Efficient Bailouts? *

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Wisconsin & NBER

October 2012

Abstract

This paper develops a non-linear DSGE model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante. During a systemic crisis, bailouts to the financial sector relax balance sheet constraints and mitigate the severity of the recession. Ex ante, the anticipation of such bailouts leads to an increase in risk-taking, making the economy more vulnerable to a financial crisis. The optimal policy requires, in general, a mix of ex-post intervention and ex-ante prudential policy. We also analyze the effects of bailouts on financial stability and welfare in the absence of ex-ante prudential policy. Our results show that the moral hazard effects of bailouts are significantly mitigated by making bailouts contingent on the occurrence of a systemic financial crisis.

Keywords: Bailouts, moral hazard, credit crunch, financial shocks
JEL classification: E32, E44, F40, G18

*I am grateful to Ufuk Akcigit, Manuel Amador, Maya Eden, Huberto Ennis, Juan C. Hatchondo, Andreas Horstein, Todd Keister, Anton Korinek, Enrique Mendoza, Andy Powell, Thomas Sargent, John Shea, Ali Shourideh, Carlos Vegh, Stijn Van Nieuwerburgh, for helpful comments. I also thanks seminar participants at Harvard, NYU, Stanford, University of Maryland, University of Pennsylvania, the Federal Reserve Bank of Richmond, Federal Reserve Bank of New York, Federal Reserve Bank of Minneapolis World Bank, Bank of Italy, Central Bank of Uruguay, Universidad de Montevideo, Society of Economic Dynamics (SED), HKUST 'International Workshop on Macroeconomics', Di Tella 'XIV Workshop in International Economics and Finance', and Dallas Fed Conference on 'Financial frictions and Monetary Policy in an Open Economy' for useful comments.

1 Introduction

A common feature of financial crises is massive government intervention in credit markets. For example, the initial Troubled Assets Relief Program (TARP) required 700 billion dollars to provide credit assistance to financial and non-financial institutions. Related measures in the ongoing European crisis continue to spark an intense debate on the desirability of government intervention in credit markets. Many argue that bailouts are often necessary to prevent a complete meltdown of the financial sector, which would bring an extraordinary contraction in output and employment. An alternative view argues that bailouts create incentives to take even more risk ex-ante, sowing the seeds for future crises. According to this view, regulations should introduce a strict limit on the government's ability to bailout the financial sector.

How does the expectation of a bailout impact the stability of the financial sector? Is it desirable to prohibit the use of public funds to bailout the financial sector? How large should bailout packages be? How critical are policies to prevent excessive risk-taking due to the anticipation of future bailouts?

This paper addresses these questions using on a non-linear DSGE model in which credit frictions generate scope for bailouts during a financial crisis, but where the anticipation of bailouts generate more risk-taking before the crisis actually hits. Recent research (e.g. Gertler and Kiyotaki, 2010) has analyzed how credit policy can mitigate a credit crunch and the resulting recession ex-post. At the same time, a growing theoretical literature investigates the moral hazard implications of government bailouts (e.g. Farhi and Tirole, 2012). However, there has been little work assessing the quantitative implications of moral hazard. This paper contributes to fill in this gap by developing a quantitative equilibrium model to assess the interaction between ex-post interventions in credit markets and the build-up of risk ex ante in a unified framework. We use this framework to investigate the optimal bailout policy and evaluate its macroeconomic and welfare effects.

The model features a representative corporate entity that faces two frictions in its capacity to finance investment. First, debt contracts are not enforceable, giving rise to a collateral constraint that limits the amount that firms can borrow. Second, there is an

equity constraint that imposes a minimum dividend payment that firms must make each period. In the stochastic steady state of the model, firms are able to finance the desired level of investment during normal economic conditions. When leverage is sufficiently high and an adverse financial shock hits the economy, however, firms are forced to cut down on investment, leading to a protracted recession. Anticipating that such episodes are costly, firms behave precautionary during normal times balancing the desire to increase borrowing and investment today with the risk of becoming financially constrained in the future.

In our model, credit crunches are socially inefficient because firms remain undercapitalized, hindering the economic recovery. From an individual point of view, households do not have an incentive to unilaterally transfer funds to firms, since this only entails costs for them. From a social point of view, however, a bailout by the government represents a Pareto improvement because the collective transfer to firms allows all households to obtain higher dividends and higher labor income in the future. Socially inefficient credit crunches also has consequences for the optimality of ex-ante risk taking decisions, as we explain below.

Our normative analysis begins by considering a benevolent social planner that takes financial decisions subject to the same financial constraints as the private economy. As in Holmstrom and Tirole (1998), the government is able to improve upon the decentralized equilibrium by extracting payments from households . In particular, by taxing households, the government can address the coordination failure that prevents an efficient recapitalization. When these benefits are sufficiently high, the government conducts these transfers at the expense of incurring efficiency costs.

Next, we turn to study the policies that can decentralize the constrained-efficient allocations. We show that the optimal policy mix requires, in general, a combination of ex-post intervention and ex-ante prudential policies. Ex-post interventions, i.e. bailouts, are necessary in order to redistribute resources from households to firms during financial crises. The scope for ex-ante intervention, i.e. macro-prudential policy, depends on the perception of how bailouts are implemented. When bailouts are implemented using lump-sum transfers conditional on the economy's aggregate states, individual financial decisions are not distorted ex-ante and there is no need for curbing borrowing before a crisis materializes. That is, although firms take more risk anticipating that the government will relax balance sheet

constraints once a crisis occurs, this is an efficient response to the insurance provided by the government. This form of non-targeted bailouts is not very practical, though, as the government needs to provide the same transfer to all entities regardless of their financial position.

In a more realistic case in which bailouts are partially targeted, the government needs to use policies that curb borrowing to implement the constrained-efficient allocations. Intuitively, targeted bailouts causing borrowing costs to decline below the social value through implicit subsidies. This gives rise to a complementarity between targeted bailouts and macro-prudential policy. Finally, we explore the importance of macro-prudential policy. In particular, we investigate how an economy with no-bailout policy differs with a decentralized economy where there are bailouts but no macro-prudential policy. In this case, bailouts create a tradeoff: bailouts ameliorate credit crunches ex-post, while they provide an implicit subsidy that causes too much risk-taking.

For our quantitative investigation, we calibrate the model using the US economy as a reference. We match targets for leverage, volatility of investment, and the frequency and duration of financial crises. In our simulations, the optimal intervention results in financial constraints becoming frequently more binding in the economy as the insurance provided by bailouts decrease the need for precautionary savings. The severity of binding constraints, however, become less severe for the economy. In fact, we find that the the impact loss in output that results from a credit crunch falls 20 percent under the optimal intervention. Moreover, the recession that follows crises become less protracted resulting in significantly smaller cumulative output losses. Hence, while the planner's intervention induces more risk-taking, it does not cause more financial fragility due to the fact that the planner can alleviate the effects of adverse financial shocks.

We then explore the importance of prudential policy to discourage excessive risk-taking. One of our key findings is that if targeted bailouts are implemented only in systemic financial crisis, there are still strictly positive welfare gains from conducting only bailouts. In fact, the magnitudes of the welfare gains come quite close to the gains obtained the fully optimal policy that allows for both prudential policy and ex-post policy. We emphasize that the welfare gains from bailouts in this setting occur not only ex-post but also ex-ante. That is,

even if bailouts generate excessive risk-taking, the benefits from better insurance outweigh the moral hazard effects. On the other hand, if bailouts are idiosyncratic and are fully determined by individual financial decisions, this causes a sharp increase in financial fragility and substantial ex-ante welfare losses.

Related Literature — This paper draws on the extensive literature on the macroeconomic effects of financial frictions, shaped by the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In particular, our model shares with Jermann and Quadrini (2012) the emphasis on financial shocks and equity financing decisions, and with Mendoza (2010) the emphasis on non-linear dynamics beyond the steady state. However, these papers do not address normative issues.

This paper is also related to a growing quantitative literature that studies the effects of credit policy during a credit crunch.[1] For reasons of tractability, most of this literature focuses on policy measures in response to unanticipated crises or on log-linear dynamics around the deterministic steady state and does not address risk considerations and the moral hazard effects of credit policy. Instead, a distinctive feature of this paper is the consideration of how expectations of future bailouts affect ex-ante risk-taking. This is crucial to assess the dynamic implications of credit intervention on financial stability and on social welfare.

This paper also builds on the theoretical literature that analyzes the effects of bailouts on risk-taking incentives and financial stability.[2] Farhi and Tirole (2012) show that bailouts generate incentives to correlate risks resulting in excessive financial fragility, and draw implications for ex-ante regulation to rule out bailouts in equilibrium. Our paper emphasizes the idea that bailouts can be welfare improving from an ex-ante point of view due to their insurance role. In this aspect, it is more closely related Keister (2012). He studies a Diamond-Dybvig economy and shows that commitment to a no-bailout policy induces banks to remain too liquid from a social point of view, and may increase the vulnerability to a bank-run. Over-

[1] See Gertler and Karadi (2011), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2010) for models of credit policy. Other recent models of credit crunches include Guerrieri and Lorenzoni (2011), Bigio (2010), Midrigin and Philippon (2011), Shourideh and Zetlin-Jones (2012).

[2] Recent examples include Farhi and Tirole (2012), Chari and Kehoe (2009), Diamond and Rajan (2009), Keister (2012), Keister and Narasiman (2011), Pastén (2011), Nosal and Ordonez (2012), Schneider and Tornell (2004). For empirical evidence on the anticipation of bailouts in the US financial crisis see Kelly, Lustig, and Van Nieuwerburgh (2011).

all, our main contribution to this literature is to provide a quantitative framework to assess the macroeconomic and welfare effects of bailouts.

In recent work, Gertler, Kiyotaki, and Queralto (2011) develop a model in which banks have access to debt and equity financing and investigate the moral hazard effects of credit policy. They restrict attention to macro dynamics around a "risk-adjusted" steady state in which financial constraints are always binding. In contrast, we study full equilibrium dynamics in a stochastic steady state in which binding financial constraints only bind occasionally. We also complement their work by characterizing and solving for the optimal bailout policy and prudential policy to avoid excessive risk-taking.

Finally, this paper is also related to a growing quantitative literature on how macroprudential policy can be used to reduce the level of financial fragility.[3] The inefficiency in this literature relates to the effects of inter-temporal reallocation of wealth across leveraged borrowers on equilibrium prices affecting funding constraint. In contrast, the scope for policy here arises because of the effects of intra-temporal reallocations of wealth between households and firms on future production capacity.

The remainder of the paper is organized as follows: Section 2 presents the analytical framework; Section 3 analyzes the optimal intervention; Sections 4 and 5 present the quantitative analysis; and Section 6 discusses the conclusions.

2 Analytical Framework

Our model economy is a small open economy populated by firms and workers who are also the firms' shareholders. We begin by describing the decisions made by different agents in the economy, and then we discuss the general equilibrium.

[3]See e.g. Bianchi (2011), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), and earlier theoretical work by Lorenzoni (2008) and Caballero and Krishnamurthy (2001). Benigno, Chen, Otrok, Rebucci, and Young (2012) also discuss ex-post policy measures to address a pecuniary externality (see also Jeanne and Korinek (2011) for a discussion of ex-ante versus ex-post policies).

2.1 Households

There is a continuum of identical households of measure one that maximize:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t - G(n_t)) \tag{1}$$

where c_t is consumption, n_t is labor supply, β is the discount factor, and $G(\cdot)$ is a twice-continuously differentiable, increasing and convex function. The utility function $u(\cdot)$ has the constant-relative-risk-aversion (CRRA) form; the composite of the utility function has the GHH form, eliminating wealth effects on the labor supply. The advantage of these preferences is that they deliver realistic responses of employment during a credit crunch without introducing frictions in labor markets that would complicate the analysis.

