Real Balance Effects in a Zero Interest Rate Environment

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Abstract

What are the economic effects of a large increase in real balances of money in a zero interest rate environment? This paper develops a computable overlapping generations model with real balance effects to answer this question. In our model there are two sources of real balance effects: finite lifespans and borrowing constraints. We find that the zero lower bound on the nominal interest rate creates an asymmetry in the welfare costs of inflation. A monetary aggregate targeting policy that is too tight has very large and negative effects on steady-state welfare. A loose monetary aggregate targeting policy has only modest effects on welfare. Results from a dynamic analysis using data from Japan finds that the effects of "quantitative easing" vary significantly depending on the timing and duration of the policy. Quantitative easing also has large and *heterogeneous* effects on consumption of cohorts of different ages.

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1 Introduction

This paper uses an economic model to assess the quantitative effects of a large increase in real balances of narrow money on the price level, economic activity and welfare in a zero nominal interest rate environment. We will refer to such a monetary policy as "quantitative easing."[1] This topic is motivated by recent experiences in Japan and the United States. In Japan after the nominal interest rate fell to zero in 1999, the Bank of Japan adopted a quantitative easing monetary policy that saw the ratio of M0 to GNP rise from 0.13 to 0.22. Since the end of 2008 monetary policy in the United States has exhibited a similar pattern: a Federal Funds rate or nearly zero and a large increase in the ratio of M0 to GDP from about 6 percent to over 12 percent.

What does economic theory say about such a monetary policy? When the nominal interest rate is zero money and short-term government bonds are perfect substitutes. In the infinite horizon models that are commonly used to analyze the effects of monetary policy on economic activity a change in the timing of total government liabilities (including money) and lump-sum taxes has no real effects on real activity. Krugman (1998) refers to this phenomenon as a liquidity trap. Ireland (2005) shows that allowing for positive population growth in a Blanchard (1985) model with infinite lived overlapping generations induces a real balance effect that breaks Ricardian equivalence. Auerbach and Obstfeld (2005) find that large open-market purchases of bonds can counteract deflationary price tendencies and lower the real value of government debt if households expect that the nominal interest rate will eventually rise above zero. Lower government debt reduces the need to tax and this raises household welfare. A limitation of this previous work is that it proposes avenues whereby real balances might matter but does not provide a quantitative assessment of the empirical magnitude of real balance effects.

The objective of this paper is to begin to bridge this gap. We consider real balance effects in an overlapping generations (OG) model. This is an attractive framework for our question. There are two factors that produce real balance effects in our model. One mechanism arises because Individuals are finite lived. With finite lifetimes the timing of government borrowing can affect their present value tax liabilities and induce wealth effects. A second channel arises from the presence of borrowing constraints. When an individual faces a binding borrowing constraint, lowering taxes increases disposable income and current consumption.

We use computational methods to solve for the equilibrium. This approach makes it possible to entertain empirically relevant model time periods and compare model prices and allocations directly with actual data. We assume the model period is a year, and that households are active for 80 years.

A steady-state analysis of the model indicates that a zero nominal interest rate is a good monetary policy that maximizes average welfare. However, the welfare costs of small deviations from this policy are asymmetric. On the one

[1] Quantitative easing was the term used by the Bank of Japan to describe the monetary policy it pursued from March 19, 2001 thru March 2006.

hand, the welfare cost of too much money growth is moderate. When the growth rate of money is too high money is dominated in rate of return. Households' efforts to economize on their holdings of money limit the crowding out effect of money on capital.

On the other hand, welfare declines very sharply as the growth rate of money is lowered from its optimal value. Once the nominal interest rate reaches zero money competes directly with capital for household savings. In this situation lowering the growth rate of money increases the real return on holding money and directly crowds out private capital.

This asymmetry in the welfare cost of inflation provides a rationale for choosing a high growth rate of money in a zero nominal interest rate environment. To see why this is the case note that the specific growth rate of money that maximizes steady-state welfare depends on the parameters of the model. If there is uncertainty about the specific values of these parameters then the value of the welfare maximizing growth rate of money is also subject to uncertainty and a benevolent monetary authority will prefer to error by setting a high growth rate of money.

We also conduct a dynamic analysis using Japanese data. Japan is interesting because it experienced a protracted period of zero nominal interest rates. During this period the central bank engineered a large and temporary increase in real balances of money. We produce a baseline simulation that reproduces some of the key empirical facts from Japan between 1984 and 2006 such as the large increase in inverse M0 velocity against a background of zero nominal interest rates and deflation. Then we compare our baseline simulation results with counterfactual simulations to assess the role of this policy.

Our dynamic analysis identifies the following effects of quantitative easing. Quantitative easing reduces deflationary pressure and thus reduces the number of periods that the nominal interest rate is zero when compared to a counterfactual with no quantitative easing. In our counter-factual the nominal interest rate is zero for 12 years as compared to 9 years under the baseline simulation.

Quantitative easing increases government debt. Higher government debt crowds out private capital which increases the real return on capital.

A higher real interest rate benefits retirees. Their savings now have a higher return. Retirees also experience a second benefit. Higher debt means temporarily lower taxes. Since they face high mortality rates they largely escape higher future taxes.

A higher real interest rate is also associated with a lower wage rate and this acts to lower consumption for working individuals. The effect of a lower wage on welfare turns out to be most pronounced for middle-aged workers who are close to their peak of lifetime labor efficiency.

Temporarily high government debt and high interest rates can ease borrowing constraints. In our model the young are borrowing constrained. They face an increasing wage profile but cannot collateralize their future human capital. Higher debt implies that taxes are lower today and this allows young agents to consume more. A higher real interest rate induces an intertemporal substitution effect that can also benefit the young by reducing demand for today's

consumption.

We find that welfare effects of monetary policy in a low interest rate environment vary significantly according to the age of the individual. The benefits and costs of quantitative easing are concentrated among older individuals. The higher real interest rates associated with quantitative easing increase consumption of retirees as much as 2.3 percent between 2001 and 2005. Workers who have the highest labor productivity, though face lower wages and experience consumption losses of as much as 1.3 percent. For younger workers the benefits of relaxed borrowing constraints are largely offset by lower wages and the consumption gains are small.

We compare the baseline specification with two other counterfactuals that are designed to assess the timing and duration of the quantitative easing policy. The counterfactual with longer quantitative easing exhibits a longer period of deflation, a larger real interest rate response and lower output than the baseline scenario. The counterfactual with earlier quantitative easing has the most interesting effects. In early periods there is more deflation but deflation ends earlier than the baseline scenario. This scenario also exhibits higher real wages, consumption and output than the baseline scenario.

