Financial Risk Capacity *

Job Market Paper

Saki Bigio†
Department of Economics
New York University

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Abstract

Financial crises appear to persist if banks fail to be recapitalized quickly after large losses. I explain this impediment through a model where banks provide intermediation services in asset markets with informational asymmetries. Intermediation is risky because banks take positions over assets under disadvantageous information. Large losses reduce bank net worth and, therefore, the capacity to bear further losses. Losing this capacity leads to reductions in intermediation volumes that exacerbate adverse selection. Adverse selection, in turn, lowers bank profits which explains the failure to attract new equity. These financial crises are characterized by a depression in economic growth that is overcome only as banks slowly strengthen by retaining earnings. The model calibrated and used to analyze several policy interventions.

JEL: E32, G01, G21

Key Words: Financial Crisis, Adverse Selection, Capacity Constraints, Capital Requirements

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†sbigio@nyu.edu.

1 Introduction

Often times financial crises originate from episodes of extreme bank losses. Declines in economic activity follow when banks retract lending after their equity is lost. This suggests that the impact and duration of crises could be mitigated if equity were quickly injected into the financial industry. Not surprisingly, the slow recovery of bank equity was a major concern for policy makers, academics, and practitioners during the financial crisis of 2008-2009. In fact, during his only television interview, the Chairman of the Federal Reserve, Ben Bernanke, was asked when he would consider the crisis to be over. He answered, "when banks start raising capital on their own."[1] Why is it then that banks cannot attract capital in times of crises?

This paper approaches this question from a novel angle. I adopt the view that banks are intermediaries in financial markets that feature asymmetric information.[2] Indeed, by dealing with a large number of parties, banks can dilute the risk of transacting under asymmetric information. However, in reality, banks cannot dilute risk entirely: Financial intermediation is risky. I also follow the literature on financial frictions in that the capacity to tolerate financial intermediation risk, *i.e.* the *financial risk capacity*, is tied to bank net worth. Consequently, lending conditions depend on the evolution of this variable. This features imply that large reductions in bank net worth magnify financial crises because they exacerbate adverse selection. The intuition is that when banks run out of net worth, they must cut back on loans per unit of collateral to decrease their exposure to future losses. In turn, borrowers respond using assets of lower quality as collateral, something that depresses lending furthermore. This form of adverse selection can reduce bank profitability. Ultimately, without profitable opportunities, the financial system cannot attract new equity. This paper shows that this transmission mechanism operates even when the information structure and the production possibility of the economy are constant.

Asymmetric information is key for this result. Absent other frictions, competition arguments suggest that banks should easily attract new equity in times when the economy most needs it. After all, like with any other product, marginal profits should be higher when supply is lower. Thus, to deliver substantial real effects, a theory that links financial intermediation to bank net worth must explain why are banks not quickly recapitalized after large losses. Here, asymmetric information breaks this relation. Asymmetric can lead to reductions in returns on equity (ROE) that preclude banks from being recapitalized despite that financial resources are readily available. This distinguishes this from other macroeconomic models that study financial intermediation.[3] In these models, banks cannot raise equity because bankers are fully-invested specialists that face other agency costs. However, these models deliver increases in bank ROE after bank losses, an opposite but testable implication. Moreover, in my model, the economy responds very differently to bank losses of different magnitude. The paper shows that equity injections are effective stabilizers of financial crises for moderate financial losses but fail to occur after large losses. This feature has important consequences for policy interventions as, I argue later.

I formalize this insight through a model that has four ingredients. (1) There is a need to trade capital goods. (2) Financial intermediation facilitates trade because there is asymmetric information in capital markets. (3) Net worth evolves through time because intermediation is risky. (4) Net worth is essential to provide intermediation because banks face limited-liability constraints. Together, these four ingredients deliver recurrent and persistent financial crises. These crises are characterized by contractions in volumes of

[1] This quote was taken from the interview titled the "The Chairman", 60 Minutes, CBS News, March 15, 2009.

[2] This is a common view in banking theory. For example, Gorton (2010) argues that "The essential function of banking is to create a special kind of debt, debt that is immune to adverse selection by privately informed agents."

[3] See the following section for a more detailed discussion about these models.

intermediation and reductions collateral quality that lead to economic declines. In parallel, banks expect low equity returns which is why they fail to attract feasible equity injections.

I model the banking system as a competitive sector that provides intermediation services for the reallocation of capital between agents in two sectors of the economy. As in Kiyotaki and Moore (2008), the first sector comprises producers of capital goods in need of funds. The second sector comprises agents that lack investment projects but, in contrast, have the resources to carry them out. The fundamental economic problem is that funds must flow from the latter to the former group and capital must flow in the opposite way. The friction that prevents the direct flow of resources is that capital producers have private information about the quality of capital units used in these transactions. Banks ameliorate this problem. They offer risk-free deposits to obtain funds from consumption producers. They transfer these funds to capital producers and take positions over part their capital units (under asymmetric information). By facilitating trade, financial intermediation is fundamental for economic growth.

When parties share common information, the environment collapses to a classic stochastic-growth economy. These economies fluctuate only in response to shocks that affect the production possibility. Moreover, there is no need for financial intermediation. With asymmetric information about the quality of capital, banks dilute the idiosyncratic risk of transacting under disadvantageous information. However, if the distribution of capital quality is known, despite asymmetric information, banking is not risky. In equilibrium, banks can infer which qualities of capital are traded and, therefore, the value of their asset positions is risk-free.[4] Without this additional source of risk, business cycles can originate from shocks that disperse the quality capital as, for example, in Bigio (2011) or Kurlat (2011). These shocks do not affect the production possibility but have real effects by aggravating adverse selection. However, fluctuations in these models are unrelated to the condition of banks. Furthermore, in these economies, severe adverse selection persists only if shocks are persistent.

This theory needs two additional ingredients to associate financial crises to low bank net worth. First, net worth must fluctuate. Second, net worth must matter. In the model, profits from intermediation are risky because, in addition to asymmetric information, the capital-quality distribution is, *a priori*, uncertain. In equilibrium, banks hold positions in the lower tail of the capital-quality distribution. Since this distribution evolves through time, the value of the banks' asset positions is risky. This induces fluctuations in bank net worth.[5]

To induce a role for net worth, I assume banks face limited-liability constraints. Under limited liability, banks must resort to their internal funds to finance any possible operational loss. This constraint makes bank net worth a key state variable: with lower net worth banks must downsize their assets and liabilities to guarantee solvency upon future contingencies. Eventually, this reaction is responsible for the feedback-loop between asymmetric information and the evolution of net worth. Furthermore, this aspect also induces an externality: banks can fail to internalize that larger positions today may lead to greater losses tomorrow that, in the aggregate, affect the quality of assets traded. This is particularly damaging if adverse selection prevents banks from raising equity, and consequently, prolongs downturns.

[4] Formally, this presumes banks can exploit the law of large numbers to wipe out financial risk: the conditional expected quality of assets bought at a given price converges to the average quality bought. Without risk, competition drives profits to 0 and bank equity plays no role. Formal arguments appear in Glosten and Milgrom (1985).

[5] Changes in the distribution of firm revenues have been found to precede recessions (see Bloom (2009) and Bloom et al. (2009) for documentation). Assuming the distribution is uncertain is a convenient, yet realistic, way of introduce risk into intermediation under asymmetric information. Very recently, a number of studies have provide several theories to explain the effects of changes in the distribution of idiosyncratic shocks to production units (see for example, Arellano et al. (2010), Gilchrist et al. (2010) or Schaal (2011)). I relate these changes to the ones in my model because capital of different qualities can be thought of as different productive units.

To summarize the mechanics of the model, consider the following hypothetical simulation. Suppose there is a sequence of shocks to the distribution of capital quality that lead to systematic financial losses. Losses are financed liquidating a substantial portion of a bank's net worth. Thus, net worth is depleted making it impossible to sustain the same magnitude of losses in the future. Banks respond by scaling down their operations thereby exacerbating adverse selection. Adverse selection decreases expected bank profits and precludes external recapitalizations. Instead, the financial system is recapitalized only through retained earnings. Yet, this process must be very slow since volumes of intermediation and marginal profits are already low. Surprisingly, to have real effects, these shocks do not need to affect the production possibility or the information structure of the economy.

Figure 1 suggests how similar mechanics could have operated during the 2008-2009 financial crisis. The top left panel plots the evolution of tangible net worth for a group of selected U.S. bank holding companies (with and without TARP injections) during the last decade. The figure shows that, excluding TARP, this variable deviated from trend in the quarters prior to the Great Recession (second gray shaded area). The bottom-left panel describes the decline in the nominal stock of capital, a symptom of the deceleration in economic activity. The top right panel shows that returns on equity (ROE) were stable during the period prior to the crisis. However, bank ROE falls during the recession and remains persistently below its historical average since. The bottom-right panel presents bank dividends and equity injections. Issuances were high in comparison to historical records but far from replenishing private bank equity.[6]

I tackle several technical and empirical challenges to solve and calibrate the model. The model features asymmetric information, limited-liability constraints, and aggregate shocks. Despite this, it is highly tractable. I also develop an algorithm that allows to study its global behavior. On the empirical side, I construct a set of aggregate banking time series that correct for institutional changes and isolate TARP injections.[7] I calibrate the model and provide rough measures of the effects of dividend taxes and capital requirements. These policy exercises show that the frequency and duration of financial crises can be reduced at the expense of economic growth in normal times.

The rest of the paper is organized in the following manner. The next section relates the paper to the literature. Section 2 introduces the model. Section 3 provides a commercial-banking interpretation of the model. Section 4 characterizes equilibria. Section 5 presents two analytic examples to underscore the role of asymmetric information in this economy. Section 6 describes the empirical strategy used to reconstruct banking measures and presents the results from the quantitative exercises. Section 7 describes the effects of several policy experiments. Section 8 discusses some extensions and Section 9 concludes. The computational algorithm, proofs, and data definitions are relegated to the appendix.

1.1 Relationship with the literature

Financial intermediation in macroeconomics. As portrayed in Figure 1, the net worth of the banking system seems to have been a driving factor during the Great Recession. The literature explains the relevance of this variable for the efficient allocation of resources from several perspectives. This paper is closely related to theories in which the financial system's net worth reduces agency costs (or leverage constraints) that constrain

[6] The figure reports reconstructed time series of banking indicators based on the empirical strategy described in Section 6.

[7] Since the beginning of the crisis, many non-bank institutions effectively became Bank Holding Companies. For example, the bank equity time series reported by Flow of Funds has substantial increments after Goldman Sachs converts into a Bank Holding Company.

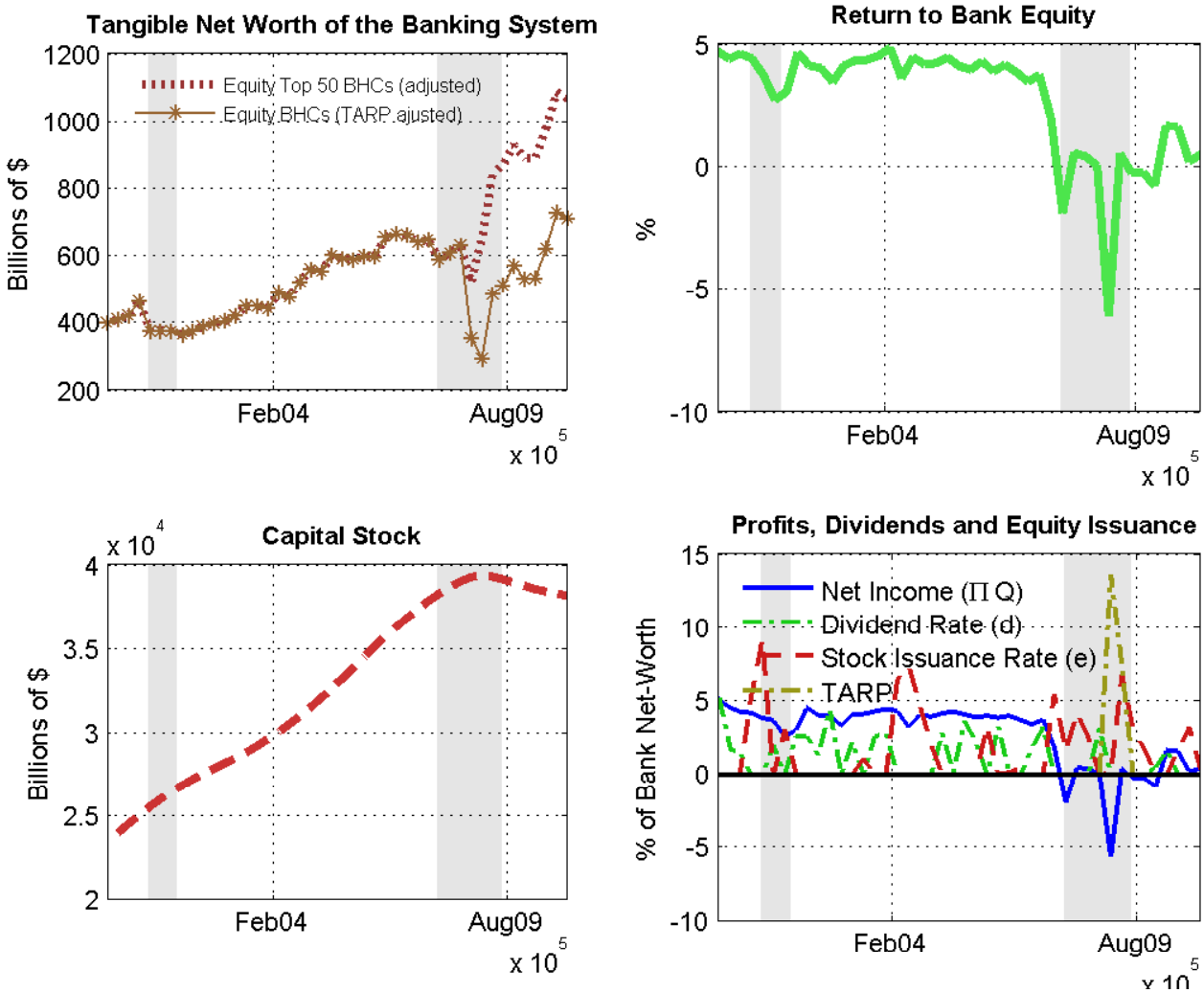

Figure 1: **U.S. Banking Indicators and Capital Accumulation.**

financial intermediation.[8] Agency costs faced by entrepreneurs were incorporated into business cycle models in the now classic financial accelerator of models of Bernanke and Gertler (1989) and Bernanke et al. (1996). Holmstrom and Tirole (1997) were among the first to incorporate similar frictions into a financial sector. In their framework, the financial sector's net worth reduces agency costs because bankers have more incentives to increase the return on loans when they have more skin in the game : with more net worth, banks can provide more intermediation. Several recent studies have introduced a financial sector with similar characteristics into quantitative business cycle models to quantify various financial stabilization policies. For example, Gertler and Karadi (2011) or Gertler and Kiyotaki (2010) study the benefits of government injections targeted to recapitalize FIs. Martinez-Miera and Suarez (2011) incorporate a similar feature into a macro model where bank net worth affects the risk structure across loans. Their model is used to study capital requirements. Similar policy exercises are performed here also.

[8] Following Diamond and Dybvig (1983), another stream of research has emphasized the role of the financial sector's net worth in bank runs caused by maturity mismatches. In these models, financial crises arise when an exogenous shock to the demand and supply of short run funds affects financial institutions. This literature underscores that the availability of funds (for the whole financial system) determines the effectiveness of internal insurance mechanisms against idiosyncratic shocks. From this perspective, the financial system's net asset position matters because it can preclude bank runs. Other seminal contributions in this area include Holmstrom and Tirole (1998) or Allen and Gale (1998). Bolton et al. (2010) relate this literature with asymmetric information. See Sargent (2011) for a recent survey.

This paper is closer to the work of Brunnermeier and Sannikov (2011) and He and Krishnamurthy (2009) because intermediary losses have an additional propagation mechanism. However, propagation in those papers occurs because financial shocks are amplified by fire-sale spirals. Fire sales occur when banks must meet obligations by liquidating capital whose price may fall rapidly if large quantities are liquidated simultaneously.[9, 10]

In common with the literature, bank net worth is key to allocating resources efficiently in this model. However, the novelty here is that financial risk is exacerbated by asymmetric information. Besides this propagation mechanism, the main distinction is in the incentives to recapitalize banks during a crises. In the literature, agency costs increase as net worth is lost. This means that the value of an additional unit of bank equity is greater in times of crisis. If in those models inside equity injections were possible, one could expect rapid recapitalizations in times of crisis.[11] This distinction is relevant because financial shocks have a mitigated impact if they are counterbalanced by equity injections. Here, in contrast, adverse selection reduces the profitability of equity injections. As a consequence, although it is always possible to inject equity, bankers choose not to do so.

The constraints on intermediation caused by low-net worth produce an externality. In that dimension, the model is also similar to Lorenzoni (2008). That paper combines features of the bank run literature with limited commitment on the side of bankers. That paper identifies a fire-sale externality that stems from excessive risky intermediation (relative to a social optimal). The externality here stems from bankers not internalizing that by leveraging excessively, they may run out of capital and cause severe adverse-selection in the future.

Asymmetric information in macroeconomics. Beginning with the seminal ideas of Stiglitz and Weiss (1981) and Myers and Majluf (1984), we know that asymmetric information in financial markets can cause credit rationing phenomena that affect economic performance. The paper also follows a recent line of these ideas into general-equilibrium. Carlstrom and Fuerst (1997) use private information in the return to investment to explain credit rationing during the cycle.[12] Eisfeldt (2004) studies a model where, like here, the quality of existing assets is private information. In this environment, assets are sold under asymmetric information for risk-insurance motives. That paper shows how asymmetric information limits risk insurance. Bigio (2011) and Kurlat (2011) study models in which assets are sold under asymmetric information with the objective of relaxing financial constraints. These papers explain how shocks that exacerbate adverse selection can generate recessions. Other models that study lemons markets, such as Hendel and Lizzeri (1999), Kurlat (2011) or Daley and Green (2011), obtain persistent effects through learning-by-holding dynamics. In the present paper, crises are persistent because the financial sector recovers only by retaining earnings.

A formal model of the interaction between financial intermediation and asymmetric information (in general equilibrium) is the main contribution of this paper. Although the formalism is new, the idea is not: Several decades of research show that net worth affects insurance markets. Liability and catastrophe insurance, for example, share a common striking feature with lending markets: Crises in this sector are recurrent and

[9] Similar feedback between losses in intermediary capital and reductions in the value of entrepreneurial capital occur in models such as Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009). Vayanos and Wang (2011) introduce asymmetric information into this framework

[10] Fire sales were first explained by Shleifer and Vishny (1992). Diamond and Rajan (2011) study strategic behavior by banks to exploit fire sales by their competitors.

[11] This channel is shut down when intermediaries are fully invested in their banks. When intermediaries are specialists without additional sources of fresh capital wealth, there are no additional sources from which to replenish bank equity.

[12] Martin (2009) compares pooling with separating equilibria in a similar context in which the quantity of collateral interest rates are used as screening devices.

characterized by large swings premia and volumes. In parallel to the 2008-2009 crisis, these events typically followed episodes of heavy losses for insurance companies.[13] Gorton and Metrick (2010) documents a similar pattern for the shadow banking sector.

Several modeling choices borrow from different papers. Banks here resemble those in O'Hara (1983) or the insurance companies in Winter (1991b). The motive for trade is follows from Kiyotaki and Moore (2008). Asymmetric information in asset qualities is introduced as in Bigio (2011).

Empirical Work During the recent crisis, there has been considerable empirical investigation that relates to this paper. Brunnermeier (2009), Krishnamurthy (2009), and Gorton and Metrick (2010) suggest that both asymmetric information and bank losses were important factors for its escalation. Harverd et al. (2011) provides evidence on the deterioration of credit quality when banks contract lending during several historical banking crisis episodes. Ivashina and Scharfstein (2010) provide evidence on the decline of volumes of financial intermediation and increased premia. Acharya et al. (2010) detail the behavior of equity injections and dividends. This work suggests that fresh private capital was not flowing to the financial system during those critical times.

2 Model

2.1 Environment

The model is formulated in discrete time with an infinite horizon. There are two goods: a perishable consumption good (the *numeraire*) and capital. Every period is divided into two stages, $s \in \{1, 2\}$. There are two aggregate shocks: a TFP shock $A_t \in \mathbb{A}$ and a shock $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, ..., \phi_N\}$ that determines the distribution of capital quality. (A_t, ϕ_t) form a joint Markov process that evolves according to a transition probability $\chi : (\mathbb{A}\times\Phi) \times (\mathbb{A}\times\Phi) \to [0, 1]$ with the standard assumptions.

Notation. I use $y_{t,s}$ to refer to the value of a variable y in period t stage s when the variable changes values between stages. Otherwise, if the variable remains constant through the period, I use the time subscript only.

Demography. There are two populations of agents: producers and bankers. Each population has a unit mass, but bankers are assumed to be bigger in a sense to be clear below. Bankers face an exogenous constant probability of exit. When an banker exits, he is immediately replaced by a newborn banker. Stochastic exits are useful to obtain analytic examples but, bankers are treated as infinitely-lived in the quantitative exercises.

Producers. Producers are identified with a number $z \in [0, 1]$ and carry their capital stock $k_t(z)$ as their individual state variable. At the beginning of the first stage, producers are randomly segmented into two groups: capital-goods producers and consumption-goods producers. I also refer to these types as k-producers and c-producers respectively. Producers become capital good producers with a probability π independent of time and z. As a consequence, every period, there are masses π of k-producers and $1 - \pi$ of c-producers.

Capital-goods producers have access to an investment technology that allows them to create new capital units using consumption goods but cannot use their capital stock for the production of consumption goods. In contrast, consumption-goods producers can use capital to produce consumption goods, but lack the possibility of augmenting their capital stock by building capital directly. This segmentation induces a need for trade:

[13] See Winter (1991a) for a survey on liability insurance market crises. Gron (1994b,a) test the effects of low net-worth on insurance premia and volumes. Duffie (2010) and Saunders and Cornett (2010) describe similar behavior in catastrophe insurance markets.

k-producers have access to the investment technology but lack the input to operate it. C-producers produce consumption goods but lack access to an investment technology that allows them to accumulate capital. Due to informational frictions, bankers provide intermediation. Randomizing across activities is introduced for tractability since, otherwise, the relative wealth of each group of producer would become a state variable.

Producers have log preferences over consumption streams and evaluate these according to an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}\beta^t \log\left(c_t\right)\right]$$

where c_t is their consumption and β their discount factor.

Bankers. Bankers are identified by some $j \in [0,1]$. Bankers own a legal institutions called banks. At the beginning of every period, they receive a large exogenous endowment of consumption goods $\bar{e}_t(j)$. In addition, they carry a stock of consumption goods $n_{t,1}(j)$ interpreted as the bank's net worth. During the first stage, bankers can alter the composition of their financial wealth by injecting equity from their personal endowment to their banks or do the opposite by paying dividends. After equity injections and dividends, a bank's net worth evolves from $n_{t,1}(j)$ to $n_{t,2}(j)$.

Bankers participate in capital markets by purchasing capital units from k-producers in the first stage and reselling them to c-producers during the second stage. They purchase capital by issuing tradeable riskless IOUs that entitle the holder to a unit of consumption in the second stage.[14] I implicitly assume that because managerial incentives cannot be met, bankers never hold on to capital to sell it in later periods.[15]

Since ϕ_t is realized between stages, the value of the pool of purchased capital is random. This randomness makes financial intermediation risky. In particular, a banker experiences losses if his purchase cost (issued IOUs) exceeds the value of his purchased capital. When the banker experiences financial losses, he is forced to draw funds from his bank's equity in order to settle these claims.