Households do not have access to bond markets, and they are the firms' shareholders. This yields the following budget constraint:

$$s_{t+1}p_t + c_t \leq w_t n_t + s_t(d_t + p_t) \tag{2}$$

where s_t represents the holdings of firm shares and p_t represents the price of firm shares.

The first-order conditions are given by:

$$w_t = G'(n_t) \tag{3}$$

$$p_t u'(t) = \beta \mathbb{E}_t u'(t+1)(d_t + p_{t+1}) \tag{4}$$

Iterating forward on (4) and imposing a no-bubble condition yields that in equilibrium, the price of shares must be equal to:

$$p_t = \mathbb{E}_t \sum_{j=1}^{\infty}\beta^j m_{t+j} d_{t+j} \tag{5}$$

where $m_{t+j} \equiv (\beta^j u'(c_{t+j} - G'(n_{t+j})))/(u'(c_t - G'(n_t)))$ represents the stochastic discount factor.

2.2 Corporate Entities

There is a measure one of identical firms with technology given by the production function $F(z_t, k_t, h_t)$ that combines capital denoted by k, and labor denoted by h to produce a final good. TFP denoted by z_t follows a first-order Markov process. Consistent with the typical timing convention, k_t is chosen at time $t-1$ and is therefore predetermined at time t. Instead, the input of labor h_t can be flexibly changed in period t.

Firms have the following technology to transform final goods into investment goods.

$$k_{t+1} = k_t(1-\delta) + i_t \tag{6}$$

where i_t is the level of investment and δ is the depreciation rate. Capital accumulation is subject to convex adjustment costs, given by $\psi(k_t, k_{t+1})$. Adjustment costs are introduced to improve the quantitative performance of the model in terms of the volatility of investment and asset pricing implications.

Firms pay dividends, denoted by d_t, and issue non-state contingent debt, denoted by b_{t+1}. Firms finance investment, including capital adjustment costs $(i_t + \psi(k_t, k_{t+1}))$, debt repayments (b_t), dividend payments (d_t) with internal cash flows $(F(z_t, k_t, h_t) - w_t n_t)$, and new debt (b_{t+1}). The flow of funds constraint for firms is then given by:

$$b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, h_t) - w_t n_t + \frac{b_{t+1}}{R_t} \tag{7}$$

where w_t is the wage rate, and R_t is the gross interest rate determined exogenously in international markets. R_t is stochastic and follows a first-order Markov process. Implicit in the flow of funds constraint is the fact that firms cannot issue new shares (we normalize the total number of shares to 1). However, they can adjust retained earnings by cutting dividend payments and servicing debt.

Firms face two types of liquidity constraints on their ability to finance investment. First, they are subject to a collateral constraint that limits the amount of borrowing to a fraction of their capital holdings:

$$b_{t+1} \leq \kappa_t k_{t+1} \tag{8}$$

This constraint is similar to those used in existing literature, and we interpret it as arising in an environment where creditors can only recover a fraction κ_t of the firms' assets.[4] Following Jermann and Quadrini (2012), κ_t represents a shock to the borrowing capacity of firms. For simplicity, this "financial shock" follows a two-state Markov chain with values given by κ^H and κ^L. In our quantitative analysis, the collateral constraint will never bind when $\kappa = \kappa^H$, so that when κ switches from high to low, this may lead to a binding constraint and a credit crunch. Note that whether the economy enters a credit crunch depends endogenously on the degree of leverage in the economy.

Without any constraints on equity financing, the shadow value of external funds would be equal to one. We assume that there is a lower bound on dividends given by $\bar{d}$, i.e., at each period firms are required to satisfy:

$$d_t \geq \bar{d} \tag{9}$$

A special case is the restriction that dividends need to be non-negative, which effectively implies that the issuance of new shares is not available. This constraint reflects the notion that dividend payments are required to reduce agency problems and information asymmetries between shareholders and managers.

We assume that firms maximize shareholder value as is standard in the dynamic corporate finance literature. Maximization of shareholder values implies that firms must discount profits at state $t+j$ the rate m_{t+j} defined above. That is, their problem is to maximize $\mathbb{E}_t \sum_{j=0}^{\infty} m_{t+j} d_{t+j}$.

2.3 Recursive Problem and Optimality Conditions

The aggregate state vector of the economy that we denote by X is given by the aggregate level of capital K, bonds B, and the aggregate shocks κ and z, i.e., $X = K, B, \kappa, z, R$. Denoting $V(k, b, X)$ the cum-dividend market value of the firm and using prime to denote

[4] We implicitly assume that the liquidation value of capital is not affected by market prices, thereby turning off a fire-sale externality mechanism (see e.g. Bianchi and Mendoza (2010) for an analysis of this channel). We make this assumption to focus on an alternative mechanism which results from reallocation of funds between shareholders and firms.

next period variables, the optimization problem for firms can be written recursively as:[5]

$$V(k,b,X) = \max_{d,h,k',b'} d + \mathbb{E}m'(X,X')V(k',b',X') \tag{10}$$

$$s.t.$$

$$b + d + k' + \psi(k,k') \leq (1-\delta)k + F(z,k,h) - wn + \frac{b'}{R}$$

$$b' \leq \kappa k'$$

$$d \geq \bar{d}$$

The optimality condition for labor demand yields a standard static condition:

$$F_h(z_t, k_t, h_t) = w_t \tag{11}$$

There are also two Euler intertemporal conditions that relate the marginal benefit from distributing one unit of dividends today with the marginal benefit of investing in the available assets and distributing the resulting dividends in the next period. Denoting by μ, the multiplier associated with the borrowing constraint, η the multiplier associated with the dividend payout constraint, the Euler equations and associated complementary slackness conditions are given by:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}) + R_t \mu_t \tag{12}$$

$$(1+\eta_t)(1+\psi_{2,t}) = \mathbb{E}_t m_{t+1} \left[1 - \delta + F_{k,t+1} - \psi_{1,t+2}\right](1+\eta_{t+1}) + \kappa_t \mu_t \tag{13}$$

$$\mu_t(\kappa_t k_{t+1} - b_{t+1}) = 0, \quad \mu_t \geq 0 \tag{14}$$

$$\eta_t(d_t - \bar{d}) = 0, \quad \eta_t \geq 0 \tag{15}$$

In the absence of financial constraints on borrowing and dividend payments, the cost of raising equity (by reducing dividends), i.e., $1/\mathbb{E}_t m_{t+1}$, would be equal to the cost of debt R_t, and firms would be indifferent at the margin between equity and debt financing.

[5] We note that since the firm is representative, in equilibrium the firm never exposes itself to the possibility of not being able to satisfy the financial constraints. Hence, there is no voluntarily or involuntary default.

However, when the collateral constraint binds, there is a wedge between the marginal benefit of borrowing one more unit and distributing it as dividends in the current period and the marginal cost of cutting dividends in the next period to repay the debt increase. In addition, when the equity constraintbinds, a positive wedge arises between the marginal benefit from investing one more unit in capital or bonds and the marginal cost of cutting one unit of dividends today.

As we will see in the model simulations, the collateral constraint and the equity constraint often bind at the same time. Intuitively, both constraints impose a limit on a firm's funding ability. A binding equity constraint forces higher levels of borrowing for given investment choices. Similarly, a tighter constraint on borrowing puts pressure on the firms to finance to reduce dividend payments.

Discussion of Financial Frictions — Our normative analysis requires a model of incomplete markets that departs from Modigliani-Miller. We discuss now the specific assumptions that we have made to deviate from Modigiliani-Miller results.

As we will show below, the key friction in our setup is that firms face an equity constraint that imposes a lower bound on dividend payments. This is a relatively standard way of capturing agency problems and information asymmetries between a firm's shareholders and its managers in the corporate finance literature, and it is in line with an extensive literature documenting the importance of agency frictions between shareholders and corporate managers (see e.g. Shleifer and Vishny (1997) for a survey). Without this constraint on dividend payments, firms would be able to raise enough equity to finance desired investments and would fail to reproduce the evolution of real and financial variables in the data.[6]

Borrowing by firms is limited by imperfect enforceability of contracts. In particular, we assume that creditors require firms to hold collateral to back promised repayments according to (8). In order to enrich the model, we introduce shocks to how much collateral firms firms are required to pledge. A possible interpretation of this shock relates to disruptions in financial intermediaries, which become either less willing to lend or more concerned about

[6]More generally, what is necessary is to assume that equity becomes relatively costly to issue in bad states of nature. One motivation for frictions on equity financing is private information about investment opportunities. For example, in Myers and Majluf (1984), good firms may find it optimal not to issue equity when they are pooled with those of lower quality (see also Bigio (2011) for recent work linking adverse selection in credit markets with banks' equity financing).

the riskiness of the corporate sector. We will show that when leverage is sufficiently high, a negative financial shock produces a credit crunch with similar features to the data. Jermann and Quadrini (2012) recently pointed out that financial shocks improve the quantitative performance of a business cycle model.

We have also assumed that asset markets are restricted to one-period non-state contingent bonds, as is standard in the literature and represents a simplification of the limited insurance that firms have access to. What is critical for our results is that firms cannot fully undo the equity constraints using contingent debt.[7] Finally, households in our model do not have access to credit markets. Households can, however, smooth consumption through dividend payments, which in turn will affect firms' financial decisions.[8]

Overall, these assumptions allow us to formulate a parsimonious analysis of optimal bailouts. Moreover, these assumptions are important for the model to produces financial and real flows that are broadly consistent with key features of the data in terms of general co-movements and financial crises dynamics.

2.4 Competitive Equilibrium

The competitive equilibrium for a small open economy that borrows from abroad at an exogenous interest rate can be constructed in the usual form. Market clearing in the labor market requires:

$$h_t = n_t \tag{16}$$

Market clearing in equity markets requires:

$$s_t = 1 \tag{17}$$

[7]Standard motivations for restrictions on state-contingent liabilities are the lack of commitment by investors to inject funds to the firms in bad times or the inability to verify the realization of the shocks. We do not model these frictions, though.

[8]At the computational cost of introducing an additional state variable, this constraint can be relaxed to some extent. What is important is that households do not have access to perfect credit markets in order to guarantees that firms do not have an incentive to deleverage in the long-run. Notice that if $\bar{d} = -\infty$, and if households have *unrestricted* access to international credit markets by borrowing and saving at the interest rate R_t, the competitive equilibrium would be unaffected by financial shocks. Combining the household's first-order condition, $(R_t E m' = 1)$ and the firm's first-order condition, $(R_t E m' + \mu = 1)$ would yield $\mu = 0$. Therefore, the Modigiliani Miller theorem would hold, and the model would become a standard RBC model.

