Optimal Fiscal and Monetary Policy in the Presence of Non-Ricardian Agents

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Abstract

This paper studies optimal monetary and fiscal policy in a New Keynesian Model under the presence of non-ricardian consumers. We embed an Overlapping Generations Model (OLG) in a fairly standard NKM with fiscal and monetary policy. We then compute the optimal policy from a timeless perspective and derive explicit rules from a linear quadratic approach.

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1. Introduction

The effects and transmission mechanisms of fiscal policy are a highly controversial issue in recent macroeconomic research. Recently, a multiplicity of countries approved massive and aggressive fiscal stimulus packages in order to prevent further output losses and facilitate the economic recovery.[1] Beside these fiscal stimulus packages, the monetary authorities - the Federal Reserve and the European Central Bank - stem against the downturn with extremely expansionary monetary policy measures.[2]

However, the massive interventions of governments and monetary authorities are not independent of each other. There are many channels through which one policy affects the other. A major risk is that governments may be tempted to inflate its debt with allowing for higher inflation in the future (Seigniorage). In this case expansionary fiscal policy would lead to expansionary monetary policy, which accelerates inflation and may result into real appreciations of the currency and - in an extreme case - to a currency or banking crisis.[3] From a forward looking perspective fiscal spending is likely to have crowding out effects of other aggregate demand components. A high government's credit demand may result in too less or too expansive credit for the private sector and therefore may lead to a decline in private consumption and private investment. With respect to this channel, monetary policy has a key role. When monetary policy is active, it will respond to boosted aggregate demand with an increase in the nominal interest rate - since it will try to close the output gap. This in turn dampens private consumption and investment and leads to crowding out effects. Another direct channel in which fiscal policy impinges on the monetary authority is the government's tax policy. Setting indirect taxes (for example a VAT) has an effect on the price level and thus on inflation. This may influence inflation expectations and lead to higher wage demands which in turn may trigger a wage-price spiral.

Along this line, most macroeconomist agree that a "good" policy design should be in terms of macroeconomic stability. In particular, a high volatility of employment and output is contrary to the mission of monetary and fiscal policy. In technical terms this implies that the underlying equilibrium reveals determinacy and therefore locally uniqueness. Monetary policy in the sense of interest rate rules with feedback to endogenous variables have been exhibit in depth according to this principle. In contrast, the implications of fiscal policy have not been scrutinized that detailed. This procedure is sufficient in an environment in which the Ricardian equivalence holds and in which taxes are non distortionary.[4] Under these assumptions the problem can be separated

[1] For instance, the American Recovery and Reinvestment Act (ARRA) in the United States is with a volume of 787 billion US-dollars in 2009 and 2010 - in addition to the 125 billion US-dollars provided by the Economic Stimulus Act already in 2008 - the largest fiscal program ever seen in world history.

[2] The Federal Reserve drastically lowered the Federal Funds Target Rate to the 0-0.5 percent level. Additionally, highly unconventional monetary policy actions have been imposed, e.g. the monetary base increased by over 200 percent and the FED purchased long-run government bonds that amount up to 300 billion US-dollars.

[3] See Hilbers (2004).

[4] Ricardian equivalence in the sense of Woodford (1995) implies that government debt has no impact on real quantities.

into two independent sub-problems, solved recursively as shown by Leeper (1991). Following this approach the monetary problem is solved isolated from fiscal issues. Under these circumstances, the Taylor principle creates conditions under which the equilibrium shows determinacy. Fiscal policy has to be passive, in the sense that government debt evolves in a stable manner. In this context fiscal policy can only be active, if and only if the solution of the monetary problem is an equilibrium which reveals one degree of indeterminacy. Then, government debt has to evolve unstable, to restore equilibrium determinacy as shown by Leith and von Thadden (2006). However, if the assumption of Ricardian equivalence is dropped non-trivial effects of fiscal policy arise, posing the question of the interaction between monetary and fiscal policy. To be precisely, we understand fiscal policy as the sequence of current and anticipated taxes, spending and debt affecting aggregate consumption. Since debt and the sequence of taxes are parts of wealth they influence consumption, while government spending has a direct effect on aggregate demand.

The baseline New Keynesian model assumes an infinitely-lived representative agent that faces the decision of consumption and investment. In this intertemporal decision problem the interest rate is the decisive factor, since a higher interest rate creates incentives to shift consumption to the future. However, if the assumption of infinitely-lived agents is relaxed, wealth affects arise causing non-trivial interactions of demand and supply, that are also a consequence of endogeneity of the capital stock.

Introducing an Overlapping Generations Model (OLG, henceforth) accounts for this approach. The basic concept is derived from the seminal contributions of Yaari (1965) and Blanchard (1985).[5] The model we use is based on that in Annicchiarico et al. (2009) where the overlapping generations assumption and capital accumulation is embedded into a New Keynesian model. The reminder of the paper is structured as follows. Section 2 presents the structure of the model. Section 3 discusses the optimal policies using (i) a Ramsey approach and (ii) a linear quadratic approach. Section 4 briefly concludes.

2. An OLG Model with Sticky Prices

The OLG model in the spirit of Blanchard/Yaari features a constant population with identical preferences and a perfectly competitive life insurance market, such that there is no incentive for intergenerational bequest. In addition, insurance companies collect financial wealth from the deceased redistributing by paying a fair premium to the survivors. Consumer's utility consists of consumption, government spending, and disutility of work. Furthermore, we introduce nominal rigidities in the sense of Calvo (1983), i.e. firms can not reset their prices in a fully flexible manner, instead they face a constant probability of being allowed to readjust. Therefore, they set prices as a weighted average of future expected marginal costs. Firms use labour and capital as inputs to produce differentiated goods. We furthermore assume convex capital adjustment costs.

[5] A different way would be to introduce distortionary taxes as shown by Schmitt-Grohé and Uribe (2006) or rule-of-thumb consumers as shown by Galí et al. (2004).

2.1. Consumer Preferences

Each agent, throughout his lifetime faces a constant probability of death, denoted by λ. In addition, at any time t, a cohort of size λ is born, and total population is normalized to one. Private markets provide insurance risklessly through life insurance companies. Free entry and a zero profit condition imply that agents will pay (receive) a rate λ to receive (pay) one good contingent on their death. Since there is no bequest motive - and since negative bequests are ruled out - agents will contract to have all of their wealth return to the life insurance company contingent on their death. The insurance company will equally distribute the wealth of the deceased to the survivors.

2.1.1. Individual Consumption

Let us denote consumption, government spending and the labor supply by $C_{s,t}, G_{s,t}, N_{s,t}$. Then, the representative agent j belonging to generation s solves the maximization problem

$$\max_{C_{s,t},B_{s,t},N_{s,t}} E_0 \sum_{t=0}^{\infty} [(1-\lambda)\beta]^t \left[\frac{(\xi_t C_{s,t}(j))^{1-\sigma}}{1-\sigma} + \frac{(\vartheta_t G_{s,t}(j))^{1-\gamma}}{1-\gamma} - \frac{(\mu_t N_{s,t}(j))^{1+\varphi}}{1+\varphi} \right], \quad (1)$$

subject to her budget constraint,

$$\frac{B_{s,t+1}(j)}{R_t} + Q_t K_{s,t+1}(j) \leq A_{s,t}(j) + W_{s,t}(j) N_{s,t}(j) + Z_{s,t}(j) - T_{s,t}(j) - P_t C_{s,t}(j). \quad (2)$$

Here, P_t is the aggregate price index, $B_{s,t}$ is a risk free per capita government bond, which pays a gross nominal interest rate $R_t = (1+i_t)$. $Z_{s,t}$ are nominal aggregate profits of the intermediate firm at time t. The agent receives labor income $W_{s,t}(j)N_{s,t}(j)$ and has to pay lump sum taxes $T_{s,t}(j)$. Furthermore, β is the households discount factor. In addition, we assume that households utility in every generation is subject to three shocks, viz. (i) a preference shock, ξ_t, (ii) a government spending taste shock, ϑ_t, and (iii) a disutility shock, μ_t.
Financial wealth is given by $A_{s,t}$,

$$A_{s,t}(j) = \frac{1}{1-\lambda} \left[B_{s,t}(j) + \left[(1-\delta)Q_t + R_t^k \right] K_{s,t}(j) \right], \quad (3)$$

where Q_t represents the price of capital, R_t^k is the nominal rental rate of capital and $K_{s,t}(j)$ denotes capital holdings.
The capital accumulation is given by

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \quad (4)$$

where $\phi'(\cdot) > 0$, $\phi''(\cdot) \leq 0$, $\phi'(\delta) = 1$ and $\phi''(\delta) = \delta$ captures convex adjustment costs. Profit maximization implies

$$Q_t \phi' \left(\frac{I_t}{K_t} \right) = P_t. \tag{5}$$

By minimizing total expenditures, we obtain the demand function, $C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} C_t$. The household chooses the path of $\{C_{s,t}(j), B_{s,t+1}(j), K_{s,t+1}(j), N_{s,t}(j)\}_{t=0}^{\infty}$, such that the first-order conditions are given by

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial C_{s,t}(j)} &= (\xi_t C_{s,t}(j))^{-\sigma} - \zeta_t P_t = 0, && (6)\\
\frac{\partial \mathcal{L}_t}{\partial B_{s,t+1}(j)} &= -\zeta_t \frac{1}{R_t} + \frac{1}{1-\lambda} \beta (1-\lambda) \zeta_{t+1} = 0, && (7)\\
\frac{\partial \mathcal{L}_t}{\partial K_{s,t+1}(j)} &= -\zeta_t Q_t + \frac{1}{1-\lambda} \beta (1-\lambda) \zeta_{t+1} \left[(1-\delta) Q_{t+1} + R^k_{t+1} \right] = 0, && (8)\\
\frac{\partial \mathcal{L}_t}{\partial N_{s,t}(j)} &= -(\mu_t N_{s,t}(j))^{\varphi} + \zeta_t W_{s,t}(j) = 0, && (9)
\end{aligned}$$

where ζ_t is the Lagrangian multiplier on the budget constraint. Combining the first two derivatives gives the standard Euler equation for consumption flows, i.e.

