Liquidity Risk and the Macroeconomic Costs of Financial Crises

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Abstract

I integrate a financial sector engaging in liquidity transformation into a New Keynesian dynamic stochastic general equilibrium model. In particular, financial firms issue demandable debt against illiquid assets. Liquidity transformation subjects financial firms to the risk of bank-run-like withdrawals of creditor support. Anticipating the possibility of withdrawals, financial firms hold government debt as a reserve of liquid assets, and issue a mix of demandable and maturity-matched debt that optimally balances the lower interest rate on demandable debt against the liquidity risk it introduces.

In the model, an unexpected increase in withdrawal risk sparks a flight to quality that reduces money velocity and nominal interest rates on government debt. Issuance of risky demandable debt declines. The macroeconomic effects of adverse shocks to the financial sector are amplified, even relative to Bernanke, Gertler, and Gilchrist (1999). Furthermore, the link between intermediation and the supply of money substitutes strengthens comovement between investment and consumption. In contrast with the standard New Keynesian model, the central bank in my model can decrease the severity of the output decline by conducting open market purchases to increase the privately available supply of *both* money and government debt, relative to the supply of illiquid assets.

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1 Introduction

i. Why do financial crises trigger sharp declines in interest rates on short-term government debt and in money velocity?

ii. How can concentrated losses in the financial sector generate a disproportionate economic contraction?

iii. Why don't households substitute towards consumption when falling intermediation capacity causes investment to contract?

iv. How do a central bank's open market purchases of risky assets influence the outcome of a financial crisis?

This paper develops a New Keynesian dynamic stochastic general equilibrium (DSGE) model that combines endogenous production of private-sector money substitutes, in the form of demandable debt, and a precautionary demand by the financial sector for reserves of highly liquid assets, such as government debt. The link between risky intermediation and the markets for money and government debt produces plausible explanations for the consequences of financial crises.

The key element of the model is that a risky financial sector engages in liquidity transformation. Financial firms issue demandable (or short-term) debt, because creditors obtain utility from holdings of demandable debt. This nonpecuniary benefit represents in reduced form the value that households place on the money-like properties of demandable debt. One possible story, offered by Diamond and Dybvig (1983), is that households face idiosyncratic consumption risk and so value the option to liquidate claims against the financial firm.

However, liquidity transformation also subjects financial firms to a risk of bank run-like withdrawals of creditor support when creditors obtain negative signals about likely payoffs to the firms' assets. If in the short-run assets cannot be liquidated at their full net present value, creditor withdrawals impose losses on remaining creditors and on the firms themselves.

Financial firms and creditors write contracts that optimally trade off liquidity on both sides of the balance sheet. On the liability side of the balance sheet, financial firms choose a mix of short-term and maturity-matched financing to optimally balance creditors' demand for money-like claims against exposure to liquidity risk. On the asset side of the balance sheet, financial firms hold reserves of liquid assets (e.g., government debt), so that withdrawals induce fewer costly liquidations of imperfectly liquid assets.

Endogenous, risky liquidity transformation generates plausible rationalizations of the macroeconomic effects of financial crises that are absent in most other macroeconomic models:

i. "Flights to quality" arise from unexpected increases in liquidity risk.

A heightened probability of early redemptions boosts the financial sector's desired ratio of liquid assets to demandable debt. Furthermore, financial firms borrow less and shift towards maturity-matched financing in future contracts, thus reducing the private production of money substitutes. Both these effects increase household demand for money balances[1] and government debt. As a result, money velocity,[2] the price level, and the equilibrating interest rate on government debt fall.

ii. Liquidity risk amplifies the costs of adverse shocks to the financial sector.

Take, for example, an unexpected decline in the average productivity of financial firms' assets. The resulting output contraction is almost three times as deep for the present model as it is for an analogous model in the mold of Bernanke, Gertler, and Gilchrist (1999) (henceforth BGG). Where does this deeper decline come from? First, the risk of early redemptions increases as more creditors obtain negative signals about the quality of their debtors' assets. This surge in liquidity risk intensifies contractions in credit. Because the model's financial firms face binding constraints on leverage, said contractions in turn reduce the price of capital, net worth, and investment. Lower net worth then reinforces the flight to quality by increasing creditors' exposure to losses from early liquidations. Second, when intermediation shrinks, households attempt to build money balances to supplement their now-smaller holdings of demandable debt. As a result, consumption falls.

iii. Endogenous liquidity transformation strengthens comovement between investment and consumption.

A slump in intermediation suppresses both investment and the private production of money substitutes. The latter effect naturally depresses consumption. In a BGG-type model, by contrast, shocks that strongly reduce net worth constrain investment and lead households to substitute towards consumption. The link between intermediation and the supply of money substitutes overturns this relationship.

[1] In the model, "money" is any riskless medium of exchange. This might correspond, e.g., to currency and insured checking deposits. Government debt and risky demandable debt are partial substitutes for money, because they possess money-like qualities.

[2] The supply of money increases due to expansionary open market operations by the Federal Reserve.

iv. Open market operations that increase the relative supply of liquid assets mitigate the liquidity effects of heightened withdrawal risk.

Traditional open market operations swap government debt for money; but because financial firms will always prefer interest-bearing liquid assets, these swaps do not increase the privately-available supply of the reserve asset. I contemplate two alternative policies. First, I consider a unified fiscal and monetary authority. This policy maker uses tax rebates to increase the aggregate supply of government debt, and expands the money supply by partially monetizing this deficit. Second, I consider a policy that the monetary authority could implement on its own: the central bank prints money, sells some of its holdings of government debt, and directly purchases risky assets. Both these policies stabilize output following an unexpected increase in redemption risk.

During the 2008-2009 financial crisis, the Federal Reserve reduced its holdings of traditional securities and purchased large volumes of Agency debt and mortgage-backed securities.[3] The Federal Reserve also lent Treasury Securities against a variety of risky securities, including mortgage-backed securities. These unconventional interventions are consistent with policy prescriptions that arise naturally from the model.

The model also provides insights for interest rate targeting, given traditional open market operations. In particular, the monetary authority should reduce its interest rate when the wedge between the return to capital and the return to government debt increases. A larger wedge indicates higher costs of the financial friction. Lowering the interest rate reduces financial firms' borrowing costs, which in turn stabilizes intermediation, investment, and output.

The story that emerges from the model has much in common with Friedman and Schwartz's (1963) analysis of the Great Depression. Friedman and Schwartz document increases in retail deposit banks' demand for precautionary reserves and declines in the supply of retail deposits. When the central bank does not appropriately expand the money base, the collapse in the money multiplier induces a deflationary contraction. I show that analogous dynamics can obtain even when the money base is flexible and the central bank follows a standard Taylor rule. This is because, as in Cochrane (2011), a financial crisis can induce heightened demand for both money and government debt; traditional central bank swaps of money for government debt fail to sate this demand. I also show how a central bank can alter its open market operations and its interest rate target to mitigate the macroeconomic

[3] Federal Reserve balance sheet data is available from the Cleveland Federal Reserve, at http://www.clevelandfed.org/research/data/credit_easing/index.cfm.

effects of financial crises.

Several authors have explored the consequences of liquidity mismatch in stylized general equilibrium models; see, e.g., Allen and Gale (1998, 2004, 2004b). The present model incorporates risky liquidity transformation in a tractable way into a medium-scale DSGE model and explores its qualitative implications for the propagation of shocks and the implementation of effective monetary policy. Because it can be integrated into a larger DSGE model, risky liquidity transformation need no longer be explored in isolation from the rest of the business cycle. Instead, it can help DSGE models replicate a rich set of observed business cycle comovements.

Authors studying episodes when the nominal interest rate hits zero often reduce rates by exogenously increasing households' time discount; see, e.g., Christiano, Eichenbaum, and Rebelo (2011). This is the classic "paradox of thrift" popularized by Keynes (1936); interest rates plunge because households suddenly want to save. By contrast, in the present model, relatively modest shocks to the financial sector sharply reduce equilibrium interest rates by sparking a flight to quality. Households substitute away from privately produced money substitutes, and towards government-backed money and debt.

In the present model, government deficits can help stabilize output without any additional spending; fiscal policy has a monetary effect because nominally risk free government debt possesses money-like properties. An interesting corollary is that empirical estimates of tax and spending multipliers (e.g., Barro and Redlick (2011) and Romer and Romer (2010)) can confound traditional incentive and aggregate demand effects with a monetary effect from deficits.

The importance of open market purchases in my model contrasts with standard New Keynesian DSGE models, in which explicitly modeling open market operations–indeed, including money at all–negligibly affects economic outcomes (see, e.g., Eggertsson and Woodford 2003, and Woodford 2007). Furthermore, augmenting the aggregate supply of financial firms' reserve asset can increase aggregate lending, without any limited participation assumptions that force financial firms to absorb the additional supply, as in Lucas (1990) and Christiano (1991). These differences arise because, in the present model, balances of government debt help intermediaries guard against costly liquidations.

The rest of the paper proceeds as follows. Section 2 outlines the model's financial sector and develops intuition for the behavior of the contract with liquidity mismatch. Financial firms' reserve ratio is increasing in the probability that creditors receive bad news about future payoffs to risky assets. Additionally, financial firms shift towards maturity-matched financing when liquidity risk is high. Section 3 specifies the rest of the macroeconomic model; the non-financial sector components

match those of a standard, medium-scale New Keynesian DSGE model in the vein of Christiano, Eichenbaum, and Evans (2005) (henceforth CEE). Section 4 specifies functional forms, steady state parameter values, and stochastic processes. Section 5 presents and discusses simulation results. Section 6 concludes.

2 Financial contract

The model economy is inhabited by a continuum of measure $\eta^h \in (0,1)$ of infinitely-lived households and a continuum of measure $\eta^b = 1 - \eta^h$ of bankers. There are two types of goods: intermediate and final. Labor and capital services are combined to build intermediate goods, which are in turn used to produce nondurable final goods. Final goods can be invested to build physical capital, or consumed by households for a utility benefit. Entrepreneurs build capital services from physical capital, which depreciates gradually. Intermediate goods firms are Calvo-type monopolists; this feature introduces short-term monetary non-neutralities.

I first introduce the financial sector, in order to highlight the novel features of the model. I then detail the other sectors of the model economy.

2.1 Description

The elements of the financial sector in the model are as follows. There is a population of entrepreneurs who have the special ability to build capital services used to make intermediate goods. Entrepreneurs need funds to purchase the physical capital from which capital services are generated. However, households in the economy do not know how to connect with entrepreneurs that need funds. Financial firms (which, for convenience, I will sometimes refer to as "banks")[4] serve as intermediaries between households and entrepreneurs.

Figure (1) illustrates the formation of the contract. Each financial sector firm uses its own net worth, n_t, and funds from households to buy equity in capital services producers, in a manner similar to Gertler and Kiyotaki (2010). I assume there is no conflict of interest between risk neutral banks and the entrepreneurs in which the banks invest. Thus, I may model the banks as directly purchasing capital K^b_{t+1} at price q_t and operating the technology for producing capital services from physical capital.[5] Other firms then combine capital services with labor to produce

[4] My subject includes any class of agents engaging in liquidity transformation. "Financial firms" and "banks" are the classes of firms that in the real world are most often associated with liquidity transformation.

[5] I focus on the relationship between financial firms and their creditors. One could easily imagine an extension of the current model that introduces a conflict of interest between capital services producers and

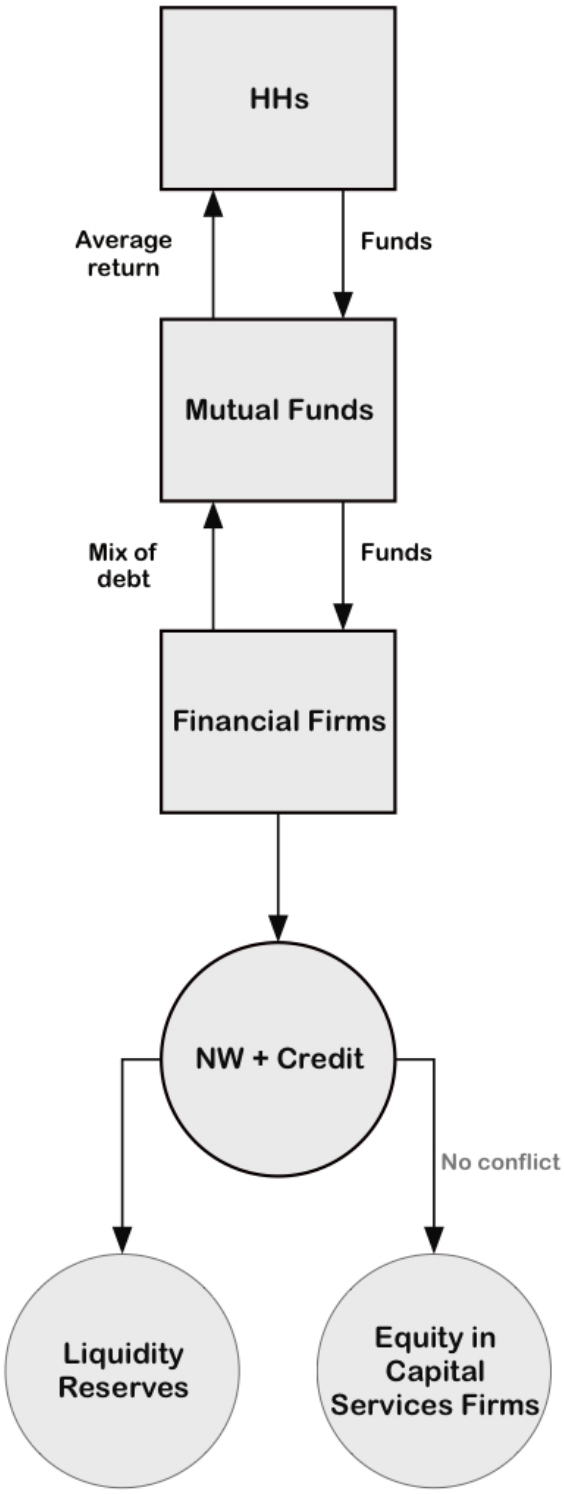

Figure 1: Contract formation

intermediate goods.

Banks' portfolios of investments have a positive expected net present value (NPV). However, banks employ proprietary methods in selecting investments, so portfolio performance randomly varies across banks. Financial firms also enjoy limited liability, which caps owners' realized losses at the value of their investment. I further assume that households must pay a monitoring cost to observe a bank's realized revenue.

As in Townsend (1979), a debt contract between households and financial firms economizes on monitoring costs.[6] In a debt contract, banks bear the first losses on their assets. When losses pierce the equity cushion, the bank defaults and households seize and monitor assets at a cost. Because insiders (bankers) must compensate outsiders (households) for these monitoring costs, external funding is more expensive than internal funding from retained earnings.

The key innovation in the model is the introduction of risky liquidity transformation into the debt contract between households and banks. A financial firm can issue maturity-matched debt and demandable debt. Under demandable debt, each creditor has the right to demand redemption at par of any funds extended at any time, including before the investments bear fruit. Creditors obtain a transactions utility from this money-like quality of demandable debt.[7] As a result, creditors are satisfied by a smaller pecuniary return on demandable debt. By issuing liquid liabilities, then, the bank can lower its borrowing costs.