Using the two equations above and combining the household budget constraint and the firms' flow of funds constraint, we obtain the resource constraint for the economy:

$$b_t + c_t + k_{t+1} + \psi(k_t, k_{t+1}) = (1-\delta)k_t + F(z, k_t, h_t) + \frac{b_{t+1}}{R_t} \tag{18}$$

The recursive competitive equilibrium can be defined as follows:

Definition 1 *A recursive competitive equilibrium is given by (i) firms' policies* $\left\{\hat{d}(k,b,X), \hat{h}(k,b,X), \hat{k}(k,b,X), \hat{b}(k,b,X)\right\}$*; households's policies* $\hat{s}(s,X), \hat{c}(s,X), \hat{n}(s,X)$*; a stochastic discount factor* $m(X,X')$*; firm's value* $V(k,b,X)$*; prices* $w(X), p(X)$*; and a law of motion of aggregate variables* $X' = \Gamma(X)$*, so that: (i) households solve their optimization problem; (ii) firms' policies and firms' value solve (10); (iii) markets clear in equity market* $(\hat{s}(1,X) = 1)$ *and labor market* $(\hat{h}(K,B,X) = \hat{n}(1,X))$*, (iv) the stochastic discount factor for firms is given by the household's marginal rate of substitution* $m(X,X') = \beta u'(\hat{c}(1,X) - G(\hat{n}(1,X)))/(u'(\hat{c}(1,X') - G(\hat{n}(1,X')))$ *(v) the law of motion* $\Gamma(\cdot)$ *is consistent with individual policy functions and stochastic processes for* κ *and* z*.*

Necessary and sufficient conditions for a competitive equilibrium can be established due to the fact that the optimization problem for households and firms are convex programs. In particular, given stochastic processes for R_t, z_t and κ_t, a set of stochastic sequences $\{c_t, k_{t+1}, i_t, b_{t+1}, d_t, h_t, n_t, w_t, p_t, \mu_t, \eta_t, s_t\}_{t\geq 0}$ is a competitive equilibrium if and only if equations (3)-(4) and (6)-(18) are satisfied.

In order to illustrate the properties of the model, it is useful to first analyze the case without uncertainty. In a deterministic steady state with $\beta R < 1$, (i) the collateral constraint is always binding, and (ii) there exists $\hat{d}$ such that the equity constraintbinds if $\bar{d} > \hat{d}$. For (i), note that in a deterministic steady state, $m_t = 1$ and (12) is simplified to $1 = \beta R + \mu$. Since $\beta R < 1$, this implies that $\mu > 0$. For (ii), one can obtain the steady state values $[k^{ss}, h^{ss},\ b^{ss},\ \mu^{ss}]$ from (3),(8),(12), and (13). Substituting these expressions, (8) and (3) in the flow of funds constraint (22) yields the value of dividends at steady state $d^{ss} = F(z^{ss}, k^{ss}, h^{ss}) - k^{ss}(\delta + \bar{\kappa}(R-1)/R) - h^{ss}G'(h^{ss})$.

In general, in a stochastic steady state, these financial constraints may or may not bind depending primarily on the magnitudes of the shocks, the discount factor, the interest rate and the tightness of the constraints.

3 Normative Analysis

3.1 Scope for Policy

We start the normative analysis by discussing the scope for government intervention in our setup. The key externality in the model is related to the undercapitalization of firms. Households are not willing to unilaterally transfer funds to firms because they only incur costs. This is a form of a free-rider externality. Instead, a social planner recognizes that transferring resources to the firm increases labor payments and dividend payments in future periods for all households in the economy.

As in the Holmström and Tiróle analysis of liquidity, there is a rationale for the government to transfer resources from consumers to producers. Three comments are in order. First, the ability of the government to improve welfare relies on the ability to extract payments from households via taxes to address the free-rider externality. The government, however, does not have a superior debt capacity than the private sector. That is, the government does not use public debt as private liquidity (Woodford, 1990). Second, in our setup, because households own firms, transfers will lead to Pareto improving transfers. Finally, to reflect the fact that these transfers are costly in practice, we assume that there is an iceberg cost φ proportional to the volume of transfers. This cost could arise from distortions in taxation or the financing of inefficient projects. However, we do not model the causes of the cost explicitly.

3.2 Constrained Social Planner's Solution

We consider a benevolent social planner who (a) directly chooses the sequence of debt, capital, and equity payout subject to the liquidity constraints and the resource constraint; (b) chooses a sequence of transfers Υ_t between firms and households at a linear cost φ; (c) lets labor markets, stock market and goods markets clear competitively.

By making the planner subject to the same financial constraints as the decentralized equilibrium, the economy is also subject to the deleveraging effects of financial shocks. Notice also that while the social planner cannot directly affect labor market outcomes, it may affect the labor market indirectly through the choice of capital as it affects wages in the next period.

Denote by $\Upsilon_t \geq 0$ the transfer from households to firms, and by $\tilde{w}_t(k,z)$ and $\tilde{h}_t(k,z)$, the market clearing wage and labor allocations. Since labor is chosen by households and firms in competitive markets, $\tilde{w}_t(k,z)$ and $\tilde{h}_t(k,z)$ satisfy (3) and (11). The problem of the social planner can be written as follows:

$$\max_{\{k_{t+1},b_{t+1},c_t,p_t,\Upsilon_t\geq 0\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t - G(\tilde{h}_t(k_t,z_t)))$$

$$\begin{aligned}
(1-\delta)k_t + F(z_t,k_t,h_t) + \frac{b_{t+1}}{R_t} - b_t - k_{t+1} - \psi(k_t,k_{t+1}) - \varphi\Upsilon_t = &\quad c_t \\
(1-\delta)k_t + F(z,k_t,h_t) - \tilde{w}_t(k_t,z_t)\tilde{h}_t(k_t,z_t) + \frac{b_{t+1}}{R_t} + \Upsilon_t - b_t - k_{t+1} - \psi(k_t,k_{t+1}) \geq &\quad \bar{d} \\
b_{t+1} \leq &\quad \kappa_t k_{t+1} \\
\beta\mathbb{E}_t u'(t+1)(d_{t+1}+p_{t+1}) = &\quad p_t u'(t)
\end{aligned}$$

where $\tilde{w}_t(k_t,z_t) = G'(\tilde{h}_t(k_t,z_t)) = F_L(z_t,k_t,\tilde{h}_t(k_t,z_t))$. We attach again η_t and μ_t to the financial constraints. To facilitate comparison with the conditions characterizing the competitive equilibrium, we normalize these multipliers by the marginal utility of consumption. Notice that the last condition is irrelevant for the planner, as the price of shares do not affect the set of feasible allocations. The resulting planner's problem is time-consistent.

First-order condition with respect to Υ_t yields:

$$\varphi \geq \eta_t \text{ with equality if } \Upsilon_t > 0 \tag{19}$$

Condition (19) is crucial to asess the tradeoffs involved in the bailout policy at the margin. This condition establishes that the planner will transfer resources from households to firms until the marginal cost given by φ equals the marginal benefits, given by the shadow value

from relaxing the equity constraint η_t (expressed in units of consumption). It also follows that $\Upsilon_t = 0$ if the equity constraint is not binding or if the shadow value from relaxing the equity constraintis small enough. Note that it is not optimal to fully relax the equity constraint, i.e. if $\Upsilon_t > 0$ for some t, it also follows that $\eta_t > 0$. We also have the following two results:

Corollary 1 *If $\varphi = 0$, the equity constraint does not bind for the social planner.*

Proof: Setting $\Upsilon_t > \bar{d} + b_t + i_t + \psi(k_t, k_{t+1}) - F(z_t, k_t, n_t) - w_t h_t + \frac{b_{t+1}}{R_t}$, the planner can completely relax the equity constraint without affecting the objective function or the rest of the constraints. Intuitively, if taxes are not distortive, the planner can use cost-free transfers as a substitute for lower dividend payments when the equity constraintbecomes binding.

Corollary 2 *If $\bar{d} = -\infty$, the competitive equilibrium and the social planner's solution coincide.*

Proof: The proof notes that $\bar{d} = -\infty$ implies $\Upsilon_t = 0$ and $\eta_t = 0$, which yields that the conditions characterizing the competitive equilibrium are identical to those characterizing the social planner. Since firms have unrestricted access to equity, implementing a transfer from households to firms has no effect. It also follows that if $\bar{d} = -\infty$ and $\varphi > 0$, bailouts are welfare reducing.

In the policy experiment below, we consider the general case where $\varphi > 0$, which implies that the government faces strictly positive efficiency costs from transferring resources from firms and households. Under these conditions, bailouts involve ex-post a trade-off between relaxing firms' balance sheet constraints and the iceberg costs associated with the transfer. When we turn attention to the decentralized solution, we will also analyze the moral hazard effects.

First-order condition with respect to capital and using the equilibrium conditions for the labor market yields:

$$(1+\eta_t)(1+\psi_{2,t}) = \frac{\beta \mathbb{E}_t u'(t+1)}{u'(t)} \left[1 - \delta + F_{k,t+1} - \psi_{1,t+2}\right] (1+\eta_{t+1}) \left(1 - \tilde{h}_{t+1} \frac{\partial \tilde{w}_{t+1}}{\partial k_{t+1}}\right) + \kappa_t \mu_t \tag{20}$$

An important difference between (20) and the analogous condition for firms (13) is that the planner internalizes how next period capital stock affects next period wages, which in turn affects the tightness of the equity constraint. In particular, firms do not internalize that one more unit of capital tightens the constraint by $\tilde{h}_{t+1}(\partial \tilde{w}_{t+1})/(\partial k_{t+1})$ which has a marginal utility cost of η_{t+1}.

3.3 Decentralization

This section analyze possible decentralization to the social planner's allocations. As we will see, the decentralization requires in general both ex-ante prudential measures and ex-post policy measures.

Debt Relief — We first analyze the role of debt relief. We consider a policy in which the government pays a fraction γ_t of private debts and finances this transfer of funds and its iceberg cost with lump sum taxes T_t to households. In addition, the government sets taxes on borrowing and capital τ_t^b and τ_t^k that is rebated by a lump-sum tax to firms, given by T_t^f. With these policies, the households' budget constraint and the firms's flow of funds constraint become respectively:

$$s_{t+1}p_t + c_t \leq w_t n_t + s_t(d_t + p_t) - T_t \tag{21}$$

$$(1-\gamma_t)b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq (F(z_t, k_t, h_t) - w_t n_t)(1-\tau^k) + \frac{b_{t+1}}{R_t}(1-\tau_t) + T_t^f \tag{22}$$

First-order condition with respect to b_{t+1} yields:

$$1 + \eta_t = R_t(1+\tau_t)\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + R_t(1+\tau_t)\mu_t \tag{23}$$

The rest of the optimality conditions remain the same. Note that from (23), the private costs of borrowing at time t are reduced by a factor of $(1-\gamma_{t+1})$ in a state t+1 in which the government provides debt relief. An examination of these first-order conditions leads to the following proposition:

Proposition 1 *The government can implement the constrained optimal allocations through a combination of debt relief, taxes on debt and capital, and lump sum taxes. These polices*

are given by:

$$\gamma_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \qquad T_t^f = \frac{b_{t+1}}{R_t}\tau_t + \tau_t^k(F(z_t,k_t,h_t) - G'(h_t)h_t)$$

$$\tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + \mu_t} - 1, \qquad \tau_t^k = \frac{\mathbb{E}_t m_{t+1}\left(\frac{\partial \tilde{w}_{t+1}}{\partial k_{t+1}}\tilde{h}_{t+1}\right)\eta_{t+1} + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t} - 1$$

where all variables are evaluated at the constrained optimal allocations.

Proof: The proof follows from noting that with the specified policy instruments the conditions characterizing the regulated competitive equilibrium are identical to those of the constrained optimal allocations.

The role of the taxes on debt and capital is Pigouvian. The tax on debt aims to correct the private cost of borrowing, which is distorted by debt relief policies. Note that the tax on debt is only strictly positive when debt relief are activated with strictly positive probability in the next period, an event that occurs when the equity constraint becomes binding in the economy. The tax on capital aims to make firms internalize how a larger amount of capital increases next period wages which tightens the equity constraint when it becomes binding. The tax on capital is also strictly positive if the equity constraint is expected to bind. Hence, both taxes are prudential.

