Collateralized Borrowing and Risk Taking at Low Interest Rates[*†]

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Abstract

A view advanced in the aftermath of the late-2000s financial crisis is that lower than optimal interest rates lead to excessive risk taking by financial intermediaries. We evaluate this view in a quantitative dynamic model where interest rate policy affects risk taking by changing the amount of safe bonds available for collateralized borrowing. If collateral is properly priced, lower than optimal interest rates reduce risk taking. However, if intermediaries can augment their collateral by issuing assets whose risk is underestimated by rating agencies, lower than optimal interest rates contribute to excessive risk taking and amplify the severity of recessions.

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1 Introduction

The recent financial crisis has renewed interest in examining the determinants of portfolio investments into safe and risky assets by financial intermediaries. A standard result in the theory of portfolio choice is that a risk averse investor's optimal investment into risky assets is decreasing in the return to safe assets. This insight suggests that low policy rates will increase the riskiness of financial intermediaries' portfolios, by altering the returns to safe assets. To the extent that increased investments in risky assets exceed the social optimum, there may be important welfare consequences.

In this paper, we examine how changes in the policy rate affect the portfolio choices of financial intermediaries, in an environment in which safe assets can be used as collateral to facilitate borrowing. Two facts motivate our decision to model collateralized borrowing: first, collateralized borrowing is a primary margin of balance sheet adjustment for intermediaries (Adrian and Shin (2010)) and, second, the cost of such borrowing is tightly linked to monetary policy rates. Our findings encompass the standard portfolio choice result and, at the same time, highlight the importance of collateral for risk taking. At low interest rates, low demand for safe assets results in a shortage of collateral, which limits borrowing and reduces risk taking by intermediaries. Relaxing the collateral constraint of intermediaries—by allowing intermediaries to use assets with misperceived safety as collateral—increases investments in risky assets and generates sizable welfare losses.

We develop a dynamic model with aggregate and idiosyncratic uncertainty in which the monetary authority controls the real interest rate on safe bonds.[1] Each period, initially

[1] Implicitly, we assume that the monetary authority is successful in ensuring price stability. In this context, we consider whether the monetary authority can control risk taking of intermediaries through the real interest rates on safe assets and examine the implications for the macroeconomy. Having nominal interest rates as a policy instrument would enrich the policy insights, but is beyond the scope of this paper.

identical financial intermediaries with limited liability receive deposits and equity investment from households and invest into safe bonds and risky projects. The latter are investments into the production technologies of small firms, and their returns are correlated with aggregate productivity.[2] After this initial portfolio decision, intermediaries find out whether they hold high-risk projects, with high variance and high expected return, or low-risk projects, with low variance and low expected return. Given this information, intermediaries reoptimize their portfolios using collateralized borrowing in the interbank market. For example, when aggregate productivity is expected to be high, intermediaries with high-risk projects—call them high-risk intermediaries—trade their bonds to invest more into their risky projects. These projects are relatively attractive from a social point of view due to their high expected return, and are even more attractive from the intermediaries' point of view because potential losses in the event of a contraction are avoided through limited liability (as in Allen and Gale (2000)). Low-risk intermediaries on the other side of the transaction accept bonds and reduce exposure to their risky projects, which have lower expected returns.

In this environment, monetary policy influences risk taking by financial intermediaries directly, through a portfolio channel, and indirectly, through a collateral channel. Changes in risk taking through the portfolio channel are similar to those discussed in Merton (1969), Samuelson (1969) and Fishburn and Porter (1976). Namely, at low interest rates, intermediaries purchase fewer safe bonds and invest more into riskier assets with a higher expected return.[3] The innovation in our paper is to consider the transmission mechanism from monetary policy to risk taking through the quantity of collateral. At low interest rates, financial intermediaries allocate few resources to safe assets and the resulting scarcity of collateral provides a safeguard against increased risk taking.

[2] In our model, the investment market is segmented in that households cannot invest directly in risky projects of small firms and are forced to use intermediaries. This is similar to Gale (2004). Noncorporate, nonfinancial firms are the data counterpart for the small firms in our model. For simplicity, we do not model loans between financial intermediaries and these firms, but rather assume that intermediaries operate their production technologies directly. To allow our model to be consistent with U.S. data, we also model a nonfinancial sector (see Section 2).

[3] This idea was also the basis of Rajan (2006), who discusses excessive risk in the financial sector.

Collateralized borrowing in our model is beneficial because it facilitates reallocation of resources between intermediaries in response to new information about the riskiness of their investments. However, borrowing against safe bonds also allows intermediaries to take advantage of their limited liability by overinvesting in risky projects. This is socially costly because financial intermediaries can go bankrupt, in which case, payments to its depositors are guaranteed by the government-funded deposit insurance.[4] The role of the monetary authority is to set interest rate policy so as to mitigate the moral hazard problem of intermediaries.[5] This is achieved by making the collateral constraint of intermediaries bind at the optimal policy.

We solve for the optimal interest rate policy and consider the implications of lower than optimal interest rates for risk taking and welfare. We say that risk taking of financial intermediaries is *excessive* if investments in high-risk projects in the decentralized economy exceed the social optimum, defined as the solution to a social planner problem. We calibrate our model's parameters to match key characteristics of economic expansions and contractions and of the financial sector in the U.S. economy. We find that, at the optimal interest rate policy, there is excessive risk taking, but welfare losses relative to the social optimum are very small. In addition, lower than optimal interest rates lead to less risk taking by financial intermediaries. The intuition for this results is that the collateral risk taking channel dominates the portfolio risk taking channel because it constrains high-risk intermediaries who have the strongest incentives to overinvest in risky projects.

In the model outlined so far, collateralized borrowings can be interpreted as repurchasing agreements (repos).[6] Empirically, repos are an important margin of portfolio adjustment, as suggested by Adrian and Shin (2010) and are largely collateralized using government bonds. It is well documented that, in the run-up to the recent financial crisis, some assets

[4] In our model, deposit insurance is provided at no cost, consistent with empirical evidence in Pennacchi (2006). For details, also see footnote 18.

[5] We note that moral hazard leads to a failure of the Modigliani and Miller (1958) theorem, see Hellwig (1981) and Myers (2003).

[6] A repo transaction is a sale of a security and a simultaneous agreement to repurchase the security at a future date. Repos are secured loans in which the borrower receives money against collateral.

used as collateral in the repo market were not truly safe (see Gorton (2010), Gorton and Metrick (2011), Krishnamurthy, Nagel, and Orlov (2011) and Hoerdahl and King (2008)).[7] We consider a version of our model in which intermediaries issue private bonds which are misrated as safe by credit rating agencies and, as a result, are accepted as collateral. In addition, there is exogenous foreign demand for the domestic assets rated as safe. This is consistent with evidence that, in the last decade, the U.S. has attracted excess world savings from countries in search of safe assets (see Krishnamurthy and Vissing-Jorgensen (2010)). These additional features allow high-risk intermediaries to relax their collateral constraint and take on more risk. In this extended model, low interest rates lead to increased risk taking by financial intermediaries, amplify the severity of recessions and generate larger welfare losses.

We conclude that the collateral channel provides a safeguard against increased risk taking, as illustrated by our benchmark model without misrated assets. This suggests that accurate risk assessment of collateral assets is essential in maintaining the protective role of the collateral risk taking channel. This may be a promising direction for regulatory changes.

Related Literature

Our paper contributes to the growing literature studying the risk taking channel of monetary policy, a term coined by Borio and Zhu (2008). Several papers find empirical evidence that, when interest rates are low for an extended period, banks take on more risks.[8] There are also theoretical explorations of this link, for example, Dell'Ariccia, Laeven, and Marquez (2010). Our paper complements this body of work, by evaluating the impact of lower than optimal interest rates on risk taking in a quantitative dynamic general equilibrium model

[7] These papers document that riskier and less liquid collateral such as private-label mortgage backed securities and asset backed securities were used in the repo market prior to the crisis. This type of collateral disappeared from the repo market as the crisis unfolded.