A comparison of these two scenarios with the baseline also reveals that the asymmetry we documented in the steadystate analysis is also operating in the dynamic analysis. In particular, the effects of higher monetary growth on economic activity differ depending on whether the initial situation is one with a positive nominal interest rate or an initial situation is a zero nominal interest rate.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 explains how we parametrize the model. Section 4 contains our results and we conclude in Section 5.

2 The Model

We consider an economy that evolves in discrete time. The structure of the real side of the economy is similar to the economy considered by Braun, Ikeda and Joines (2009). Their model accounts for many of the principal variations in macroeconomic activity in the Japanese economy between 1961 and 2002.

2.1 Demographics

Agents are born and become active at age 21. The growth rate of 21 year old individuals, n_1 is assumed to be constant in each period. Agents are subject to mortality risk in each period. If we let $N_{j,t}$ be the number of households of age j in period t, the dynamics of population are governed by a first-order Markov

process:

$$\mathbf{N}_{t+1} = \begin{bmatrix} (1+n_1) & 0 & 0 & \dots & 0 \\ \psi_1 & 0 & 0 & \dots & 0 \\ 0 & \psi_2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \psi_{J-1} & 0 \end{bmatrix} \mathbf{N}_t \equiv \mathbf{\Gamma N}_t, \tag{1}$$

where $\mathbf{N}_t$ is a $J \times 1$ vector that describes the population of each cohort in period t, ψ_j is the conditional probability that a household of age j survives to the next period and ψ_J is implicitly assumed to be zero. The aggregate population in period t, denoted by N_t, is given by

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{2}$$

The population growth rate is then given by $n = N_{t+1}/N_t$. The unconditional probability of surviving from birth in period $t-j+1$ to age $j > 1$ in period t is:

$$\xi_j = \psi_{j-1}\xi_{j-1} \tag{3}$$

where $\xi_{1,t} = 1$ for all t.

2.2 Problem for a household born into cohort s

Households are born with zero assets and retire at age 65. The maximum lifespan of an individual is $J = 100$ years. Money is introduced by assuming that households receive utility from two goods. The cash good, c_{1t}, is subject to a cash in advance constraint as in Lucas and Stokey (1987). The credit good, c_{2t}, may be purchased with cash or on credit. Households also value leisure, l_{jt}. Given these definitions expected present value utility of a household belonging to cohort j is:

$$\sum_{j=s}^{J+s} \beta^{j-s} \xi_j u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}). \tag{4}$$

The specific functional form of preferences we will consider are:

$$u(c^s_{1jt}, c^s_{2jt}, l^s_{jt}) = \gamma \ln(c^s_{1jt}) + (1-\gamma)\ln(c^s_{2jt}) + \alpha \ln(l^s_{jt}). \tag{5}$$

This choice of preferences is consistent with balanced growth.[2] A household of age s in period t, who works h^s_t hours receives nominal earnings of $P_t w_t \varepsilon_j h^s_{jt}$. In this expression P_t is the price level, w_t is the wage rate, and ε_j is an age specific efficiency. A household can save by accumulating cash, M^s_{t+1}, bonds B^s_{t+1}, or capital k^s_{t+1}.

[2] More generally preferences of the form: $\ln([\gamma(c^s_{1jt})^\sigma + (1-\gamma)(c^s_{1jt})^\sigma]^{1/\sigma}) + \alpha \ln(1-n_t)$ are also consistent with balanced growth.

At the start of each period households visit a financial market where claims from the previous period are settled. Households also receive a lump-sum transfer from the government T^s_{jt}, and adjust their holdings of money and bonds. Total holdings of assets are restricted by the following borrowing constraint:

$$k^s_{j,t+1} + B^s_{j,t+1} + M^s_{j,t+1} \geq 0. \tag{6}$$

This borrowing constraint rules out uncollateralized borrowing.

After the financial market closes households separate into a worker and shopper. The shopper's purchases of the cash good and investment goods in any period are subject to the following cash in advance constraint:

$$\frac{B^s_{j,t+1}}{1+R_t} + P_t[k^s_{j,t+1} - k^s_{j-1,t}] + P_t c^s_{1jt} \leq \\ M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} + P_t(1-\tau)(r_t - \delta)k_{j-1,t} \tag{7}$$

where δ is the depreciation rate on capital and τ is a tax on capital income.[3]The household's overall budget constraint is given by:

$$M^s_{j-1,t} + T^s_{jt} + B^s_{j-1,t} + P_t w_t \varepsilon_j h^s_{jt} + P_t(1-\tau)(r_t - \delta)k^s_{j-1,t} \geq \\ P_t(c^s_{1jt} + c^s_{2jt}) + \frac{B^s_{j,t+1}}{1+R_t} + M^s_{j,t+1} + P_t[k^s_{j,t+1} - k^s_{j-1,t}]. \tag{8}$$

Given these definitions the problem for a household born into cohort j is to choose the sequence $\{c^s_{1t}, c^s_{2t}, h^s_t, M^s_{t+1}, B^s_{t+1}, k^s_{t+1}\}^J_{t=s}$ that maximizes (4) subject to (7),(8), and (6). Some important household first order necessary conditions are:

$$\xi_j \gamma / c^s_{1jt} = P_t(\mu_t + \lambda_t) \tag{9}$$

$$\xi_j (1-\gamma)/c^s_{2jt} = P_t \lambda_t \tag{10}$$

$$\xi_j \frac{\alpha}{1-h^s_{jt}} = \lambda_t P_t w_t \varepsilon_j \tag{11}$$

$$\beta(\mu_{t+1} + \lambda_{t+1})/P_{t+1} + \phi_t = (\mu_t + \lambda_t)/\{P_t(1+R_t)\} \tag{12}$$

$$\beta(\lambda_{t+1} + \mu_{t+1})[1 + (1-\tau)(r_{t+1} - \delta)] + \phi_t = (\lambda_t + \mu_t) \tag{13}$$

$$\beta\left(\mu_{t+1} + \lambda_{t+1}\right)/P_{t+1} + \phi_t = \lambda_t / P_t \tag{14}$$

plus the CIA constraint, household budget constraint and the borrowing constraint.