$$1 = \beta R_t \frac{1}{\pi_{t+1}} \left(\frac{\xi_t C_{s,t}(j)}{\xi_{t+1} C_{s,t+1}(j)} \right)^{\sigma}. \tag{10}$$

In addition, the labor supply schedule reads as

$$\frac{W_{s,t}(j)}{P_t} = (\mu_t N_{s,t}(j))^{\varphi} (\xi_t C_{s,t}(j))^{\sigma}. \tag{11}$$

Following Woodford (2003), we define the stochastic discount factor as

$$\Lambda_{t,t+1}(s,j) = \beta \frac{1}{\pi_{t+1}} \left(\frac{\xi_t C_{s,t}(j)}{\xi_{t+1} C_{s,t+1}(j)} \right)^{\sigma}, \tag{12}$$

such that

$$\Lambda_{t,t+1}(s,j) = \frac{1}{R_t}. \tag{13}$$

The transversality condition, ruling out a Ponzi-Game, is

$$\lim_{T \to \infty} E_t (1-\lambda)^{T-t} \Lambda_{t,T}(s,j) A_{s,T}(j) = 0, \tag{14}$$

where $\Lambda_{t,T}(s,j) = \prod_{k=T+1}^{T} \Lambda_{k-1,k}(s,j)$, as well as $\Lambda_{t,t}(s,j) = 1$.
The Appendix shows that the equation for individual consumption is

$$P_T C_{s,T}(j) = [1 - \beta(1-\lambda)] \left[A_{s,t}(j) + H_{s,t}(j) \right]. \tag{15}$$

Here, we have defined $H_{s,t}(j)$ as human wealth,

$$H_{s,t}(j) = \sum_{T=t}^{\infty} E_t(1-\lambda)^{T-t}\Lambda_{t,T}(s,j)\left(W_{s,T}(j)N_{s,T}(j) + Z_{s,T}(j) - T_{s,T}(j)\right), \tag{16}$$

which is the expected, discounted stream of labor incomes and profits net of taxes.

2.1.2. Aggregate Consumption

First, notice that in the symmetric equilibrium all workers of all generation will receive the same wage and supply the same amount of work, such that

$$W_{s,t}(j) = W_t, \tag{17}$$
$$N_{s,t}(j) = N_t, \tag{18}$$

for all $j \in (0,1)$.
The aggregate value, X_t, of any individual variable, $X_{s,t}(j)$, is obtained according to

$$X_t = \sum_{s=-\infty}^{t}\left\{\int_0^{\lambda(1-\lambda)^{t-s}} X_{s,t}(j)dj\right\}. \tag{19}$$

The aggregate budget constraint, the aggregate consumption, and aggregate labor supply are then given by

$$\frac{B_{t+1}}{R_t} + Q_t K_{t+1} = A_t + W_t N_t + Z_t - T_t - P_t C_t, \tag{20}$$
$$P_t C_t = \left[1-\beta(1-\lambda)\right]\left[A_t + H_t\right], \tag{21}$$
$$\frac{W_t}{P_t} = (\mu_t N_t)^{\varphi}(\xi_t C_t)^{\sigma}. \tag{22}$$

The human wealth is then given by

$$H_t = \sum_{T=t}^{\infty} E_t(1-\lambda)^{T-t}\Lambda_{t,T}\left(W_T N_T + Z_T - T_T\right). \tag{23}$$

Then, as shown in the Appendix, aggregate consumption follows

$$P_t C_t = \frac{\psi}{1-\psi}\lambda E_t \Lambda_{t,t+1} A_{t+1} + \frac{1-\lambda}{1-\psi} E_t \Lambda_{t,t+1} P_{t+1} C_{t+1}, \tag{24}$$

where $\psi = [1-\beta(1-\lambda)]$.

2.2. Intermediate Good Firms

Along the firm side of our economy, intermediate good producers i use capital, $K_t(i)$, and labor, $N_t(i)$, as inputs for a Cobb-Douglas production function, viz.

$$Y_t(i) = \mathcal{A}_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}, \tag{25}$$

where we denote the technology shock by $\mathcal{A}_t$. Then, we can write the labor input as a CES aggregator of individual labor supply, such that

$$N_t(i) = \left(\sum_{s=-\infty}^{t} \int_0^{\lambda(1-\lambda)^{t-s}} N_{s,t}(i,j)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{26}$$

Cost minimzation by firms yields the optimality condition

$$\frac{K_t(i)}{N_t(i)} = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^k}, \tag{27}$$

and real marginal costs are given by

$$MC_t = \frac{\alpha^{-\alpha}(1-\alpha)^{\alpha-1}}{\mathcal{A}_t} \left(\frac{R_t^k}{P_t}\right)^{\alpha} \left(\frac{W_t}{P_t}\right)^{1-\alpha}, \tag{28}$$

being identical across all firms.
Intermediate good producers price setting decision is modelled through a discrete time version of Calvo (1983) staggering mechanism. In each period, a firm faces a constant probability, $1-\theta$, of being able to re-optimize its price and chooses the new price $P_t(i)$ that maximizes

$$E_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} Y_T(i) \left(P_t(i) - P_T MC_T\right), \tag{29}$$

subject to the demand schedule

$$Y_T(i) = \left(\frac{P_t(i)}{P_T}\right)^{-\epsilon} Y_T. \tag{30}$$

The first-order condition is

$$E_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} Y_T P_T^{\epsilon} \left(P_t(i) - (1+\mu^P P_T MC_T\right) = 0, \tag{31}$$

where $\mu^P = \frac{1}{\epsilon-1}$ is the equilibrium mark-up. Finally, the aggregate price index follows

$$P_t = \left[\theta (P_{t-1})^{1-\epsilon} + (1-\theta) P_t(i)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{32}$$

2.3. Final Good Firms

Perfectly competitive firms bundle type-i differentiated goods into homogenous final goods by a Dixit-Stiglitz aggregator

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{33}$$

where $Y_t(i)$ is the quantity of the intermediate good produced by intermediate good firm i. From the maximization program of the representative competitive firm, taking as given the final good price P_t and the prices of the intermediate goods $P_t(i) \, \forall \, i \in [0,1]$, the overall demand addressed to the producer of intermediate good i is

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{34}$$

while the zero profit condition in the final good sector implies

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}. \tag{35}$$

2.4. Monetary and Fiscal Policy

2.4.1. Monetary Policy

Monetary policy follows a standard interest rate rule with a feedback to the endogenous variables inflation and output. The Taylor rule is then given by

$$\left(\frac{1+R_t}{1+R} \right) = \left[\left(\frac{1+\pi_t}{1+\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_Y} \right] e^{\nu_t}, \tag{36}$$

where $\phi_\pi > 0$ is the weight on inflation and $\phi_Y > 0$ is the weight on output. ν_t is an interest rate shock.

2.4.2. Fiscal Policy

Our fiscal authority issues Bonds, provides government spending, G_t, and uses lump sum taxes for redistribution purposes. However, only two of those instruments can be set independently, while the third follows from the budget constraint. Then, the flow budget constraint in nominal terms is given by

$$\frac{B_{t+1}}{R_t} = B_t + P_t G_t - T_t. \tag{37}$$

Rewriting this equation in real terms gives

$$\frac{l_{t+1}}{R_t} = \frac{l_t}{\pi_t} + G_t - \tau_t, \tag{38}$$

where $l_t = B_t/P_{t-1}$ denotes total real government liabilities and $\tau_t = T_t/P_t$ is real tax collections.
We assume that our fiscal authority follows a debt-based tax rule and a government spending rule to conduct fiscal policy.[6] Following Leeper (1991), the tax rule can - in log-linearized terms - be written as

$$\hat{\tau}_t = \tau_l \hat{l}_t + \tau_Y \hat{Y}_t. \tag{39}$$

Here, $\tau_l \geq 0$ is the parameter governing the feedback on liabilities and $\tau_Y \geq 0$ is the coefficient on output. The former accounts for a debt stabilization goal of the fiscal authority, while the latter takes business cycle movements into account.
Then, we assume that the government spending rule - in log-linearized terms - follows

$$\hat{G}_t = -\omega_l \hat{l}_t - \omega_Y \hat{Y}_t + \Gamma_t, \tag{40}$$

where Γ_t is an exogenous spending shock. As before, $\omega_Y > 0$ accounts for the business cycle stabilization goal of our government and $\omega_l > 0$ is about to stabilize debt.

2.5. Closing the Model

In the symmetric equilibrium, factor and goods market clear, such that

$$N_t = \int_0^1 N_t(i)di, \tag{41}$$

$$K_t = \int_0^1 K_t(i)di, \tag{42}$$

for all $i \in [0, 1]$.
The resource constraint is described by

$$Y_t = C_t + I_t + G_t. \tag{43}$$

The technology shock is a standard AR(1) process, i.e.

$$\mathcal{A}_t = \mathcal{A}_{t-1}^{\rho_{\mathcal{A}}} e^{\alpha_{\mathcal{A},t}}, \tag{44}$$

where $0 < \rho_{\mathcal{A}} < 1$ determines the degree of autocorrelation and $\alpha_{A,t} \sim N(0, \sigma_A)$ is an i.i.d. error term following an univariate normal density distribution with standard

[6] See Leeper et al. (2010) for an Bayesian estimation of different tax rules within a neoclassical growth model or Rossi and Wesselbaum (2010) for an Bayesian estimation of different tax rules in a DSGE model with labor market imperfections.