[8] For example, Gambacorta (2009), Ioannidou, Ongena, and Peydró (2009), Jiménez, Ongena, Peydró, and Saurina (2009), Delis and Kouretas (2010) and Altunbas, Gambacorta, and Marques-Ibane (2010) use data from different countries to show that banks grant riskier loans and soften lending standards when interest rates are low. de Nicolò, Dell'Ariccia, Laeven, and Valencia (2010) use U.S. commercial bank Call Reports to document a negative relationship between the real interest rate and the riskiness of banks' assets.

calibrated to the U.S. economy. Through the lens of our model, low interest rates per se do not increase risk taking, but may do so in conjunction with financial imperfections that affect collateralized borrowing.

Our paper is closely related to Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2011).[9] These authors consider the effects of credit policies (e.g. discount window lending, equity injections) and macro prudential policies (e.g. subsidies to issuance of outside equity) on financial intermediation and risk taking incentives, in environments in which banks choose equity and deposits endogenously. Our work is similar to these two papers in that we build a quantitative model in which intermediaries make endogenous portfolio choices. An important difference is that we allow intermediaries to invest in safe bonds, which are later used as collateral in interbank borrowing. This allows us to highlight the role of monetary policy in affecting risk taking through the quantity of available collateral. We also complement the work in these papers by analyzing the contribution of collateral assets with misperceived safety to risk taking.

Our paper is also related to the literature studying the impact of collateral constraints on the macroeconomy. For example, Kiyotaki and Moore (1997) show that shocks to credit-constrained firms are amplified and transmitted to output through changes in collateral values. Caballero and Krishnamurthy (2001) consider the impact of a shortage in domestic and international collateral on real activity. While we do not consider valuation effects of interest rates on collateral, our paper makes an important contribution by highlighting that relaxing the collateral constraint can result in increased risk taking with adverse effects for real activity.

There is an extensive theoretical literature that examines other related aspects of financial intermediation. For example, Shleifer and Vishny (2010), consider a model in which financial intermediaries alter capital allocation based on investor sentiment, and volatility of this sentiment transmits to volatility in real activity. Stein (1998) examines the transmission

[9] These papers augment the existing quantitative macro models with financial amplification mechanism à la Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

mechanism of monetary policy in a model in which banks' portfolio choices respond to changes in the availability of financing via insured deposits. Diamond and Rajan (2009), Acharya and Naqvi (2010) and Agur and Demertzis (2010) examine the optimal policy when the monetary authority has a financial stability objective. Farhi and Tirole (2009) and Chari and Kehoe (2009) consider moral hazard consequences of government bailouts.

The paper is organized as follows. Section 2 presents the model and derives equilibrium properties. Section 3 outlines the methods we use to pin down our model's parameters. Section 4 describes the various experiments and the main results of the paper. Section 5 concludes.

2 Model Economy

The economy is populated by a measure one of identical households, a measure π_m of identical nonfinancial firms, a measure $1-\pi_m$ of financial intermediaries and a government. Financial intermediaries are initially identical and later split into into high-risk or low-risk. Time is discrete and infinite. Each period, the economy is subject to an exogenous aggregate shock which affects the productivity of all firms, as outlined in section 2.2. The aggregate state $s_t \in \{\overline{s}, \underline{s}\}$ follows a first-order Markov process. The history of aggregate shocks up to t is s^t. A summary of the timing of events in our model is presented in Section A.1 of the Appendix.

2.1 Households

At the beginning of period t, the aggregate state s_t is revealed and households receive returns on their previous period investments, wage income and lump-sum taxes or transfers from the government. Households split the resulting wealth, $w\left(s^t\right)$, into current consumption, $C\left(s^t\right)$, and investments that will pay returns in period $t+1$.

Investments take the form of deposits, nonfinancial sector equity and financial sector equity. Deposits, $D_h\left(s^t\right)$, earn a fixed return, $R^d\left(s^t\right)$, which is guaranteed by deposit insurance. Equity invested in financial intermediaries, $Z(s^t)$, is a risky investment which gives households a claim to the profits of the intermediaries. The return per unit of equity is $R^z(s^{t+1})$. Similarly, the equity investment into the nonfinancial sector, $M(s^t)$, entitles the household to state contingent returns next period, $R^m(s^{t+1})$.

Households supply labour inelastically. We assume that labour markets are segmented.[10] Fraction π_m of a household's time is spent working in the nonfinancial sector, and fraction $1-\pi_m$ is spent in the financial sector. Wage rates vary by sector, the type of firm within the sector and the aggregate state of the economy: $W_m(s^t)$ is the wage rate paid by nonfinancial firms given history s^t, while $W_j(s^t)$ is the wage rate paid by a financial intermediary of type $j \in \{h, l\}$. Throughout, h denotes high-risk and l denotes low-risk intermediaries. With these assumptions, labour supplied to each firm is normalized to one unit, for any realization of the aggregate state.

The household's problem is given by:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \varphi\left(s^t\right) \log C\left(s^t\right)$$

$$\begin{aligned}
\text{subject to} &: \\
w\left(s^t\right) &= R^m\left(s^t\right) M\left(s^{t-1}\right) + R^d\left(s^{t-1}\right) D_h\left(s^{t-1}\right) + R^z\left(s^t\right) Z\left(s^{t-1}\right) \\
&\quad + \pi_m W_m\left(s^t\right) + \left(1-\pi_m\right)\pi_l W_l\left(s^t\right) + \left(1-\pi_m\right)\pi_h W_h\left(s^t\right) + T\left(s^t\right) \\
w\left(s^t\right) &= C\left(s^t\right) + M\left(s^t\right) + D_h\left(s^t\right) + Z\left(s^t\right)
\end{aligned}$$

where β is the discount factor, $\varphi(s^t)$ is the probability of history s^t, π_j with $j \in \{h, l\}$ is the probability of working for financial intermediary of type j, where $\pi_h + \pi_l = 1$, and $T(s^t)$ are lump-sum transfers if $T(s^t) \geq 0$, or lump-sum taxes otherwise.

[10]The assumption of a labour market segmentation is done for convenience. Relaxing this assumption to allow labour to move across firms and sectors, would reinforce the risk taking channel present in our model, as both capital and labour would flow in the same direction.

2.2 Firms

Financial and nonfinancial firms differ in the way they are funded, in the types of investments they make and the productivity of these investments. *Financial firms* finance their operations through equity and deposits. The main difference between these two forms of funding is that equity returns are contingent on the realization of the aggregate state in the period when they are paid, while returns to deposits are not. In addition, equity returns are bounded below by zero due to the limited liability of intermediaries, while deposit returns are guaranteed by deposit insurance. Financial intermediaries invest into safe government bonds and risky projects. The latter are investments into the production technologies of small firms and can be of two types: high-risk projects with productivity $q_h(s_t)$ and low-risk projects with productivity $q_l(s_t)$.[11] *Nonfinancial firms* are funded through household equity only.[12] All equity raised is invested into capital whose return depends on the productivity of the production technology in the nonfinancial sector, $q_m(s_t)$. Note that, implicitly, households in our model invest directly into the risky production technology of nonfinancial firms. However, they need intermediaries to invest into the risky projects of small firms.

We assume that high-risk financial intermediaries are more productive during a good aggregate state ($s_t = \overline{s}$), and less productive during a bad aggregate state ($s_t = \underline{s}$), compared to low-risk financial intermediaries. Formally, $q_h(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_h(\underline{s})$. Moreover, we consider that the productivity of the production technology of nonfinancial firms is such that: $q_h(\overline{s}) \geq q_m(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_m(\underline{s}) > q_h(\underline{s})$. For details on the parameterization of these relative productivity levels, see section 3.

[11] We assume that financial intermediaries operate the production technologies of small firms directly. By not modeling loans between intermediaries and these firms, we abstract from information problems à la Bernanke and Gertler (1989). Also see footnote 2.

[12] In the model, the important assumption is that the nonfinancial sector is funded through state contingent claims. We use equity for simplicity, but we could also allow for state contingent corporate bonds. Our assumption is consistent with the fact that in U.S. data, corporate nonfinancial firms are mostly equity financed.

2.2.1 Financial Sector

There is a measure $1-\pi_m$ of financial intermediaries. The problem of an intermediary is to choose a portfolio that maximizes the expected value of its equity. Initially, all financial intermediaries are identical, they receive the same amount of deposits and equity from the households and make the same investments into government bonds and risky projects. Financial intermediaries are subject to capital regulation, which requires a minimum amount of equity for every unit of risky investment as a buffer for potential losses. Since our main focus is on optimal interest rate policy and risk taking, we perform several experiments without binding capital regulation.