Second, I assume that risky assets that have not yet borne fruit are imperfectly liquid; that is, claims to the assets' future income cannot be sold in advance at their full NPV. There are a variety of possible rationalizations for an early liquidation discount, including asymmetric information (see, e.g., Akerlof (1970)), uncertainty aversion by potential buyers who face Knightian uncertainty (Knight (1921)) about the quality of assets being liquidated, or management inefficiencies from the transfer of ownership and associated loss of "soft information" possessed by the manager/originator (Shleifer and Vishny (1988)).

The early liquidation of imperfectly liquid risky assets imposes costs on remaining creditors and equity holders. The more assets that must be liquidated and the deeper is the fire sales discount, the better the remaining assets must perform to yield a given amount of revenues and to avoid default.

financial firms; this would restrict capital services producers' leverage.

[6] Debt contracts have the additional advantage of being relatively information-insensitive; see, e.g., Gorton and Pennacchi (1990).

[7] Section 3 details household preferences.

Given this liquidity risk, there is a demand by financial firms for a precautionary reserve of perfectly liquid assets, such as cash or nominally-riskless government debt. Liquidity reserves permit financial firms to meet at least some early creditor redemptions without destroying value by selling imperfectly liquid assets at a discount from their full NPV. In the present model, money and government debt are the only perfectly liquid assets. As long as the nominal interest rate is strictly positive, the banker will hold liquidity reserves composed of holdings of government debt, b^b_{t+1} (the b-superscript will differentiate bank from household holdings of government debt).

The financial firm will choose the optimal contract. In exchange for lending one final good to the banker, a creditor receives a mix of demandable and maturity-matched debt. The mix is indexed by the fraction ξ^D_t of debt that is demandable. Demandable debt is characterized by the right to demand redemption and a gross interest rate R^D_{t+1} owed on all credit not-withdrawn early. Maturity-matched debt enjoys no redemption right and bears gross interest rate R^L_{t+1}. Additionally, the contract between a banker and the households specifies capital purchases K^b_{t+1}, balances of liquid assets b^b_{t+1}, and fire-sales-market transactions, which I describe below. As in BGG, the promised interest rates are contingent on the realized time $t+1$ aggregate state. Knowledge of the $t+1$ aggregate state arrives immediately after signing the contract and purchasing assets. I show in the online appendix[8] that the optimal contract in this environment will not condition on idiosyncratic risk.

2.2 Contract risk and outcomes

I now detail the risks faced by financial firms and the resolution of the contracts between households and financial firms.

2.2.1 The resolution of idiosyncratic risk

Figure (2) illustrates the resolution of bank b idiosyncratic risk. After project initiation but before project completion, either "good" or "bad" news arrives at each bank. Bad news indicates that a bank's assets are distressed in that they have a low conditional expectation. I will sometimes refer to banks receiving good news as "sound banks," and banks receiving bad news as "distressed banks."

In particular, the capital services produced by bank b capital are $\omega^b_{t+1}K^b_{t+1}$ where

$$\omega^b_{t+1} \sim \text{log-}\mathcal{N}\left(\mu + \omega^b_{t,n}, \sigma^2_t\right)$$

[8] Available at depot.northwestern.edu/~ajn371/indexjm.html.

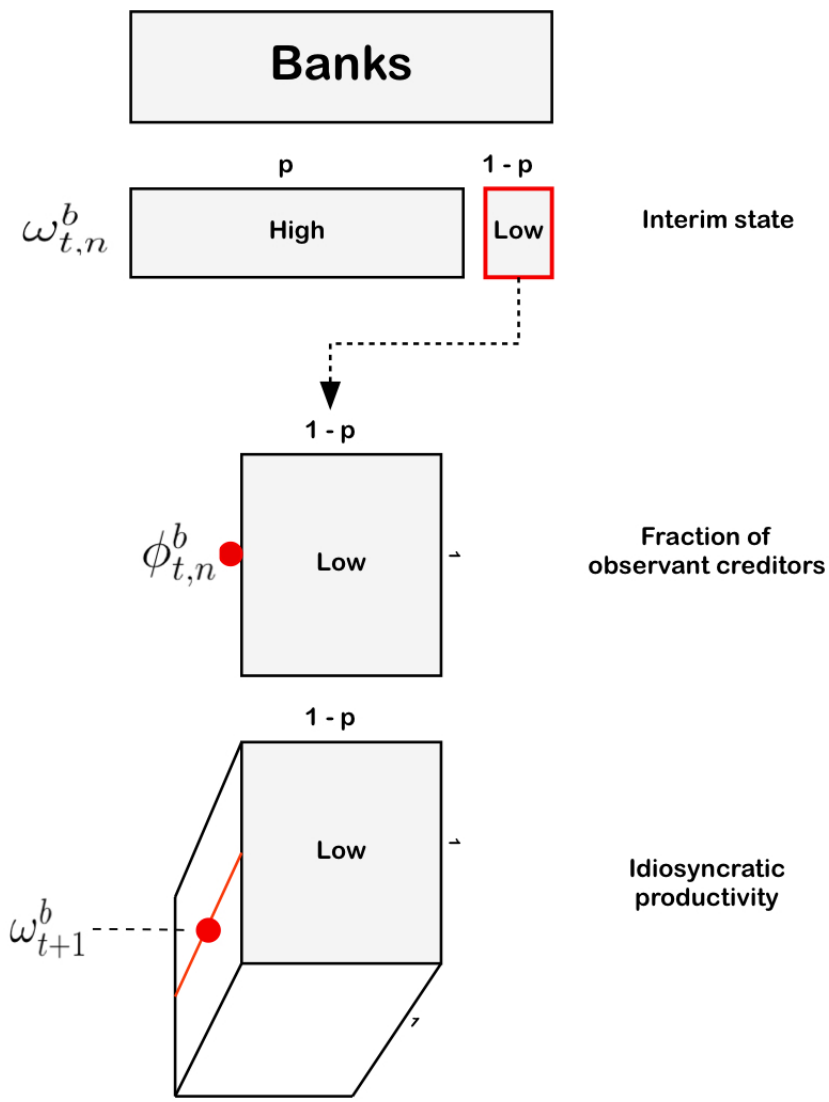

Figure 2: Resolution of uncertainty

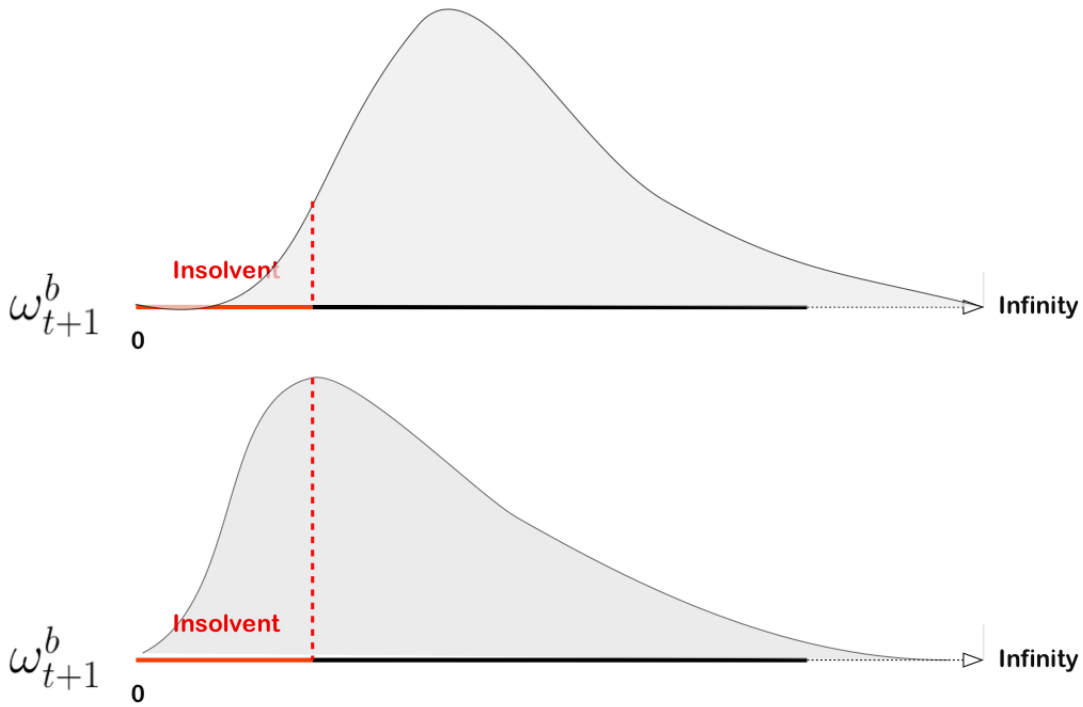

Figure 3: Good and bad news
Top panel: $\omega^b_{t,n} = \omega^H_t$ Bottom panel: $\omega^b_{t,n} = \omega^L_t$

with

$$\omega^b_{t,n} \in \left\{\omega^L_t, \omega^H_t\right\}, \ \omega^H_t > 0 > \omega^L_t, \ p_t := \Pr_t\left(\omega^b_{t,n} = \omega^H_t\right)$$

Figure (3) illustrates the two possible t, n-conditional probability density functions of idiosyncratic asset productivity. The bank b-specific mean $\mu + \omega^b_{t,n}$ of the lognormal distribution is realized before ω^b_{t+1} itself, but after the arrival of knowledge of the $t+1$ aggregate state; the t, n-subscript denotes the "night" between dates t and $t+1$.

A bank-idiosyncratic, random fraction $\phi^b_{t,n} \sim F^\phi_t(\cdot)$ of creditors[9] are notified upon resolution of the realization of asset productivity.[10] Each bank now has two kinds of creditors: "observant" creditors who know $\omega^b_{t,n}$, and not observant creditors, who do not know whether their bank is sound. Observant creditors decide how much of their holdings of demandable debt to redeem based on this information.

[9] I will assume later that $F^\phi_t(\cdot)$ is a beta distribution with range parameter ϕ_1 equal to unity. I calibrate the shape parameter, $\phi_{2,t}$.

[10] In order to avoid ex post heterogeneity across household creditors, I assume in the background that creditors invest in banks through a continuum of mutual funds. Thus, each household's investment is spread amongst different mutual funds, who in turn each invest their funds across the banks. The household's money manager at each mutual fund then observes any news and decides whether or not to redeem credit extended. I assume no conflict of interest between households and money managers.

2.2.2 Contract resolution

I focus on parameterizations such that, if the news is bad (i.e., $\omega_{t,n}^b = \omega_t^L$), each observant creditor will individually desire to redeem all monies she lent to the bank. If the news is good, observant creditors make no redemptions. Creditors are physically separated from one another for the duration of the contract, and so those creditors not notified of $\omega_{t,n}^b$ infer no information and make no redemptions. Each withdrawal is owed either one unit of currency (in a nominal contract) or one unit of final goods (in a real contract). Let R_{t+1}^W denote the final goods owed per withdrawal.

A distressed bank may sell capital services projects at price $q_{t,n}$ to sound banks in a fire sale. In equilibrium, $q_{t,n}$ equals the purchasing banks' marginal willingness to pay. The purchasers' marginal willingness to pay is less than the asset's objective NPV. The fire sale discount reflects two components. First, because purchasing banks shift their portfolio mix towards risky assets, the price must compensate purchasing banks for their increased exposure to idiosyncratic risk. Second, I assume that the purchasers pay a final goods cost proportional to the final goods value of the asset purchased; let μ^{liq} denote the constant of proportionality. The particular reason for this penalty on early liquidations does not matter much for the dynamics of the general equilibrium in a linearized economy. What matters is the existence of such a penalty. The final goods penalty used here can therefore be thought of as a generic, reduced-form representation of a variety of stories for why early liquidations of risky assets are costly.

Figure (4) summarizes the mapping from the realization of distressed bank b risk into contract outcomes. When $\phi_{t,n}^b$ is sufficiently small, distressed banks draw down reserves of perfectly liquid assets to meet withdrawals $W_{t,n}^b$. As $\phi_{t,n}^b$ rises, perfectly liquid assets are eventually exhausted, at which point the bank sells its capital services projects at a fire sales penalty. Eventually, the bank may run out of assets, at which point it must default on its obligation to meet redemptions.[11] In this case, the bank is "illiquid." The banker also defaults if, upon realization of ω_{t+1}^b, revenues are insufficient to pay creditors promised principle and interest; in this case, the bank is "insolvent."

In the event of default - due either to illiquidity or insolvency - all assets are taken into receivership. Risky assets are monitored at a cost proportional to revenues recovered. The constant of proportionality $\mu > 0$ will be sufficiently large that

[11] I preclude banks from entering ex ante into liquidity insurance schemes by assuming that it is costly for each bank to observe another bank's state.

There is also no night-time market for interbank funds. Banks with excess reserves would only be willing to lend to banks with reserve shortages if their debt claims were sufficiently senior to the claims of the bank's existing creditors. But if the bank's existing creditors were notified of this change in their rights, they would all withdraw their support.

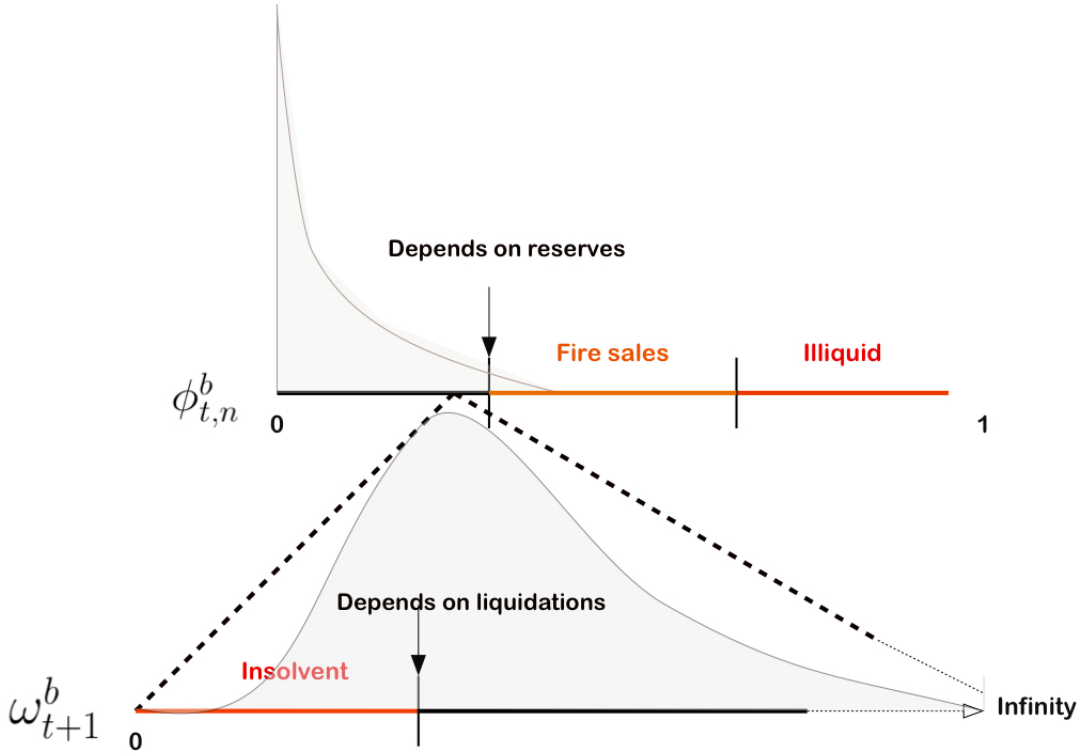

Figure 4: Contract outcomes

household creditors do not desire to engage in a contract that induces bankruptcy with probability one. All units of credit not previously redeemed share equally in recovered revenues.

