Financing Constraints and Investment: Evidence from US Business Credit Contracts

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JOB MARKET PAPER

Abstract

Standard models of financial market imperfections limit the ability to borrow to some multiple of the borrower's net worth. In macroeconomic models such as Bernanke, Gertler, and Gilchrist (1999), this type of financing constraint amplifies the response of an economy to shocks. While previous studies relied on indirect evidence to identify financing constraints in the data, this paper exploits business credit contracts to examine this financing constraints. In the Surveys of Small Business Finances (SSBF) firm owners report access to credit, the latest business credit contract and provide all information necessary to construct the net worth of firm-entrepreneur entities. I derive testable implications of financing constraints for the relationship of net worth and external finance from the standard model of financially constrained firms used in macroeconomic models. In line with the predictions of the model I find that net worth is the limiting factor in the firms' financing abilities. An additional dollar of net worth accounts for 30-36 cents of external finance. Interestingly, the net worth multiplier varies by firm age. I show that a simple modification of the financing constraint model can explain this heterogeneity. Secondly, as predicted by models with financially constrained firms, the interest rate on the last credit paid is increasing in leverage. This provides evidence for the key mechanism that amplifies of shocks in macroeconomic models with financing constraints. To assess this propagation mechanism, I calibrate the New Keynesian model of Bernanke, Gertler, and Gilchrist (1999) to the small business data. The response of output to a monetary policy shock is up to 28% larger compared to the frictionless benchmark.

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1 Introduction

The perhaps most influential way to include financial frictions in Dynamic Stochastic General Equilibrium (DGSE) was introduced by Bernanke, Gertler, and Gilchrist (1999), henceforth BGG. At the heart of the these financial frictions lies costly state verification (CSV) pioneered by Townsend (1979). Entrepreneurs invest in part with external finance in risky return projects. Lenders who provide external finance incur a cost to observe the realized returns.[1] This agency problem between entrepreneurs and lenders exhibits two key features: i) external finance is some multiple of the net worth of the entrepreneur and ii) the interest rate paid on the external finance is increasing in the leverage of the project. Both features affect the investment decisions of entrepreneurs. In general equilibrium models financial frictions can propagate shocks substantially. The literature refers to this propagation mechanism as financial accelerator.

Economists recognize the importance of financial frictions in order to explain macroeconomic data.[2] Therefore, many applied macroeconomic models include financial frictions as modeled in BGG. Yet there is little direct, micro-based evidence supporting this modeling strategy of financing constraints.[3] This paper exploits a comprehensive data set of credit data from US small businesses to directly assess the empirical plausibility of financing constraints.

In the analysis I use the Surveys of Small Business Finances (SSBF). The SSBF includes detailed information about the firms with up to 500 employees. Firm owners report the firms' balance sheets and private net assets. The latter is important as small business owners collateralize business credits with private net worth. Together this information allows the construction the net worth of the matched firm-entrepreneur entity, the crucial variable in models of financing constraints. The data also provide measures of a firm's access to credit, the credit amount, and the interest rate of the last credit obtained. One advantage of focusing on small businesses is that they are bank-dependent and have no access to bond markets. Only few small firms attract additional outside equity. In addition the firm size distribution in the SSBF is comparable to the firm size distributions reported in the literature.

In order to assess the empirical plausibility of the key features of the CSV implied financing constraints, I define the net worth multiplier as the additional units of external finance obtained by the firm due to an additional unit of net worth of the firm-entrepreneur entity. For tractability macroeconomic models abstract from firm heterogeneity. In their model all firms have the same net worth multiplier. To capture the heterogeneity in the SSBF data, I extend the model of financially constrained firms to allow the net worth multiplier to depend on firm size. This feature is common in the literature on firm growth and provides for additional testable implications for the plausibility of CSV.[4]

[1] See also Gale and Hellwig (1985).

[2] See e.g. Christiano, Motto, and Rostagno (2007), Christensen and Dib (2008), von Heideken (2008), Gilchrist, Ortiz, and Zakrajsek (2009) and Nolan and Thoenissen (2009).

[3] Thus far only indirect evidence for the existence of financing constraints was presented. Many authors identify financing constraints using cash flow sensitivity of investment. Hubbard (1998) and Stein (2003) survey this literature and point to shortcomings of this approach. Gertler and Gilchrist (1994) argue that inventory fire sales after a monetary contraction are the result of financing constraints.

[4] See e.g. Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), and Clementi and Hopenhayn (2006).

Consistent with the predictions of financing constraints induced by CSV, the small business data exhibit a statistically significant, positive net worth multiplier. An additional dollar of net worth accounts, on average, for 30 to 36 cents of additional external finance. Secondly, supporting the key mechanism that propagates shocks in macroeconomic models, the interest rate paid on the most recent credit is positively correlated with leverage. A 1% increase in leverage increases the interest rate by one basis point. The elasticity of the interest rate with respect to leverage in the data exceeds the value recently estimated in Bayesian DGSE models. Lastly, the net worth multiplier is age-dependent and ranges from 70-102 cents for firms up to three years old to 20-30 cents for firms older than 15 years. I show that this age pattern is consistent with models of firm growth with firms constrained to one period debt contracts.

In the data young, small firms are less likely to have access to credit. Together with the age pattern of the net worth multiplier this provides strong evidence for the importance of financing constraints in understanding firm growth and the evolution of firm size distribution.[5]

From a macroeconomic perspective the firm level heterogeneity only plays a minor role. Therefore, the modeling strategy and key mechanisms of the financial accelerator employed by BGG are empirically plausible. I investigate the quantitative importance of financial frictions in a DSGE model. I use leverage and the elasticity of the interest rate with respect to leverage from the small business data to calibrate the BGG model. The response of output to a monetary policy shock is up to 28% higher compared to a model without financial accelerator.

The following section presents the data, summarizes credit market outcomes, and explains the definition of net worth. Section 3 presents the one period debt contract and tests its implications for the relationship of net worth and credit. In section 4 the impact of the financial situation of the firm on the interest rate is examined. The findings are used to calibrate the model of BGG. Section 5 concludes.

2 Credit Constraints and Net Worth in the Data

This section presents the data used throughout the analysis. I discuss how firms without access to (additional) credit are identified in the data. The section concludes with a discussion of collateral offered by small business and the construction of net worth.

2.1 Firms in the Surveys of Small Business Finances (SSBF)

The Survey of Small Business Finances (until 1998 National Survey for Small Business Finances) was conducted by the Federal Reserve Board for the fiscal years ending in 1987, 1993, 1998 and 2003. The surveys aim to assess credit availability, to provide financial statement data of small firms, and to study the effects of changes through technology and

[5] Cabral and Mata (2003) argue that financing constraints tying net worth to external finance can explain the firm size distribution of Portuguese firms. Angelini and Generale (2008), using credit denial as proxy for financing constraints, however, cannot not confirm that financing constraints affect firm growth.

Table 1: Credit Decision

	1998		2003	
	all firms	firm age < 3 years	all firms	firm age < 3 years
Considered Seeking Credit	38.7%	44.2%	44.5%	43.1%
Did not apply	15.4%	21.0%	10.3%	14.1%
Applied for credit	23.3%	23.2%	34.2%	29.0%
% of application denied	19.9%	26.9%	10.3%	14.7%

Source: SSBF 1998 and 2003.

mergers in the banking sector. Within the surveys, small businesses are defined as firms with fewer than 500 employees. The samples were drawn from the Dun and Bradstreet database.[6]

The information provided includes firm and owner characteristics, an inventory of small businesses' use of financial services and of their financial service suppliers, income and the firm's balance sheet. One section of the survey asks about the most recent loan application. The data on the credit contract include the credit amount, the interest rate, and collateral requirements. The surveys distinguish between firms in need of external finance that did not apply for credit and firms whose credit application was denied. The SSBF also provides Dun and Bradstreet credit scores for each firm and indicators whether the firm had declared bankruptcy or has delinquent obligations or judgements. For the business owner the same indicators are available except for the personal credit score. The crucial variable to construct net worth of a merged firm-household entity, personal net worth, separated by net home value and other assets, is available for the 1998 and 2003 surveys only.[7] I therefore restrict my analysis to the last two cross sections. The SSBF provides a final, representative sample of 3,561 firms in 1998 and 4,240 firms in 2003. The sample weights imply that the 1998 sample represents 5,291,245 firms and the 2003 sample represents 6,298,087 firms.

The composition of small businesses in the 1998 and 2003 surveys are comparable. The average number of employees was 8.6 and the median number of employees was 3 in both samples. Firms in the manufacturing sector (SIC 20-39) have on average more than twice as many employees than the firms in the service sector (SIC 70-89) and firms in insurance and real estate (SIC 64-65). Average firm age increase from 13.3 years in the SSBF 1998 to 14.3 years in the SSBF 2003 after falling from 14.5 years in the SSBF 1993. The median also increased by one year from 11 years to 12 years. The vast majority of the firms is very small and owner-managed (94 percent). Less than 30 percent of the firms reported annual sales of more than $500,000. 35 percent of small business were located in the South, 24 percent in the West, 21 percent in the Midwest, and 20 percent in the Northeast. Roughly 4 out of 5 firms had their headquarters or main office in urban areas. 44.5% (1998: 49.4%) of firms are proprietorships, 8.7% (7%) partnerships, 31% (23.9%) C-Corporations and 15.8% (19.8%) S-Corporations.[8]

[6] A detailed description of the sampling procedure can be found in the methodology reports. Firms were asked to use tax data and previously sent worksheets to answer the questions. The data and methodology reports to all surveys can be downloaded at http://www.federalreserve.gov/Pubs/Oss/Oss3/nssbftoc.htm

[7] For a description of earlier surveys, see Berger and Udell (1998).

[8] Bitler, Robb, and Wolken (2001) and Mach and Wolken (2006) provide descriptive statistics, summarizing

Table 2: Net Value of Private Residence

	1998		2003	
	mean	median	mean	median
all firms	128,915	75,000	232,652	120,000
did not need finance	140,153	91,040	243,378	140,000
considered and did not apply	83,883	47,000	145,820	40,000
denied	84,327	22,000	143,298	60,000
received credit	140,326	77,769	235,167	140,000

Source: SSBF 1998 and 2003.

2.2 Credit Constraints I: Access to Credit

A main problem of empirical work is to identify financially constrained firms.[9] The SSBF provides two direct measures of financing constraints. The first measure is whether firms applied for credit when they were in need of credit. In particular, the both samples identify firms reporting that were in need of credit but decided against applying for a loan as they were afraid of denial. Secondly, not all credit applications were granted by financial institutions. Not being able to obtain (additional) external finance, these firms have to finance investment with internal resources only. Arguably these firms face the hardest credit constraints. Firm owners who decided not to apply for fear of denial give low credit scores, firm age, and financial situation as reasons for their decision. In the second case banks cite poor credit scores and poor financial position of the firms as reason for credit denial in both survey. Therefore I assume that firms that did not apply for credit were able to form precise expectations about denial and are actually financing constrained.

In the credit application process banks require a business financial statement and a personal financial statement. The business financial statements includes balance sheet information, information on accounts receivable, organizational form, and other business characteristics. The personal financial statement asks for a detailed description of assets and liabilities, including the net value of the private residence, total net worth, and previous bankruptcies.[10] Credit applications are costly. Banks charge application fees, and credit denial leaves a stain on the credit report that can increase the cost of future borrowing.

In total four types of firms can be distinguished. Firms that 1) indicate no need for credit, 2) were in need of credit but decided against applying for a loan as they were afraid of denial (so-called "discouraged borrowers", Jappeli (1990)), 3) applied for credit and were denied, and 4) successfully applied for credit. To avoid ambiguities, I drop observations reporting that they were "sometimes" turned down in their most recent application. Less than half of the sample considered participation or participated in the credit market (2-4). However, the sectoral composition between the participants and non-participants show only minor differences. In 1998 15% of the firms report that they were in need of credit but did

the financial services used by small businesses for the SSBF 1998 and 2003, respectively.

[9] Most commonly they use panel data to estimate the relationship between Tobin's q, cash flow, and investment. See Hubbard (1998) for a survey and Fazzari, Hubbard, and Petersen (2000) and Kaplan and Zingalis (2000) for potential problems with the methodology.

[10] For detailed application forms, see Cavalluzzo and Wolken (2005).

Table 3: Firm Size and Access to Credit by Age Class

	all firms		did not apply		denied credit		got credit	
	Number of obs.	Employees average (median)	Number of obs.	Employees average (median)	Number of obs.	Employees average (median)	Number of obs.	Employees average (median)
Age	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<3	574	5.0	100	3.6	35	6.5	113	7.4
		(2)		(2)		(4)		(4)
3-6	1508	6.5	258	4.2	91	6.3	385	10.6
		(3)		(3)		(4)		(5)
7-15	2468	8.2	277	5.4	104	7.1	762	14.0
		(3)		(3)		(4)		(6)
16-25	1758	9.9	127	4.8	46	13.2	629	16.6
		(4)		(2)		(4)		(6)
>25	1393	13	68	10.2	26	7.1	517	21.5
		(4)		(3)		(3)		(6)
Total	7649	8.5	830	5.0	302	7.6	2406	14.6
		(3)		(3)		(4)		(6)

Source: SSBF 1998 and 2003

not apply fearing denial. In 2003 only 10% of the sample had the same concern. While 19.9% of the applications were turned down (4.96% of the sample) in 1998, the denial rate dropped to 10.3% of the applications (3.52% of the sample). Young firms are less likely to apply for credit, and their applications are turned down more frequently (see Table 1).