The above expressions can be rearranged to yield the following restrictions on market clearing:

[3]It is more common to assume that investment is not subject to the cash in advance constraint. However, the expressions for the first order conditions are a bit more convenient using this formulation and since we are considering low inflation environments the distinction between this formulation and one that treats capital as a credit good are small.

$$\frac{\alpha}{\gamma}\frac{c^s_{1jt}}{1-h^s_{jt}} = w_t\varepsilon_j/(1+R_t) \tag{15}$$

$$\frac{\alpha}{1-\gamma}\frac{c^s_{2jt}}{1-h^s_{jt}} = w_t\varepsilon_j \tag{16}$$

$$\beta[1+(1-\tau_{t+1})(r_{t+1}-\delta)]\xi_{j+1}\gamma/c^s_{1j,t+1} = \xi_j\gamma/c^s_{1jt} - \phi_t \tag{17}$$

$$(1+R_t)/(1+\pi_{t+1}) = 1+(1-\tau)(r_{t+1}-\delta) \tag{18}$$

$$\phi_t(M^s_{j,t+1}+k^s_{j,t+1}+B^s_{j,t+1}) = 0, \quad \phi_t \geq 0 \tag{19}$$

$$\mu_t \left\{ \begin{array}{c} M^s_{j-1,t}+T^s_{jt}+B^s_{j-1,t}-\frac{B^s_{j,t+1}}{1+R_t}+(1-\tau)(r_t-\delta)k_{j-1,t}+ \\ P_t[k^s_{j-1,t}-k^s_{j,t+1}]-P_tc^s_{1jt} \end{array} \right\} = 0, \quad \mu_t \geq 0 \tag{20}$$

$$M^s_{j-1,t}+T^s_{jt}+B^s_{j-1,t}+P_tw_t\varepsilon_jh^s_{jt}+P_t(1-\tau)(r_t-\delta)k^s_{j-1,t} = \\ P_t(c^s_{1t}+c^s_{2t})+\frac{B^s_{j,t+1}}{1+R_t}+M^s_{j,t+1}+P_t[k^s_{j,t+1}-k^s_{j-1,t}]. \tag{21}$$

2.3 The Firm's Problem

Firms produce consumption goods with a constant returns to scale production technology. In each period firms choose labor, H_t, and capital, K_t, to maximize

$$A_tK_t^{\theta}H_t^{1-\theta} - w_tH_t - r_tK_t, \tag{22}$$

where w_t is the real wage and r_t is the real rental rate on capital, A_t evolves according to

$$A_{t+1} = g_tA_t, g_t > 0.$$

2.4 The Government and aggregate feasibility constraints

The government issues bonds, money and raises revenue through a tax on asset income. Government revenue is used to finance government purchases and lump-sum transfers:

$$P_tG_t + \sum_{j=1}^{J} N_{jt}T_{jt} = \frac{B_{t+1}}{1+R_t} - B_t + M_{t+1} - M_t + P_t\tau(r_t-\delta)K_t \tag{23}$$

The government expands (nominal) money supply at the rate σ_t by making lump-sum transfers to all households alive in a given period according to:

$$M_{t+1} = (1+\sigma_t)M_t.$$

We don't formally model a social security system. Instead we will assume that accidental bequests are lump-sum transferred back to surviving members of the same cohort.

The aggregate resource constraint for this economy is:

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c_{1jt}^s + c_{2jt}^s) + K_{t+1} - (1-\delta)K_t + G_t \tag{24}$$

2.5 Competitive Equilibrium

Definition *Competitive Equilibrium*
Given an initial population wealth distribution, $\{M_{0j}, k_{0j}, B_{0j}\}_{j=1}^{J}$, a sequence of technologies, $\{A_t\}_{t=0}^{\infty}$, and government policies, $\{\tau, M_{t+1}, B_{t+1}, G_t, \{T_{jt}^s\}_{j=1}^{J}\}_{t=0}^{\infty}$, a competitive equilibrium is a price system $\{r_t, P_t, R_t, w_t\}_{t=0}^{\infty}$ and a sequence of allocations $\{c_{jt}^s, h_{jt}^s, k_{j,t+1}^s, M_{j,t+1}^s\}_{t=0}^{\infty}$ that solves the household problem, the firms problem and satisfies the following market clearing/feasibility conditions:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} k_{j,t+1}^s \tag{25}$$

$$H_t = \sum_{j=1}^{J} N_{jt} h_{jt}^s \tag{26}$$

$$M_{t+1} = \sum_{j=1}^{J} N_{jt} M_{j,t+1}^s \tag{27}$$

$$A_t K_t^{\theta} H_t^{1-\theta} = \sum_{j=1}^{J} N_{jt}(c_{1jt}^s + c_{2jt}^s) + K_{t+1} - (1-\delta_t)K_t + G_t. \tag{28}$$

When solving the model we will specify an initial population wealth distribution and a terminal steady-state and then solve for the transitional dynamics. We thus define a steady-state equilibrium next.

Definition *Balanced Growth Equilibrium*
Suppose that technology grows at the constant rate: $g_t = g$, and that money supply grows at a constant rate: $\sigma_t = \sigma$, and the output shares of government purchases, and government debt are constant Then a balanced growth equilibrium is a competitive equilibrium in which the real wage rate grows at the rate of output, the real interest and nominal interest rates are constant and the output shares of capital and consumption are constant.

2.6 Computation of the equilibrium.

Before we compute the equilibrium we transform the economy. This is done using the transformations:

$$\begin{aligned} \hat{K}_t = \frac{K_t}{N_t A_t^{1/(1-\theta)}}, \hat{C}_t = \frac{C_t}{N_t A_t^{1/(1-\theta)}}, \hat{B}_t = \frac{B_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \\ \hat{M}_t = \frac{M_t}{P_{t-1} N_t A_t^{1/(1-\theta)}}, \hat{T}_t = T_t/P_t, \hat{H}_t = H_t/N_t, \hat{w}_t = \frac{w_t}{A_t^{1/(1-\theta)}}. \end{aligned} \tag{29}$$

We first describe computation of the steady-state equilibrium. We are interested in considering situations where the nominal interest rate is positive and also in situations where it is zero. In the later situation the cash in advance constraint (7) ceases to bind and the steady-state conditions are different. When $R > 0$ we start by guessing the aggregate values of hours, capital real balances and lump-sum transfers $(\hat{H}_0, \hat{K}_0, \hat{M}_0, \hat{T}_0)$. Given these objects we can derive the wage and rental rates $\widetilde{w}_0, r_0$ and solve the household's problem. (Note that the inflation rate can be derived from real balances using the following equation:

$$(1+\pi) = \frac{(1+\sigma)}{(1+n)(1+g_{TFP})} \tag{30}$$

where $1 + g_{TFP} = A_t^{1/(1-\theta)}/A_{t-1}^{1/(1-\theta)}$. When $R > 0$, the solution to the household's problem uniquely determines individual demand for real balances: $\hat{M}_0^{d,s}$ for each cohort $s = \{1, ...J\}$ and labor supply for each cohort $\hat{H}^s$. However, the household's problem only determines the sum of saving in the form of capital and bonds. We denote this sum as $\hat{S}_0^s$.