deviation σ_A and $cov(A_{t-1}, \alpha_{A,t}) = 0 \ \forall \ t$.
The government spending shock is

$$\Gamma_t = \Gamma_{t-1}^{\rho_\Gamma} e^{\alpha_{\Gamma,t}}, \tag{45}$$

where $\alpha_{\Gamma,t} \sim N(0, \sigma_\Gamma)$ is i.i.d. and $cov(\Gamma_{t-1}, \alpha_{\Gamma,t}) = 0 \ \forall \ t$.
Finally, the interest rate shock is AR(1),

$$\nu_t = \nu_{t-1}^{\rho_\nu} e^{\alpha_{\nu,t}}, \tag{46}$$

where $\alpha_{\nu,t} \sim N(0, \sigma_\nu)$ is i.i.d. and $cov(\nu_{t-1}, \alpha_{\nu,t}) = 0 \ \forall \ t$.
For the given stochastic processes $\{A_t, \Gamma_t, \nu_t\}_{t=0}^{\infty}$, a determined equilibrium is a sequence of allocations and prices $\left\{a_t, C_t, \tilde{C}_t, G_t, I_t, K_t, l_t, MC_t, N_t, \pi_t, q_t, r_t, r_t^k, \tau_t^w, \tau_t^k, W_t, Y_t\right\}_{t=0}^{\infty}$ which for given initial conditions satisfies equations (6) to (8), (23), (24), (26), (27), (29), (30), (34), (38), (40) to (43), (46) to (49), and the expression for adjusted consumption. Then, the set of equations forming the equilibrium is linearized around the non-stochastic steady-state with zero inflation.
We calibrate our model on a quarterly basis for the United States based upon parameter values from the recent literature.
On the household side, we set the disocunt factor to $\beta = 0.998$ as in Annicchiarico et al. (2009). The probability of death, λ, is set to 0.015 as in Leith and Wren-Lewis (2000). The weight of money in the utility function is 0.018. The inverse of the Frisch elasticity of labor supply is 0.47 as in Benigno and Benigno (2004). The steady state work time is set to 1/3.
Then, on the firm side, we set the elasticity of output to capital, α, to 0.3 as in in Christiano et al. (2005). The price re-setting probability is calibrated to be 0.75 as in Benigno and Benigno (2004). The capital depreciation rate is set to 0.025, which is equal to 10 % per annum also following Christiano et al. (2005). The elasticity of investment wit respect to asset prices is 1, as in King and Watson (1996). The elasticity of substitution between goods is set to 11.
The Taylor rule parameters are set to their respective standard values of 1.5 for ϕ_π, 0.125 for ϕ_Y. The fiscal rule parameters are taken from Leeper et al. (2010) and are set to $\omega_l = 0.23$ and $\omega_Y = 0.03$. Furthermore, the coefficients of the tax rule are set to $\tau_l = 0.5$ and $\tau_Y = 0.5$, in line with Annicchiarico et al. (2009). The autocorrelations of the three shocks are all set to 0.9.

3. Optimal Monetary and Fiscal Policy

This section presents two approaches to describe the design of optimal monetary and fiscal policy. First, we begin with optimal policy from a timeless perspective as for instance described in Woodford (2010). To be precise, we assume that our social planner minimizes her objective loss function by setting the path of the available instruments. Second, we derive explicit rules for the conduct of optimal monetary and fiscal policy

by applying a linear quadratic approach.

3.1. A Quick Detour - Productivity and Spending Shock

Before, we start to describe the optimal policy reaction, we want to analyze the effects of a negative productivity shock and an increase in government spending to better understand the role for fiscal and monetary policy.
The response of our economy to a one percent, negative, temporary technology shock is presented in Figure 1. The decrease in productivity reduces output according to the production fucntion (25). As a consequence, consumption and investment decline. The decrease in investment leads the capital stock to slowly fall. While the shock dies out, firms need to increase output which is mainly done by increasing employment, as the adjustment along the capital dimension is time consuming. Since the monetary policy's primary target is inflation, the interest rate jumps on impact and converges from above to the old steady state. The fall in tax payments and the higher interest rate payments lead government liabilities to increase such that the government is more reluctant to use its spending instrument.
Let us now consider an one percent increase in government spending. As shown in Figure 2, the increase in government spending translates into an increase in output, which is slightly smaller as one ($\frac{dY_t}{dG_t} \approx 0.87$). The reason for this is a crwoding-out of private consumption (due to the interest rate channel and a drop in financial wealth) and investment. The interest rate reacts to the underlying Taylor rule to the increased output and higher inflation. Higher government spending also implies an increase in liabilities , being also driven by higher interest rate payments.

3.2. Optimal Policy from a Timeless Perspective

We begin the discussion of optimal policy with a description of optimal monetary and fiscal policy from a timeless perspective. The solution to the optimization problem is a state-contingent evolution for our economy described by a sequence of first-order conditions. The decisive property of this solution is that the continuation of the optimal policy choosen in t_0 is exactly the solution to the problem in t. To be precise, solving for the forward path of the endogenous variabes subject to the constraint that only paths consistent with the initial pre-commitment are feasible, one finds that this solution is the forward path that conforms to the target criterion from t onward. It will also be equivalent to the solution of the unconstrained Ramsey problem in t_0.
As a starting point, the monetary authority has one instrument, namely the interest rate R_t, while the government has two instruments, viz. τ_t, taxes, and G_t, government spending.
Then, we can make the following propositions
Proposition 1
A feasible allocation is defined as a sequence that satisfies the representative household's budget constraint $\{K_t, C_t, N_t, G_t\}_{t=0}^{\infty}$.
Proposition 2

A price system is a nonnegative bounded sequence $\left\{W_t, R_t^k, R_t\right\}_{t=0}^{\infty}$.
Proposition 3
A government policy is a sequence $\{R_t, \tau_t, G_t\}_{t=0}^{\infty}$.
Then, a competitive equilibrium satisfies Propositions 1-3. To be explicit, optimal monetary and fiscal policy is the process $\{R_t, \tau_t, G_t\}_{t=0}^{\infty}$ associated with the equilibrium that yields the highest level of utility to the representative household.
Therefore, the benevolent Ramsey planner chooses contigent plans to minimize the quadratic intertemporal loss function in period t

$$\mathcal{L}_t = (1-\beta)E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\pi_{t+\tau}^2 + \upsilon y_{t+\tau}^2\right], \tag{47}$$

where υ is the relative weight on output stabilization. In the case that $\upsilon = 0$, we would have strict inflation targeting, while the more realistic case of $\upsilon > 0$ corresponds to flexible inflation targeting.
Svensson (2002) has shown that, if the discount factor approaches unity and if a quarterly model is applied, the limit of the loss function (47) is simply the weighted sum of the unconditional variances of inflation and output, i.e.[7]

$$\lim_{\beta \to 1} \mathcal{L}_t = Var(\pi_t) + \upsilon Var(y_t). \tag{48}$$

Before we start to discuss our results, we want to briefly identify the distortions in our model economy. Average mark-up distortions are caused by the assumption of monopolistic competition. Dynamic mark-up distortions follow from the introduction of sticky prices. Since our model features real money balances, we also obtain monetary distortions.
As a first step, we assume that the Ramsey planner determines optimal monetary and fiscal policy jointly. Later on, we will interpret this as a benchmark case of cooperation between monetary and fiscal authorities.
Figure 3 shows the optimal path of selected variables to a negative aggregate demand shock for four different cases. First, and plotted in black, both instruments are set optimally. Second, plotted in red, only government spending is set optimally, while monetary policy follows the Taylor rule, see eq. (36). Third, plotted in blue, the interest rate is set optimally, while government spending follows the rule determined before, see eq. (40). Finally, plotted in green, both instruments follow their respective rules.
Let us begin with the obvious case of jointly optimal policy. As pointed out by Tinbergen (1952), if the number of targets equals the number of instruments, all targets can be achieved. Our problem has two targets, viz. inflation and output, and two instruments, viz. the interest rate and government spending. Therefore, both targets can be accomplished and hence the loss is 0. As a response to the negative productivity shock, the Ramsey planner increases government spending as to offset the negative ef-

[7] In addition, it has to hold that the unconditional mean of inflation equals the inflation target, i.e. $E\left[\pi_t\right] = \pi^*$.

fect of the shock on output and overcome the self-initiated crowding-out effects. While government spending is used to stabilize output, monetary policy is left with mainly stabilizing inflation and only slightly accomodating fiscal policy in stabilizing output. Therefore, it increases the interest rate, as lower productivity and higher spending cause a higher inflation rate. In the case, where the monetary authority sets the interest rate optimal, while government spending follows its rule, we find that interest rates drop on impact, as to stimulate economic activity, which is in contrast to all other cases. As the monetary authority has to stabilize inflation and output, it can not stabilize both at the same time. It allows inflation to increase, to avoid a larger drop in output (compare to the green line, where both instruments follow rules). In addition, as spending follows its rule, this instrument stays almost on its steady state level. Consequently, liabilities fall as the nominal debt is inflated away and interest rate payments fall. However, as indicated by Table 1, we find that this case creates the second highest loss. The reason is a high volatility of output. The third case considers optimal government spending under constrained monetary policy. Here, we observe a very small loss, almost being only a fifth of the previous one. Causative for this result is the non-optimal use of spending to stabilize output over the cycle. Since monetary policy is constrained to follow a Taylor rule, the increase in the interest rate is larger as in the joint case. Here, monetary policy is not optimal such that output fluctuates around the steady state. Government spending can not be set optimally, as this would imply that inflation would increase, hence creating a larger loss. Finally, if both instruments follow their respective rules, we observe the highest loss. In this case, government spending is used to restrictively, and the interest rate increases too much. We can conclue that government spending is a key in order to stabilize the economy after the negative productivity shock. Monetary policy should be accomodative in the way it sets the interest rate, as only cooperation between both policies ensures much lower losses. From a game theoretical viewpoint, cooperation, i.e. the case in which both instruments are set optimally, is always (also in repeated games) a (subgame perfect) Nash equilibrium.

3.3. A Linear Quadratic Approach

This sections aims to find explicit rules for government spending as well as an optimal inflation targeting rule. Therefore, we follow the seminal contributions from Benigno and Woodford (2003, 2005, 2006) and solve the corresponding linear quadratic problem. In a more recent application of the LQ approach to the design of fiscal policy in the context of a standard New Keynesian model with Ricardian agents, Eser et al. (2009) show that it is optimal to not use government spending to stabilize the economy. They further show that monetary policy follows a standard inflation targeting rule as for instance derived in Clarida et al. (1999). However, and as we have seen in the precedent section, as our model features non-Ricardian agents (and capital), we find a role for active fiscal policy to stabilize the economy. We are therefore interested in finding explicit rules to identify the reaction schemes of both instruments and the potential degree of interaction between the instruments. The full derivation of the LQ can be found in the appendix.