In principle, financial losses could be financed with the banker's personal endowment. Instead, this does not happen here because financial intermediation is subject to a *limited-liability constraint* (LLC). With LLC, the banker's personal endowment is not liable to his bank's losses. This implies that losses from financial intermediation cannot exceed their bank's net worth under any contingency.[16] As a consequence, the bank's net worth will affect the banker's capacity to intermediate because net worth acts as a cushion to absorb potential losses. For this reason, there is a distinction between a banker's personal endowment and his bank's equity: net worth relaxes the LLC constraint whereas the personal endowment does not.[17]

Upon exit, bankers sell their bank to an entrant banker. There are dividend taxes which ensure that entrant bankers will rather buy a bank from an exiting banker than start a new one. This exit probability, ρ, is constant throughout. When, $\rho = 0$, bankers are characterized by an infinite-horizon problem. When $\rho = 1$,

[14] In a later section I explain how this institutional environment can be reinterpreted to resemble commercial bank practices. Introducing banks as dealers is done for pedagogical reasons.

[15] However, the model can be extended in this direction. The outcome involves a non-trivial fire-sale behavior for financial firms which in turn may have implications for the behavior of the financial system. Fire-sales may induce a different type of externality than the one caused by adverse selection which is the focus of this paper. See Brunnermeier and Sannikov (2011) or Diamond and Rajan (2011) for example.

[16] Other authors call this a *solvency* or *non-default* constraint.

[17] In He and Krishnamurthy (2009) or Brunnermeier and Sannikov (2011)), this LLC is obtained endogenously when bankers lack commitment. If bankers cannot be forced to inject equity into their banks to cover losses, this constraint shows up as an ex-post incentive-compatibility condition. Otherwise, this constraint can be obtained from a high penalty for bankruptcy or defaulting on IOUs resulting from reputation concerns. Finally, as shown later, one can take the LLC as a legal constraint (i.e., leverage constraints).

one can solve the model analytically. Bankers have linear preferences over consumption streams and evaluate these according to an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}(\beta^f)^t c_t\right]$$

where c_t is their consumption and β^f the banker's time discount factor.

Technology. The investment technology transforms one unit of consumption into an efficiency unit of capital available for production the following period. A c-producer that holds a capital stock of $k_t(z)$, produces consumption goods according to a linear technology $A_t k_t(z)$. In addition, bankers have access to a storage technology that transforms one unit of consumption into R^b units of consumption from the second stage to the following period. In principle, one can think of this technology as a risk-less government bond financed through lump-sum taxes but I leave the interpretation open.

Capital Units. At the beginning of every period, capital held by each producer is divisible into a continuum of units. Each unit is identified by a quality $\omega \in [0,1]$. There is a fixed increasing differentiable function $\lambda(\omega) : [0,1] \to \mathbb{R}_+$ that determines the efficiency units that will evolve from each quality. $\lambda(\omega)$ can also be interpreted as a quality-dependent gross depreciation (ones minus the depreciation rate).

In addition, each quality is affected by a multiplicative shock $f_{\phi_t}(\omega)$ determined by the realization of ϕ_t. During the first stage of t, each piece evolves into homogeneous *t+1* units scaling that piece by $\lambda(\omega) f_{\phi_t}(\omega)$. I assume that for any ϕ, f_ϕ is an absolutely continuous function of ω. Thus, by the end of the second stage, the *t+1* capital stock that remains from an original t-period stock k is $k\int \lambda(\omega) f_{\phi_t}(\omega)d\omega$. Therefore, f_ϕ is also a $\phi-$dependent measure across qualities. Given ϕ, the average quality under a certain cut-off ω^* is $\mathbb{E}_\phi[\lambda(\omega)|\omega \leq \omega^*] \equiv \int_0^{\omega^*} \lambda(\omega) f_\phi(\omega) d\omega$. I also use $\bar{\lambda}(X) \equiv \mathbb{E}_\phi[\lambda(\omega)]$ to denote unconditional average quality when ϕ is part of the aggregate state X (soon to be described). Thus, $\lambda(\omega)$ gives an ordinality across qualities but the eventual cardinality depends on ϕ_t.[18]

To simplify the analysis, I assume that the capital stock of every producer's is divided and depreciated in the same way: The $\omega-$qualities of every entrepreneur are hit by the same $f_{\phi_t}(\omega)-$shock. However, this process evolves through time depending on ϕ_t. Once capital units are scaled by their corresponding depreciation shocks, pieces are become homogeneous. Scaled pieces from one producer can be added to pieces that belonged to others to form a new homogeneous stock of *t+1* capital. This homogeneous capital stock is carried to the following period. In the following period, that stock is divided into its $\omega-$components and transformed in the same way regardless how it was built. The process is repeated indefinitely.

Producers do not necessarily hold on to all of their capital units: they may choose to sell particular units before the realization of ϕ in exchange for consumption goods. These decisions are summarized by an indicator function $\mathbb{I}(\omega) : [0,1] \to \{0,1\}$. $\mathbb{I}(\omega)$ takes a value of 1 when units of quality ω are sold. Because each quality has 0 measure, the restriction to all-or-nothing sales is without loss of generality. When a producer chooses $\mathbb{I}(\omega)$, he transfers $k\int \mathbb{I}(\omega) d\omega$ units of capital to a bank. These units evolve into $k\int \lambda(\omega)\mathbb{I}(\omega) f_{\phi_t}(\omega) d\omega$ units of *t+1* capital after the realization of ϕ_t. Simple accounting shows that the efficiency units that remain with the producer are $k\int \lambda(\omega)[1-\mathbb{I}(\omega)] f_{\phi_t}(\omega)d\omega$. Taking into consideration investments and purchases, a

[18] Distinguishing between ($\lambda(\omega)$ and $f_\phi(\omega)$) is useful for the interpretation. Two pairs of functions (λ,f_ϕ) yield the same product function.

producer's capital stock evolves according to:

$$k' = i + k^{b} + k \int_0^1 \lambda(\omega)\left[1 - \mathbb{I}(\omega)\right] f_{\phi_t}(\omega) d\omega. \tag{1}$$

In this expression, i is the capital created by investing (when available) and k^b are purchases of *t+1* capital from banks. I impose some structure on the set of $\{f_\phi\}$ to provide an order relation for these shocks:

Assumption 1 *The set $\{f_\phi\}_{\phi\in\Phi}$ satisfies that $\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega < \omega^*\right]$ is weakly decreasing in ϕ for any ω^*.*

The condition above states that the average quality under some cut-off ω^* gets worse with larger ϕ.[19] In equilibrium, bankers will purchase qualities under an endogenously determined threshold. Assumption 1 implies that bankers will be worse off with a larger ϕ. By construction, $\bar{\lambda}(X)$ is the depreciation of the economy's capital stock so, at the aggregate level, the depreciation rate of the entire economy becomes greater as ϕ increases.

This environment is general enough to accommodate two polar cases of interest. The first is when $\lambda(\omega)$ and $f_\phi(\omega)$ are constant across ω but $f_\phi(\omega)$ is smaller for larger values of ϕ. In this case, ϕ becomes an aggregate depreciation shock that affects each piece identically. This shock will induce risk into financial intermediation but isolates the effects of asymmetric information.[20]

The second polar case is when the unconditional depreciation, $\bar{\lambda}(X)$, is constant for any ϕ but where the condition in Assumption 1 is strict for $\omega^* < 1$. Since in this case, ϕ is a mean-preserving shock ($\bar{\lambda}(X)$ is constant), this implies that the production possibility of the economy is unchanged with ϕ. However, since $\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega < \omega^*\right]$ is decreasing in ϕ, this also corresponds to an environment with ex-ante adverse selection. This feature has the connotation that if ϕ affects outcomes, it is because it affects the *equilibrium* allocation but not the *feasible set* of allocations.

Information. There are two endogenous aggregate states: $K_t = \int k_t(z)\,dz$ and $N_{t,s} = \int n_{t,s}(j)\,dj$. These correspond to the aggregate capital stock and the net worth of the financial sector respectively. It will be shown that in order to characterize equilibria, it is only necessary to keep track of their ratio $\kappa_{t,s} \equiv N_{t,s}/K_t$ as a unique endogenous state variable. Throughout the paper, $\kappa_{t,s}$ is interpreted as the financial sector's size. Since its size determines the capacity to bear losses relative to the capital stock, I also refer to it as the *financial risk capacity*. Thus, the aggregate state of this economy is summarized by $X_{t,1} = \{A_t, \phi_{t-1}, \kappa_{t,1}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$ and $X_{t,2} = \{A_t, \phi_t, \kappa_{t,2}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. At every point, $X_{t,s}$ is common knowledge.

The source of asymmetric information stems from the $\omega-$quality behind a unit being only known to its owners. This means that bankers can only observe the volume of capital being purchased, $k\int \mathbb{I}(\omega)\,d\omega$, but cannot discern which ω's are inside that pool. After ϕ is realized, the quality of the pool is given by $\int \lambda(\omega)\,\mathbb{I}(\omega)\,f_{\phi_t}(\omega)d\omega$. Therefore, any agent buying capital faces two sources of uncertainty: not only is ϕ unknown at the time of the purchase but also the $\omega-$composition. In contrast, producers selling capital face uncertainty only about ϕ_t because they know exactly what $\omega's$ are being sold. Thus, conditioning on ϕ_t, they can compute the average quality of a pool, $\int \lambda(\omega)\,\mathbb{I}(\omega)\,f_{\phi_t}(\omega)d\omega$ sold. In equilibrium, bankers will only infer

[19] The assumption applies a first-order-dominance condition to conditional expectations. However, Assumption 1 is neither a general nor a particular case of first- or second-order stochastic dominance. The standard definitions of stochastic dominance are related to properties of the distribution of qualities, f_ϕ. Instead, here, the condition refers to the properties of functions that are the conditional expectation under a threshold quality (where the threshold is the argument of the function). One can construct counterexamples in either direction.

[20] For this case, this shock has similar effects to the financial shocks studied by Brunnermeier and Sannikov (2011) or Gertler and Karadi (2011).

what units are part of that pool. At the beginning of the second stage, the quality of that pool is common knowledge. For tractability, I assume that the producer's type is known. This assumption ensures that, in equilibrium, c-producers are excluded from selling capital. Bankers are informed if they exit the industry at the beginning of the second stage.

Markets. Markets are incomplete. There is no insurance against type-risk and k-producers cannot sell claims against the production of *t+1* capital. Instead, the only possible transactions are purchases and sales of capital from or to bankers. This assumption is not modeled formally, however, it is implicitly assumed that bankers can engage in many more transactions than producers. This scale differences allows only bankers to exploit the law of large numbers. With this, bankers can dilute the idiosyncratic risk faced by producers if they choose to buy capital directly (under asymmetric information). Implicitly, I also assume banks are fully diversified across financial contracts (or mutually insure against idiosyncratic risk). Consequently, profits or losses from financial intermediation are perfectly correlated across banks.

There are two markets for capital. The first is the one for capital units sold by k-producers and bought by banks under asymmetric information. This market opens during the first stage and satisfies the following assumption:

Assumption 2 *Banks are* ***competitive.*** *Capital markets are* ***anonymous*** *and* ***non-exclusive.***

Without anonymity, bankers could pay a different prices depending on the volume of capital sold. With exclusivity, bankers could use dynamic incentives to screen. Assumption 2 therefore implies that the market in the first stage is a pooling market. Hence, there is a unique pooling price p_t during the first-stage capital market.[21] I refer to this market as the pooling market.

The second market opens during the second stage. In this market, bankers sell back the units purchased during the first stage. At this stage, the efficiency units behind a capital are known so, effectively, this is a market for *t+1* homogeneous capital units. This market clears at a price q_t. I refer to it as the resale market.

Timing. (1) At the beginning of the period, $X_{t,1}$ is realized and k-producers choose $\mathbb{I}(\omega)$. Bankers choose the amount of capital units purchased, equity injections, and dividend pay outs. (2) During the second stage, ϕ_t is realized and $X_{t,2}$ is updated correspondingly. Bankers learn the average quality of the pool of purchased capital and resell the merged pieces as units of homogeneous *t+1* capital. K-producers and c-producers simultaneously choose over consumption and purchases of capital, and k-producers also decide on the scale of investment. By the end of the period, bankers settle all claims against all issued IOUs and realize profits.

The timing of the model is summarized by Figure 2. The following sections describes the problem faced by the agents in this economy and the corresponding market-clearing conditions that define equilibria. This economy has a recursive representation so from now on I drop time subscripts. I use x' to denote the value of a variable x in the subsequent stage. I denote by n bank equity brought from the previous period. n' is bank equity after equity injections and dividends. n'' is bank equity taken to the following stage.

2.2 First-Stage Problems

K-producer's first stage. During the first stage, a *k-producer* enters the period with a capital stock k. At this stage, his only choice is which qualities to sell:

[21] In a similar problem, Guerrieri and Shimer (2011) allow agents to trade in multiple markets for which capital is exchanged with some probability. Differences in probability and prices allow for separation.

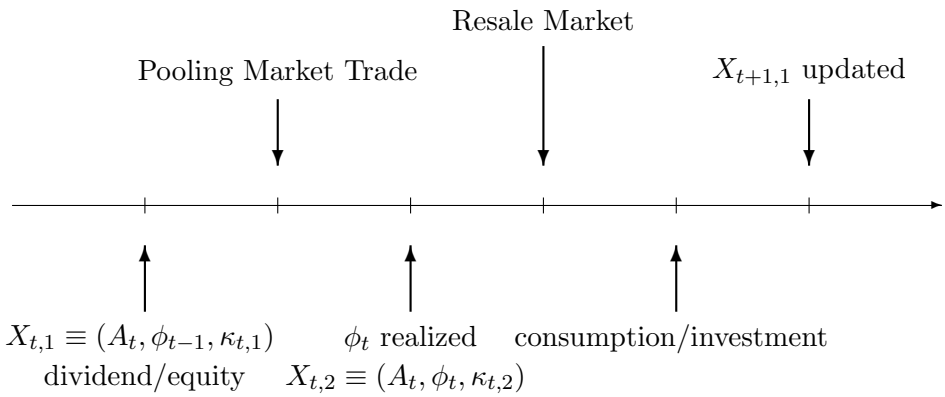

Figure 2: **Timing**

Problem 1 (*k-producer's s=1 problem*) *The k-producer's first stage problem is:*

$$V_1^k(k, X) = \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[V_2^k\left(k'\left(\phi'\right), x, X'\right)|X\right]$$

$$\textit{subject to } x = pk\int_0^1 \mathbb{I}(\omega)\, d\omega \textit{ and } k'\left(\phi'\right) = k\int \lambda(\omega)\left[1-\mathbb{I}(\omega)\right] f_{\phi'}(\omega)\, d\omega$$

The first equation is the k-producer's budget constraint. x is the quantity of consumption goods available during the second stage. This is obtained by selling $k\int_0^1 \mathbb{I}(\omega)\, d\omega$ units of capital at a pooling price p. The second equation accounts for the following period's capital stock that remains with him. The solution to this problem determines a supply schedule for capital units in the pooling market.

C-producer's first stage. Since c-producers are excluded from the pooling market, they take no actions in this stage. Their value function is the expected value of their second stage's value function:

Problem 2 (*c-producer's s=1 problem*) *The c-producer's first stage value function:*

$$V_1^c(k, X) = \mathbb{E}\left[V_2^c\left(k'\left(\phi'\right), x, X'\right)|X\right]$$

$$\textit{where } x = Ak \textit{ and } k'\left(\phi'\right) = k\int \lambda(\omega) f_{\phi'}(\omega)\, d\omega$$

Banker's first stage. A banker enters the period with n consumption goods stored in his bank and $\bar{e}$ as a personal endowment. A banker chooses an amount from his endowment e as equity injections to his bank at the expense of decreasing consumption. He can reduce his bank's net worth by transferring d consumption units as dividends to be consumed after dividend taxes τ. Equity injections and dividends are limited by their sources: $e \in [0, \bar{e}]$ and $d \in [0, n]$. In this paper I focus on equilibria in which $e \leq \bar{e}$ never binds with the interpretation that there are always resources available to recapitalize banks. The banker's consumption is $c = (\bar{e} - e) + (1-\tau)\, d$. His bank's net worth is instantaneously transformed according to $n' = n + e - d$. The presence the dividend tax is introduced to obtain inaction regions for the banker's financial policy.[22]

[22] A distinction between costly equity injections and dividend taxes is common in the dynamic corporate finance literature. See for example Hennessy and Whited (2005) or Palazzo (2010) among others. In this environment, only theratio of the cost of equity and dividend taxes matters, so I normalize the tax rate to account for this differences.

Let Q be the volume of capital units purchased by a banker in the pooling market. He funds this purchases by issuing IOUs at face value pQ. These IOUs bear no interest and are redeemed by the second stage. During the second stage, the value of the capital pool purchased is $q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right]Q$. The resale market trades following-period capital at a resale price q. The Q units are purchased in current period, so these units are scaled by their average depreciation, $\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right]$, to count them as following period capital. $\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right]$ results from ϕ and the $\mathbb{I}\left(\omega\right)$-policies that depend on X.

The LLC states that the amount of issued IOUs cannot exceed the bank's net worth plus the value of this capital under any realization of ϕ':

$$pQ \leq q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right]Q + n' \text{ for any } \left(X, X'\right) \in \mathbb{X} \times \mathbb{X}$$

Let $\Pi\left(X, X'\right) \equiv q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right] - p$ be the banker's marginal profit from intermediation. $\Pi\left(X, X'\right)$ is a function of X and X' since qualities sold depend on X but their depreciation depends on X' through ϕ'. The banker's problem is,

Problem 3 *The banker's first stage problem is:*

$$V_1^f\left(n, X\right) = \max_{Q, e\in[0,\bar{e}], d\in[0,n]} c + \mathbb{E}\left[V_2^f(n' + \Pi\left(X, X'\right)Q, X')|X\right]$$

$$\begin{aligned} \textit{subject to } -\Pi\left(X, X'\right)Q &\leq n', \ \forall X' \\ c &= \left(\bar{e} - e\right) + \left(1 - \tau\right)d \\ n' &= n + e - d \end{aligned} \tag{2}$$

The first constraint is the LLC. The second and third constraints are the banker's budget constraints and the law of motion of his net worth.

2.3 Second-Stage Problems

K-producer's second stage. During the first stage, k-producers have sold part of their capital stocks in exchange for x consumption goods brought into the current. They also bring the k capital units they did not sell. Given their idiosyncratic and the aggregate states, they solve,

Problem 4 (*i-entrepreneur's s=2 problem*) *The k-producer's problem in the second stage is:*

$$V_2^k\left(k, x, X\right) = \max_{c\geq 0, i, k^b \geq 0} \log\left(c\right) + \beta\mathbb{E}\left[V_1^k\left(k', X'\right)|X\right], \ j \in \{i, p\}$$

$$\textit{subject to } c + i + qk^b = x \textit{ and } k' = k^b + i + k$$

This budget constraint says that the k-producer uses x to consume c, invest i, or purchase k^b capital at price q. The capital accumulation equation is consistent with equation (1) since k already incorporates sales and depreciation (accounted in the previous stage).

C-producer's second stage. The c-producer's problem is identical to the k-producer's except that he is restricted to set $i \leq 0$ because he lacks the investment technology:[23]

[23] Investment reversibility is introduced for tractability.

Problem 5 (*p-entrepreneurs s=2 problem*) *The c-producer's problem at the second stage is:*

$$V_2^c(k,x,X) = \max_{c\geq 0, i\leq 0, k^b\geq 0} \log(c) + \beta\mathbb{E}\left[V_1^j\left(k',X'\right)|X\right],\ j\in\{i,p\}$$

$$\textit{subject to } c+i+qk^b = x \textit{ and } k' = k^b+i+k$$

Banker's second stage. A banker's only action during the second stage consists on reselling all the capital units purchased during the first stage after accounting for their depreciation. Thus, their value is $V_2^f(n,X) = \beta^F\mathbb{E}\left[V_1^f(R^b n, X')|X\right]$ if they remain in the industry or $V_2^f = (1-\tau)\beta^F R^b n$ if they exit.

2.4 Market-Clearing Conditions and Equilibrium

Notation. I append terms like ${}^j(k,X)$ to variables that indicate the policy function of a producer of type j in state (k,X). I use $\mathbb{I}(\omega,k,X)$ to refer to a k-producer's decision to sell an $\omega-$quality when his state is (k,X) .

Aggregation. In every period and stage, there are measures over capital holdings across the population of k and c-producers. I denote these measures by Γ^k and Γ^c respectively. By independence of the producer's activities, these satisfy:

$$\int_0^\infty \Gamma^k(dk) = \pi K \quad \text{and} \int_0^\infty \Gamma^c(dk) = (1-\pi)K. \tag{3}$$

Their evolution is consistent with individual decisions and the activity segmentation process. In addition, there is also a measure Λ for the bankers net worth.

First stage. Market clearing during the first stage requires that the demand for capital by bankers be equal to the supply of capital by k-producers. This condition is given by:

$$\int_0^\infty Q(n,X)\Lambda(dn) = \int_0^\infty k\int_0^1 \mathbb{I}(\omega,k,X)\,d\omega\Gamma^k(dk)$$

Second stage. The demands for following-period capital by c- and k-producers are respectively:

$$D^c\left(X,X'\right) \equiv \int_0^\infty k^{b,c}\left(x(k,X), k\int_0^1 \lambda(\omega) f_{\phi'}(\omega)\,d\omega, X'\right)\Gamma^c(dk)$$

and

$$D^k\left(X,X'\right) \equiv \int_0^\infty k^{b,k}\left(x(k,X), k\int_0^1 \lambda(\omega)\left[1-\mathbb{I}(\omega,k,X)\right] f_{\phi'}(\omega)\,d\omega, X'\right)\Gamma^k(dk).$$

Bankers sell all the units bought so the supply of capital in the second stage is:

$$S\left(X,X'\right) \equiv \mathbb{E}_{\phi'}\left[\lambda(\omega)|X\right]\int_0^\infty Q(n,X)\Lambda(dn).$$

The second stage market clearing condition is $S(X,X') = D^c(X,X') + D^k(X,X')$. A recursive competitive equilibrium is:

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium (RCE) is (1) a set of price functions, $\{q(X,X'), p(X)\}$, (2) a set of policy functions for c-producers $c^c(x,k,X)$, $k^{b,c}(x,k,X)$, $i^c(x,k,X)$, a set of policy functions for k-producers $c^k(x,k,X)$, $k^{b,k}(x,k,X)$, $\mathbb{I}^k(\omega,k,X)$, a set of policy functions for bankers $Q(n,X)$, $e(n,X)$, $d(n,X)$, (3) sets of value functions, $\{V_1^l(k,X), V_2^l(x,k,X)\}_{l=c,k}$*

,$\left\{V_s^f(n,X)\right\}_{s=1,2}$, and (4), a law of motion for the aggregate state X, such that for any measures Γ^c, Γ^k and Λ satisfying the consistency condition (3) the following hold: (I) The producers' policy functions are solutions to their problems taking $q(X,X')$, $p(X)$ and the law of motion for X as given. (II) $Q(n,X)$, $e(n.X)$, $d(n.X)$ are solutions to the Bankers problem taking $q(X,X')$, $p(X)$ and the law of motion for X as given. (III) Capital markets clear during the first and second stages. (IV) The law of motion X is consistent with policy functions and the transition function χ. All expectations are consistent with the law of motion for X and agent policies.