After project completion, liquid and solvent banks rent capital services to intermediate goods firms and sell the capital left over after depreciation, yielding their new net worth. Bankers are required to consume their entire own net worth and exit the economy with probability $1-\gamma$ at each date. A sufficiently high probability $1-\gamma$ of exit prevents the bankers from accumulating net worth sufficient to self-finance desired investment. Liquid and solvent banks who do not exit use own and borrowed funds to again purchase capital at the end of the period, and the process repeats. Figure (5) reviews the timeline of the contract.

The contract will be optimal under the simplifying assumptions made. For further detail, see the appendix. The intuition is that by altering its mix of liabilities and assets, a financial firm affects the distribution of capital income it faces. A debt contract still efficiently economizes on costly monitoring. Last, private information and costly information acquisition prevent indexation to $\omega^b_{t,n}$.

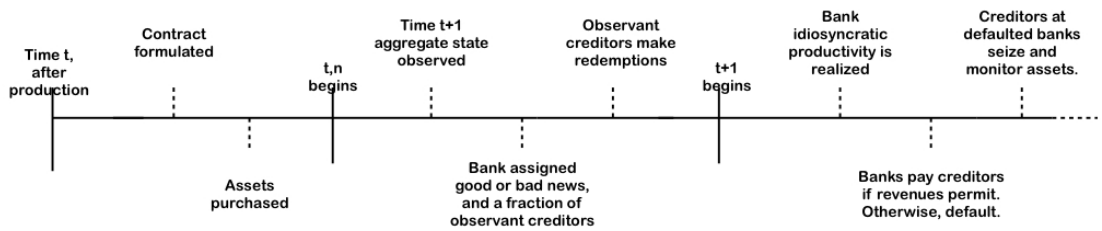

Figure 5: Contract timeline

2.3 Details of the contracting problem

The contract between financial firms and their creditors maximizes the expected value of financial firm profits. Maximization is subject to the following constraints. First, financial firms must meet withdrawal demands. Second, because financial firms are perfectly competitive, the demandable and maturity-matched credit must deliver–on average–market rates of return of $R_{t+1}^{AVG,D}$ and $R_{t+1}^{AVG,L}$, respectively. In a real contract, it is assumed that these rates of return are invariant to the aggregate state. In a nominal contract, only the corresponding nominal returns are invariant to the aggregate state. The relevant market rates are determined by household optimality conditions, covered in Section 3.

2.3.1 The requirement to meet withdrawals

Denote by $K_{t+1,n}^b$ bank b's holdings of capital after fire sale transactions have occured between periods t and $t+1$. Similarly define $b_{t+1,n}^b$. The requirement that financial firms meet any redemption requests imposes the constraint that

$$\frac{R_{t+1}^g}{\Pi_{t+1}}\left(b_{t+1}^b - b_{t+1,n}^b\right) + q_{t,n}\left(K_{t+1}^b - K_{t+1,n}^b\right) \geq R_{t+1}^W W_{t,n}^b \tag{1}$$

If bank b is a distressed bank, $b_{t+1,n}^b$ and $K_{t+1,n}^b$ reflect forced sells needed to meet early creditor withdrawals; in the optimal contract, government debt will be liquidated first. If bank b is a sound bank, there are no withdrawals. In this case, $K_{t+1,n}^b$ is chosen optimally; net purchases are financed by sales of government debt. In equilibrium, $K_{t+1,n}^b > K_{t+1}^b$ as sound banks buy up assets liquidated by distressed banks.

Next, note that net purchases in the fire sale market mean that sound banks will hold positive amounts of both sound and distressed assets. Let $\tilde{\omega}_{t+1}^b$ be a draw from log-$\mathcal{N}\left(0, \sigma_t^2\right)$. Then I assume that the mixture-productivity $\hat{\omega}_{t+1}^b$ at a sound bank b

may be written as

$$\hat{\omega}_{t+1}^{b} = \left[\left(\frac{K_{t+1}^{b}}{K_{t+1,n}^{b}} \right) \exp\left(\mu + \omega_t^H\right) + \left(1 - \mu^{liq}\right) \left(\frac{K_{t+1,n}^{b} - K_{t+1}^{b}}{K_{t+1,n}^{b}} \right) \exp\left(\mu + \omega_t^L\right) \right] \times \tilde{\omega}_{t+1}^{b}$$

Thus, I have assumed that the sound bank draws a single $\log\left(\tilde{\omega}_{t+1}^{b}\right)$ for all of its capital assets. The mix of capital then determines the mean to which this draw is added. If no net purchases were made, we would have $\hat{\omega}_{t+1}^{b} \sim \text{log-}\mathcal{N}\left(\mu + \omega_t^H, \sigma_t^2\right)$. The term $\left(1 - \mu^{liq}\right)$ subtracts out that portion of the value of assets purchased at fire sale that the buyer loses due to the illiquidity of capital. Including μ^{liq} in the definition of $\hat{\omega}_{t+1}^{b}$ will simplify all future equations.

At a distressed bank, let $\hat{\omega}_{t+1}^{b} = \omega_{t+1}^{b}$.

2.3.2 The return to credit

Because creditors experience a discontinuous drop in net income when the financial firm defaults, the instance of default is a key determinant of the average payoff to creditors. Let $\bar{\omega}_{t+1}^{b}$ denote the insolvency cutoff at bank b; i.e., the realization of risky asset productivity $\hat{\omega}_{t+1}^{b}$ such that the bank will just exhaust its income paying back its creditors. Then $\bar{\omega}_{t+1}^{b}$ satisfies

$$R_{t+1}^{*} \times \left(q_t K_{t+1}^{b} + b_{t+1}^{b} - n_t\right) = \tag{2}$$

$$\bar{\omega}_{t+1}^{b} \times R_{t+1}^{K} q_t K_{t+1,n}^{b} + \frac{R_{t+1}^{g}}{\Pi_{t+1}} b_{t+1,n}^{b}$$

where R_{t+1}^{*} is the weighted average promised interest rate:

$$R_{t+1}^{*} = \xi_t^D \left(1 - w_{t,n}^{b}\right) R_{t+1}^{D} + \left(1 - \xi_t^D\right) R_{t+1}^{L}$$

and $w_{t,n}^{b}$ denotes the *fraction of demandable debt* that is withdrawn for bank b, given the idiosyncratic state.

The first line of (2) gives the outstanding liabilities of bank b. The second line computes asset revenues at the bankruptcy cutoff. The (idiosyncratic) amount of capital services produced per unit of physical capital is $\bar{\omega}_{t+1}^{b}$. The gross return per unit of capital services is R_{t+1}^{K}.

Average *gross* income to creditors equals the sum of interest income when the financial firm is solvent; recovered income when the financial firm is insolvent; and income from early withdrawals of demandable debt. Specifically, average gross cred-

itor income, conditional on the time $t+1$ aggregate state, may be written

$$\mathrm{E}_{t+1}\left[\Pr_{t,n}\left(\hat{\omega}^b_{t+1} > \bar{\omega}^b_{t+1}\right) R^*_{t+1}\right] \times \left(q_t K^b_{t+1} + b^b_{t+1} - n_t\right)$$

$$+R^K_{t+1} q_t \times \mathrm{E}_{t+1}\left\{K^b_{t+1,n}\mathrm{PE}_{t,n}\left[\hat{\omega}^b_{t+1}|\hat{\omega}^b_{t+1} \leq \bar{\omega}^b_{t+1}\right]\right\}$$

$$+\frac{R^g_{t+1}}{\Pi_{t+1}} \times \mathrm{E}_{t+1}\left[b^b_{t+1,n}\Pr_{t,n}\left(\hat{\omega}^b_{t+1} \leq \bar{\omega}^b_{t+1}\right)\right] + \mathrm{E}_{t+1}\left[W^b_{t,n}\right]$$

where $\mathrm{E}_{t+1}[\cdot]$ denotes the expectation over $\omega^b_{t,n}$ and $\phi^b_{t,n}$, conditional on the $t+1$ aggregate state; and $\mathrm{PE}_{t,n}\left[\hat{\omega}^b_{t+1}|\hat{\omega}^b_{t+1} \leq \bar{\omega}^b_{t+1}\right]$ denotes the partial expectation of $\hat{\omega}^b_{t+1}$ over $\hat{\omega}^b_{t+1} \leq \bar{\omega}^b_{t+1}$, conditional on $\omega^b_{t,n}$ and $\phi^b_{t,n}$.

Let me describe the elements of gross creditor income, starting with the first line. The first line computes the average of interest income to creditors when the financial firm is solvent; that is, when $\hat{\omega}^b_{t+1} > \bar{\omega}^b_{t+1}$. The second line gives the average of recovered capital income to creditors when the financial firm is insolvent. The first term of the third line gives the average of recovered government debt income received by creditors when the financial firm is insolvent. The final term gives the average value of income received by creditors for their withdrawal demands.

To obtain *net* creditor income, subtract from gross income the quantity

$$\mu \times R^K_{t+1} q_t \times \mathrm{E}_{t+1}\left\{K^b_{t+1,n}\mathrm{PE}_{t,n}\left[\hat{\omega}^b_{t+1}|\hat{\omega}^b_{t+1} \leq \bar{\omega}^b_{t+1}\right]\right\}$$

equal to the monitoring costs creditors pay when the financial firm defaults.

The average income to demandable claims and to maturity-matched claims may be written in an analogous manner. Details are in the appendix.

2.3.3 Bank profit

Finally, the bank's expected profit can be written as the residual of total gross asset income, less gross income to creditors. Income to government debt will always be received by creditors; it will either form part of the income used in interest payments, or it will be seized (at no penalty) in bankruptcy court, or it will be received as income to a withdrawal. Thus, net income to the bank depends only on the share of the revenue to each unit of $K^b_{t+1,n}$ that is received by the bank. In particular, bank expected profit may be written as

$$E_t\left\{R^K_{t+1} q_t K^b_{t+1,n} \times \mathrm{PE}_{t,n}\left[\left(\hat{\omega}^b_{t+1} - \bar{\omega}^b_{t+1}\right)|\hat{\omega}^b_{t+1} > \bar{\omega}^b_{t+1}\right]\right\}$$

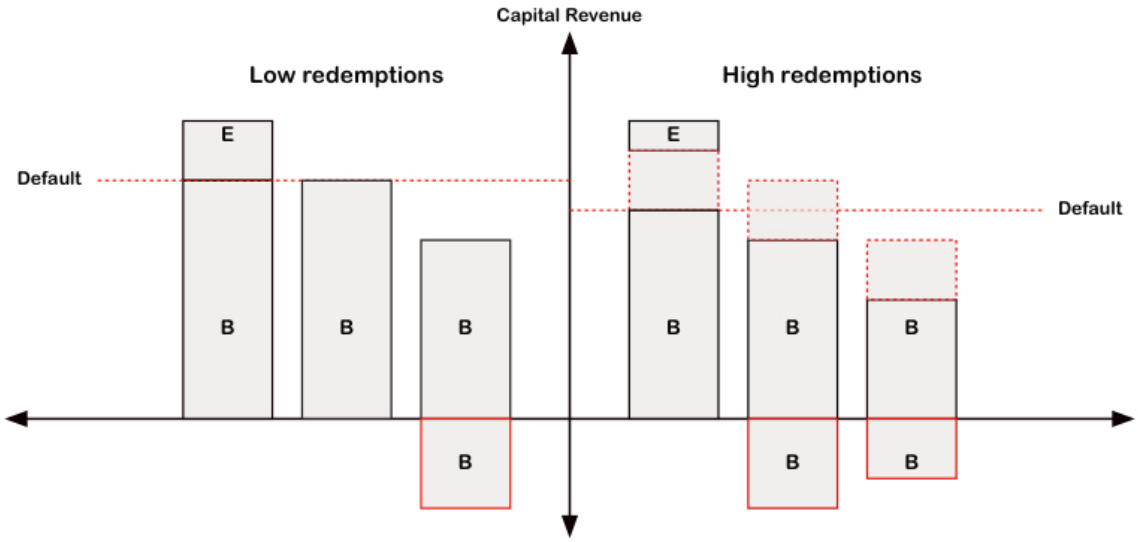

Figure 6: Income to equity holders and remaining creditors

2.4 Properties of the contract

As in BGG, because controls enter into the bounds of integration and the arguments of the nonlinear CDF and PDF functions governing the distributions of $\phi^b_{t,n}$ and ω^b_{t+1}, closed-form solutions to the nonlinear contracting problem do not exist. I here explore the qualitative properties of the contract. I also describe in the appendix why banks will hold a strictly positive liquidity reserve and issue demandable debt.

2.4.1 Intuition for the contracting problem

The bankers want to avoid early liquidations because they reduce capital income for any given asset productivity ω^b_{t+1}. Diversified creditors want borrowers to reduce liquidity risk because early liquidations reduce capital income precisely when fundamentals are weak. Early liquidations therefore concentrate losses, which amplifies the risk of costly monitoring for any given interest rate. In general equilibrium, the inefficiency arising from costly state verification (CSV) varies endogenously with liquidity risk.

Figure (6) illustrates the effect of creditor redemptions on income to equity holders (E) and not-withdrawn creditors (B) at a distressed bank for three possible values of ω^b_{t+1}. Start from the left-hand side. The first two bars illustrate that, in a debt contract, as ω^b_{t+1} falls, income to the equity holders decreases and vanishes before

the creditors' claim is reduced. The third bar illustrates the receivership revenue of creditors at an insolvent bank (above the horizontal axis) and the monitoring costs incurred (below the horizontal axis), which are for illustrative purposes set to half the income recovered.

Now consider the bars on the right-hand side of the vertical axis. These illustrate the outcomes when redemptions are sufficient to exhaust perfectly liquid assets, and the bank must liquidate some risky assets at a discount in order to satisfy withdrawals. On the right-hand side, the additional withdrawals reduce the bank's outstanding liabilities; that is why the amount owed (represented by the dashed, horizontal line labeled "default") drops. However, for illustrative purposes I assume that reducing outstanding liabilities by this amount requires a decline twice that size in revenues available for remaining creditors and equity holders. The first bar on the right shows that, as a result, even when ω^b_{t+1} is relatively high, the banker's share shrinks substantially. The second bar shows that now the bank is insolvent at a value of ω^b_{t+1} at which it would have survived in the absence of costly fire sales. The last bar shows that redemptions reduce receivership income per-unit of retained credit.

Figure (6) also illustrates the tradeoffs between the different contract variables. An increase in the average promised interest rate augments the creditors' share of capital income when ω^b_{t+1} is sufficient to avoid default; however, it also raises the default cutoff. Given the lognormal distribution of ω^b_{t+1}, there exists a threshold average interest rate such that, below this value, creditors' average income is increasing in the average promised interest rate.

Furthermore, additional borrowing grows the total revenue generated; but it also raises the default cutoff, because the fixed amount of equity now provides a smaller cushion per-unit-of-debt. Additionally, short-term borrowing is less expensive than maturity-matched borrowing, but increases the bank's exposure to outcomes like those illustrated on the right-hand side of the vertical axis in Figure (6).