Given solutions to each cohort's optimization problem we then sum over households to derive aggregate assets supplied by households: $\hat{S}_0^{'}$, aggregate labor supply: $\hat{H}_0^{'}$ and aggregate demand for real balances: $\hat{M}_0^{'}$ Given these objects we can solve for the capital stock using the fact that the stock of government bonds is exogenous and: $\hat{S}' - \hat{B} = \hat{K}'$. Then using the initial guesses of the wage rate and rental rate we can update transfers using the steady-state version of the government budget constraint:

$$\begin{aligned} \left\{ \frac{(1+g_{TFP})(1+n)}{1+R} - \frac{1}{1+\pi} \right\} \hat{B} + \left\{ (1+g_{TFP})(1+n) - \frac{1}{1+\pi} \right\} \hat{M}' \\ + \tau(r-\delta)\hat{K}' = \hat{G} + \hat{T}'. \end{aligned} \tag{31}$$

Finally, we update our guess of capital, labor, real balances and government transfers by taking a weighted average of the initial guess plus the new values derived from household optimization:

$$\hat{K}_1 = \lambda \hat{K}_0^{'} + (1-\lambda)\hat{K}_0 \tag{32}$$

$$\hat{H}_1 = \lambda \hat{H}_0^{'} + (1-\lambda)\hat{H}_0 \tag{33}$$

$$\hat{T}_1 = \hat{T}_0' \tag{34}$$

$$\hat{M}_1 = \lambda \hat{M}_0' + (1-\lambda)\hat{M}_0 \tag{35}$$

When $R = 0$, the household problem only pins down household supply of *total* assets which now consists of the sum of real balances, capital and bonds: $\hat{S}_0^{'} = \hat{K}_0^{'} + \hat{B} + \hat{M}_0^{'}$. In this case we derive real balances and the capital in the following way. First, we use the fact that:

$$(1 + \pi_0) = \frac{(1 + \sigma)}{(1 + n)(1 + g_{TFP})} \tag{36}$$

to pin down the inflation rate. Then we use

$$(1 + \pi_0) = (1 + r_0)^{-1} \tag{37}$$

to pin down the real interest rate. Given the real interest rate we derive a new guess of the capital stock, $\hat{K}_0^{'}$, from aggregate labor supply plus the marginal product pricing relationship:

$$r_0 = (1 - \tau) \left\{ \theta \left(\hat{K}_0^{'} / \hat{H}_0^{'} \right)^{\theta - 1} - \delta \right\} \tag{38}$$

Then we derive real balances from the saving identity: $\hat{S}_0^{'} - \hat{K}_0^{'} - \hat{B} = \hat{M}_0^{'}$. The updating of the guess proceeds in the same way as before.

When solving for the dynamic transition we proceed in an analogous way. The main distinction is that we now guess and update sequences of the form: $(\hat{H}_{i,t}, \hat{K}_{i,t}, \hat{M}_{i,t}, \hat{T}_{i,t})$ where i denotes the ith iterate and t indexes time.

3 Model Parameterization

The strategy for calibrating the model is similar to the strategy used in Braun, Joines and Ikeda (2009). The preference discount rate β is calibrated to reproduce the average capital output ratio between 1984 and 2000. This results in a value of 0.97. The leisure weight in preferences, α is set to reproduce the value of labor input in the Japanese economy between 1984 and 2000. This yields $\alpha = 2.5$. The capital share parameter is set to 0.362 which is the average value of capital's share of GNP between 1984 and 2000. The depreciation rate calibrated in the same way is 0.085. The average tax rate on asset income over the same period is 0.46. The labor tax rate is set to zero. This assumption is also maintained by Hayashi and Prescott (2002) who assert that the principal tax wedge in Japan is a high tax on capital income. With this choice, the remainder of the calibration turns out to be very similar to what one finds using U.S. data. We assume a constant population growth rate of 1 percent per year. We set the share weight on cash goods, $\gamma = 0.07$. This choice reproduces the ratio of real balances of monetary base to GNP which averaged 0.08 between 1984 and 1994.

4 Steady-state Analysis

Here we report results from a comparative steadystate analysis. This analysis provides intuition about the workings of our model. We will document an

asymmetry between the welfare cost of inflation and the welfare cost of deflation that provides a rationale for expanding money supply when the nominal interest rate is zero.

Table 1 reports the steady-state properties of our model for alternative settings of the growth rate of money. These results allow for age specific variation in the efficiency of work effort and assume that the population growth rate is 1 percent, the growth rate of TFP is 1.9 percent, the share of government purchases in output is 0.144 and the government debt ratio is 0.22. These correspond to the average value of these variables in Japanese data over the 1984 to 2000 sample period. Table 1 has several noteworthy features. First, observe that there are a range of monetary policies that implement a zero nominal interest rate in our economy. Interestingly, the welfare maximizing choice occurs when the nominal interest rate reaches zero and is associated with a growth rate of money that declines at a rate of 1.43 percent per year. If the Friedman Rule is defined as a monetary policy that sets the nominal interest rate to zero as in Chari, Christiano and Kehoe (1991), then the Friedman Rule is the optimal (steady-state) monetary policy in our economy too.

Bhattacharya, Haslag and Russell (2005) consider the optimality of the Friedman rule in a 2 period overlapping generations model and find that it is not optimal in their setting. The reason for this is that in their model young households have low initial wealth and yet must pay a lump-sum tax to finance contraction of the money supply when the growth rate of money is negative. We allow agents to borrow against their first period labor earnings and this mitigates the negative effect of lump-sum taxation on the youngest households.

One of the most noteworthy features of Table 1 is an asymmetry in the welfare costs of alternative growth rates of money. The welfare loss associated with large growth rates of money (e.g. 7 percent) is modest. However, lowering the growth rate of money below -1.43 percent has much larger effects on welfare. For instance, steadystate welfare when money growth is -1.6 percent is about the same as steadystate welfare when the growth rate of money is 7 percent per annum!