The social planner maximizes its objective function (which is a second-order approximation to the households utility function (1))

$$\mathcal{L}_t = U_c Y \left[\begin{array}{c} \Phi \hat{Y}_t + \frac{1}{2}\left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y}\right) Y \hat{Y}_t^2 + \frac{U_{II}}{U_c Y}\hat{I}_t + \frac{1}{2}\left(\frac{U_{II}}{U_c Y} + \frac{U_{II}}{U_c Y} I^2\right)\hat{I}_t^2 + \left(\frac{U_G G}{U_c Y} + \frac{V_G G}{U_c Y}\right)\hat{G}_t \\ + \frac{1}{2}\left(\frac{U_G G}{U_c Y} + \frac{U_{GG} G^2}{U_c Y} + \frac{V_G G}{U_c Y} + \frac{V_{GG}}{U_c Y} G^2\right)\hat{G}_t^2 - \frac{W_K K}{W_y Y}\hat{K}_t - \frac{1}{2}\left(\frac{W_K K}{W_y Y} + \frac{W_{KK} K^2}{W_y Y}\right)\hat{K}_t^2 \\ -\frac{1}{2}\left(\frac{\theta}{(1-\theta)(1-\theta\beta)}\right)(\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2 \hat{\pi}_t^2 \\ + \frac{U_{c\xi}}{U_c}\hat{Y}_t \xi_t + \frac{U_{cG}}{U_c} G \hat{Y}_t \hat{G}_t + \frac{U_{cI}}{U_c} I \hat{Y}_t \hat{I}_t + \frac{U_{I\xi}}{U_c Y} I \hat{I}_t \xi_t + \frac{U_{IG}}{U_c Y} I G \hat{I}_t \hat{G}_t + \frac{U_{G\xi}}{U_c Y} G \hat{G}_t \xi_t \\ -\frac{W_{yK}}{W_y} K \hat{Y}_t \hat{K}_t - \frac{W_{K\mu}}{W_y Y} K \hat{K}_t \mu_t - \frac{W_{yA}}{W_y}\hat{Y}_t A_t - \frac{W_{KA}}{W_y Y} K \hat{K}_t A_t - \frac{W_{y\mu}}{W_y}\hat{Y}_t \mu_t + \frac{V_{G\vartheta}}{U_c Y}\hat{G}_t \vartheta_t \end{array} \right]$$
$$+ t.i.p. + ||\mathcal{O}||^3, \tag{49}$$

s.t.

$$-\lambda_t^{PC}\left[\frac{1}{R}\hat{\pi}_{t+1} + \kappa\left(\hat{r}_t^k - \left(\hat{Y}_t - \hat{K}_t\right) - \hat{\pi}_t\right)\right], \tag{50}$$

$$-\lambda_t^{IS}\left[\begin{array}{c} -\left(\hat{r}_t - \frac{1}{1+\omega}\hat{\pi}_{t+1}\right) + \frac{1}{1+\omega}\left(\hat{Y}_{t+1} - s_I \hat{I}_{t+1} - s_G \hat{G}_{t+1}\right) \\ + \frac{\omega}{1+\omega}\left\{\frac{1}{s_a}\left(R\left[\hat{l}_t - \frac{d}{Y}\hat{\pi}_t + \hat{G}_t - \tau_l \hat{l}_t - \tau_y \hat{Y}_t\right] + s_b \hat{r}_t\right) + \frac{R}{s_a} s_K \left(\hat{r}_t + \hat{K}_{t+1} + \frac{\hat{I}_t - \hat{K}_t}{\eta}\right)\right\} \\ -\hat{Y}_t + s_I \hat{I}_t + s_G \hat{G}_t \end{array}\right] \tag{51}$$

One can show that the solution to this problem yields a rule for government spending,

$$\hat{G}_t = \frac{V_G G - U_G G - U_{cG} Y G \hat{Y}_t - U_{IG} I G \hat{I}_t - U_{G\xi} G \xi_t - V_{G\vartheta}\vartheta_t}{U_G G + U_{GG} G^2 + V_G G + V_{GG} G^2}, \tag{52}$$

and for the inflation target,

$$\hat{\pi}_t = -\left[\frac{\Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y}\right) Y \hat{Y}_t + \frac{U_{c\xi}}{U_c}\xi_t + \frac{U_{cG}}{U_c} G \hat{G}_t + \frac{U_{cI}}{U_c} I \hat{I}_t - \frac{W_{yK}}{W_y} K \hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y}\mu_t}{\left(\frac{\theta}{(1-\theta)(1-\theta\beta)}\right)(\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2}\right]. \tag{53}$$

First, let us consider the optimal rule for government spending. As we have seen in the discussion of the Ramsey optimal policy, government spending reacts negatively to output and, which is now evident, to investment. In addition, it would react to the preference and the taste shock.

Second, we find an optimal inflation targeting rule which looks standard. Inflation reacts to movements in output, as in Clarida et al. (1999). Furthermore, investment and capital influence the optimal inflation rate. In contrast, the inflation rule has feedback on all shocks and not just to the preference shocks. Finally, monetary policy takes into account the path of government spending. While government spending was independent from monetary policy actions, monetary policy itself reacts to changes in government spending. It appears that inetractions between these two rules are only one-sided.

tba

4. Conclusion

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A. Tables and Figures

Table 1: Losses.

	$Var(\pi)$	$Var(y)$	$\mathcal{L}$
R	0.1547	0.9828	0.6461
G	0.1108	0.0321	0.1268
R & G	0	0	0
Rules	0.2638	1.4832	1.0054

Notes: $\upsilon = 0.5$, R: Interest Rate set optimally, Spending follows rule. G: Spending set optimally, Interest Rate follows Taylor Rule. R & G: Both set optimally and Rules: Both follow rules.

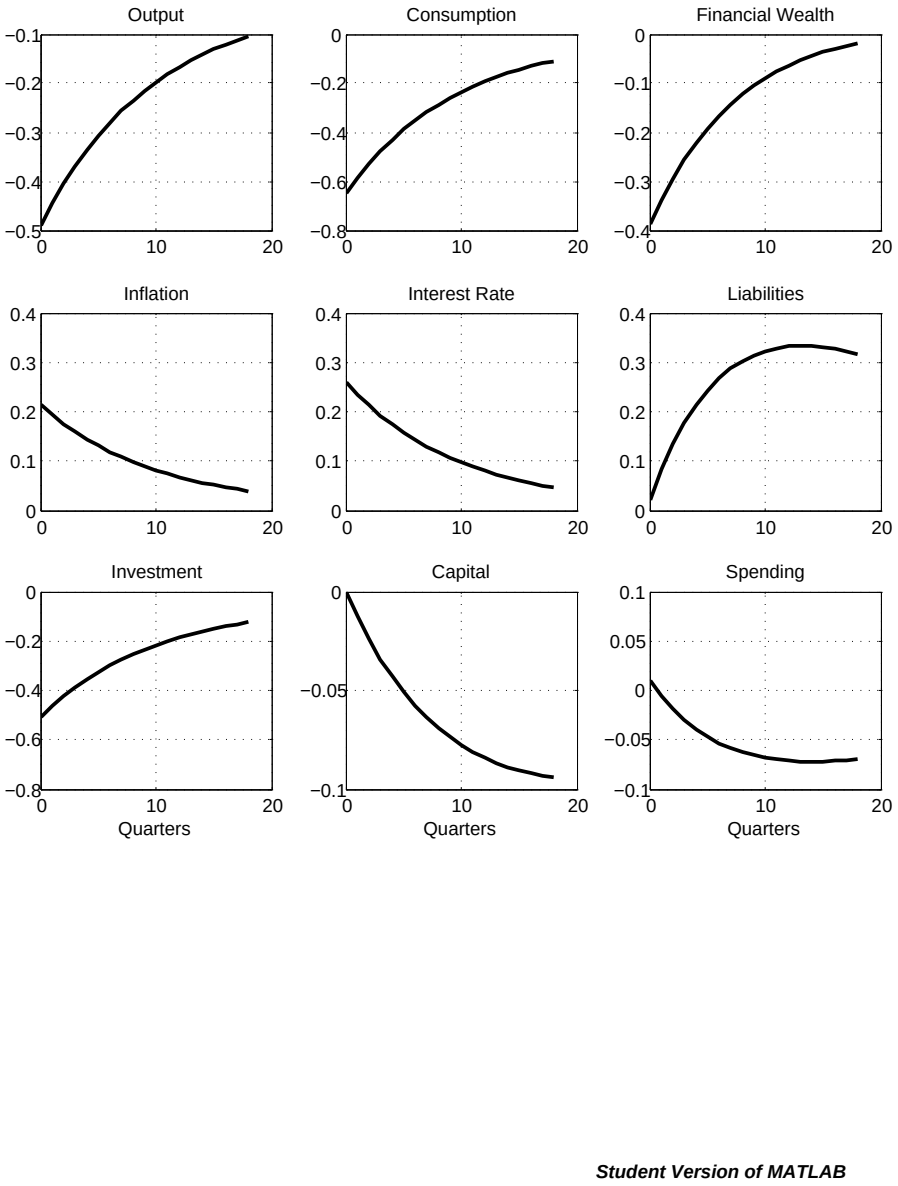

Figure 1: Impulse Responses to a 1 % negative Productivity Shock. Horizontal axes measure quarters, vertical axes deviations from steady state.