The definition of equilibrium does not depend on the measures of asset holdings because this economy admits aggregation. This is shown in the following section. However, there is an important detail. Nothing precludes multiplicity. This occurs because for a given X during the first stage, there could be two or more equilibrium triplets $(\omega, Q, p(X), q(X,X'))$ that satisfy the conditions for equilibrium. The source of multiplicity is that as prices increase, both the average quality of capital sold and the quantity increase. In consequence, bank profits are possibly non-monotone. This leads to the possibility of finding the same worst-case-scenario profits for two different equilibrium prices. Multiplicity is a common feature in static models of asymmetric information. Although multiplicity is an interesting phenomenon in itself, it is not the focus of this paper. Thus, for the rest of the paper, I introduce an equilibrium refinement:

Definition 2 (Pareto Un-improvable Equilibrium) *A RCE is Pareto un-improvable if given the law of motion for X, $\forall X$, there does not exist any $p^o > p(X)$, such that p^o satisfies market clearing in the first stage, and induces a second stage market clearing price $\tilde{q}(p,X')$ that is consistent with the producer's policy functions and the LLC.*

This refinement selects the RCE where the volume of intermediation is greatest. A later section shows that such equilibria, indeed, cannot be Pareto improved upon. We need to show some intermediate results first. Before proceeding to the characterization, I provide a description of alternative interpretations of the LLC constraint and financial intermediation.

3 Discussion

3.1 Accounting and Financial Constraints

Bank balance sheets. During the first stage, bankers have net worth n held in their banks. At the beginning of the stage, they alter their bank's balance sheet by injecting equity or paying dividends. In addition, the bank increases liabilities by issuing pQ in IOUs and accumulates assets buy purchasing pools of capital. From the beginning to the end of the first stage, their banks' balance sheets evolves in the following way:

Assets	**Liability**
n	
	Net worth
	n

Beginning-of-period Balance Sheet

$\Rightarrow$

Assets	**Liability**
$n+e-d$	pQ
pQ	**Net worth**
	$n+e-d$

End-of-stage-1 Balance Sheet

$=$

Assets	**Liability**
n'	pQ
pQ	**Net worth**
	n'

End-of-stage-1 Balance Sheet

.

After ϕ is realized, the value of the assets in the balance sheet adjusts as both, the quality and the price of capital respond to ϕ. Thus, the *mark-to-market* value of capital may differ from the amount of IOUs issued.

The issuance of IOUs is a form of inside liquidity and are effectively deposit contracts. Accounting for these changes in values, the following balance sheet changes to:

Assets	**Liability**
n'	pQ
$q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\left\|X\right]Q$	**Net worth**
	$n'+\Pi Q$

End of stage 2 Balance Sheet

In this balance sheet, net worth is adjusted by gains or losses from financial intermediation $\Pi(X,X')Q = \left(q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\left|X\right]-p\right)Q$. The evolution of these balance sheets is similar to the description of commercial banks found in any banking text book.

3.2 Commercial Banking Interpretation

The institutional environment in this model can interpreted as investment banking, since banks take positions on the capital they pool. However, the model can be reinterpreted as a model of commercial banking by introducing institutional features that render the same allocations. I omitted this representation from the description of the environment to simplify the exposition. However, it is worth discussing this point. So far, liability side resembles commercial bank deposit contracts. The operations on the asset not because commercial banks have loans, not capital units. However, variables on the asset side can be reinterpreted as *ad-hoc* collateralized loan contracts instead of capital purchases. For that purpose, assume Q represents units of capital used as a collateral that guarantees a loan, and p the corresponding amount lent per unit of collateral. A collateralized loan contract also specifies an amount R^L per unit of collateral in return for the loan. This return is settled during the second stage. Upon the borrower's failure to repay, collateral is seized and sold to c-producers. One obtains the same equilibrium allocations as in the environment with sales if the return is set to $R^L\left(X\right)=\max_{\phi'} q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\left|X\right]\right.$. The reason is if this is the repayment scheme, in states in which $\phi=\phi_1$, k-producers are indifferent between repaying loans and defaulting. In non-default states, can obtain funds to repay $R^L\omega$ by selling the capital that was used as collateral. With these sales, they obtain the face value of debt needed to settle their loans. In contrast, defaults occur if $\phi>\phi_1$ because the producer would rather loose his collateral than repay the face value of debt. In that case, banks seize the capital units and sell them for $q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\left|X\right]\right.$. For any value of ϕ, the flow of resources in this economy are the same as in the environment with purchases and sales.

Unless additional structure is introduced, such an implementation is ad-hoc. These type of contract and the terms used are chosen to obtain the same allocations as with sales of capital. Collateralized debt contracts appear in versions of the costly state verification problem of Townsend (1979) may be introduced to obtain collateralized loans as an equilibrium outcome if there are other idiosyncratic components to the model.[24]

[24] Also, DeMarzo and Duffie (1999) show that optimal collateralized debt contracts appear when the return to assets that back security issuances have a private information component. Lacker (2001) finds a similar result when borrowers value their collateral more than lenders.

3.3 Interpreting LLC as Regulatory Constraints

LLC as a leverage constraint. The LLC can be expressed as a constraint on leverage. Arranging terms in equation (2) leads to:

$$\overbrace{\underbrace{\frac{Q}{n'}}_{\text{Net worth}}}^{\text{Traded Capital}} \leq \underbrace{\frac{1}{-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right]}}_{\text{Marginal Leverage Constraint}}, \ \forall X'.$$

The term on the right is the marginal leverage: When the LLC binds, an additional unit of net worth allows the producer to increase Q in that amount.[25] Multiplying by $p(X)$ and adding 1 to both sides leads to an equivalent constraint expressed in terms of the bank's leverage:

$$\underbrace{\underbrace{\frac{\overbrace{p(X)Q+n'}^{\text{Assets}}}{n'}}_{\text{Net worth}}}_{\textit{Leverage}} \leq \underbrace{\frac{p(X)}{-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right]}+1}_{\text{Total Leverage Constraint}}, \ \forall X'.$$

The Basel Accords impose constraints on bank leverage.[26] I return to this point later on when I study the effects of capital requirements.

LLC as Value at Risk (VaR). The LLC can be manipulated to obtain a constraint in terms of the Value at Risk of bank net worth. The LLC constraint states that $n'+\Pi(X',X)Q=n''\geq 0$ for any possible realization of ϕ'. Given X, the worst outcome is realized with some probability η that depends on ϕ. Thus, the LLC states that the value of net worth at its $\eta-$percentile must be greater than 0. Letting $VaR(b,\eta)$ denote the value of a random variable b at its $\eta-$percentile, the LLC can be expressed as $VaR\left(n'',\eta\right)\geq 0$. The Basel Accords require banks to satisfy constraints in terms of the Value at Risk of their portfolios. Thus, the LLC can be adapted to capture these regulatory constraints.[27]

4 Characterization

4.1 Policy Functions

Produces' second-stage policies. I begin the characterization of equilibria by describing the policy functions of c-producers. As a result of log-preferences, the c-producer's policy functions are linear functions of his virtual wealth $W^c\left(k,x,X\right)\equiv\left(A+q\bar{\lambda}\left(X\right)\right)k$. His virtual wealth is sum of his produced consumption goods and the market value of his capital stock.

Proposition 1 *In any RCE, the c-producer's policy functions are* $k^{c,\prime}\left(k,x,X\right)=\beta\frac{W^c(k,x,X)}{q}$ *and* $c^c\left(k,x,X\right)=\left(1-\beta\right)W^c\left(k,x,X\right)$ *and his value function is of the form* $V_2^c\left(k,x,X\right)=\psi^c\left(X\right)+\log\left(W^c\left(k,x,X\right)\right)$ *where* $\psi^c\left(X\right)$ *is a function of the aggregate state.*

Policy functions for k-producers are also linear in their corresponding virtual wealth, $W^k\left(k,x,X\right)\equiv(x+q^i\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*(X)])k$. The k-producer's virtual wealth per unit of capital is the sum of the consumption

[25] Note that when $-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right]<0$, the constraint is trivially satisfied.
[26] See *Basel Committee on Banking Supervision* (2010).
[27] See *Basel Committee on Banking Supervision* (2010).

goods obtained by selling capital units, x, and the replacement value of his remaining units. The average depreciation of the units he keep is $\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*(X)]$. This is the average depreciation under that of a cut-off quality kept $\omega^*(X)$. The following section shows that, indeed, his capital sales policies are characterized by a threshold quality. The replacement cost of their capital is $q^i = \min\{1, q\}$. q^i takes this form because when $q > 1$, k-producers will produce capital at a unit cost. If $q^i \leq 1$, the k-producers may purchase capital from banks instead of investing. Hence, q^i is the lowest cost of accumulating capital for them. Their policy functions are given by:

Proposition 2 *In any RCE, the k-producer's policy functions are* $k^{k,\prime}(k, x, X) = \beta \frac{W^k(k,x,X)}{q^i}$ *and* $c^k(k, x, X) = (1-\beta) W^k(k, x, X)$ *and his value function is of the form* $V_2^k(k, x, X) = \psi^k(X) + \log\left(W^k(k, x, X)\right)$ *where* $\psi^k(X)$ *is a function of the aggregate state.*

Producers' first-stage policies. One can use Proposition 2 to obtain a closed-form expression for the k-producer's value function during the first stage. Replacing the definitions of x and $\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*(X)]$), their value function is:

$$\begin{aligned} V_1^k(k, X) &= \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\log\left(p(X)\int_0^1 \mathbb{I}(\omega)\, d\omega \ + q^i(X, X')\int \lambda(\omega)\left[1-\mathbb{I}(\omega)\right] f_{\phi'}(\omega)\, d\omega\right)|X\right] \\ &\quad + \psi^k(X) + \log(k). \end{aligned}$$

Through this expression, it is clear that quality-sales decisions have to be the same across entrepreneurs regardless of the size of their capital stock. These decisions are characterized by a portfolio problem:

Proposition 3 *In any RCE, the k-producer's policy function in the first stage is given by,* $\mathbb{I}^*(\omega, k, X) = 1$ *if* $\omega < \omega^*$ *and 0 otherwise. The cut-off quality is given by:*

$$\omega^* = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left(p\tilde{\omega} \ + q^i(X, X')\int_{\tilde{\omega}}^1 \lambda(\omega) f_{\phi'}(\omega)\, d\omega\right)|X\right]. \tag{4}$$

Moreover, ω^* *is increasing in* p.

Proposition 3 shows that the solution to the producer's problem during the first stage is characterized by a unique cut-off quality such that units of inferior quality are sold. This outcome resembles the solution to lemons problem of Akerlof (1970), but there is a distinction. In the original lemons problem, the cut off is the quality for which the seller is indifferent between selling or keeping the asset. Here, ω^* is chosen by solving a portfolio problem because the producer does not know the outcome of ϕ when selling capital. This portfolio problem has an intuitive interpretation: ω^* is the fraction of the producer's capital stock sold to banks. Once he exchanges these units, the producer loads the depreciation risk to the bank. In doing so, ω^* becomes the risk-less portion of his portfolio. The remaining fraction, $(1-\omega^*)$, is risky because ϕ' is realized after ω^* is chosen. Since, ω^* is increasing in $p(X)$, the supply of capital has a typical upward sloping form.

Remark 1 $\omega^*(X)$ *indicates the highest quality of capital traded and the volume of intermediation.*

Thus, from now on, I use threshold quality and volume of intermediation to refer to ω^* interchangeably.

Bankers' policies. At the beginning of every period, bankers choose e, d and Q to maximize expected profits. The following Proposition shows that their value function and policies are linear in n:

Proposition 4 *The banker's value functions is a linear function of n: $V_1^f(n,X) = v_1^f(X)\,n$ and $V_2^f(n,X) = v_2^f(X)\,n$ where $v_1^f(X)$ and $v_2^f(X)$ are the marginal value of financial equity in stages 1 and 2 respectively. $v_1^f(X)$ solves the following Bellman equation:*

$$v_1^f(X) = \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\,d - e + \mathbb{E}\left[v_2^f\left(X'\right)\left(\Pi\left(X,X'\right)Q + n'\right)|X\right] \tag{5}$$

subject to,

$$\begin{aligned} -\Pi\left(X,X'\right)Q &\leq n', \forall X' \\ n' &= 1+e-d. \end{aligned}$$

The Bellman equation (5) is solved by linear optimal policies, $e(X)$, $d(X)$ and $Q(X)$ given by:

$$e(n,X) = e^*(X)\,n,\ \ d(n,X) = d^*(X)\,n,\ \ Q(n,X) = Q^*(X)\left(1+e^*(X)-d^*(X)\right)n.$$

In addition, $v_2^f(X) = \beta^F R^b$ if the banker exits and $v_2^f(X) = \beta^F \mathbb{E}\left[v_1^f(X)\,R^b\right]$ otherwise.

The value function of the banker is linear in his net worth because of risk-neutrality and the linearity of the LLC. In equilibrium, there may be multiple solutions to $d(n,X)$ and $e(n,X)$ but, without loss of generality, I restrict the attention to linear policies. The following proposition describes them:

Proposition 5 *$Q^*(X)$ is given by,*

$$Q^*(X) = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)|X\right]\tilde{Q} \text{ subject to } \Pi\left(X,X'\right)\tilde{Q} \leq 1, \forall X'.$$

In equilibrium, $min_{\tilde{X}'}\Pi\left(X,\tilde{X}'\right) < 0$, and $\left(e^*(X), d^*(X)\right)$, satisfy:

$$\begin{aligned} e^*(X) &> 0 \text{ only if } \beta^F\left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)}, 0\right\}\right] \geq 1 \\ d^*(X) &> 0 \text{ only if } \beta^F\left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)}, 0\right\}\right] \leq (1-\tau). \end{aligned}$$

When the inequalities are strict, $e = \bar{e}$ and $d = 1$. $e^*(X)$ and $d^*(X)$ are indeterminate at the individual level when the relations hold with equality. $e^*(X)$ and $d^*(X)$ equal 0 when the inequalities are violated.

This proposition states that the LLC is binding whenever $\mathbb{E}\left[v_2^f(X')\,\Pi(X,X')|X\right] > 0$. $v_2^f(X')\,\Pi(X,X')$ is the marginal value of an additional unit of intermediation in state X'. This term is the product of marginal profits and the marginal value of bank equity. $v_2^f(X')$ fluctuates because it summarizes future expected returns to bank equity, which depends on X'. When, $\mathbb{E}\left[v_2^f(X')\,\Pi(X,X')|X\right] = 0$, Q is indeterminate: The banker is indifferent between any volume of intermediation. Q is 0 when this terms is negative.

The proposition also states conditions for capital injections and dividend payoffs. These financial policies

depend on the marginal value of keeping equity in the bank:

$$\tilde{v}\left(X\right) \equiv \beta^{F}\left[\mathbb{E}[v_{2}^{f}(X')] + \max\left\{\frac{\mathbb{E}[v_{2}^{f}(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right]. \tag{6}$$

$\tilde{v}\left(X\right)$ has an intuitive interpretation. β^{F} is the discount factor of future utility. The first term inside the bracket, $\mathbb{E}[v_{2}^{f}(X')]$, is the future marginal value of an additional unit of equity. In a risk-neutral environment such as this one, this term is the bank's stochastic discount factor. The second term represents the shadow value of equity obtained from relaxing the LLC. As explained earlier, the inverse of the worst-case scenario losses, $-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)$, is the bank's marginal leverage. Increasing a unit of equity allows the bank to issue this amount in IOUs to purchase capital. This additional units of capital have an expected marginal value of $\mathbb{E}[v_{2}^{f}(X')\Pi\left(X,X'\right)]$, which is the marginal value of an additional unit of intermediation. The max operator sets this marginal value to 0 when $\mathbb{E}[v_{2}^{f}(X')\Pi\left(X,X'\right)] < 0$.

When $\tilde{v}\left(X\right) < \left(1-\tau\right)$, the banker prefers to pay out dividends: the marginal value of equity is $\tilde{v}\left(X\right)$, but the after-tax marginal benefit of dividends is higher. In contrast, the banker injects equity to his bank when the value of holding equity exceeds one, the opportunity cost of equity in terms of foregone consumption. This result implies that banks have (S,s)-bands for their dividend policies with $\left[\left(1-\tau\right),1\right]$ as the inaction region. The properties of these regions are discussed in more detail in the following section.

The following section shows that in absence of asymmetric information, $\tilde{v}\left(X\right)$ is in fact monotone decreasing in κ. However, if asymmetric information is sufficiently severe, $\tilde{v}\left(X\right)$ is non-monotone. This non-monotonicity leads to multiple inaction regions. In turn, this feature explains why the financial sector is not recapitalized for low levels of $\kappa \equiv N/K$ although recapitalization does not occur at higher levels. This results from $\Pi\left(X,X'\right)$ being non-monotone in κ when asymmetric information is severe. To show this result formally, we need to obtain the market clearing conditions for $p\left(X\right)$ and $q\left(X,X'\right)$, and this is done in the following section.

4.2 Market Prices and Bank Profits

Resale-market price function. The linearity of policy functions allows for aggregation. Integrating across the measure of c-producer's capital stock yields their aggregate demand for following-period capital:

$$D^{c}\left(X,X'\right) = \left[\frac{\beta\left(A+q\left(X,X'\right)\bar{\lambda}\right)}{q\left(X,X'\right)} - \bar{\lambda}\left(X'\right)\right]\left(1-\pi\right)K.$$

The demand for capital units by k-producers is obtained similarly. In their case, their aggregate demand function is broken up into three tranches because q determines whether they buy capital or produce it:[28]

$$D^{k}\left(X,X'\right) = \begin{cases} \beta\frac{p(X)\omega^{*}(X)}{q(X,X')} & \text{if} \quad q\left(X,X'\right) < 1 \\ \left[0,\beta p\left(X\right)\omega^{*}\left(X\right)\pi K\right] & \text{if} \quad q\left(X,X'\right) = 1 \\ 0 & \text{if} \quad q\left(X,X'\right) > 1. \end{cases}$$

[28] The first tranche is downwards sloping when $q < 1$. This is a region for values of q in which k-producers find it cheaper to buy capital than to produce it. The second region is a flat demand at $q = 1$ since k-producers are indifferent between investing or buying capital. Otherwise, k-producers don't participate in the market when $q > 1$ because its less costly to produce capital directly.

Aggregate investment is the difference between the k-producer's desired capital holdings and their demand for units sold by banks:[29]

$$I\left(X,X'\right)=\beta\frac{p\left(X\right)\omega^{*}\left(X\right)}{q\left(X,X'\right)}\pi K-D^{k}(X,X').$$

Capital supplied by bankers during the second stage, $S\left(X,X'\right)$, are the units sold by k-producers during the first stage scaled by their average quality:

$$S\left(X,X'\right)=\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\omega^{*}\left(X\right)\right]\omega^{*}\left(X\right)\pi K.$$

$q\left(X,X'\right)$ is obtained by using these expressions, and clearing out the price from the second-stage market-clearing equation. This price is characterized by:[30]

Proposition 6 *In equilibrium q is given by,*

$$q\left(X,X'\right)=\max\left\{\left[\frac{\beta A}{\pi\omega^{*}\left(X\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\omega^{*}\left(X\right)\right]+\left(1-\pi\right)\left(1-\beta\right)\bar{\lambda}\left(X'\right)}\right],g\left(p\left(X\right),X'\right)\right\}$$

where

$$g\left(p\left(X\right),X'\right)\equiv\min\left\{1,\frac{\beta\left(\pi p\left(X\right)\omega^{*}\left(X\right)+A\left(1-\pi\right)\right)}{\left(\beta\omega^{*}\left(X\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\omega^{*}\left(X\right)\right]\pi+\left(1-\beta\right)\bar{\lambda}\left(X\right)\right)}\right\}.$$

There are two things worth noting about this price function. First, it is immediate to show that it is decreasing in $\omega^{*}\left(X\right)$. This is a natural outcome since larger cut offs lead to more capital supplied by banks (and capital is a normal good). Secondly, the price is decreasing in ϕ. This shock lowers the average quality of capital for any possible cut-off, which is equivalent to lowering the supply of this good.[31]

Pooling-market price function. In equilibrium, market clearing in the first stage requires:

$$Q^{*}\left(X\right)\int_{0}^{\infty}n'\left(n,X\right)d\Lambda\left(n\right)=\omega^{*}\left(X\right)\int_{0}^{\infty}k\Gamma^{k}\left(dk\right)\Longleftrightarrow Q^{*}\left(X\right)\kappa=\omega^{*}\left(X\right).$$

Note that $p(X)$ determines $\omega^{*}\left(X\right)$ through the k-producer's portfolio problem, which is independent of the capital stock. This is the reason why equilibria only depend on κ and not on the relative wealth of either sector. Conversely, any given pooling price p^{*} can be solved implicitly as a function of any ω^{*} through the k-producer's portfolio problem. To obtain the actual price in a given state one must find the largest possible ω^{*} for which, the current κ can sustain losses at such levels of intermediation. Such ω^{*} determines $p(X)$. The following proposition shows that p is well defined and monotone in the size of the financial sector.

Proposition 7 *Given a state X and a law of motion for the state, there exists some $p\left(X\right)$ satisfying market clearing. In addition, in any Pareto un-improvable equilibrium, $p\left(X\right)$ is weakly increasing, and right continuous in the financial risk capacity κ.*

[29] From the expression, it is clear that investment is 0 when $q<1$. When $q=1$, $D^{k}\left(X,X'\right)$ is obtained as the residual of the difference between the supply of used units minus purchases by k-producers.

[30] $q(X,X')$ depends not only on the second stage state X' but also on X. The dependence on X is because the supply of capital is a function of the cut-off function $\omega^{*}\left(X\right)$, which is decided in the prior stage. This price may also depend on $p\left(X\right)$ because this determines the wealth of k-producers which, in turn, purchase capital when $q\left(X,X'\right)<1$.

[31] The response of q to ϕ is also a measure of the inefficiencies that occur in this economy. The social cost of investment is 1. A larger ϕ increases the market price of capital beyond its social cost. The wedge between q and 1 is a measure is a measure of ex-post inefficiency.

This result implies that the volume of intermediation is greater when the financial risk capacity is higher. In turn, κ is determined by the bankers' financial decisions which ultimately depend on expected returns.

Bank profits. Replacing the functional form q obtained in Proposition 6, we find an analytic expression for marginal profits:

$$\Pi\left(X,X'\right)=\begin{cases}\frac{\beta(\pi p(X)\omega^*(X)+A(1-\pi))}{\beta\omega^*(X)\pi+(1-\beta)\frac{\bar{\lambda}(X)}{\mathbb{E}_{\phi'}[\lambda(\omega)|\omega<\omega^*(X)]}}-p\left(\omega^*\left(X\right)\right) & \text{if} \quad q\left(X,X'\right)<1\\ \mathbb{E}_{\phi}\left[\lambda\left(\omega\right)|\omega<\omega^*\left(X\right)\right]\ -p\left(\omega^*\left(X\right)\right) & \text{if} \quad q\left(X,X'\right)=1\\ \frac{\beta A}{\pi\omega^*(X)+(1-\pi)(1-\beta)\frac{\bar{\lambda}(X)}{\mathbb{E}_{\phi'}[\lambda(\omega)|\omega<\omega^*(X)]}}-p\left(\omega^*\left(X\right)\right) & \text{if} \quad q\left(X,X'\right)>1\end{cases}$$

Recall that $p\left(X\right)$ is determined prior to the realization of ϕ' so this price does not respond to ϕ'. One can observe that $\Pi\left(X,X'\right)$ is decreasing in ϕ' because, for any possible $\omega^*\left(X\right)$, $\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\omega^*\left(X\right)\right]$ is decreasing in the shock. Thus, ϕ' affects marginal profits on two dimensions. The first is on the quantity dimension since there is a contraction in the supply of following period capital induced by a greater depreciation. The second is the positive effect in price, $q\left(X,X'\right)$, caused by the reduction in the supply. Marginal profits decrease because the first effect always dominates since capital is a normal good. Thus, larger ϕ' are associated with worse results for the banking industry. In contrast, $\Pi\left(X,X'\right)$ is not monotone in κ. This is the main feature that determines the evolution of κ and, consequently, the dynamics of this economy.

