 2007), albeit computationally very challenging, would be of great importance.

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Tables and Figures

Table 1: Baseline Calibration for the Numerical Exercise

Name	Value	Name	Value
θ	0.65	ρ_a	0.85
ϕ_π	2.00	σ_a	0.70
ϕ_y	0.50	σ_r	0.10
ρ_r	0.50		

Table 2: Observables

Variables	**Description**	**Source**
GDP_t	Gross Domestic Product - Quarterly	BEA (GDPC96)
$POP_t^{\geq 16}$	Civilian noninstitutional population - 16 years and over	BLS (LNS10000000)
$PGDP_t$	Consumer Price Index - Averages of Monthly Figures	BLS (CPIAUCSL)
$FEDRATE_t$	Effective Federal Funds Rate - Averages of Daily Figures	Board of Governors (FEDFUNDS)
$PGDP2_t$	Mean of Expectations of current GDP price index	SPF in mean.xls (PGDP2)
$PGDP3_t$	Mean of Expectations of one-quarter-ahead GDP price index	SPF in mean.xls (PGDP3)
$PGDP6_t$	Mean of Expectations of one-year-ahead GDP price index	SPF in mean.xls (PGDP6)

Table 3: Prior Distributions

Name	Support	Density	Median	95% Credible Set
θ	$[0, 1]$	Beta	0.65	$(0.28, 0.99)$
ϕ_π	$\mathbb{R}+$	Gamma	2.0	$(1.61, 2.40)$
ϕ_y	$\mathbb{R}+$	Gamma	0.25	$(0.00, 0.65)$
ρ_r	$[0, 1]$	Beta	0.50	$(0.15, 0.90)$
ρ_a	$[0, 1]$	Beta	0.85	$(0.30, 0.99)$
ρ_g	$[0, 1]$	Beta	0.50	$(0.15, 0.90)$
σ_a	$\mathbb{R}+$	InvGamma	0.70	$(0.35, 1.70)$
$\widetilde{\sigma}_a$	$\mathbb{R}+$	InvGamma	1.40	$(0.95, 2.20)$
σ_g	$\mathbb{R}+$	InvGamma	1.00	$(0.50, 2.40)$
$\widetilde{\sigma}_g$	$\mathbb{R}+$	InvGamma	1.00	$(0.67, 1.55)$
σ_r	$\mathbb{R}+$	InvGamma	0.10	$(0.05, 0.85)$
σ_{m_1}	$\mathbb{R}+$	InvGamma	0.45	$(0.22, 1.10)$
σ_{m_2}	$\mathbb{R}+$	InvGamma	0.45	$(0.22, 1.10)$
$\ln\gamma$	$\mathbb{R}$	Normal	0.00	$(-0.20, 0.20)$
$\ln\pi_*$	$\mathbb{R}$	Normal	0.00	$(-0.20, 0.20)$

Table 4: Posterior Distributions

Name	IIM			PIM		
		95% Interval			95% Interval	
	Median	Lower	Upper	Median	Lower	Upper
θ	0.46	0.39	0.53	0.61	0.57	0.65
ϕ_π	1.07	1.03	1.12	1.33	1.22	1.45
ϕ_y	0.25	0.17	0.33	0.24	0.16	0.35
ρ_r	0.71	0.66	0.75	0.49	0.42	0.55
ρ_a	0.99	0.98	0.99	0.98	0.97	0.99
ρ_g	0.77	0.74	0.80	0.84	0.81	0.87
σ_a	1.10	0.94	1.26	1.03	0.92	1.16
$\widetilde{\sigma}_a$	1.14	0.90	1.40	NA	NA	NA
σ_g	1.21	1.05	1.31	0.81	0.67	0.95
$\widetilde{\sigma}_g$	1.57	0.94	2.52	NA	NA	NA
σ_r	0.61	0.50	0.70	0.57	0.50	0.65
σ_{m_1}	0.16	0.15	0.19	0.19	0.17	0.22
σ_{m_2}	0.16	0.14	0.18	0.18	0.16	0.21
$100\ln\gamma$	0.32	0.28	0.35	0.31	0.26	0.34
$100\ln\pi_*$	0.80	0.62	0.99	0.81	0.59	1.01

Table 5: Marginal-Data-Density Comparisons

	$\mathcal{M}_{IIM}$	$\mathcal{M}_{PIM}$
MDD	-252.3	-266.3

Table 6: Forecasting Performance of the Smoothed Estimates

	RMSEs			
	1Q-ahead SPF		4Q-ahead SPF	
	IIM	PIM	IIM	PIM
1970:3-1986:4	1.18	1.49	1.25	1.75
Full Sample	0.90	1.18	0.97	1.34

Note: The table provides the root mean squared errors (RMSEs) for the smoothed estimates of the inflation expectations

Table 7: Informative Content of the Public Signal at the Posterior Medians of the IIM

	$\varepsilon_{a,t}$	$\varepsilon_{r,t}$	$\varepsilon_{g,t}$
Informative Content of R_t	26.73%	35.13%	38.14%

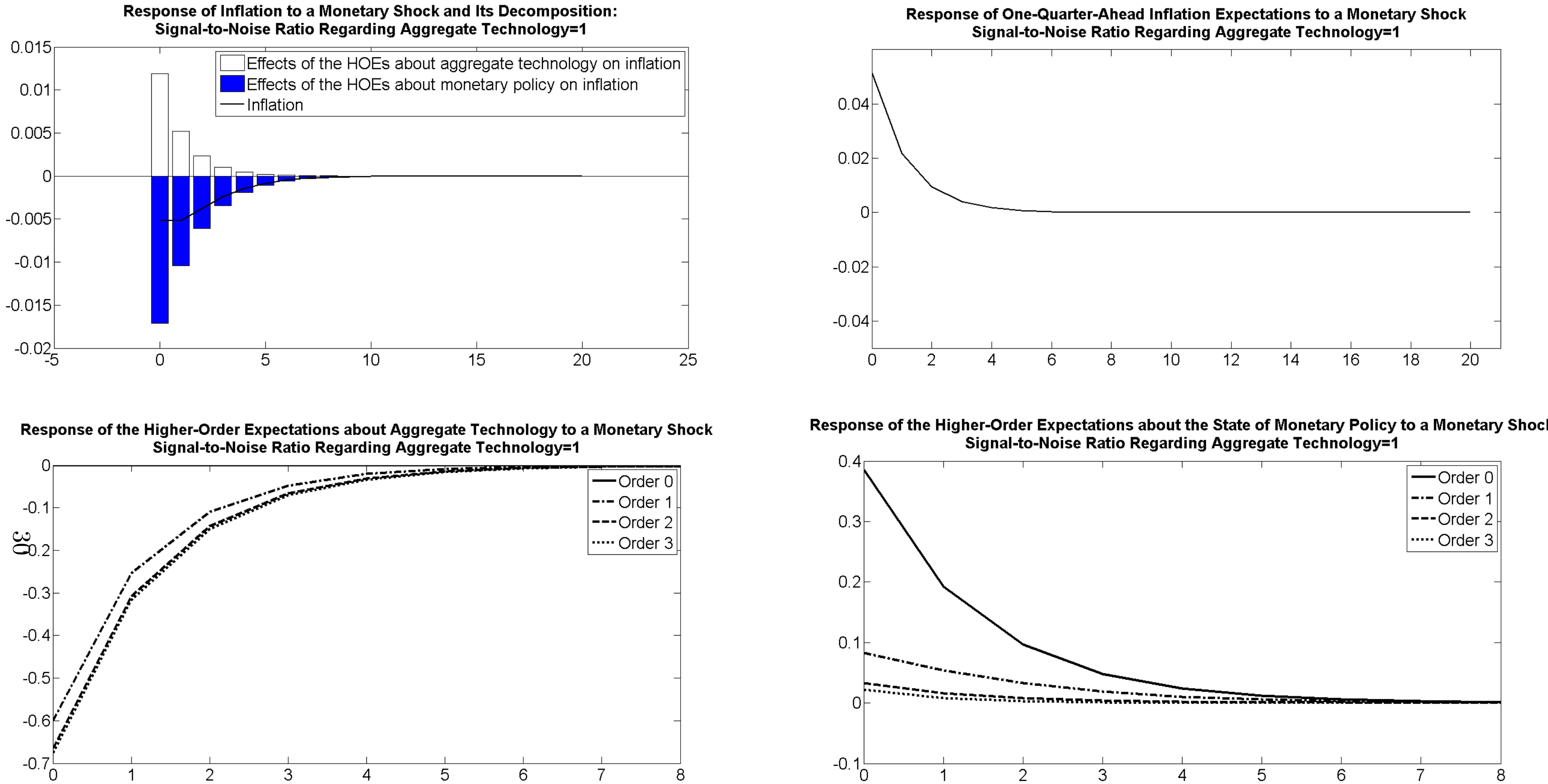

Figure 1: The top left plot reports the response of inflation to a monetary shock. The top right plot reports the response of one-quarter-ahead inflation expectations to a monetary shock. The bottom left panel reports the response of the average expectations about technology from order 0 to 3 to a monetary shock. The bottom right panel reports the response of the average expectations about the state of monetary policy from order 0 to 3 to a monetary shock. The monetary shock is normalized so that it produces a 25-basis-point rise in the interest rate. Conventionally, the average zero-order expectations about a variable are the realization of the variable itself. Hence, in the bottom left plot the average zero-order expectations about the technology shock do not respond to a monetary shock as the two shocks are orthogonal.