Lump-sum transfers — The final policy instrument we consider is a lump-sum transfer that is independent of any individual choice made by the firms. In contrast to the above polices, first-order conditions are unaffected when the government bails out firms with a lump-sum transfer, which implies that no prudential policy is required. Intuitively, the government provides insurance in such a way that firms see the benefits from the bailout as entirely exogenous from their financial decisions. As a result, the government can implement the constrained optimal allocations without resorting to a prudential tax on debt. It is important to keep in mind that lump-sum transfers are extremely impractical for reasons that are not modeled in this paper In this respect, we see the implementation of lump-sum transfers as purely illustrative.[9]

[9]The appendix also describes a decentralization using equity injections. In this case, because shareholders absorb part of the cost of new issuances, a tax on debt is only necessary when $\varphi >$, i.e., shareholders do not internalize that public resources are wasted during bailouts.

Financial Intermediaries — In practice, central banks implement a variety of policies with the aim of facilitating the corporate sector's access to credit. For example, under the Commercial Paper Funding Facility (CPFF), the Federal Reserve expanded eligible collateral to include commercial paper directly targeting the corporate sector, as in our model. Other policies included in the TARP, involved equity injections to financial institutions. To simplify the analysis, we do not model financial intermediaries and consider only direct bailouts to firms.[10] The crucial factor for our analysis is that this intervention relaxes balance sheets across the economy and mitigates the fall in credit and investment that occurs during crises.

3.4 Quantitative Policy Experiments

In our quantitative analysis, we will start by exploring the properties of the optimal policy instruments and its effects over macroeconomic dynamics. As we analyzed above, the optimal policy requires, in general, prudential policy to counteract excessive risk-taking incentives. We are also interested in examining the importance of the complementarity between bailouts and prudential policy. For this purpose, we will analyze two additional policy experiments. One experiment consists of imposing the bailout policy— implemented with debt relief— we derive from the optimal intervention, but without the use of prudential policy. We call this policy systemic bailout policy. Second, we study idiosyncratic bailouts. In this scenario, bailouts now depend entirely on firm specific choices independently on aggregate states. In particular, the government uses the debt relief policy solved above, but now the bailout is given by $\Upsilon(b_t, k_t, z, \kappa, R)$, i.e. there is no subscript in Υ associated with macro variables.

These two additional experiments allows us to analyze the trade-off involved in bailouts. Ex-post, bailouts help relax balance sheet constraints. Ex ante, there is too much risk taking relative to the social optimum. Hence, we will study whether is is possible to increase welfare using bailouts without prudential policy.

[10] We could modify the problem of the corporate entities so that instead of carrying out production activities on their own, they provide intra-period loans to firms in the form of capital goods. It is easy to see that this modified setup would be isomorphic to our baseline model. See also Bianchi (2011) for a mapping between taxes on debt, capital requirements, and margin requirements

3.5 Numerical Solution

The numerical solution to the model involves several challenges. First, there are the well-known complications of non-linearities that arise in incomplete markets and, in particular, the occasionally binding financial constraints. In addition, the state variables in the model are not confined to a narrow region of the state space. We use a policy function iteration algorithm whereby we approximate the equilibrium functions over the entire state space, interpolating outside the grid and checking that the equilibrium conditions are satisfied at all grid points. A key feature of this algorithm is that the two financial constraints are allowed to bind only in some states of nature. The algorithm also captures the high non-linearities close to these constraints.

3.6 Calibration

We calibrate the model to an annual frequency using data from the U.S. To focus on post-financial globalization period, our reference period is 1984.I-2010.II. Parameter values are summarized in Table 1.

Functional Forms— We make the following assumptions regarding functional forms for preferences and technology:

$$u(c - G(n)) = \frac{\left[c - \chi \frac{n^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right]^{1-\sigma} - 1}{1-\sigma}$$

$$F(z, k, h) = zk^{\alpha}h^{1-\alpha}$$

$$\psi(k_t, k_{t+1}) = \frac{\phi_k}{2}\left(\frac{k_{t+1} - k_t}{k_t}\right)^2 k_t$$

Stochastic Processes— We model the shocks to the interest rate and productivity as a first-order bivariate autoregressive process:

$$\begin{pmatrix} \hat{z}_t \\ \hat{R}_t \end{pmatrix} = \begin{pmatrix} \bar{z} \\ \bar{R} \end{pmatrix} + \boldsymbol{\rho} \begin{pmatrix} \hat{z}_{t-1} \\ \hat{R}_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t} \\ \epsilon_{R,t} \end{pmatrix}$$

where $\varepsilon_t = [\epsilon_{z,t}\epsilon_{R,t}]'$ follows a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix V. To construct these series, we use take the ex-post

real interest rate on the 3 month US-Treasury Bills for the interest rate, and we follow the standard Solow residuals approach to construct the series for TFP. Since the period in our model is a year, we construct Our OLS estimation yields the following values:

$$\boldsymbol{\rho}=\begin{bmatrix} 0.755972 & -0.030037 \\ -0.074327 & 0.743032 \end{bmatrix} \qquad \mathbf{V}=\begin{bmatrix} 0.0000580 & -0.0000107 \\ -0.0000107 & 0.0001439 \end{bmatrix}$$

We discretize the VAR(1) process for TFP and interest rate shocks is using Tauchen and Hussey quadrature based procedure with 9 values.

Financial shocks are modeled as an independent process following a two-state Markov chain with values given by $\left\{\kappa^L, \kappa^H\right\}$ and transition matrix:

$$P=\begin{bmatrix} P_{L,L} & 1-P_{L,L} \\ 1-P_{H,H} & P_{H,H} \end{bmatrix}$$

Parameter Values— We need to assign values to 16 parameters that we classify in two sets. The first subset includes parameters that are chosen independently of equilibrium conditions or are calibrated using steady state targets, some of which are typical in the business cycle literature. This subset is given by $\{\alpha, \delta, \omega, \beta, R, \varphi, \chi\}$. The capital share α is set to 0.33; the depreciation rate is set at 10 percent; the risk aversion σ is set to 2; the Frisch elasticity of labor supply in the GHH preference specification ω is set to 2.[11] The value of β is pinned down by setting the capital-output ratio equal to 2.5 in a deterministic steady state with $\kappa = 0$, which results in a value of 0.964.[12] $\bar{R}$ is set so that the average interest rate is equal to 1.015 percent which is the average interest rate in the period considered.

The efficiency cost φ is more specific to our framework. For this parameter, we choose a benchmark value of 50 bps. Considering that financial intermediation represent about 5 percentage points of GDP, this implies that cost of public supply of credit is 10 percent higher than the private one [13] Finally, we normalize the labor disutility coefficient χ and the average

[11]Notice that to make these preferences consistent with long-run growth, one needs to assume that technological progress increases the utility of leisure. As shown by Benhabib, Rogerson, and Wright (1991), these preferences can be interpreted as reduced form preferences for an economy with home production and technological progress in the home production sector.

[12]Due to precautionary savings, average capital is 2.6, which is still within the range of empirical estimations.

[13]We also note that given the rest of the parameter values, we find that financing the bailout with a distortionary tax on labor and setting $\varphi = 0$ yield very similar results that our benchmark value for φ.

value of the TFP shock so that employment and output equal one in the deterministic steady state.

The remaining six parameters are $\left\{\phi_k, \kappa^L, \kappa^H, \bar{d}, P_{L,L}\, P_{H,H}\right\}$ and are set to jointly match a set of six long-run moments from the economy where bailouts are absent.[14] These moments are: (1) a standard deviation of investment of 10 percent; (2) an average leverage ratio of 45 percent; (3) four credit crunches occurring every 100 years; (4) an average duration of a credit crunch of 3 years; (5) a probability of a binding dividend constraint equal to the probability of a binding collateral constraint; and (6) a probability of a binding collateral constraint conditional on $\kappa = \kappa^H$ equal to zero. While all these parameters affect all the target moments, each parameter has a more significant impact on one particular moment, as we explain below.

We set the value of κ^H high enough so that the collateral constraint never binds when κ takes this value. The value of κ^L is set to target an average leverage of 45 percent. The choice of a leverage ratio of 45 percent corresponds the ratio of credit market instruments to net worth in the years preceding the 2007 financial crisis (see Table B102 in the Flow of Funds database).

The dividend threshold $\bar{d}$ is more difficult to pin down from the data because it is difficult to identify whether constraints on equity financing or on borrowing are more pervasive. For simplicity, we set $\bar{d}$, so that the borrowing constraint and the equity constraintbind with the same probability in the long run. This yields $\bar{d} = 0.035$ and probabilities of binding constraints equal to 8 percent. The adjustment cost on capital is calibrated to match the standard deviation of investment between 1950-2010. This calibration yields $\phi_k = 2.2$, a standard value.

We calibrate the transition matrix for the financial shock to target the frequency and the duration of financial crises. We define a financial crisis as an episode in which credit falls below two standard deviations. The financial crisis begins in the period in which credit falls below one standard deviation, providing that some point within the next two years,

[14] To the extent that optimal bailouts in our model have similar features to the data, one could argue that it would be more appropriate to target the moments of the economy with the optimal bailout policy. Since we will experiment with different values of φ,, it is more practical to calibrate the economy without bailouts. In any case, this choice does not have a significant impact on the results.

Table 1: Calibration

	Value	Basis
Mean Interest rate	$R - 1 = 0.015$	3 month US-Treasury Bills
Discount factor	$\beta = 0.97$	Capital-output= 2.5
Depreciation rate	$\delta = 0.11$	Standard value
Share of capital	$\alpha = 0.34$	Standard Value
Labor disutility coefficient	$\chi = 0.66$	Normalization
Risk aversion	$\sigma = 2$	Standard value
Frisch elasticity parameter	$\omega = 2.0$	Benchmark value
Efficiency cost	$\varphi = 50bps$	Benchmark value
Parameters set by simulation	Value	Target
Financial shock		
	$\kappa_L = 0.43$	Average leverage =45 percent
	$\kappa_H = 0.54$	Non-binding collateral constraint
	$P_{HH} = 0.9$	Probability of credit crunch = 4 percent
	$P_{LL} = 0.25$	Duration of credit crunch = 3 years
Adjustment cost	$\phi_k = 2.2$	SD of investment = 9 percent
Dividend threshold	$\bar{d} = 0.035$	Equalize prob. binding constraints

the level of credit falls at least two standard deviations below its mean. The crisis ends when the level of credit exceeds one standard deviation below its mean. Consistent with the empirical literature (e.g. Reinhart and Rogoff, 2009), we target an incidence of crises of 4 every 100 years and an average duration of 3 years. This procedure yields that $P_{L,L}$, which mostly affects the duration of crises, equals 0.1. $P_{H,H}$, which primarily affects the long-run probability of a crisis, equals 0.9. With these values, the economy spends only 11 percent of the time with negative financial shocks.

4 Results of the Quantitative Analysis

4.1 Dynamics of Financial Crises

We begin by analyzing the dynamics of financial crises in the unregulated competitive equilibrium. We will later show that crises in the model are consistent with several features

of financial crises in the data, and we will analyze how bailouts affect the incidence and the severity of crises.

We construct a crisis event for the economy when the optimal policy is implemented according to the following steps. First, we run a long time-series simulation after feeding a sequence of shocks, drawn from the distribution of the stochastic processes for (κ, R, z). Second, we identify financial crises events. As explained above, these events are defined as periods in which credit falls by more than two standard deviations. Finally, we compute averages of the main macro variables of the model centered around those episodes. The result of this experiment are illustrated in Figure 1 (period t denotes the financial crisis event).

The top panels of Figure 1 show the role of the different shocks around financial crises and the evolution of the multipliers on the financial constraints. A first key result is that financial crises are always triggered by negative financial shocks while TFP shocks and interest rate shocks play a minor role. In fact,all crises episodes coincide with a negative financial shock. Moreover, crises are preceded by favorable credit conditions with $\kappa = \kappa^H$ and slack financial constraints. Because the financial shock is persistent, there a few values of negative values of the financial shock following a crisis event.