After the initial investment decisions, intermediaries acquire more information about the riskiness of their projects. With probability π_j, the project an intermediary previously invested into is of type $j \in \{h, l\}$, (i.e. j is i.i.d., for details see Section 3). We refer to intermediaries as being high-risk or low-risk intermediaries, based on the type j of their risky projects. The probabilities, π_h and $\pi_l = 1-\pi_h$, are time and state invariant and known. Once $j \in \{h, l\}$ is known, but before the realization of s_t, intermediaries trade bonds in the repo market in order to adjust the amount of resources invested into the risky projects. This timing assumption is meant to capture the idea that information about the riskiness of projects evolves over time. As a result, financial intermediaries adjust their portfolios, but may be constrained in their choices. The transactions in this market can be interpreted as bilateral repurchasing agreements and are observable only by intermediaries. As a result, financial intermediaries may violate the capital regulation constraint.

We now describe the two stages of an intermediary's problem that take place during period $t-1$. This shows how capital used for production in the financial sector in period t is determined.

Portfolio Choice in the Primary Market

After production in period $t-1$ has taken place, intermediaries receive resources from households and make investment decisions that pay off in t. Financial intermediaries don't know the type of risky projects and maximize expected profits, taking as given future trades in the repo market. Since households own all firms in the economy, firms value profits at history s^t according to the households' marginal utility of consumption weighted by the probability of history s^t. In particular, $\lambda(s^t) = \varphi(s^t)/C(s^t)$.

Taking as given $\lambda(s^t)$, the amount of equity issued by an intermediary, $z(s^{t-1})$, the future repo market activities and all prices, an intermediary chooses deposit demand, $d(s^{t-1})$, safe bonds, $b(s^{t-1})$, risky investments, $k(s^{t-1})$, and labour, $l(s^{t-1})$, to maximize the expected profits in $(P1)$:

$$\max \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P1}$$

subject to:

$$z\left(s^{t-1}\right) + d\left(s^{t-1}\right) = k\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right) \tag{1}$$

$$V_j\left(s^t\right) = \max\left\{\begin{array}{l} q_j(s_t)\left[k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})\right]^{\theta}\left[l(s^{t-1})\right]^{1-\theta-\alpha} \\ +q_j(s_t)(1-\delta)\left[k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})\right] \\ +\left[b(s^{t-1}) - \tilde{b}_j(s^{t-1})\right] - R^d(s^{t-1})d(s^{t-1}) - W_j(s^t)l(s^{t-1}), 0 \end{array}\right\} \tag{2}$$

$$z\left(s^{t-1}\right)/k\left(s^{t-1}\right) \geq \eta \tag{3}$$

where $V_j(s^t)$ are profits for intermediary $j \in \{h,l\}$ at history s^t, $p(s^{t-1})$ is the primary market bond price, $\tilde{p}(s^{t-1})$ is the secondary market or repo market price, and $\tilde{b}_j(s^{t-1})$ is the amount of bonds traded in the repo market by intermediary j.

The production technology operated by intermediary j is $q_j(s_t)\left[k_j(s^{t-1})\right]^{\theta}\left[l(s^{t-1})\right]^{1-\theta-\alpha}$, where $q_j(s_t)$ is the productivity parameter, $k_j(s^{t-1}) \equiv k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})$ is the amount of resources invested in the risky projects and $l(s^{t-1})$ is the amount of labour employed. Recall that we abstract from labour redistribution and normalize $l\left(s^{t-1}\right)$ to 1. Parameters θ and α satisfy $\alpha, \theta \in [0,1]$, $1-\alpha-\theta \geq 0$. If $\alpha > 0$ there is a fixed factor present in the production process. In the absence of bankruptcy, this factor's returns are payable to the equity holders.

In equation (2), the undepreciated capital stock of firms is adjusted by the productivity level. This allows for variation in the value of capital, similar to Merton (1973) and Gertler and Kiyotaki (2010). The idea is that while capital may not depreciate in a physical sense during contraction periods, it does so in an economic sense. In a case study of aerospace plants, Ramey and Shapiro (2001) show that the decrease in the value of installed capital at plants that discontinued operations is higher than the actual depreciation rate. In addition, Eisfeldt and Rampini (2006) provide evidence that costs of capital reallocation are strongly countercyclical.

Lastly, financial intermediaries are subject to capital regulation, which requires the amount of equity they hold per unit of risky investment to be larger than a constant η. This constraint—given in (3)—captures some aspects of the Basel II accord.[13] In most numerical experiments, we set $\eta = 0$. We examine risk taking in the presence of capital regulation in one experiment with $\eta = 8\%$, as in Basel II.[14]

Portfolio Adjustments via Repo Market

Once intermediaries find out their type $j \in \{h, l\}$, they adjust the riskiness of their portfolios by trading bonds, $\tilde{b}_j\left(s^{t-1}\right)$, amongst themselves. Intermediaries choose $\tilde{b}_j\left(s^{t-1}\right)$ to solve:

$$\max \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P2}$$

[13] There are other forms of regulation that are worthwhile contemplating in this model, including a state specific capital adequacy requirement. We leave this for future research.

[14] The interaction between capital regulation and risk is also discussed in Dubecq, Mojon, and Ragot (2009). They find that opaque capital regulation leads to uncertainty about the risk exposure of financial intermediaries, a problem which is more severe at low interest rates.

where $V_j\left(s^t\right)$ is given in equation (2) and $\tilde{b}_j\left(s^{t-1}\right) \in \left[-\frac{k\left(s^{t-1}\right)}{\tilde{p}\left(s^{t-1}\right)}, b\left(s^{t-1}\right)\right]$.

We assume that $\tilde{b}_j\left(s^{t-1}\right)$ are not observed by the regulatory authority and, as a result, the capital regulation constraint may not hold here. $\tilde{b}_j\left(s^{t-1}\right)$ can be interpreted either as sales of bonds or, alternatively, as repurchasing agreements.[15] For this reason, we use the terms secondary bond market and repo market interchangeably.

Empirically, collateralized repos are an important margin of balance sheet adjustment by intermediaries and a good indicator of financial market risk, as suggested by Adrian and Shin (2010). In our model, the redistribution of resources that takes place through the repo market allows financial intermediaries to change their risk exposure in light of new information obtained about their investments. Intermediaries who use bonds as collateral in the repo market increase the amount of resources allocated to risky investments. By the same token, intermediaries who give resources against collateral decrease their risk exposure. Intermediaries can collateralize either a subset or all of their bonds in exchange for an equal amount of resources to be invested in risky projects. We note that our model abstracts from haircuts in the repo market.[16] That is, the intermediaries' ability to increase their risky investment is limited by their primary market activities. Higher purchases of bonds in the primary market make balance sheets seem safer initially, but may lead to increased risk taking through the repo market.

2.3 Nonfinancial sector

There are π_m identical nonfinancial firms which are funded entirely through household equity. Each nonfinancial firm enters period t with equity $M\left(s^{t-1}\right)/\pi_m$ from households which is invested into capital. Hence, $M\left(s^{t-1}\right)/\pi_m = k_m\left(s^{t-1}\right)$. The problem of a nonfinancial firm

[15] While we model $\tilde{b}_j\left(s^{t-1}\right)$ as bond sales, incorporating explicitly the repurchase of bonds—which is typical in a repo agreement—would yield identical results.

[16] A repo transaction in the data may require the borrower to pledge collateral in excess of the loan received. See, for example, Krishnamurthy, Nagel, and Orlov (2011) who document that average haircuts vary between 2 and 7 percent by type of collateral. Requiring excess collateral in our model—in the magnitude consistent with the data—would reduce borrowing via the repo market and would make our results stronger.

is to choose capital and labour to produce output:

$$\max\left\{y_m\left(s^t\right)+q_m\left(s_t\right)\left(1-\delta\right)k_m\left(s^{t-1}\right)-R^m\left(s^t\right)k_m\left(s^{t-1}\right)-W_m\left(s^t\right)l_m\left(s^{t-1}\right)\right\}$$
$$\text{subject to: } y_m\left(s^t\right)=q_m\left(s_t\right)\left(k_m\left(s^{t-1}\right)\right)^{\theta}\left(l_m\left(s^{t-1}\right)\right)^{1-\theta}.$$

We introduce this sector in order to bring our model closer to U.S. data. Specifically, this allows our model to be consistent with a high equity to deposit ratio observed for U.S. households, a low equity to deposit ratio in the U.S. financial sector and the relative importance of the two sectors in U.S. production. Moreover, a large nonfinancial sector—as observed in U.S. data—reduces the quantitative importance of the financial intermediation sector for welfare and risk taking in our model. Excluding it, would overstate the impact of policy on our results.