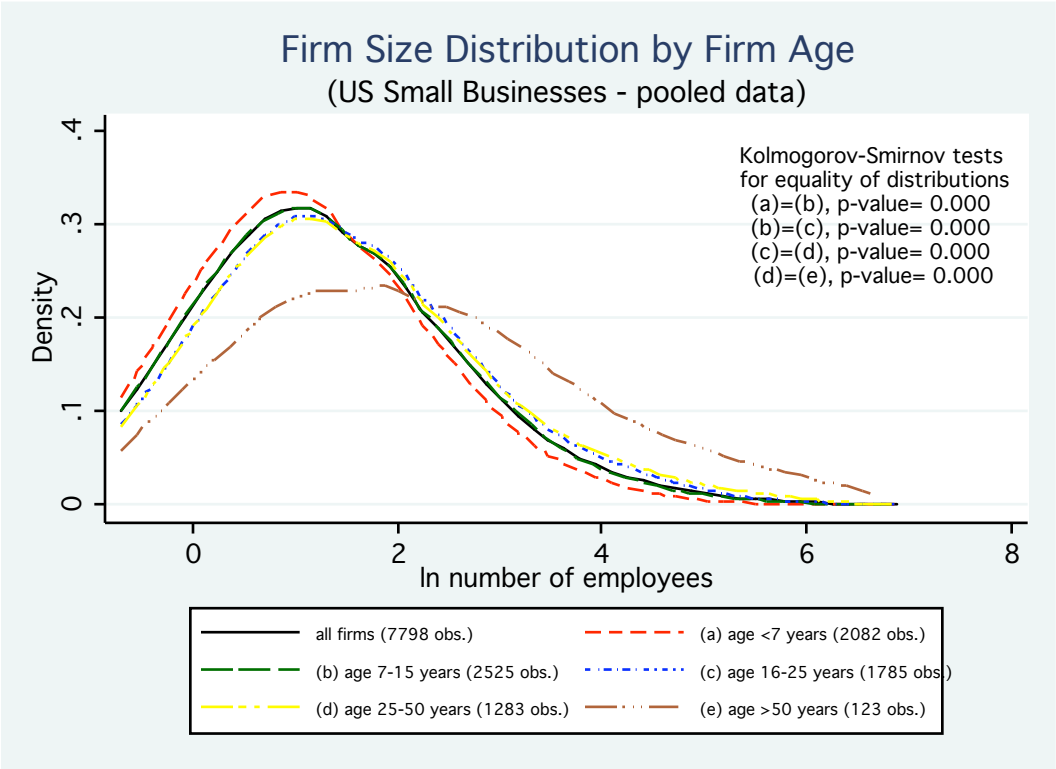
With perfect financial markets all firms would immediately produce at their optimal scale. Under this condition firm size and growth would be independent, the so-called Gibrat's law. The firm size distribution (FSD) is expected to be lognormal as processes that follow Gibrat's law converge to a lognormal distribution. Current studies have shown that the FSD of younger firms is skewed to the right and only the distribution of old firms appears to be symmetric.[11] Underinvestment due to financing constraints provides a possible explanation for these stylized facts.

First, I compare the firm size distribution by age. The firm size distribution in Figure 1 is comparable to the Portuguese data used by Cabral and Mata (2003) and the Italian data used by Angelini and Generale (2008) even though the SSBF is censoring the right tail of the firm size distribution. The size distribution of younger firms is strongly skewed to the right and becomes more symmetric with firm age.[12] Performing Kolmogorov-Smirnov tests for equality of distributions, I reject that the size distribution across age groups are the same (See Figure 1). The median and the average firm applying for credit is larger and possesses more resources than the median and the average firm considering participation in the credit market. This can also be seen in the net value of the residence (Table 2). Owners of firms

[11] Sutton (1997) surveys the older literature.

[12] I use the Epanechnikov kernel function with a bandwidth of 0.7 in all estiamtions.

Figure 1:



that successfully applied for credit have a higher equity in their private residence. Firm owners deciding against credit application have significantly lower equity in their privates houses than those who applied.

Table 3 shows the average and median employment by age group. Younger firms are smaller than their older counterparts. Firms that did not apply for credit have fewer employees than firms that were denied credit. In general firms without access to credit are smaller than firms with access to credit. Figure 2 illustrates differences in firm size by 1) the need of credit, 2) the decision whether to apply for credit, and 3) whether a firm got credit graphically. The first criterion distinguishes firms with no need for (additional) external finance and firms that report a need for external finance. Less than one half of small businesses are in need of external finance. Those that report to be in need of credit are larger. Businesses successfully applying for credit are larger than those whose credit applications were rejected. Firms that did to not apply for credit despite being in need of external finance are smaller than firms that were denied or received credit.

Next, I divide the sample by age groups. The results stated above hold in the subsamples. In Figure 2 I only report the results for the youngest age group which is affected most by financing constraints. One important feature of the SSBF is that it only surveys firms up to 500 employees, and hence the age distribution is biased towards young, supposably more financially constrained firms. Therefore, the data contain relatively many young firms. Secondly, the absolute number of firms whose of credit applications that were denied and the absolute number of firms that did not apply for credit fearing denial is large. Contrary

Figure 2:

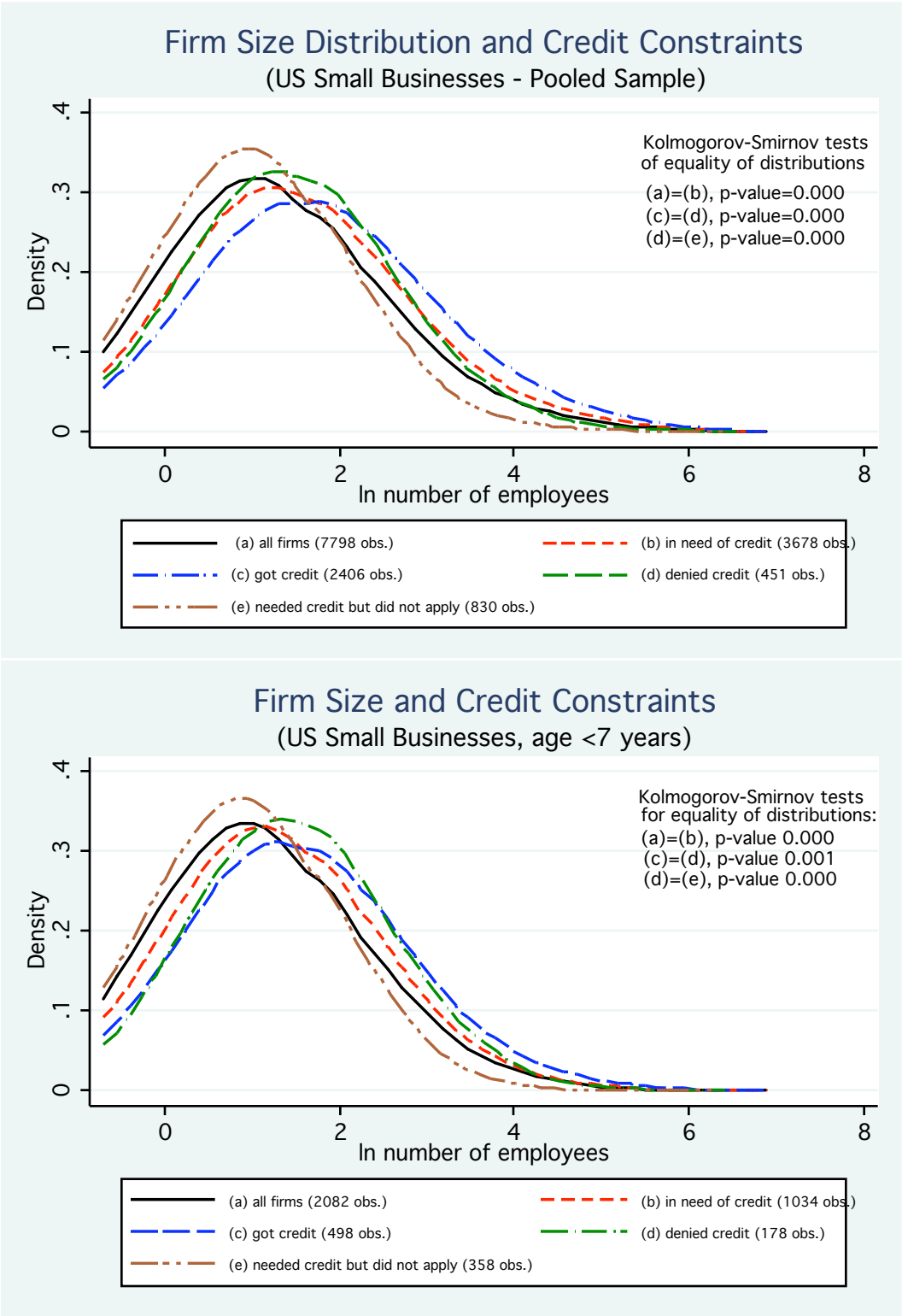

Firm Size Distribution and Credit Constraints
(US Small Businesses - Pooled Sample)
Kolmogorov-Smirnov tests
of equality of distributions
(a)=(b), p-value=0.000
(c)=(d), p-value=0.000
(d)=(e), p-value=0.000
Density
0
.1
.2
.3
.4
0
2
4
6
8
ln number of employees
(a) all firms (7798 obs.)
(b) in need of credit (3678 obs.)
(c) got credit (2406 obs.)
(d) denied credit (451 obs.)
(e) needed credit but did not apply (830 obs.)
Firm Size and Credit Constraints
(US Small Businesses, age <7 years)
Kolmogorov-Smirnov tests
for equality of distributions:
(a)=(b), p-value 0.000
(c)=(d), p-value 0.001
(d)=(e), p-value 0.000
Density
0
.1
.2
.3
.4
0
2
4
6
8
ln number of employees
(a) all firms (2082 obs.)
(b) in need of credit (1034 obs.)
(c) got credit (498 obs.)
(d) denied credit (178 obs.)
(e) needed credit but did not apply (358 obs.)

Table 4: Average Equity Share of Assets by Firm Age Class

	all firms		did not apply		denied credit		got credit		
	Number of obs.	Share average (median)	Number of obs.	Share average (median)	Number of obs.	Share average (median)	Number of obs.	Share average (median)	Share excl. last credit
Age	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<3	574	63.7%	100	50.4%	35	39.2%	113	54.3%	61.8%
		(86.6%)		(57.4%)		(17.0%)		(61.8%)	(86.5%)
3-6	1508	60.1%	258	53.1%	91	44.1%	385	54.9%	64.7%
		(76.4%)		(65.6%)		(42.4%)		(64.4%)	(81.4%)
7-15	2525	65.7%	277	49.0%	104	46.3%	761	54.8%	63.8%
		(85.0%)		(56.8%)		(47.6%)		(58.7%)	(80.5%)
16-25	1785	70.7%	127	59.9%	46	45.6%	630	61.2%	69.3%
		(85.4%)		(69.7%)		(47.5%)		(70.0%)	(84.3%)
>25	1406	76.3%	68	55.4%	26	62.5%	517	66.3%	76.0%
		(92.0%)		(63.6%)		(78.4%)		(78.5%)	(92.1%)
Total	7798	66.6%	830	52.6%	302	45.7%	2406	57.6%	67.5%
		(84.7%)		(64.5%)		(45.8%)		(65.8%)	(84.4%)

Source: SSBF 1998 and 2003

to Angelini and Generale (2008) the hypothesis of equality of distributions can be rejected also for credit denial only, their baseline definition of financially constrained firm.[13]

Sales provide an alternative measure of firm size. In the full sample the impact of financing constraints on the FSD are in line with the evidence presented using employment as measure of firm size (see Figure 7 in the Appendix). For the subsample of young firms, however, the hypothesis of equality of distributions between firms that were denied credit and all firms cannot be rejected.

2.3 Credit Constraints II: Collateral and Net Worth

Agency models of the credit market tie the abilities to borrow to the net worth of the borrower. Net worth can be used to collateralize the credit. Alternatively the owner can provide a personal guarantee. Personal guarantees apply only to the business owner. The owners declares an individual pledge to repay the business loan. Depending on the credit contract, the owner can be held responsible for the business loan even if the business is protected by limited liability laws or gets dissolved. In the pooled 1998 and 2003 SSBF 52% of the firms reported that collateral was required to receive credit. 54% gave personal guarantees. 30% of business owners provided both. This is in line with Sullivan, Warren, and Westbrook (1989) who, using bankruptcy data, show that entrepreneurs are more likely

[13]The highest p-value was 0.012 for the age group 1-6 years excluding "discouraged borrowers". While in the Italian data set of Angelini and Generale (2008) only 55 out of 1,009 firms up to 6 years of age were denied credit, we observe 178 out of 2,082 firms being financially constrained by this definition.