The medium panels of Figure 1 show the evolution of leverage and investment measured as percentage deviations from the mean values. A key result is that there is a substantial deleveraging following a financial crisis. In fact, leverage is above trend during the periods preceding financial crises and average around -5 percentage points following the crises. Investment collapses during a crisis and then it recovers as the economy becomes unconstrained again in period t+1.

The bottom panels of Figure 1 show the evolution of output and employment. In line with the evolution of investment, output and employment drops significantly following a crisis. Notice also that the presence of adjustment cost in capital implies that employment and output do not recover immediately despite the fact that financial constraints are not binding.[15]

[15]These dynamics are consistent with Mendoza (2010). An important difference, however, is that in our model a crisis is caused by financial shock that triggers binding equity and collateral constraints.

4.2 Financial Crises Comparison

Table 2: Financial Crises Comparison

	No Intervention [b]	Optimal Policy [c]	Great Recession [d]
Output	-8.2	-7.7	-7.7
Hours	-5.3	-4.9	-9.80
Investment	-50.7	-43.0	-38.8
Consumption	-5.8	-7.3	-6.4
Debt-Repurchases	13.0	12.0	7.1
Equity-Payouts	-0.7	-0.7	-1.0

We now compare financial crises in the unregulated competitive equilibrium to the crises that correspond to the constrained-efficient allocations—in addition, we also compare these allocations with the Great Recession. Following the methodology described above, we simulate the constrained-efficient economy and select from the resulting distribution of crises a subset of those events. In particular, we extract episodes where the fall in GDP following the crisis is close to 7.7 percent, which is the drop in GDP during the US Great Recession for the 2009:2-2010:1 period, with shocks to TFP and interest rate close to the mean. We then compute the mean of macro variables of all these episodes. The second column of Table 2illustrates these results.

In order to gauge the importance of bailouts, we conduct a counterfactual policy experiment in which the government does not conduct bailouts. This experiment consists of taking the crisis episodes identified in the economy with the optimal bailout policy and ask what would be the response of the economy without bailout policy. That is, given the initial state X_t at the subset of crises identified above and the policy rules of the unregulated economy, we simulate what would be the response of macro variables.

The results of the counterfactual experiment shows that without intervention the credit crunch is substantially more severe. The fall in output and employment with the optimal policy is respectively -7.7 and -4.9 percent versus -8.2 and 5.3 percent for the unregulated competitive equilibrium. What explains the differences in the deepness of the recession is

the behavior of investment when the negative financial shock hits. Without the bailout, which averages 1.2 percentage points of GDP, the economy experiences a decline in investment which is about 8 percentage points higher. Having less capital following the financial crises, the economy reduces employment and output. On the financial side, both financial constraints are binding during the crisis event, but the deleveraging measured by the amount of debt repurchases is stronger for the economy with no intervention.

It is important to note that this counterfactual experiment does not take into account explicitly the anticipation effects of bailouts, as both economies start with the same initial state variables. Moreover, this analysis also focuses on the immediate macro responses and does not consider persisting effects. We will analyze these two important effects below.

The last column in Table 2 shows the empirical counterpart of the model variables during the Great Recession. The results in Table 2 shows that the model performs relatively well when compared to the Great Recession. On the real side, there is an important contraction in employment, consumption and investment. On the financial side, there is also significant deleveraging as well as an important contraction in equity payouts.

4.3 Optimal Bailout Policy

How does the optimal bailout policy look like? Figure 2 shows the optimal bailout policy $\Upsilon(X)$ when a negative financial shock hits the economy. In particular, it displays the optimal bailout for different values of capital and bonds and average values for TFP and interest rate shocks.

Figure 2 shows that bailouts are strictly positive when the economy is highly leveraged. As the level of debt is reduced or the level of capital is increased, bailouts are reduced. Moreover, bailouts are zero in a wide range of the state space, which is the "low leverage region". That is, in this region, even if a negative financial shock hits the economy, the planner does not conduct bailouts. Recall that condition (19) indicates that bailouts are conducted only when the equity constraint is sufficiently binding.

There is also a region with large bailouts in the order of 15 percentage points of GDP, but the economy never reaches these states with strictly positive probability. In fact, the largest bailout observed in the simulation is about 3 percentage points of GDP. Conditional

on a crisis, the average bailout is 0.9 percentage points of GDP. If the economy is not in a crisis, the average bailout is zero.

In our model simulations, bailouts occur when the economy is under deep financial distress. As Figure **??** shows, these are rare events. This figure plots a scatter diagram with the drop in investment in the x-axis and the bailout in the y-axis. Notice that only when investment falls more than 10 percent, it is possible to observe bailouts taking place.

4.4 Policy Functions and Laws of Motion

In order to illustrate the workings of the model, we now analyze the equilibrium policy functions of the model with no intervention and in the constrained-efficient allocations.

Figure 4 shows the laws of motion for debt, capital, and leverage in the economy as a function of the current level of debt, for a value of capital approximately equal to the average value and an average TFP and interest rate shock. Since mean output is approximately one, all variables can be interpreted as ratios with respect to the average output. The left (right) panel corresponds to a positive (adverse) financial shock. The straight lines correspond to the competitive equilibrium with no-bailout policy, and the dashed lines correspond to the economy with the optimal bailout policy, i.e. the constrained efficient allocations. Let us first describe the behavior of the economy without intervention.

The two upper panels show that for the same initial values, the economy accumulates more debt when there are favorable credit conditions. Another important feature of the model is that the occasionally binding collateral constraint produces a non-monotonic law of motion for debt, which is apparent in the top right panel of Figure 4. For low values of current debt, the collateral constraint is not binding. In this region, next period level of debt is increasing in the current period debt. When the current level of debt reaches 1.18, the collateral constraint becomes binding. Since firms need to cut down on investment, the borrowing capacity shrinks. As a result, for $b > 1.18$ in the inferior panel, the next period debt holdings decrease in current debt holdings. For this law of motion, the equity constraint becomes binding approximately at the same value of debt. For the law of motion for debt when $\kappa = \kappa^H$, which corresponds to upper left panel, none of the financial constraints bind and as a result next period debt is increasing in current debt. Moreover,

The medium panels show the book-value leverage (b_{t+1}/k_{t+1}), again for different values of current debt. In the right panel, the leverage ratio is bounded by κ^L, which becomes binding at $b = 1.18$. The law of motion for capital, which correspond to the lower panels, is decreasing in the amount of debt. This relationship becomes more pronounces when the collateral constraint become binding in the right panel. When the collateral constraint is not binding, however, there is still a negative relationship because two reasons. First, a higher level of debt reduces the willingness to invest in a risky asset, which results in a higher risk premium. Second, a higher level of debt reduces the desired level of savings.

We turn attention now to the effects of bailouts over the laws of motion. Bailouts allow firms to borrow more during periods in which financial constraints become binding, as firms that receive transfers, allocate these funds to invest more in capital, which boosts their capacity to borrow. The increase in the ability to borrow is apparent in the shaded region, i.e., the region where the planner conducts bailouts.

A central aspect to our analysis is how these ex-post response modify the ex-ante financial decisions. In the region where the financial constraints are not binding, firms also borrow more in the competitive equilibrium with bailouts because there is a lower incentive to accumulate precautionary savings during normal times since crises become less severe. This effect is markedly stronger when the economy has a positive financial shock and has a relatively large amount of debt, so that a future financial crisis is relatively likely.

The effects of bailouts in capital accumulation when the collateral constraint is not binding is generally ambiguous due to two opposing forces. On one hand, the fact that with bailouts, the economy has effectively more insurance available increases the demand for risky assets, in this case, capital. On the other hand, since capital is also a form of savings, the demand for capital decreases. Overall, the lower panels illustrate that the first effect dominates for high values of debt.

4.5 Non-Linear Impulse Responses

This section conducts simulations of the economy with and without bailouts to demonstrate how the time-series are affected by the presence of bailouts.

Our first experiment is a non-linear impulse response. In particular, we simulate the economy with a constant TFP and interest rate equal to the average values and a financial shock that has a high value at the beginning of the simulations, then falls for one period and returns to the high value. To pick the initial endogenous states, we start the economy at a level of bonds and capital at which the competitive equilibrium converges after a long sequence of κ^H and the mean value for the other shocks. For illustrative purposes, the negative financial shock hits after 60 periods so that the economy with the optimal bailout remains approximately constant in the absence of any other shock.

We compute the simulations for the economy with no intervention, the economy with the optimal bailout policy and a third economy where government intervention is unanticipated. That is the economy behaves from $t = 0$ to $t = 60$, as if bailouts have a zero probability from occurring, i.e., at $t = 60$, agents are "surprised". From $t = 61$, we assume that agents again do not expect bailouts in the future. This means that the government switches from No-Bailout Policy to Optimal Bailout Policy in period 60 and then returns to No-Bailout Policy the following period. Each of these switches is unanticipated. The purpose of this experiment is to illustrate the effects of risk-taking decisions on the effectiveness and the magnitudes of bailout ex-post.[16]

The result of the non-linear impulse response is illustrated in Figures 6. This figure shows the evolution of leverage, output, employment, investment, and the exogenous shocks. When the bailout is unanticipated, the economy does not experience as sharp a decline in investment, output and employment. Moreover, the recovery is also quite faster than in the absence of bailouts. Considering that the bailout is 0.5 percentage points of GDP and that the cumulative gain in output as a result of the bailout is about 1.5 percentage point, this yields a bailout-multiplier of 3.

When the bailout policy is anticipated, there is a clear increase in leverage in the run-up to the crisis as the economy perceives that the government will intervene ex-post in case the economy enters a crisis. As a result, the overall effects during a crises become more ambiguous: The increase in the amount of debt makes the economy more vulnerable to

[16] As discussed above, the constrained-efficient allocations are time-consistent by construction. This means that the government does not have any incentive to "persuade" agents that it is not going to intervene. This will not be the case when we examine bailouts without prudential policy.

a negative shock, while bailouts relax balance sheet constraints ex-post. For the optimal bailout policy, however, the effects of the credit crunch are less protracted. In this case the bailout-multiplier is 1.2 (as opposed to 3) since the bailout is 1 percentage point of GDP and the cumulative output gain is 1.3 percentage points of GDP.

4.6 Moral Hazard

We not turn to the final step and central aspect of our normative analysis: How critical are policies to prevent excessive risk taking?

As we explain above, the increase in leverage and risk-taking in the previous section is efficient in the sense that this is the optimal response to the higher level of insurance provided by bailouts. That is, during the optimal intervention, the cost of borrowing is still the socially optimal one because the planner can control the level of borrowing. To see the importance of the prudential policy, we consider the two cases described above: systemic bailout and idiosyncratic bailout policy. We reconstruct the non-linear impulse response from Figure 6, but now we compare the economy with the optimal bailout policy with the systemic bailout policy and the idiosyncratic bailout policy.

Figure 7 shows the results of this experiment. The two economies without prudential policy experience an increase in borrowing relative to the optimal bailout policy and experience a larger crash in period 60. As Figure 7 shows the systemic bailout policy, however, does not cause a substantial increase in financial fragility. On the other hand, the economy with the idiosyncratic bailout policy causes the financial crisis to be several times deeper than the systemic bailout policy or the optimal bailout policy. Moreover, this occurs despite the government implementing a bailout which is 5 times larger than in the economy with the optimal bailout policy.