Last, higher liquidity reserves suffer a low return. But they increase the probability mass on outcomes like those illustrated on the left-hand side of the vertical axis, and reduce the probability mass on outcomes like those illustrated on the right-hand side.

2.5 An aside on assumptions

2.5.1 Fundamentals-based runs, versus random panics

I assume that sunspots never cause a run. See Gorton (1988) for evidence against the view that banking panics are unrelated to underlying fundamentals.

By ignoring random panics, I am not denying the role of strategic complementarity in creditors' decisions to withdraw. In the present environment, any non-sunspot withdrawal decision is trivial. In a richer environment, I could assume a continuum of $\omega^b_{t,n}$, and gift depositors with idiosyncratic, noisy signals of $\omega^b_{t,n}$. One could then develop a unique threshold equilibrium determining a signal value below which creditors would withdraw, as in Goldstein and Pauzner (2005).

2.5.2 Why assume costly default?

I assume costly state verification, or costly default, for the following reasons. I want feedback between banks' realized net worth and the price of capital, because this feedback generates larger declines in net worth and investment, as in BGG. Costly default naturally limits overall outside financing relative to bank net worth. By contrast, in the absence of costly default or some other conflict of interest between banks and their creditors, there would exist no leverage constraint on maturity-matched debt.

Why not use a simple moral hazard model to limit outside financing relative to net worth? Liquidity risk means the financial firm may not be able to pay creditors any fixed promised rate. Hence, the model must include a default concept. But then why not assume default is costly? Costly default is a realistic source of value destruction for creditors. Additionally, costly default means that sharper increases in financial firm leverage strengthen the flight to quality described in the results in Section 5. When default is costly, the losses to creditors from any given amount of fire sales rise as the ratio of net worth to borrowing falls. This is because there is a greater likelihood of losses eating through a bank's capital cushion. And when default is costly, the resulting bank failure induces a discontinuous decline in creditors' net income. Thus, when leverage increases, banks shift more towards maturity-matched debt and higher liquidity reserves.

Last, costly state verification assists steady state calibration. Costly default permits a smaller liquidation penalty to rationalize a given target ratio of reserves to demandable debt. Additionally, in the absence of a discontinuous cost at default it would be difficult to rationalize a reasonably-small equilibrium probability of default at substantially leveraged firms. This is because default would impose no costs on any parties to the contract, and so the optimal contract would not economize on the likelihood of such an event.

3 Macroeconomic model

3.1 Financial firms in the macroeconomy

First, recall Figure (1), which summarizes the financial firms' relationship with the households in the model. Next, reviewing the contracting problem, the reader can see that if we divide all constraints and all asset-choices by n_t, then net worth does not affect the solution to the optimal contracting problem. Thus, I can aggregate across financial firms with different net worths.

The banks in aggregate must purchase all claims to future capital, so

$$\int_{\eta^b} K^b_{t+1} db = K_{t+1}$$

The market clearing condition for the fire sales market simply requires that for every (forced) seller there is a (willing) buyer at price $q_{t,n}$, or

$$\int_{\eta^b} \left(K^b_{t+1,n} - K^b_{t+1}\right) db = 0$$

Aggregate net worth invested in time t contracts is:

$$n_t = \gamma \times E_t \left\{ R^K_t q_{t-1} K^b_{t,n} \times \mathrm{PE}_{t-1,n} \left[\left(\hat{\omega}^b_t - \bar{\omega}^b_t\right) | \hat{\omega}^b_t > \bar{\omega}^b_t \right] \right\} + (1-\gamma)\, \eta^b x^b \tag{3}$$

The term in brackets equals the profit to financial firms from time $t-1$ contracts. A fraction γ of this profit is left after bankers randomly exit. Exiting bankers are replaced by new entrants each provided a transfer x^b.

Because the financial friction constrains bank leverage, new investment and the value of existing capital are tied to net worth. Equation (3) shows that net worth in turn depends (through inherited capital) on past investment. Thus, the leverage constraints embed a form of intertemporal investment adjustment costs.

Consumption by exiting bankers is

$$C^b_t := (1-\gamma) \times E_t \left\{ R^K_t q_{t-1} K^b_{t,n} \times \mathrm{PE}_{t-1,n} \left[\left(\hat{\omega}^b_t - \bar{\omega}^b_t\right) | \hat{\omega}^b_t > \bar{\omega}^b_t \right] \right\}$$

3.2 Capital production and the price of capital

Future capital evolves according to

$$K_{t+1} = (1-\delta)\,\tilde{K}_t + \Phi\left(\frac{I_t}{\tilde{K}_t}\right)\tilde{K}_t$$

where $\tilde{K}_t := E_t\left[\omega_t^b\right] K_t$ and $\Phi(\cdot)$ is increasing and concave, with $\Phi(0) = 0$, as in BGG. The equilibrium price q_t of future capital is the marginal final goods cost of building new capital; that is,

$$q_t = \left[\Phi'\left(\frac{I_t}{\tilde{K}_t}\right)\right]^{-1}$$

As in BGG, a decline in the net worth of the leverage-constrained agent feeds on itself. When net worth declines, the aggregate value of financial sector asset purchases must decline. For this to occur, investment must decline. The marginal cost of building new capital falls as investment drops. The resulting decline in q_t further damages the leverage-constrained bank's net worth.

3.3 Production sector

3.3.1 Intermediate goods producers

Intermediate firm producer $j \in [0,1]$ produces type-j intermediate goods y_{jt} according to Cobb-Douglas production with a fixed cost

$$y_{jt} = A_t k_{jt}^{\theta_k} \left(\ell_{jt}\right)^{1-\theta_k} - \Theta$$

where $\theta_k \in (0,1)$ and A_t is total factor productivity at time t. Intermediate goods producers compete monopolistically and periodically choose an optimal price $\tilde{p}_{jt}$ in the manner of CEE to solve

$$\max_{\tilde{p}_{jt}} E_t\left\{\sum_{k=0}^{\infty}\left(\beta\phi_p\right)^k \lambda_{t+k}\left[\frac{p_{jt+k}y_{jt+k}}{P_{t+k}} - s_{t+k}y_{jt+k}\right]\right\}$$

where ϕ_p is the probability of an intermediate goods producer not choosing a new price, λ_{t+k} is the marginal value of real income at time $t+k$ to the firm's household shareholders, and s_{t+k} denotes time $t+k$ marginal costs. The price p_{jt+k} evolves

according to a lagged inflation indexation rule

$$p_{jt+k} = \prod_{s=0}^{k-1} (\Pi_{t+s})^{\zeta} \, \tilde{p}_{jt}$$

where $\Pi_{t+s} := \frac{P_{t+s}}{P_{t+s-1}}$ denotes gross inflation between periods $t+s-1$ and $t+s$ and $\zeta \in [0,1]$ denotes the degree of indexation of not-reoptimizing monopolists' prices to lagged inflation.

Intratemporal cost minimization together with the aggregate conditions

$$\int \ell_{jt} dj = \eta^h \ell_t^h$$

$$\int k_{jt} dj = \tilde{K}_t$$

yield factor prices

$$w_t^h = s_t \theta_h \frac{Y_t + \Theta}{\eta^h \ell_t^h} \tag{4}$$

$$r_t = s_t \theta_k \frac{Y_t + \Theta}{\tilde{K}_t} \tag{5}$$

3.3.2 Final goods producers

Final goods firm $j \in [0,1]$ produces final goods Y_t via a Dixit-Stiglitz aggregation of intermediate goods

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\xi_p - 1}{\xi_p}} dj \right)^{\frac{\xi_p}{\xi_p - 1}} \tag{6}$$

for $\xi_p > 1$.

The final producer's problem is to choose quantities $\{y_{jt}\}$ given intermediate goods prices $\{p_{jt}\}$ and final goods price P_t to solve

$$\max_{\{y_{jt}\}_{j \in [0,1]}} P_t \left(\int_0^1 y_{jt}^{\frac{\xi_p - 1}{\xi_p}} dj \right)^{\frac{\xi_p}{\xi_p - 1}} - \int_0^1 p_{jt} y_{jt} dj$$

3.3.3 New Keynesian Phillips Curve

The log-linear approximation of the pricing rule for the intermediate goods firms about a zero inflation steady state yields the New Keynesian Phillips Curve. The New Keynesian Phillips Curve shows the relationship between nominal and real variables that results from the assumed price-setting rigidities of the intermediate goods firms:

$$\hat{\Pi}_t - \zeta\hat{\Pi}_{t-1} = \frac{(1-\phi_p\beta)(1-\phi_p)}{\phi_p}\hat{s}_t + \beta E_t\left(\hat{\Pi}_{t+1} - \zeta\hat{\Pi}_t\right) \tag{7}$$

Here, $\hat{Z}_t$ denotes $\frac{Z_t - Z^{SS}}{Z^{SS}}$ where Z^{SS} is the steady state value of generic variable Z. Additionally, s_t is the cost of the marginal unit of intermediate goods production, equal to the ratio of the real wage to the marginal productivity of labor.

3.4 Households

Households maximize expected discounted utility

$$E\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t, \ell_t^h\right) + u^{trans}\left(c_t, m_{t+1}^h, b_{t+1}^h, d_t^D\right)\right]$$

where $u\left(c_t, \ell_t^h\right)$ is a household's time t momentary utility over its consumption c_t and labor ℓ_t^h, and $u^{trans}\left(c_t, m_{t+1}^h, b_{t+1}^h, d_t^D\right) < 0$ is the household's utility from transaction costs, given consumption c_t, real money balances m_{t+1}^h, real balances of government debt b_{t+1}^h, and holdings of demandable bank debt d_t^D.

Why does the household derive utility from its holdings of government debt? Krishnamurthy and Vissing-Jorgensen (2010) show that short-term US government debt is a partial substitute for money. In particular, it is a highly liquid and nominally certain investment. The "transactions utility" of b_{t+1}^h represents the positive value assigned to government debt's money-like qualities by any agent **other** than firms engaging in liquidity transformation and accumulating liquid assets as a precautionary reserve against creditor redemptions.

The optimizing household faces the final goods budget constraint

$$c_t + d_{t+1}^L + d_{t+1}^D + b_{t+1}^h + m_{t+1}^h = R_t^{AVG,L}d_t^L + R_t^{AVG,D}d_t^D + \frac{1}{\Pi t}m_t^h + \frac{R_t^g}{\Pi_t}b_t^h + w_t^h\ell_t^h + o_t + x_t^h$$

The right-hand side of the budget constraint shows the average interest income $R_t^{AVG,L}d_t^L$ and $R_t^{AVG,D}d_t^D$ from previous-period long and short term loans d_t^L and d_t^D

to banks, respectively; the time t final goods value of time $t-1$ real money balances m_t^h; interest income from previous-period nominal purchases b_t^h of government debt; wages $w_t^h \ell_t^h$ from the contemporaneous labor decision; the household's share of intermediaries' profits o_t; and lump sum household transfers (or taxes, if negative), x_t^h. Each household chooses between consuming final goods c_t; making long and short term loans, d_{t+1}^L and d_{t+1}^D, to banks; buying nominally risk-free, one-period government debt b_{t+1}^h; and accumulating real money balances m_{t+1}^h. The household also supplies labor ℓ_t^h.

The first-order conditions for household optimization are

$$\lambda_t = \frac{\partial u_t}{\partial c_t} + \frac{\partial u_t^{trans}}{\partial c_t} \tag{8}$$

$$\lambda_t - \frac{\partial u_t^{trans}}{\partial m_{t+1}^h} = \beta E_t \left[\lambda_{t+1} \times \frac{1}{\Pi_{t+1}} \right] \tag{9}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \times R_{t+1}^{AVG,L} \right] \tag{10}$$

$$\lambda_t - \frac{\partial u_t^{trans}}{\partial d_{t+1}^D} = \beta E_t \left[\lambda_{t+1} \times R_{t+1}^{AVG,D} \right] \tag{11}$$

$$\lambda_t - \frac{\partial u_t^{trans}}{\partial b_{t+1}^h} = \beta E_t \left[\lambda_{t+1} \times \frac{R_{t+1}^g}{\Pi_{t+1}} \right] \tag{12}$$

$$\frac{\partial u_t}{\partial \ell_t^h} = \lambda_t w_t^h \tag{13}$$

where f_t is the value of the (generic) function f at time t; and λ_t is the household's Lagrangian multiplier on her time t budget constraint, interpreted as the individual household's marginal value of real income at that date.

The household optimality conditions determine the required market rates of return on demandable and maturity-matched bank loans. Specifically, conditions (10) and (12) are satisfied if the time $t+1$ average return on maturity-matched loans to banks satisfies[12]

$$\beta \lambda_{t+1} R_{t+1}^{AVG,L} = \beta \lambda_{t+1} \frac{R_{t+1}^g}{\Pi_{t+1}} + \frac{\partial u_t^{trans}}{\partial b_{t+1}^h} \tag{14}$$

Conditions (11) and (12) are satisfied if the time $t+1$ average return on demandable credit satisfies

$$\beta \lambda_{t+1} R_{t+1}^{AVG,D} = \beta \lambda_{t+1} \frac{R_{t+1}^g}{\Pi_{t+1}} + \left(\frac{\partial u_t^{trans}}{\partial b_{t+1}^h} - \frac{\partial u_t^{trans}}{\partial d_{t+1}^D} \right) \tag{15}$$

[12] Recall the participation constraints are assumed to bind in each time $t+1$ aggregate state.

Since $\frac{\partial u_t^{trans}}{\partial d_{t+1}^D} \geq 0$, it must be that $R_{t+1}^{AVG,D} < R_{t+1}^{AVG,L}$.

3.5 Monetary policy

Monetary policy sets R_{t+1}^g , the short-term nominal interest rate on risk-free government debt, according to the following rule:

$$\hat{R}_{t+1}^g = \rho_R \hat{R}_t^g + (1 - \rho_R) \left(\rho_\pi \hat{\Pi}_t \right) \tag{16}$$

The scaling $\rho_R \in [0, 1)$ summarizes interest rate inertia and $\rho_\pi > 1$ gives the sensitivity of interest rates to inflation.

3.6 Aggregation and market clearing conditions

The clearing condition for the market for money is

$$m_{t+1}^h \eta^h + \int_{\eta^b} m_{t+1}^b db = m_{t+1}$$

The market clearing condition for government debt is

$$b_{t+1}^h \eta^h + \int_{\eta^b} b_{t+1}^b db = b_{t+1}$$

In the baseline model, the law of motion for government debt is implied by traditional open market transactions that exchange money for government debt:

$$m_{t+1} - \frac{m_t}{\Pi_t} = - \left(b_{t+1} - \frac{b_t}{\Pi_t} \right) \tag{17}$$

I will change this assumption when I explore alternative open market operations, below.

