Preliminary Draft
NA for Quotation

Trends in the Labor Force Participation of Married Women

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1 Introduction

This study attempts to quantify determinants of female labor–force participation. We construct a life–cycle model that can jointly analyze a married household's consumption–saving choices, female labor force participation at different life stages, and male age of retirement. We then estimate the model's key parameters from pseudo panel data from the Consumer Expenditure Survey (CEX) and (actual) panel data from the Health and Retirement Study (HRS). Our goal is to utilize the framework and parameter estimates to investigate (a) whether the rapidity of the increase in U.S. married women's labor supply in the last quarter of the twentieth century was due to the speed of exogenous change, or whether it might have been preordained by the structure of the life–cycle model; (b) what the present and future impact of increased female earnings might be on male retirement ages; and, (c) what role decreasing fertility, vis-a-vis higher relative wages for women, might have played in determining recent patterns of female labor force participation.

This paper's life–cycle model has a number of familiar components. For example, each household desires, cet. par., the highest expenditure when it has the most members, and an adult's labor in one period raises his/her subsequent earning ability. (We refer to the latter process as "accumulation of experiential human capital.") A more novel assumption of our specification is that a household can produce a greater "service flow," which is the ultimate argument of its utility function, from a given expenditure on market goods when its adult male and/or female have more time at home.

In an environment with constant trend growth in both male and female wages, this paper's preference ordering delivers the simple implication that the overall labor supply is stationary. On the other hand, suppose that changes in fertility and/or relative wages make female participation more attractive. The model might predict a continuous response in married women's participation. Or, the response might be discontinuous. In the second case, our life–cycle formulation might have multiple local utility maxima, say, one at low lifetime participation for women, and one at high participation; the economy might start out with the low–participation critical point being the global maximum for most households, but following a period of change in the relative wage, the high–participation critical point might become best; and, as the second critical point supplants the first, the economy's labor supply from married women might increase very quickly. This paper investigates the role of human capital in making the "tipping point" scenario plausible.

If a household has female retirement age R^F and male retirement age R^M, a common empirical outcome is $R^F < R^M$. Our model implies that when this is true, rising relative female earnings will tend to lower the optimal age of male retirement. This suggests that an era of rising relative wages for women and declining fertility would tend to cause

falling retirement ages for married men. If age differences within couples eventually cause $R^F \geq R^M$, we show that the trend for R^M could disappear (or reverse).

Finally, the model allows us to estimate both the direct cost to a household of raising children and the indirect cost — through lost female earnings and lost investments in human capital — and it enables us to assess their relative importance for female labor force participation.

The organization of this paper is as follows. Section 2 presents CPS evidence on male and female participation in recent decades. Section 3 presents our model and several characterizations of its solution. Section 3 discusses our data and derives parameter estimates. Section 4 discusses an important generalization. Section 5 describes preliminary parameter estimates.

2 CPS Data

This section presents labor–force participation data from the Current Population Survey (CPS) March 1984 and 2004. For married couples, "age" refers to wife's age. "Singles" include adults who never married and those who are divorced or widowed.[1] Each graph categorizes a household's labor–force participation as (i) zero; (ii) "part–time," which is 1-20 hrs/week; or, (iii) "full–time," which is taken to be 21+ hrs/week.

Figures 1 presents cross-sectional data for married males. The patterns for 1984 and 2004 are similar. Full–time labor–force participation is high (90-95 percent) until household age 45-50. Thereafter it drops, with the drop becoming quite steep by age 55. The median household age at which the curves for full–time and zero participation cross is about 60. Part–time work plays little role — though it rises around the age at which the other curves cross. Profiles for single males (not shown) are quite similar .

Figure 2 presents cross-sectional data for married women. As with men, part–time work (i.e., 1-20 hrs/week) is an order of magnitude less prevalent than zero or 21+ hrs/week, although it is roughly twice as prevalent for married women as for married men. For simplicity, this paper concentrates on zero participation versus positive participation.

Three observations on the female data are as follows. (i) The labor–force participation level for married women is noticeably lower than for married men: 50-55 percent in 1984 versus 90-95 percent for ages 25-45; by 2004, 60-70 percent versus 90-95 percent. Single women's behavior is much closer to men's. (ii) In 1984, the full–time and zero participation curves for married women cross at age 50-52 — versus 60 for married men. However, by 2004 the curves for women cross at 60. Single women's crossing age is 60 even in 1984. It seems possible that the trend increase in the level of married women's labor–force participation provides at least part of the explanation for the 1984 outcome: the graphs come from cross–sectional data; older ages correspond, therefore, to earlier birth cohorts — for which participation was lower. (iii) There is little evidence in 1984 of an increase

[1] In 1984, for example, our usable sample had 28,890 couples and, among singles, 6800 widow(er)s, 5173 divorced, 1585 separated, and 5365 never married. In 2004, there are 26,382 married and 5316 widow(er)s, 7204 divorced, 1287 separated, and 7290 never married.

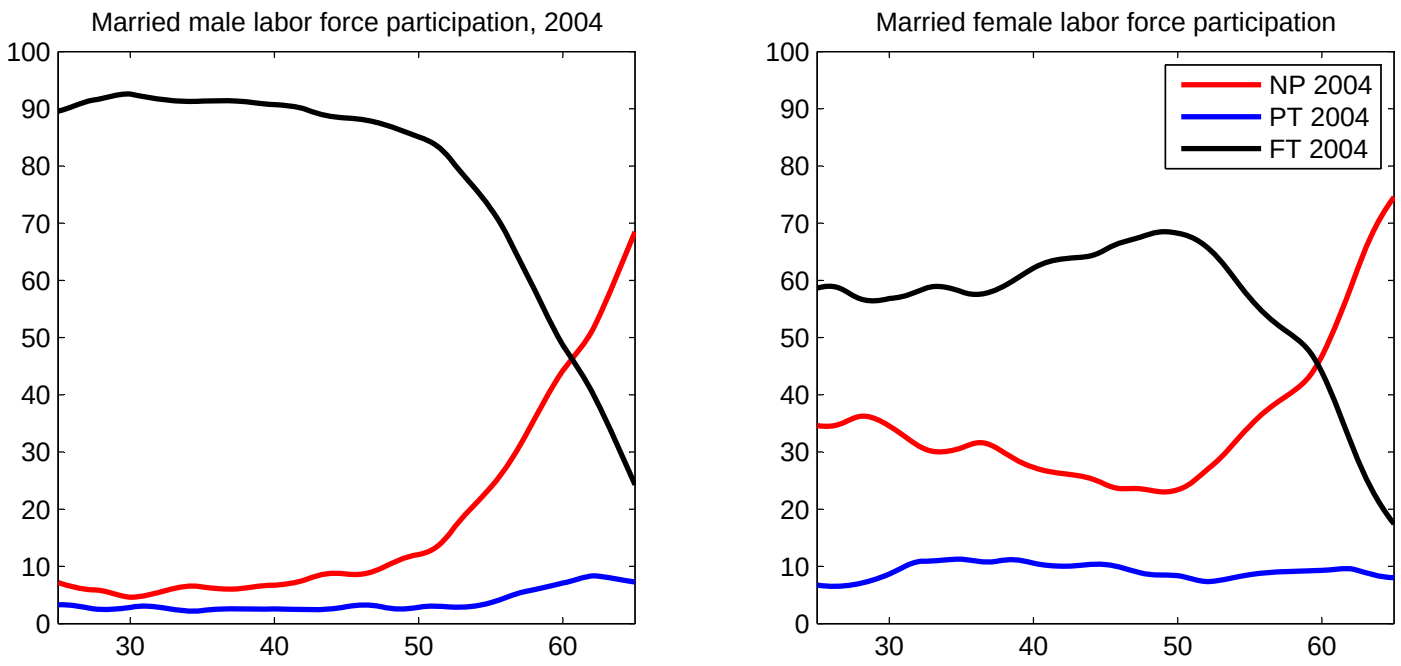

Figure 1. Labor force participation of married males and females, 2004, by wife's age.

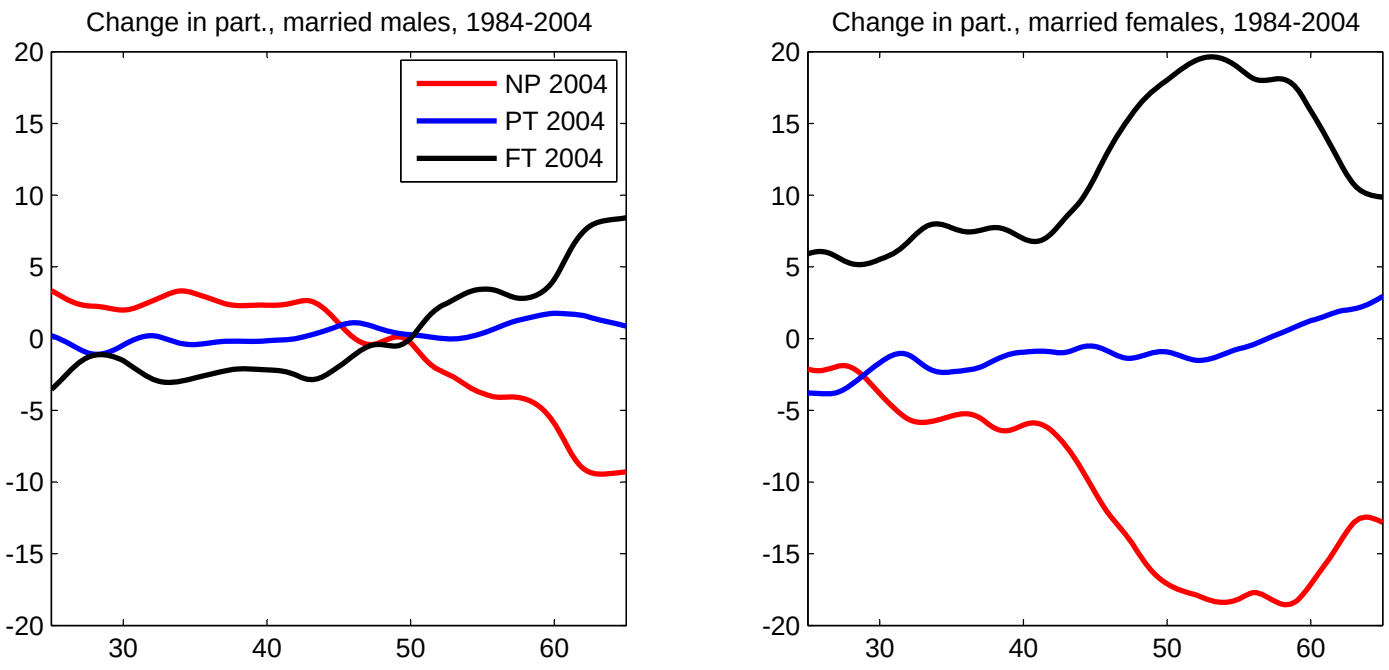

Figure 2. Change in labor force participation of married males and females, 1984-2004, by wife's age.

in full–time participation for married women after their children grow up. This paper's graphs exclude earlier ages because of worries about selection: women who obtain more education tend to marry and enter the labor force later; so, data for earlier ages will heavily manifest changes in educational composition.