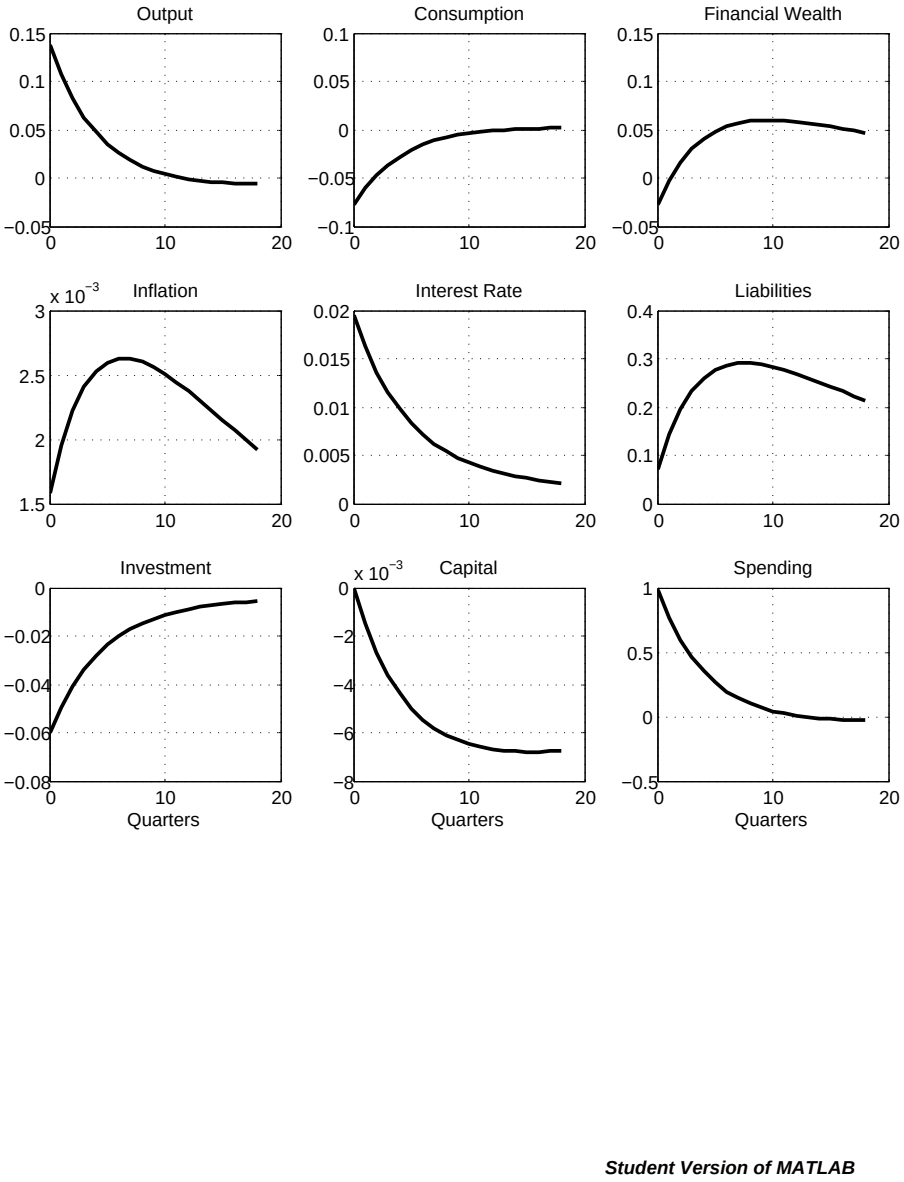

Figure 2: Impulse Responses to a 1 % increase in Government Spending. Horizontal axes measure quarters, vertical axes deviations from steady state.

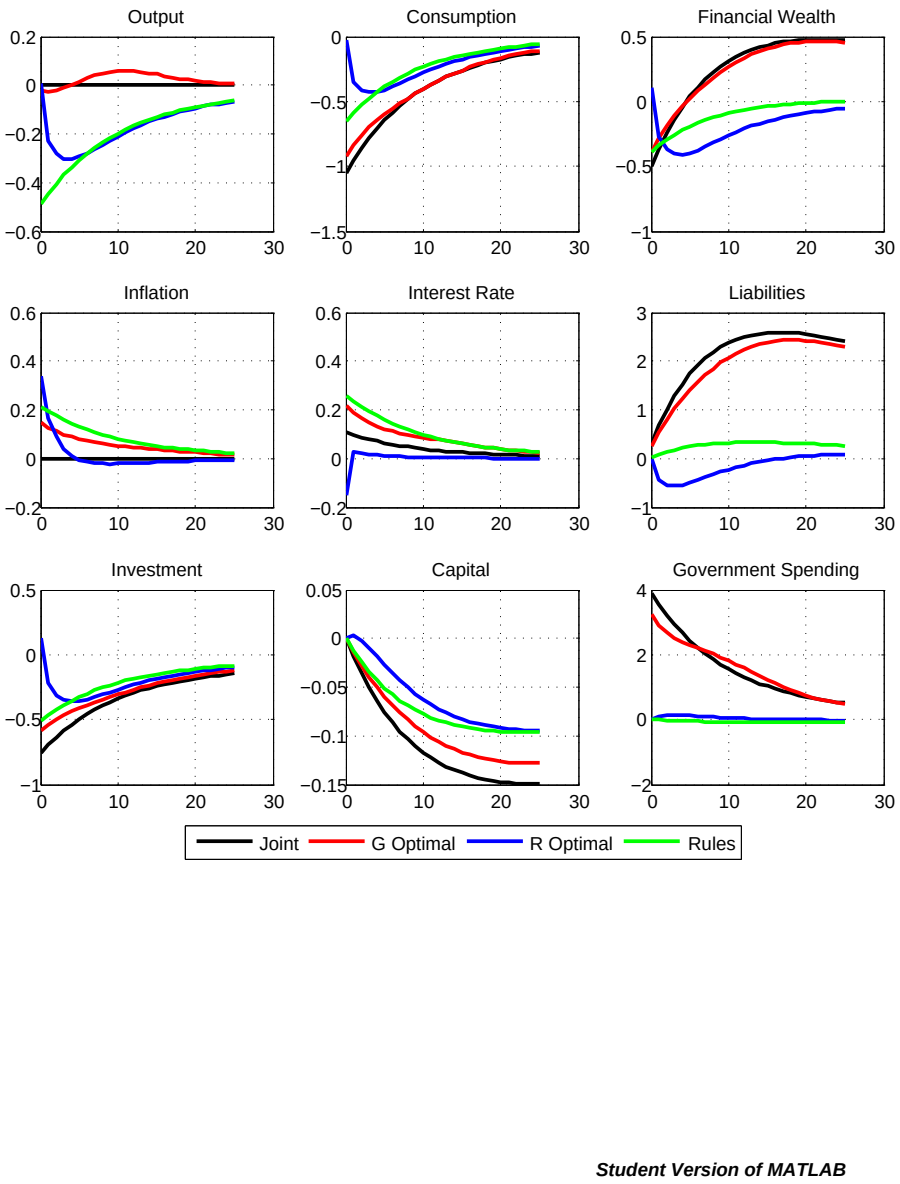

Figure 3: Optimal Policies. Joint: Spending and Interest Rates are set optimally. G Optimal: Spending is set optimally, interest rate follows a Taylor Rule. R optimal: Interest Rate is set optimally, Spending follows rule. Horizontal axes measure quarters, vertical axes deviations from steady state.

B. Individual Consumption

The households utility function is given by

$$U_{s,t} = \sum_{t=0}^{\infty} [(1-\lambda)\beta]^t \left[\frac{(\xi_t C_{s,t}(j))^{1-\sigma}}{1-\sigma} + \frac{(\vartheta_t G_{s,t}(j))^{1-\gamma}}{1-\gamma} - \frac{(\mu_t N_{s,t}(j))^{1+\varphi}}{1+\varphi} \right], \tag{54}$$

which is maximized subject to the intertemporal budget constraint

$$\frac{B_{s,t+1}(j)}{R_t} + Q_t K_{s,t+1}(j) \leq A_{s,t}(j) + W_{s,t}(j) N_{s,t}(j) + Z_{s,t}(j) - T_{s,t}(j) - P_t C_{s,t}(j), \tag{55}$$

where financial wealth is

$$A_{s,t}(j) = \frac{1}{1-\lambda} \left[B_{s,t}(j) + \left[(1-\delta) Q_t + R_t^k \right] K_{s,t}(j) \right], \tag{56}$$

and j is the representative agent belonging to generation s.
Maximization gives

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial C_{s,t}(j)} &= (\xi_t C_{s,t}(j))^{-\sigma} - \zeta_t P_t = 0, && (57)\\
\frac{\partial \mathcal{L}_t}{\partial B_{s,t+1}(j)} &= -\zeta_t \frac{1}{R_t} + \frac{1}{1-\lambda} \beta (1-\lambda) \zeta_{t+1} = 0, && (58)\\
\frac{\partial \mathcal{L}_t}{\partial K_{s,t+1}(j)} &= -\zeta_t Q_t + \frac{1}{1-\lambda} \beta (1-\lambda) \zeta_{t+1} \left[(1-\delta) Q_{t+1} + R_{t+1}^k \right] = 0, && (59)\\
\frac{\partial \mathcal{L}_t}{\partial N_{s,t}(j)} &= -(\mu_t N_{s,t}(j))^{\varphi} + \zeta_t W_{s,t}(j) = 0, && (60)
\end{aligned}$$

where ζ_t is the Lagrangian multiplier on the budget constraint.
Combining the first two derivatives gives the standard Euler equation for consumption flows

$$1 = \beta R_t \frac{1}{\pi_{t+1}} \left(\frac{\xi_t C_{s,t}(j)}{\xi_{t+1} C_{s,t+1}(j)} \right)^{\sigma}. \tag{61}$$

In addition, the labor supply schedule reads as

$$\frac{W_{s,t}(j)}{P_t} = (\mu_t N_{s,t}(j))^{\varphi} (\xi_t C_{s,t}(j))^{\sigma}. \tag{62}$$

Now, let the stochastic discount factor be

$$\Lambda_{t,t+1}(s,j) = \beta \frac{1}{\pi_{t+1}} \left(\frac{\xi_t C_{s,t}(j)}{\xi_{t+1} C_{s,t+1}(j)} \right)^{\sigma}, \tag{63}$$

such that

$$\Lambda_{t,t+1}(s,j) = \frac{1}{R_t}. \tag{64}$$

The FOC for capital can be written as

$$1 = \beta \left(\frac{\xi_t C_{s,t}(j)}{\xi_{t+1} C_{s,t+1}(j)} \right)^{\sigma} \frac{1}{\pi_{t+1}} \frac{\left[(1-\delta) Q_{t+1} + R^k_{t+1} \right]}{Q_t}, \tag{65}$$

$$1 = \Lambda_{t,t+1}(s,j) \frac{\left[(1-\delta) Q_{t+1} + R^k_{t+1} \right]}{Q_t}, \tag{66}$$

which also is

$$\Lambda_{t,t+1}(s,j) R_t = \Lambda_{t,t+1}(s,j) \frac{\left[(1-\delta) Q_{t+1} + R^k_{t+1} \right]}{Q_t} = 1. \tag{67}$$