Why are bank returns non-monotone? From Proposition 7 we know that $p\left(X\right)$ is increasing in κ. Hence, non-monotonicity must follow from the value of the capital pool, $q\left(X,X'\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\omega^*\left(X\right)\right]$. The reason for this non-monotonicity is that two effects oppose each other as the volume of intermediation increases. On the one hand, there is a quantity effect: Since capital is a normal good, the greater the volume of capital supplied, the lower its price $q\left(X,X'\right)$. However, the value of the capital pool is also affected its quality. Thus, on the other hand, as the volume $\omega^*\left(X\right)$ increases with κ, so does its average quality. The two effects interact in a way that marginal profits are non-monotonic if $\lambda\left(\omega\right)$ is sufficiently sensitive to ω. The non-monotonicity of profits in κ makes $\tilde{v}\left(X\right)$ non-monotone alsoe. This will ultimately is the critical factor that prevents the recapitalization of the financial system in times of low κ.

As explained earlier, $q\left(X,X'\right)$ is decreasing in κ regardless of f_ϕ. This feature implies that if $\lambda\left(\omega\right)$ is constant across qualities, that is, if asymmetric information is not present, $\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega>\omega^*\left(X\right)\right]$ is constant. Hence, marginal profits are decreasing in κ. This discussion proves the following proposition.

Proposition 8 *Without asymmetric information, marginal profits $\Pi\left(X,X'\right)$ are decreasing in the financial risk capacity κ. For sufficiently severe asymmetric information $\Pi\left(X,X'\right)$, is non-monotone.*

4.3 Evolution of Financial Risk Capacity

Equity injections and dividends. In equilibrium, $\tilde{v}\left(X\right)\in\left[\left(1-\tau\right),1\right]$ for any X. If it were the case that $\tilde{v}\left(X\right)>1$ for a given X, equity would be injected into the banking system until the value of equity is one. This always happens because, eventually, competition effects dominate qualitiy effects when κ is increased. If this did not happen, bankers would have incentives to alter the financial composition of their banks. In turn, this would imply that κ is not in equilibrium. Hence, states where $\tilde{v}\left(X\right)>1$ are instantaneously reflected into a new state where $\tilde{v}\left(A\times\phi\times\kappa'\right)=1$ for some $\kappa'>\kappa$.

The opposite occurs when $\tilde{v}\left(X\right)<\left(1-\tau\right)$. In such states, dividends are payed until $\tilde{v}\left(X\right)=\left(1-\tau\right)$. This means that there is an inaction region characterized by states where $\tilde{v}\left(X\right)\in\left[1-\tau,1\right]$. Thus, at the

beginning of the first stage, κ evolves according to:

$$\kappa' = (1 + e^*(X) - d^*(X))\,\kappa$$

Between the first and the second stages, κ evolves depending on realized profits and the growth rate of the capital stock:

$$\kappa' = R^b\left[1 + \Pi\left(X, X'\right) Q^*(X)\right]\frac{\kappa}{\gamma\left(X, X'\right)}.$$

In this expression, $\gamma\left(X, X'\right)$ is the growth rate of the capital stock:

$$\gamma\left(X, X'\right) = \pi\beta\frac{[p(X)\omega^*(X) + q^c\left(X, X'\right)\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega > \omega^*(X)\right]\omega^*(X)]}{q^c\left(X, X'\right)} + (1-\pi)\,\beta\frac{[A + q\left(X, X'\right)\lambda(X)]}{q\left(X, X'\right)}$$

Marginal value of equity functional equation. We can obtain a recursive expression for $\tilde{v}(X)$ and $v_1^f(X)$ that depend on the transition function from X to X'' by evaluating the Bellman equation at the optimal policies. The marginal value of bank equity at any state X is:

$$v_1^f(X) = \min\left\{\max\left\{\beta^f R^b\mathbb{E}\left[\tilde{v}(X)\,|X\right], (1-\tau)\right\}, 1\right\}. \tag{7}$$

Combining this expression with the definitions of $\tilde{v}(X)$ and $v_2^f(X)$ defines a self-map for $v_1^f(X)$.[32] In any RCE, the state space maybe divided into several regions depending on the shape of $\tilde{v}$. The following section describes the possible states of the financial industry.

4.4 States of the Financial Industry

The states of the financial industry are defined in terms of the incentives to inject equity or pay dividends. These policies affect the volume of intermediation by increasing or decreasing the financial risk capacity. For a given exogenous state, (A, ϕ), these are summarized by the marginal value of equity and how it changes with κ. Thus, the state space can be separated into 4 regions depending on $\tilde{v}(X)$ and $\tilde{v}_\kappa(X)$. At a given state X equity injections or dividends can instantaneously change κ which in turn alters $\tilde{v}(X)$ and $\tilde{v}_\kappa(X)$. Since this analysis is static, the following description of leaves (A, ϕ) fixed.

Dividend-Payoff and Reflecting Barrier. As explained earlier, dividends are payed whenever $\tilde{v}(X) < (1-\tau)$. When κ falls within this region, dividends payments instantaneously reduce κ to its closest reflecting barrier: The closest κ satifying $\tilde{v}(X) = (1-\tau)$. A dividend-payoff region associated with a sufficiently large κ always exists. This is because expected profits must be decreasing for large enough ω^*.[33] In equilibrium, expected profits are positivive, so κ enters this region infinitely often. There may be dividend-payoff regions for intermediate values of κ if the quality effect brings down marginal profits at those levels.

Equity-Injection and Reflecting Barrier. When κ falls in states where $v(X) > 1$, expected discounted profits are high enough to attract equity injections. Injections reflect κ to the closest point satisfying $v(X) = 1$, so this region is the counterpart of the dividend pay-off region. Similarly, there may also be multiple intervals of κ with this property.

[32] Blackwell's conditions cannot be checked immediately because it may fail to satisfy the discounting property. The operator is monotone though. All numerical iterations lead to the same outcome. When $\rho = 1$, analytical expressions can be obtained.

[33] This occurs because the quantity effect always dominates the quality effect as ω^* approaches 1 because volumes have less impact on the conditional expectation at the end of the support. As argued earlier, ω is also increasing κ.

Competitive Inaction Regime. The the other two possible regions correspond to inaction regions. These can be subdivided into two. The first is a competitive inaction region:

Definition 3 (Competitive Inaction Region) *A state X is in a competitive inaction region if (a) $\tilde{v}(X) \in [(1-\tau), 1]$ and (b) when $v_\kappa(X)$ exists, $v_\kappa(X) \leq 0$ and, if defined, $v_\kappa(X)|_{\tilde{\kappa}} \leq 0$ for any $\tilde{\kappa} > \kappa$.*

Condition (a) guarantees this is an inaction region. Condition (b) implies that expected discounted marginal profits are decreasing in the financial risk capacity for small increases from a given level of κ, onwards. This implies that there are ever less incentives to recapitalize banks as the financial risk capacity increases. In equilibrium $\tilde{v}(X)$ may jump as a consequence of the Pareto refinement. For this reason, condition (b) is not equivalent to $v(X)$ being decreasing. Instead, the condition says that it is decreasing from that level of κ onwards, except at the finitely many points where the Pareto refinement applies. This definition captures the idea that in a competitive inaction region the quantity effect dominates the quality (adverse selection). Therefore, expected discounted profits decrease with more intermediation. Although a Pareto refinement may change these incentives at a given point, it doesn't modify the direction of this incentives in the vecinity of that point. I label this region as competitive because it is consistent with competition arguments by which marginal profits are decreasing in the volume. We can say more in fact,

Proposition 9 *Every equilibrium has an competitive inaction region. Moreover, the inaction region with largest volumes of intermediation is always competitive.*

Financial Crisis (inaction) Regime. The remaining region is the financial crises regime. It is defined by,

Definition 4 (Financial Crises Region) *A state X is in a financial crises region if (a) $\tilde{v}(X) \in [(1-\tau), 1]$ and is not a competitive inaction region.*

In a financial crises regime, κ is low in the sense that it triggers adverse selection, since banks don't have the net worth to expose to large losses. With low volumes of intermediation, market clearing requires a low $p(X)$, but this also brings down the quality of capital traded. Moreover, expected profits are low, and this discourages equity injections. By definition, higher levels of κ increase the value equity in this region. This happens as more financial risk capacity leads to more intermediation, ameliorates adverse selection, and increases expected profits altogether. However, in these region, bankers choose not to recapitalize because they lack the incentives at this level of marginal profits. The situation could be more dramatic: it is possible that at higher levels of κ, expected profits attract equity injections. This means bankers face a coordination problem. If they would all inject equity for higher levels of κ, they would benefit from injecting equity in a financial crisis regime. After all, injections drive $\tilde{v}(X)$ to 1, a value higher that is higher than in a financial crisis. However, when profits are low, bankers fail to synchronize over the recapitalization that would end the financial crisis.

Financial crises are therefore regions characterized by low ω^* and expected profits. Hence, exit times out of this regions may be long. This feature may feedback by depressing the return to equity furthermore.

Efficient Intermediation. For the policy discussions to come, it is useful to define what is the efficient level of intermediation. A planner facing the same information constraints but without the LLC would purchase capital so that expected profits are zero in every period. To be precise,

Definition 5 (Efficient Intermediation) *The volume of intermediation in state X if is efficient if it is the largest volume such that $\mathbb{E}[\Pi(X', X)|X] = 0$.*

The economy described above may also have states in which intermediation is efficient. There are some conditions required for the occurrence of efficient states.

Proposition 10 *If $(1-\tau) > \beta^F R^b$, intermediation is never efficient. States with efficient intermediation may occur if $\beta^F R^b$ is sufficiently close to 1.*

A natural conjecture is that a planner facing the LLC would try to be as close as possible to the efficient levels of intermediation through time. For this purpose, governments may want to impose several government policies. I return to this point in a later section. The following section explains how to compute equilibria.

4.5 Solving Equilibria

This section outlines the strategy to compute Pareto un-improvable equilibria. The solution method involves 2 steps. The first is to compute first- and second- stage prices and expected profits for any (possibly off-equilibrium) volume of intermediation given exogenous states (A, ϕ, ϕ'). The second step uses these calculations to find equilibrium volumes given κ and with this, obtain $\tilde{v}(X)$, and the inaction regions for κ.

Notation. Equilibrium objects are functions of the aggregate state. However, to compute equilibria, one needs to obtain prices and profits for any off-equilibrium ω. I use bold letters to distinguish equilbrium from non-equilibrium objects. Thus, I use $\mathbf{p}(\omega, \phi)$ to indicate the first-stage supply schedule given ϕ and a value of ω. In addition, $\mathbf{q}(\omega, \mathbf{p}, A, \phi')$ denotes the price consistent with second-stage market clearing given $(\omega, \mathbf{p}, A, \phi')$. Finally, $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$ are the corresponding profits given these prices, volumes, and exogenous states.

Step 1 - Volumes, Prices, and Profits off equilibrium. Through Proposition 3, we can find a first-stage price $\mathbf{p}(\omega, \phi)$ associated with ω by inverting the solution to the k-producer's portfolio problem. We can also use Proposition 6 to obtain $\mathbf{q}(\omega, \mathbf{p}, A, \phi')$ for any pair $(\omega, \mathbf{p}, A, \phi')$. With this, we can compute $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$, including worst-case losses. These calculations are performed only once.

Step 2.1 - Volumes, Prices, and Profits at equilibrium. We begin with an initial guess for the marginal value for bank equity $\tilde{v}(X)$. This value is updated in the following step. Using the calculations in Step 1, we find the equilibrium volume of intermediation given this guess. Thus, for each X, we look for the volumes yielding non-negative expected discounted profits and the largest ω such that the worst-case losses are at most κ :

$$\omega^*(X) = \max\left(\omega : \kappa \leq \min_{\phi} \mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')\, \omega \text{ and } \mathbb{E}\left[\tilde{v}(X')\, \mathbf{\Pi}(\omega, \mathbf{p}, A, \phi') \,|X\right] \geq 0\right).$$

Since $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$ is continuous and $\omega \in [0, 1]$, this quantity is well defined. $\omega^*(X)$ is the largest volume of intermediation yielding non-negative expected profits such that there is enough capacity to sustain losses in the worst state. Thus, $\omega^*(X)$ is consistent with the banker's problem and is the Pareto un-improvable equilibrium volume, the largest volume satisfying market clearing and optimal decisions by all agents.

Step 2.2 - Equilibrium $\tilde{v}(X)$. Given this $\omega^*(X)$, one can compute $\Pi(X, X') = \mathbf{\Pi}(\omega^*(X), \mathbf{p}(\omega^*(X), \phi), A, \phi')$. We use the functional equation (7) and the definition of $v_1^f(X)$ to update $\tilde{v}(X)$. Steps 2.1 and 2.2 are iterated until convergence. When $\rho = 1$, $v_2^f(X) = (1-\tau)$, so steps 2.1 and 2.2 are done only once. Appendix A provides details for the implementation of this strategy in a computer. The following section describes two examples of equilibria to compare economies with and without asymmetric information. In doing so, these examples illustrates the solution method.

5 Analytic Examples

This section provides two examples that illustrate how equity injections and dividends are stabilizing forces for this economy. The first example describes a version of the model under symmetric information. The second example contrasts this with a version in which asymmetric information is severe. The point of the section is to show that in presence of severe asymmetric information, recapitalization no longer stabilizes financial markets in response to large shocks. For the rest of this section, I assume $\rho = 1$ to exploit closed analytic expressions.

5.1 Example 1 - Risky intermediation without asymmetric information.

The first example is an economy where financial intermediation is risky but asymmetric information is not present. Here, I assume that $\lambda(\omega) = \lambda^*$, so all units are of the same quality. I also assume that $f_\phi(\omega)$ are constant in ω but decrease in ϕ. As explained earlier, this case corresponds to a version of the model with an aggregate depreciation shock $\bar{\lambda}(X)$. Here, ϕ takes one of two possible values: a good value, ϕ_G, and a bad value ϕ_B. Draws are i.i.d. and $\phi_B > \phi_G$. In addition, A is constant. In this environment, we have the following:

Proposition 11 *In any economy without asymmetric information, κ' fluctuates within a unique equilibrium interval $[\underline{\kappa}, \overline{\kappa}]$. If $\kappa \leq \underline{\kappa}$, then $e^*(X)$ is such that $\kappa' = \underline{\kappa}$. If $\kappa \geq \overline{\kappa}$, $d^*(X)$ is such that $\kappa' = \overline{\kappa}$. $v(X)$ is decreasing and $\omega(X)$ is increasing in κ.*

The proof to this proposition is straight forward. From Proposition 6, we know that $\mathbf{\Pi}(\omega, \mathbf{p}, \phi)$ is decreasing in ω since quality effects are not present without asymmetric information. Also, as noted earlier, $\rho = 1$, $v_2^f(X) = (1-\tau)$. We can use $\mathbf{\Pi}(\omega, \mathbf{p}, \phi)$ and equation (6) to obtain an expression for the marginal value of equity in terms of any arbitrary ω. Call that value $\tilde{\mathbf{v}}(\omega, \mathbf{p})$. Without asymmetric information, $\tilde{\mathbf{v}}(\omega, \mathbf{p}, A, \phi)$ is decreasing in ω. Consequently, by Propositions 5, there is a unique interval for ω such that $\tilde{\mathbf{v}}(\omega, \mathbf{p}) \in [(1-\tau), 1]$. Correspondingly, since $\mathbf{\Pi}$ is decreasing in ω, there is a unique equilibrium interval for κ that determines a unique competitive inaction region.

Figure 3 shows the construction of equilibria graphically. The upper-left panel depicts four curves associated with any arbitrary off- equilibrium ω. These correspond to the capital-supply schedule, $\mathbf{p}(\omega, \phi)$, the marginal value of bank assets in good and bad states, $\mathbf{q}(\omega, \mathbf{p}, \phi_H)\lambda(\phi_H)$ and $\mathbf{q}(\omega, \mathbf{p}, \phi_L)\lambda(\phi_L)$, and their expected value $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi)\lambda(\phi)]$. The difference between $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi)\lambda(\phi)]$ and $\mathbf{p}(\omega)$ is the expected marginal profit from financial intermediation. Multiplying this amount by ω yields the total expected bank profits $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi)\lambda(\phi) - \mathbf{p}(\omega)]\omega$ normalized by the capital stock. Total expected bank profits are plotted at the bottom-left panel. The bottom-right panel plots worst-case profits, $[\mathbf{q}(\omega, \mathbf{p}, \phi_L)\lambda(\phi_L)] - \mathbf{p}(\omega)]\omega$, also normalized. In equilibrium, κ must be sufficient to sustain the losses induced by a volume of intermediation associated with it. The panel in the top right plots the expected value of bank equity $\tilde{\mathbf{v}}(\omega, \mathbf{p})$ as a function of ω.

The horizontal lines in the top-right panel are the marginal costs of injecting equity and the marginal benefit of dividend pay-offs, 1 and $(1-\tau)$ respectively. In equilibrium, if a given ω is the equilibrium volume of intermediation, bankers will not alter the net worth of their banks at that volume of intermediation. The set of possible equilibrium ω is characterized by volumes for which the value of equity is within the marginal cost of injections and the benefit of dividend payouts. The shaded areas in the graphs correspond to this set. Since, $\tilde{\mathbf{v}}(\omega, \mathbf{p})$ is decreasing in ω, the equilibrium set is a unique interval. For each ω in that interval, there

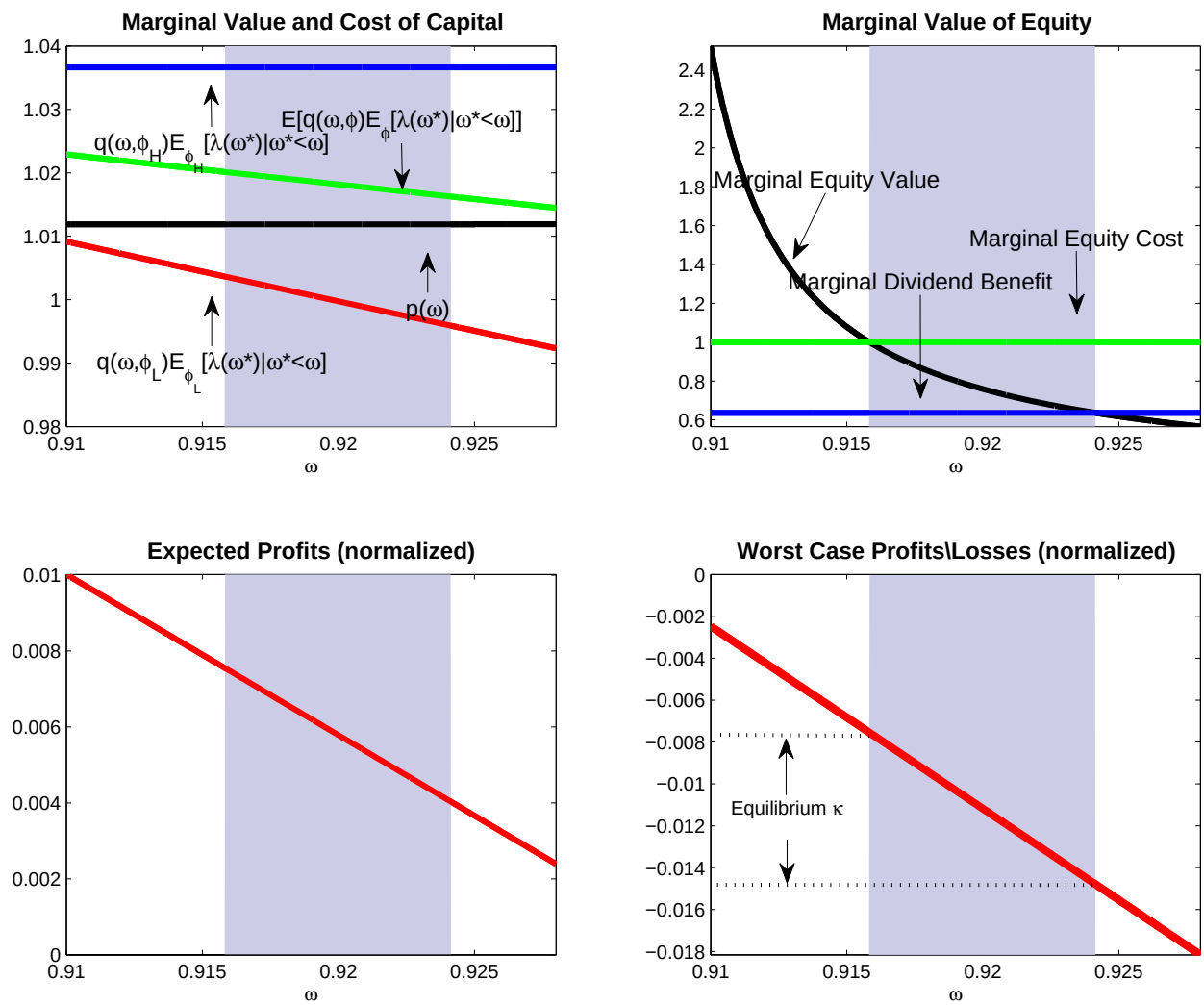

Figure 3: **Model without asymmetric information: Equilibrium objects as functions of ω. Parameters are set to:** $\pi = 0.1$, $\beta = 0.99$, $\beta^f = 0.99$, $A = 1.42$, $\tau = 0.6364$, $\rho = 1$.

is an equilibrium κ corresponding to it. Following step 2 in the previous section, we obtain this equilibrium set by computing the maximal losses given each ω in the equilibrium set. The bottom-right panel shows this interval for κ is obtained as the image of worse case losses for the equilibrium ω-set.

Figure 4 plots the equilibrium objects. These are obtained by following the second step of the solution method described earlier. The top-left panel plots ω^* as a function of κ (that is, κ before equity injections or dividends). The panel on the top right depicts $\tilde{v}$. In equilibrium, κ' must be in the inaction region where $\tilde{v}(\kappa) \in [(1-\tau), 1]$. The bottom panel depicts equity injections, dividends and κ' as functions of κ. In equilibrium, e and d adjust to bring κ' to the equilibrium set depicted in Figure 3. The shaded area in the figure is the competitive inaction region. The regions to the right and left of the shaded area are the dividend payoff and equity injection regions, respectively.