2.4 Government

The government issues bonds that financial intermediaries can use either as an asset or as a medium of exchange on the repo market. At the end of period $t-1$, the government sells bonds, $B\left(s^{t-1}\right)$, at price, $p\left(s^{t-1}\right)$. These bonds pay off during period t. Part of the proceeds from the bond sales is used to cover a proportional cost, τ, of issuing bonds, while the remainder is deposited into financial intermediaries.[17] Each financial intermediary receives $D_g\left(s^{t-1}\right)/\left(1-\pi_m\right)$ of government deposits, where

$$D_g\left(s^{t-1}\right)=\left(1-\tau\right)p\left(s^{t-1}\right)B\left(s^{t-1}\right).$$

To guarantee the fixed return on deposits the government provides deposit insurance at zero price which is financed through household taxation.[18] The government balances its

[17] Alternatively, the proceeds from the bond sales could be handed to the households via transfers. Our results would be unaffected by such a change.

[18] See Pennacchi (2006, pg. 14), who documents that since 1996, deposit insurance has been essentially free for U.S. banks. In our model, the assumption of a zero price of deposit insurance is not crucial. What matters is that the insurance is not priced in a way that eliminates moral hazard. This means, for example,

budget after the production takes place at the beginning of period t :[19]

$$T\left(s^{t}\right)+B\left(s^{t-1}\right)+\Delta\left(s^{t}\right)=R^{d}\left(s^{t-1}\right)D_{g}\left(s^{t-1}\right).$$

Here, $\Delta\left(s^{t}\right)$ is the amount of deposit insurance necessary to guarantee the fixed return on deposits, $R^{d}\left(s^{t-1}\right)$. Given the limited liability of intermediaries, if they are unable to pay $R^{d}\left(s^{t-1}\right)$ on deposits, they pay a smaller return on deposits which ensures they break-even. The rest is covered by the deposit insurance.

The main policy instrument is the price of government bonds on the primary market, $p\left(s^{t-1}\right)$. The government satisfies any demand for bonds given this price. The key decision from the government's perspective is to choose the bond price $p\left(s^{t-1}\right)$ that maximizes the welfare of the households in the decentralized economy.

2.5 Market clearing

There are eight market clearing conditions. The *labour market clearing conditions* state that labour demanded by financial intermediaries and nonfinancial firms equals labour supplied by households:

$$\begin{aligned}(1-\pi_m)\,l\left(s^{t-1}\right) &= 1-\pi_m\\ \pi_m l_m\left(s^{t-1}\right) &= \pi_m\end{aligned}$$

The *goods market clearing condition* equates total output produced with aggregate consumption and investment. Output produced by nonfinancial firms is $\pi_m q_m\left(s^{t}\right)\left(k_m\left(s^{t-1}\right)\right)^{\theta}$, while output produced by financial firms is $(1-\pi_m)\sum_{j\in\{l,h\}}\pi_j q_j\left(s^{t}\right)\left(k_j\left(s^{t-1}\right)\right)^{\theta}$, where

that the deposit insurance can not be made contingent on the portfolio decisions of the intermediaries.

[19] We concentrate on new issuance of bonds only and abstract from outstanding bonds for computational reasons. Considering the valuation effects of current policy in the presence of outstanding bonds may be an interesting extension of the model.

$k_j\left(s^{t-1}\right)$ are resources allocated to the risky projects after repo market trading.

$$\begin{aligned} C\left(s^t\right)+M\left(s^t\right)+D_h\left(s^t\right)+Z\left(s^t\right) &= \pi_m q_m\left(s_t\right)\left[\left(k_m\left(s^{t-1}\right)\right)^\theta+(1-\delta)\,k_m\left(s^{t-1}\right)\right] \\ &\quad +\left(1-\pi_m\right)\sum_{j\in\{l,h\}}\pi_j q_j\left(s_t\right)\left[\left(k_j\left(s^{t-1}\right)\right)^\theta+(1-\delta)\,k_j\left(s^{t-1}\right)\right] \end{aligned}$$

Financial markets clearing conditions ensure that the deposit markets, equity markets and bond markets clear. Deposits demanded by financial intermediaries equal deposits from the households and the government:

$$D_h\left(s^{t-1}\right)+D_g\left(s^{t-1}\right)=D\left(s^{t-1}\right)=\left(1-\pi_m\right)d\left(s^{t-1}\right)$$

In the primary bond market, total bond sales by the government equal the bond purchases by financial intermediaries.

$$B\left(s^{t-1}\right)=\left(1-\pi_m\right)b\left(s^{t-1}\right)$$

In the repo market, trades between the different types of intermediaries must balance.

$$\sum_{j\in\{l,h\}}\pi_j\tilde{b}_j\left(s^{t-1}\right)=0 \tag{4}$$

Total equity invested by households in the financial and nonfinancial sectors are distributed over the firms.

$$\begin{aligned} M\left(s^{t-1}\right) &= \pi_m k_m\left(s^{t-1}\right) \\ Z\left(s^{t-1}\right) &= \left(1-\pi_m\right)z\left(s^{t-1}\right) \end{aligned}$$

2.6 Social Planner Problem

We consider the following social planner's problem as a reference point for our decentralized economy. For ease of comparison between the two environments, we abuse language and refer to the existence of financial and nonfinancial sectors even in the context of the social planner's problem. At the beginning of period t, the aggregate state, s_t, is revealed and production takes place using capital that the social planner has allocated to the different technologies of production: $k_m(s^{t-1})$ for the nonfinancial sector, $k_h(s^{t-1})$ and $k_l(s^{t-1})$ for the high-risk and low-risk technologies of the financial sector. The resulting wealth is then split between consumption and capital to be used in production at $t+1$. At the time of this decision, the social planner does not distinguish between the high-risk and low-risk technologies of the financial sector used in production next period, and simply allocates resources, $k_b(s^t)$, to both of them. Once their type is revealed, the social planner can reallocate resources between the two technologies, at a cost.

The social planner solves:

$$\max E \sum_{t=0}^{\infty} \beta^t \log C\left(s^t\right)$$

$$\begin{aligned}
\text{subject to} \quad &: \\
C\left(s^t\right) + \pi_m k_m\left(s^t\right) + \left(1-\pi_m\right) k_b\left(s^t\right) &= \pi_m q_m\left(s_t\right)\left[\left(k_m\left(s^{t-1}\right)\right)^{\theta} + (1-\delta) k_m\left(s^{t-1}\right)\right] \\
&\quad + \left(1-\pi_m\right)\pi_l q_l\left(s_t\right)\left[\left(k_l\left(s^{t-1}\right)\right)^{\theta} + (1-\delta)\left(k_l\left(s^{t-1}\right)\right)\right] \\
&\quad + \left(1-\pi_m\right)\pi_h q_h\left(s_t\right)\left[\left(k_h\left(s^{t-1}\right)\right)^{\theta} + (1-\delta) k_h\left(s^{t-1}\right)\right] \\
k_l\left(s^t\right) &= k_b\left(s^t\right) - \iota_n\left(s^t\right)\tau n\left(s^t\right) - \frac{\pi_h}{\pi_l} n\left(s^t\right) \\
k_h\left(s^t\right) &= k_b\left(s^t\right) - \iota_n\left(s^t\right)\tau n\left(s^t\right) + n\left(s^t\right)
\end{aligned}$$

where $n(s^t)$ is the amount of resources given to (or taken from) each high-risk production technology. To achieve this reallocation, $\frac{\pi_h}{\pi_l} n(s^t)$ resources need to be taken from (or given to) each low-risk technology. In addition, τ is a proportional cost of reallocating resources and is identical to that of issuing bonds in the competitive equilibrium (for details, see Appendix A.2). Reallocation costs are split equally over all technologies of production in the financial sector, which is facilitate by the indicator function $\iota_n(s^t) = \begin{cases} 1 & \text{if} \quad n(s^t) \geq 0 \\ -1 & \text{if} \quad n(s^t) < 0 \end{cases}$.