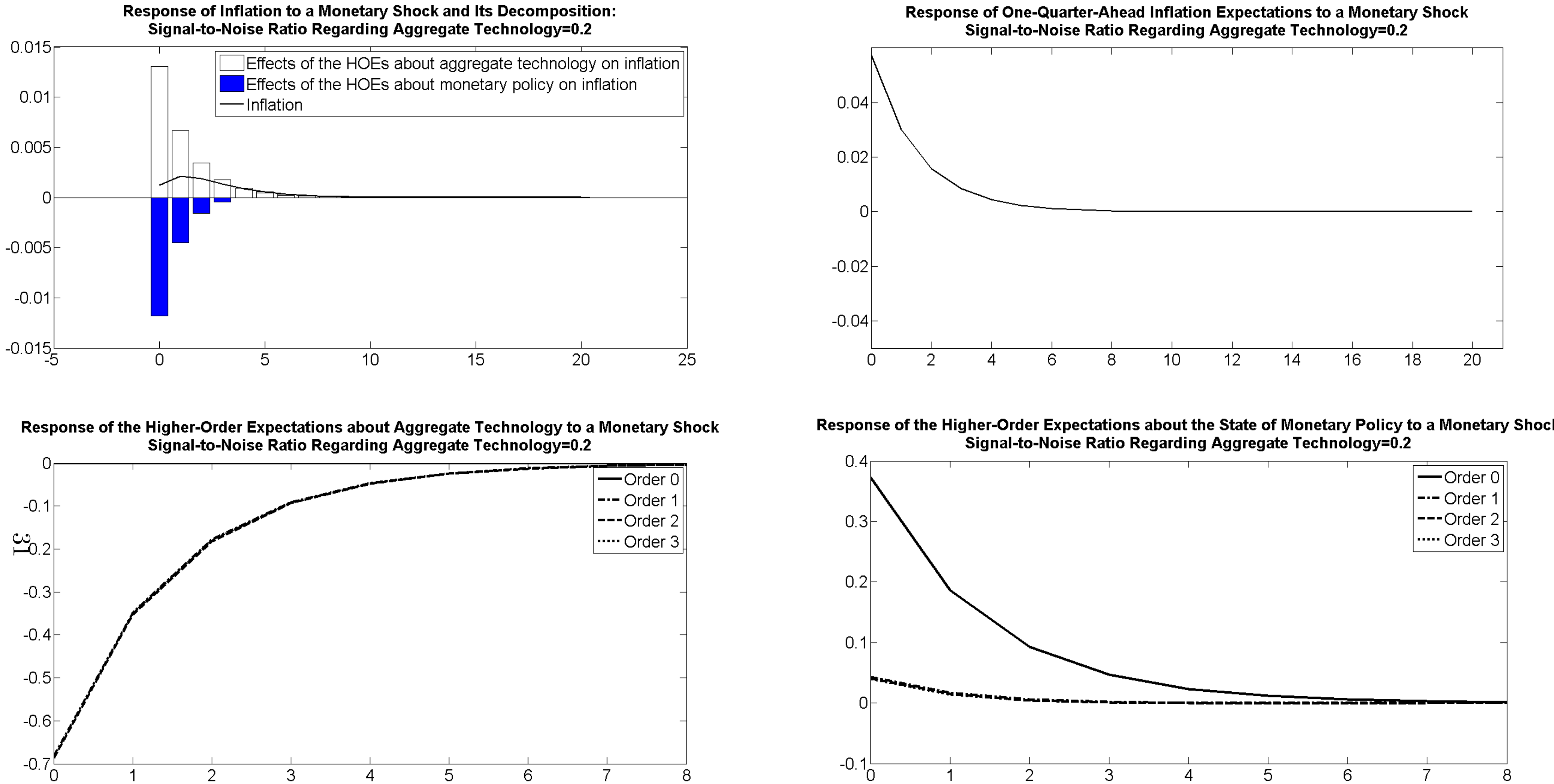

Figure 2: The top left plot reports the response of inflation to a monetary shock. The top right plot reports the response of one-quarter-ahead inflation expectations to a monetary shock. The bottom left panel reports the response of the average expectations about technology from order 0 to 3 to a monetary shock. The bottom right panel reports the response of the average expectations about the state of monetary policy from order 0 to 3 to a monetary shock. The monetary shock is normalized so that it produces a 25-basis-point rise in the interest rate. Conventionally, the average zero-order expectations about a variable are the realization of the variable itself. Hence, in the bottom left plot the average zero-order expectations about the technology shock do not respond to a monetary shock as the two shocks are orthogonal.

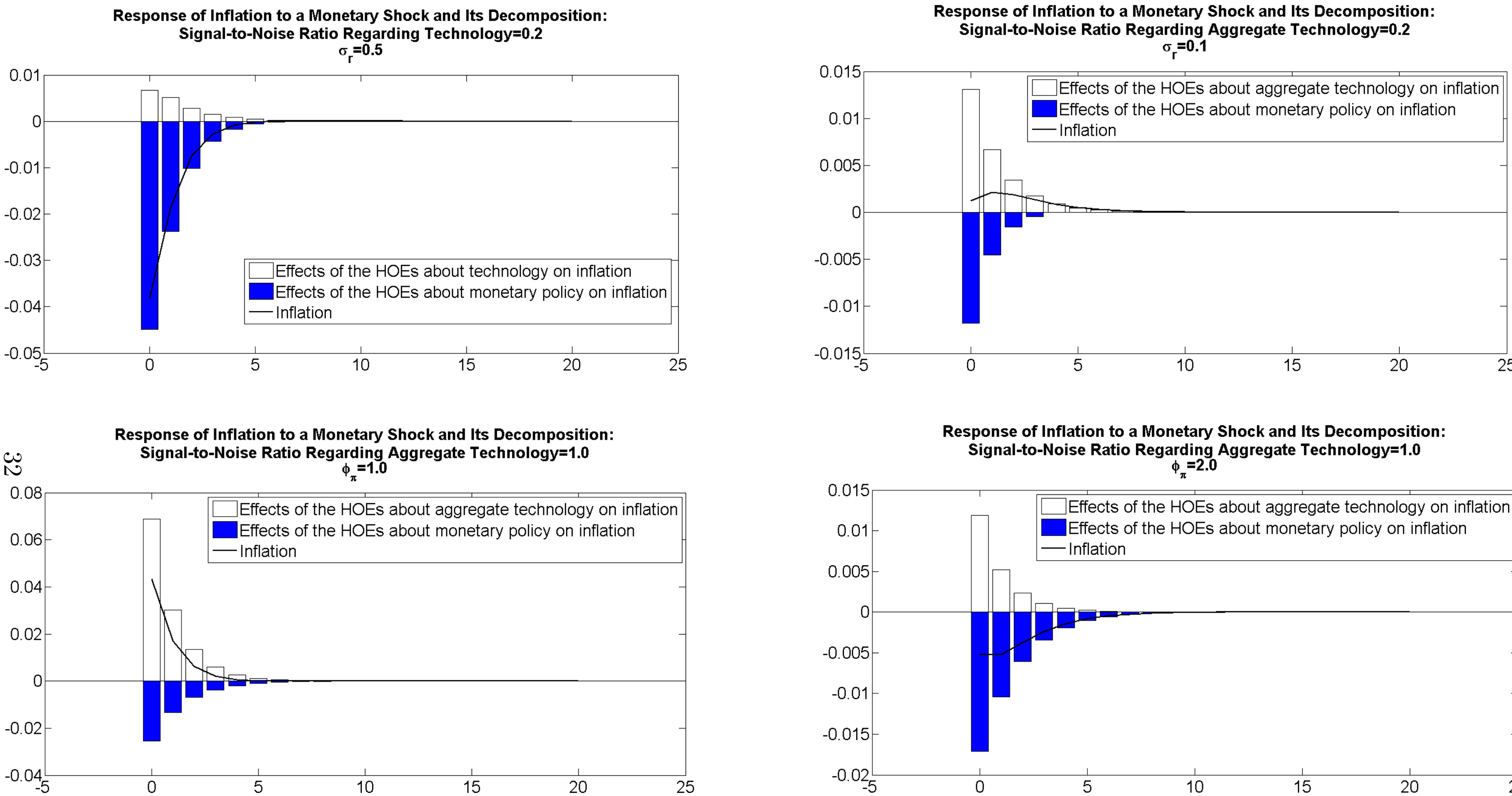

Figure 3: The plots report the response of inflation and its decomposition to a monetary policy shock for the specified parameterization. The monetary shock is normalized so that it produces a 25-basis-point rise in the interest rate.