This paper's model assumes discrete labor–supply choices — zero or full time — for both men and women, and the figures show this may be a reasonable approximation.[2] This paper assumes men work at all ages until they retire; it allows heterogeneity of male retirement age. It allows a woman to enter and leave the labor force numerous times, with different women not constrained to make identical decisions.

3 Model

Our model studies the behavior of married households, which we take to be the modal living arrangement. Consider the life cycle of household i. The household includes a woman, a man (if surviving), and dependent children. The household's size at age s is N_{is} "equivalent adults" (see below). Let the "household's age" be the wife's age. The man retires at $s = R_i$. We assume that men work full time until retirement, and not at all afterward. Women also work full time or not at all, though a woman may enter and leave the labor force several times. Given the evidence in Section 2, we assume, for simplicity, that married men retire no earlier than their wives. Households take prices and wages as given. Time is continuous. There is no uncertainty. At inception, household i chooses its life–cycle expenditure profile for market consumption goods, x_{is}; its male retirement age, R_i; and, its female labor–force participation profile, $p_{is} \in \{0\,,\,1\}$, where $p_{is} = 1$ means the wife works.

Households derive utility from a final consumption aggregate, which is, in turn, produced from a combination of market consumption goods, leisure, and home production. Let x_{is} denote the age s expenditures of household i on market consumption goods. Let λ_{is} denote the weight the household places on such expenditures. We think of λ_{is} as reflecting the household's ability to enjoy market expenditures; thus, λ_{is} depends positively on leisure and home production, and a household, cet. par., "produces" more utility from the same expenditure during periods when either or both parents can spend more time at home. A household's utility flow, $u(.)$, is an isoelastic function of $c_{is} \equiv \lambda_{is} \cdot x_{is}$. Letting the intertemporal elasticity of substitution be $1/(1-\gamma)$,

$$u(\lambda \cdot x) \equiv \begin{cases} \frac{1}{\gamma} \cdot [\lambda \cdot x]^{\gamma}\,, & \text{for } \gamma < 1 \text{ and } \gamma \neq 0\,, \\ \ln(\lambda \cdot x)\,, & \text{for } \gamma = 0\,. \end{cases} \tag{1}$$

A simple specification of λ_{is} might set

[2] See the discussion, for example, in Rust and Phelan [1997, p.786], Hurd [1996].

$$\lambda_{is} = \begin{cases} 1 & \text{if both spouses work in the labor market,} \\ \lambda^F \geq 1 & \text{if only the husband works in the labor market,} \\ \lambda^M \geq 1 & \text{if only the wife works in the labor market,} \\ \lambda^M \cdot \lambda^F & \text{if neither spouse works in the labor market,} \end{cases} \tag{2}$$

For a more detailed motivation, suppose a household produces service flows from consumption expenditure and time at home — with the latter valued either as leisure or the opportunity for home production — using a Cobb–Douglas technology

$$c_{is} = f(x_{is}\,,\,\ell^F_{is}\,,\,\ell^M_{is}) \equiv [x_{is}]^{\alpha_1} \cdot [\ell^F_{is}]^{\alpha_2} \cdot [\ell^M_{is}]^{\alpha_3}\,, \quad \alpha_j \in (0\,,\,1)\,, \quad \sum \alpha_j = 1\,, \tag{3}$$

where ℓ^F (ℓ^M) denotes female (male) time at home. Auerbach and Kotlikoff [1987] and Altig et al. [2001], for example, use a similar formulation — assuming $f(x\,,\,\ell)$ is CES; Nishiyama and Smetters [2005], Cooley and Prescott [1995], and French [2005], on the other hand, make $f(x\,,\,\ell)$ Cobb–Douglas. Normalize $\ell^F_{is} = 1$ for ages with labor–force participation; set $\ell^F_{is} = \bar{\ell}^F > 1$ otherwise; and, similarly for ℓ^M_{is}. Let "$\sim$" mean "induces the same preference ordering." Then the relationship of (2)-(3) is

$$\frac{1}{\bar{\gamma}} \cdot [f(x_{is}\,,\,\ell^F_{is}\,,\,\ell^M_{is})]^{\bar{\gamma}} = \frac{\alpha_1}{\alpha_1 \cdot \bar{\gamma}} \cdot \left[[x_{is}] \cdot [\ell^F_{is}]^{\frac{\alpha_2}{\alpha_1}} \cdot [\ell^M_{is}]^{\frac{\alpha_3}{\alpha_1}}\right]^{\alpha_1 \cdot \bar{\gamma}} \sim \frac{1}{\gamma} \cdot \left[\lambda_{is} \cdot x_{is}\right]^{\gamma}$$
$$\text{provided} \quad \gamma = \alpha_1 \cdot \bar{\gamma}\,, \quad \lambda^F = [\bar{\ell}^F]^{\frac{\alpha_2}{\alpha_1}}\,, \quad \text{and} \quad \lambda^M = [\bar{\ell}^M]^{\frac{\alpha_3}{\alpha_1}}\,. \tag{4}$$

Other possible interpretations of (2) include: (i) household utility from market expenditures rises upon retirement because a household no longer incurs wardrobe and transportation costs of going to work (Cogan [1981]); or, (ii) a household's ability to enjoy market expenditure rises at retirement because a household can relocate and completely control its schedule (Hamermesh [2005]). In terms of likely parameter magnitudes, interpretation (i) *a priori* suggests $\lambda^M = 1.0$ to 1.1. Expressions (3)-(4), on the other hand, imply a larger value. If time available for market and home production and leisure is, say, $7 \times 12 = 84$ hrs/wk, then $\ell^M = 84/44 \approx 1.9$; hence, under (4), λ^M could easily be 2.0 or more. Our results (see below) tend, on this basis, to point away from interpretation (i). Interpretation (ii) suggests that a couple may gain most from the retirement of its second adult, and we return to that possibility below.

To identify the role of family composition in household behavior, our analysis must include comparisons of singles and couples; in practice, moreover, virtually all couples eventually face widowhood. In the spirit of (3)-(4), suppose the technology for an unmarried man or woman is

$$c^{single}_{is} = x_{is} \cdot \lambda^{single}_{is}\,, \tag{5}$$

$$\lambda^{single}_{is} \equiv \begin{cases} 1\,, & \text{if working in the labor market,} \\ \lambda^* > 1\,, & \text{otherwise.} \end{cases}$$

Upon marriage, suppose market consumption is a public good, nonmarket time is not, and household utility depends on the average

$$\begin{aligned} c_{is} &= \sqrt{(x_{is} \cdot \lambda_{is}^{single,\ male}) \cdot (x_{is} \cdot \lambda_{is}^{single,\ female})} \\ &= x_{is} \cdot [\lambda_{is}^{single,\ male}]^{1/2} \cdot [\lambda_{is}^{single,\ female}]^{1/2} \,. \end{aligned}$$

Then for the couple,

$$c_{is} = x_{is} \cdot \lambda_{is}^{couple} \,, \tag{6}$$

$$\lambda_{is}^{couple} \equiv \begin{cases} 1 \,, & \text{if both spouses work in labor market,} \\ \lambda^* \,, & \text{if only one spouse works in labor market,} \\ [\lambda^*]^2 \,, & \text{if neither spouse works in labor market.} \end{cases}$$

Formulation (5)-(6) has symmetry across households — in the sense that fully working or fully non-working households have the same λ_{is} regardless of marital status — and it has symmetric treatment of spouses within couples. In the notation of (2), it imposes

$$\lambda^F = \lambda^M = \lambda^* \,.$$

Hamermesh's argument that a household enjoys extra gains from having both spouses free of market–work obligations suggests a slight generalization:

$$\lambda_{is}^{single} = \begin{cases} 1 \,, & \text{if working,} \\ \lambda > 1 \,, & \text{if not working,} \end{cases} \tag{7}$$

$$\lambda_{is}^{couple} = \begin{cases} 1 \,, & \text{if both spouses working,} \\ \lambda^* \geq 1 \,, & \text{if only one spouse works,} \\ \lambda > \lambda^* \,, & \text{if neither works.} \end{cases} \tag{8}$$

If retirement for both spouses together provides a premium, then

$$[\lambda^*]^2 < \lambda \,.$$

For a second interpretation of the generalization, suppose that a couple "retires" when both spouses cease market work, but prior to that time the wife might engage in either market work or home production, with the latter yielding benefits to the whole family. In Laitner and Silverman [2008], work and home production are taken to be interchangeable in this circumstance and $\lambda^* = 1$; House, Laitner, and Stolyarov [2008] model a continuous time–allocation choice for married women between market work and home production with nonlinear losses from time away from home. With indivisible market workdays, this paper seeks to capture the home–production tradeoff using (7)-(8) with

$$\lambda^* \geq 1 \,, \quad \lambda > \lambda^* \,, \quad \text{and} \quad [\lambda^*]^2 > (<) \, \lambda \,. \tag{9}$$

In the end, our favored specification is (7)-(9).

Dynamic Model I. We consider two life–cycle model specifications. This preliminary draft emphasizes the first, which has perfect capital markets.

When the wife works, she accumulates experiential human capital H_{is}:[3]

$$\dot{H}_{is} = p_{is} \quad \text{with} \quad H_{i,S_i} = 0\,. \tag{10}$$

If g is the economy's rate of labor–augmenting technological progress, A_i registers the wife's idiosyncratic earning ability, and the household reached its starting age S_i at B_i, we assume

$$y^F_{is} = A_i \cdot e^{g\cdot(s-S_i+B_i)} \cdot y(H_{is})\,, \tag{11}$$

where $y(.)$ is a known, nondecreasing function.[4]

In our first life–cycle specification, household i solves

$$U(S_i\,,\,B_i) \equiv \max_{x_{is},p_{is},R_i} \int_{S_i}^{R_i} e^{-\rho\cdot s} \cdot N_{is} \cdot u(\lambda_{is} \cdot \frac{x_{is}}{N_{is}})\,ds\,, \tag{12}$$

$$\text{subject to:} \quad \dot{a}_{is} = r \cdot a_{is} + y^M_{is} + p_{is} \cdot y^F_{is} - x_{is}\,,$$

$$\dot{H}_{is} = p_{is}\,,$$

$$\lambda_{is} = \lambda^* \cdot p_{is} + (1 - p_{is}) \quad \text{for} \quad s < R_i\,,$$

$$\lambda_{is} = \lambda \quad \text{for} \quad s \geq R_i\,,$$

$$y^F_{is} = A_i \cdot e^{g\cdot(s-S_i+B_i)} \cdot y(H_{is})\,,$$

$$y^M_{is} = 0 \quad \text{for} \quad s > R_i\,,$$

$$a_{i0} = 0 \quad \text{and} \quad H_{i,S_i} = 0\,.$$

Actual female earnings are

$$\bar{y}^F_{is} = p_{is} \cdot y^F_{is}\,.$$

[3] We ignore male human capital accumulation since it adds nothing to the analysis.

[4] One could, without invalidating the procedure below, also multiply the right–hand side of (11) by a function of age — perhaps reflecting declining productivity at advanced age. Such a function might capture the typical concave quadratic shape of most estimated earning dynamics profiles, and it might provide an additional reason for retirement.