Consider the government's flow of funds

$$\eta^h x_t^h + \eta^b (1 - \gamma) x^b + \left(m_{t+1} - \frac{m_t}{\Pi_t} \right) + \left(b_{t+1} - \frac{b_t}{\Pi_t} \right) - \frac{\left(R_{t+1}^g - 1 \right)}{\Pi_t} b_t = 0$$

where I have assumed that there is no government consumption. The open market condition (17) implies the government balances its budget each period, including net interest expenses:

$$\eta^h x_t^h + \eta^b (1 - \gamma) x^b - \frac{\left(R_{t+1}^g - 1 \right)}{\Pi_t} b_t = 0$$

The market clearing conditions for households' loans to financial firms are

$$\eta^h d_{t+1}^D = \xi_t^D \left(b_{t+1}^b + q_t K_{t+1} - n_t\right)$$

and

$$\eta^h d_{t+1}^L = \left(1 - \xi_t^D\right) \left(b_{t+1}^b + q_t K_{t+1} - n_t\right)$$

The economy's resource constraint is

$$Y_t = C_t + C_t^b + I_t + G_t$$

where $C_t := \eta^h c_t$; G_t is the final goods bankruptcy cost from the resolution at time t of contracts initiated in time $t-1$; and

$$Y_t \approx A_t \tilde{K}_t^{\theta_k} \left(\eta^h \ell_t^h\right)^{1-\theta_k} - \Theta$$

4 Functional forms and calibration

4.1 Functional forms

The fraction of creditors that observe the interim state, $\phi_{t,n}^b$, is distributed according to a beta distribution with range parameter $\phi_1 = 1$ and shape parameter $\phi_{2,t} > 1$, which will be subject to exogenous perturbations.

Intratemporal investment adjustment costs take the form

$$\Phi\left(x\right) = x - \frac{\sigma_{adj}}{2} x^2$$

I choose σ_{adj} so that for every 1% increase in the ratio of investment to capital, the price of capital increases by 0.25%, as in BGG. As discussed in section 3, the financial friction endogenously generates sluggishness in intertemporal changes in investment.

Household utility takes the form

$$u\left(c_t, \ell_t\right) := \frac{\left[c_t^{1-\psi} \left(1 - \ell_t^h\right)^{\psi}\right]^{1-\sigma_c}}{1 - \sigma_c}$$

for $\sigma_c >= 1$ and $\psi \in (0, 1)$. Because the marginal utility of consumption is increasing in labor, a decline in labor will reduce the incentive to consume. This will help induce contractions in consumption when output falls.

ϕ_2	8.805	μ	0.050	σ	0.155
μ^{liq}	0.2951	ω^L	0.816	p	0.99
ρ_R	0	ρ_π	1.5		
β	0.995	σ_c	2	ψ	0.678
ψ_M	0.01	σ_M	-19.84	θ_{bh}	0.6531
θ_d	0.3207	ξ_u	1.50	η^h	0.9
ξ_p	6	ϕ_p	0.75	ζ	1
θ_k	0.36	σ_{adj}	9		

Table 1: Baseline parameter values

Additionally

$$u^{trans}\left(c_t, m_{t+1}^h, b_{t+1}^h, d_t^D\right) := -\psi_M \frac{\left(\frac{c_t}{\text{trans}\left(m_{t+1}^h, b_{t+1}^h, d_t^D\right)}\right)^{1-\sigma_M}}{1-\sigma_M}$$

where

$$\text{trans}\left(m_{t+1}^h, b_{t+1}^h, d_t^D\right) :=$$

$$\left[(1-\theta_{bh})\left(m_{t+1}^h\right)^{\frac{\xi_u}{\xi_u-1}} + \theta_{bh}(1-\theta_d)\left(b_{t+1}^h\right)^{\frac{\xi_u}{\xi_u-1}} + \theta_{bh}\theta_d\left(d_t^D\right)^{\frac{\xi_u}{\xi_u-1}}\right]^{\frac{\xi_u-1}{\xi_u}}$$

for $\theta_{bh},\ \theta_d \in [0,1]$ and $\xi_u \geq 1$. The form of $u^{trans}(\cdot)$ emulates Christiano, Motto, and Rostagno (2009), except that I permit a generally higher degree of substitutability between money, government debt, and demandable bank debt than would obtain under a Cobb Douglas aggregate. A slightly higher degree of substitutability will help generate a marginal utility from holdings of demandable debt that is sufficiently large to hit the calibration target for the optimal mix of demandable and maturity-matched debt. The assumption of a fair amount of substitutability seems eminently plausible.

4.2 Calibration

I choose contract-related parameters γ, σ, μ, ϕ_2, ω^L, and θ_d to match the following steady state statistics. I target a steady state, annualized net return to capital of 5%, in the range of Mulligan (2002)'s post-World War II (after tax) estimates. Gross leverage is set to four, as in Gertler and Kiyotaki (2010); as the authors note, this is a very rough target.

Just as it is difficult to match the very high ratios of borrowing to equity observed in the data, it is difficult to match the extremely low credit spreads observed before the crisis at many highly leveraged financial firms. In the baseline model I use an annualized credit spread of 200 basis points. This is equal to the lower bound on

the range of borrowing costs for nonfinancial companies employed by Christiano, Motto, and Rostagno (2009). Although this coincides with the lower bound for *nonfinancial* firms, because I am assuming much higher leverage than in either BGG or Christiano, Motto, and Rostagno (2009), an annualized spread of 200 basis points is actually quite modest. In separate calculations, I find that the model can rationalize lower credit spreads when the government partially protects financial firm creditors from losses.

The steady state ratio of perfectly liquid assets (i.e., money and government debt) to financial firms' short term liabilities is set to 3.38%. To obtain this figure, I use flow of funds data for security brokers and dealers to compute the average value from 2000:1-2007:2 of the ratio to liabilities of checkable deposits, plus currency, plus Treasury securities, plus Agency and Government Sponsored Enterprise (GSE)-backed securities. I also compute the same ratio, only excluding Agency and GSE-backed securities. I then take a weighted average of these ratios, because it is unclear how to map Agency and GSE-backed securities into my model. Although Agency and GSE-backed securities were ultimately transformed into liabilities of the federal government, this assumption of liability was not perfectly anticipated ex ante, as evidenced by increases in credit spreads on these securities during 2007 and the first half of 2008.

Next, I target a value for the fraction of debt that is demandable of $\xi_t^D = 0.4$. Setting the target for the fraction of outside financing from short-term or demandable debt issuance is challenging. For the contracting problem, what ultimately matters is the ratio of average fire sales costs to the optimal contract's equity cushion. But, just as the empirical ratio of overall borrowing to equity is extremely high, so the ratio of short-term financing to equity is very high. Between 1990 and 2007, the ratio of federal funds and security repurchase agreements to assets at security brokers and dealers averaged about 0.30. Given the large fraction of assets purchased with borrowed money, this translates to a very high ratio of short term borrowing to equity. Furthermore, this figure does not include the short-term borrowing of the special purpose vehicles (SPVs) sponsored by these firms; Tirole (2011) documents considerable short-term borrowing by the SPVs, relative to the equity of their sponsors. As a rather conservative approximation, I assume that 40% of all outside financing is borrowed short. I do not employ data on commercial banks, because a large fraction of these liabilities are explicitly insured by the US government; incorporating partial government protection of model financial firms is left to future work.

The steady state probability of financial firm distress is 0.01. To calibrate ω^L, the conditional expected value of equity at a distressed financial firm is set equal

to 65% of the unconditional expected value of net worth, a not-unreasonable target given the substantial leverage of the financial firms I have in mind. Then ω^H is chosen to satisfy $p\omega^H + (1-p)\,\omega^L = 1$. Additionally, μ^{liq} is chosen so that the steady state fire sale discount is 30%. This is consistent with Campbell, Giglio, and Pathak (2011)'s finding of an average forced-sale discount on foreclosed homes of 27% for the period 1987 to March 2009. For public equity, Hau and Lai (2011) explore the effect of forced sales by equity funds on the value of nonfinancial stocks during 2007-2009. They find that, during the crisis, US nonfinancial stocks with owners that were relatively exposed to the financial sector traded at a discount of 35% to their less-exposed industry peers; the discount grows even larger for higher quantiles of the authors' measure of exposure. Recall that, in the present model, μ^{liq} applies only to the most distressed 1% of financial firms, and even then only to those distressed firms experiencing large withdrawals. Last, a key determinant of the model dynamics is the risk that losses from fire sales will eat through firm equity and induce a costly default. My model's modestly leveraged financial firms are therefore likely *less* sensitive to increases in redemption risk than are real world financial firms, for any given μ^{liq}.

The other parameter values are chosen inside the range of standard values in the New Keynesian DSGE literature; see, e.g., CEE. For intermediate goods, $\xi_p = 6$ gives a steady state markup over marginal cost of 20%. The probability of an intermediate goods firm not re-optimizing $\phi_p = 0.75$ implies that on average an intermediate goods firm re-optimizes once every four quarters.

I calibrate the households' time discount β to support an annualized net interest rate of 1.97%, which is equal to the 1926-2008 average of the real interest rate on the three-month US Treasury bill. I choose θ_b and steady state household holdings of government debt b^h to match two steady state targets. First, the interest rate would be 72 basis points higher in the absence of the transactions value to households from government debt, which corresponds to Krishnamurthy and Vissing-Jorgensen's (2010) estimates of the liquidity and safety premia on Treasury yields for 1926-2008. Second, the steady state elasticity of these premia with respect to the aggregate supply of bonds matches Krishnamurthy and Vissing-Jorgensen's (2010) estimates of the response of premia to changes in the supply of Treasuries.

The weight on the momentary disutility of labor ψ is chosen so that steady state labor equals 30% of the (unit) time endowment. Fixed costs Θ extinguish steady state profits. The weight on inflation in the Taylor rule ρ_π is 1.5.

4.2.1 Stochastic processes

In simulations below, I explore the macroeconomic effects of unexpected changes

in the probability that creditors observe a bank's interim state, the likelihood of financial firm distress, and the penalty on early liquidations of risky assets. In particular, I perturb the shape parameter $\phi_{2,t}$ governing the distribution of $\phi^b_{t,n}$, the probability p_t of the high interim state, and the standard deviation σ_t of log-ω^b_{t+1}. Each of these random quantities is realized at time t, before time t contracts are formed. The contemporaneous aggregate state is common knowledge.

I assume that the stochastic processes $\{\phi_{2,t}\}$, $\{p_t\}$, and $\{\sigma_t\}$ are independent AR(1) processes, each with a lag coefficient of 0.75. The half-life of a shock is thus approximately 2.4 quarters. The qualitative conclusions below are robust to variations in the lag coefficient.

5 Simulation results

The results in this section demonstrate that the model can explain large macroeconomic effects of reasonably-sized shocks to the financial sector. First, I explore the consequences of an unexpected surge in redemption risk. Next, I show that the presence of liquidity risk powerfully amplifies the contractionary consequences of other adverse shocks to the financial sector.

All simulation results are for a first order Taylor approximation of the true, nonlinear economy.

5.1 Anatomy of a redemption risk shock

Recall that $\phi_{2,t}$ is the shape parameter of the beta distribution of $\phi^b_{t,n}$. I call an unexpected deviation in $\phi_{2,t}$ from its baseline value a "redemption risk shock." A negative perturbation in $\phi_{2,t}$ elevates the mean and standard deviation of the distribution of $\phi^b_{t,n}$. Thus, on average a greater fraction of creditors receive advance information about future asset payoffs. Because it directly affects *only* liquidity risk, the redemption risk shock will develop understanding of the role of risky liquidity transformation in the general equilibrium model, as well as illustrate a flight to quality.

This shock has multiple possible interpretations. First, creditors could exogenously obtain public information. For example, Gorton and Metrick (2011) argue that the January 2006 launch of the ABX index–an index of 20 equally weighted tranches of subprime residential-mortgage-backed securities–generated public signals about the likely payoffs to comparable assets. Second, creditors might intensify due diligence, thereby making themselves more likely to obtain advance information about future asset performance. Under either interpretation, risky assets become more information sensitive. This heightened sensitivity increases the risks of liquidity transformation.

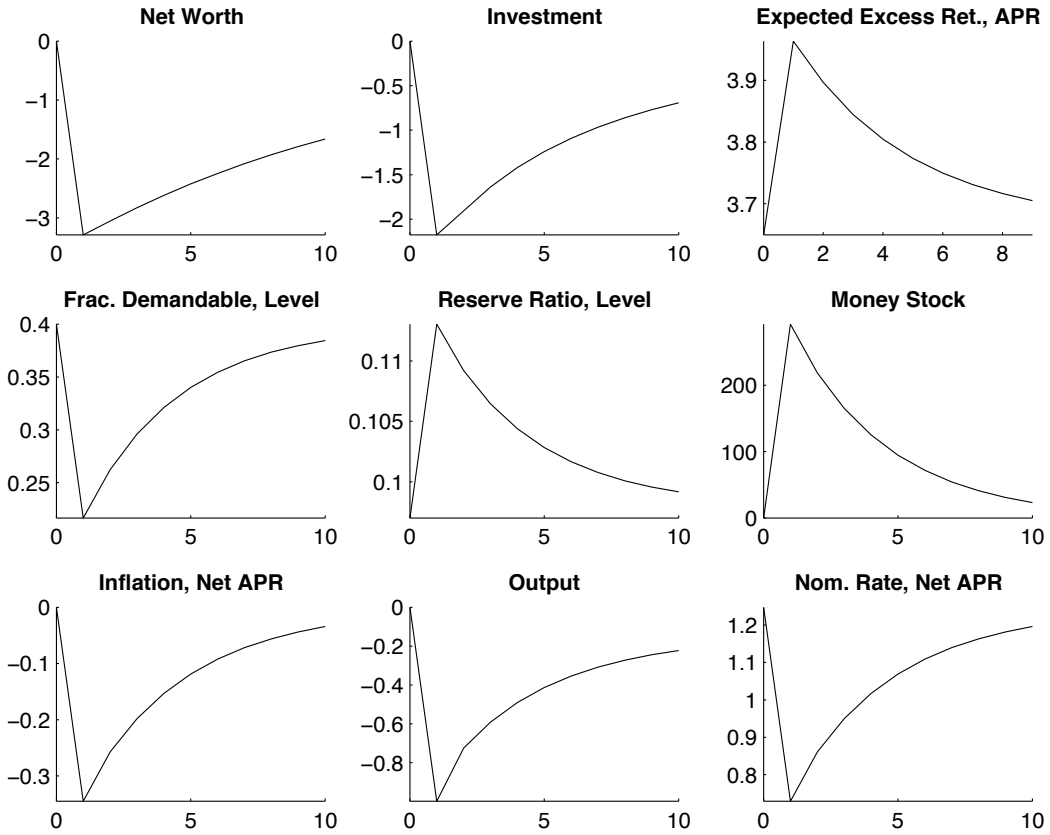

Figure 7: Redemption risk shock

All plots are annualized percent deviations from steady state, unless otherwise noted

Figure (7) displays responses to a redemption risk shock that increases the average value of $\phi^b_{t,n}$ by ten percentage points. Given that the probability of financial firm distress is a mere 0.01, this shock represents a modest absolute increase in exposure.

Financial firms respond to the heightened liquidity risk by shifing towards maturity-matched debt and increasing desired reserves. To understand why, recall Figure (6): the redemption risk shock increases the weight on outcomes illustrated on the right-hand side of the vertical axis. Limiting dependence on short term financing reduces financial firm exposure to early creditor withdrawals. Additionally, a higher ratio of liquidity reserves to demandable debt permits financial firms to satisfy additional redemptions without costly liquidations of imperfectly liquid assets that would erode the value of equity and remaining creditors' claims.

Because banks issue less demandable debt, households substitute towards money balances and government debt. The price level falls. Under a New Keynesian Phillips curve (7), this deflation is consistent with output contraction.