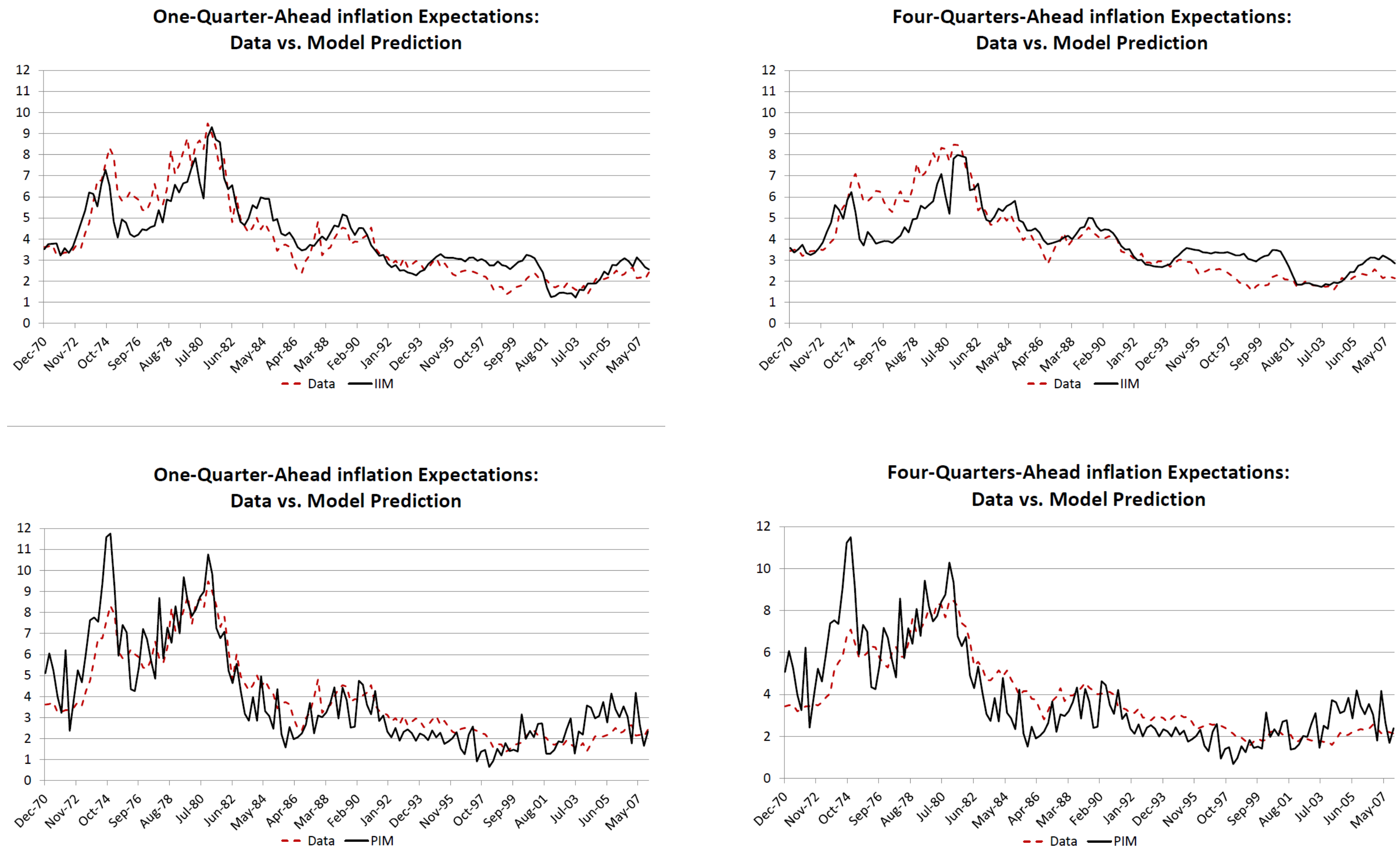

Figure 4: Comparison of the smoothed estimates of the one-quarter-ahead and four-quarters-ahead inflation expectations of the IIM (top plots) and the PIM (bottom plots), which are estimated to a data set that does not include the SPF, with the data

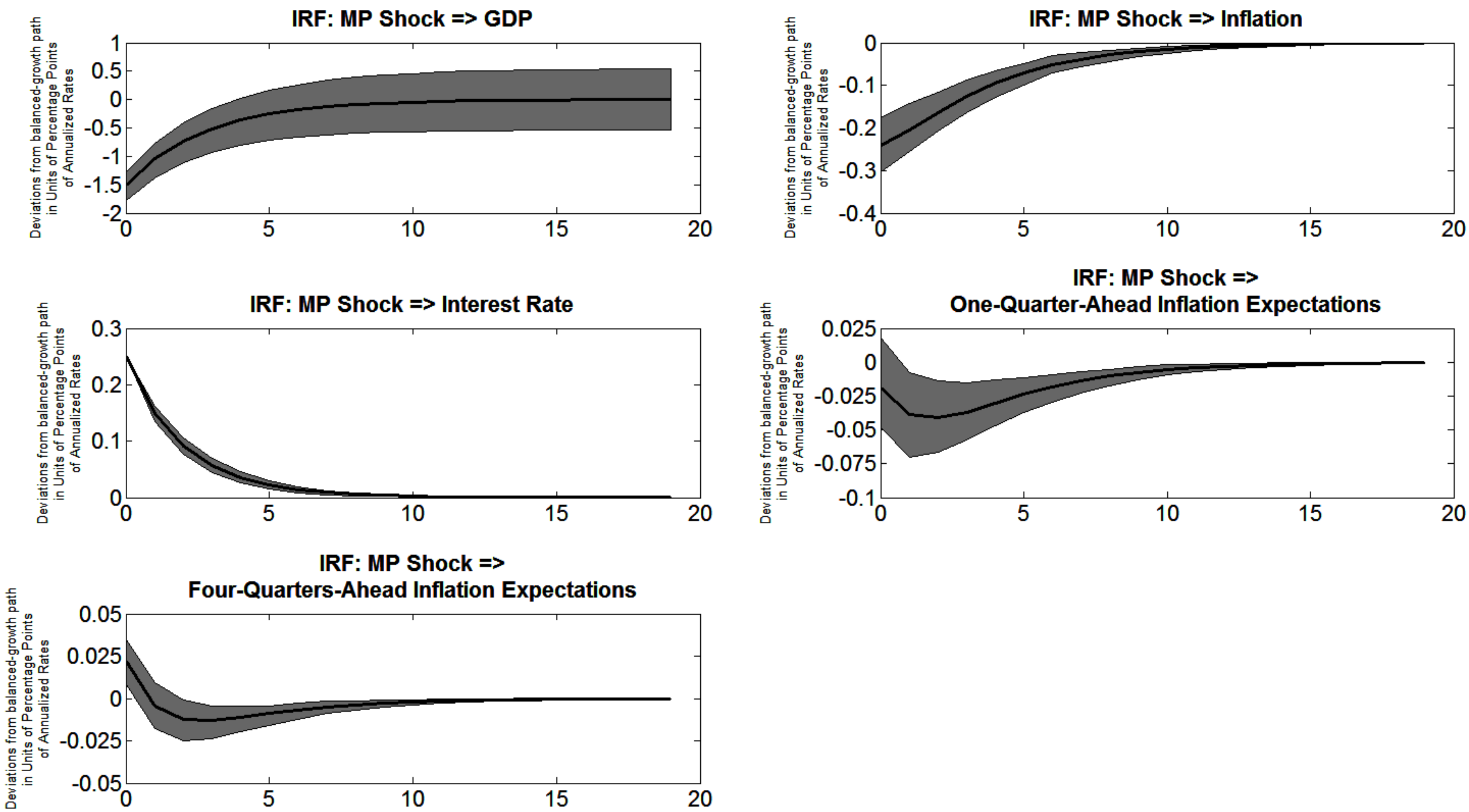

Figure 5: Impulse response functions of the observables to a one-standard-deviation monetary policy shock and their 95 percent posterior credible sets.

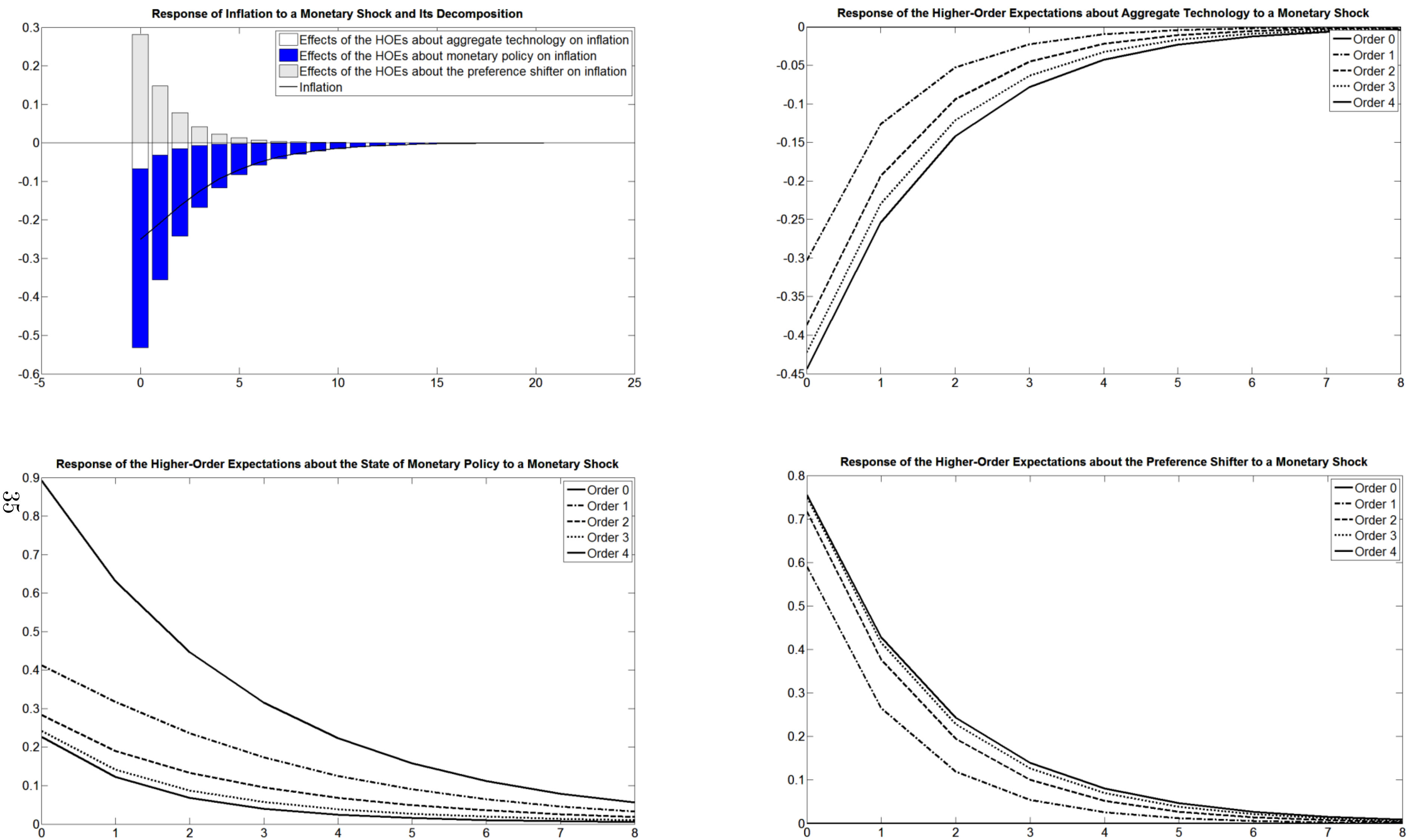

Figure 6: Impulse Response Functions of Inflation to a Monetary Shock and its Decompositions. The top left plot reports the response of inflation and its decomposition to the effects of the HOEs about the three exogenous state variables on inflation. The top right plot reports the response of the higher-order expectations about the aggregate technology to the monetary shock. The bottom left plot reports the response of the higher-order expectations about the state of monetary policy to the monetary shock. The bottom right plot reports the response of the higher-order expectations about the preference shifter to the monetary shock. The size of the initial shock is normalized so that it produces a 25-basis-point rise in the interest rate. All the figures are obtained by evaluating the model at the posterior median.