This asymmetry between the welfare cost of inflation and the welfare cost of deflation reflects the fact that monetary policy affects real economic activity in a different way when the nominal interest rate is zero. When the average growth rate of money is higher than the optimal level, monetary policy acts as a tax on labor supply and capital. Households act to limit their holdings of cash and this limits the incidence of this tax. This can readily be seen in Table 1. Higher growth rates of money are associated with lower consumption of cash goods. However, cash goods only constitute 7 percent of total consumption under the Friedman rule. The effect on the capital output ratio and thus the real interest rate is also modest when the growth rate of money exceeds -1.43 percent.

To understand why the welfare losses increase rapidly when the growth rate of money is too low, recall that when the nominal interest rate is zero the cash in advance constraint ceases to bind and money and capital earn the same real return. As the steadystate growth rate of money is lowered from the welfare maximizing level, the inflation rate falls and this increases the real return on

money. Holdings of private capital must then fall in order to insure that the capital stock continues to earn the same return as money. There is also a second channel operating here. A lower growth rate of money is also associated with higher lump-sum taxes which is costly to households who are borrowing constrained. Table 1 indicates that the combination of these two mechanisms produces a sharp decline in welfare when money supply contracts at a more rapid rate than 1.43 percent per annum.[4]

This asymmetry has implications for the conduct of monetary policy. Suppose we assume that the monetary authority knows the model but that there is uncertainty about the values of the model parameters including the long run average values or growth rates of the exogenous variables. To be specific suppose that the policy maker estimates the growth rate of TFP is 4 percent rather than 2 percent. An estimate of this magnitude would arise if the policy maker were to estimate the growth rate of TFP using Japanese data from 1960 to 1990. In this scenario welfare is maximized when the growth rate of money is -0.61 percent and also falls rapidly if the money supply is contracted more rapidly. Setting the growth rate of money to -1.43 percent, which is the *optimal* monetary policy in Table 1, induces very large welfare losses. This property of the model provides a rationale for a monetary authority to pursue an expansionary monetary policy when it finds itself in a zero interest rate environment. The welfare costs associated with too much monetary expansion are much smaller than the welfare costs of a monetary policy that is too tight.

And important limitation of the steady-state analysis is that it is difficult to produce an empirically plausible calibrated specification of the model with a steadystate in which a zero nominal interest rate is associated with deflation and a positive growth rate of money. Thus it is difficult to use a comparative steadystate analysis to understand Japan's experience from the mid 1990s to 2006 when the growth rate of money was positive and yet there was a protracted period of deflation. We turn next to describe the results from a dynamic analysis that reproduces these outcomes.

5 A Dynamic Analysis of "Quantitative Easing"

Japan is an interesting case for analyzing the quantitative effects of a large increase in real balances of money in a zero interest rate environment. In Japan slower real economic growth during the 1990s was associated with a steady decline in the uncollateralized call rate on overnight loans from 7.4 percent in 1990 to 0.06 percent in 1999. The nominal interest rate remained at effectively zero (except for a brief interlude in 2000) until 2006. Once the nominal interest rate reached zero policy makers considered a variety of options for using monetary policy to stimulate the economy. The outcome of these deliberations was the "Quantitative Easing"policy that was adopted on March 19, 2001. This policy

[4]The asymmetry we are documenting here would be even larger if investment was treated as a credit good. If investment is a credit good the welfare cost of 7 percent inflation is smaller. However, the welfare cost of too much deflation remains essentially unchanged.

which targeted the level of bank deposits at the Bank of Japan was effectively an excess reserve targeting policy. The Bank of Japan announced an end to the quantitative easing policy in March 9, 2006. But, it kept the call rate at zero until July 14, 2006 at which point the call rate was increased to 0.25 percent.

We investigate the effects of this policy using dynamic perfect foresight simulations. Chen, Imrohoroglu and Imrohoroglu (2007) and Braun, Ikeda and Joines (2009) have previously found that computable general equilibrium models that allow for variation in TFP and demographics can account for some of the principal movements in real economic activity in Japan from 1960 through 2002. Here we abstract from demographic variation and model only variation in TFP and government debt. Our government debt series is taken from Braun, Joines and Ikeda (2009). They construct a government debt series following the methodology of Broda and Weinstein (2005). The initial period of our simulation is taken to be 1984. The initial wealth distribution is taken from the terminal steady-state but is rescaled to reproduce the capital stock in Japanese data in 1984. We set that the initial values of the nominal interest rate, government purchases and government bonds to their values in Japanese data in 1984. The terminal nominal interest rate is 5.9 percent, terminal government debt is 22% of output, terminal government purchases are 14.4 percent of output and terminal TFP growth is 1.9 percent.

We would like the model to reproduce variations in the Japanese call rate and real balances of money during the period 1986-2006.[5] There are two issues that arise in doing this. First, when the nominal interest rate is zero the composition of government liabilities is indeterminate. Open market operations that exchange money for bonds have no real effects when the nominal interest rate is zero. Monetary policies that alter the total amount of outstanding government debt do have real effects. However, it is hard to ascertain directly what fraction of quantitative easing should be interpreted as having altered the amount of outstanding government debt. During this period the Bank of Japan purchased equities of private companies, accepted a broader range of assets as collateral and purchased long-term bonds. Throughout most of this period the Bank of Japan was also the sole provider of funds in the overnight call money market. The reason for this was that the interest rate was so low that the return from lending over-nite was dwarfed for all but very large loans my even moderate costs of orginating an overnight loan. This effectively killed one side of the over-nite loan market. This implies that each loan was provided at a subsidized rate.

Valuing all of these transactions directly is beyond the scope of this paper. Instead we use our model to help us to infer the fraction of the total debt accumulation during this period that was associated with monetary policy. This is accomplished in the following way. First, we treat the sequences of government debt and the nominal interest rate as exogenous and solve for the equilibrium allocations and prices.

The resulting sequence of real balances and lump-sum transfers does a good

[5] Our measure of money is the monetary base.

job of reproducing the path of inverse M0 velocity (the ratio of M0/P to GNP) in the period up to 1997. In the period after that though the model understates this ratio. Our model has the property that once the nominal interest rate is zero the overall level of government liabilities is determinant but not its composition. We use this property of the model to next adjust the composition of government liabilities to reproduce the actual trajectory of M0/P to GNP during the period 1997-2006.