From a social planner's perspective, it is optimal for resources to flow to high-risk intermediaries during expansion periods and to low-risk intermediaries during contractions.[20] To induce these reallocation flows in the decentralized economy, bond prices, $p(s^t)$, need to be appropriately chosen by the monetary authority (see results Section 4 for details).

2.7 Competitive Equilibrium Properties

In this section, we discuss equilibrium properties of our model and present results on the relationship between equilibrium bond prices and the return to deposits. In addition, we define what we mean by risk taking behavior of financial intermediaries and provide intuition for how interest rate changes affect risk taking.

2.7.1 Constrained and Unconstrained Equilibria

Financial intermediaries maximize expected returns to equity, but benefit from limited liability. When a bad aggregate shock has occurred, intermediaries of type j who are unable to pay the promised rate of return to depositors declare bankruptcy. Equity holders receive no return on their investments, while the returns to depositors are covered by deposit insurance. Limited liability introduces an asymmetry in that it allows the high-risk intermediary to make investment decisions that bring large profits in good times, while being shielded from losses in bad times. In our numerical experiments, only the high-risk intermediaries go bankrupt.

For a given policy, $p(s^t)$, multiple equilibria exist. A common situation is the coexistence of an equilibrium with positive government bond holdings and one with zero bond holdings. We focus our analysis on the former, since trading in the repo market is always desirable given a sufficiently low cost of issuing bonds. Furthermore, equilibria can be of two types. When financial intermediaries choose to pledge only a fraction of bonds as collateral in the repo market, i.e. $\tilde{b}_j(s^t) < b(s^t)$, we refer to equilibria as having an *unconstrained repo*

[20] For very high values of τ, there is no reallocation in the social planner problem and no bonds issued in the competitive equilibrium.

market. Equilibria with a *constrained repo market* are ones in which either high-risk or low-risk intermediaries pledge all their bond holdings as collateral. When the interest rate policy is chosen optimally, the equilibrium has a constrained repo market. The intuition is that optimal policy aims to restrict risk taking of high-risk financial intermediaries, who otherwise may take advantage of their limited liability and overinvest in risky projects. An effective way to restrict risk taking and potential bankruptcy is to limit the amount of bonds, so that collateral for future trading in the repo market is scarce.

Due to the limited liability of financial intermediaries and the possibility of a constrained repo market, we need to employ non-linear techniques to solve our model. We use a collocation method with occasionally binding non-linear constraints (for details, see Appendix $A.3$).

2.7.2 Bond Prices and the Return to Deposits

Proposition 1 *Consider an economy with positive government bond holdings. In the absence of capital regulation or if this regulation does not bind, the equilibrium bond prices and the return to deposits satisfy:* $p\left(s^{t-1}\right)=\tilde{p}\left(s^{t-1}\right)$ *and* $R^{d}\left(s^{t-1}\right)\geq\frac{1}{p\left(s^{t-1}\right)}$. *The last inequality is strict in the case of a constrained repo market. Moreover, in an equilibrium with binding capital regulation, bond prices and the return to deposits are such that:* $p\left(s^{t-1}\right)>\tilde{p}\left(s^{t-1}\right)$ *and* $R^{d}\left(s^{t-1}\right)\geq\frac{1}{p\left(s^{t-1}\right)}$.

Proof. These results follow from the first order conditions of the financial intermediaries' problems. Appendix A.4 outlines the proof. ■

The intuition for these results is as follows. In the absence of capital regulation, there are no frictions in the model that would make primary and secondary bond prices different. When capital regulation binds, intermediaries are required to hold a minimum share of safe assets, and they are only willing to acquire additional bonds in the repo market if the price is lower than in the primary market.

In addition, returns to deposits are weakly greater than returns to bonds, since otherwise there would be a profit opportunity. Namely, an intermediary would have incentives to pay a slightly higher deposit return to attract additional deposits and be able to invest more into bonds. The result $R^d\left(s^{t-1}\right) \geq \frac{1}{p(s^{t-1})}$ can also be interpreted in terms of the option value provided by bonds in this economy. Bonds have value to intermediaries because they can be retraded on the repo market. Whenever some intermediaries are constrained in the amount of collateral they hold, bonds carry a discount: $R^d\left(s^{t-1}\right) > \frac{1}{p(s^{t-1})}$. However, in an unconstrained equilibrium, both high-risk and low-risk intermediaries have sufficient bonds and the option value of bonds is zero: $R^d\left(s^{t-1}\right) = \frac{1}{p(s^{t-1})}$.

Proposition (1) is important for two reasons. First, it shows that interest rate policy has a direct effect on the repo market. In fact, the close relationship we obtain between policy, $p\left(s^{t-1}\right)$, and the repo rate, $\tilde{p}\left(s^{t-1}\right)$, is supported by U.S. evidence, as shown in Bech, Klee, and Stebunovs (2010). Second, the return to depositors is bounded below by the implicit interest rate of government bonds. Thus, the interest rate policy not only affects the choices financial intermediaries make, but also affects the investment choices of households. In quantitative experiments, we find the latter effect to be weaker than the former.

2.7.3 Risk Taking: Measurement and Impact of Policy

We use our model to assess whether and how interest rate policy influences risk taking of intermediaries. To this end, we make the notion of risk taking precise. We define risk taking as the percentage deviation in resources invested in the high-risk projects in a competitive equilibrium relative to the social planner. Formally,

$$r\left(s^{t-1}\right) = \frac{k_h^{CE}\left(s^{t-1}\right) - k_h^{SP}\left(s^{t-1}\right)}{k_h^{SP}\left(s^{t-1}\right)} \tag{5}$$

where superscripts $\{CE, SP\}$ denote whether the variable is part of the solution to the competitive equilibrium for a given interest rate policy or part of the social planner's problem. Here, $k_h^{SP}(s^t) = k^{SP}(s^t) + \left(1 - \iota_n^{SP}(s^t)\,\tau\right) n^{SP}(s^t)$ is the capital that the social planner invests in the high-risk technology and $k_h^{CE}(s^{t-1}) \equiv k^{CE}(s^{t-1}) + \tilde{p}^{CE}(s^{t-1})\,\tilde{b}_h^{CE}(s^{t-1})$ is the capital invested in the high-risk projects in the competitive equilibrium.

A positive value of $r(s^{t-1})$ in equation (5) tells us that there is excessive risk taking in the competitive equilibrium, while a negative value indicates too little risk taking. In numerical results, we plot the cyclical behaviour of risk taking, but also report an aggregate measure defined as an average over expansions and contractions, $r \equiv E\left[r(s^{t-1})\right]$.

In what follows, we provide some intuition on how interest rate changes affect risk taking during an expansion or a contraction. For illustration purposes, we consider a static, partial equilibrium setting of the financial intermediation sector in our model. The bond prices are exogenously fixed and the aggregate shock is either high $(\overline{s})$ or low $(\underline{s})$. We examine the portfolio choices of intermediaries in the primary market and the repo market.

When the economy is in an expansion, resources are optimally redistributed from the low-risk intermediary to the more productive high-risk intermediary. Figure 1 illustrates the impact that lower returns to safe bonds have on investments in risky projects. Purchases of bonds in the *primary market* are negatively related to bond returns, which means that *all* intermediaries invest more capital into risky projects at low interest rates. Then, in an expansion, high-risk intermediaries use the repo market to lower their holdings of bonds and invest extra resources in their risky projects (as illustrated by the fact that the dotted line is below the solid line). In Figure 1, the squares to the right of the kink on the dotted line mark equilibria in which the high-risk intermediaries are unconstrained in the repo market. In these equilibria, they collateralize only a subset of their bond holdings in order to borrow on the repo market. Then, as the return to bonds decreases—say, from 1.08 to 1.06 in the figure—high-risk intermediaries allocate more resources to risky projects. While the following result is not visible from our illustration, we note that, in our full model, such an increase in high-risk investments exceeds the social optimum. Hence, risk taking goes up as safe returns decline, whenever intermediaries are unconstrained in their repo activities.