Table 5: Net Worth and Access to Credit by Firm Age Class

	all firms		did not apply		denied credit		got credit	
	Number of obs.	Net Worth (in mill.) average (median)	Number of obs.	Net Worth (in mill.) average (median)	Number of obs.	Net Worth (in mill.) average (median)	Number of obs.	Net Worth (in mill.) average (median)
Age	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
<3	574	0.276	100	0.120	35	0.177	113	0.224
		(0.095)		(0.042)		(0.072)		(0.114)
3-6	1508	0.301	258	0.191	91	0.361	385	0.434
		(0.107)		(0.080)		(0.100)		(0.140)
7-15	2525	0.358	277	0.138	104	0.148	761	0.535
		(0.150)		(0.063)		(0.087)		(0.225)
16-25	1785	0.514	127	0.220	46	0.429	630	0.734
		(0.238)		(0.107)		(0.165)		(0.300)
>25	1406	0.798	68	0.367	26	0.446	517	1.238
		(0.269)		(0.112)		(0.207)		(0.401)
Total	7798	0.431	830	0.181	302	0.276	2406	0.642
		(0.160)		(0.068)		(0.099)		(0.231)

Source: SSBF 1998 and 2003

to file personal bankruptcy as firm credits are collateralized or guaranteed by the owner's private net worth.

Giambona and Schwienbacher (2008) argue that only "hard" tangible assets, namely land and buildings, can be collateralized. Yet, almost 45% used equipment as collateral and only 23% their business real estate. The private residence served as collateral in 24% of the cases. Accounts receivable and inventory was offered as security in 23% of the cases.[14] The main difference between the two surveys is the shift from equipment (1998: 55%, 2003: 39%) to the private residence (1998: 15%, 2003: 29%) as the source of collateral. The increase in the net value of private residence caused by the increase in house prices over this period may explain this shift (see Table 2). Only about 6% use "other" personal asset beyond the private residence to collateralize their credit.

The patterns of collateral strongly suggest a wider definition of net worth than the one advocated by Giambona and Schwienbacher (2008). The literature defines net worth as maximum (collateralizable) assets; the sum of liquid and collateralizable, illiquid assets less all obligations. I construct net worth from the data as follows: Using the balance sheet information I subtract all liabilities from the total assets to calculate firm equity. In line with the patterns of collateral, for firm owners, who own a house, I add the net value of the private residence. Some of the firm-household entities have no positive equity. Instead of using the standard measure of leverage, assets divided by net worth, I construct the equity share of total assets. I divide total equity by the total assets (firm+residence) to attain $\frac{net\ worth}{capital}$,

[14] About 25% of the firms provided more than one type of collateral.

the inverse of leverage. The equity share of assets of a firm-household entity with negative equity is 0. Table 3 summarizes the results. Younger firms have a lower equity share of asset than older firms. Similarly, firms with no access to credit are relatively stronger indebted than firms with access to credit. On average this translates into an leverage, a capital to net worth ratio, of 1.5 or a equity to debt ratio of 2 and lies in the range of 1.3 to 2 reported in Masulis (1988). Table 5 shows that younger firms have less net worth. Firm successfully applying for credit have, on average, more resources.

3 Net Worth Multiplier

This section describes the standard optimal one period debt contract as used in the literature on firm growth and in macroeconomic models with a financial accelerator. I discuss two versions of the model, one with a linear and one with a concave production technology, and derive testable implications for the relationship of net worth and credit amount from the optimal contract. Before taking the model to the data, I lay out some sources of heterogeneity and their implications. I find that the corresponding empirical results are in line with the prediction of the model. Lastly, I calibrate the model of firms constraint by the optimal debt contract and show that the simulated data are consistent with the empirical facts.

3.1 The Optimal Debt Contract

Asymmetric information between the lender and the borrower leads to an agency problem that underlies the financing friction. The lender can not observe the outcome of a leveraged project. In case of bankruptcy the lender incurs a cost to verify the outcome. This costly state verification ties the ability to obtain credit to the net worth of an entrepreneur (Townsend (1979)). Gale and Hellwig (1985) show that in a world with costly state verification the optimal, incentive-compatible debt contract is the standard one period debt contract.

To derive the optimal one period debt contract, I first describe the preferences and technology of both, borrowers and lenders. Risk neutral entrepreneurs own a risky return technology. Entrepreneurs can finance their capital stock with equity and debt. If an entrepreneur borrows, she signs the standard one period debt contract with costly state verification.[15]. To better understand the implications of the underlying financing constraint, I abstract for now from aggregate uncertainty, differences in productivity, and equity issuance.[16] Let k be the capital stock used in production. Capital depreciates at rate δ. The entrepreneur can only use a fraction ξ of her net worth n in production. The fraction $(1-\xi)$ of net worth is pure collateral such as land or buildings. The entrepreneur can borrow to produce at scale k. She signs a one period debt contract with a financial intermediaries, the lenders, specifying the credit amount $d = k - \xi n$ and the gross interest rate Z. The payoff of production with external financing is

$$\Pi = \omega R^k k^\alpha + (1-\delta)k + (1-\xi)n - Zd, \tag{1}$$

[15]The first general equilibrium models with this type of financing constraint are by Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999).

[16]For models with debt/equity choice using the costly state verification, see Cooley and Quadrini (2001), Covas and DenHaan (2006), and Hennessy and Whited (2007).

where ω is an idiosyncratic productivity shock and R^k the price of output. The productivity shock ω is unknown at the time when the debt contract is signed and i.d.d. across time. The random variable has a continuous differentiable c.d.f $F(\omega)$ and $E(\omega) = 1$. The distribution satisfies the following regularity condition: $\frac{\partial h(\omega)}{\partial \omega} > 0$, where $h(\omega)$ is the hazard rate. This assumption guarantees that a interior solution is a non-rationing outcome. The property holds for, among others, the lognormal distribution.

The financial intermediaries, offering loans to entrepreneurs, are risk neutral, perfectly competitive, and hold perfectly diversified portfolios. The opportunity cost of funds for financial intermediaries is the economy's riskless gross rate of return $1 + r$. Financial intermediaries do not observe the realization of the idiosyncratic productivity shock. Lenders have to pay a cost to be able to observe the borrower's productivity shock. As in Carlstrom and Fuerst (1997) monitoring cost in case of bankruptcy are assumed to be a fraction μ of the expected output.

The entrepreneur defaults on the debt if after the realization of the idiosyncratic shock she does not have enough resources on hand to pay back the loan. Let $\bar{\omega}$ be the cutoff productivity for which the entrepreneur can just repay her debts.

$$\bar{\omega} R^k k^\alpha + (1-\delta)k + (1-\xi)n = Zd \tag{2}$$

In case of default the financial intermediary pays the monitoring cost $\mu R^k k^\alpha$ and seizes the firm's remaining assets. The default payoff is

$$\omega R^k k^\alpha + (1-\delta)k + (1-\xi)n - \mu R^k k^\alpha. \tag{3}$$

The financial intermediaries are perfectly competitive. Their zero profit condition has to hold in expectation on each contract. Using equation (3) the zero profit condition of the lender can be written as

$$\int_{\bar{\omega}}^{\infty} \left(\bar{\omega} R^k k^\alpha\right) dF(\omega) + \int_{0}^{\bar{\omega}} \left(\omega R^k k^\alpha - \mu R^k k^\alpha\right) dF(\omega) + (1-\delta)k + (1-\xi)n = (1+r)(k-\xi n). \tag{4}$$

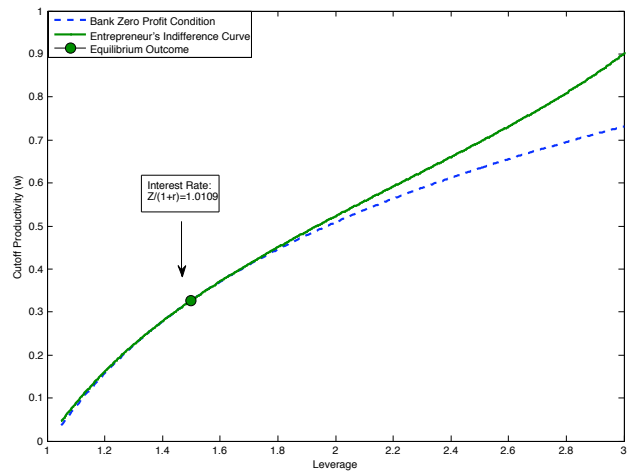
and the entrepreneur's expected income is

$$\int_{\bar{\omega}}^{\infty} \omega R^k k^\alpha dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega} k^\alpha. \tag{5}$$

For simplicity, define the firm's share of output as

$$\Gamma(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}))\bar{\omega}. \tag{6}$$

Figure 3: The Optimal Contract



and the lender's share of output as

$$G(\bar{\omega}) = (1 - F(\bar{\omega}))\bar{\omega} + \int_0^{\bar{\omega}} \omega dF(\omega) - \mu F(\bar{\omega}). \tag{7}$$

The combined shares of the lender and the borrower are together less than 1: $\Gamma(\bar{\omega}) + G(\bar{\omega}) = 1 - \mu F(\bar{\omega})$. This illustrates the distortion caused by asymmetric information about the idiosyncratic shock. An increase in the cutoff productivity $\bar{\omega}$, equivalent with more bankruptcies, or an increase in the monitoring cost lead to larger distortions. The loan contracts, in general, specify a pair of $(k - \xi n, Z)$. Here it is convenient to use the definition of the cutoff productivity $\bar{\omega}$ and to rewrite the problem as a choice over $(k, \bar{\omega})$. The solution to the optimal contract maximizes over these two variable the expected payoff to the borrower subject to the zero profit condition on the lender.

$$\max_{k,\bar{\omega}} \Gamma(\bar{\omega}) R^k k^\alpha \tag{8}$$

subject to

$$G(\bar{\omega}) R^k k^\alpha + (1 - \delta)k + (1 - \xi)n = (1 + r)(k - \xi n) \tag{9}$$

For simplicity, assume $\delta = 1$. In the linear case ($\alpha = 1$) the problem can be scaled by n and solved for the optimal leverage, the capital to net worth ratio, $L = k/n$ for all entrepreneurs. In other words, net worth matters for the amount of credit but, in equilibrium, all entrepreneurs have the same leverage and the same cutoff productivity $\bar{\omega}$. Figure 3 illustrates the solution in terms of indifference curves of the entrepreneur, the combinations of $(L, \bar{\omega})$ yielding the same expected profit, and the lenders zero profit condition. Let λ be the multiplier on the lender's zero profit constraint. For an interior solutions the first order

conditions are

$$\bar{\omega} : \Gamma'(\bar{\omega})R^k k + \lambda(G'(\bar{\omega})R^k k) = 0 \tag{10}$$

$$k : \Gamma(\bar{\omega})R^k + \lambda\left(G(\bar{\omega})R^k - (1+r)\right) = 0 \tag{11}$$

$$\lambda : \; G(\bar{\omega})R^k k + (1-\xi)n = (1+r)(k - \xi n) \tag{12}$$

Note that $\Gamma'(\bar{\omega}) = -(1 - F(\bar{\omega}))$ and $G'(\bar{\omega}) = -\Gamma'(\bar{\omega}) - \mu F'(\bar{\omega})$. In an interior optimum $G'(\bar{\omega}) > 0$ otherwise the bank could increase profits by lowering $\bar{\omega}$. The value of the Lagrange multiplier depends only on the cutoff productivity,

$$\lambda = -\frac{\Gamma'(\bar{\omega})}{G'(\bar{\omega})} \geq 1. \tag{13}$$

Intuitively, without the presence of bankruptcy cost, $\mu = 0$, the credit would not be risky and the Modigliani-Miller theorem holds. The marginal unit of capital can be financed with either net worth or credit without changing the payoff of the lender and $\lambda = 1$. If the entrepreneur has to take out a risky loan, then the multiplier reflects the shadow value of agency cost which are increasing in $\bar{\omega}$.