4.7 Welfare

Next, we compute the welfare gains from policy intervention. We consider the welfare effects of the three cases analyzed above: (a) optimal intervention; (b) systemic bailout, and (c) idiosyncratic bailout. We compute for every possible state the percentage increase in consumption that leaves a household indifferent between living in an economy with the

corresponding government policy and remaining in an economy without government intervention.

The results from the calculations for constrained-efficient allocations are shown in Figure 8 for different values of bonds and capital and for average TFP and interest rate shocks and an adverse financial shock. Welfare gains are positive in all states and reach the maximum levels when the economy is highly leveraged.[17]

Figure 9 shows the comparison of the welfare gains for the same shock realizations and taking an initial value of capital equal to the mean. This figure shows that systemic bailout policy delivers welfare gains which are quite close to the optimal policy. On the other hand, idiosyncratic bailouts deliver substantial welfare losses. These welfare results are consistent with the non-linear impulse responses we showed in the previous section.

5 Conclusion

We developed a quantitative framework to examine the effects of bailouts. The key novelty of our work is that we investigate what is the optimal bailout policy considering both the effects of relaxing balance sheet constraints ex-post and the increase in leverage that occurs ex-ante.

Our findings are particularly relevant to the ongoing debates about the appropriate role of the government during financial crises. Our analysis suggests that systemic insurance provided by the government should be part of the financial system's safety net. The design of this systemic insurance, however, should be generally complemented by macro-prudential policy during good times to offset incentives for excessive risk-taking.

One important refinement of our analysis would be to study the interaction between bailouts and the degree of market incompleteness in the economy. In practice, it is possible that government bailouts affect the financial sector's incentives to supply hedging instruments. To the extent that there are clear imperfections and costs associated with private provision of liquidity, our analysis can also be seen as providing support for the government's role in supporting the development of the market for private insurance.

[17] For graphical purposes, we attach a value of zero for those states that are non feasible, i.e., those with very high levels of debt and low values of capital.

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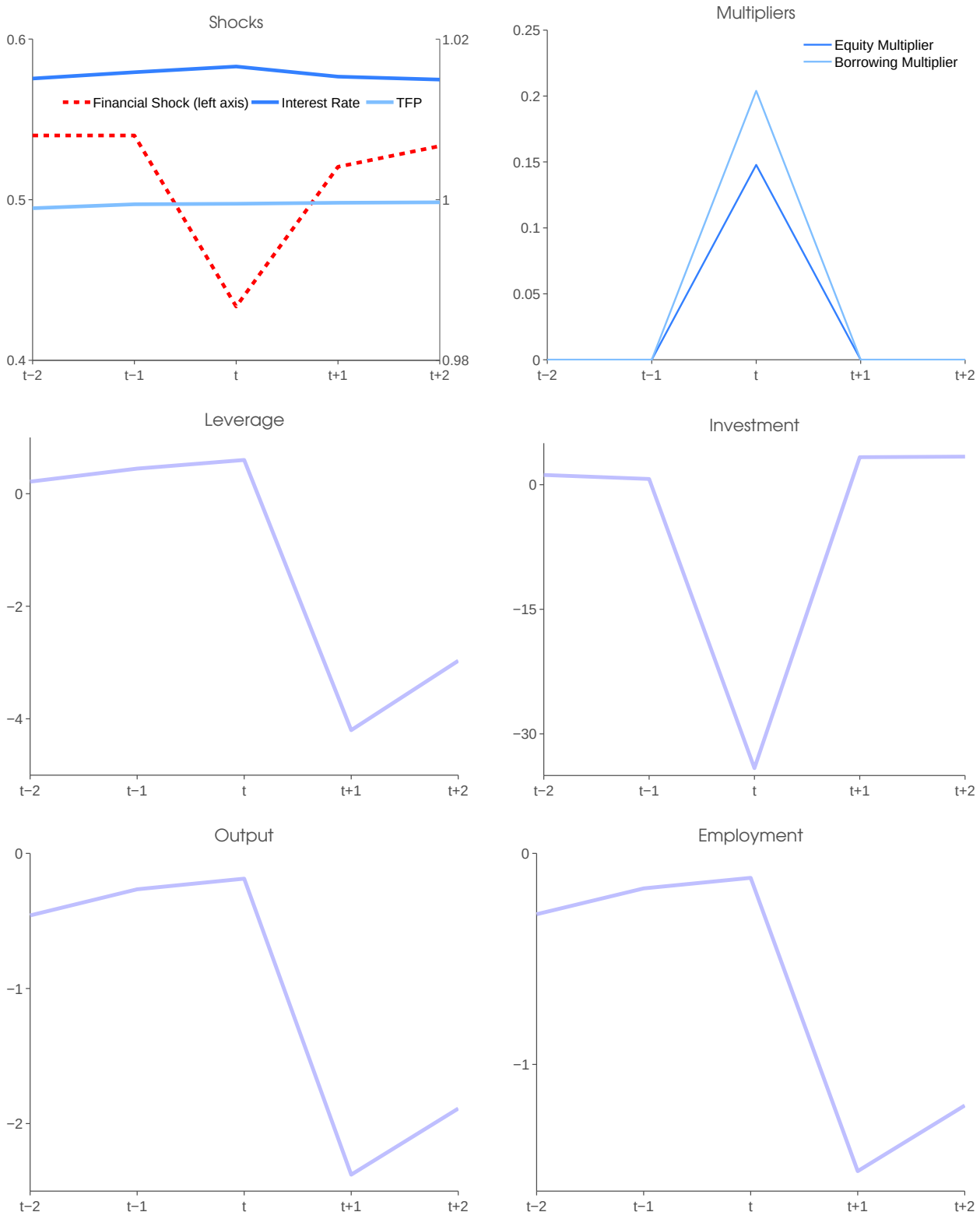

Figure 1: Dynamics of Financial Crises

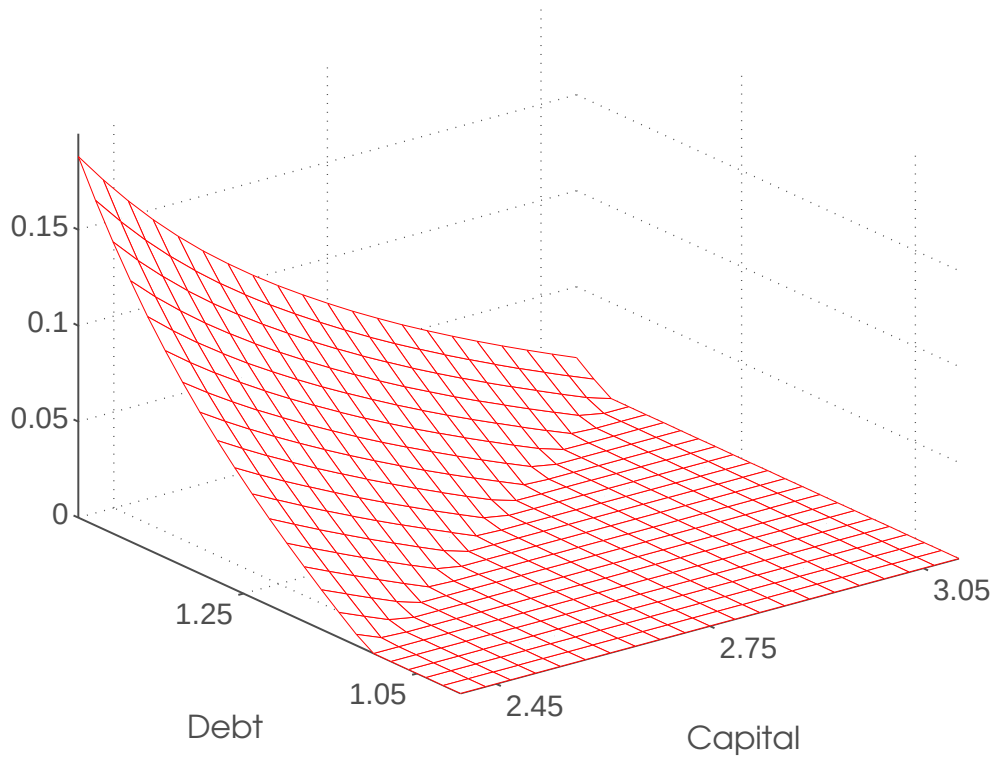

Figure 2: Optimal Bailout Policy for $\kappa = \kappa^L$ and average TFP and interest rate shocks.

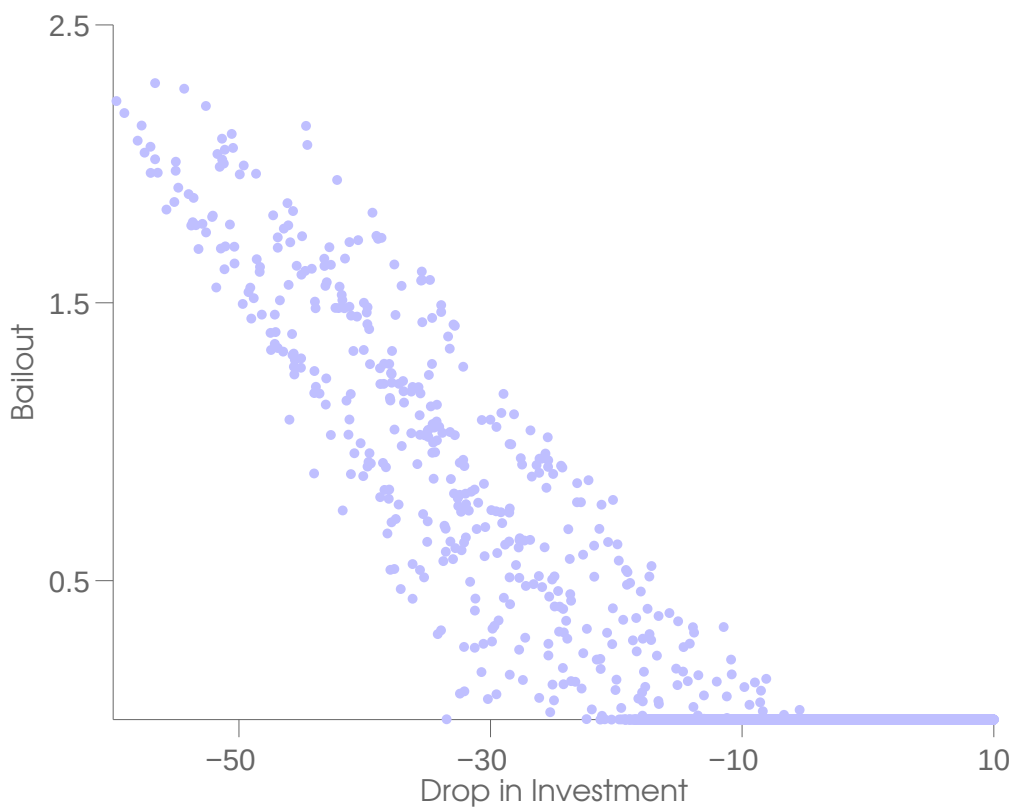

Figure 3: Model Simulations. Bailouts coincide with investment collapses.