In addition, in an expansion, intermediaries may be constrained in their repo market transactions, if they purchased few bonds in the primary market. In Figure 1, constrained equilibria are marked by the squares to the left of the kink on the dotted line. In this example, if the return to bonds decreases—say from 1.03 to 1.02 in the figure—reallocation between intermediaries is restricted due to scarce collateral. In the full model, this leads to a reduction in risk taking relative to the social optimum.

In contrast, when the economy is in a contraction, resources are optimally distributed from the high-risk intermediary to the low-risk intermediary. As before, lower rates on safe assets push more capital into risky projects in the *primary market*. In the *repo market*, in an *unconstrained equilibrium*, the low-risk intermediaries receive extra resources and risk taking reduces. However, in a *constrained repo market* equilibrium, due to fewer bond purchases in the primary market, there is limited retrading and less resources are given from the high-risk to the low-risk intermediary, thus increasing risk taking.

Empirically, expansion periods are longer than contractions. Our calibrated model is consistent with this fact. This means that, in our benchmark model with a constrained repo market, lowering interest rates leads to less risk taking, on average, relative to the social planner problem. The opposite is true in our benchmark model with an unconstrained repo market.

3 Calibration

This section outlines our approach for determining the various parameters of the model and describes the data we use. We calibrate the following parameters: β, θ, τ, the aggregate shock transition matrix Φ, and π_h. We determine π_m, α, δ, $q_h(\overline{s})$, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$ using a minimum distance estimator. All parameter values are summarized in Tables 1 and 2.

The utility discount factor, β, is calibrated to ensure an annual real interest rate of 4%

in our quarterly model. We obtain $\beta = 0.99$. The capital income share is determined using data from the U.S. National Income and Product Account (NIPA) provided by the Bureau of Economic Analysis (BEA) for the period 1947 to 2009. We find $\theta = 0.29$ for the business sector.[21] The cost of issuing government bonds, τ, is determined from existing literature. Stigum (1983, 1990) reports brokerage fees for U.S. Treasury bills between 0.0013% and 0.008% of the amount issued. Green (2004) reports fees around 0.004%. A higher cost of issuing bonds has negative consequences in our paper, since it reduces welfare and it makes the use of bonds as a medium of exchange less desirable. To stress the robustness of our results, we choose the highest estimate, $\tau = 0.008\%$.

To calibrate the transition matrix for the aggregate state of the economy, we use the Harding and Pagan (2002) approach of identifying peaks and troughs in the real value added of the U.S. business sector, from $1947Q1$ to $2010Q2$.[22] We find 11 contractions with an average duration of 5 quarters. Hence, the probability of switching from a bad realization of the aggregate shock at time $t-1$ to a good realization at time t is $\phi\left(s_t = \overline{s}|s_{t-1} = \underline{s}\right) = 0.20$. Moreover, the probability of switching from an expansion period to a contraction is $\phi\left(s_t = \underline{s}|s_{t-1} = \overline{s}\right) = 0.0553$. The calibrated transition matrix is

$$\Phi = \left[\begin{array}{cc} \phi\left(s_t = \overline{s}|s_{t-1} = \overline{s}\right) & \phi\left(s_t = \underline{s}|s_{t-1} = \overline{s}\right) \\ \phi\left(s_t = \overline{s}|s_{t-1} = \underline{s}\right) & \phi\left(s_t = \underline{s}|s_{t-1} = \underline{s}\right) \end{array}\right] = \left[\begin{array}{cc} 0.9447 & 0.0553 \\ 0.2 & 0.8 \end{array}\right].$$

The idiosyncratic shock in the economy—the type of risky projects financial intermediaries invest in—is assumed to be i.i.d. to retain tractability of the numerical solution. The motivation behind the i.i.d. assumption is that the financial sector in the U.S. economy is complex and the subset of financial intermediaries who are considered the most risky changes

[21] For the corporate business sector—where income is split into capital and labor by the BEA—we find $\theta = 0.29$. For noncorporate businesses which include proprietors, we need to split proprietor's income into capital and labor income in order to compute the capital income share. We attribute 0.788 percent of proprietor's income to labor income and find a capital share for the noncorporate sector of 0.29. While 0.788 might seem high, it is not unreasonable.

[22] The business cycles we identify closely mimic those determined by the NBER.

considerably over time.

For this reason, it is difficult to determine the share of high risk financial intermediaries in the data. In our benchmark model, we set π_h equal to 15%. In the context of the recent financial crisis, one can think of brokers and dealers as a proxy for high-risk intermediaries in the U.S. Under this assumption and using U.S. Flow of Funds data from 2000 to 2007, we find that financial assets of brokers and dealers were, on average 4% of the financial assets of all financial institutions and 20% of the financial assets of depository institutions.[23] Our benchmark value of π_h is between these two estimates. It should be noted that, while the assumption that brokers and dealers are high-risk intermediaries seems reasonable for the recent crisis, the widespread use of off-balance sheet activities among other institutions suggests that this definition may be too narrow. We perform sensitivity analysis with respect to π_h.

Next, we determine the following 9 parameters: the importance of the nonfinancial sector, π_m, the fixed factor in the production function of the financial sector, α, the depreciation rate, δ, and the productivity parameters, $q_h(\overline{s})$, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$. The absolute level of productivity is not important in our model. As a result, we normalize the productivity of the high-risk intermediary in the good aggregate state, $q_h(\overline{s}) = 1$. We estimate the remaining eight parameters using eight data moments described below. Unless otherwise noted, we use quarterly data from $1987Q1$ to $2010Q2$. We focus on this time period because U.S. inflation was low and stable.

1. The first moment we target in our estimation procedure is the share of output produced by the nonfinancial sector. This pins down the value of π_m in our model. We identify our model's total output with the U.S. business sector value added published by the BEA. In addition, we identify the nonfinancial sector in our model with the U.S. corporate nonfinancial sector.[24] We aim to match the average value added share of the corporate nonfinancial

[23] We note that the 20% average masks a large variation, from 18% in early 2000s to 28% in the eve of the recent crisis.

[24] Note that we treat the remainder of the U.S. business sector, namely the corporate financial businesses and the noncorporate businesses, as the model's financial intermediation sector. In U.S. data, noncorporate

sector of 66.9% observed in the U.S. since 1987.

2. The parameter α influences the returns to equity in our model's financial sector, which, in turn, depend on the equity to total assets ratio of the intermediaries. We use the equity to asset ratio for corporate financial businesses as a second data moment to target in our estimation. Using data from the U.S. Flow of Funds from $1994Q1$ to $2010Q2$, we find this ratio to be on average 7.6%. In performing this calculation, we exclude mutual funds.[25] We choose the time period beginning in 1994, because the Basel I capital regulation had been implemented by then.

3. In our model, the depreciation rate is stochastic and is given by:

$$\frac{\pi_m q_{m,t} \delta k_{m,t} + (1-\pi_m)\left(\pi_h q_{h,t} \delta k_{h,t} + \pi_l q_{l,t} \delta k_{l,t}\right)}{\pi_m k_{m,t} + (1-\pi_m)\left(\pi_h k_{h,t} + \pi_l k_{l,t}\right)}$$

We determine the value of δ to ensure that the average depreciation rate in the model matches the data, namely 2.5% per quarter.

4. We target the maximum decline in real output in the business sector, averaged across all contraction periods since 1947. We detrend output by a constant growth trend to make it stationary. Then, using the turning points approach in Harding and Pagan (2002), we find the average decline in output to be 6.48%.

5. We aim to match a coefficient of variation for the U.S. business sector output of 3.75%. We calculate this statistic after removing a linear trend from the logarithm of output.

6. We target a coefficient of variation for U.S. household net worth of 8.17%. To obtain this statistics, we use U.S. Flow of Funds data and detrend the logarithm of household net worth using a polynomial of order three. We focus on net-worth because it is closely related to the state variable $w(s^t)$ in our model.

businesses are strongly dependent on the financial sector for funding. In the past three decades, bank loans and mortgages were 60 to 80 percent of noncorporate businesses' liabilities. For simplicity, we do not model these loans, but rather assume that the financial intermediary is endowed with the technology of production of noncorporate businesses.

[25] The equity to asset ratio of depository institutions only—commercial banks, savings institutions and credit unions—is essentially identical to the ratio computed for the corporate financial sector excluding mutual funds.