Dynamics. Proposition 11 is useful to understand the dynamics of this economy. Recall that in equilibrium worst-case losses are always negative because the converse would imply infinite leverage and infinite expected

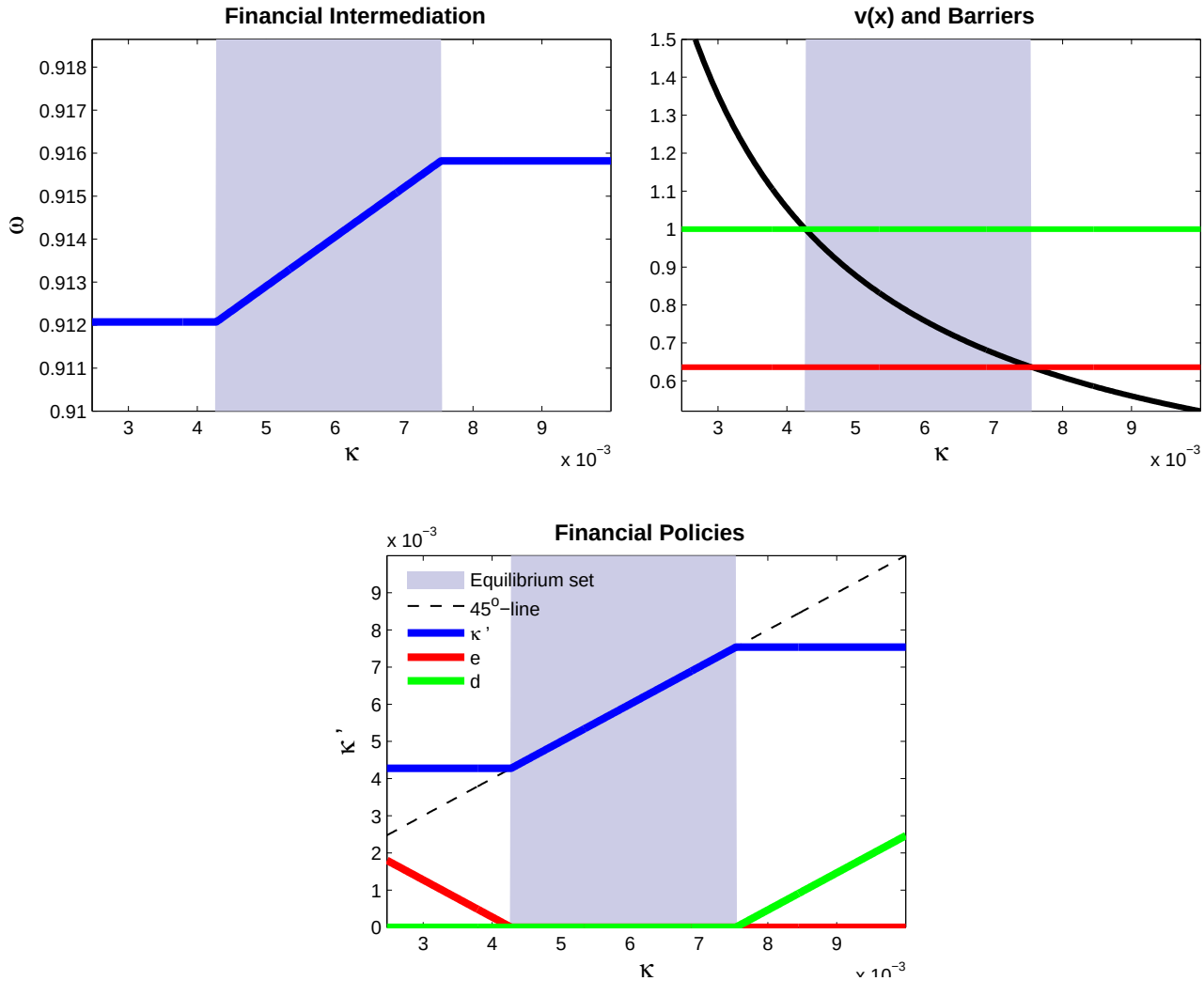

Figure 4: **Model without asymmetric information: Equilibrium objects the as functions of κ.**

profits. In contrast, expected profits must be non-negative since otherwise no intermediation would be provided. This implies that κ will increase or decrease depending on the realization of ϕ. When ϕ_B is realized, profits are negative and drag κ down. Below $\underline{\kappa}$, profits attract equity injections that recapitalize banks and increase the financial risk capacity. Thus, injections stabilize a system with low financial risk capacity. When ϕ_G is realized, dividends work in the opposite direction and κ increases in expectation. When it increases beyond $\bar{\kappa}$, dividend payoffs reflect the financial risk capacity downwards. Hence, without asymmetric information κ fluctuates within a unique interval. The next example shows how asymmetric information changes incentives to inject equity in a way that precludes this stabilizing force.

5.2 Example 2 - Risky intermediation with asymmetric information.

I modify $\lambda(\omega)$ and $\{f_\phi\}$ to introduce asymmetric information by altering the functional form for $\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*]$. I fix values for the lower and upper bounds of $\lambda(\omega)$, λ_L and λ_H. Then, I use the following functional form $\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*] = \lambda_L + (\lambda_H - \lambda_L) F_\phi(\omega^*)$. I assume F_ϕ is the CDF of a Beta distribution where ϕ indexes

its parameters in a way that satisfies Assumption 1. The rest of the calibration is similar to the one in the previous example.

Figure 5 is the asymmetric-information analog of Figure 3. The upper-left panel shows four curves that correspond to $\mathbf{p}(\omega,\phi)$, $\mathbf{q}(\omega,\mathbf{p},\phi_H)\,\lambda(\phi_H)$, $\mathbf{q}(\omega,\mathbf{p},\phi_L)\,\lambda(\phi_L)$, and $\mathbb{E}[\mathbf{q}(\omega,\mathbf{p},\phi)\,\lambda(\phi)]$. Note that $\mathbf{q}(\omega,\mathbf{p},\phi_H)\,\lambda(\phi_H)$ and $\mathbf{q}(\omega,\mathbf{p},\phi_L)\,\lambda(\phi_L)$ are no longer decreasing in ω after asymmetric information is introduced. As volumes increase, the price of capital falls, but the quality improves. The relative strength of either effect governs the shape of the value of bank assets. These forces cause total expected and worst-case profits to be non-monotonic (bottom-left and right panels). Same levels of worst-case losses can result from multiple values of ω. This implies that a given κ can possibly sustain multiple levels of intermediation. The Pareto refinement implies that, in equilibrium, the largest volume consistent with bank optimality and the capacity constraint is intermediated. The top-right panel plots the marginal value of bank equity. There is an important difference

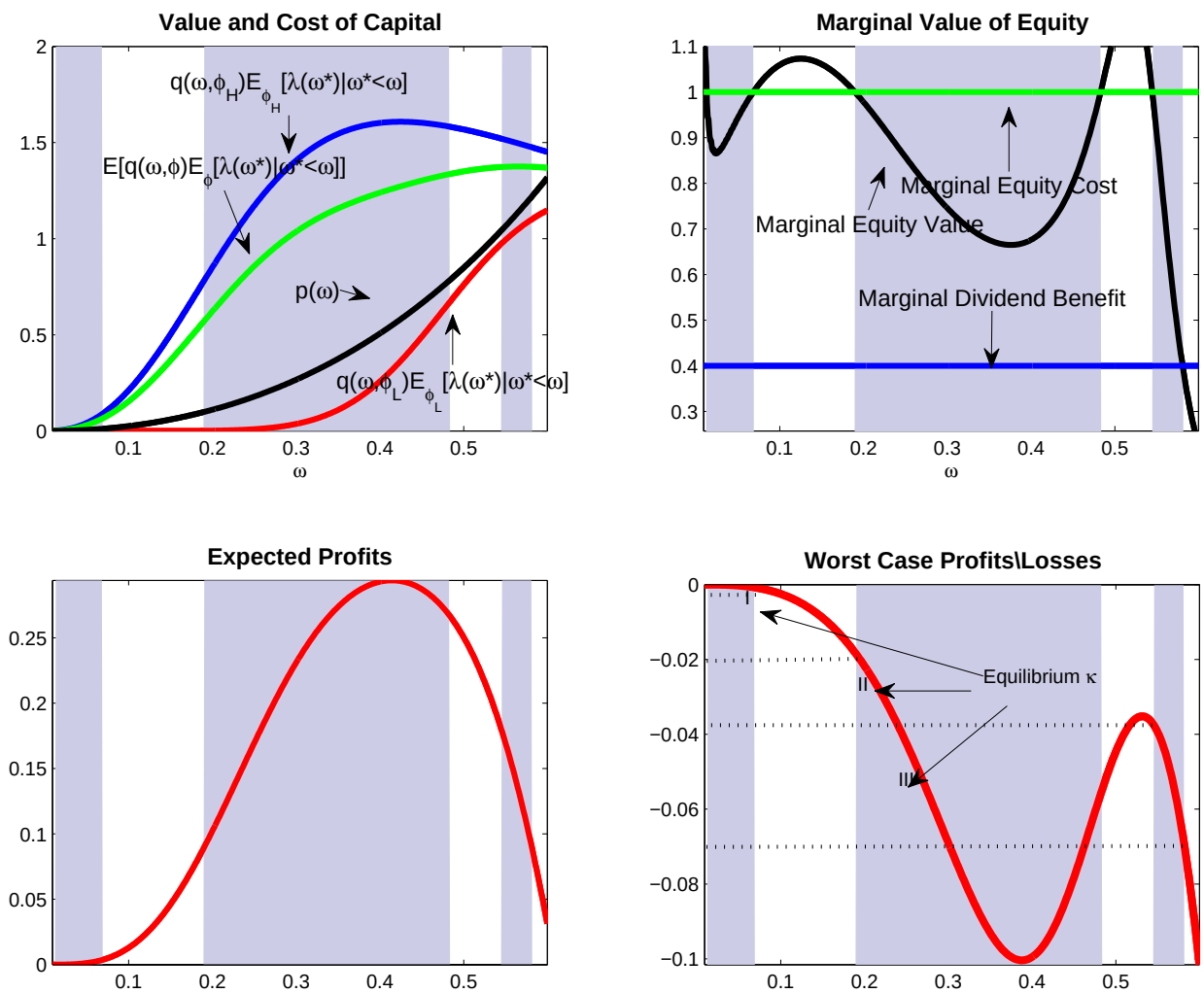

Figure 5: **Model with asymmetric information: Equilibrium objects as functions of ω. Parameters are set to: $\pi = 0.1$, $\beta = 0.98$, $\beta^f = 0.6$, $A = 1.45$, $\tau = 0.6364$, $\rho = 1$.**

in the shape of the value of bank equity when asymmetric information is present. With asymmetric informa-

tion, the marginal value of equity inherits the non-monotonic behavior of profits. Once again, the horizontal lines correspond to the marginal cost equity and benefit of dividends. Non-monotonic profits lead to multiple inaction regions. In particular, for low levels of intermediation expected profits are low and equity injections are not profitable.

This example also illustrates the wide range of phenomena that may occur when asymmetric information is incorporated. Not surprisingly, this system will have much richer dynamics in this case. For this particular example, there are three equilibrium inaction intervals identified by the shaded areas. As before, each volume in this interval is identified with a single κ that sustain potential losses. Again, the equilibrium intervals for the financial risk capacity are obtained as the image of the worst-case losses for each equilibrium ω-interval. The intervals corresponding to the largest sizes of κ (intervals I and II in the figure) are competitive inaction regions.[34] The upper bound of interval II in the figure has a distinctive property: If κ is increased slightly at that point, it can support a much larger level of intermediation. This happens because there are much larger level of intermediation that only increases worst-case losses infinitesimally. The Pareto refinement will push intermediation towards those levels.

Interval III is a financial crises regime. It is associated with low levels of financial intermediation. This region is an inaction region since bankers do not inject equity at these levels. Note that for larger values of ω, equity injections are profitable since $\tilde{\mathbf{v}}$ is above 1. As discussed earlier, this underscores the nature of the coordination failure faced by banks. Banks choose to keep the their net worth low, engage in less intermediation, and, by this, only bad quality capital is traded.

Figure 6 plots the equilibrium objects as functions of κ. The upper-left panel plots the equilibrium financial intermediation. We can observe a discrete jump in the volume of intermediation from the second to the first region. This jump is the result of the fact that slightly larger κ can support a much larger volume of intermediation and the Pareto refinement selects the equilibrium with largest volumes. The equity injection region between regions I and II is very small: regions I and II are close to each other. Note that to the left of the second region, the marginal value of equity $\tilde{v}$ is increasing for very lows values of κ. The financial crisis regime can be barely observed because the volume of intermediation and losses associated with it are very small. Hence, κ is very small in that region. This regime occurs when $\tilde{v}$ crosses the cost of equity injections. Thus, in presence of asymmetric information, e and d may sease to adjust κ' for low values.

Figure 8 in the appendix, illustrates equilibrium banking and economic activity indicators as functions of κ. Among other things, the figure shows the financial crises regime is characterized lower growth and investment rates. By means of an example, we have shown:

Proposition 12 *For sufficiently severe asymmetric information, the return to financial intermediation is non-monotone in κ and there exists a financial crises regime.*

Dynamics. The response of κ to ϕ is similar as before. When $\phi = \phi_B$, κ is reduced. The opposite occurs when $\phi = \phi_G$, increases. The difference in the dynamics is given by the incentives to recapitalize banks after large negative shocks. A realization of $\phi = \phi_B$, drives the financial system to the financial crises regime. As adverse selection effects kick-in the profitability no longer justifies the injection of capital to the system, and the economy may take a while to recover from this regime. This economy eventually recovers as banks slowly build equity through retained earnings. Once κ reaches the equity injection regions between intervals III and

[34] Note that the condition for an inaction competitive regions is with respect to κ and not ω. Although $\mathbf{v}$ is non-monotone for larger values of ω, the sign of v_κ does not change for larger values of κ. Had I defined inaction competitive regions in terms of monotonic properties, region II would be a financial crisis region also.

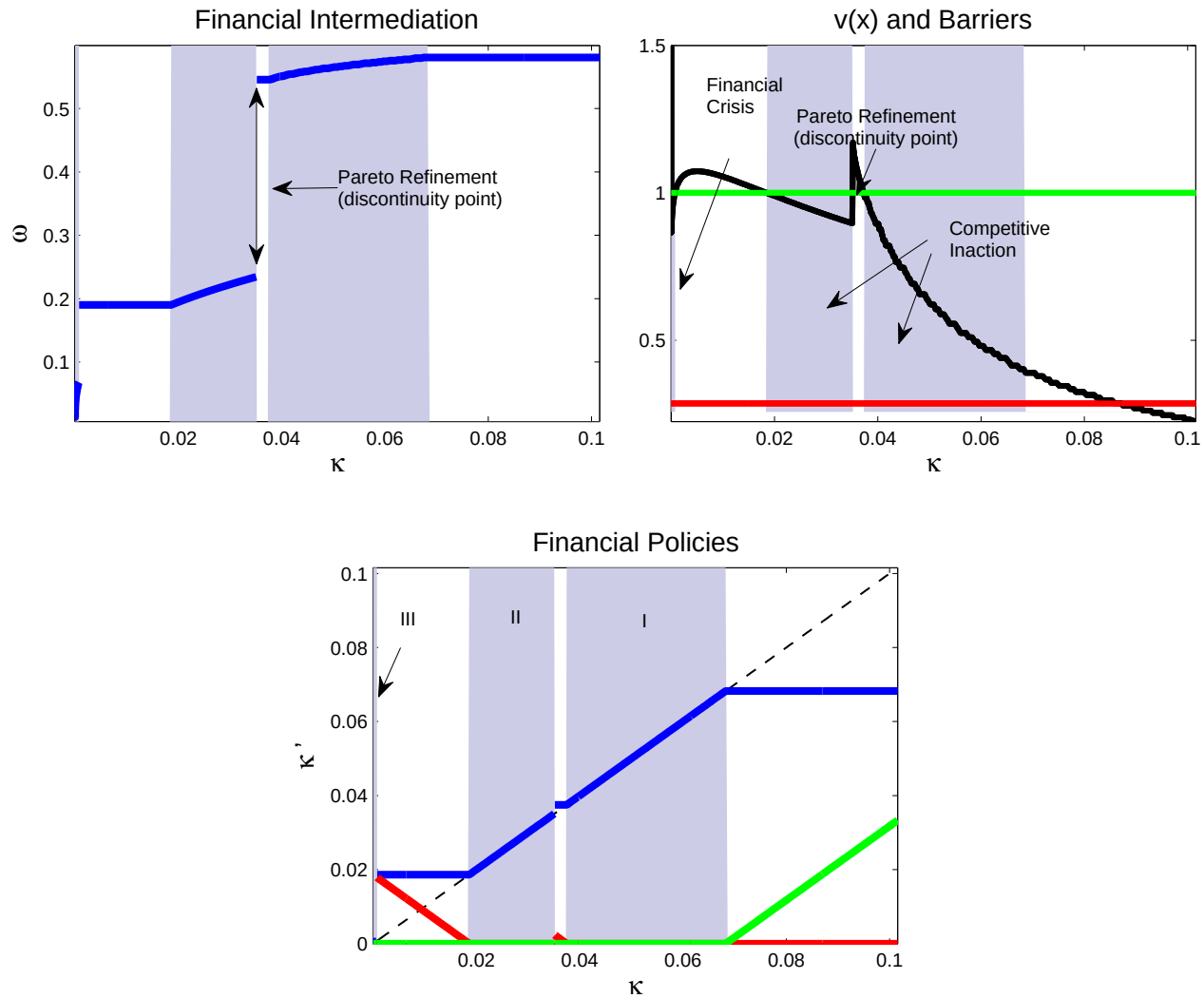

Figure 6: **Model with asymmetric information: Equilibrium objects the as functions of κ.**

II, the banking system attracts equity injections, and κ reaches the competitive inaction region II. The rest of the paper studies the quantitative properties of a richer version of this model introducing some extensions. The intuition provided in this section is the same. Policy implications are discussed later.

6 Quantitative Exercises

6.1 Issues with U.S. Bank National Accounts

Several issues must be resolved before confronting the model with banking data. This section describes these challenges and the empirical strategy I undertake to reconstruct several measures of aggregate banking indicators. Details are left for the appendix.

Accounting procedures, regulatory reform, and M&A activity. For regulatory purposes, the largest commercial banks in the U.S. are organized into legal institutions called bank holding companies (BHCs).[35] The

[35] See for example Saunders and Cornett (2010).

main assets of BHCs are equity of commercial banks. This detail is important because various banking indicators can be constructed from aggregate or disaggregate balance sheets. The relevance of these indicators depends on the application. For example, BHCs also borrow funds directly and indirectly through the liability of their Commercial Bank subsidiaries.[36] This implies that banking indicators such as leverage differ for the consolidated and individual balance sheets. To build an estimate of the relevant bank leverage for this paper, I consider the consolidated balance sheet of the BHCs. Unfortunately, Flow of Funds does not report consolidated BHC balance sheets. Instead, individual consolidated balance sheet are available and aggregates must be constructed.[37]

Consolidation issues aside, aggregate banking data has been affected by changes in the regulatory framework. These changes have lead many non-BHCs to become BHCs during the last decade. Moreover, these have promoted a considerable amount of merger and acquisition (M&A) activity within the industry.[38] Unfortunately, aggregate banking time series reported in the Flow of Funds do not adjust for mergers and redefinitions. This leads to spurious increments in various aggregate balance sheet accounts. This issue is particularly problematic after 2008 when many investment banks and finance corporations became BHCs.[39]

Shadow banking. BHC take positions not reported on their balance sheets. Nonetheless, these positions may lead to substantial operating losses. For example, banks offer substantial amounts of loan commitments and financial insurance contracts. The majority of the losses suffered by the largest institutions during the crisis originated from Special Investment Vehicles and Investment Conduits issuing these derivatives. These vehicles were explicitly or implicit guaranteed by liquidity or bailout facilities funded by the assets of commercial bank under control of the same parent BHC.[40].

Equity Measures. Bank equity typically includes a substantial component of intangible assets such as goodwill. Such items cannot be easily liquidated upon bankruptcy. Goodwill accounts include items such as the bank's trademark which has no value outside the firm. In fact, the Basel agreements specify constraints based on tangible equity (common equity - goodwill - positive tax accounts) measures. Tangible equity is not reported by Flow of Funds accounts.[41] The relevance of bank equity measures depends also on the assumptions about the impact of TARP equity injections. TARP injections were in the form of preferred equity. In terms of the model, preferred equity may or may not be interpreted as a perfect substitute to common private equity for relaxing the LLC.[42]

Cash Reserves. Banks hold non-negligible amounts of cash and other riskless assets (e.g., treasuries) as part of their portfolio. After the crisis, there has been a large shifts towards riskless investments. The model is not explicitly about the role for cash. Therefore, I must take a stance on the role of riskless investments to provide a measure of leverage.[43]

[36] The typical leverage of a BHC is 1. Liabilities are often obtained indirectly via money markets by issuing short term instruments that are not deposits (e.g., REPO and Commercial paper facilities) or via corporate bond issuances.

[37] One can find though the balance sheet of the consolidated financial sector. Commercial banks and BHC balance sheets are reported individually.

[38] The enactment of the Gramm-Leach-Bliley Act of 1999 allowed commercial banks, investment banks, securities firms, and insurance companies to consolidate as bank-holding companies.

[39] Goldman Sachs, Morgan Stanley, American Express, CIT Group and General Motors Acceptance Corporation successfully converted to bank holding companies. M&A effects includes Wells Fargo's acquisition of Wachovia in 2008, by then the 4th largest bank in the U.S. and J.P. Morgan's acquisition Washington Mutual assets.

[40] The *Shadow Banking* has been widely documented. See for example, Acharya and Richardson (2010), Pozsar et al. (2010), Gorton (2010), or Saunders and Cornett (2010).

[41] Authors such as Duffie (2011) argues that this is the right measure to account for bank leverage.

[42] In principle, preferred equity could be seen as subordinated debt that does not act as a buffer to bank losses. See for example Duffie (2011).

[43] Keister (2011) argues that this increase in cash holdings has been the result of TARP and that these injections have had

6.2 Reconstructing U.S. Bank National Accounts

Empirical Strategy. To obtain measures of aggregate banking activity from consolidated balance sheets, I aggregated the consolidated balance sheets of the largest 50 U.S. BHCs since 2000 to 2010. I compare the time series of equity for this group to the historical time series of BHCs reported by the Flow of Funds.[44] Prior to 2008, both series are highly correlated, but the relationship breaks down as large institutions are incorporated into the BHC sector. To obtain an appropriately scaled version for the reconstructed series, I rescale it so that the time series of equity for both groups are roughly the same.[45] I use the same scaling factor in order to obtain measures of consolidated assets and liabilities not available from Flow of Funds. By doing this, I assume treat leverage as constant across the industry.

To isolate the effect of M&A activity, I adjust the sample by keeping only those BHCs that did not experience any substantial changes. Since the equity size distribution of BHCs is fat tailed, I treat the M&A activity for these institutions as if it were the entire M&A activity of the industry. This isolates an important part of the M&A activity but cannot account for the entire effect of M&A activity. The final sample includes 44 BHSs for which I construct M&A-adjusted banking indicators. The adjusted and non-adjusted series are highly correlated until 2008 also. The difference between the them gives a sense of the measurement problem caused by redefinitions and mergers. To isolate the effects of TARP injections, I use institution-specific information on equity injections and extract them to obtain ex-TARP measures for the model's targets. This is again a lower estimate.[46]

Figure 7 plots these series. The bottom-right panel of the figure presents the evolution of BHC tangible equity for the sample and the industry aggregate (both with and without TARP injections). One can observe that both series behaved very similarly prior to the crisis. Without controlling for TARP injections, equity increases during the period. The top-left panel shows the behavior of loans for the top 50 banks and the sample that excludes M&A activity. A similar pattern can be observed. Prior to 2008, both series look alike, but due to redefinitions, lending increases for the non-adjusted sample. In contrast, loans reported by commercial banks behave similarly as the adjusted sample.

The top-right panel presents total assets of all commercial banks and that of BHCs in the reconstructed sample and the aggregate non-consolidated assets reported by the flow of funds. These series are reported adjusted for M&A activity. One can observe two things from this panel. First, that there is an order of magnitude between the consolidated BHC assets and the non-consolidated aggregate time series reported in the Flow of Funds. Second, both time series comove throughout the sample prior to 2008. The fact that the adjusted sample series of BHCs and commercial banks comoves implies that off-balance sheet items where close to being in zero net supply within the sample of BHCs: Derivative assets such as Mortgage Back Securities are accounted in the BHC's balance sheet but are not part of the Commercial Banks subsidiary assets. If both series had the same scale, the difference in the two would correspond to the net supply of off-balance-sheet items. The difference in the scale is because I am only using a subsample of all the BHCs. Once the sample

minor effects on the bank lending policies.