Dynamic Model II. Our second life–cycle specification assumes no borrowing without tangible collateral. In other words, we maintain (12) but incorporate into it a liquidity constraint

$$a_{is} \geq 0 \quad \text{all} \quad s\,. \tag{13}$$

Life–cycle Consumption Expenditure. The first proposition characterizes a household's optimal expenditure path given p_{is} and R_i — but absent (13).

Proposition 1: *Let $(x_{is}\,,\, p_{is}\,,\, R_i)$ be the optimal solution to household problem (12). Let Y_{is} be the present value of lifetime earnings of household i at age s, so that*

$$Y_{i0} = \int_{S_i}^{T} e^{-r\cdot s} \cdot (y_{is}^M + p_{is} \cdot y_{is}^F)\, ds \quad \textit{with} \quad y_{is}^M = 0 \quad \textit{for} \quad s \geq R_i\,.$$

Then

$$x_{is} = \Big[\frac{Y_{i0}}{\int_{S_i}^{T} e^{\sigma\cdot s} \cdot N_{is} \cdot [\lambda_{is}]^{\frac{\gamma}{1-\gamma}}\, ds}\Big] \cdot N_{is} \cdot [\lambda_{is}]^{\frac{\gamma}{1-\gamma}} \cdot e^{(r+\sigma)\cdot s}\,, \quad s \in (S_i\,,\, R_i)\,, \tag{14}$$

where

$$\sigma \equiv -r + \frac{r-\rho}{1-\gamma}\,.$$

Proof: See Appendix I.

Proposition 1 shows that optimal expenditure per adult equivalent is piecewise exponential, with discontinuities occurring at ages when λ_{is} or N_{is} shift due to participation, retirement, or household–composition changes. The optimal x_{is} will tend to jump at the latter ages. Let Δ_{is} be the expenditure jump at age s for household i. From (14),

$$\Delta_{is} = \frac{c_{i,s+0}/N_{i,s+0}}{c_{i,s-0}/N_{i,s-0}} = \Big[\frac{\lambda_{i,s+0}}{\lambda_{i,s-0}}\Big]^{\frac{\gamma}{1-\gamma}}\,, \tag{15}$$

where the notation $s+0$ $(s-0)$ denotes ages arbitrarily after (before) s. When λ_{is} rises (from retirement or non–participation), expenditure per adult equivalent decreases (increases) when $\gamma < 0$ $(\gamma > 0)$. Most empirical studies point to a value $\gamma < 0$. In that range, the implication of (15) seems consistent with other evidence. For example, provided $\gamma < 0$, the model predicts a decline in household expenditure at retirement, which accords with some recent observations (e.g., Hurd and Rohwedder [2003], Aguiar and Hurst [2005], Laitner and Silverman [2005, 2008], Haider and Stephens [2004], and others[5]). Similarly, examining household balance sheets at retirement, House et al. [2008] find evidence that couples with working wives consume more prior to retirement.

[5] See also business–cycle frequency analysis in Hall [2008].

An intuitive explanation for the nature of the relationship between Δ_{is} and λ_{is} is as follows. In a period of life with a higher λ, a household can utilize market goods more effectively. That tends to make it want to allocate more expenditure to such ages. The curvature of the utility function, on the other hand, makes the household want to equalize $\lambda \cdot x$ at different ages. If $\gamma > 0$, the former tendency predominates. If, however, $\gamma < 0$, the severity of curvature is such that the household's desire to smooth $\lambda \cdot x$ leads it to desire to cut x when λ is high.[6]

Female Labor–Force Participation. Overall, labor force participation increases a household's earnings but reduces utility generated per unit of expenditure. The next proposition derives a necessary condition for optimal female labor–force participation that captures this tradeoff.

Proposition 2: *Let $(x_{is}\,,\,p_{is})$ be the optimal solution to household life–cycle problem (12) given R_i. Assume a wife changes her labor–force participation a finite number of times. At each age of change s,*

$$\Delta p_{is} \equiv |p_{i,s+0} - p_{i,s-0}| = 1\,.$$

Otherwise, $\Delta p_{is} = 0$. Let

$$w_{is}^F \equiv \int_s^T e^{-r\cdot(t-s)} \cdot p_{it} \cdot \frac{\partial\, y_{it}^F}{\partial\, H_{it}}\, dt\,.$$

If $\Delta p_{is} = 0$ and $s < R_i$, we have

$$p_{is} = 1\ (0) \Longleftrightarrow$$
$$\int_{S_i}^T e^{\sigma\cdot s} \cdot N_{is} \cdot [\lambda_{is}]^{\frac{\gamma}{1-\gamma}}\, ds \geq\ (\leq) \frac{1-\gamma}{\gamma} \cdot N_{is} \cdot e^{(\sigma+r)\cdot s} \cdot \frac{Y_{i0}}{y_{is}^F + w_{is}^F} \cdot ([\lambda^*]^{\frac{\gamma}{1-\gamma}} - 1)\,. \quad (16)$$

When $\Delta p_{is} = 1$ and $s < R_i$, there is equality in (16).

Proof: See Appendix I.

We can use Proposition 1 to simplify the expressions.

Corollary 2.1: *Let $(x_{is}\,,\,p_{is})$ be the optimal solution to household life–cycle problem (12) given R_i. Assume a wife changes her labor–force participation a finite number of times. Then*

[6] Heckman [1974] provides an interesting contrast. He assumes a general temporally non–separable utility specification. He argues as follows: suppose that time at home and consumption expenditure are substitutes. Suppose that an individual's wages peak at ages 40-60. Then work hours will peak at the latter ages, and households may consume more at the same ages to compensate themselves for less time at home. His reasoning depends on variable hours of work, whereas we assume discrete work options.

$$\Delta p_{is} = 0\,, \quad s < R_i\,, \quad p_{is} = 1 \Longrightarrow \frac{y_{is}^F + w_{is}^F}{x_{is}} \geq \frac{1-\gamma}{\gamma} \cdot ([\lambda^*]^{\frac{\gamma}{1-\gamma}} - 1)\,, \tag{17}$$

$$\Delta p_{is} = 0\,, \quad s < R_i\,, \quad p_{is} = 0 \Longrightarrow \frac{y_{is}^F + w_{is}^F}{x_{is}} \leq \frac{1-\gamma}{\gamma} \cdot \frac{[\lambda^*]^{\frac{\gamma}{1-\gamma}} - 1}{[\lambda^*]^{\frac{\gamma}{1-\gamma}}}\,, \tag{18}$$

$$\Delta p_{is} = 1\,, \quad s < R_i\,, \quad p_{i,s-0} = 1 \Longrightarrow \frac{y_{is}^F + w_{is}^F}{x_{i,s-0}} = \frac{1-\gamma}{\gamma} \cdot ([\lambda^*]^{\frac{\gamma}{1-\gamma}} - 1)\,, \tag{19}$$

$$\Delta p_{is} = 1\,, \quad s < R_i\,, \quad p_{i,s-0} = 0 \Longrightarrow \frac{y_{is}^F + w_{is}^F}{x_{i,s-0}} = \frac{1-\gamma}{\gamma} \cdot \frac{[\lambda^*]^{\frac{\gamma}{1-\gamma}} - 1}{[\lambda^*]^{\frac{\gamma}{1-\gamma}}}\,. \tag{20}$$

Proof: See Propositions 1-2.

To interpret Proposition 2 and Corollary 2.1, notice that w_{is}^F is the present value at age s of the marginal product of experiential human capital. Setting $p_{is} = 1$ over $[s\,,\, s + ds)$ shifts up the woman's earning profile thereafter, raising her human capital by ds. The woman benefits from this increment to her human capital at every future time that she works.

Consider (17), for example. Female participation at age s yields a dollar reward at rate $y_{is}^F + w_{is}^F$, where the first component measures explicit, current earnings and the second the shadow value of current investment. Multiplying by marginal utility, $[N_{is}]^{1-\gamma} \cdot [\lambda_{is}]^{\gamma} \cdot [x_{is}]^{\gamma-1}$, values the reward in utility units. Expression (17) compares the value of more market work against the value of more time at home. If the woman stops working in the market, her household can produce more utility from a given amount of expenditure on consumption, and the gain is proportional to current utility — $[N_{is}]^{1-\gamma} \cdot [\lambda_{is}]^{\gamma} \cdot [x_{is}]^{\gamma}/\gamma$. Thus, if

$$\frac{(y_{is}^F + w_{is}^F) \cdot [N_{is}]^{1-\gamma} \cdot [\lambda_{is}]^{\gamma} \cdot [x_{is}]^{\gamma-1}}{[N_{is}]^{1-\gamma} \cdot [\lambda_{is}]^{\gamma} \cdot [x_{is}]^{\gamma}/\gamma} = \frac{y_{is}^F + w_{is}^F}{x_{is}/\gamma}$$

exceeds a threshold (dependent on λ and γ), the woman will want to work in the market.

We can see the key role of λ^* in determining female labor force participation as follows. We have

$$\ln([\lambda]^{\frac{\gamma}{1-\gamma}}) \approx \ln(1) + 1 \cdot ([\lambda]^{\frac{\gamma}{1-\gamma}} - 1) = [\lambda]^{\frac{\gamma}{1-\gamma}} - 1\,.$$

So,

$$\frac{1-\gamma}{\gamma} \cdot ([\lambda]^{\frac{\gamma}{1-\gamma}} - 1) \approx \frac{1-\gamma}{\gamma} \cdot \ln([\lambda]^{\frac{\gamma}{1-\gamma}}) = \ln(\lambda)\,.$$

Thus, condition (i) of Corollary 2.1, for example, implies

$$\frac{y_{is}^F + w_{is}^F}{x_{is}} \approx \ln(\lambda^*) \approx \lambda^* - 1\,. \tag{21}$$

Another potentially important factor is current family size. A higher N_{is} at a given age s makes x_{is} higher — according to (14). A high x_{is} makes participation less likely in each condition of Corollary 2.1. In our life–cycle formulation, household's allocate the most resources to ages with the largest family size. Proposition 2 shows the same applies to allocation of women's time at home.

In the end, a woman's participation is more likely, *cet par*, if her market earnings are high. And, she is more likely to participate at one age if she anticipates participation in the future. Her participation is less likely if the family's total lifetime resources (i.e., Y_0) are high or if the current family size (i.e., N_{is}) is high. Finally, if $r > \rho$ and $\gamma < 0$, a household's optimal consumption expenditure rises with age. That makes the marginal utility of earnings fall with s, reducing women's incentives to participate in the labor force at older ages.

Implementing Corollary 2.1 requires computing w_{is}^F. Fortunately, the next corollary shows that we can use integration by parts to derive an expression for w_{is}^F that depends only on observable variables.