7. We aim to match a ratio of household deposits to total financial assets of 17.2%, as observed in U.S. Flow of Funds data.

8. Finally, we aim to match the recovery rate during bankruptcy. We use an estimate provided by Acharya, Bharath, and Srinivasan (2003), which states that, the average recovery rate on corporate bonds in the United States during 1982 to 1999 was 42 cents on the dollar.

We determine all eight parameters jointly using a minimum distance estimator to match the target moments above. Let Ω_i be a model moment and $\tilde{\Omega}_i$ be the corresponding data moment. Our procedure makes use of the problems given in (6) and (7) below. Notice that in (6) we impose restrictions on the ordering of productivity parameters across the different technology types. For our benchmark calibration, we are abstracting from capital adequacy requirement and set $\eta = 0$.

$$\begin{aligned} Q^* &= \arg \min_{\substack{Q=\{q_m(\overline{s}),q_m(\underline{s}),q_l(\overline{s}),\\ q_l(\underline{s}),q_h(\underline{s}),\delta,\alpha,\pi_m\}}} \sum_{i=1}^{8} \left(\frac{\Omega_i - \tilde{\Omega}_i}{\tilde{\Omega}_i} \right)^2 \qquad (6) \\ \text{s.t.} &: \; q_h(\underline{s}) < q_m(\underline{s}) < q_l(\underline{s}) \leq q_l(\overline{s}) < q_m(\overline{s}) \leq q_h(\overline{s}) \text{ and} \\ & \quad \Omega_i \text{ is implied in a competitive equilibrium given policy } p^* \end{aligned}$$

$$\begin{aligned} p^* &= \arg \max_p E \sum_{t=0}^{\infty} \beta^t \log C\left(s^t\right) \qquad (7) \\ \text{s.t.} &: \; \left\{C\left(s^t\right)\right\} \text{ is part of a competitive equilibrium given } Q^* \end{aligned}$$

We start out with a guess Q_1^* and solve the problem in (7) for an optimal policy p^*. Next, we take this optimal policy as given and choose parameters to minimize the distance between our model moments and the corresponding data moments, as shown in (6). This step yields Q_2^*. We continue the procedure till convergence is achieved. The reason for choosing this two-step procedure is because our model is highly nonlinear and the initial guess is very important in finding a competitive equilibrium solution. The guess we start with is the social planner's solution.

The estimated parameters are presented in Tables 2. Notice that despite the assumption that depreciation is stochastic, the model is able to perfectly match the average depreciation observed in the data. Table 3 shows that the model matches the targeted data moments well. Some moments—such as the capital depreciation rate, or the coefficient of variation of output—are matched very well, while others—the recovery rate after bankruptcy, or the deposits to asset ratio for households—are still a bit far from the data. Regarding the recovery rate in bankruptcy, one aspect to keep in mind is that the data target taken from Acharya, Bharath, and Srinivasan (2003) was for corporate bonds only, while the model considers recovery rates for small business bankruptcies. In addition, there is a tight relationship between the model's recovery rate, deposit and equity ratios. The reason for the low recovery rate is a low equity to asset ratio of financial intermediaries and a very strong decline of output during contractions. Given a low recovery rate in bankruptcy, households desire safe assets and choose to hold a relatively high proportion of their wealth in deposits.

4 Results

First, we present results from our benchmark model. Then, we consider a version of our model that allows for issuance of private misrated bonds, and for foreign demand for these bonds. The latter experiment is meant to shed light on some aspects of the recent financial crisis.

4.1 Risk Taking and Welfare in the Benchmark Model

We present most of the results from the competitive equilibrium by contrasting them with the optimal social planner solution. Our first finding is that the social planner allocation cannot be implemented as a competitive equilibrium.

We aim to find prices, including the interest rate policy, that would implement the social planner allocation as a competitive equilibrium in our model with financial and nonfinancial sectors. This would require that, in a bad aggregate state, the returns to deposits and bonds satisfy: $R^d < 1/p$, which violates the competitive equilibrium result derived in Proposition 1. The intuition for our finding is as follows. In a bad aggregate state, it is optimal to shift resources from high-risk to low-risk intermediaries, which are now relatively more productive. Implementing the social planner optimal allocation has two implications for competitive equilibrium prices. First, high-risk intermediaries would need to buy a large value of bonds in the repo market, so as to shift their portfolio away from their risky projects. To provide these incentives, bond returns need to be sufficiently high implying that bond prices need to be sufficiently low in a bad aggregate state. Second, returns to deposits need to be relatively low so that intermediaries can pay back depositors. In combination, prices would have to satisfy $R^d < 1/p$, which contradicts Proposition 1. Therefore, the social planner allocation cannot be implemented, since the moral hazard problem of the high-risk financial intermediaries is so severe, that interest rate policy alone cannot resolve it.

Given that the social planner allocation is not implementable, we find the optimal bond price, $p^*(s^{t-1})$, that maximizes the unconditional welfare of the representative consumer. We solve $(P3)$ numerically, taking the function $p(\cdot)$ from the space of linear spline functions.

$$p^*\left(s^{t-1}\right) = \arg\max_{p(s^{t-1})} E\left[\sum_{t=0}^{\infty} \beta^t \log \tilde{C}\left(s^t\right)\right] \tag{P3}$$

$$\text{subject to: } \tilde{C}\left(s^t\right) \text{ is part of a competitive equilibrium given policy } p\left(s^{t-1}\right)$$

We use two metrics to compare competitive equilibrium results to the social planner allocation. First, we use the risk taking measure defined in Section 2.7.3 to determine whether a particular interest rate policy implies too much or too little risk taking relative to the social planner. In addition, we consider a standard welfare measure. We define the lifetime consumption equivalent (LTCE) as the percentage decrease in the optimal consumption from the social planner allocation required to give the consumer the same welfare as the consumption from the competitive equilibrium with a given interest rate policy.

We conduct four experiments and report welfare and risk taking results from 5000-quarter simulations in Table 4. Unless otherwise noted, we abstract from capital regulation, i.e. we set $\eta = 0$ in equation (3).

Experiment 1: Equilibrium without repo market reallocation. We consider the solution to our benchmark economy when the monetary authority sets a very high primary market bond price. This leads to no purchases of bonds, and, as a result, intermediaries cannot adjust their portfolios in the repo market. The shutdown of the repo market leads to a substantial welfare loss, namely 0.88% in LTCE (see Table 4). Put differently, each time period, the competitive equilibrium consumption is 0.88% lower relative to the social planner. One of the reasons for the welfare loss is the excessive risk taking observed in the competitive equilibrium due to a suboptimal allocation of resources across financial intermediaries. Reallocation of resources via the repo market is optimal and would bring the economy closer to the social planner allocation.

Experiment 2: Optimal interest rate policy, $\left[p^{*}\left(s^{t-1}\right)\right]^{-1}$. Next, we optimize over the policy function in our benchmark economy. We find that at the optimal policy the welfare is very close to the social planner. There is less reallocation compared to what the social planner chooses, and this entails of small welfare loss of 0.04% in LTCE. Yet, even for the best interest rate policy, the risk taking is elevated exceeding the one found in the planner's problem by 23.6%.

Figure 2 presents simulation results for our benchmark model and comparisons to the allocation from the planner's problem. We plot results for a sequence of one hundred random draws of the aggregate shock. As seen in the two bottom subplots, the excessive risk taking in the competitive equilibrium is mainly due to periods with good realizations of the aggregate state, when resources in the repo market are reallocated from the low-risk to the high-risk projects. Risk taking in contractions is lower than in expansions, but still in excess of the social planner optimum.

Figure 2 also shows that, at the optimal policy, returns to safe bonds are high in good times and low in bad times. Here is a brief intuition for these results. Overall, returns to bonds are linked to expected returns to equity through non-arbitrage conditions. In addition, bond returns in a contraction need to be low in absolute terms, so that the returns to deposits are low (recall Proposition 1) and potential bankruptcy costs are minimized.

Another important insight from experiment 2 is that, at the optimal interest rate policy, government transfers to households are positive, on average, due to net revenues from issuing bonds. This is true despite the fact that the government provides deposit insurance at no cost, and it needs to tax households to guarantee deposit returns when the high-risk intermediaries become bankrupt.