Define the external finance premium as rate of return over the risk free rate: $s = R^k/(1+r)$. Combing the first order condition for $\bar{\omega}$ and k gives:

$$s = \frac{1}{G(\bar{\omega}) - \Gamma(\bar{\omega})\frac{G'(\bar{\omega})}{\Gamma'(\bar{\omega})}} \tag{14}$$

By inverting the function, BGG show that $\bar{\omega}$ is increasing in s. Rewriting the zero profit condition of the lender yields

$$G(\bar{\omega})sL + \frac{1-\xi}{1+r} = L - \xi \tag{15}$$

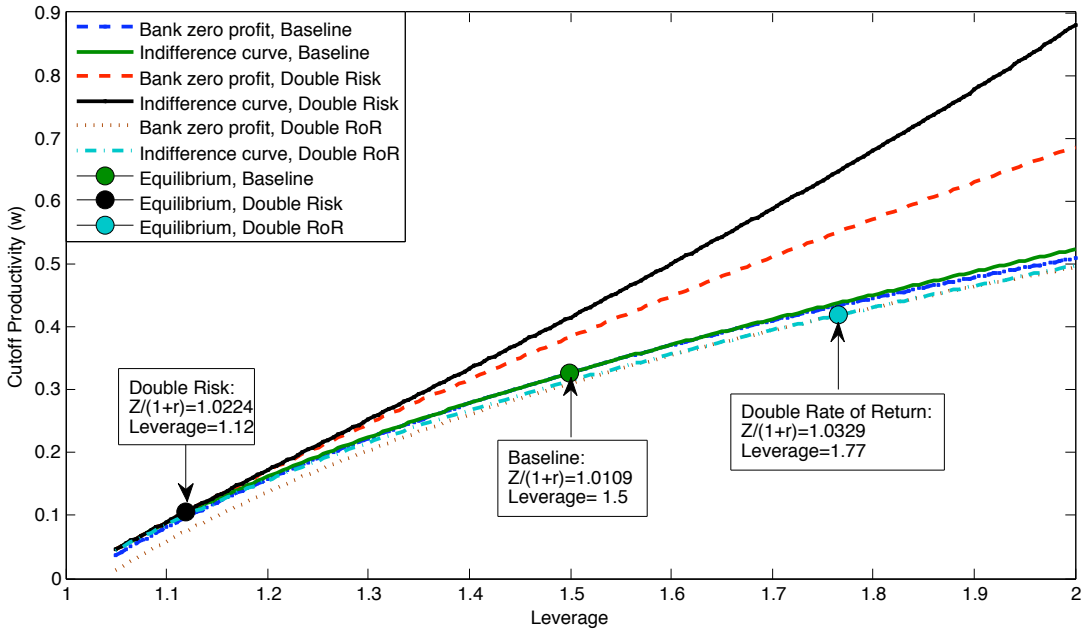
or

$$L = \frac{1 + r\xi}{(1+r)(1 - G(\bar{\omega})s)}. \tag{16}$$

The optimal contract trades off the return on an additional unit external finance against a rise in bankruptcy states, i.e. higher agency costs. The assumption of linearity in the production function is appealing for macroeconomic models as all entrepreneurs have the same leverage, face the same probability of default, and pay the same interest rate. The latter is implicitly defined in equation (3). Capital in production k is proportional to n. The equilibrium contract, defined as a pair of $(L, \bar{\omega})$, depends on four parameters: 1) the external finance premium s, 2) the variance of the idiosyncratic shock σ which measures risk, 3) the bankruptcy cost μ, and 4) the fraction of net worth used in production ξ. The zero profit condition of the lender implies that the amount of credit covered by collateral, $(1-\xi)n$, needs to earn the risk free rate. This introduces a additional wedge in the choice of leverage.[17]

[17] Any additional cost of repossessing collateral reduces leverage further. To see this, assume that in case of bankruptcy the bank only receives $(1-\mu)(1-\xi)n$. The optimal leverage now is $L = \frac{1+r\xi-\mu F(\bar{\omega})(1-\xi)}{(1+r)(1-G(\bar{\omega})s)}$

Figure 4: Comparative Statics



In the data firms are heterogeneous. I consider two sources of heterogeneity: i) differences in the rate of return R^k and ii) in risk σ. An increase in the rate of return increases the returns to external finance. A higher leverage is therefore optimal. This also implies a higher cutoff productivity and thereby higher interest rate paid on the loan Z. An increasing in variance of the idiosyncratic shock makes low productivity states more likely to occur as $E(\omega) = 1$. With a higher variance optimal contract exhibits a lower leverage and a higher interest rate (see Figure 4). Incomplete depreciation does not change these results but scales up leverage and reduces the probability of bankruptcy. The testable implications of the linear model are 1) the credit amount is a multiple of net worth, and 2) controlling for differences in risk, leverage and the interest rate are positively correlated.

The linear model predicts the same leverage for all entrepreneur regardless of firm age or net worth of the entrepreneur. However, Table 3-5 show a clear firm age-firm size pattern and difference is leverage by firm age. Therefore, I consider the case of decreasing returns to scale, $\alpha < 1$, with imperfect depreciation as in Cooley and Quadrini (2001) and Covas and DenHaan (2006). I derive the implications for leverage and interest rates on loan and show that the net worth-leverage pattern emerges when employing with decreasing returns to scale technology and the one period debt contract. In the case of $\alpha < 1$ for an interior

which is lower than without additional repossession cost. One interpretation of this friction are homestead exemption in bankruptcy laws. Berkowitz and White (2004) find that higher exemptions reduce access to credit significantly.

solutions the first order conditions are

$$\bar{\omega} : \Gamma'(\bar{\omega})R^k k^\alpha + \lambda(G'(\bar{\omega})R^k k^\alpha) = 0 \tag{17}$$

$$k : \alpha\Gamma(\bar{\omega})R^k k^{\alpha-1} + \lambda\left(\alpha R^k G(\bar{\omega})k^{\alpha-1} - (\delta + r)\right) = 0 \tag{18}$$

$$\lambda : \; G(\bar{\omega})R^k k^\alpha + (1-\delta)k + (1-\xi)n = (1+r)(k - \xi n) \tag{19}$$

Combing first order condition for $\bar{\omega}$ and k yields the optimal capital stock

$$k = \left(\frac{r+\delta}{-\frac{G'(\bar{\omega})}{F'(\bar{\omega})}\alpha R^k \Gamma(\bar{\omega}) + G(\bar{\omega})}\right)^{\frac{1}{\alpha-1}}. \tag{20}$$

In the case of $\alpha < 1$ the relationship between the optimal capital stock and net worth is not immediately clear from this equation. The entrepreneur's profit's are increasing in k and decreasing in the $\bar{\omega}$. For a given value of n an increase in k implies that the share of the lender $G(\bar{\omega})$ has to increase to make zero profits. Again, the optimal credit contract trades off the increase in payoff to the borrower and an increase in agency cost. The following proposition fleshes out this trade off and the relationship between k and n.

Proposition 1: *If $\alpha < 1$ then for an interior solution of the optimal debt contract*

1. *the bankruptcy rate, $F(\bar{\omega})$, is decreasing in n (Covas and DenHaan (2006)),*
2. *the debt to asset ratio $(k-n)/k$ is decreasing in n and increasing in ξ, and*
3. *k is strictly concave in n.*

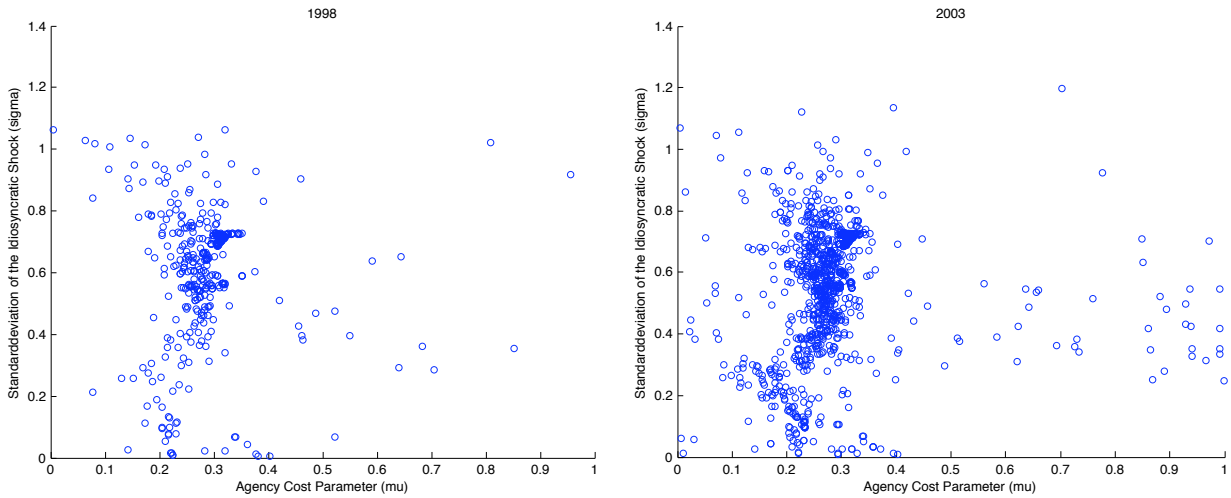
Proof: See Appendix.

One implication of proposition 1 is that the largest firms are those with the highest equity to asset ratios, where size is measure as units of capital in production. In the data, firm size is correlated with age. Young firms are on average smaller and with less net worth, as shown in Table 4. The credit amount $k - n$ is increasing in n but the debt to asset ratio is decreasing in n. In other words, the marginal effect of an additional unit of net worth, the net worth multiplier, is lower for large, old firms. Lastly, it follows from the first part of proposition 1 that leverage and the interest rate are positively correlated even without heterogeneity in the rate of return R^k or in risk σ. Differences in these two variable have the same implication as in the linear model.

3.2 Fitting the One Period Debt Contract

Before testing the implications of CSV in a regression analysis, I perform a first plausibility test in the spirit of Levin, Natalucci, and Zakrajsek (2004). I solve for the parameters of the credit contract, the agency cost μ, the standard deviation of the underlying log-normal distribution σ, the rate of return R^k, and the cutoff productivity $\bar{\omega}$ for each observation in the SSBF. I solve for these four parameters of a quarterly credit contract using (i) the definition of the cutoff productivity (equation (2)), (ii) the optimality conditions (equations

Figure 5: Implied Agency Cost and Risk Parameters



(14) and (16)), and (iii) the probability of bankruptcy $F(\bar{\omega})$. Since 90% of firms report to use the credit for working capital, I set $\delta = 1$. In line with financing constraints employed in macroeconomic models I set $\alpha = 1$ and $\xi = 0$.

In constructing leverage, I use the credit amount on the most recent credit and net worth as discussed in section 2.3. The interest rate on the credit in the data is used calibrate Z in equation (2). The risk free rate r is set to the Fed Funds Rate at the time the credit as contracted. For the 1998 SSBF the risk free rate is about 5% annually and 1% annually for the 2003 observations. The bankruptcy rate is taken from the Dun and Bradstreet credit scores. The credit scores provided in the SSBF indicate the probability of delinquency (payments are at least 90 days late) rather than bankruptcy.[18] Therefore, I assume that firms that are more like to miss payments are also more likely to file for bankruptcy. I scale the probability of delinquency incidents to match annual bankruptcy rate of 3% as reported in BGG. On a quarterly basis, this yields that firms with the highest score (6) have an quarterly failure probability of 0.19% while 4.44% firms with the lowest credit score (1) fail in the 2003 sample. In 1998, the probability of failure for the best (worst) risks is 0.15% (3.52%). In some cases, for instance when net worth is equal to zero, there exists no solution to the system of equations describing the credit contract or the solution involves implausible values such as negative agency cost or an agency cost share larger than 1. However, for 70% of the observations in 1998 (463 out of 660) and 77% of the observations in 2003 (1225 out of 1598) the system of equations can be solved with plausible parameter values. The distribution of parameters are displayed in Figure 5.

In 1998, the average agency cost μ is estimated to be 0.289 (stddev 0.08) which corresponds to 28.9% of output. For the 2003 sample the agency cost are 0.287 (stddev 0.11). This lies in the plausible range of 0.2-0.36 reported in Carlstrom and Fuerst (1997). BGG argue for lower values (0.12). Lower values for the bankruptcy cost are consistent with higher bankruptcy rates. The mean standard deviation of the idiosyncratic shock, σ, measuring the

[18]For a detailed discussion, see http://www.dnb.com.

Table 6: Dependent Variable: Credit Amount

	1998 Sample			2003 Sample		
	OLS	OLS	Selection[a]	OLS	OLS	Selection[a]
Net Worth	0.320**	0.298**	0.318**	0.360**	0.334**	0.318**
	(0.084)	(0.083)	(0.077)	(0.055)	(0.057)	(0.055)
Ln Number of Employees		0.066*			0.139**	
		(0.028)			(0.032)	
Intercept	0.051†	-0.052†	0.084†	0.057†	-0.189**	0.159**
	(0.028)	(0.030)	(0.046)	(0.034)	(0.043)	(0.057)
controls	no	no	no	no	no	no
N	711	711	959	1693	1693	1856
R^2	0.29	0.30	-	0.27	0.28	-

Significance levels: : † : 10% ∗ : 5% ∗∗ : 1%

[a] Heckman selection model using information on whether the firm's credit application was accepted. The probability of receiving credit is estimated using net worth, ln(emploment), ln(length of relationship with the lender), ln(firmage), credit score, and dummy variables for industry, region, gender, minority status, organizational form of the firm, previous bankruptcies and judgements against the firm.

riskiness of projects is 0.62 (stddev 0.2) in 1998. In 2003 the average standard deviation is with 0.57 (stddev 0.2) slightly lower.

3.3 Empirical Evidence

I now turn to the implications of the optimal debt contract. One implication of the one period standard debt contract is that the credit amount is some multiple of net worth. I now examine the relationship between the credit amount firms received and the net worth of the firm-entrepreneur entity.

Entrepreneurs borrow some multiple of their net worth n: $d = \varphi n$, where d is the credit amount and φ the net worth multiplier.[19] This implies the following estimation equation:

$$d_i = \alpha + \beta n_i + \epsilon_i \tag{21}$$

The agency model suggests $\beta > 0$. In the regressions credit amount and net worth are measured in millions. Table 6 summarizes the results for the full sample 1998 and 2003. The variation in net worth explains the variation in credit amounts well (R^2 of 0.29 and 0.27). The second and the fifth column show the results after controlling for firm size as proxy for the differences in the rate of return. The point estimates for the net worth multiplier barely change. In line with the prediction of heterogeneity in the rate of return, firm size affect the credit amount positively. Table 1 in section 2.2 shows that not all credit applications were accepted. Hence, the OLS estimators are potentially subject to a selection bias. The

[19] Reduced form financing constraints of this form are also commonly used in the literature on entrepreneurship, see e.g. Evans and Jovanovic (1989) and Cagetti and de Nardi (2006).