Corollary 2.2: *Let $(x_{is}\,,\,p_{is})$ be the optimal solution to household life–cycle problem (12) given R_i. Let $\mathcal{J}(p_{is}\,,\,t)$ index the ages at which the wife changes her labor–force participation. Assume the set of changes is finite. Let*

$$\mathcal{J}(p_{is}\,,\,t) = \{j = 0, 1, ..., J \,|\, s_0 = t;\; s_J = T\,, s_{j+1} > s_j, \Delta p_{is} = 1 \Leftrightarrow s = s_j\}\,,$$

so that $\mathcal{J}(p_{is}\,,\,t)$ provides a partition

$$[s_0\,,\,s_1) \cup [s_1\,,\,s_2) \cup ... \cup [s_{J-1}\,,\,s_J) = [t\,,\,T)$$

with, for each j, either $p_{is} = 1$ or $p_{is} = 0$ for all $s \in [s_j\,,\,s_{j+1})$. Let $\mathcal{J}^+(p_{is}\,,\,t) \subset \mathcal{J}(p_{is}\,,\,t)$ be such that $j \in \mathcal{J}^+(p_{is}\,,\,t) \Longrightarrow p_{is} = 1$ all $s \in [s_j\,,\,s_{j+1})$. Then

$$w_{is} = e^{r\cdot s} \cdot \Big[\sum_{j \in \mathcal{J}^+(p_{is}\,,\,s)} [e^{-r\cdot s_{j+1}} \cdot \bar{y}_{i,s_{j+1}}^F - e^{-r\cdot s_j} \cdot \bar{y}_{i,s_j}^F] + (r-g) \cdot \int_s^T e^{-r\cdot z} \cdot \bar{y}_{iz}^F \, dz \Big]\,. \quad (22)$$

Proof: See Appendix I.

<u>Male Retirement</u>. The male retirement–decision problem resembles the female participation choice, though the male problem is somewhat simpler because the shadow price of human capital is zero at retirement. We have

Proposition 3. *Let R_i be the optimal male retirement age for couple i. For simplicity, assume that N_{is} is continuous in s at $s = R_i$. Let $R = R_i$.*
(i) If $\Delta p_{iR} = 0$ and $p_{iR} = 0$, then

$$\int_{S_i}^T e^{\sigma\cdot s} \cdot N_{is} \cdot [\lambda_{is}]^{\frac{\gamma}{1-\gamma}}\, ds = \frac{1-\gamma}{\gamma} \cdot N_{iR} \cdot e^{(\sigma+r)\cdot R} \cdot \frac{Y_0}{y_{iR}^M} \cdot \big[[\lambda]^{\frac{\gamma}{1-\gamma}} - [\lambda^*]^{\frac{\gamma}{1-\gamma}}\big]\,. \quad (23)$$

Or, using Proposition 1,

$$\frac{y_{iR}^M}{x_{i,R-0}} = \frac{1-\gamma}{\gamma} \cdot \left[[\frac{\lambda}{\lambda^*}]^{\frac{\gamma}{1-\gamma}} - 1 \right].$$

(ii) If $\Delta p_{iR} = 1$ *and* $p_{i,R-0} = 1$*, then*

$$\int_{S_i}^{T} e^{\sigma \cdot s} \cdot N_{is} \cdot [\lambda_{is}]^{\frac{\gamma}{1-\gamma}} \, ds = \frac{1-\gamma}{\gamma} \cdot N_{iR} \cdot e^{(\sigma+r)\cdot R} \cdot \frac{Y_0}{y_{iR}^M + y_{iR}^F + w_{iR}^F} \cdot \left[[\lambda]^{\frac{\gamma}{1-\gamma}} - 1 \right]. \quad (24)$$

Or, using Proposition 1,

$$\frac{y_{iR}^M + y_{iR}^F + w_{iR}^F}{x_{i,R-0}} = \frac{1-\gamma}{\gamma} \cdot \left[[\lambda]^{\frac{\gamma}{1-\gamma}} - 1 \right].$$

Proof: See Appendix I.

Using the same approximation as in (21), condition (i) of Proposition 3, for example, implies

$$\frac{y_{iR}^M}{x_{i,R-0}} \approx \ln(\frac{\lambda}{\lambda^*}) \approx \frac{\lambda}{\lambda^*} - 1. \quad (25)$$

Our model of retirement works as follows. Prior to retirement, the left–hand side of either expression in (25) exceeds the right. With maturity, growth of y_{is}^M moderates. Eventually, male earnings may level off or decline. If $r > \rho$, it is also true that x_{is} grows with age. Provided the denominator of the left side above grows faster than the numerator, eventually a household reaches an optimal retirement age (or dies).

Condition (25) illustrates the homotheticity of our model: raising men and women's wage rates at all ages by z, changes the numerator and denominator of (25) by the same percentage — leaving the optimal R unaffected. Furthermore, a comparison of (21) and (25) shows why married women tend to retire before their husbands even if $\lambda^M = \lambda^F$ (or $[\lambda^*]^2 = \lambda$): a lower wage with the same household consumption yields a lower optimal retirement age. (Parenthetically, (25) also that single men and women will tend to retire at the same age if their wage profiles are parallel — regardless of the levels of their profiles.)

4 Liquidity Constraints

We can distinguish two broad categories of life–cycle–model specification in the recent literature.

One resembles (12). Examples include Auerbach and Kotlikoff [1987] and Altig et al. [2001]. The second adds moment–by–moment liquidity constraints (13). Examples include Mariger [1986], Zeldes [1989], and Hubbard and Judd [1986]. Many models in the second vein also incorporate stochastic earnings. See, for example, Deaton [1991], Hubbard et al. [1995], and Gourinchas and Parker [2002]. The frequency and possible duration of very low earning samplings, and the nature of fall–back resources — e.g., public transfers (Hubbard et al. [1995]) or family line inter vivos gifts (Laitner [1992]) — in the event of bad

luck, become topics of potentially great importance. Additional complications can arise. At a minimum level, a household with no liquid assets may have to pay check cashing fees, etc. In the middle class, lack of net worth may preclude access to the mortgage market or lead to higher interest rates on credit, etc. And, households may face uncertainty about expenses as well as earnings.

Coincidentally, parameter values in the first two literatures tend to differ rather substantially. Laitner and Silverman [2008] provides estimates for a formulation of the first type. Proposition 1 shows that consumption expenditure in the absence of family composition or work–status changes tends to grow at rate $(r-\rho)/(1-\gamma)$. Laitner and Silverman's estimate is $(r-\rho)/(1-\gamma) \approx 0.025$. This is roughly in line with calibrations in Auerbach and Kotlikoff [1987], Altig et al. [2001], and Nishiyama and Smetters [2005]. Gourinchas and Parker [2002], in contrast, estimate a formulation with (13) and earnings uncertainty, and they find $(r-\rho)/(1-\gamma) < 0$. This is similar to many calibrations in the second literature. Empirical evidence seems to point to an average consumption expenditure profile that rises with age until mid adulthood and then falls. Laitner and Silverman's estimates track the rise with $(r-\rho)/(1-\gamma) > 0$, and the fall with children leaving home and declining expenditure due to retirement. Gourinchas and Parker attribute the rise in youth to liquidity constraints, which cause expenditure to mimic rising (with age) earning profiles. Thereafter $(r-\rho)/(1-\gamma) < 0$ dominates.

The model of this paper potentially has a bearing on the importance of liquidity constraints. (i) Our specification of λ_{is} can, if $\gamma < 0$, explain a general drop in household consumption at retirement. Otherwise, $(r-\rho)/(1-\gamma) < 0$ may be necessary — see the discussion above. With our specification, one cannot dismiss the unconstrained model simply on the basis of the general shape of the aggregative consumption expenditure profile with respect to age. (ii) If married women typically participate in the labor market for only a fraction of their lives, they may adjust their dates of participation to alleviate (13).[7]

Discussion. The discussion of parameter values here has a potentially important bearing on the reason people retire. Consider (25). If $(r-\rho)/(1-\gamma) < 0$, the denominator on the left–hand side is falling with age. In that case, the impetus for retirement must be an even faster decline in y_{is}^M. Presumably the idea would be that aging eventually causes productivity to fall, which impels retirement. On the other hand, if $(r-\rho)/(1-\gamma) > 0$, the left–hand side denominator rises with age. Even if male productivity only levels off, rather than declines, with age, a rising x_{is} can then impel retirement. Put differently, it seems that a specification with $(r-\rho)/(1-\gamma) < 0$ almost forces declining health to govern retirement, whereas under the alternative, older households may choose retirement to make optimal use of their high consumption late in life.

In historical perspective, the second case in the preceding sentence suggests that retirement is a threshold phenomenon. Suppose that male earnings tend to rise to a peak

[7] Unfortunately, this does not necessarily mean that low participation women optimally must participate early in life if they participate at all — if they do not participate when they have young children, for instance, but participate heavily after their children have matured, the shadow price w_{is}^F in Corollary 2.1 may make participation attractive only late in life.

and then level off. With short lifespans, almost no one may reach the trigger for retirement in (25). Increasing longevity, however, eventually allows many men to reach a high x_{is} relative to y_{is}^M. Then "retirement" should become a common occurrence.

5 Estimation

We propose to estimate our model's parameters from two data sources. The first is consumption expenditure data from the annual Consumer Expenditure Survey (CEX). The second is data on year–by–year household earnings, family composition, and labor supply from the Health and Retirement Study (HRS).

Block-1 Estimation Procedure. Proposition 1 yields one set of estimation conditions as follows.

Consider household i at age s and time t, where s is the age of the adult female or the single adult male. Let its consumption expenditure be x_{ist}. Define

$$\chi^{female}(i,s,t) \equiv \begin{cases} 1 & \text{if adult female works 0 hrs/week at age } s, \\ 0 & \text{otherwise,} \end{cases}$$

$$\chi^{male}(i,s,t) \equiv \begin{cases} 1 & \text{if adult male is retired at household age } s, \\ 0 & \text{otherwise.} \end{cases}$$

Condition (14) implies

$$\begin{aligned} \ln(x_{ist}) - \ln(x_{i,s-1,t-1}) = \frac{r-\rho}{1-\gamma} + \ln(n_{ist}) - \ln(n_{i,s-1,t-s}) + \\ \frac{\gamma}{1-\gamma} \cdot \ln(\lambda^F) \cdot [\chi^{female}(i,s,t) - \chi^{female}(i,s-1,t-1)] + \\ \frac{\gamma}{1-\gamma} \cdot \ln(\lambda^M) \cdot [\chi^{male}(i,s,t) - \chi^{male}(i,s-1,t-1)] \,. \end{aligned}$$

Treating the influence of family composition as in Tobin [1967], let

$$N_{ist} = 1 + \chi^{spouse}(i,s,t) \cdot \xi^{spouse} + \chi^{kids}(i,s,t) \cdot \xi^{kids} \,, \tag{26}$$

where

$$\chi^{spouse}(i,s,t) \equiv \begin{cases} 1\,, & \text{if the household includes a spouse when it is age } s, \\ 0\,, & \text{otherwise,} \end{cases}$$

$$\chi^{kids}(i,s,t) \equiv \text{number of "kids" ages 0-25 present.}$$

To be more precise, with our CPS data below we include kids up to age 25 who are listed as members of their parents' household; with our HRS data, we assume kids leave home at age $\max\{18, \text{yrs education} + 6\}$, though not later than 25. We also follow the Social Security system in only counting kids currently present up to a maximum of 2. And, χ^{kids}

"weights" kids 0-5 with 1/3, kids 6-17 with 2/3, and kids 18+ with 1 — see, for example, Attanaio et al. [2008]. Approximate (26) with

$$\ln(n_{ist}) \approx \chi^{spouse}(i,s,t) \cdot \xi^{spouse} + \chi^{kids}(i,s,t) \cdot \xi^{kids} \,. \tag{27}$$