The resulting trajectory for M0/P to GNP from the model and Japanese data are reported in Figure 1. This same figure also reports plots of the capital output ratio, the deviation of output from a 1.9 percent trend and the inflation rate as measured by the growth rate of the GNP price deflator. The general fit of the model is reasonably good. The model reproduces the increase in the capital-output ratio and the decline in output relative to trend that Japan experienced after 1990. However, the model understates the average value of the inflation rate in Japanese data. Broda and Weinstein (2007) argue that problems in price measurement induce an upward bias of about 2 percentage points in the Japanese inflation rate. If we subtract 2 percent from the actual data, the model reproduces the overall level of the inflation rate and also some of its principal movements between 1986 and 2005.

We evaluate the effects of quantitative easing by comparing the baseline simulation with three counterfactual simulations. The *no quantitative easing* scenario assumes that the ratio of real balances of M0 to GNP rises at the rate of 2 percent per year between 2000 and 2006. The *longer quantitative easing* scenario allows the ratio of real balances to output to rise to 0.24 in 2011. The *earlier quantitative easing* scenario assumes that quantitative easing is started in 1995 instead of 2002. Figure 2 shows the trajectory of inverse M0 velocity for each of these scenarios.

Table 2 summarizes of properties of the model for prices. A comparison of the baseline simulation with the no quantitative easing simulation reveals two effects of quantitative easing. The quantitative easing simulation exhibits less deflation during the 1990s and a higher nominal interest rate than the no quantitative easing simulation. Expectations are clearly playing an important role. The baseline specification shows higher nominal interest rates and higher average inflation rates between 1991 and 2000 which is before quantitative easing was undertaken. After 2000 the two policies are very similar. Movements in the Inflation rate are large and of primary importance in determining the evolution of the nominal interest rate. However, there are also some small but discernible effects on the real interest rate. The real interest rate is higher under quantitative easing in all sub-periods.

An interesting property of the model is that longer quantitative easing increases deflationary pressure. The average inflation rate is lower under longer quantitative easing as compared to the baseline in each of the last three subsamples. One consequence of more deflation is a longer period of zero nominal interest rates. We will describe the economics underlying this result in more detail below but we want to mention that the basic mechanism operating here is the fiscal effect we discussed in the steadystate analysis. Longer quantitative

easing is associated with higher growth in money supply and with the nominal interest rate at zero the real return on money has to rise in order for households to be willing to hold it. A higher real return on money crowds out private capital and the real return on capital increases. In Table 2 we see that this effect is very persistent. The real interest under longer quantitative easing is higher than the baseline simulation in each of the final four sub-periods or a period of twenty years in total. The average magnitude of the difference is 16 basis points.

Earlier quantitative easing, induces more deflation during the 1990s but less deflation after 2001 as compared to the baseline scenario. Interestingly, this simulation shows a lower real interest rate and thus a higher wage rate than the baseline simulation. The value of the real interest rate under earlier quantitative easing is lower than the baseline in all but the first sub-sample and the real wage rate is correspondingly higher.

Table 3 reports simulation results for aggregate allocations. Quantitative easing depresses output when compared with the no quantitative easing scenario and longer quantitative easing depresses output more. Interestingly, earlier quantitative easing produces higher output than any of the other three scenarios between 1991 and 2005.

The fact that early quantitative easing acts to lower the real interest rate and raise output while longer quantitative easing acts to increase the real interest rate and lower output might appear to be puzzling. However, these results can be attributed to the same two distortions that we discussed in the steadystate analysis. Starting from a situation with a positive nominal interest rate, inflation acts as a tax on labor and capital. From the steady-state analysis in Table 1 we know that lower steady inflation rates are associated with lower monetary base growth, a lower real interest rate, higher wages and higher output. These same mechanisms are operating in the dynamic simulations. The early quantitative easing scenario exhibits lower average monetary based growth than the baseline scenario from 1991-2006 and this accounts for the fact that the earlier quantitative easing scenario has lower average inflation rates, higher output and lower real interest rates than the baseline scenario in the earlier sub-periods.

The dynamic effects of quantitative easing are quite different though once the nominal interest rate is zero. This can most readily be observed by comparing the baseline with the longer quantitative easing scenario. The longer quantitative easing scenario exhibits higher money growth and real balances after 2001, a higher real interest rate and lower output. In the steady-state analysis above we saw that once the nominal interest rate was zero a monetary policy that increased the real return on money increased real balances and crowded out private capital.

To further explore the nature of this crowding out effect Table 4 reports the ratio of real balances to output and the capital output ratio for the four scenarios. Before discussing these results it should be pointed that in the dynamic analysis we are limiting attention to transitory changes in monetary policy. The ratio of real balances to output and the debt output ratios are the same in both the initial and terminal steady-states in all four scenarios. The results in Tables

2 and 4 indicate that this is an important distinction. Comparing the baseline scenario with the longer quantitative easing scenario, we see from Table 2 that longer quantitative easing produces more deflation. The reason for this can be seen in Table 4. Longer quantitative easing increases real balances and temporarily increases total government debt. Temporarily higher government debt increases the real interest rate and crowds out private capital. This is why the longer quantitative easing simulation exhibits lower capital output ratios, higher real interest rates, lower inflation and lower output than the baseline scenario after 2001. In other words, starting from a situation with zero nominal interest rates, the anticipated inflation effects of temporarily higher money growth are dominated by the fiscal effects of monetary policy on total government debt.

Next we turn to consider the distributional effects of quantitative easing. We allow labor productivity to vary with age. In the presence of age-specific earnings young agents face binding borrowing constraints They would like to shift consumption forward from future periods when their labor income will be high but are unable to collateralize their future high human capital. Quantitative easing temporarily lowers taxes and this, in principal, can relax borrowing constraints. It also raises the real interest rate which reduces the incentive to consume today. These two effects can be seen in Table 5 which reports lump-sum taxes, the real interest rate and the average number of constrained cohorts for each sub-period. Notice that the baseline scenario has lower average lump-sum taxes as compared with the no quantitative easing scenario between 2001 and 2005 but that the difference is small. The value of the real interest rate is higher under quantitative easing but again the difference is small. As a result the effect of quantitative easing in relaxing borrowing constraints of the young is small. The number of cohorts that face binding borrowing constraints is a bit lower during the period of quantitative easing (14.0 as versus 14.6 cohorts) but the pattern of borrowing constraints in other periods is very similar in the two simulations.