Table 7: Dependent Variable: Credit Amount

	1998 Sample				2003 Sample			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
age 1-2 x	0.725†				1.061*			
Net Worth	(0.380)				(0.486)			
age 3-6 x		0.648**				0.400**		
Net Worth		(0.132)				(0.064)		
age 7-15 x			0.294**				0.503**	
Net Worth			(0.088)				(0.119)	
> age 16 x				0.215**				0.304**
Net Worth				(0.042)				(0.082)
Intercept	0.065	-0.054	0.088*	0.073†	-0.136	0.134†	-0.002	0.054
	(0.077)	(0.032)	(0.043)	(0.039)	(0.114)	(0.032)	(0.060)	(0.063)
N	39	163	249	260	74	221	512	885
R^2	0.09	0.71	0.23	0.21	0.58	0.55	0.26	0.20

Significance levels : $\dagger$: 10% $*$: 5% $**$: 1%

selection model estimates in Table 6 indicate that selection not a big concern. The point estimates for the net worth multiplier are positive and significant in all regressions.

The data provide a rich set of additional control variables for sources of heterogeneity: the credit score of the firm, previous bankruptcy, census region, industry, and organizational form. The credit score and judgements against the firm, previous private and firm bankruptcies proxy for differences in risk. Regional and industry dummies control for difference in risk and rate of return in these dimensions. The impact of credit score is estimated to be zero in both samples. Due to lack a variation in the 1998 sample, judgements, previous private and firm bankruptcies can only be used in the 2003 sample. Judgements have a strong negative effect while previous bankruptcies exhibit no significant effect. Firms with judgements already pending are more risky. The negative effect of judgements on the credit amount is in line with the prediction of lower leverage for higher risk firms. While I find no significant differences between industries in the 1998 sample, construction (SIC 15-19) and transportation (SIC 40-49) dummies affect the credit amount negatively and the dummy for financial services (SIC 60-69) has a positive effect in the 2003 sample. Due to differences in liability, the variation in the legal form of the firms proxies for differences in bankruptcy cost. However, in both samples the legal form has no significant effect. This is consistent with the fact that have of the sample provided personal guarantees, and therefore limited liability of the firm has little effect. Employing this large set of controls does not yield significantly different results (see Appendix). An additional dollar of net worth accounts for 30 to 36 cents of additional external finance in the full sample.

Next, I cut the samples by firm age. Table 4 shows that the *average* leverage is decreasing in firm age which is consistent with the decreasing return to scale technology model but not with the linear model. However, the one period debt contract should be applied to the

latest credit as long run debt is taken into account when calculating net worth. Section 3.1 argues that the positive correlation between firm age and firm size implies that the net worth multiplier is decreasing firm age, as older firm get closer to their optimal size. For these firms it is optimal to reduce leverage and thereby reduce agency cost. I investigate whether the net worth multiplier exhibits the described pattern by firm age. In fact, in the data the point estimates of net worth are decreasing in firm age (see Table 7). Again the result is robust to controlling for firm size, including additional controls, and accounting for selection as discussed above (see Appendix). This is consistent with Gertler and Gilchrist (1994) who find that smaller firms respond more sensitively to a monetary contraction than large firms.[20] The model also implies that firms of the same size have to same net worth multiplier independent from age. Stratifying the sample by age and net worth, the null hypothesis of the same net worth multiplier across age groups for a given net worth category cannot be rejected in any subsample.[21]

The results are not driven by differences in collateral requirements. The percentage of credits that required collateral or a personal guarantee is constant across all firm ages. The main difference occurs in the type of collateral. Older firms tend to offer business real estate as collateral. Young firm typically own no or less business real estate.[22]

Section 2.3 argues that in the context of small businesses a wider definition of net worth is appropriate. To ensure that our results are not solely the result of the definition of net worth, I consider three additional definitions consistent with the pattern of collateral offered. Since equipment accounts for almost half of collateral offered, we always include depreciable assets in our definitions: 1) firm assets and net value of private residence less firm's loans, 2) firm's depreciable assets, land, building and net value of private residence less firm's loans, and 3) firm's depreciable assets, land, buildiung, and net value of private residence less firm's liabilities. The narrower definition of net worth also reduces the explanatory power of net worth in all regressions substantially. Some point estimates change, however the age-dependence pattern described above reoccurs in all estimations.

In addition the age pattern of the net worth multiplier, Figure 2 shows that firms without access to credit are smaller. Not being able to finance additional investment, these firms should also have slower employment growth compared to firms with access to credit. The 2003 SSBF asked only whether employment grew, remained the same, or declined over the last year. The 1998 SSBF does not allow us to make any inference on the change of firm size measure in the number of employees. I estimate the following regression:

$$\begin{aligned} \textit{change in employment} =& \beta_0 + \beta_1 \ln(firm\ age) + \beta_2 D^{constraint} \\ &+ \sum_j \gamma_j D_j^{sector} + \sum_k \alpha_k D_k^{region} + \epsilon, \end{aligned} \tag{22}$$

[20] It is also consistent with the evidence on asset growth by Carpenter and Peterson (2002) and on the cash flow sensitivity of investment summarized in Hubbard (1998).

[21] Due to small sample size all coefficients were estimated with wide confidence intervals. Nevertheless, most coefficients are statistically different from zero.

[22] While the decreasing coefficients on net worth are consistent with mitigation of asymmetric information over time in lending relationships (Petersen and Rajan (1994), Berger and Udell (2002)), the fact that collateral requirement are constant across age groups is not.

Table 8: Dependent Variable: Change in Employment

	Ordered Probit - all firms				Ordered Probit - firm age <10 years			
	Full Sample	Needed Credit	Full Sample	Needed Credit	Full Sample	Needed Credit	Full Sample	Needed Credit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Denied only	-0.09	-0.14			-0.03	-0.08		
	(0.17)	(0.15)			(0.23)	(0.21)		
no access to credit			-0.13	-0.25**			-0.14	-0.32**
			(0.08)	(0.08)			(0.11)	(0.12)
ln firm age	-0.19**	-0.17**	-0.19**	-0.21**	-0.20**	-0.17	-0.20*	-0.18†
	(0.03)	(0.04)	(0.03)	(0.04)	(0.08)	(0.11)	(0.08)	(0.12)
N	4028	2101	4028	2101	1338	693	1338	693
R^2	0.01	0.01	0.02	0.02	0.01	0.02	0.01	0.03

Significance levels : † : 10% ∗ : 5% ∗∗ : 1%

where change in employment is measured as an ordered outcome, shrunk, same or grew, in the 2003 sample. I employ two definitions of "constraint" in this regression: 1) only firms that were denied credit, and 2) firms with no access to credit (firm that were denied credit and firms in need of external finance that did not apply fearing denial).

Coefficients on the measures of financing constraints exhibit the expected negative sign (see Table 8). In line with the visible differences between firms with and without access to credit in section 2.1, the estimated effects of financing constraints are negative and, in part, significant. The effect sharply increases when considering firms that expressed the need for external finance only. The results do not change when restricting the sample to firms that are in business for at most 9 years.

Firm age significantly decreases the growth in employment. This is consistent with models of firm dynamics with financing constraints (e.g. Cooley and Quadrini (2001)). The estimated effect might be overstating the true parameter as the sample is censoring firms with more than 500 employees. The regression analysis confirms that firms without access to credit differ from firms that received external finance in employment growth. Of course, this estimation could be subject to reverse causality. However, growth in employment over the last year and self-reported credit constraints are observed within the same year. Hence, the credit decision was made before the actually growth rate of employment could be observed. Limiting the sample to firms seeking credit in the first months of the year does not change the results. This is consistent with the view that financing constraints can explain the skewness of the firm size distribution and its evolution as firms reach maturity (Cabral and Mata (2003)). Together with the age pattern of the net worth multiplier the results highlight the importance of financing constraints for firm dynamics.

In line with the predictions of the one period debt contract the data exhibit a strong, robust relationship between net worth and the amount of credit. The variation in net worth has high explanatory power for the variation in credit amount. The age pattern of

Table 9: Parameter Values

Parameter	Value	Moment
r	0.04	
α	0.7	Hennessy and Whited (2005)
μ	0.3	D&B bankruptcy rate 1% (Cooley and Quadrini(2001))
σ	0.27	average risk premium 250 basis points
c	0.31	average firm age 14.5 years in SSBF
δ	0.5	leverage of 2 for unconstraint firms
ξ	0.67	share of fixed assets in SSBF

Hennessy and Whited (2005) report an estimate of 0.693 coefficient on capital in the production function α for small firms. I set the bankruptcy cost parameter $\mu = 0.3$ to match a bankruptcy rate of 1% as reported by Cooley and Quadrini (2001) for Dun and Bradstreet data. This is within the range of 0.2-0.36 reported in Carlstrom and Fuerst (1997). The variance of the idiosyncratic productivity shock $\sigma = 0.27$ is set to match an average premium of 250 basis points, the middle of the range of the values suggested by Bernanke, Gertler, and Gilchrist (1999), Levin, Natalucci, and Zakrajsek (2004) and de Fiore and Uhlig (2005) (200-298 basis points). The annual risk free interest rate is set to 4%. The fixed cost parameter c was set to 0.3 to match the average firm age of 14.5 years in the SSBF. I choose depreciation to be high but incomplete $\delta = 0.5$, which yields a leverage of 2 for unconstrained firms. The share of fixed assets, such as land and buildings, of total firm assets in SSBF is 0.33. Hence, the fraction of the net worth used in production ξ is set to 0.67. The initial wealth of the new entrepreneur is drawn from a log normal distribution with mean equal to -1.9 and a variance of 0.25 of the underlying normal distribution. These parameter values imply that young firms start, on average, with one third of the average capital stock.

the net worth multiplier cannot be explained with a constant returns to scale technology, however assuming the same underlying friction with a diminishing return technology yields results consistent with this pattern. To be clear, the age dependency of the net worth multiplier is not driven by concavity of the production function alone. In the model the age dependency pattern is the result of the optimal contract with costly state verification. Without a financing friction the capital structure would be indetermined and no age pattern of the net worth multiplier would arise. The following section illustrate the implications of the agency problem for firm dynamics.

3.4 Repeated One Period Debt Contracts

Infinitely lived entrepreneurs maximize lifetime consumption. As in Cooley and Quadrini (2001) entrepreneurs produce with a decreasing return to scale technology and sign one period debt contracts described in section 3.1 in each period. At the beginning of the period they invest k financed by net worth and debt. Next, the idiosyncratic shock ω, unobservable to the financial intermediaries, realizes. Non-bankrupt entrepreneurs pay a fixed cost c to enter the next period and decide on consumption. Empirically, bankruptcy rates are too low to account for exit of firms. In the calibration c is set to match average firm age in the data. An alternative interpretation of this assumption is a minimum consumption requirement. Entrepreneurs discount the future at the gross risk free rate. The evolution of net worth of a firm depends on the realization of the idiosyncratic shock. The end-of-period net worth of non-bankrupt entrepreneur i is

$$n_{i,t+1} = \omega_{i,t} k_{i,t}^{\alpha} + (1-\delta)k_{i,t} + (1-\xi)n_{i,t} - Z(k_{i,t} - n_{i,t}) - c. \tag{23}$$

Table 10: Simulated Data: Credit Amount

	(1)	(2)	(3)	(4)
age 1-2 x	0.83			
n	(0.02)			
age 3-6 x		0.57		
n		(0.01)		
age 7-15 x			0.48	
n			(0.01)	
> age 16 x				0.47
n				(0.01)
Intercept	0.83	0.90	0.95	0.95
	(0.00)	(0.00)	(0.01)	(0.01)
R^2	0.99	0.95	0.92	0.92

Standard errors from 500 simulations. See text.

The optimal savings decision of the entrepreneur trades off a marginal unit of consumption in this period and the expected return of the marginal unit of net worth in the next period. Let $\Pi(n)$ be the ex ante value of the optimal debt contract. Then

$$\frac{\partial \Pi}{\partial n} = \lambda(1 + \xi r) \tag{24}$$

In contrast with the literature, which ensures that all entrepreneurs are in fact financing constrained by assuming either impatient entrepreneurs or tax advantages of debt, entrepreneurs in this model will never accumulate enough net worth to self finance because a fraction of net worth is pure collateral and is not used in production. The lower the fraction of working capital ξ, the less incentive entrepreneurs have to accumulate net worth. As argued in section 3.1, $\lambda \geq 1$ and decreasing in net worth. Richer entrepreneurs have less incentive to accumulate additional net worth since the agency cost in their loan contracts are already low. Let $k(n^*)$ denote the capital stock where the discounted expected return of the marginal unit of net worth is equal to the entrepreneur's discount rate $\frac{1}{1+r}$. Entrepreneurs with $n^i < n^*$ save all their net worth for the next period. Entrepreneurs with $n^i \geq n^*$ consume $n^i - n^*$. Consistent with the literature, the model implies that only large firms pay dividends.