Define

$$\beta^{CEX} \equiv (\beta_1^{CEX}\,,\, \beta_2^{CEX}\,,\, \beta_3^{CEX}\,,\, \beta_4^{CEX}\,,\, \beta_5^{CEX}) \equiv \\ \big(\frac{r-\rho}{1-\gamma}\,,\, \xi^{spouse}\,,\, \xi^{kids}\,,\, \frac{\gamma}{1-\gamma} \cdot \ln(\lambda^F)\,,\, \frac{\gamma}{1-\gamma} \cdot \ln(\lambda^M)\big)\,.$$

Employing (26)-(27), the equation that we estimate is

$$\begin{aligned} &\ln(x_{ist}) - \ln(x_{i,s-1,t-1}) = \\ &\quad \beta_1^{CEX} + \beta_2^{CEX} \cdot [\chi^{spouse}(i,s,t) - \chi^{spouse}(i,s-1,t-1)] + \\ &\quad \beta_3^{CEX} \cdot [\chi^{kids}(i,s,t) - \chi^{kids}(i,s-1,t-1)] + \\ &\quad \beta_4^{CEX} \cdot [\chi^{female}(i,s,t) - \chi^{female}(i,s-1,t-1)] + \\ &\quad \beta_5^{CEX} \cdot [\chi^{male}(i,s,t) - \chi^{male}(i,s-1,t-1)]\,. \end{aligned} \tag{28}$$

We use combined diary and survey CEX cross–sectional data.[8] Although this does not provide observations on individual households (since the diaries and surveys apply to different respondents), the sample size is large enough to provide means $\bar{x}_{st}$ for different (age , $year$) cells. Letting $\bar{\chi}^{spouse}(s\,,\,t)$, etc., be corresponding averages, we estimate (28) using

$$\begin{aligned} &\ln(\bar{x}_{ist}) - \ln(\bar{x}_{i,s-1,t-1}) = \beta_1^{CEX} + \beta_2^{CEX} \cdot [\bar{\chi}^{spouse}(i,s,t) - \bar{\chi}^{spouse}(i,s-1,t-1)] + \\ &\quad \beta_3^{CEX} \cdot [\bar{\chi}^{kids}(i,s,t) - \bar{\chi}^{kids}(i,s-1,t-1)] + \\ &\quad \beta_4^{CEX} \cdot [\bar{\chi}^{female}(i,s,t) - \bar{\chi}^{female}(i,s-1,t-1)] + \\ &\quad \beta_5^{CEX} \cdot [\bar{\chi}^{male}(i,s,t) - \bar{\chi}^{male}(i,s-1,t-1)] + \epsilon_{st}^1 - \epsilon_{s-1,t-1}^1\,, \end{aligned} \tag{29}$$

where we will think of ϵ_{st}^1 as measurement error in $\ln(\bar{x}_{st})$.

<u>Block-1 Estimation Results</u>. As described above, the CEX data yields a pseudo panel. It is difficult to eliminate non–married households because of selection issues and because we have no way of determining past and future marital status for a given household. Fortunately, Proposition 1 should hold regardless of marital status. See the discussion of preferences above.

Tables 1-4 present preliminary estimates from (29) alone. The dependent variable comes from CEX data for ages 25-69 and years 1984-2001. We use CEX data on spouse present. The remaining independent variables come from Section 2's CPS data — the

[8] See Laitner and Silverman [2005].

fraction of households with children present, of adult females each year with zero labor–force participation, and of adult males retired.[9] As above, "age" is wife's age in the case of couples. As stated, although our theoretical analysis focuses exclusively on married couples, the CEX data is a random sample of all households. As in Section 2, we avoid ages under 25 because of selection problems.[10]

The tables consider ages 25-69, 30-69, ... , and 50-69, and 25-64, ... , 50-64. Older starting ages lessen the chance of inconsistent estimates due to liquidity constrained behavior; younger starting ages yield a larger sample. It is also the case that children's birth ages tend to be easily observable whereas children's age of separation from their parent household may well be ambiguous; therefore, there is a possible quality advantage for youthful households in this respect. In both the CPS and HRS (see below) we drop observations above age 69 because a disconcerting fraction of males remain unretired, calling into question the meaning of "retirement." Since we have classified our Block-1 data by female age, a top age of 64 eliminates many 66-69 year old men.

Table 1 estimates β_1^{CEX} to β_5^{CEX}; Table 2 imposes $\lambda = [\lambda]^2$. A likelihood ratio test never rejects the second specification. In general, β_4^{CEX} and β_5^{CEX} are imprecisely estimated. Table 2 often finds $\beta_5^{CEX} \approx 0$, which is consistent with $\lambda > 1$ only when $\gamma = 0$. Tables 3-4 are similar, though they generally suggest β_5^{CEX} is -0.10 to -0.15. A Chow test never rejects maintaining the five youngest ages in columns 2-6; thus, the test does not provide reason to think that liquidity constraint (13) binds at youthful ages. Likewise, the point estimates of β_1^{CEX} are as likely to increase with exclusion of the youngest ages as to decrease. If (13) binds at first and then becomes slack, one might expect β_1^{CEX} to fall as we move to the right.

Looking at the first three columns of Table 4, we find estimates $(r-\rho)/(1-\gamma) \approx 0.023$ to 0.031. This yields optimal expenditure profiles that rise with age — as is characteristic of the Auerbach–Kotlikoff strand of literature reviewed in Section 4. The spousal weight is about 0.35, which implies more increasing returns from household size than the treatment of widows in the Social Security system. The parameter estimates imply optimal household expenditure rises when men and women have less time at home. Propositions 2-3 show this is consistent with (7)-(9) provided $\gamma < 0$.

For comparison, Laitner and Silverman [2008,Tab.4] estimates $\beta_1^{CEX} = 0.0262$, $\beta_2^{CEX} = 0.3632$, $\beta_3^{CEX} = 0.1345$, and $(\gamma/(1-\gamma)) \cdot \ln(\lambda) = -0.2562$ — with $(\gamma/(1-\gamma)) \cdot \ln(\lambda^*)$ constrained to equal 0.

To attempt to obtain more precise estimates, we turn to Blocks 2-3. They will also allow us to recover the underlying structural parameters ρ, γ, λ^*, and λ.

<u>Block-3 Estimation Procedure</u>. Propositions 2-3 yield two other sets of estimation conditions. Begin with Proposition 3.

Let

[9] Laitner and Silverman [2005, 2008] discuss problems with CEX measures of retirement. We use CPS female participation data out of the same concern with accuracy.

[10] In practice, college educated men and women may typically form their households in their mid 20s or beyond. At earlier ages, household heads have less education and hence earn and consume less on average.

Table 1A. "Kids" Regressor Covers Child Ages 0-25
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Estimating λ^* and λ; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 25-69	Ages 30-69	Ages 35-69
$\beta_1^{CEX}=\frac{r-\rho}{1-\gamma}$	0.0232 *** [0.0009/ 26.4221]	0.0232 *** [0.0012/ 19.4395]	0.0276 *** [0.0017/ 15.7730]
$\beta_2^{CEX}=\xi^{spouse}$	0.3338 *** [0.0539/ 6.1934]	0.3304 *** [0.0597/ 5.5385]	0.3816 *** [0.0641/ 5.9486]
$\beta_3^{CEX}=\xi^{kids}$	0.3227 *** [0.0174/ 18.5083]	0.3335 *** [0.0219/ 15.2485]	0.4010 *** [0.0293/ 13.6656]
$\beta_4^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda^*)$	-0.0443 [0.1238/ -0.3580]	-0.1256 [0.1434/ -0.8756]	-0.1049 [0.1585/ -0.6616]
$\beta_5^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)$	-0.0372 [0.0444/ -0.8387]	-0.0475 [0.0475/ -0.9981]	-0.0757 [0.0508/ -1.4913]
Summary Statistics [transformed data]			
SSR	6.0568	5.5997	5.0348
Observations	765.0000	680.0000	595.0000
MSE	0.0080	0.0083	0.0085
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	2.53	2.53	3.01
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2)=[(SSR(J-1)-SSR(J))/N1]/MSE(J)$ where $N1=OBS(J-1)-OBS(J)\,,\,N2=OBS(J)-5$			
F-Stat	NA	0.6482	0.7789
P-Value	NA	0.9933	0.9242

Source: see text. Significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 1B. "Kids" Regressor Covers Child Ages 0-25
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Estimating λ^* and λ; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 40-69	Ages 45-69	Ages 50-69
$\beta_1^{CEX}=\frac{r-\rho}{1-\gamma}$	0.0307 *** [0.0030/ 10.1971]	0.0253 *** [0.0044/ 5.7126]	0.0186 *** [0.0048/ 3.8410]
$\beta_2^{CEX}=\xi^{spouse}$	0.3929 *** [0.0715/ 5.4989]	0.3450 *** [0.0802/ 4.3020]	0.3474 *** [0.0850/ 4.0893]
$\beta_3^{CEX}=\xi^{kids}$	0.4397 *** [0.0404/ 10.8814]	0.4080 *** [0.0467/ 8.7337]	0.4486 *** [0.0634/ 7.0799]
$\beta_4^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda^*)$	-0.1143 [0.1813/ -0.6304]	-0.0355 [0.2084/ -0.1705]	0.1506 [0.2376/ 0.6340]
$\beta_5^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)$	-0.1200 * [0.0695/ -1.7265]	-0.0003 [0.1079/ -0.0030]	0.2565 ** [0.1291/ 1.9863]
Summary Statistics [transformed data]			
SSR	4.6735	4.0919	3.3093
Observations	510.0000	425.0000	340.0000
MSE	0.0093	0.0097	0.0099
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	3.41	2.76	2.11
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2)=[(SSR(J-1)-SSR(J))/N1]/MSE(J)$ where $N1=OBS(J-1)-OBS(J)\,,\,N2=OBS(J)-5$			
F-Stat	0.4592	0.7024	0.9319
P-Value	1.0000	0.9761	0.6454

Source: see text. Significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 2A. "Kids" Regressor Covers Child Ages 0-25 GLS Coefficients Equation (29); CEX Data 1984-2001; Specification Constraining $\lambda^* = \sqrt{\lambda}$; No Time Dummies [Estimated Parameter (Std. Error/T Stat.)]			
Parameter	Ages 25-69	Ages 30-69	Ages 35-69
$\beta_1^{CEX} = \frac{r-\rho}{1-\gamma}$	0.0233 *** [0.0007/ 35.8760]	0.0239 *** [0.0008/ 28.5945]	0.0281 *** [0.0014/ 20.2103]
$\beta_2^{CEX} = \xi^{spouse}$	0.3313 *** [0.0527/ 6.2817]	0.3269 *** [0.0595/ 5.4974]	0.3806 *** [0.0641/ 5.9402]
$\beta_3^{CEX} = \xi^{kids}$	0.3234 *** [0.0172/ 18.8503]	0.3392 *** [0.0206/ 16.4935]	0.4060 *** [0.0273/ 14.8864]
$\beta_5^{CEX} = \frac{\gamma}{1-\gamma} \cdot \ln(\lambda)$	-0.0332 [0.0407/ -0.8157]	-0.0303 [0.0421/ -0.7198]	-0.0643 [0.0443/ -1.4501]
Summary Statistics [transformed data]			
SSR	6.0572	5.6047	5.0366
Observations	765.0000	680.0000	595.0000
MSE	0.0080	0.0083	0.0085
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	2.54	2.60	3.07
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2) = [(SSR(J-1) - SSR(J))/N1]/MSE(J)$ where $N1 = OBS(J-1) - OBS(J)\,,\, N2 = OBS(J) - 5$			
F-Stat	NA	0.6420	0.7843
P-Value	NA	0.9942	0.9184
Likelihood Ratio Test: $F = [SSR(Tab\ 2) - SSR(Tab\ 1)]/MSE(Tab\ 1)$			
F-Stat	0.0515	0.6063	0.2145
P-Value	0.8206	0.4364	0.6435