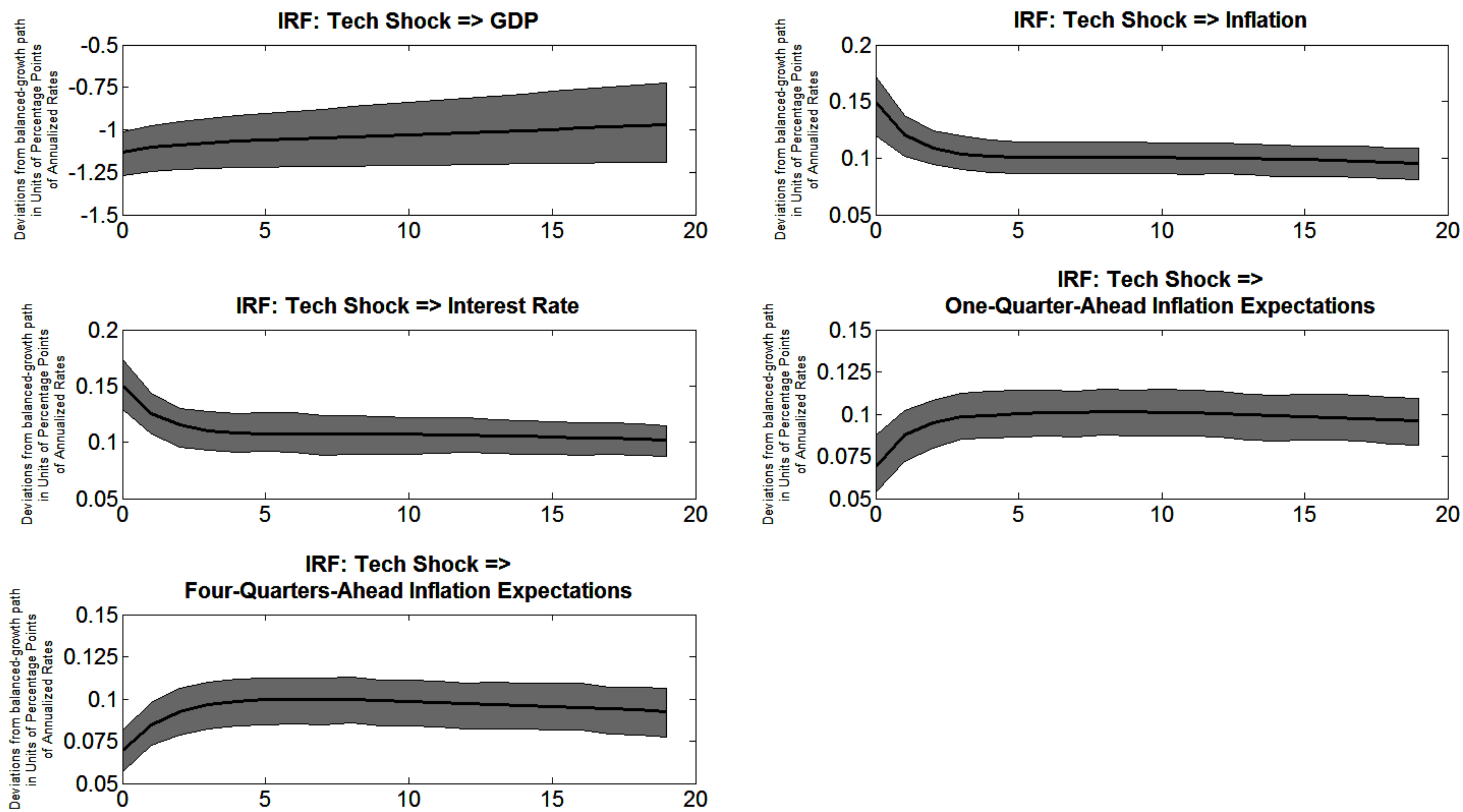

Figure 7: Impulse Response Functions of the Observables to a One-Standard-Deviation Aggregate Technology Shock and Their 95 Percent Posterior Credible sets.

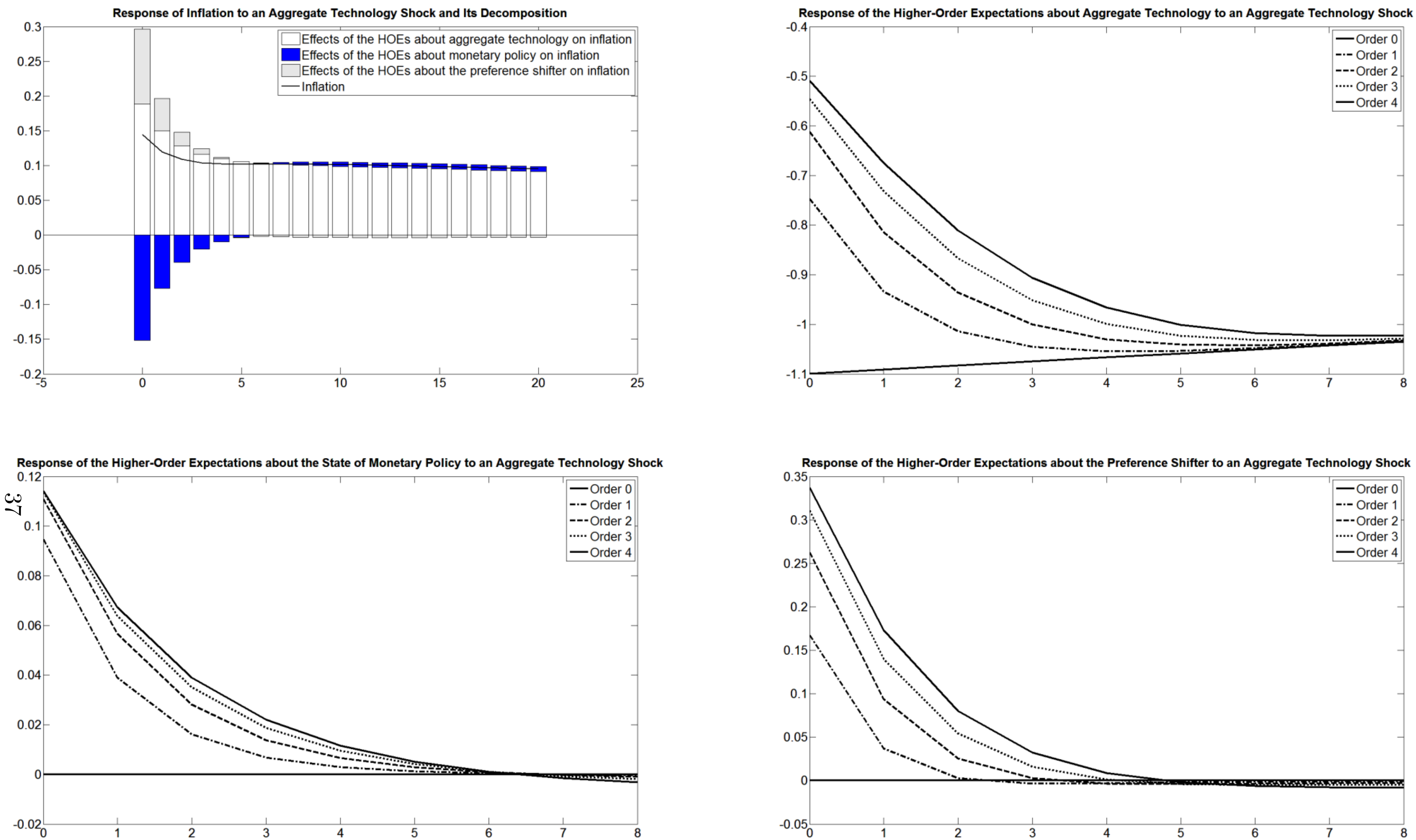

Figure 8: Impulse Response Functions of Inflation to a Two-Standard-Deviation Aggregate Technology Shock and its Decompositions. The top left plot reports the response of inflation and its decomposition to the effects of the HOEs about the three exogenous state variables on inflation. The top right plot reports the response of the higher-order expectations about the aggregate technology to the technology shock. The bottom left plot reports the response of the higher-order expectations about the state of monetary policy to the technology shock. The bottom right plot reports the response of the higher-order expectations about the preference shifter to the technology shock. All the figures are obtained by evaluating the model at the posterior median.