We observe larger differences in the number of borrowing constrained cohorts when we compare the baseline with the early and the longer quantitative easing simulations. The differences are particularly large when comparing the baseline with the earlier quantitative easing simulation. A substantially smaller number of cohorts are borrowing constrained in the first four sub-samples with earlier quantitative easing. In this simulation the young face a lower interest rate and higher lump-sum taxes but also higher wages.

The effects of quantitative easing on consumption can vary significantly with the age of the cohort. Quantitative easing benefits older individuals most. For retirees a higher real interest rate increases the value of their saving and consumption increases. Moreover, older retirees also enjoy the benefits of temporarily low taxes and escape most of the future burden of higher taxes by passing away before taxes rise. The magnitude of these benefits can be substantial. To illustrate these points we compare the baseline specification with the longer quantitative easing and the early quantitative easing scenarios.[6] The consump-

[6]The consumption differences are small when comparing the baseline with the no quanti-

tion benefits to the old are largest in the longer quantitative easing simulation. In this simulation the old benefit from both lower taxes and a higher real return on their saving. If we index cohorts by their age as of 2001, all cohorts that are of age 74 and older experience consumption gains in this scenario as compared to the baseline simulation. The maximum gain occurs for the cohort this is of age 84 in 2001. This cohort enjoys an annualized increase in consumption of 4.2 percent between 2001 and 2005. All cohorts younger than 74 years as of 2001 experience consumption losses. The biggest loss is for the 64 year old cohort who see their consumption fall by 1.3 percent per annum. The consumption loss for the 21 year old cohort is moderate and about 0.6 percent per annum.

The pattern of gains and losses changes when we compare the early quantitative easing simulation with the baseline. In this simulation both the young and the old enjoy more consumption in the baseline and the middle aged enjoy more consumption under earlier quantitative easing. Considering first the young, all cohorts aged 36 or younger as of 2001 experience higher consumption under the baseline. The consumption benefits of the baseline are highest for the cohort aged 21 in 2001. They experience an annual consumption loss of 0.8 percent under earlier quantitative easing. For these cohorts the benefits of higher wages are offset by higher taxes and consumption falls. Cohorts aged 80 and over also experience consumption losses under early quantitative easing and these losses increase monotonically with age but are always less than 1 percent per year. The remaining cohorts enjoy higher consumption on higher real wages. The biggest beneficiaries are workers with high labor productivity. The 58 year old cohort, for instance, experiences a consumption gain of 2.1 percent per year.

6 Conclusion

In this paper we have developed a model with real balance effects due to finite lifespans and binding borrowing constraints and used it to analyze the quantitative effects of a large increase in real balances of money in a low interest rate environment.

According to our model quantitative easing as pursued in Japan was an effective measure for limiting deflationary pressure but only had a small effect on the evolution aggregate economic activity. However, other policies that have been proposed in the literature such early quantitative easing or longer quantitative easing have larger effects on economic activity. Our results suggest that how quantitative easing effects the economy varies depending on whether the nominal interest rate is zero. Starting from an initial situation with a positive nominal interest rate, quantitative easing lowers the real interest rate and increases the wage rate and this stimulates economic activity. Starting from a situation of a zero nominal interest rate quantitative easing has the opposite effect. It crowds out private capital and depresses wages.

tative easing simulation for the reasons discussed above. So we omit a comparison of these two simulations.

The decision of whether to pursue quantitative easing at all and if so when to pursue it is complicated further by the fact that this policy has large distributional effects. Early quantitative easing benefits middle aged workers most and lowers consumption for the old and the young. Longer quantitative easing reduces consumption for most cohorts and only the oldest cohorts benefit.

In future work we plan to relax our current assumption that the government budget constraint is met by altering lump-sum taxes and instead make the more realistic assumption that a distortionary tax is adjusted instead. This will likely introduce stronger non-neutralities. Our model generates borrowing and lending in equilibrium. It is consequently a good framework for modeling financial intermediation and central bank lending. In future work we plan to pursue these extensions as well.

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Figure 1
Baseline Model and Japanese Data