Source: see text. Test significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 2B. "Kids" Regressor Covers Child Ages 0-25
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Constraining $\lambda^* = \sqrt{\lambda}$; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 40-69	Ages 45-69	Ages 50-69
$\beta_1^{CEX} = \frac{r-\rho}{1-\gamma}$	0.0311 ***	0.0255 ***	0.0185 ***
	[0.0028/ 11.0781]	[0.0043/ 5.8840]	[0.0047/ 3.9219]
$\beta_2^{CEX} = \xi^{spouse}$	0.3918 ***	0.3438 ***	0.3482 ***
	[0.0713/ 5.4941]	[0.0799/ 4.3046]	[0.0846/ 4.1172]
$\beta_3^{CEX} = \xi^{kids}$	0.4438 ***	0.4104 ***	0.4478 ***
	[0.0385/ 11.5335]	[0.0451/ 9.1059]	[0.0629/ 7.1247]
$\beta_5^{CEX} = \frac{\gamma}{1-\gamma} \cdot \ln(\lambda)$	-0.1096 *	0.0078	0.2520 **
	[0.0620/ -1.7660]	[0.0994/ 0.0786]	[0.1218/ 2.0684]
Summary Statistics [transformed data]			
SSR	4.6746	4.0922	3.3094
Observations	510.0000	425.0000	340.0000
MSE	0.0092	0.0097	0.0098
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	3.46	2.78	2.10
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2) = [(SSR(J-1) - SSR(J))/N1]/MSE(J)$ where $N1 = OBS(J-1) - OBS(J)\,,\,N2 = OBS(J) - 5$			
F-Stat	0.4611	0.7048	0.9350
P-Value	1.0000	0.9750	0.6384
Likelihood Ratio Test: $F = [SSR(Tab\ 2) - SSR(Tab\ 1)]/MSE(Tab\ 1)$			
F-Stat	0.1102	0.0380	0.0111
P-Value	0.7400	0.8456	0.9161

Source: see text. Test significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 3A. "Kids" Regressor Child Ages 0-25, Half Weight Ages 0-5 GLS Coefficients Equation (29); CEX Data 1984-2001; Specification Estimating λ^* and λ; No Time Dummies [Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 25-64	Ages 30-64	Ages 35-64
$\beta_1^{CEX}=\frac{r-\rho}{1-\gamma}$	0.0235 *** [0.0009/ 26.8911]	0.0233 *** [0.0012/ 19.4881]	0.0276 *** [0.0018/ 15.3091]
$\beta_2^{CEX}=\xi^{spouse}$	0.3506 *** [0.0610/ 5.7465]	0.3329 *** [0.0683/ 4.8739]	0.3625 *** [0.0734/ 4.9370]
$\beta_3^{CEX}=\xi^{kids}$	0.2987 *** [0.0187/ 15.9642]	0.3083 *** [0.0233/ 13.2095]	0.3839 *** [0.0327/ 11.7411]
$\beta_4^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda^*)$	-0.0702 [0.1241/ -0.5652]	-0.1700 [0.1448/ -1.1737]	-0.1527 [0.1613/ -0.9461]
$\beta_5^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)$	-0.1222 *** [0.0471/ -2.5956]	-0.1302 ** [0.0506/ -2.5714]	-0.1430 *** [0.0536/ -2.6695]
Summary Statistics [transformed data]			
SSR	5.1511	4.7086	4.1577
Observations	680.0000	595.0000	510.0000
MSE	0.0076	0.0080	0.0082
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	2.56	2.54	3.02
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2)=[(SSR(J-1)-SSR(J))/N1]/MSE(J)$ where $N1=OBS(J-1)-OBS(J)\,,\,N2=OBS(J)-5$			
F-Stat	NA	0.6522	0.7873
P-Value	NA	0.9924	0.9133

Source: see text. Significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 3B. "Kids" Regressor Child Ages 0-25, Half Weight Ages 0-5
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Estimating λ^* and λ; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 40-64	Ages 45-64	Ages 50-64
$\beta_1^{CEX}=\frac{r-\rho}{1-\gamma}$	0.0344 ***	0.0368 ***	0.0240 **
	[0.0037/ 9.3038]	[0.0077/ 4.7839]	[0.0093/ 2.5814]
$\beta_2^{CEX}=\xi^{spouse}$	0.3536 ***	0.3054 ***	0.3166 ***
	[0.0816/ 4.3312]	[0.0933/ 3.2718]	[0.1026/ 3.0871]
$\beta_3^{CEX}=\xi^{kids}$	0.4757 ***	0.4921 ***	0.4488 ***
	[0.0534/ 8.9134]	[0.0813/ 6.0515]	[0.1043/ 4.3020]
$\beta_4^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda^*)$	-0.1885	-0.2322	0.0012
	[0.1861/ -1.0127]	[0.2249/ -1.0326]	[0.2684/ 0.0046]
$\beta_5^{CEX}=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)$	-0.2239 ***	-0.2820 *	0.0521
	[0.0721/ -3.1047]	[0.1473/ -1.9148]	[0.1882/ 0.2767]
Summary Statistics [transformed data]			
SSR	3.7728	3.2096	2.4980
Observations	425.0000	340.0000	255.0000
MSE	0.0090	0.0096	0.0100
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	3.97	4.36	2.61
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2)=[(SSR(J-1)-SSR(J))/N1]/MSE(J)$ where $N1=OBS(J-1)-OBS(J)\,,\,N2=OBS(J)-5$			
F-Stat	0.5041	0.6916	0.8379
P-Value	0.9999	0.9789	0.8291

Source: see text. Significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 4A. "Kids" Regressor Child Ages 0-25, Half Weight Ages 0-5
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Constraining $\lambda^* = \sqrt{\lambda}$; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 25-64	Ages 30-64	Ages 35-64
$\beta_1^{CEX=\frac{r-\rho}{1-\gamma}}$	0.0235 ***	0.0239 ***	0.0282 ***
	[0.0006/ 36.7049]	[0.0008/ 28.8897]	[0.0014/ 19.5547]
$\beta_2^{CEX=\xi^{spouse}}$	0.3496 ***	0.3288 ***	0.3606 ***
	[0.0598/ 5.8464]	[0.0681/ 4.8294]	[0.0733/ 4.9193]
$\beta_3^{CEX=\xi^{kids}}$	0.2989 ***	0.3141 ***	0.3901 ***
	[0.0185/ 16.1750]	[0.0222/ 14.1717]	[0.0307/ 12.7041]
$\beta_5^{CEX=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)}$	-0.1209 ***	-0.1130 **	-0.1291 ***
	[0.0441/ -2.7418]	[0.0458/ -2.4699]	[0.0472/ -2.7335]
Summary Statistics [transformed data]			
SSR	5.1511	4.7137	4.1602
Observations	680.0000	595.0000	510.0000
MSE	0.0076	0.0080	0.0082
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	2.56	2.61	3.09
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2) = [(SSR(J-1) - SSR(J))/N1]/MSE(J)$ where $N1 = OBS(J-1) - OBS(J)\,,\,N2 = OBS(J) - 5$			
F-Stat	NA	0.6453	0.7919
P-Value	NA	0.9935	0.9078
Likelihood Ratio Test: $F = [SSR(Tab\ 2) - SSR(Tab\ 1)]/MSE(Tab\ 1)$			
F-Stat	0.0052	0.6290	0.3049
P-Value	0.9423	0.4281	0.5811

Source: see text. Test significance: * 10 percent, ** 5 percent, *** 1 percent.

Table 4B. "Kids" Regressor Child Ages 0-25, Half Weight Ages 0-5
GLS Coefficients Equation (29); CEX Data 1984-2001;
Specification Constraining $\lambda^* = \sqrt{\lambda}$; No Time Dummies
[Estimated Parameter (Std. Error/T Stat.)]

Parameter	Ages 40-64	Ages 45-64	Ages 50-64
$\beta_1^{CEX=\frac{r-\rho}{1-\gamma}}$	0.0350 ***	0.0371 ***	0.0240 ***
	[0.0035/ 9.9464]	[0.0077/ 4.8435]	[0.0093/ 2.5961]
$\beta_2^{CEX=\xi^{spouse}}$	0.3511 ***	0.3012 ***	0.3156 ***
	[0.0814/ 4.3140]	[0.0928/ 3.2446]	[0.1019/ 3.0969]
$\beta_3^{CEX=\xi^{kids}}$	0.4818 ***	0.4967 ***	0.4490 ***
	[0.0517/ 9.3275]	[0.0807/ 6.1571]	[0.1041/ 4.3125]
$\beta_5^{CEX=\frac{\gamma}{1-\gamma}\cdot\ln(\lambda)}$	-0.2093 ***	-0.2585 *	0.0583
	[0.0646/ -3.2393]	[0.1388/ -1.8627]	[0.1792/ 0.3256]
Summary Statistics [transformed data]			
SSR	3.7747	3.2118	2.4981
Observations	425.0000	340.0000	255.0000
MSE	0.0090	0.0096	0.0100
Addendum: 40-Year Growth Factor Implied by Estimated β_1^{CEX}			
Growth Factor	4.05	4.41	2.61
Chow Test: current column (col J) vs preceding column $F(N1\,,\,N2) = [(SSR(J-1) - SSR(J))/N1]/MSE(J)$ where $N1 = OBS(J-1) - OBS(J)\,,\,N2 = OBS(J) - 5$			
F-Stat	0.5058	0.6927	0.8437
P-Value	0.9999	0.9784	0.8194
Likelihood Ratio Test: $F = [SSR(Tab\ 2) - SSR(Tab\ 1)]/MSE(Tab\ 1)$			
F-Stat	0.2093	0.2317	0.0120
P-Value	0.6476	0.6306	0.9129

Source: see text. Test significance: * 10 percent, ** 5 percent, *** 1 percent.

$$\Lambda \equiv [\lambda]^{\frac{\gamma}{1-\gamma}}\,, \quad \Lambda^* \equiv [\lambda^*]^{\frac{\gamma}{1-\gamma}}\,, \quad \Gamma \equiv \frac{1-\gamma}{\gamma}\,.$$

Looing at (23), for example, define

$$f_i \equiv \frac{y_{iR}^M}{x_{i,R-0}} - \Gamma \cdot [\frac{\Lambda}{\Lambda^*} - 1]\,. \tag{30}$$

We can construct a value for f_i for each household i in our HRS data. In theory, since the male retires at age $R = R_i$, we should have $f_i = 0$ — if $f_i > 0$, the male should have stayed at work; if $f_i < 0$, he should have retired earlier. Suppose that in practice, however, the male's action depends upon F_i where

$$F_i = f_i + \nu_i\,, \quad \nu_i \sim N(0\,,\,\sigma_\nu^2)\,. \tag{31}$$

Possible interpretations: (i) employer wishes (including proffered buyout), conditions in the male's occupation and industry, etc., imply the trigger for change in participation status is $f_i + \nu_i = 0$ rather than $f_i = 0$, where ν_i may be positive or negative; (ii) we have

$$\ln(\lambda_i) = \ln(\overline{\lambda}) + \nu_i\,,$$

where $\overline{\lambda}$ is the population average, on which f_i is based, but λ_i is the actual parameter for household i; or, (iii) we have

$$y_{is}^{M,actual} = y_{is}^{M,measured} + y_{is}^{M,measured} \cdot \nu_{is}$$

with $y_{is}^{M,measured} \cdot \nu_{is}$ = measurement error and ν_{is} uncorrelated with $y_{is}^{M,measured}$. Under any of these interpretations, ν_i is orthogonal to f_i.