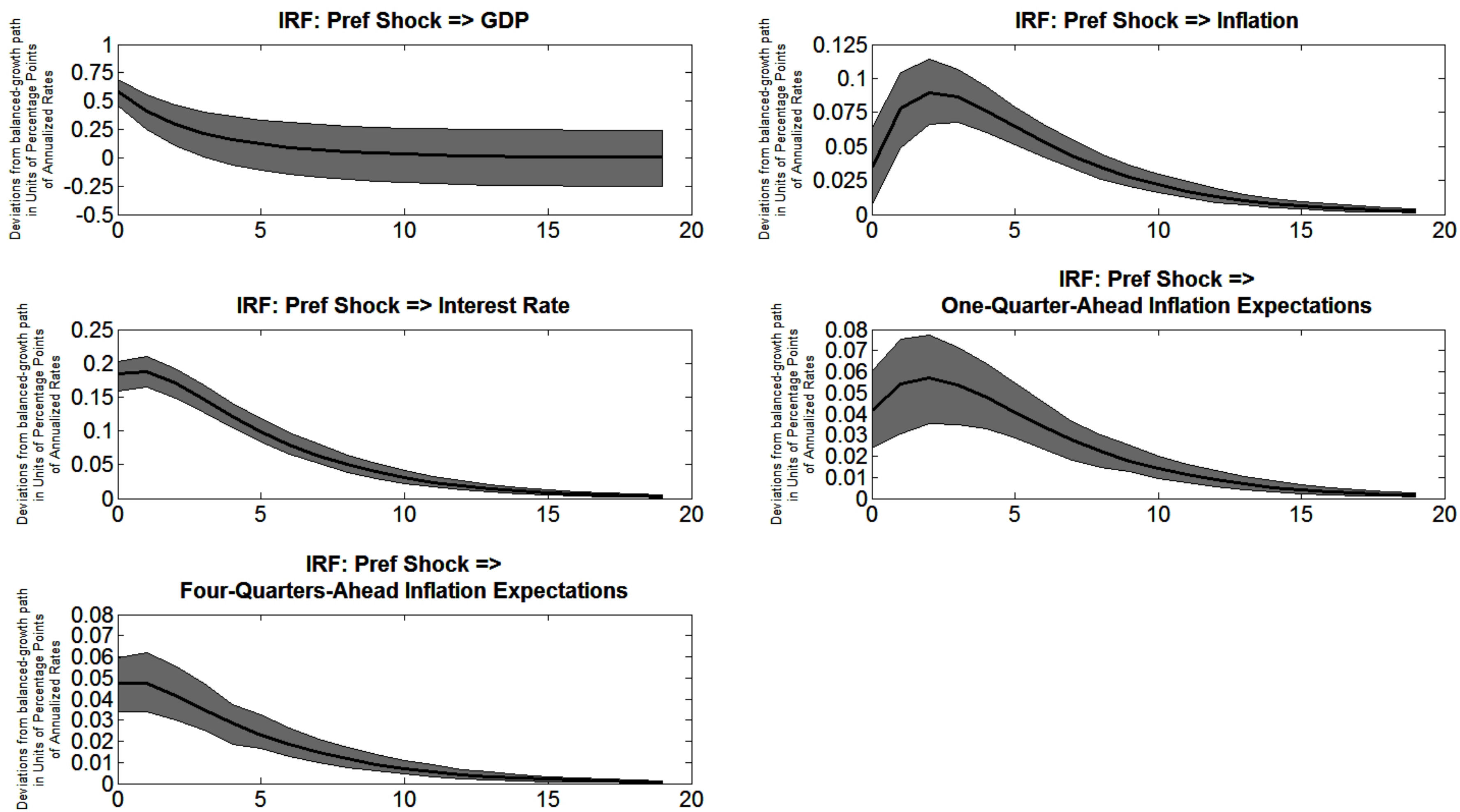

Figure 9: Impulse response functions of the observables to a one-standard-deviation preference shock and their 95 percent posterior credible sets.

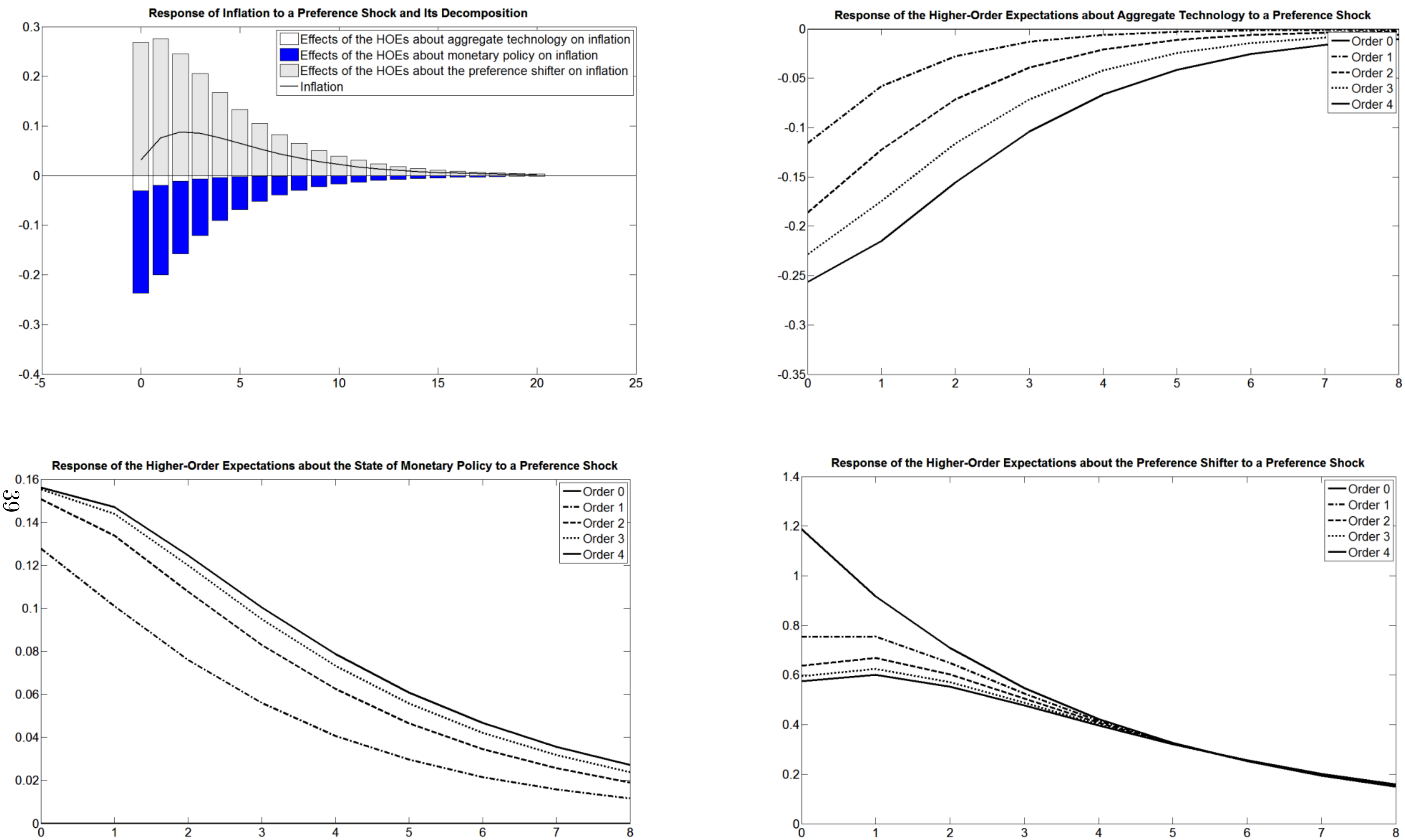

Figure 10: Impulse Response Functions of Inflation to a Two-Standard-Deviation Preference Shock and its Decompositions. The top left plot reports the response of inflation and its decomposition to the effects of the HOEs about the three exogenous state variables on inflation. The top right plot reports the response of the higher-order expectations about the aggregate technology to the preference shock. The bottom left plot reports the response of the higher-order expectations about the state of monetary policy to the preference shock. The bottom right plot reports the response of the higher-order expectations about the preference shifter to the preference shock. All the figures are obtained by evaluating the model at the posterior median.

Appendix

We solve the model assuming common knowledge of rationality (Nimark, 2008) and focusing on equilibria where the higher-order expectations about the exogenous state variables, $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{\eta}_{r,t|t}^{(s)}, \widehat{g}_{t|t}^{(s)} : 0 \leq s \leq k\right]'$ follow the VAR(1) process in equation (17), where $\mathbf{M}$ and $\mathbf{N}$ are matrices that are yet to be determined. Note that we have truncated the order of the average expectations at $k < \infty$. Furthermore, we guess the matrix $\mathbf{v}_0$ which determines the dynamics of the endogenous variables $\mathbf{s}_t \equiv \left[\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t\right]$ in equation (16). Given the guessed matrices $\mathbf{M}$, $\mathbf{N}$, and $\mathbf{v}_0$, the structural equations of the model can be written as

$$\Gamma_0 \mathbf{s}_t = C + \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)} \tag{20}$$

For a given parameter set Θ_{IIM}, take the following steps:

Step 0 Set $i = 1$ and guess the matrices $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$.

Step 1 Solve the model (17) and (20) through a standard linear rational expectations model solver (e.g., Blanchard and Kahn, 1980, or Sims 2002). The solver delivers the matrix $\mathbf{v}_0^{(i)}$, such that $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$.

Step 2 Given the law of motion (17) for $X_{t|t}^{(0:k)}$, equation (15) for the signal concerning the aggregate technology, equation (14) for the signal concerning the preference shifter, and

$$\widehat{R}_t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$$

for the endogenous policy signal $\widehat{R}_t \in \mathbf{s}_t$, solve firms' signal extraction problem through the Kalman filter. This delivers the law of motion of the higher-order expectations $X_{t|t}^{(1:k)}$ that are used to work out the matrices $\mathbf{M}^{(i+1)}$ $\mathbf{N}^{(i+1)}$.

Step 3 If $\left\|\mathbf{M}^{(i)} - \mathbf{M}^{(i+1)}\right\| < \varepsilon_m$ and $\left\|\mathbf{N}^{(i)} - \mathbf{N}^{(i+1)}\right\| < \varepsilon_n$ with $\varepsilon_m > 0$ and $\varepsilon_n > 0$ and small, STOP or else set i=i+1 and go to STEP 1.