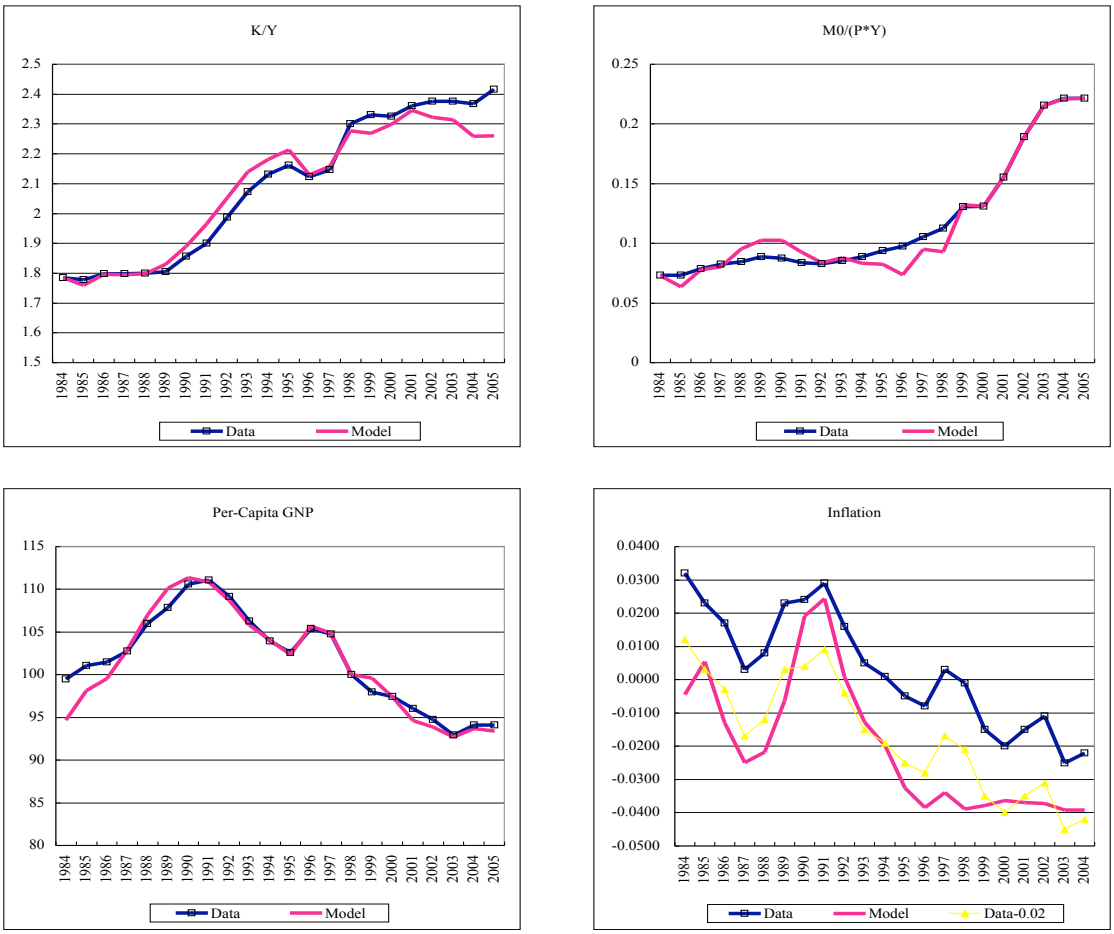

Figure 2
M0/(P*GNP) Alternative Scenarios

Table 1
Model Steadystates for Alternative Growth Rates of Money

Specification with Age Specific Efficiency Units

Growth rate of money (Percentage)	Cash Good Consumption*	Credit Good Consumption*	Money Output ratio	Capital Output Ratio	Output*	Real Interest Rate (Percentage)	Inflation Rate (Percentage)	Nominal Interest rate (Percentage)	Welfare
7.00	91.7	99.7	0.06	2.12	98.2	4.6	3.9	8.8	-68.86
3.00	95.4	99.8	0.07	2.14	99.0	4.5	0.1	4.6	-68.75
0.00	98.5	100.0	0.07	2.16	99.7	4.5	-2.9	1.5	-68.67
-1.43	100.0	100.0	0.08	2.17	100.0	4.4	-4.2	0.0	-68.63
-1.60	99.6	99.6	0.25	2.13	98.8	4.6	-4.4	0.0	-68.83
-1.80	99.2	99.2	0.48	2.08	97.5	4.8	-4.6	0.0	-69.10
-2.00	98.9	98.9	0.73	2.04	96.4	5.0	-4.8	0.0	-69.42
-2.20	98.6	98.6	1.01	1.99	95.3	5.3	-5.0	0.0	-69.80

* Cash good consumption, credit good consumption and output are expressed as a percentage of the respective variable under the Friedman rule.

Table 2

Simulation Results: Prices

	Inflation				Nominal Interest rate				Real Interest rate				Wage rate			
Period	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing*	Baseline	Longer Quantitative Easing*	Earlier Quantitative Easing*
1991-1995	-1.38	0.23	3.15	-4.13	2.10	3.73	6.88	0.15	4.71	4.72	4.71	4.56	100.0	100.0	100.0	101.0
1996-2000	-3.96	-3.63	-1.95	-3.83	0.00	0.19	1.42	0.00	4.12	4.22	4.35	3.98	100.6	103.1	99.2	101.5
2001-2005	-3.72	-3.78	-3.90	-3.42	0.00	0.00	0.00	0.66	3.87	3.93	4.06	3.73	100.4	105.0	99.2	101.3
2006-2010	-3.27	-3.28	-3.78	-0.12	0.61	0.64	0.00	4.42	3.77	3.80	3.93	3.54	100.2	105.9	99.2	101.8
2011-2015	-1.15	-1.13	-3.52	2.90	3.31	3.34	0.24	6.13	3.61	3.62	3.89	3.63	100.1	107.2	98.2	100.0

* Percentage relative to the baseline scenario

Table 3

Simulation results: Allocations

	Consumption				labor input				Output				Money Growth			
Period	No Quantitative Easing*	Baseline	Longer Quantitative Easing*	Earlier Quantitative Easing*	No Quantitative Easing*	Baseline	Longer Quantitative Easing*	Earlier Quantitative Easing*	No Quantitative Easing*	Baseline	Longer Quantitative Easing*	Earlier Quantitative Easing*	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	99.7	100.0	100.8	100.1	100.9	100.0	99.3	101.7	100.9	100.0	99.3	102.7	-3.14	-3.14	-3.14	1.33
1996-2000	100.6	96.5	99.7	101.4	99.9	98.3	99.4	100.1	100.5	95.9	98.5	101.7	11.35	14.08	14.08	4.37
2001-2005	100.4	91.7	99.4	100.8	100.0	97.2	99.7	101.4	100.5	88.8	98.9	102.7	1.00	4.37	7.86	-15.23
2006-2010	100.3	86.8	99.8	102.6	99.8	99.9	98.5	98.2	100.0	86.6	97.7	100.0	-12.90	-15.23	2.73	-6.29
2011-2015	100.1	84.9	97.9	100.8	100.0	98.3	101.2	98.4	100.1	82.6	99.3	98.4	-6.29	-6.29	-16.18	5.91

* Percentage relative to the baseline scenario.

Table 4

Real Balances, Capital and Total Debt as a Fraction of GNP

	Ratio of real balances to output				Ratio of capital to output			
Period	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	0.09	0.09	0.08	0.13	2.11	2.11	2.11	2.15
1996-2000	0.12	0.11	0.08	0.20	2.25	2.23	2.19	2.29
2001-2005	0.17	0.20	0.16	0.13	2.32	2.30	2.27	2.35
2006-2010	0.12	0.13	0.21	0.07	2.34	2.33	2.30	2.41
2011-2015	0.07	0.07	0.19	0.05	2.39	2.39	2.31	2.38

Table 5

Taxes, interest rate and borrowing constraints

	Lump-sum taxes*				Real interest rate				Number of borrowing constrained cohorts			
Period	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing	No Quantitative Easing	Baseline	Longer Quantitative Easing	Earlier Quantitative Easing
1991-1995	0.07	0.06	0.07	0.07	4.71	4.72	4.71	4.56	19.0	19.2	19.8	18.0
1996-2000	0.01	0.02	0.01	0.02	4.12	4.22	4.35	3.98	15.4	15.8	17.8	12.4
2001-2005	0.09	0.08	0.07	0.14	3.87	3.93	4.06	3.73	14.6	14.0	14.6	13.4
2006-2010	0.24	0.25	0.19	0.22	3.77	3.80	3.93	3.54	20.8	20.8	17.6	14.8
2011-2015	0.18	0.18	0.22	0.16	3.61	3.62	3.89	3.63	21.6	21.6	19.4	22.6

* Lump-sum taxes are expressed as a fraction of total average consumption