[44] Note that if one consolidates the balance sheet of a BHC with its commercial bank subsidiary, the consolidated net-worth is close to that of the parent company so long as the subsidiary has no external owners.

[45] This data is available from FRY-9 fillings reported by BHCs. It is available from the Federal Reserve Bank of Chicago and http://www.chicagofed.org/webpages/banking/financial_institution_reports/index.cfm. The list of the top 50 BHCs is described by the National Information Center, Federal Reserve System: http://www.ffiec.gov/nicpubweb/nicweb/Top50Form.aspx and it is formatted by the Wharton Research Data Services. In terms of assets, the Top 50 BHCs hold 50% of the assets the entire baking industry.

[46] The details of TARP are available at: http://www.treasury.gov/initiatives/financial-stability/about/Pages/default.aspx

series is scaled, both series are very close to each other, which reveals that off-balance-sheet items add to a small fraction of aggregate assets (but not in terms of individual bank assets).

Once one adjusts for M&A activity, one can get an idea of the reduction in bank assets: this understates the effect on intermediation since much of TARP injections was converted into cash reserves that banks have not lent out. The effects on real activity can be more clearly understood from the top-left panel. This panel shows how lending froze immediately at the beginning of the crisis, reversing the constant upward trend that prevailed prior to the crisis. The bottom-left panel presents measures of total equity.

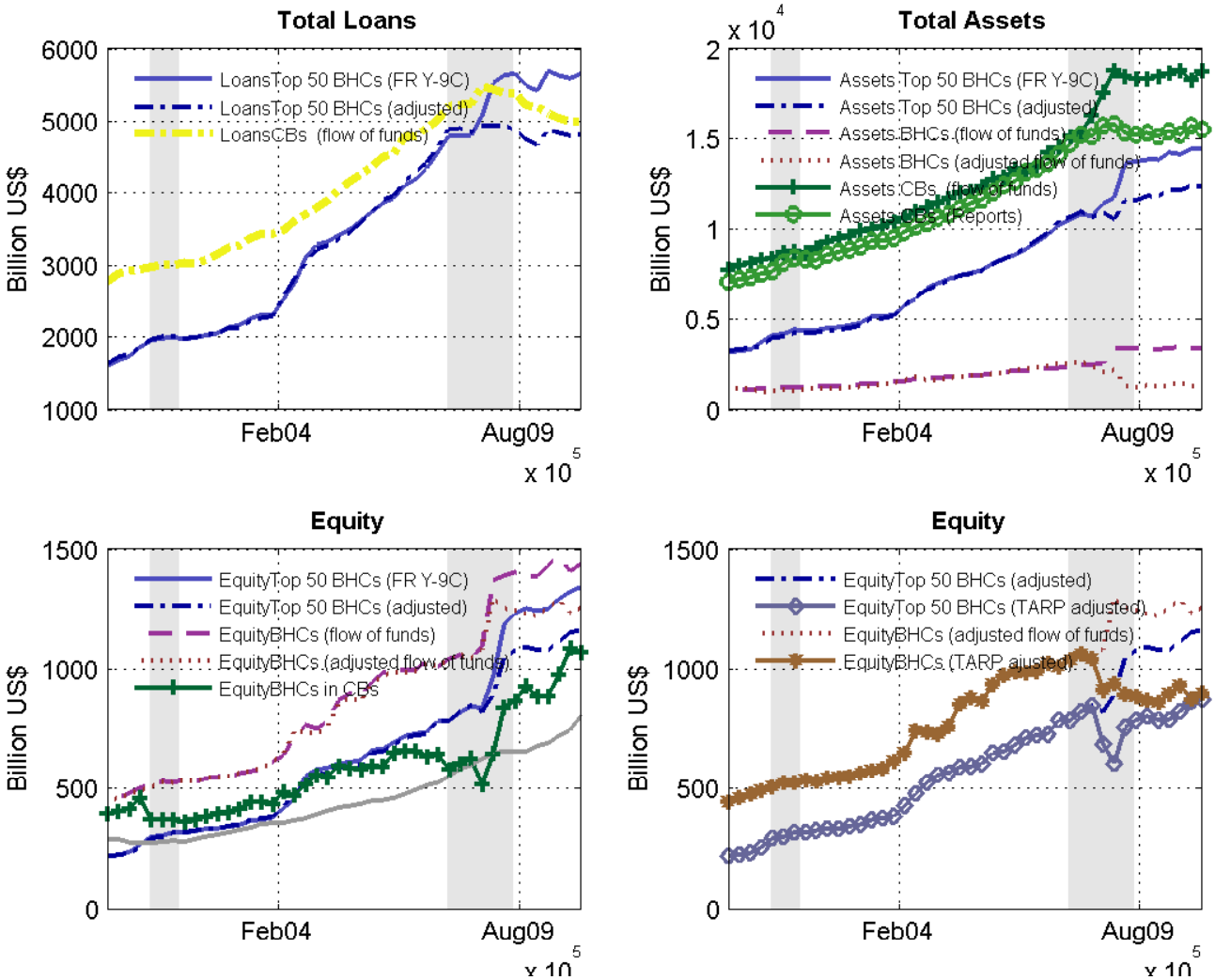

Figure 7: **Relationship between adjusted sample and Flow of Funds banking aggregates.**

I reconstruct aggregate time series that have a theoretical counterpart in the model using individual balance sheets of the adjusted groups. Table 1 summarizes the equivalence between variables in the model and data time series. Individual balance sheets report total commercial and industrial, real estate, leases, and consumer loans (or the total loans from banks to the private sector). I add these entries and treat this sum as my measure for total loans. I also subtracted goodwill and positive tax accounts from equity to construct a measure of tangible equity.[47] I use information on the difference between equity issuances and dividend payoffs

[47] Tangible measure over the physical capital stock in the model features no trend during the las decade. This is not true about other measures of equity, so also in terms of the model, it is more convenient to use tangible equity aside from the issues described in the previous section.

Variable	Model Equivalent	Data Notes
Deposits / Loans	$p(\omega)\,\omega K$	C&I + real estate + leases + consumer loans
Loans + Interest Income	$q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,\vert\omega<\omega^*(X)\right]\omega K$	Loans+ net interest income
Net Interest	$q(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,\vert\omega<\omega^*(X)\right]-p(\omega)$	Net interest income / loans
Equity	N	Tangible net-worth
Size of Financial Sector	κ	Tangible net-wort/ capital stock
Fixed costs	ψ	Average non interest net-expenses / N
Leverage	$\frac{p(\omega)\omega}{\kappa}$	Loans / κ
Bank ROE	$\frac{\left[q(X)\mathbb{E}_\phi[\lambda(\omega)\vert\omega<\omega^*(X)]-p(\omega)\right]}{\kappa}-\psi$	
Bank ROA	$\frac{\left[q(X)\mathbb{E}_\phi[\lambda(\omega)\vert\omega<\omega^*(X)]-p(\omega)\right]-\psi\kappa^{-1}}{p(\omega)}$	
Dividends	d	$(\Delta$ N -profits$)^-$
Equity Injections	e	$(\Delta$ N -profits$)^+$

Table 1: **Data and Model equivalences.**

to obtain a measure of net equity injections and dividend payoffs. I use net interest and non-interest income plus expenses such as salaries to compute bank profits and fixed expenses. I can obtain all the moments I am looking for from these measures.

6.3 Additional Features

To obtain a better quantitative performance of the model, I incorporate additional featuresthat do not alter its qualitative behavior.

Financial management costs. Bank returns are influenced by various non-interest expenditures. Most of these expenditures are salaries. To improve the realism of the model, I assume bankers pay a constant amount of their equity every period and a constant bonus when profits are positive. I interpret this cost as financial management costs. This parameter corresponds to ψ in Table 1.

Capital requirements. Capital requirements can be modeled by introducing a wedge into the LLC. Assuming that the capital requirement is such that banks, are not allowed to lose more than a fraction θ of equity in a given period, the LLC reads:

$$-\Pi\left(X, X'\right) Q \leq (1-\theta)\, n'.$$

Physical capital cost. I introduce an additional parameter so that the cost of unit of capital is not one. Since production is linear, this parameter is observationally equivalent with the mean of TFP. I introduce it so that I can use the estimated process for TFP described in the next section.

6.4 Parameters

Calibrated Parameters. This section describes the calibration chosen for the quantitative exercise provided in the subsequent sections. I set β is set so that the risk free rate in the deterministic frictionless version of the model is *2.5%* in annual terms. The discount factor for the financial sector's discount factor, β^f, is set to the same number. The return to equity in stage 2, R^b, is set to 1, assuming that banks can hold reserves that earn no interest. I assume that the aggregate depreciation rate of the economy is a constant $\bar{\lambda}$ equal to 0.9756. This value leads to an annualized depreciation rate of 10% which is commonly found in the literature. In this economy, this parameter gives a lower bound to the growth rate of the economy. The fraction of capital good producers π is set to 0.1. This number is consistent plant investment frequencies found in Cooper et

al. (1999). This is a standard calibration for models where investment activities are segmented. I set the banker's exit rate, ρ, to 0 so that they face infinite horizon problems. τ is calibrated to be consistent with a marginal cost of equity of 10% and tax-rate on dividends 30%. These values follow from the estimates of the dynamic corporate finance model of Hennessy and Whited (2005). I set the wedge in the LLC, θ, to 8% so that it is made consistent with Basel-II requirements.

Estimated Parameters. I estimate an $AR(1)$ process for $log\left(A_t\right)$. I obtain I time series for the model's TFP dividing U.S. output by the capital stock reported by NIPA. The auto-correlation coefficient is estimated to be 0.993, its mean to -0.885 and the standard deviation of innovations to 0.0083. I assume that ϕ follows a two state Markov chain. This Markov chain is estimated using a property of the model: in the model expected profits are positive and worst-case profits are negative. Hence, when ϕ takes only two values, positive profits are associated with the lowest value of ϕ and negative profits with its largest value. I can use the series for bank results to estimate the transition probabilities for ϕ in that case. This transition probability is crucial to determine expected profits and therefore, the value of equity injections.

Matched parameters. Calibrating the quality and dispersion shocks represent the biggest challenge. To calibrate the distribution of asset qualities, I use the same parametric form as in the examples introduced in Section 5:

$$\mathbb{E}_{\phi}\left[\lambda\left(\omega\right)|\omega \leq \omega^{*}\right] = \lambda_L + \left(\bar{\lambda} - \lambda_L\right) F_{\phi}\left(\omega^{*}\right).$$

As explained in that section, this parametric form is consistent with Assumption 1. λ_L is the lowest capital quality, and $F_{\phi}\left(\omega^{*}\right)$ is any CDF with support in $[0,1]$. I set $\lambda_L = 0$ to have the interpretation that so capital units are worthless. As before, I use the Beta-CDF for F_{ϕ}. The Beta distribution has two parameters. In these applications, these parameters are indexed by ϕ. These parameters are denoted by (A_{ϕ}, B_{ϕ}). A_{ϕ} and B_{ϕ} are uniformly spaced between $[A_L, A_H] = [2,5]$ and $[B_L, B_H] = [1,1]$ respectively. Thus, there is a total of 4 parameters that characterize the quality distributions: $[A_L, A_H, B_L, B_H]$. These parameters are set so that they match 4 moments: historical and great recession levels for κ and bank ROA. Other banking indicators such as financial GDP, leverage, fancying premia and bank ROE will follow from this parameters. In addition, I set the non-interest expense parameter, ψ, to 4%. This matches the non-interest expenditures to equity ratios for the selected sample of BHCs. Table 2 summarizes the parameter values used in the following quantitative exercises.

6.5 Results

6.5.1 Invariant Distribution and Historical Histograms

Figure 9 reports 4 distributions for κ and the growth rate of the capital stock in the model and in the data. The top left panel presents the marginal distribution of κ from obtained from the stationary distribution of the model. The heights of the bars represent the frequency of occupation times at different intervals for κ. The top-right panel is the analog constructed from the reconstructed time series for κ in the data.

The model's invariant distribution shows that κ is bounded by the largest and smallest reflecting barriers corresponding to the limits of the dividend pay-off and equity injection regions. A salient feature of the data and the model's distribution is their bimodal shape. In the model, the financial system's size fluctuates between 0.025 to 0.035 most of the time. With some probability though, the financial system is reduced to the range between 0.001 and 0.01. A similar bump is also found at the lower end of the historical distribution in the data. Both histograms also show an empty region between the concentrated probability mass at the

Parameters	Value	Note
Calibrated		
β	0.987259	2.5% discount rate
β^f	0.945742	2.5% discount rate
R^b	1	0 interest on reserves
π	0.097342	Cooper et al. (1999)
τ	0.08	Hennessy and Whited (2005)
ψ	0.08	Bank non-interest expense per net-worth
θ	0.08	Basel-II capital requirements
Estimated		
μ_A	-0.885	Estimated TFP process
ρ_A	0.993	Estimated TFP process
σ_A	0.0083	Estimated TFP process
Matched		
A_L	3.9	To match Bank ROA and premiums
A_h	4	To match Bank ROA and premiums
B_L	6.2	To match Bank ROA and premiums
B_H	5.2	To match Bank ROA and premiums
Φ	[0.99 0.3; 0.01 0.7]	To match profit transition of Banks

Table 2: **Parameter Values**

lower end, and at intermediate levels.

In the model, the occupation times at the lower end of the distribution occur when a sufficiently bad combination of shocks leads the economy to the financial crises regimes. In particular, the red colored (darker) bars are the conditional probabilities on the event of being in a financial crisis regime. This is a consequence of the long exit times in the financial crisis regions.

The intervals for which κ has no occurrence are equity injection regions: states in this region are transient because equity injections induce a reflect κ towards the mass in the middle region. Dividends are payed at the right extremes of the distribution.

The two panels at the bottom of the figure show the behavior of the growth rate of the physical capital stock capital stock. As with the occupation times of κ, both histograms also feature of two regions with concentrated mass. It is clear from the model that with less κ, capital stock grows at a lower rate. Figure 1 in the introduction describes how these variables comoved in the time dimension.

6.5.2 Model and Historical Moments

Table 3 reports the moments delivered by the model and those obtained through the empirical time series. These moments for the model are computed from the invariant distribution of X. I describe them in this section and try to relate them with some references in the literature.

The occupation time on financial crises regimes in the model is close to 12% of the whole sample. The Great Recession represents 14.6 of the time in the sample. This figure is very doubtful given that the financial system was very stable before the great recession. As reference, Reinhart and Rogoff (2009) calculate that, during the national banking era banking crises occurred during 13% of the years in their sample (according to different definition to mine). The average time the economy takes to leave a financial crisis region in the model is roughly 9 quarters. The great recession lasted 6 quarters. In the model, the exit time from a financial

crises regime is 7 quarters.

The average growth rate of the economy is, by construction, close to the historical growth rate of the U.S. economy. During a financial crises, output falls to -2.5%, a similar figure as during the recession. Because, this is a model with endogenous growth, 100% of the growth rate is explained by capital accumulation (TFP fluctuates around a constant). During a financial crises, TFP is responsible for some portion of growth because TFP is mean reverting so since financial crisis are most likely to occur in periods of low TFP, TFP is expected to grow during these episodes.

The variance decomposition shows that in normal times, most of the volatility of output growth is due to movements in lending conditions. During a financial crisis, financial intermediation is responsible for a smaller share of this volatility because financial crises occur more often in low TFP states. These states are mean reverting, and hence, in a financial crises, are expected to drive growth.

Financial crises episodes are also associated with a strong reduction in the volumes of financial intermediation. The volume of intermediation falls from an average ω^* of 0.73 in normal times to 0.58 during crisis episodes. In the model, intermediation is key to provide funds to investing producers. Without these funds, they cannot carry out investment projects so capital is lost due to depreciation. Thus, the reduction in growth is close explained by the fall in the volume o f intermediation. Cerra and Saxena (2008) report a reduction in the growth rate of about 8% for a cross-country sample and while output in the U.S. fell about 4.1% during the great financial recession.

The investment to GDP ratio in the model is very high in comparison to the data. There are no further margins of improvement here. The process for A, and the calibration of π and β affect this ratio. So does the calibration of F_ϕ, but I use bank indicator targets. A similar shortcoming is found when observing the investment to capital ratio.

In addition, the model also delivers predictions about the GDP contribution of the financial sector. In the model this is figure is close to 5% in normal times. This contribution falls to 0.7% during the crisis regime, because the profits of the sector are very low. Financial GDP during the Great Recession actually increased.

The second block of moments reports indicators for financial variables. The most important of this is κ. This figure is close to the data in historical times but fall much more during a financial crises regime. During a financial crises, κ can fall up to 92% in a given period given the nature of the LLC. During the recession, κ decreased by 25%. There are many explanations as to why the model is more extreme than the data. The return to assets in the model and in the data are very similar and show a decline during the crises regime in both cases. The model fails to match the leverage as there are no additional parameters left. Since leverage is substantially larger in the model than in the data, the return to bank equity is orders of magnitude larger.

The unconditional financing premia in the model and in the historical financial premia in the data are very similar. In both cases, these are close to 5%. Financing premia during the crises fall both in the model and in the data, but the drop in the model is much more substantial. This contrasts with the behavior of corporate bond spreads are typically around 6% in the data reported in Gorton and Metrick (2010). During financial crises though, spreads rose to almost 30% in REPO markets. .

The following sections illustrate the dynamics of the model through a simulation path and then I describe the behavior of the model through an impulse response analysis.

Variable	Unconditional	Crisis	Historical	Great Recession
Occupation Times				
Occupation Time	100%	35.7%	100%	14.6%
Duration (quarters)	23	6	20.8	6
Economic Activity				
Average Growth Rate	2.98%	3.09%	2.99%	-2.36%
Growth Explained by K	95.3%	0.971%	101%	-46.5%
Growth Volatility	0.19%	0.192%	6.76%	2.34%
Volatility explained by K	2.88%	7.97%	101%	46.5%
Investment/Output	31%	34.1%	8.67%	5.76%
Investment/Capital	1.99%	2.03%	6.2%	4%
Financial GDP/GDP	5.48%	-1.59%	4.4%	5.44%
Financial Intermediation Indicators				
Average κ	0.0183	0.00187	0.0162	0.0127
Financial Leverage	20.2	39.6	9.87	11.3
Loans Output	3.94	4.01	NaN	NaN
Return to Assets (ROA)	2.06%	0.654%	1.18%	-0.0839%
Return to Assets (ROE)	2.68e+003%	7.42e+003%	16.4%	-1.07%
Financing Premia	5.92%	2.4%	6.25%	5.89%
Financial Equity Indicators				
Average Dividend Rate	-0.163%	-0.434%	1.12%	-18.8%
Financial Stock Index	100%	8.93%	100%	42.9%

Table 3: **Model moments and reference statistics.**

6.5.3 Simulated Path

Figure 10 presents a typical sample path for the economy studied in this paper. The simulations cover 80 periods corresponding to a span of 20 years. The figure contains 9 plots. The episodes of financial crisis are identified by the gray areas behind of each plot. The top-left panel describes the evolution of, κ and κ', the size of the financial system before and after equity injections and dividends. In normal times, κ fluctuates between 60% and 90% as a fraction of the rest of the economy's capital. As one can see, whenever κ reaches some large value, dividends are payed-out (bottom-left panel). In crisis episodes, the financial system's size is reduced severely shrinking almost to a negligible fraction. The middle left panel plots expected profits and actual profits. One can see from that figure that expected profits are on average above zero. Actual profits fluctuate around this quantity. During a financial crisis regime, expected profits shrink to almost zero. The large drops in κ are explained by the leverage ratio of FIs.

Financial crises are triggered when the purchase cost of capital exceeds the value of the collateral. This relation is plotted in the middle panel at the top of the figure. The plot to the right shows how the volume of intermediation falls during a crisis episode. During the periods after the surprise, the financial system looses the capacity to intermediate. Acting competitively, financial institutions demand large discount rates. The volume and the quality of collateral are reduced affecting both the margins. Because the margins of intermediation are large, the value of an additional unit of net worth is above the average but not enough to justify the injection of capital to the financial system (see middle panel). We see that episodes of crises are characterized by inactivity in dividend pay outs and equity injections.

The growth rate of the economy falls dramatically during crises episodes. This reduction shows seems like a break in the trend of output.

6.5.4 Response to Dispersion Shocks

Figure 11 shows the response of the economy to an increase in the dispersion of asset qualities. The initial effect is depicted in the top-right panel. The solid line corresponds to the change in the purchase price of capital (bank liabilities). The dashed line presents the value of assets. When the shock is realized, the value of assets falls below the value of liabilities. This discrepancy induces a reduction in profits. Whereas expected profits (middle panel) remained unchanged, actual profits fall substantially. The reduction in profits causes a reduction in the relative size of the financial sector (top right panel). In the subsequent periods, the price of capital falls below its average and consequently, there is a reduction in financial intermediation (middle left panel). The marginal value of a unit of equity in the financial system increases because the financial system is experiencing an increase in markups but since κ falls, the relative value of the financial industry also drops. On average, dividend payouts fall after the shock because equity becomes more valuable on the margin.

The real effects of the dispersion shock are depicted in the bottom row. The growth rate of the economy shrinks with the reduction in intermediation. The middle panel in the bottom shows the level of output with and without the shock. Because this is a linear growth model, this reduction in growth has a permanent effect. The bottom-right panel shows the likelihood of falling into the financial crises regime after the shock hits. This probability increases immediately after the shock hits, and slowly returns to its expected value.

7 Financial Stability Policies

This section discusses some potential policies aimed at restoring financial stability.

7.1 The Externality

Externality. The behavior of financial firms induces an externality. By increasing the scale of their intermediation activities, bankers face a larger risk of losing net worth. They do this rationally considering the risks involved but fail to internalize that by reducing their own financial risk capacity, they induce market responses on prices, volumes of intermediation and the average quality capital. This happens because in equilibrium, $p(X)$ is taken as given. If banks could coordinate, they would take into account the reduction in capacity has additional effects on the profitability of the sector. Thus, there is a coordination failure whereby there are alternative government policies may improve welfare of every agent in the economy. This externality motivates the need for policy intervention.

7.2 Capital requirements

Effects of capital requirements. Increasing capital requirements θ, has two effects. First, they increase the volume of intermediation in a given period, which may be desirable if banks are lending excessively (although, in general, a reduction in lending will have adverse effects on growth). The second effect is that they also reduce the profitability of the financial system which, in turn, will affect the distribution of κ. This effects

show up in the conditions that determine the bank's policies:

$$\begin{aligned} e &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \geq 1 \\ d &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \leq (1-\tau). \end{aligned}$$

The expression shows that θ reduces the marginal benefit of equity: $\theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\}$. Thus, reducing the equilibrium intervals of κ. On the other hand, the volume of financial intermediation:

$$Q = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f\left(X'\right) \Pi\left(X,X'\right) |X \right] \tilde{Q} \text{ subject to } \Pi\left(X,X'\right) Q \leq (1-\theta)\, n', \forall X'.$$

A social planner will face trade-off between reducing probability of a financial crises by introducing capital requirements and at the expense of a reduction in financial intermediation. An optimal government policy potentially involves a state dependent θ.

Invariant distribution under Basel-II and III. Section to be completed.

Timing of Basel-III right. Section to be completed.

Countercyclical Capital Requirements. Section to be completed.