Given equation (17) and equation $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$ obtained in step 1, the law of motion of the model variables follows

$$\begin{bmatrix} X_{t|t}^{(0:k)} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{(i)} & \mathbf{0} \\ -\mathbf{v}_0^{(i)} \mathbf{M}^{(i)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1}^{(0:k)} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(i)} \\ -\mathbf{v}_0^{(i)} \mathbf{N}^{(i)} \end{bmatrix} \boldsymbol{\varepsilon}_t \tag{21}$$

Other Appendices

In Section A, I derive of the imperfect-common-knowledge Phillips curve (10). In Section B, I show how to characterize the laws of motion for the three endogenous state variables (i.e., inflation $\hat{\pi}_t$, real output $\hat{y}_t$ and the interest rate $\hat{R}_t$) in equation (20). In Section C, I characterize the transition equations for the average higher-order expectations about the exogenous state variables, that is, equation (17).

A The Imperfect Common Knowledge Phillips Curve

The log-linear approximation of the labor supply can be shown to be given by $\widehat{c}_t = \widehat{w}_t$. Recalling that the resource constraint implies that $\hat{y}_t = \hat{c}_t$, then the labor supply can be written as follows:

$$\hat{y}_t = \hat{w}_t \tag{22}$$

Log-linearizing the equation for the real marginal costs yields

$$\widehat{mc}_{j,t} = \hat{w}_t - \widehat{a}_t - \varepsilon^a_{j,t}$$

Recall that $(\ln A_{j,t} - \ln \gamma \cdot t) \in \mathcal{I}_{j,t}$ and write:

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{w}_{j,t}\underbrace{-\widehat{a}_t - \varepsilon^a_{j,t}}_{\ln A_{j,t} - \ln \gamma \cdot t}$$

where $\mathbb{E}_{j,t}$ are expectations conditioned on firm j's information set at time t, $\mathcal{I}_{j,t}$, defined in (7). Using the equation (22) for replacing $\hat{w}_t$ yields:

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{y}_t - \widehat{a}_t - \varepsilon^a_{j,t}$$

By integrating across firms, we obtain the average expectations on marginal costs:

$$\widehat{mc}^{(1)}_{t|t} = \hat{y}^{(1)}_{t|t} - \widehat{a}_t$$

The linearized price index can be written as:

$$0 = -\theta\hat{\pi}_t + (1-\theta)\int \hat{p}^*_{j,t} dj$$

By rearranging:

$$\int \hat{p}^*_{j,t} dj = \frac{\theta}{1-\theta}\hat{\pi}_t$$

Recall that we defined $\hat{p}^*_{j,t} = \ln P^*_{j,t} - \ln P_t$ and $\hat{\pi}_t = \ln P_t - \ln P_{t-1} - \ln \pi_*$,

$$\int \ln P^*_{j,t} dj - \ln P_t = \frac{\theta}{1-\theta}\left(\ln P_t - \ln P_{t-1} - \ln \pi_*\right)$$

and then

$$\int \ln P^*_{j,t} dj = \frac{1}{1-\theta}\ln P_t - \frac{\theta}{1-\theta}\left(\ln P_{t-1} + \ln \pi_*\right)$$

By rearranging:

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln \pi_*\right) + (1-\theta)\int \left(\ln P^*_{j,t}\right) dj \tag{23}$$

The price-setting equation is:

$$\mathbb{E}\left[\sum_{s=0}^{\infty}(\beta\theta)^s \frac{\Xi_{j,t+s}}{P_{t+s}}\left[(1-\nu)\pi_*^s + \nu\frac{MC_{j,t+s}}{P^*_{j,t}}\right] Y_{j,t+s}|\mathcal{I}_{j,t}\right] = 0$$

Define

$$\begin{aligned} y_t &= \frac{Y_t}{\gamma^t},\ c_t = \frac{C_t}{\gamma^t};\ p^*_{j,t} = \frac{P^*_{j,t}}{P_t},\ y_{j,t} = \frac{Y_{j,t}}{\gamma^t} \\ w_t &= \frac{W_t}{\gamma^t P_t},\ a_t = \frac{A_t}{\gamma^t}, R_t = \frac{R_t}{R_*},\ mc_{j,t} = \frac{MC_{j,t}}{P_t} \\ \xi_{j,t} &= \gamma^t \Xi_{j,t} \end{aligned}$$

Hence, write:

$$\mathbb{E}\left\{\xi_{j,t}\left[1-\nu+\nu\frac{mc_{j,t}}{p^*_{j,t}}\right]y_{j,t} + \sum_{s=1}^{\infty}(\beta\theta)^s \xi_{j,t+s}\left[(1-\nu)\pi_*^s + \nu\frac{mc_{j,t+s}}{p^*_{j,t}}\left(\Pi_{\tau=1}^s \pi_{t+\tau}\right)\right] y_{j,t+s}|\mathcal{I}_{j,t}\right\} = 0 \tag{24}$$

First realize that the square brackets are equal to zero at the steady state and hence we do not care about the terms outside them. We can write

$$\mathbb{E}\left[\left[1-\nu+\nu mc_{j,*}e^{\widehat{mc}_{j,t}-\widehat{p}^*_{j,t}}\right] + \sum_{s=1}^{\infty}(\beta\theta)^s\left[(1-\nu)\pi_*^s + \nu mc_{j,*}e^{\widehat{mc}_{j,t+s}-\widehat{p}^*_{j,t}+\sum_{\tau=1}^s \hat{\pi}_{t+\tau}}\right]|\mathcal{I}_{j,t}\right] = 0$$

Taking the derivatives yield:

$$\mathbb{E}\left[\widehat{mc}_{j,t}-\widehat{p}^{*}_{j,t}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left[\left(\widehat{mc}_{j,t+s}-\widehat{p}^{*}_{j,t}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)\right]|\mathcal{I}_{j,t}\right]=0$$

We can take the term $\widehat{p}_{j,t}$ out of the sum operator in the second term and gather the common term to obtain:

$$\mathbb{E}\left[\widehat{mc}_{j,t}-\frac{1}{1-\beta\theta}\widehat{p}^{*}_{j,t}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]=0$$

Recall that $\widehat{p}^{*}_{j,t}\equiv\ln P^{*}_{j,t}-\ln P_{t}$ and cannot be taken out of the expectation operator. We write:

$$\ln P^{*}_{j,t}=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}+\frac{1}{1-\beta\theta}\ln P_{t}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau}\right)|\mathcal{I}_{j,t}\right]\tag{25}$$

Rolling this equation one step ahead yields:

$$\ln P^{*}_{j,t+1}=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1}+\frac{1}{1-\beta\theta}\ln P_{t+1}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s+1}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t+1}\right]$$

Take firm j's conditional expectation at time t on both sides and apply the law of iterated expectations:

$$\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)=(1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t+1}+\frac{1}{1-\beta\theta}\ln P_{t+1}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\left(\widehat{mc}_{j,t+s+1}+\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}\right)|\mathcal{I}_{j,t}\right]$$

We can take $\widehat{mc}_{j,t+1}$ inside the sum operator and write:

$$\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)=(1-\beta\theta)\,\mathbb{E}\left[\frac{1}{1-\beta\theta}\ln P_{t+1}+\frac{1}{\beta\theta}\sum_{s=1}^{\infty}(\beta\theta)^{s}\,\widehat{mc}_{j,t+s}+\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]$$

Therefore,

$$\sum_{s=1}^{\infty}(\beta\theta)^{s}\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]=\frac{\beta\theta}{1-\beta\theta}\left[\mathbb{E}\left(\ln P^{*}_{j,t+1}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-\beta\theta\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\tag{26}$$

The equation (25) can be rewritten as:

$$\begin{aligned}\ln P_{j,t}^{*} &= (1-\beta\theta)\left\{\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right]+\sum_{s=1}^{\infty}(\beta\theta)^s\,\mathbb{E}\left[\widehat{mc}_{j,t+s}|\mathcal{I}_{j,t}\right]\right\}\\ &\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right]\end{aligned}$$

By substituting the result in equation (26) we obtain:

$$\begin{aligned}\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right]\right]\\ &\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\\ &\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right]\end{aligned}$$

Consider the last term:

$$\begin{aligned}(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+(1-\beta\theta)\sum_{s=2}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right]\\ &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+\\ &\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\left(\sum_{\tau=1}^{s}\left[\left(\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\right)\right]+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\right)\end{aligned}$$

Therefore we can write that

$$\begin{aligned}(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\\ &\quad +(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]\\ &\quad +(1-\beta\theta)\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]\end{aligned}$$

Note that

$$\left(\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\right)=\frac{(\beta\theta)^2}{1-\beta\theta}$$

Hence,

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= (1-\beta\theta)\,\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&\quad + (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&\quad + (\beta\theta)^{2}\,\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]
\end{aligned}
$$

and by simplifying:

$$
\begin{aligned}
(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau}|\mathcal{I}_{j,t}\right] &= \beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \\
&\quad + (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right]
\end{aligned}
$$

We substitute this result into the original equation to get:

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]\right] \\
&\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]-(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \\
&\quad +\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]+(1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^{s+1}\sum_{\tau=1}^{s}\mathbb{E}\left[\hat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}\right] \qquad (27)
\end{aligned}
$$

After simplifying we get:

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\left[\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\frac{1}{1-\beta\theta}\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right]\right] \\
&\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right]+\beta\theta\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right] \qquad (28)
\end{aligned}
$$

We can rearrange:

$$
\begin{aligned}
\ln P_{j,t}^{*} &= (1-\beta\theta)\,\mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right]+\mathbb{E}\left[\ln P_{t}|\mathcal{I}_{j,t}\right] \\
&\quad +\beta\theta\left[\mathbb{E}\left(\ln P_{j,t+1}^{*}|\mathcal{I}_{j,t}\right)+\mathbb{E}\left[\hat{\pi}_{t+1}|\mathcal{I}_{j,t}\right]-\mathbb{E}\left(\ln P_{t+1}|\mathcal{I}_{j,t}\right)\right] \qquad (29)
\end{aligned}
$$

Note that by definition $\hat{\pi}_{t+1}\equiv\ln P_{t+1}-\ln P_{t}-\ln\pi_{*}$. Hence we can show that

$$\begin{aligned} \ln P^*_{j,t} &= (1-\beta\theta)\cdot \mathbb{E}\left[\widehat{mc}_{j,t}|\mathcal{I}_{j,t}\right] + (1-\beta\theta)\,\mathbb{E}\left[\ln P_t|\mathcal{I}_{j,t}\right] \\ &\quad +\beta\theta\cdot\mathbb{E}\left(\ln P^*_{j,t+1}|\mathcal{I}_{j,t}\right) - \beta\theta\ln\pi_* \end{aligned} \tag{30}$$

We denote the firm j's average k-th order expectation about an arbitrary variable $\hat{x}_t$ as

$$\mathbb{E}^{(k)}\left(\hat{x}_t|\mathcal{I}_{j,t}\right) \equiv \int \mathbb{E}\left(\int \mathbb{E}\left(\ldots\left(\int \mathbb{E}\left(\hat{x}_t|\mathcal{I}_{j,t}\right)dj\right)\ldots|\mathcal{I}_{j,t}\right)dj|\mathcal{I}_{j,t}\right)dj$$

where expectations and integration across firms are taken k times.