Additionally, equilibration of the market for privately available government debt requires a decline in the expected rate of return on government debt. This can be seen from equation (12), reproduced here for convenience:

$$\lambda_t - \frac{\partial u_t^{trans}}{\partial b_{t+1}^h} = \beta E_t \left[\lambda_{t+1} \times \frac{R_{t+1}^g}{\Pi_{t+1}} \right]$$

Traditional expansionary open market operations shrink the privately available supply of government debt. Because both demandable debt and privately available government debt shrink, $\frac{\partial \mathbf{u_t^{trans}}}{\partial \mathbf{b_{t+1}^h}}$ rises. As the nonpecuniary value to households of the marginal unit of government debt rises, the equilibrating rate of return drops.

Financial firms fail to fully offset the exogenous increase in redemption risk with additional liquidity reserves. The financial firm must compensate creditors for the foregon nonpecuniary benefit of holding government debt. Thus, the surge in $\frac{\partial \mathbf{u_t^{trans}}}{\partial \mathbf{b_{t+1}^h}}$ augments the wedge between the average rate of return R_{t+1}^{AVG} owed financial firm creditors and the pecuniary return $\frac{R_{t+1}^g}{\Pi_{t+1}}$ to firm holdings of government debt. The higher cost to financial firms of liquidity reserves discourages financial firms from accumulating sufficient additional reserves. On net, therefore, equilibrium liquidity risk increases sharply.

As a result of the heightened liquidity risk, the optimal level of intermediation shrinks. The resulting slump in the demand for capital reduces the price of capital, which erodes net worth and further softens the demand for capital.

In summary, the degree of liquidity transformation is sensitive to liquidity risk. Traditional open market operations do not help financial firms offset the heightened

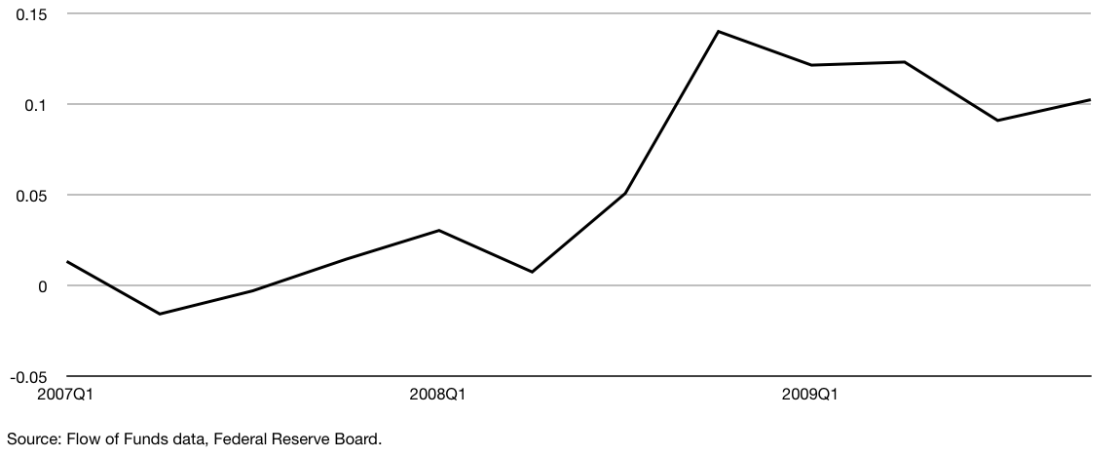

Figure 8: Ratio of highly liquid assets to total financial assets, securities brokers and dealers

liquidity risk resulting from the redemption risk shock. Consequently, net worth and the privately optimal level of intermediation decline substantially. There is a flight to quality, as households substitute away from risky private debt and towards money and government debt. The price level drops and output contracts. The deflation, in turn, rationalizes the lower interest rate under the Taylor rule.

Note that the aforementioned fragility of privately optimal liquidity transformation arises in the model even though government debt and demandable bank debt are only imperfectly substitutable. That is, the nonpecuniary value to households of the marginal unit of demandable debt rises as the supply of demandable debt contracts. A higher degree of substitutability would induce an even sharper reduction in the privately optimal issuance of demandable debt.

Comparison to developments in the shadow banking system[13]

The simulated increase in financial firms' reserve ratio is consistent with data for the 2008-2009 financial crisis. Figure (8) displays the fraction of security brokers and dealers' assets consisting of checkable deposits, currency, and Treasury securities. These holdings of liquid assets rise sharply during 2008. In fact, the magnitude of the increase far exceeds that in the model. Some of this gap can be explained by the Federal Reserve's efforts to support market liquidity during the crisis. In simulations below, I show that unconventional open market operations substantially amplify the simulated response of the reserve ratio, by increasing the availability of liquid assets.

Additionally, the sign and magnitude of the change in the fraction of debt that

[13]For a description of the shadow banking system, see Brunnermeier (2009) and Gorton and Metrick (2011).

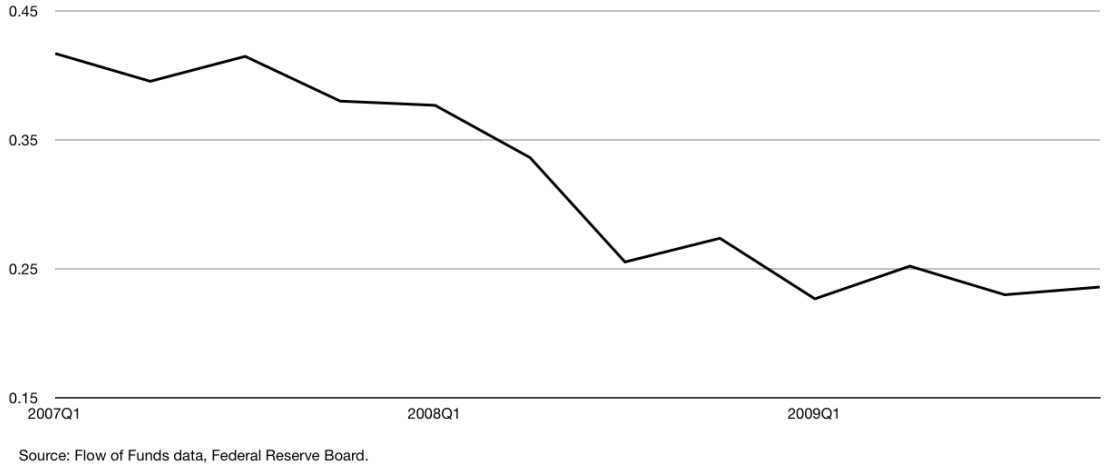

Figure 9: Ratio of repurchase agreements to total liabilities, securities brokers and dealers

is demandable is qualitatively consistent with flow of funds data on the liabilities of security brokers and dealers. Figure (9) displays the ratio of securities brokers and dealers' repurchase agreement liabilities to total liabilities. The magnitude of the decline from 2007Q1 to 2009Q1 is similar to that in the model simulation, although in the data the decline is more gradual and persistent, with a majority of the decline occurring between 2008Q1 and 2008Q3.[14]

A similar shift occurs in the financing of special purpose vehicles (SPVs) holding securitized assets. Before the crisis, commercial paper financed almost 20% of the assets of issuers of asset backed securities (ABS). By the first quarter of 2009, this fraction had declined to less-than 12%, before falling further to under 5% in 2010.[15]

What happened to these SPVs? He, Khang, and Krishnamurthy (2010) find that, during the crisis, the commercial banks that sponsored many of the SPVs brought Agency and GSE-backed ABS onto their balance sheets. These purchases were financed with expansions in FDIC-insured deposits and corporate debt.[16] Although

[14] The magnitude of the ratio is not precise, because many repurchase agreements are between broker-dealers and other broker-dealers, rather than between broker-dealers and, say, money market mutual funds (see Krishnamurthy, Nagel, and Orlov (2011)). But any such double-counting symmetrically affects both the numerator and the denominator, so it does not affect the qualitative trend. As a check, I examine the SEC filings of major US broker-dealers for 2007Q4 and 2009Q1. Goldman Sachs, Morgan Stanley, and Citigroup all shift away from risky short-term financing. The financing mix at JP Morgan is largely unchanged.

[15] Source: Flow of funds data.

[16] Banks' corporate debt was eligible for FDIC insurance under the Temporary Liquidity Guarantee

deposits are short-term, they are explicitly insured and, hence, not a source of liquidity risk. The Federal Reserve also absorbed Agency and GSE-backed ABS. The shift away from financial entities subject to liquidity risk, and towards those with more stable financing, is also consistent with the model.

Despite the model's agreement with data on risky forms of liquidity transformation, the foregoing discussion also highlights the model's limitations. Pervasive government efforts to shore up the financial sector likely had a decisive impact on the outcome of the crisis, as did the shift in financing towards explicitly-insured intermediaries. I capture some of these factors in simulations of purchases of risky assets by the monetary authority. But accurately depicting the crisis likely requires a model with partial government protection of financial firm investors and creditors. I leave this important task to future work.

Finally, the present model does not reproduce the observed shortening of the maturity structure *within* classes of debt; for example, for repurchase agreements (as documented by Krishnamurthy, Nagel, and Orlov (2011)) or for asset-backed commercial paper. This is because the model does not feature a continuum of maturities within the class of maturity-mismatched debt. In the model, there is no reason for such a continuum, because only one piece of information arrives about a bank's assets before the end of a contract.

5.2 Amplification of financial sector distress

Next, I show that liquidity risk amplifies the contractionary effects of heightened financial sector distress. In particular, suppose the probability that $\omega_{t,n}^b = \omega_t^L$ unexpectedly jumps from 0.010 to 0.015. This constitutes an unexpected increase in the fraction of the financial sector's assets that are distressed. Figure (10) plots the impulse responses to the baseline model against the responses for a no-liquidity-risk model.

Liquidity risk greatly amplifies the adverse effects of heightened financial sector distress. The troughs in investment and net worth are almost twice as deep under the baseline model. The troughs in inflation and output are nearly than three times as deep.

To understand this, consider that in both models, heightened financial sector distress reduces average payoffs to capital services projects. As a result, the optimal contract size drops, which reduces bank net worth, the price of capital, and investment.

Program.

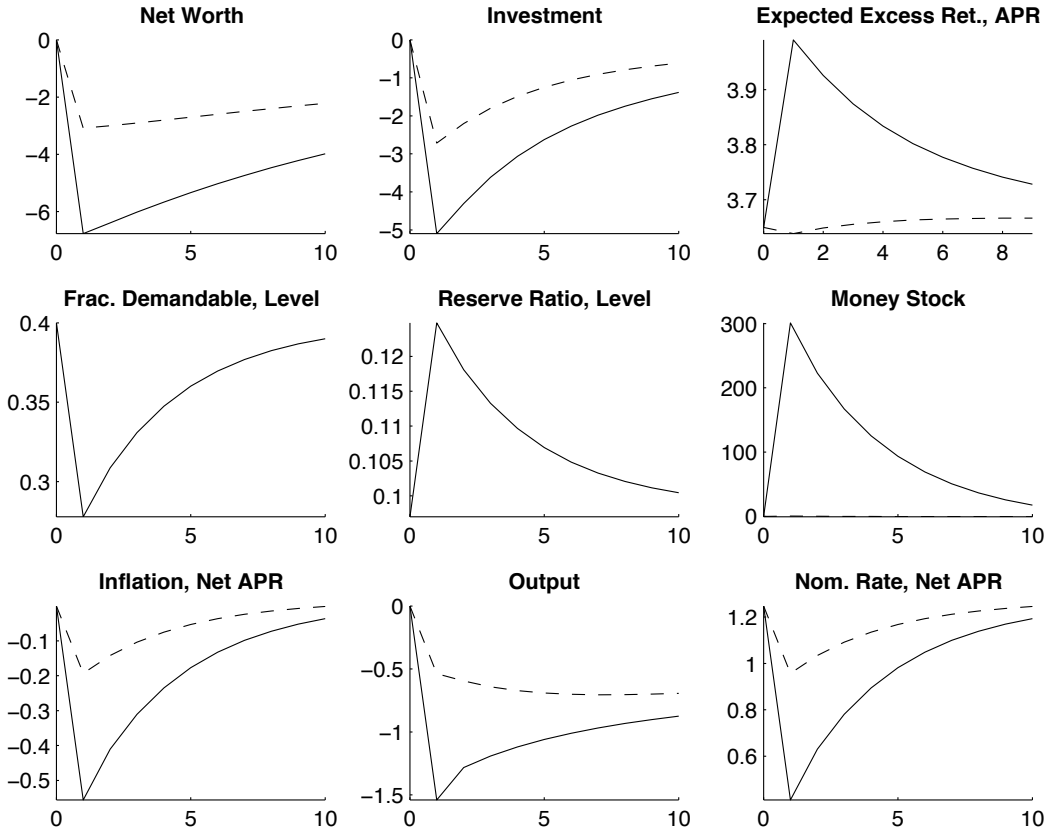

Figure 10: Increase in the probability of bad news

All plots are annualized percent deviations from steady state, unless otherwise noted.

Solid: Baseline specification. Dotted: Low steady state liquidity risk

In the baseline model, however, financial sector distress also greatly magnifies exposure to bank runs, because more creditors receive negative signals about bank asset quality. Banks respond by reducing their dependence on demandable debt and by augmenting their precautionary reserves. These forces set in motion a deflationary contraction, as described in the discussion of the redemption risk shock. Furthermore, declining net worth reinforces the flight to quality. This is because the smaller is net worth, the more the losses from early liquidations increase the likelihood of default, which is costly to creditors.

Importantly, the presence of liquidity risk reverses the increase in consumption that otherwise arises when the drop in net worth drags down the financial sector's capacity for intermediation. This reversal obtains despite the fact that intermediation contracts even more in the presence of liquidity risk than in its absence.

To understand this, recall that financial firms dramatically reduce their production of partial substitutes for money, both because overall borrowing shrinks, and because banks shift towards maturity-matched financing. Why don't expansionary open market operations offset this contraction? Under traditional open market operations, when the central bank provides households with additional money balances, it also reduces the privately available supply of government debt. As a result, the equilibrium nonpecuniary value to households of the marginal unit of government debt rises.

But as discussed in the results for the redemption risk shock, the boost to $\frac{\partial \mathbf{u_t^{trans}}}{\partial \mathbf{b_{t+1}^h}}$ discourages the accumulation of liquidity reserves by financial firms by widening the wedge between the average return owed creditors and the pecuniary return on liquidity reserves. A smaller liquidity reserve exacerbates liquidity risk. Thus, overall borrowing drops, net worth contracts, and financial firms shift towards maturity-matched debt. In sum, the issuance of demandable debt dwindles further.

The central bank therefore fails to effectively stabilize the supply of money and money substitutes. In order to build up desired money balances, households reduce consumption relative to what it otherwise would be, and prices fall.

A similar dynamic emerges in simulated responses to other shocks. In the case of an unexpected increase in σ_t–which Christiano, Motto, and Rostagno (2010) call a "risk shock"–consumption falls in the presence of liquidity risk, but rises sharply in its absence. Also, the contraction in investment is almost 50% larger in the present model than in its BGG-type analogue.

In summary, heightened liquidity risk results in reductions in the private production of money substitutes, expansions in financial firms' desired liquidity cushion, and contractions in the privately optimal level of intermediation. These effects powerfully amplify the macroeconomic costs of financial sector distress.