The LHS of the budget constraint can also be written as

$$\frac{B_{s,t+1}(j)}{R_t} + Q_t K_{s,t+1}(j) \leq ..., \tag{68}$$

$$B_{s,t+1}(j) + R_t Q_t K_{s,t+1}(j) \leq .... \tag{69}$$

Using (67),

$$\Lambda_{t,t+1}(s,j) \left[B_{s,t+1}(j) + \left[(1-\delta) Q_{t+1} + R^k_{t+1} \right] K_{s,t+1}(j) \right] \leq .... \tag{70}$$

Using the definition for financial wealth, the LHS equals

$$\Lambda_{t,t+1}(s,j) A_{s,t+1}(j) \leq .... \tag{71}$$

Then, the budget constraint can be written as

$$(1-\lambda) \Lambda_{t,t+1}(s,j) A_{s,t+1}(j) + P_t C_{s,t}(j) \leq A_{s,t}(j) + W_{s,t}(j) N_{s,t}(j) + Z_{s,t}(j) - T_{s,t}(j). \tag{72}$$

The transversality condition is given by

$$\lim_{T \to \infty} E_t (1-\lambda)^{T-t} \Lambda_{t,T}(s,j) A_{s,T}(j) = 0. \tag{73}$$

Applying this to the budget constraint yields

$$\sum_{T=t}^{\infty} E_t (1-\lambda)^{T-t} \Lambda_{t,T}(s,j) P_T C_{s,T}(j) \leq A_{s,t}(j) + \sum_{T=t}^{\infty} E_t (1-\lambda)^{T-t} \Lambda_{t,T}(s,j) \left(W_{s,T}(j) N_{s,T}(j) + Z_{s,T}(j) - T_{s,T} \right. \tag{74}$$

Notice that

$$E_t \lambda^{T-t} \Lambda_{t,T}(s,j) P_T C_{s,T}(j) = \beta^{T-t} P_T C_{s,T}(j). \tag{75}$$

It follows

$$\sum_{T=t}^{\infty} E_t (1-\lambda)^{T-t} \beta^{T-t} P_T C_{s,T}(j) \leq A_{s,t}(j) + \sum_{T=t}^{\infty} E_t (1-\lambda)^{T-t} \Lambda_{t,T}(s,j) \left(W_{s,T}(j) N_{s,T}(j) + Z_{s,T}(j) - T_{s,T}(j) \right) \tag{76}$$

Using the expression for a geometric row yields

$$\frac{1}{1-\beta(1-\lambda)}P_T C_{s,T}(j) \leq A_{s,t}(j)+\sum_{T=t}^{\infty} E_t(1-\lambda)^{T-t}\Lambda_{t,T}(s,j)\left(W_{s,T}(j)N_{s,T}(j)+Z_{s,T}(j)-T_{s,T}(j)\right). \tag{77}$$

Define

$$H_{s,t}(j)=\sum_{T=t}^{\infty} E_t(1-\lambda)^{T-t}\Lambda_{t,T}(s,j)\left(W_{s,T}(j)N_{s,T}(j)+Z_{s,T}(j)-T_{s,T}(j)\right), \tag{78}$$

as human wealth.
Then, individual consumption follows

$$P_T C_{s,T}(j)=\left[1-\beta(1-\lambda)\right]\left[A_{s,t}(j)+H_{s,t}(j)\right]. \tag{79}$$

C. Aggregate Consumption

First, the aggregate value of any variable X_t is obtained by

$$X_t=\sum_{s=-\infty}^{t}\left\{\int_0^{\lambda(1-\lambda)^{t-s}} X_{s,t}(j)dj\right\}. \tag{80}$$

The aggregate budget constraint, the aggregate consumption, and aggregate labor supply are then given by

$$\frac{B_{t+1}}{R_t}+Q_tK_{t+1} = A_t+W_tN_t+Z_t-T_t-P_tC_t, \tag{81}$$

$$P_tC_t = \left[1-\beta(1-\lambda)\right]\left[A_t+H_t\right], \tag{82}$$

$$\frac{W_t}{P_t} = (\mu_tN_t)^{\varphi}(\xi_tC_t)^{\sigma}. \tag{83}$$

Remember that

$$H_t=\sum_{T=t}^{\infty} E_t(1-\lambda)^{T-t}\Lambda_{t,T}\left(W_TN_T+Z_T-T_T\right). \tag{84}$$

For simplicity, let us define

$$\psi=\left[1-\beta(1-\lambda)\right]. \tag{85}$$

We know that the following has to hold

$$\frac{B_{t+1}}{R_t}+Q_tK_{t+1}=E_t\Lambda_{t,t+1}\left[B_{t+1}+\left[(1-\delta)Q_{t+1}+R^k_{t+1}\right]K_{t+1}\right], \tag{86}$$

such that the aggregate budget constraint is given by

$$E_t\Lambda_{t,t+1}\left[B_{t+1}+\left[(1-\delta)Q_{t+1}+R^k_{t+1}\right]K_{t+1}\right]=A_t+W_tN_t+Z_t-T_t-P_tC_t, \tag{87}$$

or

$$E_t\Lambda_{t,t+1}A_{t+1}+P_tC_t=A_t+W_tN_t+Z_t-T_t. \tag{88}$$

Here, aggregate financial wealth is

$$A_t=B_t+\left[(1-\delta)Q_t+R^k_t\right]K_t. \tag{89}$$

Then, the aggregate consumption equation can be written as (by using (88) to substitute into (82), while noticing that the period t human wealth has to be subsituted out such that the sum now starts at $t+1$.)

$$P_tC_t=\psi\left[E_t\Lambda_{t,t+1}A_{t+1}+P_tC_t+E_t\sum_{T=t+1}^{\infty}(1-\lambda)^{T-t}\Lambda_{t,T}\Omega_T\right], \tag{90}$$

where

$$\Omega_t=W_tN_t+Z_t-T_t. \tag{91}$$

Now, if we iterate the aggregate consumption equation one period forward, we obtain

$$P_{t+1}C_{t+1}=\psi\left[A_{t+1}+H_{t+1}\right], \tag{92}$$

equivalently,

$$P_{t+1}C_{t+1}=\psi\left[A_{t+1}+E_{t+1}\sum_{T=t+1}^{\infty}(1-\lambda)^{T-(t+1)}\Lambda_{t+1,T}\Omega_T\right]. \tag{93}$$

Multiplying by $(1-\lambda)\Lambda_{t,t+1}$ and taking expectations, we arrive at

$$(1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}=\psi\left[(1-\lambda)E_t\Lambda_{t,t+1}A_{t+1}+E_t\sum_{T=t+1}^{\infty}(1-\lambda)^{T-t}\Lambda_{t,T}\Omega_T\right]. \tag{94}$$

Solving for the expression in Ω_T

$$E_t\sum_{T=t+1}^{\infty}(1-\lambda)^{T-t}\Lambda_{t,T}\Omega_T=\frac{1}{\psi}(1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}-(1-\lambda)E_t\Lambda_{t,t+1}A_{t+1}. \tag{95}$$

Inserting (95) into (90) gives

$$
\begin{aligned}
P_tC_t &= \psi\left[E_t\Lambda_{t,t+1}A_{t+1} + P_tC_t + E_t\sum_{T=t+1}^{\infty}(1-\lambda)^{T-t}\Lambda_{t,T}\Omega_T\right], && (96)\\
P_tC_t &= \psi\left[E_t\Lambda_{t,t+1}A_{t+1} + P_tC_t + \frac{1}{\psi}(1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1} - (1-\lambda)E_t\Lambda_{t,t+1}A_{t+1}\right], && (97)\\
P_tC_t &= \psi\left[E_t\Lambda_{t,t+1}A_{t+1} + P_tC_t - (1-\lambda)E_t\Lambda_{t,t+1}A_{t+1}\right] + (1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}, && (98)\\
P_tC_t &= \psi\left[P_tC_t + \lambda E_t\Lambda_{t,t+1}A_{t+1}\right] + (1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}, && (99)\\
(1-\psi)P_tC_t &= \psi\lambda E_t\Lambda_{t,t+1}A_{t+1} + (1-\lambda)E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}. && (100)
\end{aligned}
$$

Finally, aggregate consumption follows

$$
P_tC_t = \frac{\psi}{1-\psi}\lambda E_t\Lambda_{t,t+1}A_{t+1} + \frac{1-\lambda}{1-\psi}E_t\Lambda_{t,t+1}P_{t+1}C_{t+1}. \qquad (101)
$$