In the calibration, I use standard parameter values (see Table 9). I simulate 500 panels with 4000 firms for 1000 periods. As in Cooley and Quadrini (2001) the length of a period is one year. An entrepreneur leaving the sample is replaced immediately. After 1000 periods the simulated data are divided by firm age, and credit amount is regressed on net worth in the cross section. In the simulated data the only source of heterogeneity is differences in net worth and firm age. The regression results on simulated data in Table 10 reflect the implications of proposition 1 and match the coefficients reported in Table 7 for the SSBF data. Young firms are on average smaller. The firm size increases with firm age. Younger firms are, on average, smaller and face higher agency cost. The marginal effect of an additional unit

of net worth is therefore higher for those firms and decreasing with firm age. Intuitively, old firms have reached or are close to reaching their optimal scale. These firms prefer to operate and grow with relatively less additional external finance to reduce the agency cost (i.e. larger firms have a smaller value of λ). Financing constraints therefore are a plausible explanation for differences in firm growth and finance choices. In addition, Cooley and Quadrini (2001), using a model with debt/equity choice, show that financing constraints are crucial to match additional stylized facts of firm growth.[23]

4 The Balance Sheet Effect

In this section I estimate the impact of the financial situation of the firm on the interest rate to capture the "balance sheet effect". In a macroeconomic model the logic of the "balance sheet effect" is as follows: A worsening of the economic conditions results in weaker balance sheets, i.e. higher debt to asset ratios. This drives up interest rates and reduces the demand for investment goods as financing becomes more expensive. The financial situation of the firms creates a feedback effect and is the key mechanism that amplifies shocks in models with financial accelerator. In the model higher risk leads to higher interest rates and lower leverage. Higher rate of returns on the other hand lead to higher leverage and higher interest rates. The identification of the balance sheet effect comes from the cross sectional variation in interest rates and debt to asset ratios, controlling for selection and controls to capture differences in risk.

4.1 Estimation

In the estimation I use the SSBF 2003 data only. A shortcoming of the 1998 survey is that it does not provide the same controls for risk. In particular, previous bankruptcies and judgements lack sufficient variation. The credit scores are also less detailed. This significantly reduces the ability to properly account for differences in risk in the 1998 data. In addition, with only about one third of the number of observations, it is considerably smaller and provides no proxy for cost of application. As in the estimation of the net worth multiplier I include industry dummies, census region, and organizational form of the firm as control variables. The Herfindahl-Hirschman Index (HHI), a measure of market concentration, of banks is used to assess the plausibility of the assumption of competitive financial intermediaries. Some firms have no assets. Therefore I construct the debt to asset ratio for the most recent credit. The debt to asset ratio on the last credit is defined as credit amount divided by credit amount plus net worth to avoid problems of infinite leverage. I drop 22 observations which are likely to be to subject to typos or misreporting. These firms report a credit amount less than 5% of the net worth yet paid an interest rate of at least 350

[23] If the optimal contract is used repeatedly then proposition 1 implies that the local sensitivity of investment to (past period's) internal cash flow is decreasing (as the agency problem is reduced) and converges monotonically to zero as n goes to infinity. The model satisfies the conditions of DeMarzo and Fishman (2007) and therefore the dynamics exhibit the cash flow sensitivities of investment patterns documented by Fazzari, Hubbard, and Petersen (1988). That is smaller, younger firms have higher sensitivities of investment to innovation to cash flow than older, larger firms.

Table 11: Dependent Variable: Interest Rate

	OLS	Selection I[a]		Selection II[b]		
	Interest Rate	Banks Decision	Interest Rate	Firm's Decision	Bank's Decision	Interest Rate
	(1)	(2)	(3)	(4)	(5)	(6)
Debt to Asset Ratio	1.16*	-0.29	1.18*	1.51**	0.15**	1.17**
	(0.57)	(0.22)	(0.56)	(0.17)	(0.03)	(0.04)
ln Number of Employees	-0.26*	0.21**	-0.26*	0.42**	0.12**	-0.24**
	(0.10)	(0.05)	(0.10)	(0.02)	(0.01)	(0.01)
Credit Score	-0.11	0.23**	-0.12		0.17**	-0.11**
	(0.08)	(0.01)	(0.06)		(0.03)	(0.01)
Firm Bankruptcy	2.55*	-0.01	2.56*		0.10	2.56*
	(1.17)	(0.58)	(1.16)		(0.10)	(1.19)
Net Worth in mill.	-0.07**		-0.07**			-0.09*
	(0.03)		(0.03)			(0.03)
Interest Rate				-0.08**		
				(0.01)		
Probability of credit				3.55**		
				(0.01)		
Controls	yes	yes	yes		yes	yes
R^2	0.12	-			-	
Log Likelihood	-	-4767952			-5853354	
N	1598	1728			2016	

Significance levels: : † : 10% ∗ : 5% ∗∗ : 1%. Standard errors of the Selection II model are approximated with the inverse of the outerproduct of the first derivate.
[a] Heckman selection model whether the firm's credit application was accepted. Additional control variables: dummy variables for industry, region, organizational form of the firm, previous bankruptcies, and judgements against the firm.
[b] Three stage Tobit model: Credit application, bank acceptance, and the outcome equation. For details, see Appendix. Additional control variables on the second stage as above.

basis points above the average interest rate in the sample. For some of these observations the reported cost of obtaining the credit exceeds the amount of credit.

Stiglitz and Weiss (1981) argue that credit markets are subject to self selection. I consider two ways to account for selection: 1) the standard selection model taken the bank's decision into account and 2) a two step selection process in which firm's choose to apply for loans in addition to the bank's decision. The last specification therefore takes potential demand for loans as laid out in section 2.2 into account. The specification of selection goes beyond the simple model presented in the previous section, yet when estimating the parameters of interest it has to be taken into account.[24] The selection models are estimated as multistage Tobit models. Entrepreneurs form expectations about the probability of obtaining credit and the interest rate and apply only if the expected profits from applying are higher than

[24]Manove, Padilla, and Pagano (2001) and de Fiore and Uhlig (2005) model selection explicitly.

the cost of applying. For banks the zero profit condition has to hold. Banks report that their decision is made on the debt to asset ratio and not on the level of net worth (Cavalluzzo and Wolken (2005)). A detailed description of the decision processes of entrepreneurs and banks and econometric procedure is provided in the Appendix.

Table 11 summarizes the results of the estimation. The financial situation of the firm, measured as the debt to asset ratio on the latest credit, has a strong positive, statistically significant effect on the interest rate paid. In line with the prediction of the model of financially constrained firms, the measures of risk, credit score and bankruptcies exhibit the expected signs. Firms with previous bankruptcies have to pay a 250 basis points premium.[25] Firms with higher credit scores pay lower interest rates; however, the effect is small. In the model net worth, firm size, and debt to asset ratio are multicollinear. In the decreasing return to scale case when looking at the impact of each of these three variables on the interest rate separately, proposition 1 predicts net worth should exhibit a negative sign, the debt to asset ratio a positive sign and firm size, measured as capital in production, a negative sign. The effect of net worth is negative but estimated close to be zero economically. The point estimate for firm size is small, negative, and significant. Lastly, the estimates for banking concentration (not reported in Table 11) imply lower interest rates by up to 33 basis point in concentrated markets. However, the effect is not statistically significant and the assumption of competitive financial intermediary cannot be rejected.

The results of the Selection II model are consistent with the reasons given for not applying for credit (section 2.2). The expected probability of getting credit has a strong, positive effect on the probability of applying. Small firms are less likely to apply. The expected interest rate has a small negative effect on the probability of applying.

Overall the results strongly support the model with financing constraints presented in section 3.1. In particular, the estimated effect of the marginal debt to asset ratio on the interest rate, the balance sheet effect that underlies the propagation mechanism in macroeconomic model such as BGG, is large, positive and significant.

4.2 Implications for the Propagation of Shocks

BGG embed financing constraints in a New Keynesian Model by introducing a one period debt contract with costly state verification between financial intermediaries and entrepreneurs. Entrepreneurs finance their capital stock partly with loans in period t. At the end of each period they rent out the capital stock k_{t+1} for the next period and earn a stochastic return ωR^k_{t+1}. The loan contract is the one period debt contract with linear production technology described in section 3.1. The loan contracts are state contingent, that is the interest rate is determined after production in $t+1$ was carried out. Equations (15) and (17) show leverage can be expressed as a function of the expected external finance premium, the rate of return over the risk free rate. Inverting this function and defining ψ as the elasticity of the external finance premium with respect to leverage, the log-linearization of this relationship between external finance premium and leverage yields

$$E_t\left(r^k_{t+1} - r_{t+1}\right) = \psi(k_{t+1} + q_t - n_{t+1}), \tag{25}$$

[25] Personal bankruptcies and judgements also exhibit positive signs but are not statistically significant.

where percentage deviation q_t of the price of capital and r_{t+1} the percentage deviation of the gross risk free rate. The crucial parameter is ψ. With $\psi = 0$ the external finance premium does not respond to changes in leverage, and the model does not exhibit any propagation of shocks. BGG proxy the external finance premium with the risk premium paid on external finance. They argue that entrepreneurs equalize the cost and return to external finance rather than calibrating the model to the interest rate of the debt contract Z. In the remainder of this section I will maintain this assumption.

I use the point estimate of the debt to asset ratio reported in Table 11 to derive the elasticity of the external finance premium with respect to leverage. The debt to asset ratio is defined as $(k-n)/k = 1 - n/k \in [0,1]$. To infer the elasticity of interest rate with respect to leverage, note that the derivative of the debt to asset ratio with respect to leverage is $1/\left(k/n\right)^2$. The average value of $(k-n)/k = 0.279$ implies an average leverage $k/n = 1.39$ on the latest credit contract. The average interest rate in the sample is $i = 6.53$. The elasticity of the interest rate, the proxy for the external finance premium, with respect to leverage is

$$\psi = \frac{\partial i}{\partial \frac{k-n}{k}} \frac{\partial \frac{k-n}{k}}{\partial \frac{k}{n}} \frac{\frac{k}{n}}{i} = 0.129.$$

BGG assume that all firms have to refinance every period. In this case the elasticity of the whole sample would be 0.129. However, weighted by net worth, only 43% of the sample take out credit. Scaled by the weighted percentage of firms that obtain credit, the elasticity of the whole sample is 0.0554. This is in line with recent findings. Gilchrist, Ortiz, and Zakrajsek (2009) proxy the external finance premium with bond spreads in a Bayesian DSGE estimation and report this elasticity to be 0.04. The authors also note that their estimate is considerably lower than the mean of their prior (0.07). Christensen and Dib (2008), using no financial data, estimate the elasticity to be 0.042. Meier and Mueller (2006) estimate the elasticity by matching impulse response functions and report an even higher, but insignificantly estimated value of 0.067.

Before calibration the BGG model with this parameter values, I briefly summarize the full general equilibrium model and present it in its log-linearized form. Entrepreneurs, constraint by the one period debt contract, buy capital from a capital good sector and rent it to monopolistic competitive intermediate good firms for the next period. In the next period, after production takes place, entrepreneurs receive a risky return on their capital, sell their non-depreciated capital to the capital good sector and settle their debt. A second friction in the model is sticky prices. Households consume, provide labor, and save. Lastly, the monetary authority sets the nominal interest rate. In the following equations lower case letters denote percent deviations from the steady state and upper case letters without time subscript denote the steady state values. The aggregate resource constraint is

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e + \phi^y, \tag{26}$$

where Y is output, C consumption, I investment, C^e entrepreneurial consumption, and ϕ^y total bankruptcy cost in the economy.[26] BGG assume log utility in consumption. Using the

[26] BGG argue that the effect of bankruptcy costs are second order. For details, see BGG.

intertemporal Euler equation yields

$$c_t = -r_{t+1} + E_t(c_{t+1}), \tag{27}$$

where r_{t+1} is the risk free interest rate. Departing entrepreneurs consume their net worth N.

$$c_t^e = n_{t+1} + \log\left(\frac{1 - C_{t+1}^e/N_{t+1}}{1 - C^e/N}\right). \tag{28}$$

As discussed in section 4.1. The expected external finance premium is a function of leverage. ψ denotes the elasticity of the external finance premium with respect to leverage.

$$E_t\left(r_{t+1}^k - r_{t+1}\right) = \psi(k_{t+1} + q_t - n_{t+1}) \tag{29}$$

The realized rate of return is given by

$$r_{t+1}^k = \left(1 - \frac{1-\delta}{(1-\delta) + \alpha Y/(NX)}\right)(y_{t+1} - k_{t+1} - x_{t+1}) + \frac{1-\delta}{(1-\delta) + \alpha Y/(NX)} q_{t+1} - q_t, \tag{30}$$

where δ is the depreciation rate, α the capital share in production, and X the markup. The price of capital is derived from the investment equation:

$$q_t = \varphi(i_t - k_t), \tag{31}$$

where φ is the elasticity of investment cost. Production is given by

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t, \tag{32}$$

where A is technology, H household labor, and Ω the household labor share. Entrepreneurial labor is inelastic and normalized to 1. The labor market equilibrium is

$$y_t - h_t - x_t - c_t = \frac{1}{\eta} h_t, \tag{33}$$

where η is the Frisch elasticity of labor supply. The presence of sticky prices yields the following Philips curve

$$\pi_t = E_{t-1}\left(-\frac{1-\theta}{\theta}(1-\theta\beta)x_t + \beta\pi_{t+1}\right), \tag{34}$$