Furthermore, given that the magnitudes of the shocks in each of the above simulations are quite modest, endogenous liquidity transformation by risky financial firms can easily rationalize very low nominal interest rates on highly liquid assets. The sharp decline in interest rates does not result from a "paradox of thrift" in the usual sense of that phrase; households do not suddenly wish to save more. Instead, adverse shocks to the financial sector induce a flight to quality wherein households substitute away from privately produced, risky money substitutes, and towards money and government-backed money substitutes (here, government debt). Traditional open market operations that swap money for government debt do not sate the increased demand for liquidity.

5.3 Unconventional open market policy improves responses to heightened liquidity risk

The central bank can substantially improve macroeconomic performance under the simple Taylor rule by altering its implementation of that target. Traditional expansionary open maket operations increase money and reduce privately available government debt. I contemplate two alternative policies.

First, I consider a unified fiscal and monetary authority. In Figure (11), the policy maker responds to a redemption risk shock by issuing lump sum tax rebates and thereby increasing government debt. At the same time, she expands the money supply by partially monetizing the debt. In particular, I replace the open market equation, (17), with

$$\theta^{ops}\left(m_{t+1} - \frac{m_t}{\Pi_t}\right) = -\left(b_{t+1} - \frac{b_t}{\Pi_t}\right) \tag{18}$$

In the baseline model, $\theta^{ops} = 1$. Here, I set $\theta^{ops} = -3$. Thus, for each unit of time t final goods that the government obtains from printing money, it reduces lump sum taxes by four final goods, one of which is monetized by open market operations.

The expansion in the privately available supply of government debt substantially mitigates the macroeconomic costs of the redemption risk shock. The initial deflation is cut by over two-thirds, and the contraction in output shrinks by almost three-quarters. Troughs in net worth and investment are over two-thirds greater than they are under traditional open market operations.

To understand the efficacy of this alternative policy, consider that, as a result of the increase in government debt, financial firms augment their liquidity reserves more than they do under the baseline specification. Additional reserves moderate

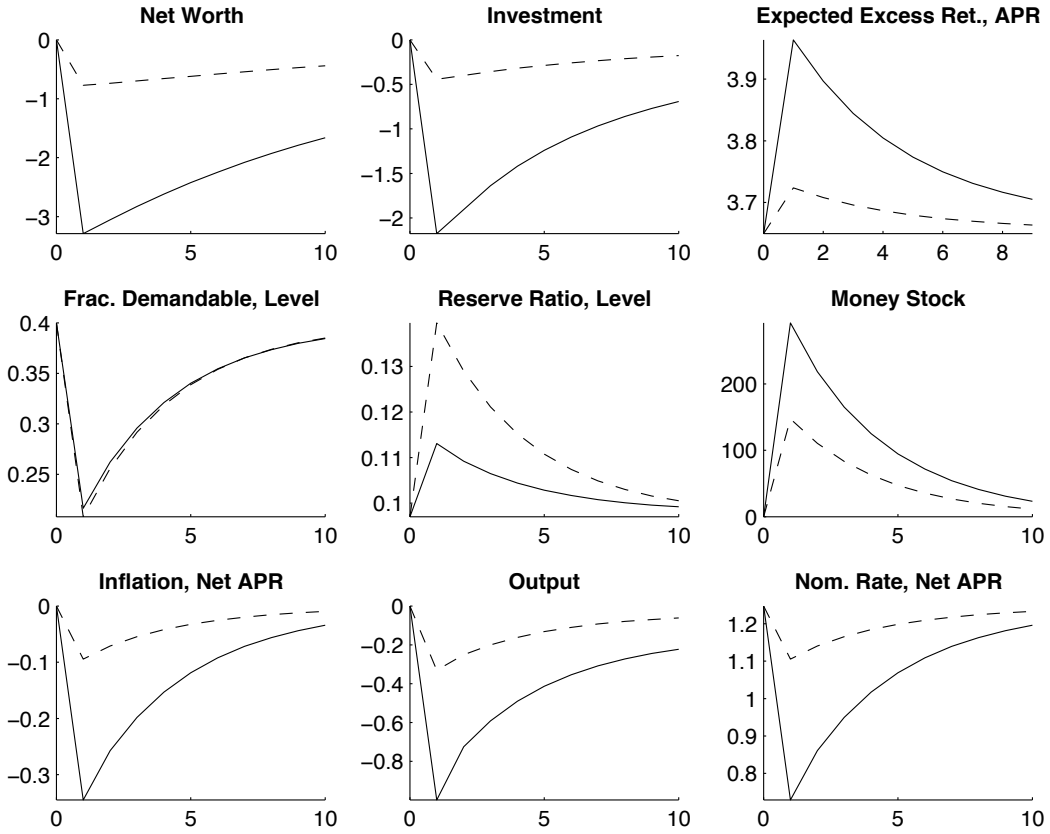

Figure 11: Rebates and monetization policy

All plots are annualized percentage deviations from steady state, unless otherwise noted.

Solid: Baseline specification. Dotted: Rebates and monetization policy

the jump in equilibrium liquidity risk. Intermediation therefore stabilizes at a higher level, which is reflected in the much-reduced peak in the excess return to capital.

Furthermore, because the aggregate supply of government debt expands substantially, households can hold more government debt, too. Additionally, although financial firms still shift towards maturity-matched debt, overall financial firm borrowing declines by a smaller amount, with the result that the private production of money substitutes does not fall as far. Households' demand for money balances therefore rises less than under traditional open market operations, which mitigates the drop in consumption.

In summary, this combined fiscal and monetary policy fortifies the level of financial intermediation and mitigates downward pressures on prices and the interest rate. Because this policy is more effective at meeting heightened liquidity demand and, hence, stabilizing the economy, the Taylor rule prescribes much smaller interest rate cuts.

Importantly, Ricardian equivalence fails in this model; the choice of government financing matters greatly. When demand for liquidity is high relative to the supply of liquidity, debt financing becomes more preferable to taxation. Notice, however, that I have assumed that the money-like properties of government debt do not deteriorate, no matter how large is the debt of the government. As evidenced by the ongoing sovereign debt crisis in Europe, real world political constraints may prevent countries from credibly commiting to pay off large, incremental deficits, especially given a large existing stock of government debt and economic contraction or stagnation. As a result, countries face a limit on how much they can borrow before government debt starts to lose its money-like liquidity and safety properties.

Last, notice that here government deficits do not arise from any additional government spending. Rather, the stimulative effect of government borrowing arises from its effect on the aggregate supply of (imperfect) money substitutes, and the role of said substitutes in risky intermediation. The increase in the supply of liquid assets improves the ability of outside investors to coordinate their resources with financial firms, despite the higher costs of the underlying financial friction. Additionally, the improved availability of liquidity reduces the magnitude of the disinflation needed to accommodate household demand for money and money substitutes. Estimates of the stimulative effects of debt-financed government spending or tax cuts may capture not only traditional incentive and aggregate demand effects, but also the effects of an expansion of the supply of money-substitutes.[17]

Next, I consider a policy that can be implemented by the central bank acting alone. In particular, the central bank prints money, sells some of its holdings of

[17] Of course, both these effects are conditional on the policy function of the monetary authority.

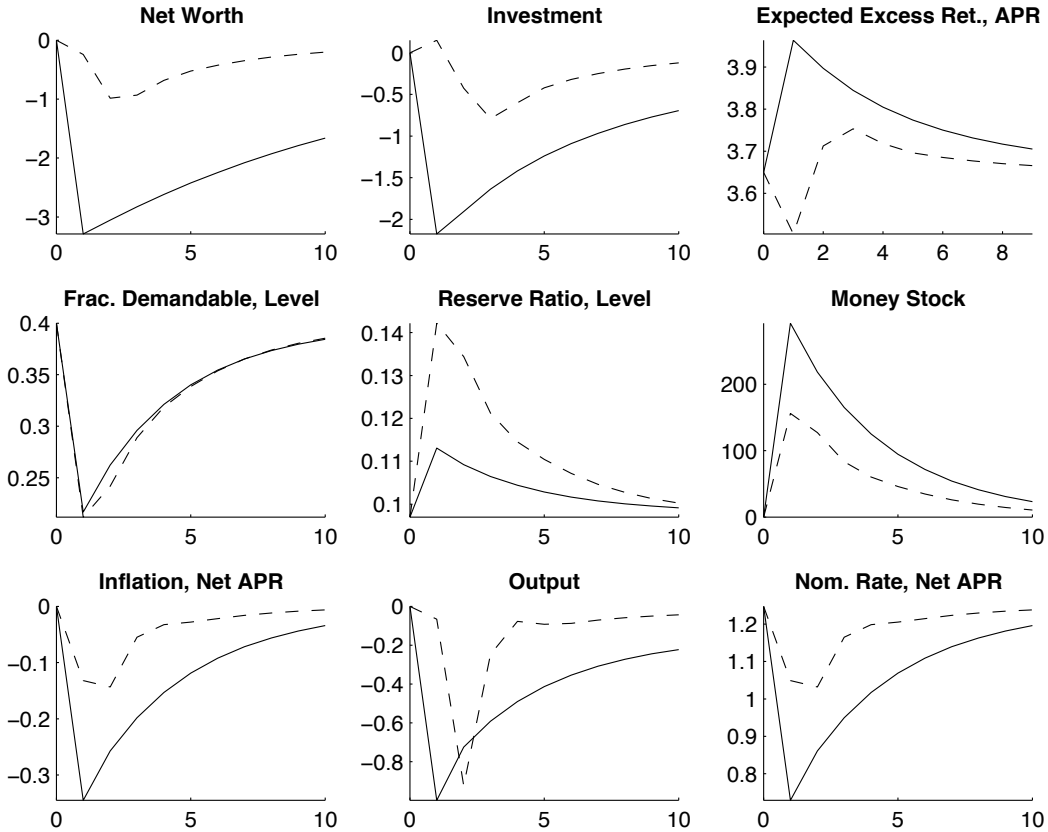

Figure 12: Open market purchases of risky assets

All plots are annualized percentage deviations from steady state, unless otherwise noted.
Solid: Baseline specification. Dotted: Open market purchaes of risky assets

government debt, and directly purchases risky assets. Total revenues from printing money and selling bonds equal total expenditures on purchases of risky assets. In the model setup, I keep $\theta^{ops} = -3$ and retain relation (18) to ensure comparability with the previous experiment, and require further that

$$(1 - \theta^{ops}) \left(m_{t+1} - \frac{m_t}{\Pi_t} \right) = q_t K^m_{t+1}$$

where K^m_{t+1} denotes purchases of risky assets by the central bank. Given this value of θ^{ops}, the central bank prints one final goods' worth of money; sells to the public three final goods' worth of the central bank's holdings of government debt; and uses the receipts to purchase risky assets.

Figure (12) displays responses to the redemption risk shock under this policy. As with the tax rebate policy, printing money and increasing the supply of privately

available government debt together lower nominal interest rates and increase the supply of liquid assets. Financial firms can boost their liquidity reserves more than they can under baseline policies, and household demand for money balances increases less than it otherwise would.

Furthermore, because the central bank purchases risky assets, financial firms in equilibrium need not hold as many imperfectly liquid assets. As a result, liquidity risk is essentially flat, while solvency risk falls. These effects together induce an initial decline in the wedge between the return to capital and the return to government debt. Open market purchases of risky assets thus mute the initial drops in net worth, investment, and output. But these variables exhibit delayed troughs because, as the central bank shrinks the money supply after the first period, it disgorges its holdings of risky assets, which the financial sector must then absorb.

During the financial crisis of 2008-2009, the Federal Reserve altered its implementation of monetary policy in a manner consistent with policy prescriptions that arise naturally from my model. In particular, the Federal Reserve's balance sheet expanded sharply, traditional security holdings fell, and the Federal Reserve purchased large volumes of Agency debt and mortgage-backed securities.[18] The Federal Reserve also lent Treasury Securities against a variety of risky securities, including mortgage backed securities.[19] Thus, the Federal Reserve effectively reduced the supply of then-illiquid risky assets that financial markets had to absorb, while augmenting the money supply and the privately available supply of government debt.

I do not model efficiency costs from intermediation of capital services projects by the monetary authority. My objective is to show the general equilibrium benefits of such intermediation. These interventions will be justified only if the benefits exceed whatever efficiency costs exist.

5.4 The central bank should lower interest rates when the excess return to capital rises

Liquidity risk dynamically affects the optimal borrowing of financial firms. First, when liquidity risk surges, a given amount of borrowing entails more costly fire sales that reduce capital revenues-per-liability. Second, because liquidity risk bites precisely when fundamentals are weak, heightened liquidity risk boosts the average

[18] Available from the Cleveland Federal Reserve, at http://www.clevelandfed.org/research/data/credit_easing/index.cfm.

[19] A timeline of policy actions is available from the St. Louis Federal Reserve, at http://timeline.stlouisfed.org/index.cfm?p=timeline#. In 2008, the Term Securities Lending Facility (TSLF) lent Treasury Securities "for 28-day terms against federal agency debt, federal agencyresidential mortgage-backed securities (MBS), non-agency AAA/Aaa private label residential MBS, and other securities."

monitoring costs creditors face for any given interest rate. Both these effects magnify the cost to the bank of external financing. This cost is summarized by the wedge between the return to capital and the return to government debt. By lowering its target interest rate when this wedge widens, the central bank can substantially improve welfare performance following a redemption risk shock.

To show this, I replace the baseline Taylor rule (16) with

$$\hat{R}^g_{t+1} = \rho_R \hat{R}^g_t + (1-\rho_R)\left(\rho_\pi \hat{\Pi}_t - \rho_e E_t\left[\mathrm{d}\left(\frac{R^k_{t+1}}{R^g_{t+1}/\Pi_{t+1}}\right)\right]\right) \tag{19}$$

where $\mathrm{d}\left(\frac{R^k_{t+1}}{R^g_{t+1}/\Pi_{t+1}}\right)$ is the deviation from steady state of the wedge between the return to capital and the return to government debt. Figure (13) displays responses to the redemption risk shock when the monetary authority almost-fully offsets changes in this wedge; i.e., when $\rho_e = 0.985$. (It turns out that the welfare loss from the redemption risk shock decreases in magnitude as ρ_e increases. But as ρ_e approaches unity, the unique nonexplosive equilibrium vanishes. Thus, I choose a value of ρ_e near - but not exactly equal - to one.)

Targeting an interest rate adjusted for changes in the excess return to capital substantially improves macroeconomic performance in the face of a redemption risk shock. The output contraction shrinks by roughly three-quarters, and the deflation is almost eliminated. Banker net worth actually tics up slightly, as does investment.

To understand this, consider that, as discussed above, under the baseline model growth in the excess return to capital captures the costs to the economy of heightened liquidity risk. By lowering the interest rate target in response to this higher excess return, the monetary authority reduces financial firms' cost of borrowing. The size of the optimal contract rises, as do net worth and investment.

Notice that both the modified Taylor rule and the alternative open market transactions stabilize the economy at a higher nominal interest rate. Large reductions in interest rates can reflect ineffective monetary policy. Ineffective policy fails to offset contractionary forces; the monetary authority responds to the resulting, deeper declines in output with larger rate cuts and expansions in the money supply. By supporting investment and liquidity transformation, an effective central banker need not cut rates as sharply.