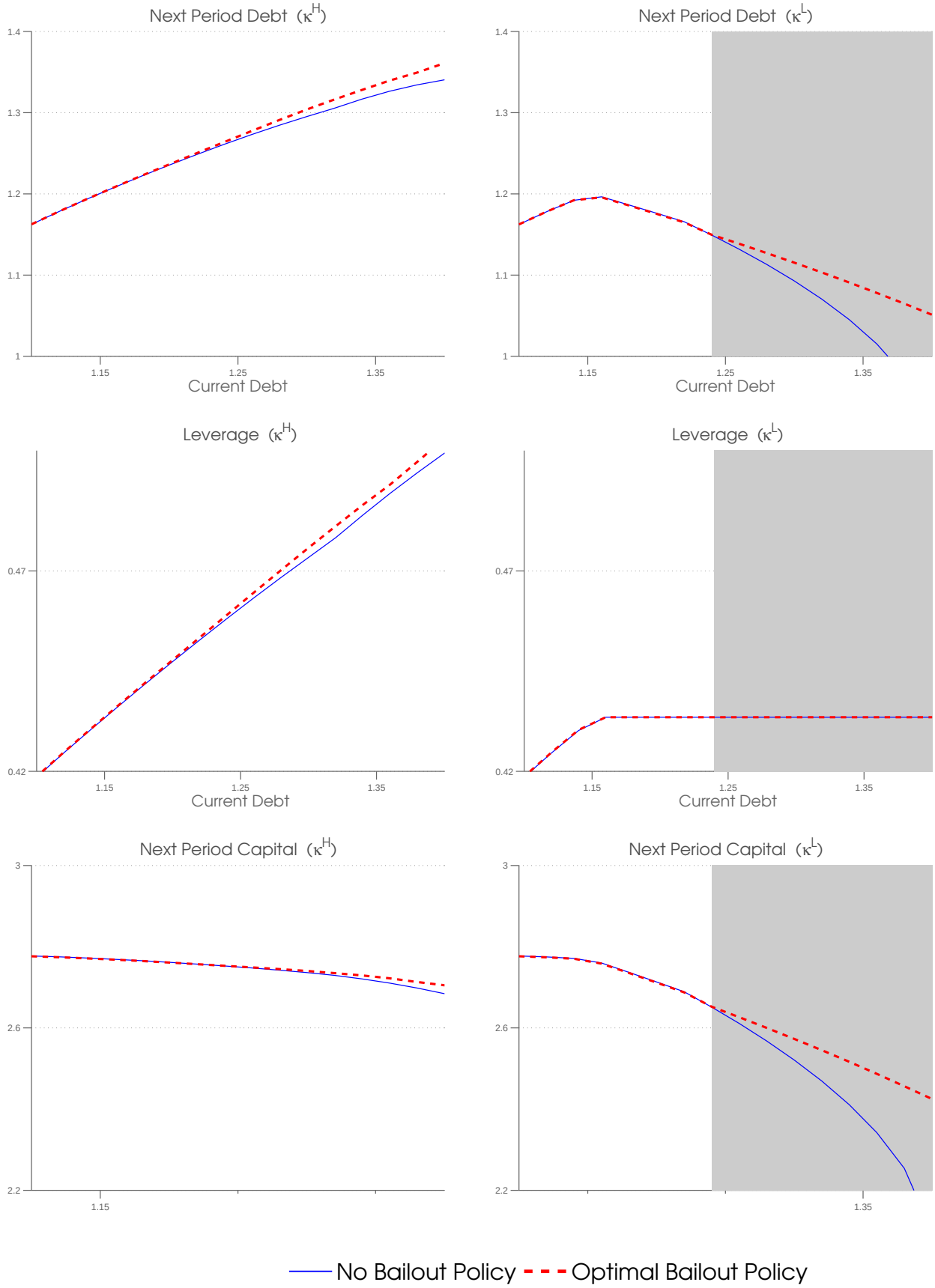

Figure 4: Laws of Motion for average value of capital and average TFP and interest rate shocks. The left (right) column corresponds to $\kappa^H(\kappa^L)$. The shaded region represents the area where bailouts are strictly positive in the allocations with the Optimal Bailout Policy.

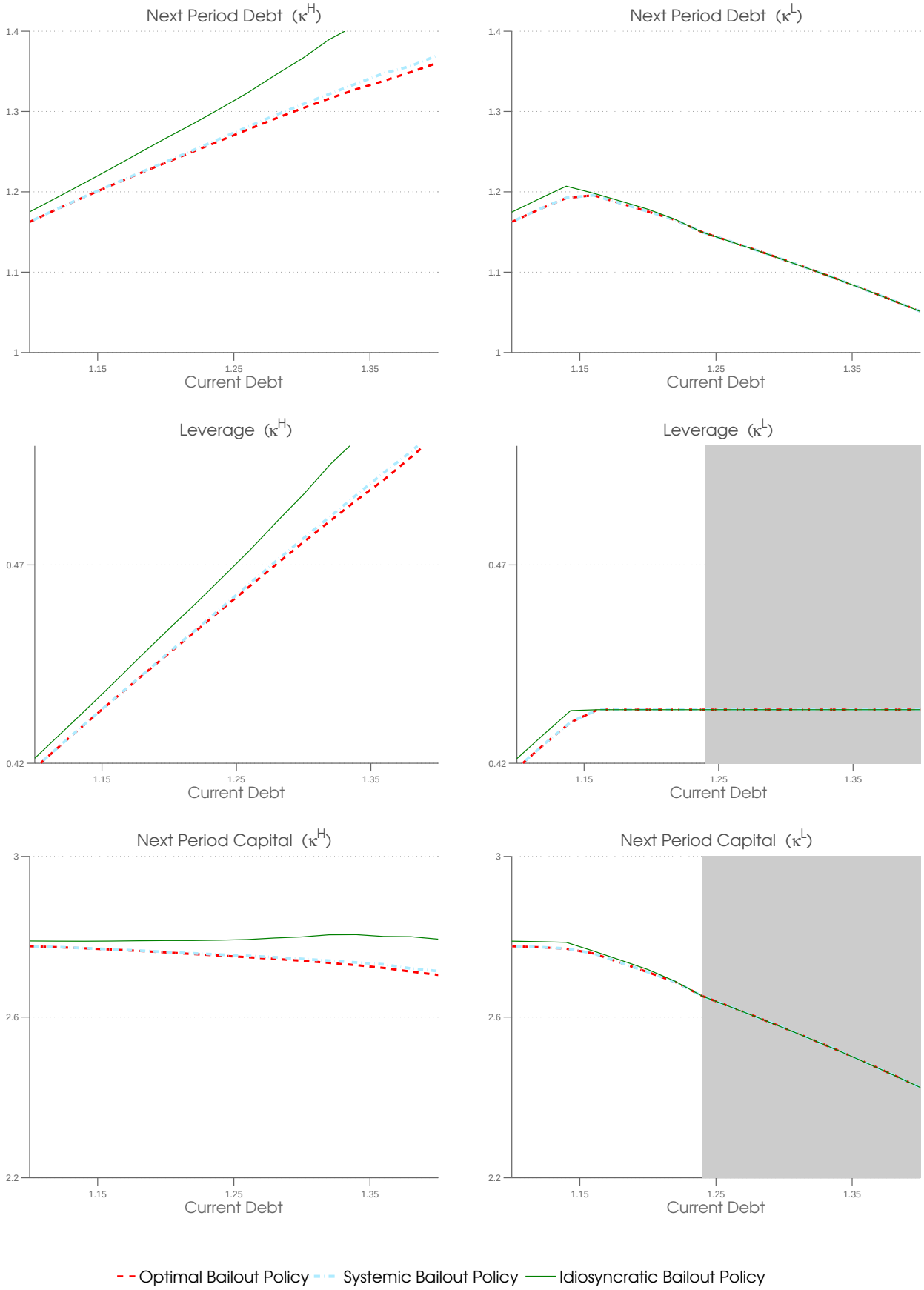

Figure 5: Laws of Motion for average value of capital and average TFP and interest rate shocks. The left (right) column corresponds to $\kappa^H(\kappa^L)$. The shaded region represents the area where bailouts are strictly positive.

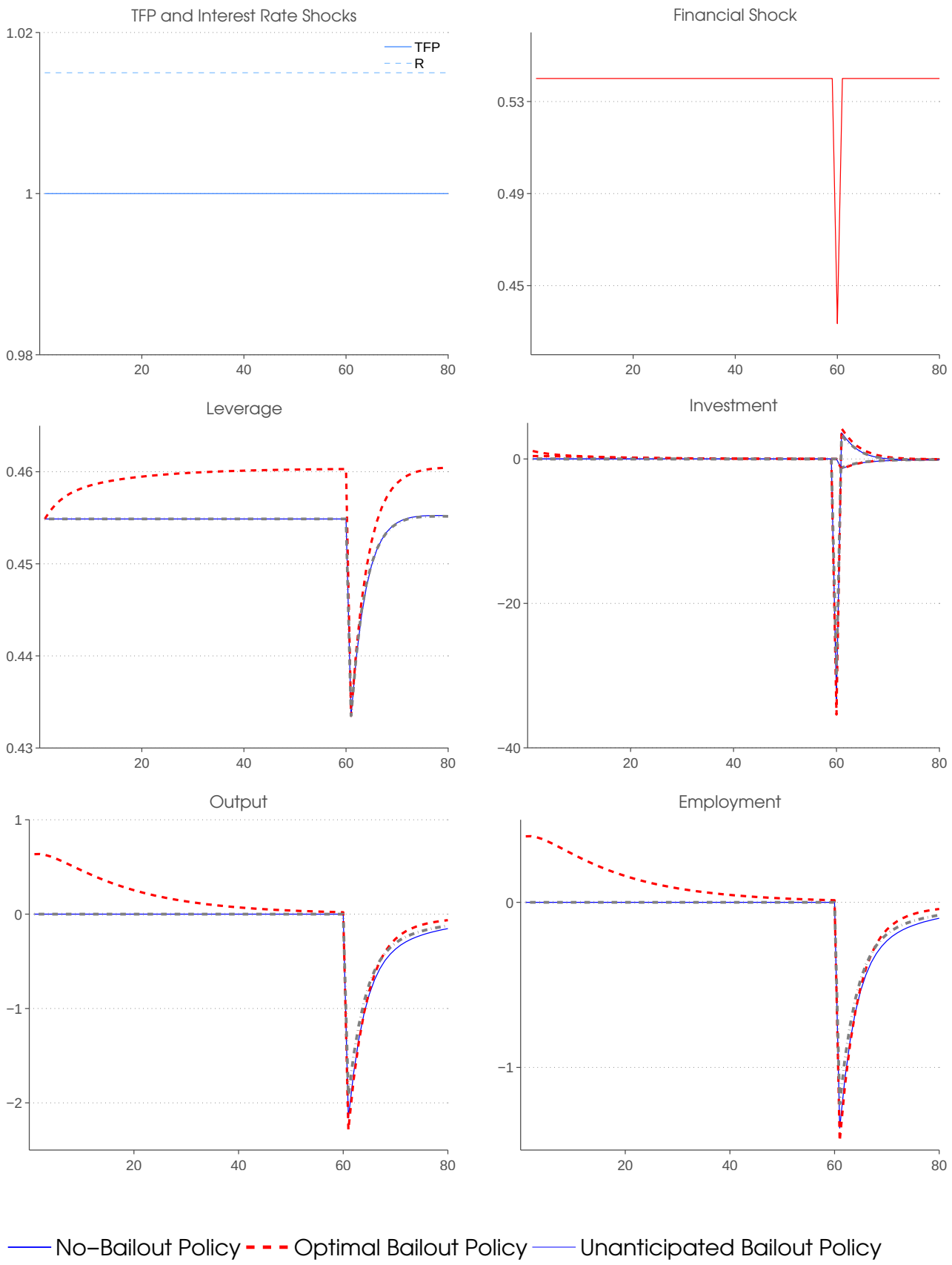

Figure 6: Non-Linear Impulse Response. Output, employment and investment are expressed as percentage deviation from pre-crisis values.