In the case of voluntary retirement,

$$F_i = 0 \Longleftrightarrow \nu_i = -f_i\,.$$

Disability or death may cause "involuntary retirement." Or, a male may not attain his optimal retirement age in sample, in which case let R correspond to the last observable age. With either involuntary retirement or no retirement in sample, optimization implies $F_i \geq 0$. We have

$$F_i \geq 0 \Longleftrightarrow f_i + \nu_i \geq 0 \Longleftrightarrow \nu_i \geq -f_i\,.$$

Letting $\phi(.,\sigma^2)$ be the normal density (as above), define

$$m_i^3 \equiv 1_{\{\substack{voluntary\\ retirement}\}} \cdot [-f_i] + 1_{\{\substack{involuntary\ retirement\ or\\ never\ retires\ in\ sample}\}} \cdot \frac{\int_{-f_i}^{\infty} e \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-f_i}^{\infty} \phi(e\,,\,\sigma_\nu^2)\,de}\,,$$

$$m_i^{3*} \equiv 1_{\{\substack{voluntary\\ retirement}\}} \cdot [-f_i]^2 + 1_{\{\substack{involuntary\ retirement\ or\\ never\ retires\ in\ sample}\}} \cdot \frac{\int_{-f_i}^{\infty} e^2 \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-f_i}^{\infty} \phi(e\,,\,\sigma_\nu^2)\,de} - \sigma_\nu^2\,.$$

Then

$$E_i[m_i^3] = E[\nu] = 0 \quad \text{and} \quad E_i[m_i^{3*}] = E\big[[\nu]^2\big] - \sigma_\nu^2 = 0\,. \tag{32}$$

<u>Digression</u>. Although Proposition 3 considers cases in which males and females adjust their participation together, our specification can always break simultaneous actions into separate components. In Proposition 3, case (ii), for instance,

$$\begin{aligned}
&\frac{y_{iR}^M + y_{iR}^F + w_{iR}^F}{x_{i,R-0}} = \Gamma \cdot [\Lambda - 1] \Longleftrightarrow \\
&\frac{y_{iR}^M}{x_{i,R-0}} + \frac{y_{iR}^F + w_{iR}^F}{x_{i,R-0}} = \Gamma \cdot [\frac{\Lambda}{\Lambda^*} - 1] \cdot \Lambda^* + \Gamma \cdot [\Lambda^* - 1] \Longleftrightarrow \\
&\{\frac{y_{iR}^M}{\tilde{x}_{i,R-0}} - \Gamma \cdot [\frac{\Lambda}{\Lambda^*} - 1]\} \cdot \Lambda^* + \{\frac{y_{iR}^F + w_{iR}^F}{x_{i,R-0}} - \Gamma \cdot [\Lambda^* - 1]\} = 0\,,
\end{aligned}$$

where $\tilde{x}$ is household expenditure after the woman retires, i.e.,

$$\tilde{x}_{i,R-0} \equiv x_{i,R-0} \cdot \Lambda^*\,.$$

When FOC's hold with equality for each spouse, then Proposition 3 is satisfied.

<u>Block-2 Estimation Procedure</u>. Continuing as above, define

$$g_{is} \equiv \begin{cases} \frac{y_{is}^F + w_{is}^F}{x_{i,s-0}} - \Gamma \cdot [\Lambda^* - 1] & \text{if } p_{i,s-0} = 1\,, \\ \frac{y_{is}^F + w_{is}^F}{x_{i,s-0}} - \Gamma \cdot \frac{\Lambda^* - 1}{\Lambda^*} & \text{if } p_{i,s-0} = 0\,. \end{cases}$$

Let female decisions be based upon

$$G_{is} = g_{is} + \eta_{is}\,, \quad \eta_{is} \sim N(0\,,\,\sigma_\eta^2)\,. \tag{33}$$

As above, assume g_{is} and η_{is} are orthogonal.

Define

$$m_{is}^2 \equiv 1_{\{\substack{\Delta p_{is}=0,\\ p_{is}=1}\}} \cdot \frac{\int_{-g_{is}}^{\infty} e \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-g_{is}}^{\infty} \phi(e\,,\,\sigma_\nu^2)\,de} + 1_{\{\substack{\Delta p_{is}=0,\\ p_{is}=0}\}} \cdot \frac{\int_{-\infty}^{-g_{is}} e \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-\infty}^{-g_{is}} \phi(e\,,\,\sigma_\nu^2)\,de} + 1_{\{\Delta p_{is}=1\}} \cdot [-g_{is}]\,,$$

$$\begin{aligned}
m_{is}^{2*} \equiv\; & 1_{\{\substack{\Delta p_{is}=0,\\ p_{is}=1}\}} \cdot \frac{\int_{-g_{is}}^{\infty} [e]^2 \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-g_{is}}^{\infty} \phi(e\,,\,\sigma_\nu^2)\,de} + 1_{\{\substack{\Delta p_{is}=0,\\ p_{is}=0}\}} \cdot \frac{\int_{-\infty}^{-g_{is}} [e]^2 \cdot \phi(e\,,\,\sigma_\nu^2)\,de}{\int_{-\infty}^{-g_{is}} \phi(e\,,\,\sigma_\nu^2)\,de} + \\
& 1_{\{\Delta p_{is}=1\}} \cdot [-g_{is}]^2 - \sigma_\eta^2\,.
\end{aligned}$$

Then

$$E_{is}[m_{is}^2] = E[\eta] = 0 \quad \text{and} \quad E_{is}[m_{is}^{2*}] = E\big[[\eta]^2\big] - \sigma_\eta^2 = 0\,. \tag{34}$$

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Appendix I

Proof of Proposition 1: For notational simplicity, suppress i. Set up a Hamiltonian

$$\mathcal{H}_s = e^{-\rho \cdot s} \cdot N_s \cdot u(\lambda_s \cdot \frac{x_s}{N_s}) + \mu_s \cdot [r \cdot a_s + y_s^M + p_s \cdot y_s^F - x_s] + \nu_s \cdot p_s \,,$$

with y_s^F obeying (11). Proposition 1 optimizes with respect to x_s, treating p_s and R as fixed.

First–order conditions imply

$$\dot{\mu}_s = -\frac{\partial \mathcal{H}_s}{\partial a_s} \Longleftrightarrow \dot{\mu}_s = -r \cdot \mu_s \Longleftrightarrow \mu_s = \mu_0 \cdot e^{-r \cdot s} \,,$$

$$e^{-\rho \cdot s} \cdot \lambda_s \cdot u'(\lambda_s \cdot \frac{x_s}{N_s}) = \mu_s \Longleftrightarrow e^{-\rho \cdot s} \cdot \lambda_s \cdot [\lambda_s \cdot \frac{x_s}{N_s}]^{\gamma - 1} = \mu_0 \cdot e^{-r \cdot s} \Longleftrightarrow$$
$$\frac{x_s}{N_s} = e^{\frac{r-\rho}{1-\gamma} \cdot s} \cdot [\lambda]^{\frac{\gamma}{1-\gamma}} \cdot [\mu_0]^{\frac{-1}{1-\gamma}} \,. \qquad (A1)$$

Integrating the budget constraint,

$$Y_0 = \int_S^T e^{-r \cdot s} \cdot x_s \, ds \,.$$

The last two expressions yield

$$[\mu_0]^{\frac{-1}{1-\gamma}} = \frac{Y_0}{\int_S^T N_s \cdot [\lambda_s]^{\frac{\gamma}{1-\gamma}} \cdot e^{\sigma \cdot s} \, ds} \,. \qquad (A2)$$

(A1)-(A2) yield (14). ∎

Proof of Proposition 2: For simplicity, suppress i. Write the Hamiltonian for problem (12), with R_i fixed, as

$$\mathcal{H}_s = e^{-\rho \cdot s} \cdot N_s \cdot u(\lambda_s \cdot \frac{x_s}{N_s}) + \mu_s \cdot [r \cdot a_s + y_s^M + p_s \cdot y_s^F - x_s] + \nu_s \cdot p_s \,.$$

The co-state equations are

$$\dot{\mu}_s = -\frac{\partial \mathcal{H}_s}{\partial a} \Longleftrightarrow \mu_s = \mu_0 \cdot e^{-r \cdot s} \,, \qquad (A3)$$

$$\dot{\nu}_s = -\frac{\partial \mathcal{H}_s}{\partial H} = -\mu_s \cdot p_s \cdot \frac{\partial y_s^F}{\partial H_s} \,. \qquad (A4)$$

The first–order condition for control variable x_s is

$$e^{-\rho \cdot s} \cdot \lambda_s \cdot [\lambda_s \cdot \frac{x_s}{N_s}]^{\gamma - 1} = \mu_s \Longleftrightarrow x_s = x^*(s \,, \lambda_s) \equiv N_s \cdot \left[\mu_s \cdot e^{\rho \cdot s} \cdot [\lambda_s]^{-\gamma}\right]^{\frac{1}{\gamma - 1}} \,. \qquad (A5)$$

Step 1. For any $s < R$, we compare $\mathcal{H}_s|_{p_s=0}$ and $\mathcal{H}_s|_{p_s=1}$.