Finally, I confirm that the presence of liquidity risk amplifies the welfare benefits of replacing the simple Taylor rule with (19). I set $\rho_e = 0.5$ and compute responses to heightened financial sector distress (i.e., a decrease in p_t) under the simple rule and (19), for models with and without liquidity risk. In the model without liquidity risk, including the excess return to capital in the monetary policy rule actually hurts welfare performance. In the presence of liquidity risk, however, the adjusted target

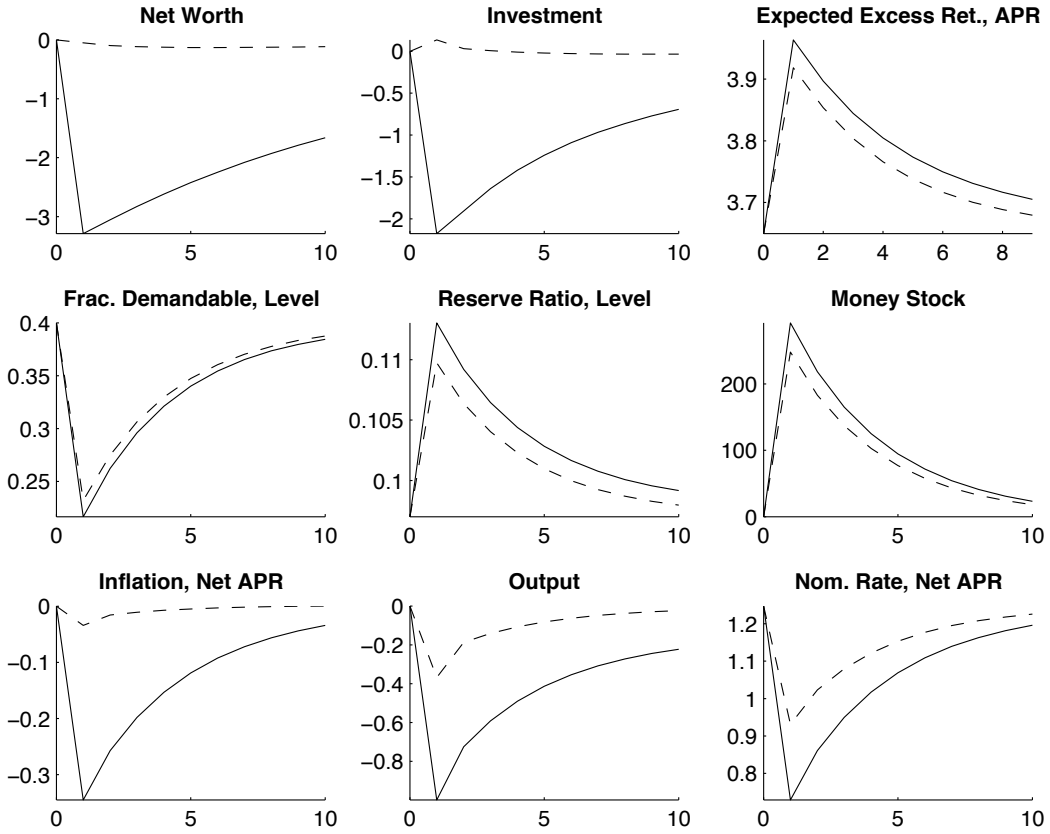

Figure 13: Modified Taylor Rule

All plots are annualized percentage deviations from steady state, unless otherwise noted.

Solid: Baseline specification. Dotted: Modified Taylor rule

improves welfare.

Under a risk shock, adjusting for the excess return to capital improves welfare, with or without liquidity risk. But the amount of welfare improvement is substantially greater in the presence of liquidity risk. The welfare benefits of stabilizing the financial sector are magnified precisely because endogenous liquidity transformation amplifies the macroeconomic costs of adverse shocks to the financial sector.

5.4.1 Adjusting for liquidity demand

Another way to understand the stabilizing effect of adjusting for the excess return to capital is as follows. I consider employing an alternative target that adjusts the desired interest rate for non-pecuniary premia on government debt. In particular, recall equation (12), which I reproduce here for convenience:

$$\lambda_t - \frac{\partial u_t^{trans}}{\partial b_{t+1}^h} = \beta E_t \left[\lambda_{t+1} \times \frac{R_{t+1}^g}{\Pi_{t+1}} \right]$$

In the absence of a transactions value to government debt, equation (12) would read

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \times \frac{R_{t+1}^g}{\Pi_{t+1}} \right]$$

Let

$$R_{t+1}^{g,target} := R_{t+1}^g + \frac{\partial u_t^{trans}}{\partial b_{t+1}^h} \times \frac{\Pi_{t+1}}{\beta \lambda_{t+1}}$$

Then, given (12), $R_{t+1}^{g,target}$ satisfies

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \times \frac{R_{t+1}^{g,target}}{\Pi_{t+1}} \right]$$

I then assume that the monetary authority follows the Taylor rule (16) as before, but replaces R_{t+1}^g with $E_t R_{t+1}^{g,target}$.

It turns out that the simulated responses to the redemption risk shock under this target are nearly identical to the responses under (19). Understanding why adjusting for liquidity demand is effective will therefore shed light on why adjusting for the excess return to capital is effective.

Consider the households' intertemporal Euler equation, (12). Suppose liquidity demand intensifies, and, therefore, $\frac{\partial u_t^{trans}}{\partial b_{t+1}^h}$ increases. Then in order for (12) to be satisfied, λ_t must increase, λ_{t+1} must fall, or the real interest rate under the Taylor

rule must fall. It turns out that the real interest rate falls. But a decline in the real interest rate under the unadjusted Taylor rule requires a decline in the current rate of deflation. Under short-term monetary nonneutralities, this is consistent only with output contraction.

By contrast, using $E_t R_{t+1}^{g,target}$ lowers R_{t+1}^{g} for any given rate of inflation when liquidity demand is elevated. Thus, when $\frac{\partial u_t^{trans}}{\partial b_{t+1}^h}$ rises, current inflation and output need not decline for (12) to be satisfied. The smaller decline in output is in turn consistent with the salutary effects on the optimal financial contract of targeting $E_t R_{t+1}^{g,target}$. In particular, because the central bank lowers R_{t+1}^{g} to the extent that $\frac{\partial u_t^{trans}}{\partial b_{t+1}^h}$ increases, financial firms do not suffer an increase in the average rate of return R_{t+1}^{AVG} owed creditors. As a result, the optimal contract size stabilizes, which in turn fortifies net worth, investment, and output.

When the monetary authority adjusts for liquidity demand, any *additional* adjustment for the excess return to capital will induce explosivity in the linearized equilibrium. Thus, the above equivalence naturally raises the question: should the monetary authority ideally adjust for liquidity demand or for the excess return to capital?

To answer this question, consider that the costs of financial frictions can fluctuate for reasons other than changes in liquidity demand. The excess return to capital captures all these fluctuations, whether they arise from elevated liquidity risk or from some other source.

Shocks to the cross-sectional dispersion σ_t of log-ω_{t+1}^b, for example, affect not only liquidity demand, but also the risk of costly monitoring under the debt contract. As a result, adjusting the target interest rate for liquidity demand is less effective at reducing the welfare costs of risk shocks than are adjustments for the excess return to capital.

Figure (14) illustrates the simulated responses to the risk shock under each of the alternative Taylor rules. Liquidity demand fails to capture the full increase in the costs of intermediation. By contrast, adjusting for the excess return to capital results in an aggressive cut to the nominal interest rate, relative to the rate of inflation. Borrowing costs decline, which offsets the higher monitoring costs and mitigates initial declines in investment, net worth, and output.

5.4.2 Adjusting for credit spreads

The excess return to capital in the real world depends on higher order moments and comovements with households' marginal value of real income. The linearized

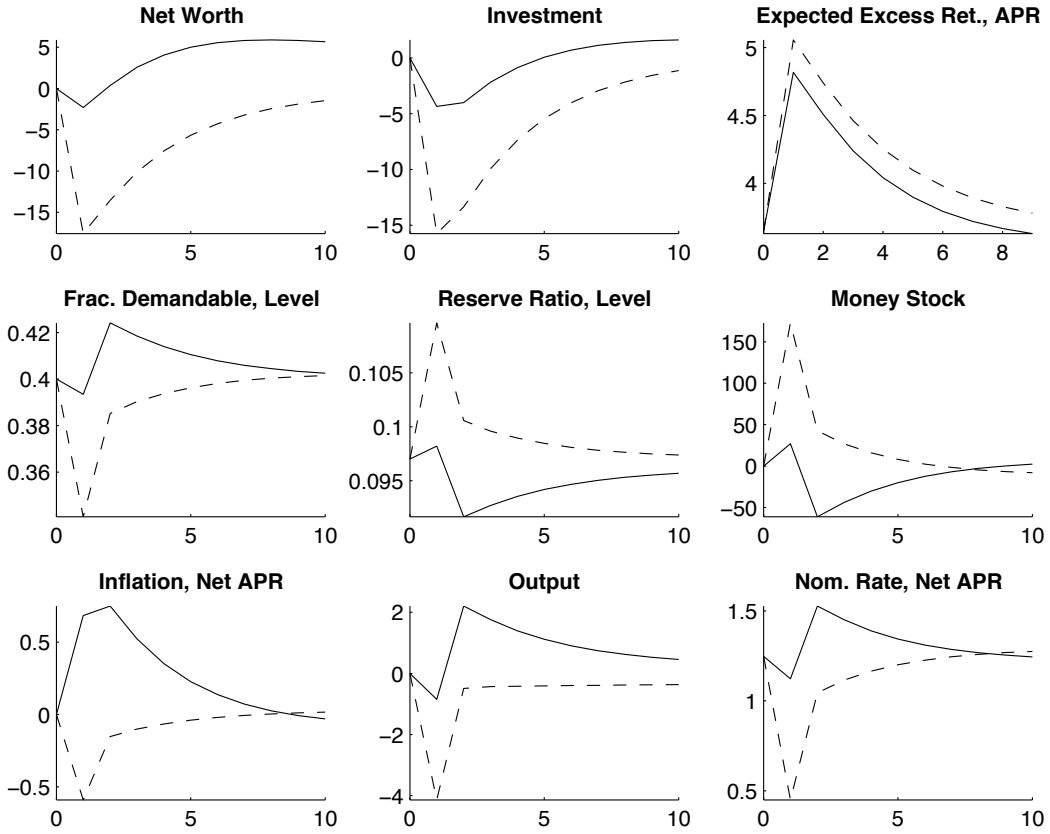

Figure 14: Excess Return vs Liquidity Demand

All plots are annualized percentage deviations from steady state, unless otherwise noted.

Solid: Excess return adjustment. Dotted: Liquidity demand adjustment

economy does not capture these relationships. Thus, the real world excess return to capital is a potentially very noisy measure of financial frictions, making it a less-than-ideal input for the monetary policy rule.

Credit spreads might provide a more reliable real world measure of these frictions. I therefore follow Curdia and Woodford (2010) and include credit spreads in the monetary policy rule. In particular, I specify that the target interest rate decreases one-for-one with increases in the spread between the average promised interest rate, $R^*_{t+1} = \xi^D_t R^D_{t+1} + \left(1 - \xi^D_t\right) R^L_{t+1}$, and the rate on government debt. The rule is therefore the same as (19), only replacing the expected excess return with the expected credit spread. I then examine responses to redemption risk shocks.

I find that adjusting the interest rate for changes in credit spreads is, in the linearized model, not as effective as adjusting for the wedge between the return to capital and the return to government debt. In fact, adjusting for credit spreads has very little effect on the response to a shock to redemption risk. This is because heightened liquidity risk primarily manifests itself in declines in overall borrowing and the fraction of debt that is demandable, and increases in the ratio of liquidity reserves to demandable debt. The surge in credit spreads is comparatively small. Because the optimal contract adjusts along dimensions besides the credit spread, said spread does not capture the full increase in the costs of financial frictions.

In summary, the welfare improvement obtained in the linearized model from adjusting for changes in the excess return to capital demonstrates the potential benefits of offsetting fluctuations in the costs of financial frictions. However, the foregoing results suggest that additional work is required to develop an observable market indicator that accurately captures these fluctuations in the real world, and to identify the set of corresponding weights that optimize expected welfare. These findings complement those of Curdia and Woodford (2010), who find that no single weighting on credit spreads effectively improves welfare performance under different types of shocks and different shock persistences.

6 Concluding thoughts and future work

Financial sector risk powerfully affects the macroeconomy through variation in financial firms' production of money substitutes and in their demand for precautionary reserves of highly liquid assets. Under traditional monetary policy, heightened liquidity risk induces a flight to quality and a deflationary contraction. Liquidity risk thus magnifies the macroeconomic costs of adverse shocks to the financial sector to a far greater extent than BGG. Furthermore, this deflation means the model can

also rationalize very low nominal interest rates following reasonably-sized shocks to the financial sector, without an exogenous increase in households' desire to save.

In stark contrast with standard results in the New Keynesian DSGE literature, open market operations powerfully influence economic outcomes in my model. Short term government debt enjoys money-like qualities and provides an interest-bearing liquidity buffer for risky financial firms. Thus, central bank injections of money in exchange for government debt fail to dampen the costs to financial firms of heightened liquidity risk. However, by altering its open market operations to increase the supply of liquid assets relative to imperfectly liquid, risky assets, a central bank can stabilize the economy. The model thus neatly rationalizes the Federal Reserve's unconventional open market operations during 2008-2009. Furthermore, the model implies that the Federal Reserve can improve macroeconomic performance even when nominal interest rates are fixed near zero, as long as there exists an unsatiated financial sector demand for liquidity. I leave to future work a more detailed exploration of liquidity transformation and monetary policy when money and government debt are perfectly substitutable.

Because Ricardian equivalence fails, tax rebates can have a stimulative effect without any additional government spending. Empirical studies may therefore capture not only the incentive and aggregate demand effects of changes in government spending and taxes, but also the effect of deficits on the supply of partial substitutes for money. However, there surely exist real world limits to this effect. Large existing stocks of debt, low economic growth, and political constraints may cause large deficits to degrade the money-like properties of government debt by introducing doubt about the likelihood (or real value) of repayment.

Alternative interest rate targets can also improve welfare in the face of shocks to the financial sector. In particular, the central bank should reduce its interest rate target when the wedge between the return to capital and the return to government debt increases. Such a policy substantially reduces the costs of adverse shocks to the financial sector, by stabilizing optimal contract size when liquidity or solvency risk unexpectedly rises.

An important challenge to all work on the macroeconomic amplification of shocks to the financial sector is that, when a model rationalizes larger responses to shocks of reasonable magnitude, the first-order Taylor expansion becomes a less accurate approximation of the true, nonlinear economy. On the other hand, nonlinear models are most tractable when they employ few state variables. But this restriction in turn limits the scale of the macroeconomic model and the potential for exploring the interactions between different model components in the DSGE literature. Striking the right balance and exploring tractable ways to include nonlinearities are important

areas for future work.

Including active government support of the financial sector will be an essential step towards understanding the events of 2008-2009. Increased protection of financial firms' creditors and investors may stabilize intermediation during a crisis. Additionally, the likelihood and degree of government protection is itself a risk factor; fluctuations in the probability that the government will shield creditors from losses should significantly impact intermediation and investment. Last, political constraints may introduce nonlinear dynamics. In particular, if political constraints on the aggregate level of support for the financial sector bind when losses are large, imperfect government insurance of creditors may amplify financial crises precisely when they are most severe.

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