7.3 Dividend Taxes

Effects of Dividend Taxes. Dividend taxes have two effects in the model. From a static perspective, increasing dividend taxes reduces the benefits of dividend payoffs. bankers will cut-back on dividends, as long as they believe the increase is temporal. If they expect a relative increase in dividend taxes in the future, the value of bank equity will fall. Thus, the second effect of dividend taxes is to affect the value of equity, ex-ante.

These effects can be analyzed by considering the bank's dividend policy. Assume that the dividend tax is increased by Δ, for a given period. Then,

$$d > 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \leq (1-\tau(1+\Delta)).$$

From this expression, it is clear that once and for all increase in dividend taxes reduce the incentives to payout dividends, which in turn, may be desirable to increase κ. Nevertheless, if tax increases are anticipated by markets, this would show up in the marginal value of equity:

$$\tilde{v}(X) = \beta^F \mathbb{E}\left[\min\left\{ \max\left\{ \mathbb{E}\left[\tilde{v}\left(X''\right) |X' \right], 1-\tau(1+\Delta) \right\}, 1 \right\} |X \right].$$

It is clear that $\tilde{v}(X)$ would decrease upon a permanent increase in taxes. Perceived dividend taxes thus, have the effect of reducing the equilibrium financial risk capacity but it is not clear though, whether this policy is desirable.

7.4 Government sponsored equity injections

Effects of Equity Injections. Equity injections have also two counterbalancing effects. On the one hand, in times of crises, equity injections allow the government to recapitalize the financial system. This policy may prevent the negative consequences of adverse selection. This policy has two costs. In the model, privately financed equity injections do not occur in equilibrium when expected returns to financial intermediation is low. This follows from a coordination failure by which bankers fail to coordinate to inject equity simultaneously all the way to increase κ to the point where the industry becomes profitable again. Government sponsored equity injections serve this purpose.

The drawback of this policy is that if is anticipated by financial institutions it would, ex-ante, induce increases in the volumes of intermediation. The reason is that this government policy would increase the marginal value of equity which would, in turn, attract more capital to Bankers. With more capital on the banks balance-sheets, the volume of intermediation of financial firms would also increase. Thus, these policies would lead the system to grow beyond its *natural* size. This may or may not be efficient.

8 Discussion of Extensions

This section discusses two extensions to the basic model with the purpose of enriching the discussion.

8.1 Spill-overs

So far, capital is ex-ante homogeneous so financial intermediation is a single activity. The model is unrealistic in that this dimension because in reality banks have a number of product lines and are engaged providing intermediation services in different sectors. Intermediation strategies in distinct sectors are differentiated according to information about profitability and risks involved. In reality, profitability and risks are assessed according to firm specific features or specific features of the collateral used. In fact, multidimensional features can explain why financial institutions are not willing to take certain forms of capital as collateral or why several product lines of the financial system froze during the aftermath of the sub-prime crisis. In principle, there is no reason for volumes of intermediation to fall in the same magnitude upon a shock that affects the financial system's balance sheet.

A version of the model with different sectors may be used to analyze how shocks to the value of collateral in one sector, such as housing for example, may end-up spilling over to manufacturing sectors by affecting the balance-sheet of FIs.

In addition, the strategy for financial institutions may also depend on the type of financial instrument employed. A version of the model that also studies different financial products, like investment banking for example, can induce spill-over effects by affecting the financial system's balance-sheet. This feature can be easily incorporated into the model by analyzing a portfolio choice problem for FIs in the interim period. In terms of the model, allowing banks to invest their equity in risky assets, is like making R^b stochastic. Thus, affecting the evolution of bank to the bank's equity.

8.2 Fire-sales

In the model, banks are forced to sell assets once they suffer losses form intermediation. So far, they sell the storage good to raise funds. The price is 1 since the bank's equity is held in the commodity that plays the

role of the numeraire. The model can be altered to introduce fire-sales. For example, the bank's equity may be held in units of capital. In this case, if financial contracts are specified in unit of consumption, upon a shock, banks may be forced to sell capital at the same time. When this is the case, banks must sell capital units at a lower price, creating a negative spiral effect as in Brunnermeier and Sannikov (2011).

8.3 Outside and Convertible Equity (CoCos)

Section to be completed.

9 Conclusions

This paper provides a theory about risky financial intermediation under asymmetric information. The main message of the paper is that financial markets where asymmetric information is a relevant friction are likely to be more unstable than others. The source of this instability is caused by the non-monotonicity in bank profits as a function of the aggregate net-worth of the financial industry. This non-monotonicity implies that banks fail to recapitalize during bad times. There are many other theories that explain why banks can fail to do so. However, their policy implications can be very different. Testing this implication and streamlining what this means for optimal financial regulation is an important task left for future research.

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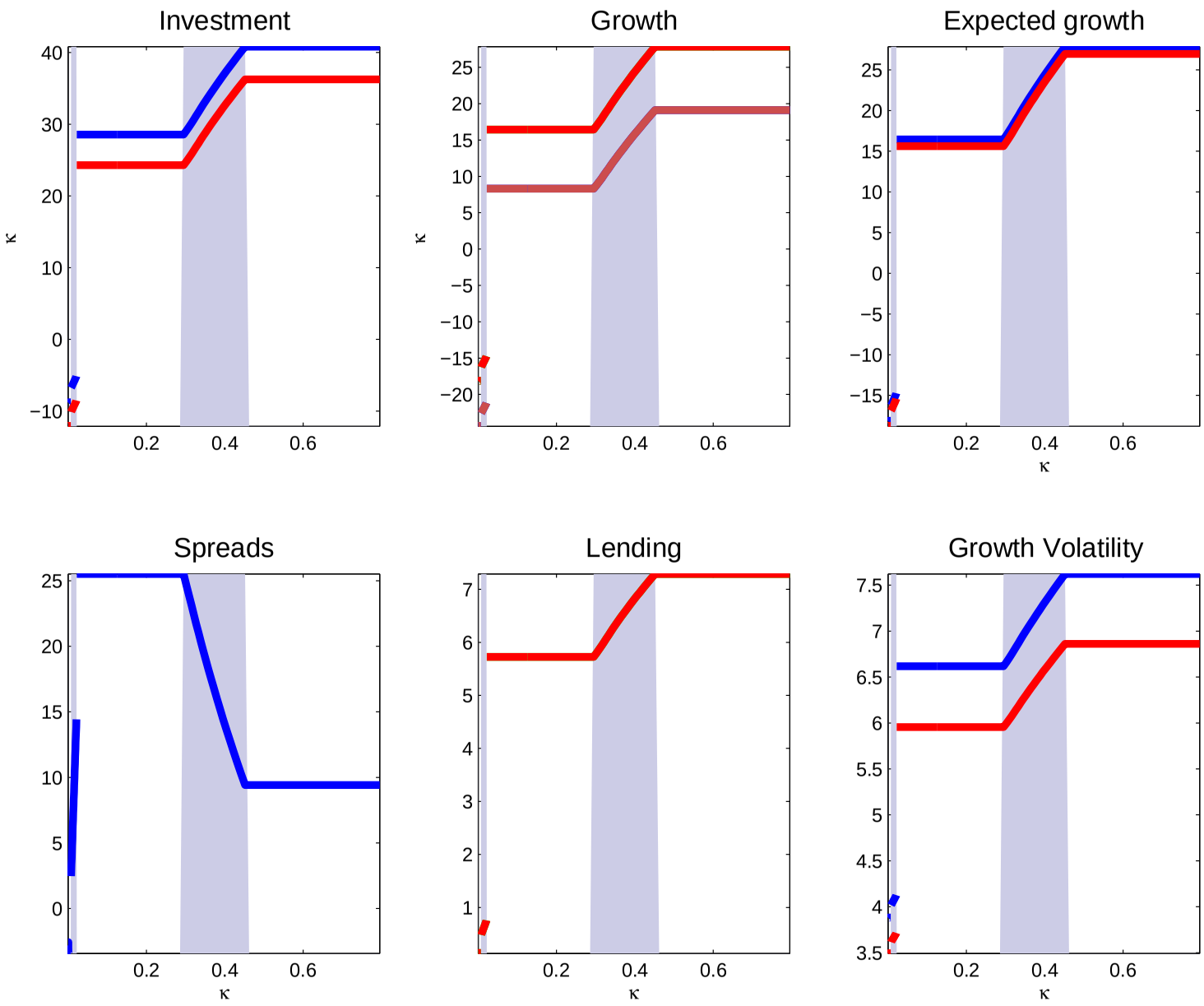

Figure 8: Equilibrium real side in the model with asymmetric information as functions of κ. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

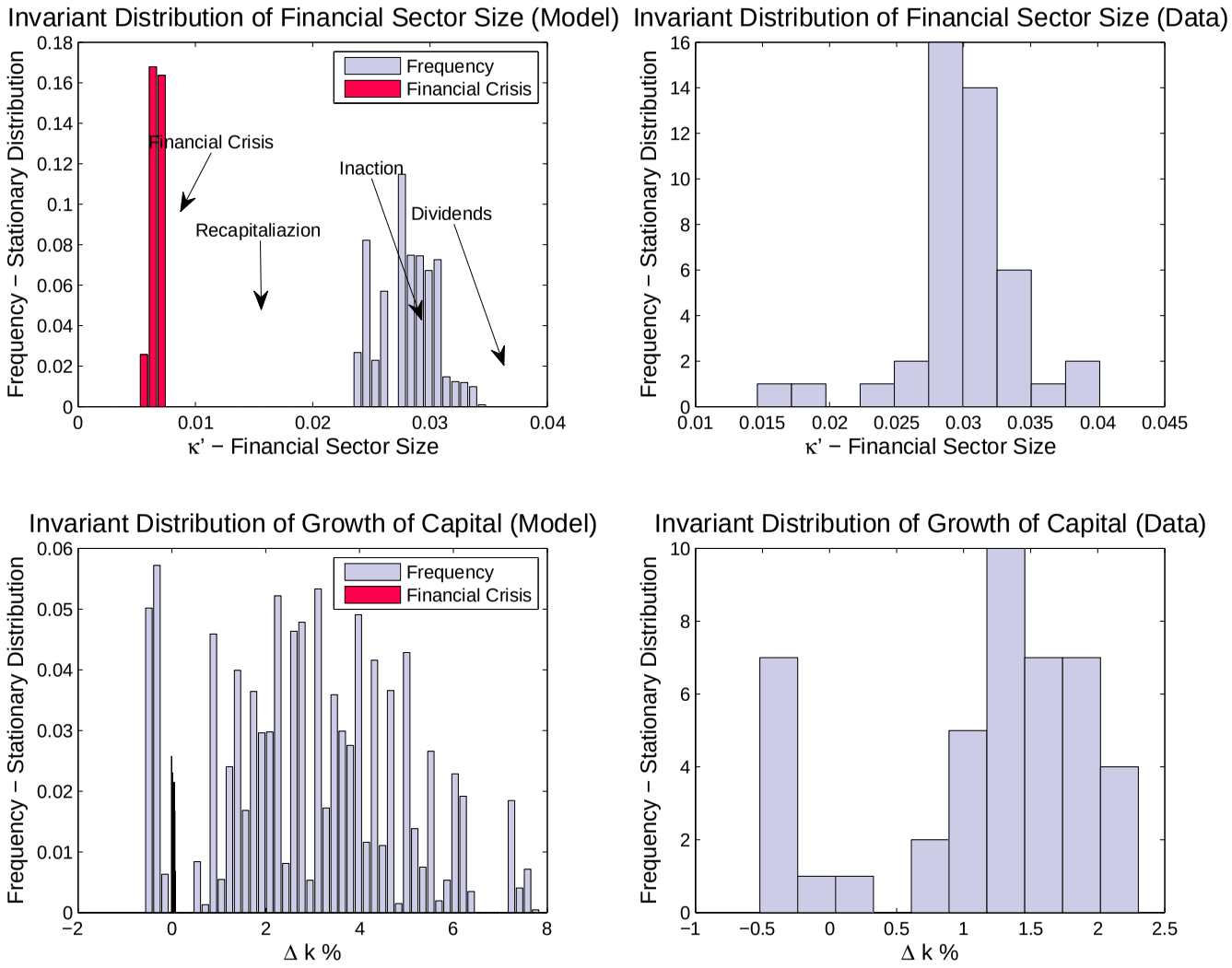

Figure 9: **Invariant distribution and empirical distributions.** The figure reports the invariant distribution of κ obtained from the model and its empirical counterpart. The analog for the growth rate of the capital stock is also included.

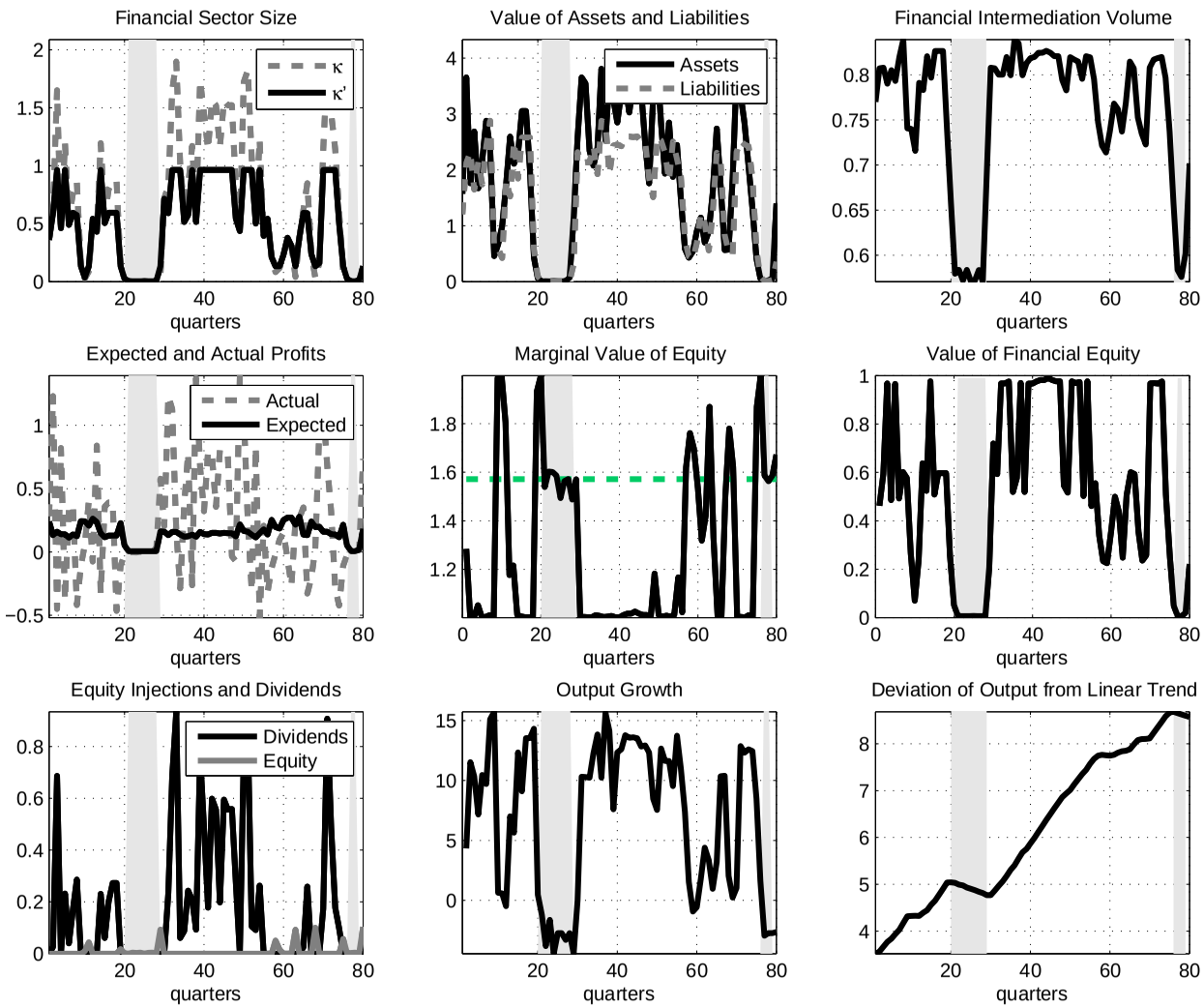

Figure 10: **Simulations.** The figure plots a simulated paths for the economy described in the paper. The shaded areas correspond to the financial crisis regimes corresponding to the definition in the paper.

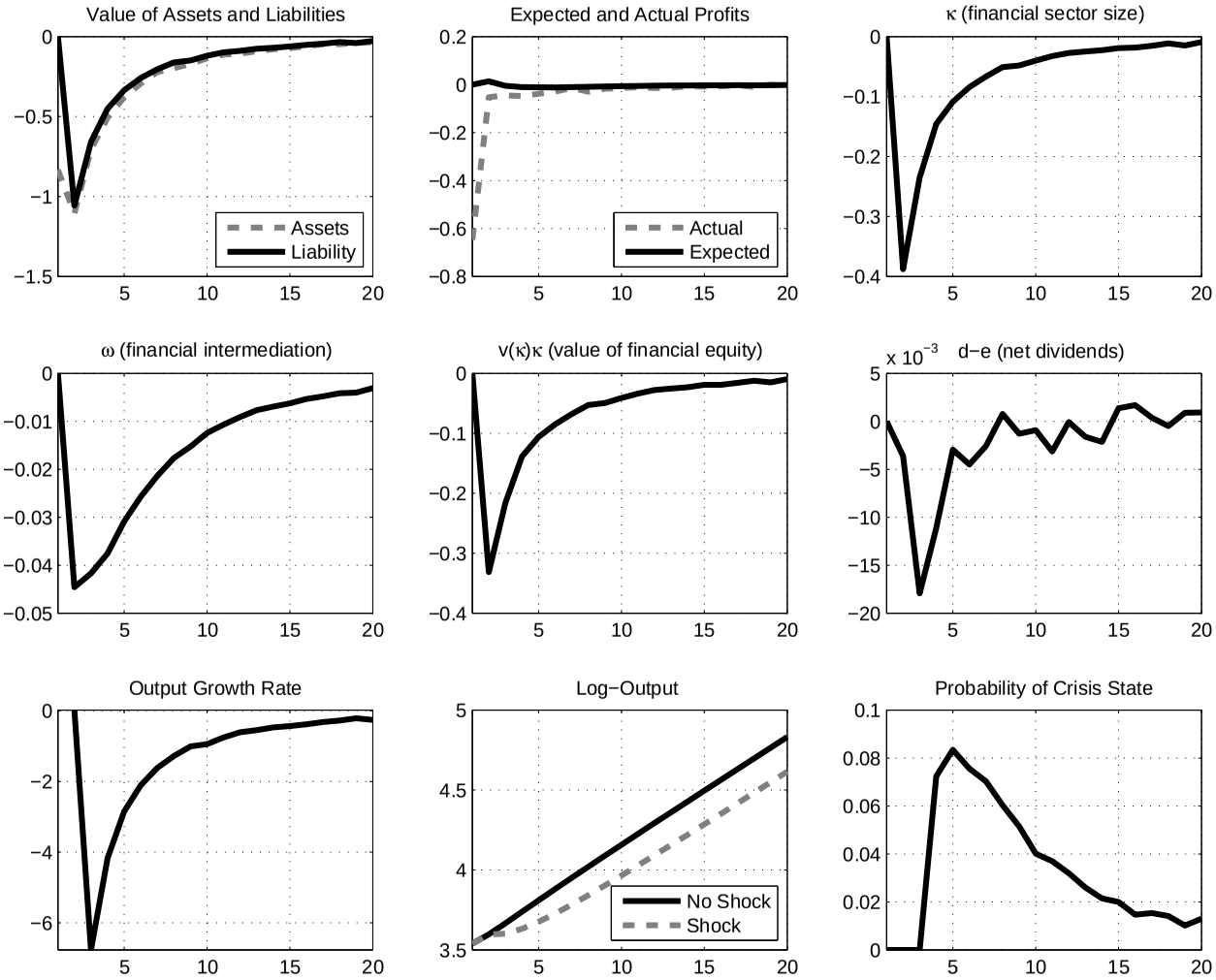

Figure 11: **Impulse response function.** The figure plots the response to an increase in dispersion.

A Algorithm

The following algorithm was used to compute equilibria in the model.

Algorithm 1 *1. Discretize the state space of $A \times \phi$ and transition function χ.*

- *I use a grid of 20 points for each variable.*

2. Discretize the unit interval for the volumes of intermediation.

- *I use a grid size of 1000 points.*

3. For all possible realizations $A \times \phi$ and ω on the grid solve $\{\mathbf{p}(\omega, \mathbb{A}, \phi), \mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi), \mathbf{\Pi}(\omega, \mathbf{A}, \phi)\}$.

- *$\mathbf{p}(\omega, \mathbf{A}, \phi)$ is discretizing the space of all possible values $p_L = \lambda(0) \min_\phi f_\phi(0)$ and $p_H = \lambda(1) \max_\phi f_\phi(1)$ and solving the optimal portfolio problem assuming $\mathbf{q}^c(\omega, \mathbf{A}, \phi) = 1$.*
- *Using this $\mathbf{p}$, one finds $\mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi)$, $\mathbf{\Pi}(\omega, \mathbf{p}, \mathbf{A}, \phi)$ and checks whether $\mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi) \geq 1$.*
- *For values where the condition fails, one solves $\mathbf{p}(\omega, \mathbf{A}, \phi)$, $\mathbf{q}(\omega, \mathbf{A}, \phi)$ jointly and then finds $\mathbf{\Pi}(\omega, \mathbf{A}, \phi)$.*

4. Guess a candidate function for $\tilde{v}$.

- *I use an initial guess of $\tilde{v} = (1 - \tau)$, corresponding to the case when $\rho = 1$.*

5. Compute the set ω^o using the candidate function $\tilde{v}$.

- *To do this I interpolate over the upper contour of $\mathbb{E}[\tilde{v}(X')\mathbf{\Pi}(\omega, \mathbf{p}, \phi)|X]$.*

6. Compute the set ω^κ.

- *This is done by computing for each ω in the grid, $\kappa = \min_\phi \mathbf{\Pi}(\omega, \mathbf{p}, \phi)\, \omega[\mathbf{mis}\sin\mathbf{g}$ part]*

7. Compute $\omega^*(X)$ for this iteration. Then, define $p(X) = \mathbf{p}(\omega^*(X), \mathbf{A}, \phi)$, $\Pi(X) = \mathbf{\Pi}(\omega^*(X), \mathbf{A}, \phi)$ and $q(X) = \mathbf{q}(\omega^*(X), \mathbf{p}(\omega^*(X), \mathbf{A}, \phi), \mathbf{A}, \phi)$.

8. Compute the transition function for X.

9. Update the $\tilde{v}(X)$.

- *This is done by iterating the Bellman equation for $\tilde{v}(X)$.*

10. Iterate steps 4-9 until convergence.

- *I used a tolerance of 0.001% for the value of equity $\tilde{v}(X)$.*

11. Compute $v(X), d(X), e(X)$.

Computation time. The algorithm runs in 5 minutes in Matlab.

B Proofs

B.1 Proof of Propositions 1, 2 and 3

The proof of propositions 1, 2 and 3 is presented jointly. The idea of the proof is to transform the producer's problem into a consumption-savings problem with log-preferences and linear constraints. For this, one has to deal with the asymmetric information problem and the randomization across producer types first. Once this is done, one can use the dynamic programming arguments for homogenous objectives in Alvarez and Stokey (1998) to argue that all the Bellman equations here have unique solutions. I thus, proceed by guess and verify to find these solutions. A similar proofs appear in Bigio (2011).