Let us denote the **average reset price** as $\ln P^*_t = \int \ln P^*_{j,t} dj$. We can integrate equation (30) across firms to obtain an equation for the average reset price:

$$\begin{aligned} \ln P^*_t &= (1-\beta\theta)\cdot\widehat{mc}^{(1)}_{t|t} + (1-\beta\theta)\ln P^{(1)}_{t|t} \\ &\quad +\beta\theta\ln P^{*(1)}_{t+1|t} - \beta\theta\ln\pi_* \end{aligned} \tag{31}$$

where we use the claim of the proposition above. Keep in mind that the price index equation can be manipulated to get equation (23)

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln\pi_*\right) + (1-\theta)\ln P^*_t \tag{32}$$

Let us plug the equation (31) into the equation (32):

$$\begin{aligned} \ln P_t &= \theta\ln P_{t-1} + \left(\theta - (1-\theta)\,\beta\theta\right)\ln\pi_* \\ &\quad + (1-\theta)\left[(1-\beta\theta)\cdot\widehat{mc}^{(1)}_{t|t} + (1-\beta\theta)\ln P^{(1)}_{t|t} + \beta\theta\ln P^{*(1)}_{t+1|t}\right] \end{aligned} \tag{33}$$

Use the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln\pi_*$ and from the price index (23):[16]

$$\ln P^*_{t+1} = \frac{\hat{\pi}_{t+1}}{1-\theta} + \ln P_t + \ln\pi_*$$

[16] This last result comes from observing that

$$\ln P_t = \theta\left(\ln P_{t-1} + \ln\pi_*\right) + (1-\theta)\ln P^*_t$$

By using the fact that $\ln P_t = \widehat{\pi}_t + \ln P_{t-1} + \ln\pi_*$:

$$\widehat{\pi}_t + \ln P_{t-1} + \ln\pi_* = \theta\left(\ln P_{t-1} + \ln\pi_*\right) + (1-\theta)\ln P^*_t$$

Rolling one period forward:

$$\widehat{\pi}_{t+1} = (\theta-1)\left(\ln P_t + \ln\pi_*\right) + (1-\theta)\ln P^*_{t+1}$$

and finally by rearranging we get the result in the text.

Furthermore, the following fact is easy to establish:

$$\ln P_{t+1} = \widehat{\pi}_{t+1} + \ln P_t + \ln \pi_*$$

Applying these three results to equation (33) yields:

$$\begin{aligned}\widehat{\pi}_t + \ln P_{t-1} + \ln \pi_* &= \theta \ln P_{t-1} + (\theta - (1-\theta)\,\beta\theta)\ln \pi_* \\ &\quad + (1-\theta)\left[(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta)\ln P_{t|t}^{(1)} + \beta\theta\left(\frac{\hat{\pi}_{t+1|t}^{(1)}}{1-\theta} + \ln P_{t|t}^{(1)} + \ln \pi_*\right)\right]\end{aligned} \tag{34}$$

and after simplifying:

$$\widehat{\pi}_t = (1-\theta)\,(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(1)} + (1-\theta)\,\widehat{\pi}_{t|t}^{(1)} + \beta\theta\left(\hat{\pi}_{t+1|t}^{(1)}\right) \tag{35}$$

By repeatedly taking firm j's expectation and average the resulting equation across firms:

$$\widehat{\pi}_{t|t}^{(k)} = (1-\theta)\,(1-\beta\theta)\cdot \widehat{mc}_{t|t}^{(k)} + (1-\theta)\,\widehat{\pi}_{t|t}^{(k+1)} + \beta\theta\left(\hat{\pi}_{t+1|t}^{(k+1)}\right)$$

Repeatedly substituting these equations for $k \geq 1$ back to equation (35) yields: the imperfect-common-knowledge Phillips curve:

$$\hat{\pi}_t = (1-\theta)\,(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^k\,\widehat{mc}_{t|t}^{(k)} + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^k\,\widehat{\pi}_{t+1|t}^{(k)}$$

B The Laws of Motion for the Endogenous State Variables

In this section I, first, introduce some useful results and, second, characterize the law of motion for the endogenous state variables $(\widehat{\pi}_t, \widehat{y}_t, \widehat{R}_t)$, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$. It will be shown that this law of motion depends on model parameters and the coefficient matrices, $\mathbf{M}$ and $\mathbf{N}$, of the transition equation for the average higher-order expectations about the exogenous variables.

B.1 Preliminaries

Recall that the *assumption of common knowledge in rationality* ensures that agents use the actual law of motion of higher-order expectations to forecast the dynamics of the higher-order expectations. The following claims turn out to be useful:

Proposition 1 *If one neglects the effect of average beliefs of order larger than* k*, then the following is approximately true:*

$$X_{t|t}^{(s:k+s)} = \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$$

where

$$\mathbf{T}^{(s)} \equiv \begin{bmatrix} \mathbf{0}_{3(k-s+1)\times 3s} & \mathbf{I}_{3(k-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times(k+1-s)3} \end{bmatrix}$$

Proof. It is straightforward but help to fix some notation. Since we neglect the average beliefs of order larger than k

$$X_{t|t}^{(s:k+s)} \equiv \begin{bmatrix} X_{t|t}^{(s:k)} \\ X_{t|t}^{(s:k+s)} \end{bmatrix}_{3(k+1)\times 1} = \begin{bmatrix} X_{t|t}^{(s:k)} \\ \mathbf{0}_{3s\times 1} \end{bmatrix}_{3(k+1)\times 1}$$

Note that

$$X_{t|t}^{(s:k+s)} = \begin{bmatrix} \mathbf{0}_{3(k-s+1)\times 3s} & \mathbf{I}_{3(k-s+1)} \\ \mathbf{0}_{3s\times 3s} & \mathbf{0}_{3s\times(k+1-s)3} \end{bmatrix} \underbrace{\begin{bmatrix} X_{t|t}^{(0:s-1)} \\ X_{t|t}^{(s:k)} \end{bmatrix}}_{X_{t|t}^{(0:k)}}$$

■

Proposition 2 $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k+s)}$*, for any* $0 \leq s \leq k$*.*

Proof. We conjectured that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$. Then common knowledge in rationality implies:

$$\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$$

■

Since we truncate beliefs after the k-th order we have that

$$\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k)}, \text{for any } 0 \leq s \leq k$$

Proposition 3 *The following holds true for any* $h \in \{0 \cup \mathbb{N}\}$

$$\mathbf{s}_{t+h|t}^{(1)} = \mathbf{v}_0 \mathbf{M}^h \mathbf{T}^{(1)} X_{t|t}^{(0:k)}$$

Proof. Consider

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$$

Then it is easy to see that by taking agents' expectations and then averaging across them we obtain by the *assumption of common knowledge in rationality*:

$$\mathbf{s}_{t|t}^{(1)} = \mathbf{v}_0 X_{t|t}^{(1:k+1)}$$

and by neglecting the contribution of beliefs of order higher than k we can write: $\mathbf{T}^{(1)} X_{t|t}^{(0:k)} = X_{t|t}^{(1:k+1)}$. This leads to write:

$$\mathbf{s}_{t|t}^{(1)} = \mathbf{v}_0 \mathbf{T}^{(1)} X_{t|t}^{(0:k)} \tag{36}$$

Furthermore, consider $\mathbf{s}_{t+1}$:

$$\mathbf{s}_{t+1} = \mathbf{v}_0 X_{t+1|t+1}^{(0:k)}$$

By taking agents' expectations and then averaging across them we obtain:

$$\mathbf{s}_{t+1|t}^{(1)} = \mathbf{v}_0 X_{t+1|t}^{(1:k+1)}$$

First note that by the assumption of common knowledge in rationality we can write: $X_{t+h|t}^{(1:k+1)} = \mathbf{M}^h X_{t|t}^{(1:k+1)}$. Second, recall that we neglect the contribution of beliefs of order higher than k. These two facts lead us to

$$\mathbf{s}_{t+1|t}^{(1)} = \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(1)} X_{t|t}^{(0:k)}$$

Consider now $\mathbf{s}_{t+2}$. By taking agents' expectations and then averaging across them we obtain:

$$\mathbf{s}_{t+2|t}^{(1)} = \mathbf{v}_0 X_{t+2|t}^{(1:k+1)}$$

and substituting $\mathbf{s}_{t+1|t}^{(1)}$ that we have characterized above yields:

$$\mathbf{s}_{t+2|t}^{(1)} = \mathbf{v}_0 \mathbf{M}^2 \mathbf{T}^{(1)} X_{t|t}^{(0:k)}$$

Keeping on deriving $\mathbf{s}_{t+h|t}^{(1)}$ for any other $h \in \{0 \cup \mathbb{N}\}$ as shown above leads at the formula in the claim. ■