For $p_s = 0$, we have $\lambda_s = \lambda^F$, $x_s = x^*(s\,,\,\lambda^F)$, and

$$\begin{aligned}
\mathcal{H}_s|_{p_s=0} &= \frac{e^{-\rho\cdot s}\cdot N_s}{\gamma}\cdot[\lambda^F\cdot\frac{x_s}{N_s}]^\gamma + \mu_s\cdot[r\cdot a_s + y_s^M - x_s] \\
&= \frac{e^{-\rho\cdot s}\cdot[N_s]^{1-\gamma}}{\gamma}\cdot[\lambda^F]^\gamma\cdot[N_s]^\gamma\cdot\left[\mu_s\cdot e^{\rho\cdot s}\cdot[\lambda^F]^{-\gamma}\right]^{\frac{\gamma}{\gamma-1}} + \mu_s\cdot[r\cdot a_s + y_s^M] \\
&\qquad - \mu_s\cdot N_s\cdot\left[\mu_s\cdot e^{\rho\cdot s}\cdot[\lambda^F]^{-\gamma}\right]^{\frac{1}{\gamma-1}} \qquad (see\ A5) \\
&= \frac{1-\gamma}{\gamma}\cdot e^{\frac{-\rho\cdot s}{1-\gamma}}\cdot N_s\cdot[\lambda^F]^{\frac{\gamma}{1-\gamma}}\cdot[\mu_s]^{\frac{-\gamma}{1-\gamma}} + \mu_s\cdot[r\cdot a_s + y_s^M]\,.
\end{aligned}$$

Similarly, if $p_s = 1$, then $\lambda_s = 1$, $x_s = x^*(s\,,\,1)$, and

$$\mathcal{H}_s|_{p_s=1} = \frac{1-\gamma}{\gamma}\cdot e^{\frac{-\rho\cdot s}{1-\gamma}}\cdot N_s\cdot[\mu_s]^{\frac{-\gamma}{1-\gamma}} + \mu_s\cdot[r\cdot a_s + y_s^M] + \mu_s\cdot y_s^F + \nu_s\,.$$

So,

$$\begin{aligned}
&\mathcal{H}_s|_{p_s=1} \geq \mathcal{H}_s|_{p_s=0} \\
&\quad\Longleftrightarrow \mu_s\cdot y_s^F + \nu_s \geq \frac{1-\gamma}{\gamma}\cdot e^{\frac{-\rho\cdot s}{1-\gamma}}\cdot N_s\cdot[\mu_s]^{\frac{-\gamma}{1-\gamma}}\cdot\left[[\lambda^F]^{\frac{\gamma}{1-\gamma}} - 1\right] \\
&\quad\Longleftrightarrow y_s^F + \frac{\nu_s}{\mu_s} \geq \frac{1-\gamma}{\gamma}\cdot e^{-\frac{\rho\cdot s}{1-\gamma}}\cdot N_s\cdot[\mu_s]^{-\frac{1}{1-\gamma}}\cdot\left[[\lambda^F]^{\frac{\gamma}{1-\gamma}} - 1\right] \\
&\quad\Longleftrightarrow y_s^F + \frac{\nu_s}{\mu_s} \geq \frac{1-\gamma}{\gamma}\cdot e^{-\frac{\rho\cdot s}{1-\gamma}}\cdot N_s\cdot e^{\frac{r\cdot s}{1-\gamma}}\cdot[\mu_0]^{-\frac{1}{1-\gamma}}\cdot\left[[\lambda^F]^{\frac{\gamma}{1-\gamma}} - 1\right] \quad (see\ A3) \\
&\quad\Longleftrightarrow y_s^F + \frac{\nu_s}{\mu_s} \geq \\
&\qquad \frac{1-\gamma}{\gamma}\cdot e^{\frac{(r-\rho)\cdot s}{1-\gamma}}\cdot N_s\cdot\frac{Y_0}{\int_S^T N_s\cdot[\lambda_s]^{\frac{\gamma}{1-\gamma}}\cdot e^{\sigma\cdot s}\,ds}\cdot\left[[\lambda^F]^{\frac{\gamma}{1-\gamma}} - 1\right]. \quad (see\ A2)
\end{aligned}$$

Step 2. For an optimum, $\nu_T = 0$. Thus, (A4) implies

$$\begin{aligned}
\nu_T - \nu_0 &= -\int_t^T \mu_s\cdot p_s\cdot\frac{\partial y_s^F}{\partial H_s}\,ds \\
\Longleftrightarrow \nu_t &= \mu_0\cdot\int_t^T e^{-r\cdot s}\cdot p_s\cdot\frac{\partial y_s^F}{\partial H_s}\,ds\,. \qquad (see\ A3)
\end{aligned}$$

Step 3. Steps 1-2 establish (16).

If $s > R_i$, we revise Step 1. We drop y_s^M. When $p_s = 0$, $\lambda_s = \lambda^F\cdot\lambda^M$. When $p_s = 1$, $\lambda_s = \lambda^M$. Step 2 remains the same. This establishes (17). ∎

Proof of Corollary 2.2: Recalling the definition of y_s^F in (12), we have

$$\begin{aligned} w_s^F &= \sum_{j \in \mathcal{J}^+(p_{it}\,,\,s)} \int_{s_j}^{s_{j+1}} e^{-r\cdot(t-s)} \cdot \frac{\partial y_t^F}{\partial H_t}\, dt \\ &= \sum_{j \in \mathcal{J}^+(p_{it}\,,\,s)} \int_{s_j}^{s_{j+1}} e^{-r\cdot(t-s)} \cdot A \cdot e^{g\cdot(t-S+B)} \cdot y'(H_t)\, dt\,. \end{aligned}$$

Integrating by parts,

$$\int_{s_j}^{s_{j+1}} e^{-(r-g)\cdot t} \cdot y'(H_t)\, dt = e^{-(r-g)\cdot t} \cdot y(H_t)|_{s_j}^{s_{j+1}} + (r-g) \cdot \int_{s_j}^{s_{j+1}} e^{-(r-g)\cdot t} \cdot y(H_t)\, dt\,.$$

Combining the last two expressions, we have (22). ▮

Proof of Proposition 3: For simplicity, suppress i. Let $p_s^M = 1$ for ages s when the male works in the market, and $p_s^M = 0$ at other ages. If the male works, his earnings are y_s^M. His actual age-s earnings are $p_s^M \cdot y_s^M$. Write the Hamiltonian for problem (12)

$$\mathcal{H}_s = e^{-\rho\cdot s} \cdot N_s \cdot u(\lambda_s \cdot \frac{x_s}{N_s}) + \mu_s \cdot [r \cdot a_s + p_s^M \cdot y_s^M + p_s \cdot y_s^F - x_s] + \nu_s \cdot p_s\,.$$

(A3)-(A5) are as in the proof of Proposition 2.

<u>Step 1</u>. Suppose $R = R_i$, $\Delta p_{iR} = 0$, and $p_{iR} = 0$. Then letting $x_s = x^*(x\,,\,\lambda_s)$ as in the proof of Proposition 2, we have

$$\begin{aligned} \mathcal{H}_s|_{p_s^M=1} &= \frac{e^{-\rho\cdot s} \cdot N_s}{\gamma} \cdot [\lambda^F \cdot \frac{x_s}{N_s}]^\gamma + \mu_s \cdot [r \cdot a_s + y_s^M - x_s] \\ &= \frac{e^{-\rho\cdot s} \cdot [N_s]^{1-\gamma}}{\gamma} \cdot [\lambda^F]^\gamma \cdot [N_s]^\gamma \cdot \left[\mu_s \cdot e^{\rho\cdot s} \cdot [\lambda^F]^{-\gamma}\right]^{\frac{\gamma}{\gamma-1}} + \\ &\qquad \mu_s \cdot [r \cdot a_s + y_s^M] - \mu_s \cdot N_s \cdot \left[\mu_s \cdot e^{\rho\cdot s} \cdot [\lambda^F]^{-\gamma}\right]^{\frac{1}{\gamma-1}} \qquad (see\ A5) \\ &= \frac{1-\gamma}{\gamma} \cdot e^{\frac{-\rho\cdot s}{1-\gamma}} \cdot N_s \cdot [\lambda^F]^{\frac{\gamma}{1-\gamma}} \cdot [\mu_s]^{\frac{-\gamma}{1-\gamma}} + \mu_s \cdot [r \cdot a_s + y_s^M]\,. \qquad (A6) \end{aligned}$$

Similarly,

$$\mathcal{H}_s|_{p_s^M=0} = \frac{1-\gamma}{\gamma} \cdot e^{\frac{-\rho\cdot s}{1-\gamma}} \cdot N_s \cdot [\lambda^F \cdot \lambda^M]^{\frac{\gamma}{1-\gamma}} \cdot [\mu_s]^{\frac{-\gamma}{1-\gamma}} + \mu_s \cdot [r \cdot a_s]\,. \qquad (A7)$$

The following are necessary conditions:

$$\mathcal{H}_{R-0}|_{p^M_{R-0}=1} \geq \mathcal{H}_{R-0}|_{p^M_{R-0}=0}\,, \tag{A8}$$

$$\mathcal{H}_{R+0}|_{p^M_{R+0}=1} \leq \mathcal{H}_{R+0}|_{p^M_{R+0}=0}\,. \tag{A9}$$

(A6)-(A7) and (A3) and our assumption about N_s imply that the right and left–hand sides, respectively, of (A8)-(A9) are continuous in s; hence,

$$\mathcal{H}_{R-0}|_{p^M_{R-0}=1} = \mathcal{H}_{R+0}|_{p^M_{R+0}=0} \tag{A10}$$

is necessary. Using (A6)-(A7), and noting that N_s and μ_s are continuous at R,

$$\begin{aligned}
&\mathcal{H}_{R-0}|_{p^M_{R-0}=1} = \mathcal{H}_{R+0}|_{p^M_{R+0}=0} \Longleftrightarrow \\
&\quad \mu_R \cdot y^M_{R-0} + N_R \cdot [\mu_R]^{-\frac{\gamma}{1-\gamma}} \cdot e^{-\frac{\rho\cdot R}{1-\gamma}} \cdot [\lambda^F]^{\frac{\gamma}{1-\gamma}} \cdot \frac{1-\gamma}{\gamma} = \\
&\qquad N_R \cdot [\mu_R]^{-\frac{\gamma}{1-\gamma}} \cdot e^{-\frac{\rho\cdot R}{1-\gamma}} \cdot [\lambda^F \cdot \lambda^M]^{\frac{\gamma}{1-\gamma}} \cdot \frac{1-\gamma}{\gamma} \Longleftrightarrow \\
&\quad \mu_R \cdot y^M_{R-0} = \frac{1-\gamma}{\gamma} \cdot N_R \cdot [\mu_R]^{-\frac{\gamma}{1-\gamma}} \cdot e^{-\frac{\rho\cdot R}{1-\gamma}} \cdot \left[[\lambda^F \cdot \lambda^M]^{\frac{\gamma}{1-\gamma}} - [\lambda^F]^{\frac{\gamma}{1-\gamma}}\right] \Longleftrightarrow \\
&\quad [\mu_R]^{\frac{1}{1-\gamma}} \cdot y^M_{R-0} = \frac{1-\gamma}{\gamma} \cdot N_R \cdot e^{-\frac{\rho\cdot R}{1-\gamma}} \cdot \left[[\lambda^F \cdot \lambda^M]^{\frac{\gamma}{1-\gamma}} - [\lambda^F]^{\frac{\gamma}{1-\gamma}}\right] \Longleftrightarrow \\
&\quad \frac{y^M_{R-0}}{Y_0 / \int_S^T N_s \cdot [\lambda_s]^{\frac{\gamma}{1-\gamma}} \cdot e^{\sigma\cdot s}\, ds} = \\
&\qquad \frac{1-\gamma}{\gamma} \cdot N_R \cdot e^{\frac{r-\rho}{1-\gamma}\cdot R} \cdot \left[[\lambda^F \cdot \lambda^M]^{\frac{\gamma}{1-\gamma}} - [\lambda^F]^{\frac{\gamma}{1-\gamma}}\right]. \qquad (see\ A2-A3)
\end{aligned}$$

<u>Steps 2-4</u>. Cases (ii)-(iv) are analogous. ∎