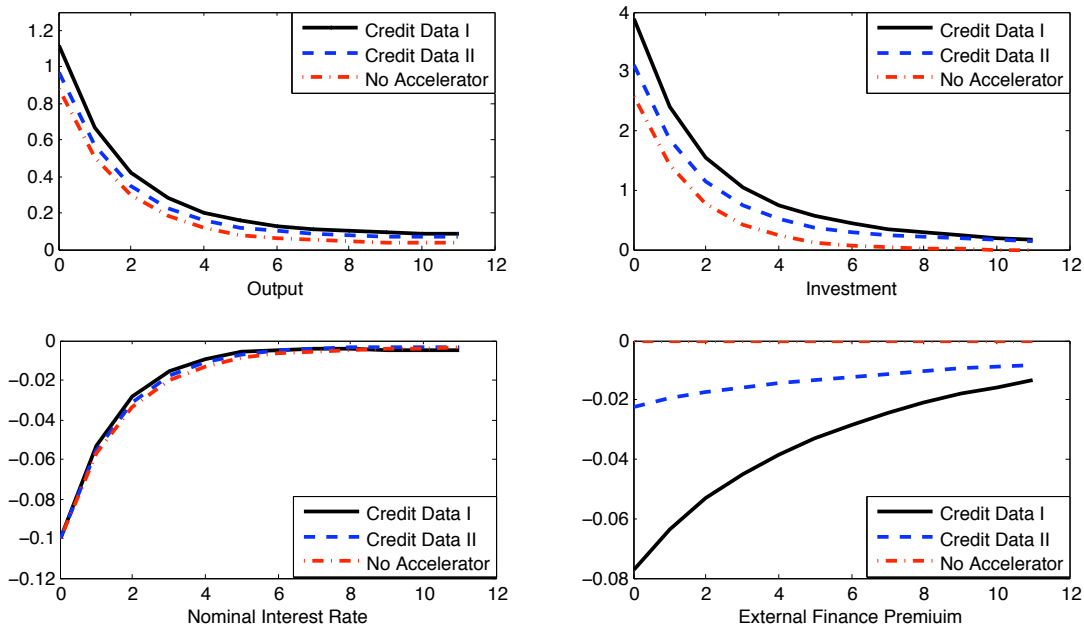
where π is inflation, $1-\theta$ is the fraction of price changers and β the households' discount rate. Capital evolves according to

$$k_{t+1} = \delta i + (1-\delta)k_t. \tag{35}$$

The evolution of entrepreneurial wealth is

$$n_{t+1} = \frac{\gamma RK}{N}(r_t^k - r_t) + r_t + n_t + \frac{(R^k/R - 1)K}{N}(r_t^k + q_{t-1} + k_t) + \frac{(1-\alpha)(1-\Omega)Y/X}{N} y_t - x_t. \tag{36}$$

Figure 6: Impulse Responses to a Monetary Policy Shock



Monetary policy follows a Talyor rule with weights ρ on the nominal interest rate R^n in the last period and ξ on past inflation.

$$r_t^n = \rho r_{t-1}^n + \xi \pi_{t-1} + \epsilon_t^n, \tag{37}$$

where ϵ_t^n is the monetary policy shock. The government consumption and technology follow AR (1) processes.

$$g_t = \rho^g g_{t-1} + \epsilon_t^g, \tag{38}$$

and

$$a_t = \rho^a a_{t-1} + \epsilon_t^a. \tag{39}$$

Next, I use the parameter values derived from the SSBF to calibrate the model of BGG.[27] A period in the model is one quarter. In the first calibration, denoted Credit Data I, I maintain the assumption of BGG that all firms have to refinance the whole capital stock. From Table 3, average leverage is set to 1.5 and the elasticity of the external finance premium is set to 0.129. The second calibration, Credit Data II, uses the average leverage on the latest

[27]Following BGG, I set the consumption to output ratio $C/Y = 0.60$ and the share of entrepreneurial consumption $C_e/Y = 0.01$. Government spending is calibrated to 20% of output. The quarterly discount factor is fixed to 0.99. The labor supply elasticity η is set 3. As is also within convention, the capital share, α, is 0.35, and the household labor share, $(1-\alpha)(1-\Omega)$, is 0.64. It follows that the share of income accruing to entrepreneurial labor is equal to 0.01. The capital depreciates at a quarterly rate $\delta = 0.025$. The authors report a value of 0.25 for the elasticity of the price of capital with respect to the investment capital ratio, φ. The survival rate of entrepreneurs is $\gamma = 0.975$. The parameters in the policy rule policy rule are set $\rho = 0.9$ and the coefficient on inflation equal to 0.11. Lastly, θ, the probability a firm does not change its price, is equal to 0.75.

credit contract 1.39 and the adjusted elasticity of 0.0554. The difference in the responses of an economy with financial accelerator can be seen in Figure 5. Both calibration can be visibly distinguished from the case without financing constraints. The response to output is 28% higher in the Credit Data I than in the calibration without financing constraints. In the second calibration, Credit Data II, the response is still 11.5% higher. The easing of financing constraints amplifies the response of the economy in particular in investment. To be clear, the SSBF data are representative for only one half the economy. However, the Bayesian DSGE estimation discussed above are consistent with my results. Secondly, the balance sheet effect is only one channel through which financial constraints affect the economy. Christiano, Motto, and Rostagno (2007) and Nolan and Thoenissen (2009) argue that shocks to the parameters of the financial sector help to explain the data.

5 Conclusion

Using US small business data this papers examine in detail the nature and the impact of financing constraints. The data allow a plausible definition and construction of net worth of the firm-entrepreneur entity. I find broad empirical support for financing constraints as implied by the optimal one period debt contract with costly state verification. In particular, the data exhibit a strong, positive relationship between net worth and credit. An additional dollar of net worth accounts for 30 to 36 cents of additional external finance. Additional control variables provide further support for this type of financing constraints. The net worth multiplier decreases with firm age. While the model of financing constraint firm with a constant returns turn scale production technology cannot account for this pattern, a model assuming the same financing constraint with a decreasing returns turn scale production technology matches the data. With this modification the model also matches stylized facts of firm dynamics. In addition younger and smaller firms are less likely to have access to credit. Firms without access to credit exhibit slower employment growth. This highlights the importance of financing constraints for firm growth and the evolution of the firm size distribution.

From a macroeconomic point of view the heterogeneity by firm age can be neglected as old firms represent the vast majority of firms weighted by net worth. Nevertheless the empirical results for these firms are still consistent with the model of financially constrained firms. The modeling strategy of BGG based a on representative entrepreneur is therefore empirically plausible. The central parameter underlying the balance sheet effect in macroeconomic models with financial accelerator, the elasticity of the external finance premium with respect to leverage, is large, significant and exceeds current estimates from Bayesian DSGE models. The calibration of the New Keynesian model with financial accelerator by Bernanke, Gertler, and Gilchrist (1999) to the small business data exhibit an amplification of the response of output to a monetary policy shock by up to 28%. Therefore, using firm level data, in this paper provides strong evidence for the existence and impact of financing constraints in both, explaining patterns of firm growth and providing an important channel for the propagation of shock to the economy.

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Appendix A - Proofs

Proposition 1
The first part of the proposition follows Covas and DenHaan (2006). Note that

$$\Gamma''(\bar{\omega}) = F'(\bar{\omega}) \tag{40}$$
$$G''(\bar{\omega}) = \Gamma''(\bar{\omega}) - \mu F''(\bar{\omega}) \tag{41}$$

Generally the sign of $G''(\bar{\omega})$ is not determined. Rewriting equation (X), the definition of the multiplier, yields $\lambda = \frac{1}{1-\mu h(\bar{\omega})}$. Together with the regularity condition $\frac{\partial h(\omega)}{\partial \omega} > 0$ gives

$$\frac{\partial \lambda}{\partial \bar{\omega}} = -\frac{\Gamma''(\bar{\omega})}{G''(\bar{\omega})} < 0. \tag{42}$$

Rewriting the first order condition with respect to capital gives

$$k^{\alpha-1} = \left(\frac{\lambda(r+\delta)}{\alpha R^k(\Gamma(\bar{\omega}) + \lambda G(\bar{\omega}))} \right). \tag{43}$$

From the calculations above it follows that the right hand side is decreasing in $\bar{\omega}$. Suppose the $\frac{\partial \bar{\omega}}{\partial n} \geq 0$. This would imply that an increase in net worth would lead to a decrease in the k and higher agency cost as $F(\bar{\omega})$ increases. But the old pair of $(k, \bar{\omega})$ is still feasible and has higher payoffs for the lender and the borrower. The new pair would never be optimal. Hence, $\frac{\partial \bar{\omega}}{\partial n} < 0$ and by the same argument $\frac{\partial k}{\partial n} > 0$.

The second part of the proposition states that debt to asset ratio, $(k-n)/k$, is decreasing in net worth. Combining the zero profit condition of the bank (20) and the optimal capital stock (21) yields

$$\frac{G(\bar{\omega})\lambda(r+\delta)}{\alpha(\Gamma(\bar{\omega}) + \lambda G(\bar{\omega}))} = (r+\delta) - (1+r\xi)\frac{n}{k} \tag{44}$$

or

$$\frac{n}{k} = \frac{(r+\delta)}{(1+r\xi)} \left((1 - \frac{1}{\alpha(\frac{1}{\lambda}\frac{\Gamma(\bar{\omega})}{G(\bar{\omega})} + 1)} \right). \tag{45}$$

Again, leverage is increasing in ξ. The interior solution always has $k/n > 1$. To show that the left hand side is increasing in n, differentiate the right hand side with respect to n. Define $\Lambda = \frac{1}{\lambda}\frac{\Gamma(\bar{\omega})}{G(\bar{\omega})}$. Note that Λ is decreasing in $\bar{\omega}$. Together with $\frac{\partial \bar{\omega}}{\partial n} \geq 0$ this implies that an increase in n reduces the denominator in equation (31). The right hand side is increasing in n. Hence, the debt to asset ratio is strictly decreasing in n.

The third part of the proposition immediately follows from the second part. As argued in first part, an increase in n leads an increase in k. This together second part yields $\frac{\partial k}{\partial n} < \frac{k}{n}, \forall n$ and implies $\frac{\partial^2 k}{\partial n^2} < 0$.

Appendix B - Estimation

The data show that credit applications are costly. In the first stage entrepreneurs trade off the profits $\hat{\Pi}$ from applying against the cost c. In their decision entrepreneurs internalize the expected interest rate $E(r)$ and the probability of receiving credit $E(P^{credit})$. Entrepreneurs only apply if $E[\hat{\Pi}_i(x_1, E(P^{credit}), E(r)) - c] \geq \bar{\Pi}$, where $\bar{\Pi}$ is the value of their outside option, and x_1 are firm characteristics. If the entrepreneur chooses to apply, the lender screens the project. Credit denial may occur in equilibrium if the bank receives a signal from screening that the project is likely to fail. The lender offers credit if she makes at least zero profits on the contract, $\Pi^b(x_2) \geq \bar{\Pi}^b$, where Π^b denotes the lender's profit and $\bar{\Pi}^b$ the opportunity cost of the lender.

Lastly, the interest rate r is a function of observables x_3. I assume that the interest rate equation is known to the entrepreneur. This assumption is plausible since the entrepreneur has to supply a large amount of information in the application process.

Firm's decision:
$\hat{\Pi}_i(x_1, E(r)) = x_1\beta_1 + \gamma E(r) + \theta E(P^{credit}) + \lambda c + \epsilon_1$

$$y_1 = \begin{cases} 1 & \text{if } E[\hat{\Pi}_i(x_1, E(r), E(P^{credit})) - c] \geq \bar{\Pi} \\ 0 & \text{otherwise} \end{cases}$$

Bank's decision:
$\Pi^b(x_2) = x_2\beta_2 + \epsilon_2$

$$y_2 = \begin{cases} 1 & \text{if } \Pi^b(x_2) \geq \bar{\Pi}^b \ \& \ y_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

Observed interest rate:
$r = x_3\beta_3 + \epsilon_3$

$$y_3 = \begin{cases} r & \text{if } y_2 = 1 \ \& \ y_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

The indicator variable for the first step y_1 is equal to 1 if a firm applied for credit and 0 if she states the need for external finance but never applied.[28] The covariates used in the first step (x_1) are ln(employment), debt to asset ratio on the last credit, and distance to the bank. I forecast the debt to asset ratio for firm that did not apply by estimating a selection model on the credit amount. Distance to the bank serves as proxy for application cost.