B.2 The Laws of Motion of the Endogenous State Variables

The laws of motion of the three endogenous state variables, which are inflation $\hat{\pi}_t$, real output $\hat{y}_t$, and the (nominal) interest rate $\hat{R}_t$, are given by the IS equation (12), the Phillips curve (10), and the Taylor Rule (13). We want to write this system of linear equations as:

$$\Gamma_0 \mathbf{s}_t = C + \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)} \tag{37}$$

where $\mathbf{s}_t \equiv \left[\hat{\pi}_t, \hat{y}_t, \hat{R}_t\right]'$. It is obvious how to write equations (12) and (13) in the form (37). However, how to write Phillips curve (10) in the form (37) is not obvious and requires a bit of work. However, note that given the results derived in the previous section and the definition (11), this equation can be rewritten as:

$$
\begin{aligned}
&\mathbf{a}_0 X_{t|t}^{(0:k)} = (1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_2^T \left[\mathbf{v}_0 \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right] + \\
&-(1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s \left[\boldsymbol{\gamma}_a^{(s)\prime} X_{t|t}^{(0:k)}\right] \\
&+\beta\theta\sum_{s=0}^{k-1}(1-\theta)^s \mathbf{1}_1^T \left[\mathbf{v}_0 \mathbf{M}\mathbf{T}^{(s+1)} X_{t|t}^{(0:k)}\right]
\end{aligned}
$$

where $\mathbf{1}_1^T = [1,0,0]$, $\mathbf{1}_2^T = [0,1,0]$, and $\boldsymbol{\gamma}_a^{(s)} = \left[\mathbf{0}_{1\times 3s}, (1,1,0,0,0,0), \mathbf{0}_{1\times 3(k-s)}\right]'$. The following restrictions upon vectors of coefficients $\mathbf{a}_0$ and $\mathbf{a}_1$ can be derived from the Phillips curve above:

$$
\widehat{\pi}_t = \left[(1-\theta)(1-\beta\theta)\left[\boldsymbol{\nu}\mathbf{m}_1 - \left(\sum_{s=0}^{k-1}(1-\theta)^s \boldsymbol{\gamma}_a^{(s)\prime}\right)\right] + \beta\theta\boldsymbol{\nu}\mathbf{m}_2\right] X_{t|t}^{(0:k)} \tag{38}
$$

where I define:

$$
\begin{aligned}
\mathbf{m}_1 &\equiv \begin{bmatrix} \left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(1)}\right] \\ (1-\theta)\left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(2)}\right] \\ (1-\theta)^2\left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(3)}\right] \\ \vdots \\ (1-\theta)^{k-1}\left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(k)}\right] \end{bmatrix}, \quad \mathbf{m}_2 \equiv \begin{bmatrix} \left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(1)}\right] \\ (1-\theta)\left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(2)}\right] \\ (1-\theta)^2\left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(3)}\right] \\ \vdots \\ (1-\theta)^{k-1}\left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M}\mathbf{T}^{(k)}\right] \end{bmatrix}, \\
\boldsymbol{\nu} &\equiv \mathbf{1}_{1\times k}
\end{aligned}
$$

Equation (38) is a linear function of the vector of average higher-order expectations $X_{t|t}^{(0:k)}$.

C Transition Equation of High–Order Expectations

In this section, we show how to derive the law of motion of the average higher-order expectations of the exogenous variables (i.e., $\widehat{a}_t$, $\varepsilon_{r,t}$, g_t,) for given parameter values and the matrix of coefficients $\mathbf{v}_0$. We focus on equilibria where the HOEs evolve according to:

$$X_{t|t}^{(0:k)} = \mathbf{M} X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t \tag{39}$$

where $\boldsymbol{\varepsilon}_t \equiv \left[\begin{array}{ccc} \varepsilon_{a,t} & \eta_{r,t} & \varepsilon_{g,t} \end{array} \right]'$. Denote $\mathbf{X}_t \equiv X_{t|t}^{(0:k)}$, for notational convenience. Firms' reduced-form state-space model can be concisely cast as follows:

$$\mathbf{X}_t = \mathbf{M}\mathbf{X}_{t-1} + \mathbf{N}\boldsymbol{\varepsilon}_t \tag{40}$$

$$\mathbf{Z}_t = \mathbf{D}_1 \mathbf{X}_t + \mathbf{Q} e_{j,t} \tag{41}$$

where

$$\mathbf{D}_1 = \left[\begin{array}{ccc} \mathbf{d}_1' & \mathbf{d}_2' & \left(\mathbf{1}_3^T \mathbf{v}_0\right)' \end{array} \right]'$$

with and $\mathbf{1}_3^T = [0, 0, 1]$, $\mathbf{d}_1' = \left[1, \mathbf{0}_{1\times 3(k+1)-1}\right]$, $\mathbf{d}_2' = [\mathbf{0}_{1\times 2}, 1, \mathbf{0}_{1\times 3k}]$, and $e_{j,t} = \left[\varepsilon_{j,t}^a, \varepsilon_{j,t}^g\right]'$ and

$$\mathbf{Q} = \left[\begin{array}{cc} \widetilde{\sigma}_a & 0 \\ 0 & \widetilde{\sigma}_g \\ 0 & 0 \end{array} \right]$$

.

Solving firms' signal extraction problem requires applying the Kalman filter. The Kalman equation and the conditional variance and covaraince matrix can be easily derived and read:

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{P}_{t|t-1}\mathbf{D}_1'\mathbf{F}_{t|t-1}^{-1}\left[\mathbf{Z}_t - \mathbf{Z}_{t|t-1}(j)\right] \tag{42}$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}_1'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}_1\mathbf{P}_{t|t-1}' \tag{43}$$

where

$$\mathbf{P}_{t|t-1} = \mathbf{W}\mathbf{P}_{\mathbf{t-1|t-1}}\mathbf{W}' + \mathbf{U}\mathbf{U}' \tag{44}$$

Therefore, combining equation (43) with equation (44) yields:

$$\mathbf{P}_{t+1|t} = \mathbf{W}\left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}_1'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}_1\mathbf{P}_{t|t-1}'\right]\mathbf{W}' + \mathbf{U}\mathbf{U}' \tag{45}$$

Denote the Kalman-gain matrix as $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1}\mathbf{D}_1'\mathbf{F}_{t|t-1}^{-1}$. Recall equation (41) and write the law of motion of the firm j's first-order beliefs about $\mathbf{X}_t$ as

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{K}_t\left[\mathbf{D}_1\mathbf{X}_t + \mathbf{Q}e_{j,t} - \mathbf{D}_1\mathbf{X}_{t|t-1}(j)\right]$$

where we have combined equations (42) and (41). By recalling that $\mathbf{X}_{t|t-1}(j) = \mathbf{W}\mathbf{X}_{t-1|t-1}(j)$, we have:

$$\mathbf{X}_{t|t}(j) = \mathbf{W}\mathbf{X}_{t-1|t-1}(j) + \mathbf{K}_t\left[\mathbf{D}_1\mathbf{X}_t + \mathbf{Q}e_{j,t} - \left(\mathbf{D}_1\mathbf{W}\mathbf{X}_{t-1|t-1}(j)\right)\right]$$

By rearranging one obtains:

$$\mathbf{X}_{t|t}(j) = (\mathbf{W} - \mathbf{K}\mathbf{D}_1\mathbf{W})\mathbf{X}_{t-1|t-1}(j) + \mathbf{K}\left[\mathbf{D}_1\mathbf{W}\cdot\mathbf{X}_{t-1} + \mathbf{D}_1\mathbf{U}\cdot\boldsymbol{\varepsilon}_t + \mathbf{Q}e_{j,t}\right] \tag{46}$$

The vector $\mathbf{X}_{t|t}(j)$ contains firm j's first-order expectations about model's state variables. Integrating across firms yields the law of motion of the average expectation about $\mathbf{X}^{(1)}_{t|t}$:

$$\mathbf{X}^{(1)}_{t|t} = (\mathbf{W} - \mathbf{K}\mathbf{D}_1\mathbf{W})\mathbf{X}^{(1)}_{t-1|t-1} + \mathbf{K}\left[\mathbf{D}_1\mathbf{W}\cdot\mathbf{X}_{t-1} + \mathbf{D}_1\mathbf{U}\cdot\boldsymbol{\varepsilon}_t\right]$$

Note that $X^{(0:\infty)}_{t|t} = \left[X_t, X^{(1:\infty)}_{t|t}\right]'$ and that:

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & \mathbf{0} \\ 0 & \rho_r & 0 & \mathbf{0} \\ 0 & 0 & \rho_g & \mathbf{0} \end{bmatrix}}_{\mathbf{R}_1} X^{(0:k)}_{t-1|t-1} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_g \end{bmatrix}}_{\mathbf{R}_2} \cdot\boldsymbol{\varepsilon}_t$$

So by using the assumption of common knowledge in rationality, we can fully characterize the matrices $\mathbf{M}$ and $\mathbf{N}$:

$$\mathbf{M} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3k} \\ \mathbf{0}_{3k\times 3} & (\mathbf{W} - \mathbf{K}\mathbf{D}_1\mathbf{W})|_{(1:3k,1:3k)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}(\mathbf{D}_1\mathbf{W})|_{(1:3k,1:3(k+1))} \end{bmatrix} \tag{47}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K}\mathbf{D}_1\mathbf{U}|_{(1:3k,1:3)} \end{bmatrix} \tag{48}$$

where $\cdot|_{(n_1:n_2,m_1:m_2)}$ denotes the submatrix obtained by taking the elements lying between the n_1-th row and the n_2-th row and between the m_1-th column and the m_2-th column. Note that $\mathbf{K}$ in the above equation denotes the steady-state Kalman gain matrix, which is obtained by iterating the equations (43)-(45) and the equation for the Kalman-gain matrix below:

$$\mathbf{K} = \mathbf{P}_{t|t-1}\mathbf{D}_1'\mathbf{F}^{-1}_{t|t-1}$$

until convergence.

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