D. Equation System - Log-Linear

$$
\begin{aligned}
C\hat{C}_t &= \frac{\psi\lambda}{1-\psi}\frac{1}{R}a\left(\hat{a}_{t+1}-\hat{R}_t\right)+\frac{1-\lambda}{1+\psi}C\frac{1}{R}\left(\hat{C}_{t+1}+\hat{\pi}_{t+1}-\hat{R}_t\right), && (102)\\
(1-\lambda)a\hat{a}_t &= B\hat{l}_t+(1-\delta)KQ\hat{q}_t+r^kK\hat{r}^k_t+((1-\delta)Q+r^k)K\hat{K}_t, && (103)\\
\hat{q}_t &= \frac{1}{R}(1-\delta)\hat{q}_{t+1}+\left[1-\frac{1}{R}(1-\delta)\right]\hat{r}^k_{t+1}-(\hat{r}_t-\hat{\pi}_{t+1}), && (104)\\
\hat{I}_t-\hat{K}_t &= \eta\hat{q}_t, && (105)\\
\hat{w}_t &= \psi\hat{N}_t+\sigma\hat{C}_t, && (106)\\
\hat{\pi}_t &= \frac{1}{R}\hat{\pi}_{t+1}+\kappa\hat{MC}_t, && (107)\\
\hat{MC}_t &= \hat{r^k}_t-(\hat{Y}_t-\hat{K}_t), && (108)\\
\hat{w}_t+\hat{N}_t &= \hat{r}^k_t+\hat{K}_t, && (109)\\
\hat{Y}_t &= \hat{F}_t+\alpha\hat{K}_t+(1-\alpha)\hat{N}_t, && (110)\\
\hat{K}_t &= \delta\hat{I}_{t-1}+(1-\delta)\hat{K}_{t-1}, && (111)\\
\hat{Y}_t &= s_C\hat{C}_t+s_I\hat{I}_t+s_G\hat{G}_t, && (112)\\
\hat{l}_t &= R\left(\hat{l}_{t-1}-\frac{ds}{Y}\hat{\pi}_{t-1}+\hat{G}_{t-1}-\hat{\tau}_{t-1}\right)+s_b\hat{r}_{t-1}, && (113)\\
\hat{\tau}_t &= \tau_l\hat{l}_t+\tau_Y\hat{Y}_t, && (114)\\
\hat{r}_t &= \rho_r\hat{r}_{t-1}+(1-\rho_r)\left(\phi_\pi\hat{\pi}_t+\phi_Y\hat{Y}_t\right), && (115)\\
\hat{G}_t &= -\omega_l\hat{l}_t-\omega_Y\hat{Y}_t+\Gamma_t, && (116)
\end{aligned}
$$

E. Linear Quadratic Approach

First, we define the utility function of our households as

$$U_t = \sum_{t=0}^{\infty} \left[(1-\lambda)\beta\right]^t \left[\frac{(\xi_t C_t)^{1-\sigma}}{1-\sigma} + \frac{(\vartheta_t G_t)^{1-\gamma}}{1-\gamma} - \frac{(\mu_t N_t)^{1+\varphi}}{1+\varphi}\right]. \tag{117}$$

We can separate this function into three parts, viz.

$$U^c = \frac{(\xi_t C_t)^{1-\sigma}}{1-\sigma}, \tag{118}$$

$$V = \frac{(\vartheta_t G_t)^{1-\gamma}}{1-\gamma}, \tag{119}$$

$$W = \frac{(\mu_t N_t)^{1+\varphi}}{1+\varphi}. \tag{120}$$

Then, we use

$$Y_t = C_t + I_t + G_t, \tag{121}$$

to substitute for consumption in U^c and approximate it up to second order, such that

$$U^c \approx U_c Y \left[\begin{array}{c} \hat{Y}_t + \frac{1}{2}\left(1 + \frac{U_{cc}}{U_c}Y\right)\hat{Y}_t^2 + \frac{U_I I}{U_c Y}\hat{I}_t + \frac{1}{2}\left(\frac{U_I I}{U_c Y} + \frac{U_{II}}{U_c Y}I^2\right)\hat{I}_t^2 + \frac{U_G G}{U_c Y}\hat{G}_t + \frac{1}{2}\left(\frac{U_G G}{U_c Y} + \frac{U_{GG}G^2}{U_c Y}\right)\hat{G}_t^2 \\ + \frac{U_{c\xi}}{U_c}\hat{Y}_t\xi_t + \frac{U_{cG}}{U_c}G\hat{Y}_t\hat{G}_t + \frac{U_{cI}}{U_c}I\hat{Y}_t\hat{I}_t + \frac{U_{I\xi}}{U_c Y}I\hat{I}_t\xi_t + \frac{U_{IG}}{U_c Y}IG\hat{I}_t\hat{G}_t + \frac{U_{G\xi}}{U_c Y}G\hat{G}_t\xi_t \end{array}\right] \tag{122}$$

$$+t.i.p. + ||\mathcal{O}||^3. \tag{123}$$

Next step is a second order approximation to V,

$$V \approx V_G G\left[\hat{G}_t + \frac{1}{2}\left(1 + \frac{V_{GG}}{V_G}G\right)\hat{G}_t^2 + \frac{V_{G\vartheta}}{V_G}\hat{G}_t\vartheta_t\right] + t.i.p. + ||\mathcal{O}||^3. \tag{124}$$

Furthermore, using the production function

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{125}$$

yields the second order approximation of the disutility of work (W)

$$W \approx U_c Y \left[\begin{array}{c} (1-\Phi)\hat{Y}_t + \frac{1}{2}\left(1 + \frac{W_{yy}}{W_y}Y\right)\hat{Y}_t^2 + \frac{1}{2}\left(\frac{\theta}{(1-\theta)(1-\theta\beta)}\right)(\epsilon^{-1} + \frac{W_{yy}}{W_y}Y)\epsilon^2\hat{\pi}_t^2 + \frac{W_K K}{W_y Y}\hat{K}_t \\ + \frac{1}{2}\left(\frac{W_K K}{W_y Y} + \frac{W_{KK}K^2}{W_y Y}\right)\hat{K}_t^2 + \frac{W_{yK}K}{W_y}K\hat{Y}_t\hat{K}_t + \frac{W_{K\mu}K}{W_y Y}K\hat{K}_t\mu_t + \frac{W_{yA}}{W_y}\hat{Y}_t A_t \\ + \frac{W_{KA}}{W_y Y}K\hat{K}_t A_t + \frac{W_{y\mu}}{W_y}\hat{Y}_t\mu_t \end{array}\right] \tag{126}$$

$$+t.i.p. + ||\mathcal{O}||^3, \tag{127}$$

where $\Phi = \frac{1}{\theta}$, θ is the elasticity of substitution, and $\dot{\varpi}$ is the Calvo staggering parameter. Combining the three approximations yields the maximization problem

$$\mathcal{L}_t = U_c Y \left[\begin{array}{c} \Phi \hat{Y}_t + \frac{1}{2}\left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y}\right) Y \hat{Y}_t^2 + \frac{U_I I}{U_c Y}\hat{I}_t + \frac{1}{2}\left(\frac{U_I I}{U_c Y} + \frac{U_{II}}{U_c Y} I^2\right)\hat{I}_t^2 + \left(\frac{U_G G}{U_c Y} + \frac{V_G G}{U_c Y}\right)\hat{G}_t \\ + \frac{1}{2}\left(\frac{U_G G}{U_c Y} + \frac{U_{GG} G^2}{U_c Y} + \frac{V_G G}{U_c Y} + \frac{V_{GG}}{U_c Y} G^2\right)\hat{G}_t^2 - \frac{W_K K}{W_y Y}\hat{K}_t - \frac{1}{2}\left(\frac{W_K K}{W_y Y} + \frac{W_{KK} K^2}{W_y Y}\right)\hat{K}_t^2 \\ - \frac{1}{2}\left(\frac{\theta}{(1-\theta)(1-\theta\beta)}\right)(\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2 \hat{\pi}_t^2 \\ + \frac{U_{c\xi}}{U_c}\hat{Y}_t \xi_t + \frac{U_{cG}}{U_c} G \hat{Y}_t \hat{G}_t + \frac{U_{cI}}{U_c} I \hat{Y}_t \hat{I}_t + \frac{U_{I\xi}}{U_c Y} I \hat{I}_t \xi_t + \frac{U_{IG}}{U_c Y} I G \hat{I}_t \hat{G}_t + \frac{U_{G\xi}}{U_c Y} G \hat{G}_t \xi_t \\ - \frac{W_{yK}}{W_y} K \hat{Y}_t \hat{K}_t - \frac{W_{K\mu}}{W_y Y} K \hat{K}_t \mu_t - \frac{W_{yA}}{W_y}\hat{Y}_t A_t - \frac{W_{KA}}{W_y Y} K \hat{K}_t A_t - \frac{W_{y\mu}}{W_y}\hat{Y}_t \mu_t + \frac{V_{G\vartheta}}{U_c Y}\hat{G}_t \vartheta_t \end{array} \right] \tag{128}$$

$$+ t.i.p. + ||\mathcal{O}||^3, \tag{129}$$

s.t.

$$-\lambda_t^{PC}\left[\frac{1}{R}\hat{\pi}_{t+1} + \kappa\left(\hat{r}_t^k - \left(\hat{Y}_t - \hat{K}_t\right) - \hat{\pi}_t\right)\right], \tag{130}$$

$$-\lambda_t^{IS}\left[\begin{array}{c} -\left(\hat{r}_t - \frac{1}{1+\omega}\hat{\pi}_{t+1}\right) + \frac{1}{1+\omega}\left(\hat{Y}_{t+1} - s_I \hat{I}_{t+1} - s_G \hat{G}_{t+1}\right) \\ + \frac{\omega}{1+\omega}\left\{\frac{1}{s_a}\left(R\left[\hat{l}_t - \frac{d}{Y}\hat{\pi}_t + \hat{G}_t - \tau_l \hat{l}_t - \tau_y \hat{Y}_t\right] + s_b \hat{r}_t\right) + \frac{R}{s_a} s_K \left(\hat{r}_t + \hat{K}_{t+1} + \frac{\hat{I}_t - \hat{K}_t}{\eta}\right)\right\} \\ -\hat{Y}_t + s_I \hat{I}_t + s_G \hat{G}_t \end{array}\right] \tag{131}$$