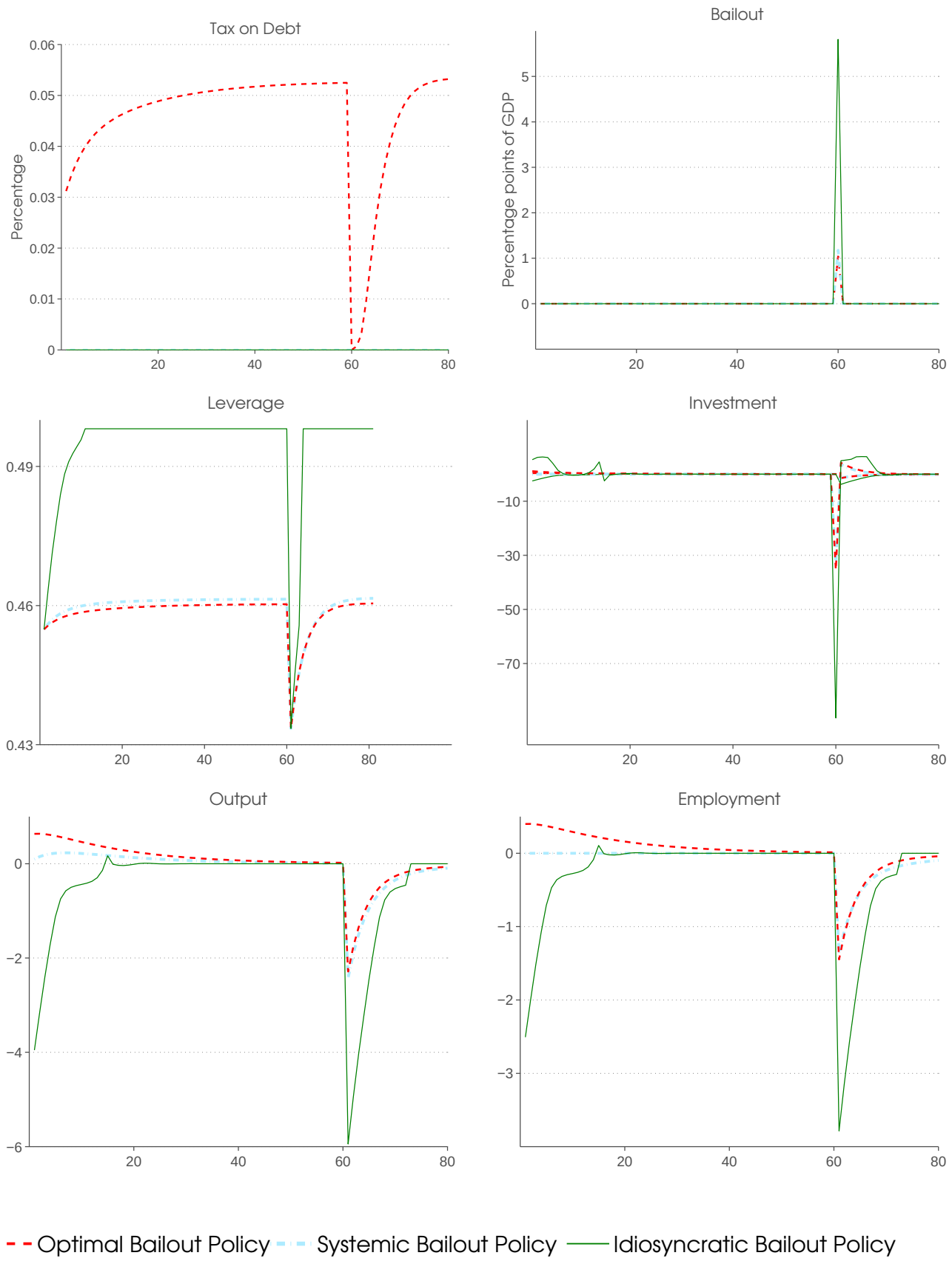

Figure 7: Non-Linear Impulse Response. Output, employment and investment are expressed as percentage deviation from pre-crisis values.

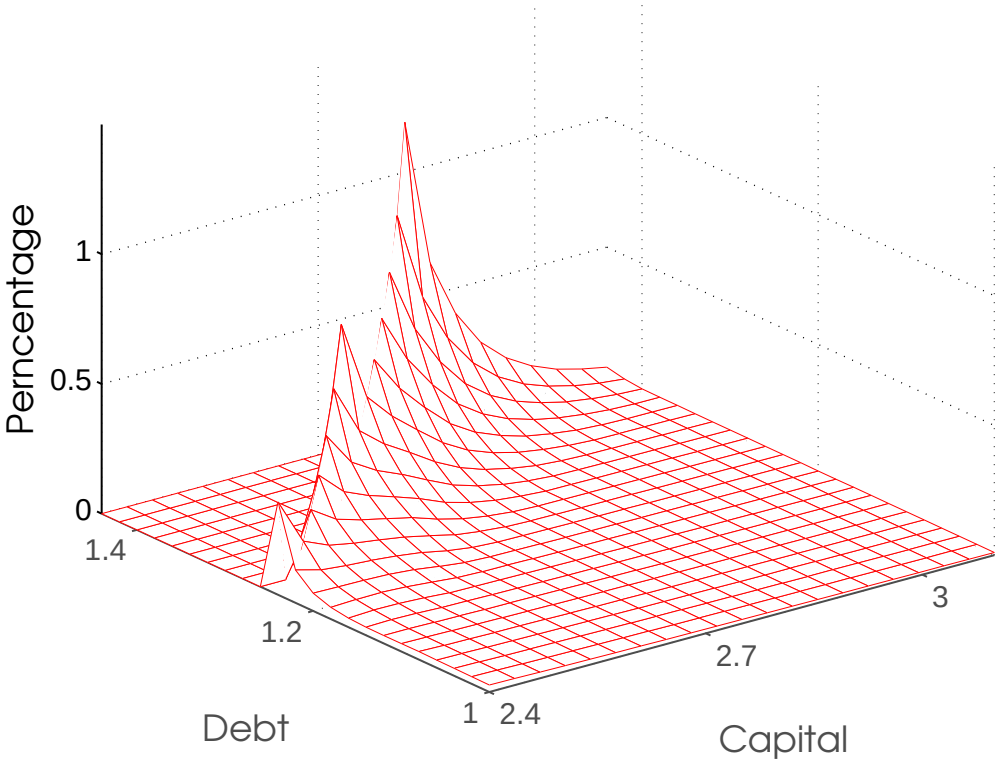

Figure 8: Welfare Gains from optimal policy for $\kappa = \kappa^L$ and average TFP shock

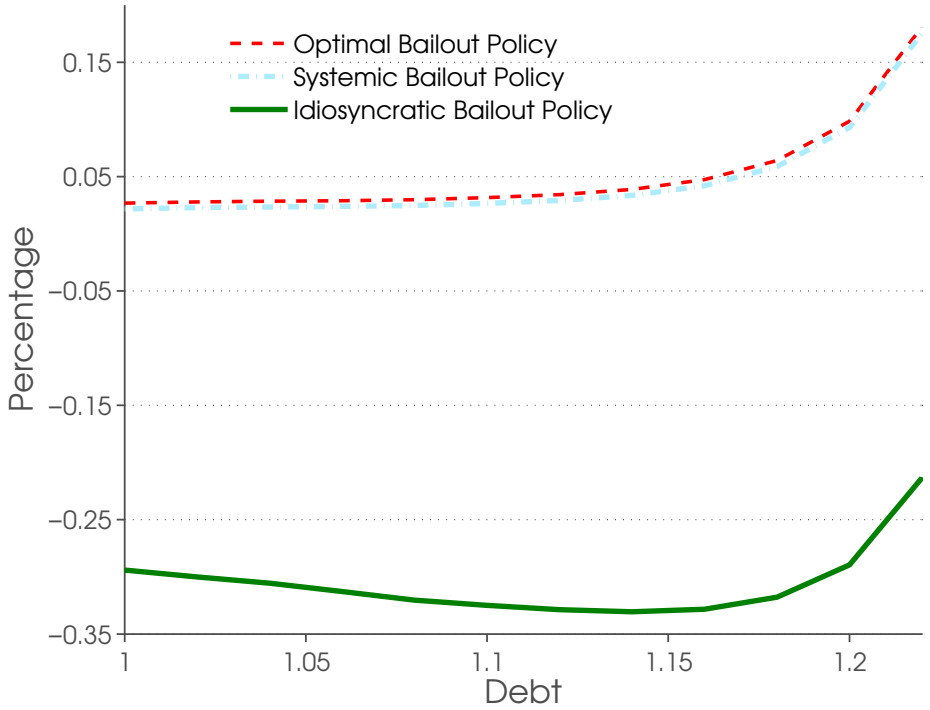

Figure 9: Welfare Gains for different policies for an initial value of capital equal to the mean in the simulation, $\kappa = \kappa^L$ and an average TFP shock

A Decentralization based of planner's problem with equity injections.

An alternative policy to the debt relief policy analyzed in the text is the injection of equity. In order to facilitate a comparison with debt relief policies, we consider an equity injection that is a fraction of the amount of individual debt held by the firm, i.e., the number of new shares issued is such that the value of equity injections equals $e_t b_t$. In particular, the government mandates firms to issue new shares and transfer those shares to the households, which then receive the future dividend payments.

The firms' objective can be expressed as maximize $\mathbb{E}_t \sum_{j=0}^{\infty} m_{t+j}(d_{t+j} - e_t b_t)$ subject to the flow of funds constraint $b_t + d_t + i_t + \psi(k_t, k_{t+1}) \leq F(z_t, k_t, h_t) - w_t n_t + \frac{b_{t+1}}{R_t} + e_t b_t(1-\tau_t) + T_t^f$ and the same financial constraints in problem (10). The first-order condition with respect to debt yields:

$$1 + \eta_t = R_t \mathbb{E}_t m_{t+1}(1 + \eta_{t+1}(1 - e_{t+1})) + R_t \mu_t \tag{24}$$

At the beginning of each period, the total number of shares can be renormalized to one. Hence, the rest of the equilibrium conditions remain the same yielding a similar proposition to the one stated for debt relief.

Proposition 2 *The government can implement the constrained optimal allocations through a combination of equity injections, taxes on debt and capital, and lump sum taxes. In particular:*

$$\gamma_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \qquad T_t^f = \frac{b_{t+1}}{R_t}\tau_t + \tau_t^k(F(z_t, k_t, h_t) - G'(h_t)h_t)$$

$$\tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1})(1-\gamma_{t+1}) + \mu_t} - 1, \tau_t^k = \frac{\mathbb{E}_t m_{t+1}\left(\frac{\partial \tilde{w}_{t+1}}{\partial k_{t+1}}\tilde{h}_{t+1}\right)\eta_{t+1} + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t} - 1$$

$$e_t = \frac{\Upsilon_t}{b_t}, \quad T_t = \Upsilon_t(1+\varphi), \quad \tau_t^b = \frac{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}) + \mu_t}{\mathbb{E}_t m_{t+1}(1+\eta_{t+1}(1-e_{t+1})) + \mu_t} - 1, \quad T_t^f = \frac{b_{t+1}}{R_t}\tau_t \tag{25}$$

where all variables are evaluated at the constrained optimal allocations.

The proof follows the same steps as Proposition 1. Note from (24) that the shadow cost of tightening the equity constraint in a state $t+1$, in which the government is recapitalizing firms, is reduced by a factor of $(1 - e_{t+1})$. Comparing this condition with (23) yields that the tax on debt is smaller than the one required for debt relief. Intuitively, equity injections also involve a cost for shareholders because they perceive a reduction in their ownership of the firm. In addition, the tax on debt is strictly positive only if $\varphi > 0$. Intuitively, firms do not internalize that when they are bailed-out, the social costs of the bailout are paid by other agents in the economy. As a result, they would take too much debt in the absence of a tax on debt.