The indicator variable for the first step y_2 is equal to 1 if the firm successfully applied for credit. The lender's decision is a function of the credit score, ln(employment), industry, region, business type, and marginal debt to asset ratio. Note that all three stages include different sets of covariants. These exclusion restrictions allow identification of the key parameters without relying on functional form assumptions only. In addition, I assume that expectation errors are uncorrelated with the error in the application decision ($E(r) = x_3\beta_3$ and $E(P^{credit})$=$P(x_2\beta_2)$). This assumption allows the identification the covariances of the

[28] Same firms report that they applied for credit and that at another occasion when they needed credit, they did not apply. These firms were coded 1 since they applied.

three error terms. I estimate the full model using maximum likelihood. In this case the likelihood function is

$$\begin{aligned}
&\mathbb{L}(\beta_1, \beta_2, \beta_3, \gamma, \Sigma) = \\
&\prod_i \left[P(\hat{\Pi}_i(x_1, E(r), c) \leq \bar{\Pi})\right]^{1[y_1=0]} \\
&\left[P(\Pi^b(x_2)] \leq \bar{\Pi}^b, \hat{\Pi}_i(x_1, E(r), c)] \geq \bar{\Pi})\right]^{1[y_2=0, y_1=1]} \\
&\left[P(\Pi^b(x_2)] \geq \bar{\Pi}^b, \hat{\Pi}_i(x_1) E(r), c] \geq \bar{\Pi}, y_3 = r)\right]^{1[y_2=1, y_1=1]}.
\end{aligned} \tag{46}$$

The first part, representing the decision not to apply, can be written as

$$P(\hat{\Pi}_i(x_1, E(r), c) \leq \bar{\Pi} | x_1, x_2, x_3) = \Phi\left(\frac{-X_1\beta_1 - \gamma E(r) - \theta E(P^{credit}) - \lambda c}{\sqrt{\sigma_{11}}}\right). \tag{47}$$

The case of no equilibrium contract satisfying the participation constraint of the firm conditioning on the (positive) application decision of the firm.

$$\begin{aligned}
&P(\Pi^b(x_2) \leq \bar{\Pi}, \Pi_i(x_1, E(r), c) > \bar{\Pi}) \\
&= P(\Pi^b(x_2) \leq \bar{\Pi}^b | \hat{\Pi}_i(x_1, E(r), c) > \bar{\Pi}) P(\hat{\Pi}_i(x_1, E(r), c) > \bar{\Pi}) \\
&= \int_{-x_1\beta_1 - \gamma E(r) - \theta E(P^{credit}) - \lambda c}^{\infty} \Phi\left(\frac{-x_2\beta_2 - \frac{\sigma_{12}}{\sigma_{11}}\epsilon_1}{\sqrt{\sigma_{22}\left(1 - \frac{\sigma_{12}^2}{\sigma_{11}\sigma_{22}}\right)}}\right) \frac{1}{\sqrt{\sigma_{11}}} \phi\left(\frac{\epsilon_1}{\sqrt{\sigma_{11}}}\right) d\epsilon_1.
\end{aligned} \tag{48}$$

Lastly, contracts are only observed if firms decide to apply and an agreement is reached.

$$\begin{aligned}
&P(\hat{\Pi}_i(x_1, E(r), c) > \bar{\Pi}, \Pi^b(x_2) > \bar{\Pi}^b, y_3 = r) \\
&= P(\hat{\Pi}_i(x_1, E(r), c) > \bar{\Pi} | \Pi^b(x_2) > \bar{\Pi}^b, y_3 = r) P(\Pi^b(x_2) > \bar{\Pi}^b | y_3 = r) P(y_3 = r) \\
&= \int_{-x_2\beta_2}^{\infty} \left(1 - \Phi\left(\frac{-x_1\beta_1 - \gamma E(r) - \theta E(P^{credit}) - \lambda c - \begin{pmatrix}\sigma_{12} & \sigma_{13}\end{pmatrix} \begin{pmatrix}\sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33}\end{pmatrix}^{-1} \begin{pmatrix}\epsilon_2 \\ r - x_3\beta_3\end{pmatrix}}{\sqrt{(\sigma_{11} - \begin{pmatrix}\sigma_{12} & \sigma_{13}\end{pmatrix} \begin{pmatrix}\sigma_{22} & \sigma_{23} \\ \sigma_{32} & \sigma_{33}\end{pmatrix}^{-1} \begin{pmatrix}\sigma_{12} \\ \sigma_{13}\end{pmatrix}}}\right)\right) \\
&\frac{1}{\sqrt{\sigma_{22}\left(1 - \frac{\sigma_{23}^2}{\sigma_{22}\sigma_{33}}\right)}} \phi\left(\frac{\epsilon_2 - \frac{\sigma_{23}}{\sigma_{33}}(r - x_3\beta_3)}{\sqrt{\sigma_{22}\left(1 - \frac{\sigma_{23}^2}{\sigma_{22}\sigma_{33}}\right)}}\right) d\epsilon_2 \frac{1}{\sqrt{\sigma_{33}}} \phi\left(\frac{r - x_3\beta_3}{\sqrt{\sigma_{33}}}\right)
\end{aligned} \tag{49}$$

First, we derive the initial values[29]:

[29]The data set only provides different imputations. The variable with the most imputed values was the value of the house (6%). However, the difference between impputations are minor. Excluding observations

1. estimate a Heckman selection model to get the loan amount for firms not applying and construct leverage at the last credit
2. estimate a Heckman Probit model on the probability of getting credit and predict probabilities.
3. estimate a Heckman selection model and predict interest rates
4. re-estimate a Heckman Probit model using predicted interest rates and acceptance probabilities.
5. use values point estimates as initial conditions.

(166) with imputed variables does not yields significantly different results.

Appendix C - Robustness Checks

Table 12: Dependent Variable: Credit Amount

	1998 Sample			2003 Sample		
	OLS	OLS	Selection[a]	OLS	OLS	Selection[a]
Net Worth	0.311**	0.295**	0.310**	0.350**	0.330**	0.347**
	(0.080)	(0.083)	(0.079)	(0.057)	(0.058)	(0.056)
Ln Number of Employees		0.059†			0.153**	
		(0.031)			(0.034)	
Controls[b]	yes	yes	yes	yes	yes	yes
N	708	708	873	1652	1652	1782
R^2	0.30	0.30	-	0.27	0.28	-

Significance levels: : † : 10% ∗ : 5% ∗∗ : 1%

[a] Heckman selection model using information on whether the firm's credit application was accepted.. The probability of receiving credit is estimated using net worth, ln(emploment), ln(length of relationship with the lender), ln(firmage), credit score, and dummy variables for industry, region, gender, minority status, organizational form of the firm, previous bankruptcies and judgements against the firm.

[b] Additional controls include ln(firmage), credit score, and dummy variables for industry, region, gender, minority status, organizational form of the firm, previous bankruptcies and judgements against the firm.

Table 13: Dependent Variable - Credit Amount

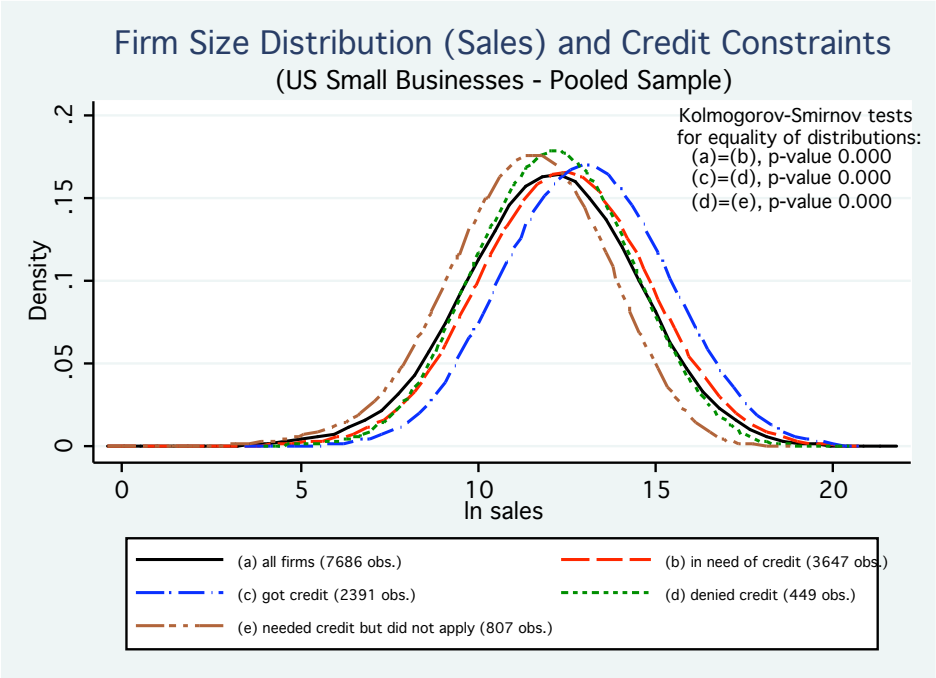
	1998 Sample		2003 Sample	
	OLS	Selection[a]	OLS	Selection[a]
age 1-2 x	0.896†	0.707*	1.038*	0.984*
Assets	(0.501)	(0.333)	(0.475)	(0.456)
age 3-6 x	0.661**	0.630**	0.576**	0.395**
Assets	(0.142)	(0.143)	(0.138)	(0.063)
age 7-15 x	0.273**	0.282*	0.471**	0.490**
Assets	(0.096)	(0.139)	(0.131)	(0.118)
age > 15	0.200**	0.195**	0.276**	0.283**
Assets	(0.045)	(0.041)	(0.071)	(0.088)
Controls[b]	yes	yes	yes	yes
N	542	872	1520	1829
R^2	0.46	-	0.30	-

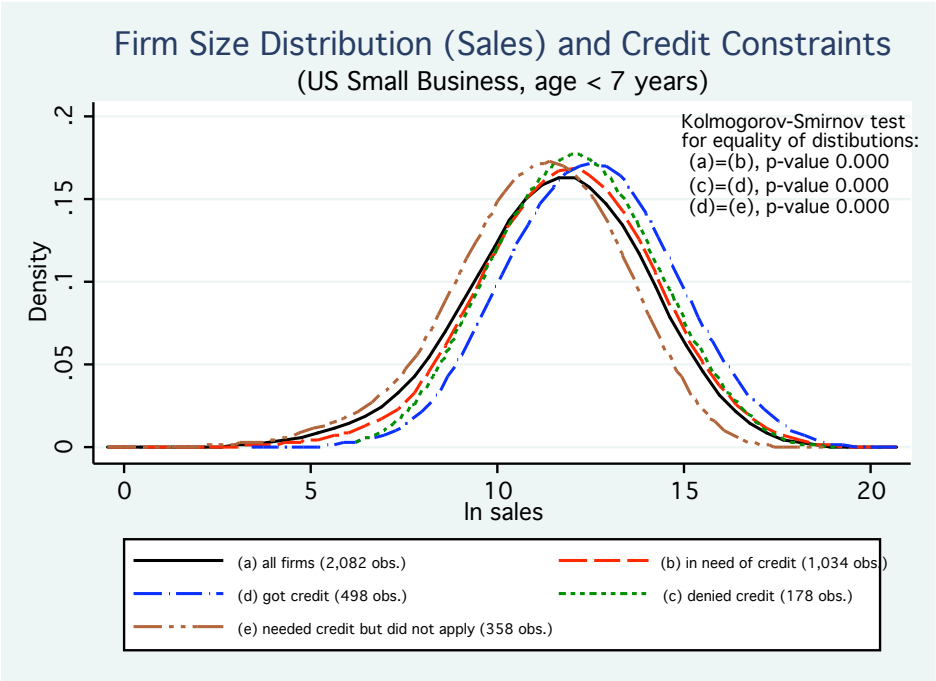
Significance levels: : $\dagger$: 10% $*$: 5% $**$: 1%

[a] Heckman selection model using information on whether the firm's credit application was accepted. The probability of receiving credit is estimated using net worth, ln(emploment), ln(length of relationship with the lender), ln(firmage), credit score, and dummy variables for industry, region, gender, minority status, organizational form of the firm, previous bankruptcies and judgements against the firm.

[b] Additional controls include ln(firmage), credit score, and dummy variables for industry, region, gender, minority status, organizational form of the firm, previous bankruptcies and judgements against the firm.

Figure 7:


Firm Size Distribution (Sales) and Credit Constraints
(US Small Businesses - Pooled Sample)
Kolmogorov-Smirnov tests
for equality of distributions:
(a)=(b), p-value 0.000
(c)=(d), p-value 0.000
(d)=(e), p-value 0.000
Density
0
.05
.1
.15
.2
0
5
10
15
20
ln sales
(a) all firms (7686 obs.)
(b) in need of credit (3647 obs.)
(c) got credit (2391 obs.)
(d) denied credit (449 obs.)
(e) needed credit but did not apply (807 obs.)


Firm Size Distribution (Sales) and Credit Constraints
(US Small Business, age < 7 years)
Kolmogorov-Smirnov test
for equality of distibutions:
(a)=(b), p-value 0.000
(c)=(d), p-value 0.000
(d)=(e), p-value 0.000
Density
0
.05
.1
.15
.2
0
5
10
15
20
ln sales
(a) all firms (2,082 obs.)
(b) in need of credit (1,034 obs.)
(d) got credit (498 obs.)
(c) denied credit (178 obs.)
(e) needed credit but did not apply (358 obs.)