The derivatives are given by

$$\frac{\partial \mathcal{L}_t}{\partial \hat{Y}_t} = U_c Y \left\{ \Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y} \right) Y \hat{Y}_t + \frac{U_{c\xi}}{U_c} \xi_t + \frac{U_{cG}}{U_c} G \hat{G}_t + \frac{U_{cI}}{U_c} I \hat{I}_t - \frac{W_{yK}}{W_y} K \hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y} \mu_t \right\} \tag{132}$$

$$+\lambda_t^{PC} \kappa + \lambda_t^{IS} + \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{1}{s_a} R \tau_y, \tag{133}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{I}_t} = U_c Y \left\{ \frac{U_I I}{U_c Y} + \left(\frac{U_I I}{U_c Y} + \frac{U_{II}}{U_c Y} I^2 \right) \hat{I}_t + \frac{U_{cI}}{U_c} I \hat{Y}_t + \frac{U_{I\xi}}{U_c Y} I \xi_t + \frac{U_{IG}}{U_c Y} I G \hat{G}_t \right\} \tag{134}$$

$$-\lambda_t^{IS} \frac{R}{s_a} s_K \frac{1}{\eta} \frac{\omega}{1+\omega} - s_I \lambda_t^{IS}, \tag{135}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{G}_t} = U_c Y \left\{ \begin{array}{c} \left(\frac{U_G G}{U_c Y} + \frac{V_G G}{U_c Y} \right) + \left(\frac{U_G G}{U_c Y} + \frac{U_{GG} G^2}{U_c Y} + \frac{V_G G}{U_c Y} + \frac{V_{GG}}{U_c Y} G^2 \right) \hat{G}_t + \frac{U_{cG}}{U_c} G \hat{Y}_t + \frac{U_{IG}}{U_c Y} I G \hat{I}_t \\ + \frac{U_{G\xi}}{U_c Y} G \xi_t + \frac{V_{G\vartheta}}{U_c Y} \vartheta_t \end{array} \right\} \tag{136}$$

$$-s_G \lambda_t^{IS} - \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{1}{s_a} R, \tag{137}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{K}_t} = U_c Y \left\{ -\frac{W_K K}{W_y Y} - \left(\frac{W_K K}{W_y Y} + \frac{W_{KK} K^2}{W_y Y} \right) \hat{K}_t - \frac{W_{yK} K}{W_y} K \hat{Y}_t - \frac{W_{K\mu}}{W_y Y} K \mu_t - \frac{W_{KA}}{W_y Y} K A_t \right\} \tag{138}$$

$$+\lambda_t^{PC} \kappa + \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{1}{s_a} R s_K \frac{1}{\eta}, \tag{139}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{\pi}_t} = -U_c Y \left(\frac{\theta}{(1-\theta)(1-\theta\beta)} \right) (\epsilon^{-1} + \frac{W_{yy}}{W_y} Y) \epsilon^2 \hat{\pi}_t + \kappa \lambda_t^{PC} + \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{1}{s_a} R \frac{d}{Y}, \tag{140}$$

$$\frac{\partial \mathcal{L}_t}{\partial \xi_t} = U_c Y \frac{U_{c\xi}}{U_c} \hat{Y}_t + U_c Y \frac{U_{I\xi}}{U_c Y} I \hat{I}_t + U_c Y \frac{U_{G\xi}}{U_c Y} G \hat{G}_t, \tag{141}$$

$$\frac{\partial \mathcal{L}_t}{\partial \mu_t} = -U_c Y \frac{W_{K\mu} K}{W_y Y} K \hat{K}_t - U_c Y \frac{W_{y\mu}}{W_y} \hat{Y}_t, \tag{142}$$

$$\frac{\partial \mathcal{L}_t}{\partial \vartheta_t} = U_c Y \frac{V_{G\vartheta}}{U_c Y} \hat{G}_t, \tag{143}$$

$$\frac{\partial \mathcal{L}_t}{\partial A_t} = -U_c Y \frac{W_{yA}}{W_y} \hat{Y}_t - U_c Y \frac{W_{KA}}{W_y Y} K \hat{K}_t, \tag{144}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{r}_t} = \lambda_t^{IS} - \lambda_t^{IS} \frac{R}{s_a} \frac{\omega}{1+\omega} s_K - \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{1}{s_a} s_b, \tag{145}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{r}_t^K} = -\lambda_t^{PC} \kappa, \tag{146}$$

$$\frac{\partial \mathcal{L}_t}{\partial \hat{l}_t} = -\lambda_t^{IS} \frac{\omega}{1+\omega} \frac{R}{s_a} + \lambda_t^{IS} \frac{\omega}{1+\omega} \frac{R}{s_a} \tau_l. \tag{147}$$

Rearranging terms for $\frac{\partial \mathcal{L}_t}{\partial \hat{G}_t}$ gives

$$U_cY\left\{\begin{array}{c}\left(\frac{U_GG}{U_cY}+\frac{V_GG}{U_cY}\right)+\left(\frac{U_GG}{U_cY}+\frac{U_{GG}G^2}{U_cY}+\frac{V_GG}{U_cY}+\frac{V_{GG}}{U_cY}G^2\right)\hat{G}_t+\frac{U_{cG}}{U_c}G\hat{Y}_t+\frac{U_{IG}}{U_cY}IG\hat{I}_t \\ +\frac{U_{G\xi}}{U_cY}G\xi_t+\frac{V_{G\vartheta}}{U_cY}\vartheta_t\end{array}\right\} \tag{148}$$

$$-s_G\lambda_t^{IS}-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}R, \tag{149}$$

$$(U_GG+V_GG)+\left(U_GG+U_{GG}G^2+V_GG+V_{GG}G^2\right)\hat{G}_t+U_{cG}YG\hat{Y}_t+U_{IG}IG\hat{I}_t+U_{G\xi}G\xi_t+V_{G\vartheta} \tag{150}$$

$$-s_G\lambda_t^{IS}-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}R \tag{151}$$

$$\hat{G}_t = \frac{\lambda_t^{IS}\left(s_G+\frac{\omega}{1+\omega}\frac{1}{s_a}R\right)-U_GG+V_GG-U_{cG}YG\hat{Y}_t-U_{IG}IG\hat{I}_t-U_{G\xi}G\xi_t-V_{G\vartheta}\vartheta_t}{U_GG+U_{GG}G^2+V_GG+V_{GG}G^2}. \tag{152}$$

Using

$$\frac{\partial \mathcal{L}_t}{\partial \hat{r}_t}=\lambda_t^{IS}-\lambda_t^{IS}\frac{R}{s_a}\frac{\omega}{1+\omega}s_K-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}s_b, \tag{153}$$

monetary policy makes the IS constraint non-binding,

$$\lambda_t^{IS}-\lambda_t^{IS}\frac{R}{s_a}\frac{\omega}{1+\omega}s_K-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}s_b = 0, \tag{154}$$

$$\lambda_t^{IS}\left(1-\frac{R}{s_a}\frac{\omega}{1+\omega}s_K-\frac{\omega}{1+\omega}\frac{1}{s_a}s_b\right) = 0, \tag{155}$$

$$\lambda_t^{IS} = 0. \tag{156}$$

Then, the optimal government spending rule follows

$$\hat{G}_t=\frac{V_GG-U_GG-U_{cG}YG\hat{Y}_t-U_{IG}IG\hat{I}_t-U_{G\xi}G\xi_t-V_{G\vartheta}\vartheta_t}{U_GG+U_{GG}G^2+V_GG+V_{GG}G^2}. \tag{157}$$

Government spending is negatively related to output and investment. Furthermore, it decreases in the preference shock as well as in the government spending taste shock. Then, by having a closer look at

$$\frac{\partial \mathcal{L}_t}{\partial \hat{l}_t}=-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a}+\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a}\tau_l, \tag{158}$$

we can draw the conclusion that the debt coefficient, τ_l, in the tax rule should move one-for-one with debt, as

$$\begin{aligned}-\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a}+\lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a}\tau_l &= 0,\\ \lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a}\tau_l &= \lambda_t^{IS}\frac{\omega}{1+\omega}\frac{R}{s_a},\\ \tau_l &= 1.\end{aligned} \tag{159}$$

In contrast, we do not find a corresponding relationship for the output coefficient τ_y. (technically: IS non-binding)
The optimal targeting rule for inflation is given by

$$U_c Y \left\{ \Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y} \right) Y\hat{Y}_t + \frac{U_{c\xi}}{U_c}\xi_t + \frac{U_{cG}}{U_c} G\hat{G}_t + \frac{U_{cI}}{U_c} I\hat{I}_t - \frac{W_{yK}}{W_y} K\hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y}\mu_t \right\} \tag{160}$$

$$+\lambda_t^{PC}\kappa + \lambda_t^{IS} + \lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}R\tau_y, \tag{161}$$

$$U_c Y \left(\frac{\theta}{(1-\theta)(1-\theta\beta)} \right) (\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2 \hat{\pi}_t = \kappa\lambda_t^{PC} + \lambda_t^{IS}\frac{\omega}{1+\omega}\frac{1}{s_a}R\frac{d}{Y}, \tag{162}$$

$$U_c Y \left(\frac{\theta}{(1-\theta)(1-\theta\beta)} \right) (\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2 \hat{\pi}_t = -U_c Y \left\{ \begin{array}{c} \Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y} \right) Y\hat{Y}_t + \frac{U_{c\xi}}{U_c}\xi_t + \frac{U_{cG}}{U_c} G\hat{G}_t \\ + \frac{U_{cI}}{U_c} I\hat{I}_t - \frac{W_{yK}}{W_y} K\hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y}\mu_t \end{array} \right\}, \tag{163}$$

$$\left(\frac{\theta}{(1-\theta)(1-\theta\beta)} \right) (\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2 \hat{\pi}_t = -\left(\begin{array}{c} \Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y} \right) Y\hat{Y}_t + \frac{U_{c\xi}}{U_c}\xi_t + \frac{U_{cG}}{U_c} G\hat{G}_t \\ + \frac{U_{cI}}{U_c} I\hat{I}_t - \frac{W_{yK}}{W_y} K\hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y}\mu_t \end{array} \right) \tag{164}$$

Finally,

$$\hat{\pi}_t = -\left[\frac{\Phi + \left(\frac{U_{cc}}{U_c} - \frac{W_{yy}}{W_y} \right) Y\hat{Y}_t + \frac{U_{c\xi}}{U_c}\xi_t + \frac{U_{cG}}{U_c} G\hat{G}_t + \frac{U_{cI}}{U_c} I\hat{I}_t - \frac{W_{yK}}{W_y} K\hat{K}_t - \frac{W_{yA}}{W_y} A_t - \frac{W_{y\mu}}{W_y}\mu_t}{\left(\frac{\theta}{(1-\theta)(1-\theta\beta)} \right) (\epsilon^{-1} + \frac{W_{yy}}{W_y} Y)\epsilon^2} \right]. \tag{165}$$

In this case, the monetary authority should react to government spending (interaction of GP and FP), output (as usual), investment, the capital accumulation, and the taste shock (related to G), the shock to disutility, and the productivity shock.

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