Define $W^p \equiv w^p k \equiv \left(A + q\bar{\lambda}\left(X\right)\right) k$ and $W^i \equiv w^i k \equiv (p\omega^* + q^c \mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*])k$ as in the main text. The guess for the c-producer's policy function is $k^{p,\prime} = \beta \frac{W^p}{q}$ and $c^p = (1-\beta) W^p$ and that his value function is of the form $V_2^p = \psi^p\left(X\right) + \frac{1}{(1-\beta)} \log W^p$ where $\psi^p\left(X\right)$ is a function of the aggregate state. For k-producers the guess is that $k^{i,\prime} = \beta \frac{W^i}{q^c}$ and $c^i = (1-\beta) W^i$ and that their value function is of the form $V_2^i = \psi^i\left(X\right) + \frac{1}{(1-\beta)} \log W^i$ where $\psi^i\left(X\right)$ is, again, a function of the aggregate state.

Consider the k-producer's problem during the first stage then. Substituting the guess for V_2^i and his constraints yields:

$$\begin{aligned} V_1^i &= \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\psi^i\left(X\right) + \log\left(\left(p\int_0^1 \mathbb{I}\left(\omega\right) d\omega \ + q^c \mathbb{E}_{\phi'}[\lambda(\omega)|\omega < \omega^*(X)]\right) k\right) |X\right] \\ &= \psi^i\left(X\right) + \log k + \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\log\left(\left(p\int_0^1 \mathbb{I}\left(\omega\right) d\omega \ + q^c \mathbb{E}_{\phi'}[\lambda(\omega)|\omega < \omega^*(X)]\right)\right) |X\right] \end{aligned}$$

From this expression, we observe that choosing $\mathbb{I}\left(\omega\right)$ is identical to choosing a cut-off ω^* under which all units of quality lower than this cut-off are sold. This follows from the fact that a solution to the problem above can be attained by an optimal $\mathbb{I}^*\left(\omega\right)$ which is monotone decreasing. Suppose not and assume the optimal plan is given by some $\mathbb{I}'\left(\omega\right)$ whose value is cannot attained by any monotone decreasing policy. It is enough to show that the producer can find another $\mathbb{I}\left(\omega\right)$ that integrates to the same number, that is monotone decreasing and that makes his value weakly greater.

Since $\mathbb{I}'\left(\omega\right)$ and $\mathbb{I}\left(\omega\right)$ integrate to the same number, the amount of IOUs obtained by the k-producer during the first stage, is the same: $p\int_0^1 \mathbb{I}\left(\omega\right) d\omega = p\int_0^1 \mathbb{I}'\left(\omega\right) d\omega$. Now, since $\mathbb{I}\left(\omega\right)$ is monotone decreasing and $\lambda\left(\omega\right)$ is monotone increasing,

$$\int \lambda\left(\omega\right)\left[1 - \mathbb{I}'\left(\omega\right)\right] f_{\phi'}\left(\omega\right) d\omega \leq \int \lambda\left(\omega\right)\left[1 - \mathbb{I}\left(\omega\right)\right] f_{\phi'}\left(\omega\right) d\omega$$

implying that any optimal can be attained by some $\mathbb{I}^*\left(\omega\right)$ monotone decreasing. This implies that, as in the textbook lemons problem, solving for $\mathbb{I}^*\left(\omega\right)$ is equivalent to choosing a threshold ω^*. Substituting this threshold into the objective yields and expression for the optimal cutoff rule:

$$\omega^*\left(X\right) = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left[p\tilde{\omega} \ + q^c\left(X, X'\right)\int_{\tilde{\omega}}^1 \lambda\left(\omega\right) f_{\phi'}\left(\omega\right) d\omega\right] |X\right]. \tag{8}$$

This proofs Proposition 3. I now return to the second stage problems. Taking the solution to (8) as given, we know that the optimal plan for an k-producers sets $x = p\left(X\right)\omega^*\left(X\right)k$. Using the optimal policy for ω^* and these definitions, one can write the second stage Bellman equation without reference to the first stage. To

do this, one can substitute in for x and k in the second stage Bellman equation to rewrite the k-producer's problem as,

$$\max_{c\geq 0, i, k^b \geq 0} \log(c) + \beta \mathbb{E}\left[\pi V_1^i + (1-\pi) V_1^p | X\right]$$
$$c + i + qk^b = p\omega^*(X) k \text{ and } k' = k^b + i + k \int_{\omega^*(X)}^1 \lambda(\omega) f_{\phi'}(\omega) d\omega.$$

Now, since $\left(V_1^i, V_1^p\right)$ are increasing in k', and k^b and i are perfect substitutes, an optimal solution will set $i > 0$ only if $q \geq 1$ and $k^b > 0$ only if $q \leq 1$. This implies that substituting the k-producer's capital accumulation equation into his budget constraint simplifies his problem to:

$$\max_{c\geq 0, k'} \log(c) + \beta \mathbb{E}\left[\pi V_1^i\left(k', X'\right) + (1-\pi) V_1^p\left(k', X'\right) | X\right] \text{ s.t. } c + q^c k' = w^i k.$$

The same steps allow one to write the c-producer's problem as,

$$\max_{c\geq 0, k'} \log(c) + \beta \mathbb{E}\left[\pi V_1^i + (1-\pi) V_1^p | X\right] \text{ s.t. } c + qk' = w^p k.$$

Replacing the definitions of V_1^i and V_1^p into the objective above, and substituting our guess yields V_2^i and V_2^p, we obtain:

$$\max_{c\geq 0, k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^i(X) \text{ s.t. } c + q^c k' = w^i k$$

and

$$\max_{c\geq 0, k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^p(X) \text{ s.t. } c + qk' = w^p k.$$

respectively. In this expressions, $\tilde{\psi}^i(X)$ and $\tilde{\psi}^p(X)$ are functions of X and don't depend on the current periods choice. Taking first order conditions for (k', c) in both problems leads to:

$$c^i = (1-\beta) w^i(\omega^*, X) k \text{ and } k^{i,\prime} = \frac{\beta}{q^c} w^i(\omega^*, X) k$$
$$c^p = (1-\beta) w^i(X) k \text{ and } k^{p,\prime} = \frac{\beta}{q} w^p(X) k$$

These solutions are consistent with the statement of Propositions 1 and 2. To verify that the guess for our value functions is the correct one, we substitute in the optimal policies:

$$\log(1-\beta) w^i(\omega^*, X) k + \frac{\beta}{(1-\beta)} \log \frac{\beta}{q^c} w^i(\omega^*, X) k + \tilde{\psi}^i(X)$$
$$= \frac{\log w^i(\omega^*, X) k}{(1-\beta)} + \psi^i(X) = \frac{\log W^i(k, \omega^*, X)}{(1-\beta)} + \psi^i(X)$$

for some function $\psi^i(X)$. The same steps lead to a similar expression for c-producers. This verifies the initial guess.

B.2 Proof of Lemma 4 and Proposition 5

Lemma 4 and Proposition 5 are proven jointly here. We begin by guessing that $V_1^f(n, X) = v_1^f(X) n$, and $V_2^f(n, X) = v_2^f(X) n$ where $v_2^f(X) = \beta^F \mathbb{E}\left[v_1^f(X) R^b\right]$ if the banker remains alive and $v_2^f(X) = \beta^F R^b n$ if he

dies.

Plugging this guess into the bankers problem yields:

$$\begin{aligned} & \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\, d - e + \mathbb{E}\left[v_2^f\left(X'\right)\left(\Pi\left(X,X'\right)Q + n'\right)|X\right] \\ = & \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\, d - e + \beta^F R^b \mathbb{E}\left[v_1^f\left(X\right)\left(\Pi\left(X,X'\right)Q + n'\right)|X\right] \end{aligned}$$

subject to,

$$\begin{aligned} -\min_{X'} \Pi\left(X,X'\right) Q &\leq n' \\ n' &= n + e - d \end{aligned}$$

Assume that the optimal solution to this problem is characterized by some $e^*(n,X)$ and $d^*(n,X)$ to be determined. In equilibrium, $\Pi\left(X,X'\right)$ is finite. Hence, $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)\right]$ is also finite, provided that the problem has a finite solution. If $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)\right] > 0$ and $-\min_{X'}\Pi\left(X,X'\right) \leq 0$, the banker would set $Q^* = \infty$. But this would imply that in equilibrium $\Pi\left(X,X'\right) \leq 0$ for any X' because there cannot be a future state where firms provide infinite intermediation and there are positive profits. Hence, it is the case that if $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)\right] > 0 \rightarrow -\min_{X'}\Pi\left(X,X'\right) > 0$. Now if this is the case,

$$Q^* = \frac{n'}{-\min_{X'}\Pi\left(X,X'\right)} > 0. \tag{9}$$

If $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)\right] < 0$, the producer optimally sets Q^*. If $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)\right] = 0$, Q^* is indeterminate (but finite). Thus, in either case, $\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)Q^*\right] = 0$.

This implies that for any optimal policy,

$$\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)Q^*\right] = \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)n'}{-\min_{X'}\Pi\left(X,X'\right)}, 0\right\}.$$

Thus, one can substitute this expression into the objective of the firm and express it without reference to Q :

$$\begin{aligned} & \max_{e\in[0,\bar{e}], d\in[0,n]} (1-\tau)\, d - e + n'\beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)}, 0\right\}|X\right] \\ = & \max_{e\in[0,\bar{e}], d\in[0,n]} (1-\tau)\, d - e + (n+e-d)\beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)}, 0\right\}|X\right] \end{aligned}$$

where the second line uses the definition of n'. Now, it is clear from this expression that any optimal financial

policy satisfies,

$$\begin{aligned} e &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max\left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \geq 1 \\ d &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max\left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \leq (1-\tau). \end{aligned}$$

If the inequalities are strict, it is clear that, $e = \bar{e}$ and $d = n$. By linearity, e and d are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied. By assumption $e = \bar{e}$ is never binding. If $d = n$, for some state, d is linear in n. This implies that $e(n, X) = e^*(X)\, n$, $d(n, X) = d^*(X)\, n$ are solutions to the producer's problem.

We now use this results to show that the value function is linear in n. Plugging in the optimal policies into the objective we obtain:

$$\left[(1-\tau)\, d^*(X) - e^*(X) + (1 + e^*(X) - d^*(X))\, \beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{ \frac{v_2^f(X')\,\Pi\left(X,X'\right)}{-\min_{X'} \Pi(X,X')}, 0 \right\} |X \right] \right] n$$

which is a linear function of n.

Returning to the optimal quantity decision, then it is clear that Q can be written as,

$$Q = \frac{1 + e^*(X) - d^*(X)}{-\min_{X'} \Pi(X,X')} n =: Q^*(X)\, n.$$

and clearly, $Q^*(X) = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f(X')\,\Pi(X,X')\,|X \right] \tilde{Q}$ subject to $\Pi\left(X,X'\right)\tilde{Q} \leq n'$. This proves, Proposition 6.

We are ready to show that $v_1^f(X)$ solves a functional equation. Define

$$\tilde{v}(X) = \beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + v_2^f\left(X'\right) \max\left\{ \frac{\Pi\left(X,X'\right)}{-\min_{X'} \Pi(X,X')}, 0 \right\} |X \right]$$

as the marginal value of equity in the bank and note that:

$$v_1^f(X) = \max_{d^*(X)\in[0,1], e\geq 0} (1-\tau)\, d^*(X) - e^*(X) + (1 + e^*(X) - d^*(X))\, \tilde{v}(X).$$

If $\tilde{v}(X) \in ((1-\tau), 1)$, then $v_1^f(X) = \tilde{v}(X)$ because $(d^*(X), e^*(X)) = 0$. If $\tilde{v}(X) \leq (1-\tau)$, then $\mathrm{e}^*(X) = 0$ and we have that,

$$(1-\tau)\, d^*(X) + (1 - d^*(X))\, \tilde{v}(X) = (1-\tau).$$

Finally, if $\tilde{v}(X) = 1$, then, $v_1^f(X) = 1$. This information is summarized in the following functional equation for $v_1^f(X)$:

$$v_1^f(X) = \min\left\{ \max\left\{ \beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) \left\{ 1 + \max\left\{ \frac{\Pi\left(X,X'\right)}{-\min_{X'} \Pi(X,X')}, 0 \right\} \right\} |X \right], (1-\tau) \right\}, 1 \right\}$$

which equals,

$$\min\left\{\max\left\{\mathbb{E}\left[\left(\rho+(1-\rho)\,v_1^f\left(X''\right)\right)\beta^F R^b\left\{1+\max\left\{\frac{\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}\right\}|X\right],(1-\tau)\right\},1\right\}.$$

This functional equation determines the slope of the bankers value function, $v_1^f(X)$. It can be shown that the solution to this functional equation is unique. Assumptions 9.18-9.20 of Stokey et al. (1989) are satisfied by this problem. It remains to show that Assumption 9.5 (part a) is also satisfied. By assumption, X is compact so the only piece left is that X is countable. Because the transition function for the state is an endogenous object, as it depends on an aggregate state, κ. It will be shown that although (d,e) are not uniquely defined, there is unique mapping from ϕ to κ'. By exercise 9.10 in Stokey et al. (1989), together, these assumptions ensures that there is a unique solution to the this functional equation.

B.3 Proof increasing $p(x)$

Let $q(X,X')$, is the market price that solves the market clearing condition given a price under asymmetric information of p. In equilibrium, this price is a function of the previous state, $p(X)$. Thus, through the equilibrium price $p(X)$, $q(X,X')$ defines an equilibrium q, implicitly, as function of the current state $X\prime$ and the previous state X: $q(X',X)\equiv\tilde{}\,(p(X),X')$. Given, X, and the law of motion for $X\prime$, $\tilde{q}(X',X)$ determines the profits for the financial sector given and amount of trade.

Proof. To characterize the key objects $Q(X), p(X)$ and $q(X)$. we need some we need to define some objects. The supply for financial contracts is $Q^s(p,X)=\tilde{\omega}(p,X)\,\kappa$ and $Q^d=\{[]$ and $\tilde{\Pi}(p,X')=\tilde{q}(p,X')\,\mathbb{E}_{\phi'}[\lambda(\omega)\,|\omega>\bar{\omega}(p,X)]$ p. I provide some further characterization of this supply.

$$Q_p^s>0, Q^s(0,X)=0$$

and

$$Q^s\left(\mathbb{E}\left[q^c\left(X'\right)\mathbb{E}_{\phi'}[\lambda(\omega)]\right],X\right)=k.$$

Thus, the supply schedule is invertible and bounded, and thus, we define:

$$P(Q)\equiv p \text{ such that } Q^s(p,X)=Q$$

and

$$P_Q(Q)>0, P(0)=0 \text{ and } P(X)=\mathbb{E}\left[q^c\left(X'\right)\mathbb{E}_{\phi'}[\lambda(\omega)]\right]$$

This proposition establishes the existence of a well behaved supply function: $P(Q)$ is bounded, differentiable and increasing. □

B.4 Proof of Proposition 6

To pin down q, fix any sequence of states (X,X'), and let $\bar{\omega}=\omega(X)$. We begin the proof assuming $q>1$ so that $D^i=0$. Market clearing in stage 2 requires $D^p(X,X')=S(X)=\mathbb{E}_{\phi'}[\lambda(\omega)\,|\omega\leq\bar{\omega}]\,\bar{\omega}\pi K$. By Proposition 1, we can integrate across the c-producer's policy functions to obtain an expression for $D^p(X,X')$ as a function of q:

$$\beta \int \left[\frac{W^p(k,x,X)}{q} - \bar{\lambda} k \right] \Gamma^c(dk) = \beta \frac{A + q\bar{\lambda}}{q} (1-\pi) K$$

By market clearing, q be such that:

$$\left[\beta \frac{A + q\bar{\lambda}}{q} - \bar{\lambda} \right] (1-\pi) K = \mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega \leq \bar{\omega} \right] \bar{\omega} \pi K.$$

Manipulating this expression leads to the value of q that satisfies market clearing:

$$q = \frac{\beta A (1-\pi)}{\mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega \leq \bar{\omega} \right] \bar{\omega} \pi + (1-\pi)(1-\beta) \bar{\lambda}}$$

Recall now that this expression is valid only when $q > 1$, because capital good producer's are not participating in the market. Thus, the expression is only true for value of

$$\beta A \left[\frac{\pi}{(1-\pi)} \mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega} + (1-\beta) \lambda \right]^{-1} > 1. \tag{10}$$

If $q = 1$, then it must be the case that the total demand for capital must be larger than the supply provided by financial firms. $D^i(X, X')$ in this case is obtained also by integrating across the demand for capital of k-producer's given in Proposition 1. Thus, for a stage one price p, this demand is given by

$$D^i + I = \beta p \bar{\omega} \pi K - (1-\beta) \mathbb{E}_{\phi'} \left[\lambda(\omega) | \lambda\omega > \bar{\omega} \right] (1-\bar{\omega}) \pi K \text{ for } q = 1$$

The corresponding condition is that,

$$\beta p \bar{\omega} - (1-\beta) \mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega > \bar{\omega} \right] (1-\bar{\omega}) \pi + \left[\beta A - (1-\beta) \bar{\lambda} \right] (1-\pi) \geq \pi E_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega} \tag{11}$$

where, the aggregate capital stock has been canceled from both sides. If the condition is satisfied, then $q = 1$, and

$$D^i(q,p) = \pi E_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega} - \left[\beta R - (1-\beta) \lambda \right] (1-\pi)$$

and

$$I = \left[\beta p^i \bar{\omega} - (1-\beta) \mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega > \bar{\omega} \right] (1-\bar{\omega}) \right] \pi - D^i(q,p)$$

If (10) and (11) are violated, this implies $q < 1$ and that $I = 0$. The corresponding market clearing condition is obtained by solving q from:

$$\begin{aligned} & \left[\frac{\beta p \bar{\omega}}{q} - (1-\beta) \mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega > \bar{\omega} \right] (1-\bar{\omega}) \right] \pi + \left[\frac{\beta A}{q} - (1-\beta) \bar{\lambda} \right] (1-\pi) \\ \geq \quad & \pi E_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega}. \end{aligned}$$

We can collect the terms where q shows in the denominator to obtain,

$$\frac{\beta (p \bar{\omega} \pi + A)}{q} = \pi E_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega} + (1-\beta) \left[-\mathbb{E}_{\phi'} \left[\lambda(\omega) | \omega < \bar{\omega} \right] \bar{\omega} \pi + \bar{\lambda} \right]$$

The solution is given by:

$$q = \frac{\beta\left(\pi p^i \bar{\omega} + A\right)}{\left(\beta E\left[\lambda\left(\omega\right)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}\pi + \left(1-\beta\right)\bar{\lambda}\right)}$$

The formula in Proposition 6 corresponds. . Moreover, the demand function is weakly decreasing so for each p, X there will be a unique q satisfying the market clearing condition.

From (**??**) we observe that the shock affects the $\tilde{q}\left(X', X\right)$ and $\mathbb{E}\left[\lambda\left(\omega\right)|\omega < \bar{\omega}\right]$. From Proposition (**??**), we can express the profit function in the following way:

$$\Pi\left(X, X'\right) = \max\left\{\left(1-\pi\right)\beta A \frac{\pi\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}}{\pi\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\bar{\omega} < \bar{\omega}\right]\bar{\omega} + \left(1-\beta\right)\lambda}, \tilde{\pi}(X, X')\right\}$$

where

$$\tilde{\Pi}(X, X') = \min\left\{1, \left(\pi p^i + A\left(1-\pi\right)\right)\frac{\pi\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}}{\left(\frac{(1-\beta)\lambda}{\beta} + \pi\bar{\omega}\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\bar{\omega} < \bar{\omega}\right]\right)}\right\}$$

Since both functions are increasing in $\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega < \bar{\omega}\right]$ in the conditional expectation, we know by Assumption **A1,** that this functions are decreasing in the shock ϕ'. Thus, $\Pi\left(X, X'\right)$ is decreasing in ϕ'.

B.5 Proof of monotone relation between κ and ω

Observe that $\mathbb{E}\left[\Pi\left(X', p\right) - p\right] > 0$, is continuous in p.Thus, there is a sufficiently small $\varepsilon > 0$ increase in p such that the inequality still holds. Since the inequality implies that capacity constraints bind in equilibrium, then, market clearing implies that there exists some $\epsilon\left(\varepsilon\right)$, such that $p(X) + \varepsilon = P\left(\frac{s(\kappa+\epsilon(\varepsilon))}{\pi\left(p(X)+\varepsilon, X'_{\min}\right)}\right)$. In an un-improvable, it must be the case that $\pi\left(p(X) + \varepsilon, X'_{\min}\right)$ is decreasing, because otherwise there would have existed a larger equilibrium with a higher price. Thus, $\epsilon\left(\varepsilon\right)$ must increase. This, implies that for a given κ, we can find a small enough increase in κ, so that the equilibrium price increase.

If $\mathbb{E}\left[\Pi\left(X', p\right) - p\right] = 0$ and constraint does not bind, then, there is always a small enough increase in κ, such that capacity constraints don't bind, and therefore, the equilibrium price remains constant. If $\mathbb{E}\left[\Pi\left(X', p\right) - p\right] = 0$ and capacity constraints bind, then increase in κ will relax the binding constraint. Either Q remains the same or increases. Thus, p must increase.

B.6 Proof of efficiency

Proof. The necessary condition: $(1-\tau_d) \leq \beta^F R^b$. Suppose not, then for any state $\Pi\left(X', X\right) = 0$, then, it is convenient to pay dividends.

$$\begin{aligned}
\tilde{v}\left(X\right) &= \beta^F R^b\left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max\left\{\frac{\mathbb{E}[\tilde{v}\left(X'\right)\Pi\left(X, X'\right)]}{-\min_{\tilde{X}}\Pi\left(X, \tilde{X}\right)}, 0\right\}\right] \\
&< \beta^F R^b\left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max\left\{\frac{\mathbb{E}[\Pi\left(X, X'\right)]}{-\min_{\tilde{X}}\Pi\left(X, \tilde{X}\right)}, 0\right\}\right] \\
&= \beta^F R^b \mathbb{E}[\tilde{v}\left(X'\right)] \\
&< \beta^F R^b
\end{aligned}$$

where the first line follows from the definition of $\tilde{v}(X)$ and v_2^f, the second follows from the fact that $\tilde{v}(X') \leq 1$ and the third from the assumption that an efficient equilibrium satisfies $\mathbb{E}[\Pi(X, X')] = 0$ and the last inequality uses $\tilde{v}(X') \leq 1$ once more. Then, if the condition is not satisfied, $\tilde{v}(X) < \beta^F R^b < (1 - \tau_d)$. This fact in turn implies that any state with financial risk capacity κ consistent with efficient intermediation is reflected to another state with $\mathbb{E}[\Pi(X, X')] > 0$.

The sufficient condition is obtained reversing the equalities.

$$\begin{aligned}
\tilde{v}(X) &= \beta^F R^b \left[\mathbb{E}[\tilde{v}(X')] + \max \left\{ \frac{\mathbb{E}[\tilde{v}(X')\,\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&> \beta^F R^b \left[\mathbb{E}[\tilde{v}(X')] + \max \left\{ \frac{(1-\tau_d)\,\mathbb{E}[\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&> \beta^F R^b \mathbb{E}[\tilde{v}(X')]
\end{aligned}$$

The first line follow from the definition of $\tilde{v}(X)$. The second uses that $(1 - \tau_d)\,\tilde{v}(X)$ has an upper bound. The third uses The hypothesis that $\mathbb{E}[\Pi(X, X')] = 0$. With this inequality, it is enough to argue that there will always exist some κ such that $\mathbb{E}[\tilde{v}(X')]$ is sufficiently above $(1 - \tau_d)$ such that state is not reflected. $\square$

C Data Appendix

To be completed.