Experiment 3: Level shifts in the optimal interest rate policy. We consider uniform upward or downward shifts in interest rates relative to those under the optimal policy. Namely, the schedules of bond returns we consider are: $\left[p^*\left(s^{t-1}\right)\right]^{-1} \pm \psi$, where $p^*(\cdot)$ is the optimal bond price and ψ is a constant, say 0.1 percentage points. We compute the benchmark model's results under these alternate policies.

We examine the extend to which lower than optimal interest rates contribute to increased risk taking of financial intermediaries and to lower welfare. Figure 3 shows our results for a wide range of values of ψ. In both subplots, the x-axis gives deviations from the optimal policy in the competitive equilibrium in percentage points at annual rates. The optimal policy is at the zero mark on the x-axis. The upper subplot of Figure 3 shows the welfare losses in the competitive equilibrium relative to the social planner, for a range of policies. The lower subplot shows risk taking relative to the social planner, for the same policies. We find that small deviations from the optimal policy, say 50 basis points, entails relatively small welfare losses, but sizable changes in risk taking. Notice that, in both figures, the lines have a kink. On the left side of the kink, the economies with the specified policies display a constrained repo market in which either the high-risk or the low-risk intermediaries pledge all bonds as collateral. To the right of the kink, we have economies with unconstrained repo market equilibria.

As conjectured in Section 2.7, the equilibrium with optimal interest rate policy (the square dots in Figure 3) features a constraint secondary market. In proximity to the optimum, an increase in bond returns leads to more risk taking and a decrease in bond returns leads to less risk taking. Notice that when the repo market becomes unconstrained the relationship between variations in bond returns and risk taking changes sign (see bottom panel of Figure 3).

Experiment 4: Capital requirement ratio, $\eta = 0.08$, **and optimal interest rate policy** $\left[p^*\left(s^{t-1};\eta\right)\right]^{-1}$. Finally, we consider the impact of a capital requirement constraint, as given by equation (3). We choose $\eta = 8\%$, which is the current level implemented in the United States in accordance with Basel II, and reoptimize the interest rate policy. We find that risk taking decreases substantially, from 23.6% in our benchmark model without regulation (experiment 2) to 0.25% in the presence of optimal policy and regulation (experiment 4). Moreover, at the optimal policy, $p^*\left(s^{t-1};\eta\right)$, welfare is comparable to the one in experiment 2. We note that, while capital regulation is successful in reducing risk taking, changes in policy in the presence of regulation are more costly in welfare terms. For example, small upward deviations from the optimal policy can lead to huge welfare losses in the order of 1% LTCE.

Implications of Variation in the Share of High-Risk Intermediaries Up to now, we reported experiments for $\pi_h = 0.15$. Next, we report some of the main implications of our model for a smaller and a larger fraction of high-risk intermediaries, $\pi_h \in \{0.13, 0.17\}$. The results from this sensitivity analysis are reported in Table 5. While the quantitative results change, we find that the qualitative results remain intact.

A first interesting result is that potential welfare gains from having a repo market are increasing with the share of high-risk intermediaries. This result is quite intuitive since the gains from reallocation arise due to the presence of high-risk intermediaries. A larger fraction of high-risk intermediaries increases the benefits of resource reallocation as the economic state varies.

Focusing on the results concerning the respective second best policy, we find that welfare losses and risk taking under the second best policy are relatively insensitive to a variation in π_h. Also the relationship between π_h and the amount of risk taking is non-linear.

Regarding variations of the interest rate around the second best, we find that lower rates lead to less risk taking regardless of the value of π_h and that indeed for the lowest considered value of π_h an upward deviation in the policy rate can become quite costly (-0.44% $LTCE$), relative to a downward deviation of similar magnitude (-0.0536% $LTCE$).

4.2 Model with Rating Agencies, Private Bonds and Foreign Investment

In this section, we modify the benchmark model to allow for some features that were prominent in the run-up to the late 2000s crisis. Our goal is to shed light on how these features, in interaction with the interest rate policy, affect risk taking of intermediaries.

In our augmented model, financial intermediaries can issue private bonds during an expansion period, and sell these bonds to other financial intermediaries, or to foreign investors. In addition, credit rating agencies "stamp" the private bonds as being safe, in exchange for a cost which is proportional to the value of the bonds.[26] Once stamped, private bonds appear to be as safe as government bonds and are traded at the same bond price $\tilde{p}$.[27] During an expansion, the private bonds are fully repaid every period by their issuers. However, when a bad aggregate state realizes, the high-risk financial intermediaries default on their private bonds. In this case, the government bails out domestic bond holders by fully guaranteeing

[26] Relative to the benchmark model, the only crucial assumption is the presence of rating agencies that misrepresent the riskiness of private bonds.

[27] We could instead allow for some yield spread between the returns to private and government bonds. A fixed spread is not going to change our qualitative results. An endogenous risk spread is beyond the scope of this paper.

their returns on the stamped private bonds. This public guarantee justifies our assumption that domestic bond purchasers are indifferent between government and private bonds. In contrast, foreign investors are surprised to learn that their allegedly safe bonds return only 90 percent of what was due. Thus, we assume that the rating agencies give foreigners the erroneous impression that the returns to private bonds are safe. The foreign investors are forced to take a 10 percent haircut on their bond values. In numerical experiments, we vary the size of the haircut and find that our risk taking results are robust.

We introduce these new features into our benchmark model, since they are believed to be at the root of the late-2000s financial crisis (for model details see Appendix A.5). It has been often argued that, credit rating agencies contributed to the propagation of asset mispricing by giving top ratings to derivative securities, which should have been assigned to a much riskier category. Moreover, the demand for top rated assets from domestic pension funds and foreign wealth funds was fueling the incentive to overlook risks and devise complex derivative securities, which would appear to be much safer than their underlying assets. For background, see Taylor (2009), Krishnamurthy, Nagel, and Orlov (2011) and Hoerdahl and King (2008).

In our extension, foreign demand for domestic bonds contributes to relaxing the collateral constraints faced by financial intermediaries.[28] Specifically, we assume that in any given period during an expansion phase, when financial intermediaries buy government bonds, they are not completely sure whether foreign investors will be willing to buy domestic private bonds. In the model, the existence or absence of foreign demand is revealed after domestic intermediaries trade government bonds among themselves in the domestic repo market. At this point, with probability π_F the foreigners are willing to buy domestic private bonds, in which case financial intermediaries who want to borrow more capital issue their own bonds, in the amount a_j for $j \in \{h, l\}$. Before they can sell these bonds, however, they must receive the approval stamp from the rating agencies. This stamp is given at the real cost of ξa_j

[28] Using Flow of Funds data, it can be shown that the rest of the world holds a significant portion of US securities: Treasury and agency backed securities, commercial paper and corporate bonds.

($\xi = 0.01$), which is due after the production takes place. Once the bonds are stamped, they can be sold at the same price as the government bonds, $\tilde{p}$.

Alternatively, with probability $(1 - \pi_F)$ the foreigners do not want to buy domestic bonds, in which case there are no private bonds issued and, at least some intermediaries, are constrained in their repo market bond sales. If the probability $(1 - \pi_F)$ is large enough, the intermediaries still want to buy government bonds from the primary bond market, to facilitate their repo transactions. In the simulations, we set π_F at 0.1 percent. Lastly, the government uses lump-sum taxes on the households to guarantee full returns to domestic private bond holders and partial returns to foreigners in case of a bad shock.

Implications of our model with rating agencies

We consider a very particular simulation of our model extension, in which there is positive foreign demand during every expansion period. The results from this simulation are meant to capture a rare event. Figure 4 plots 100 periods from this simulation. The foreign demand allows financial intermediaries to relax their collateral constraint. As a result, we find higher levels of excessive risk in comparison to the benchmark economy simulation, shown in Figure 2.

We also report results from simulations that have 24 quarters of expansion and 6 quarters of contraction, similar to the U.S. business cycle dated by the NBER to have lasted from November 2001 to June 2009. We consider the very low likelihood event that, foreign demand is positive in every period of the expansion phase. These results are not thought of as representing a regular functioning financial market, but are meant to capture extreme circumstances, such as those recently experienced in the United States. Our results are shown in Figure 5.

The red dashed curve in the top panel of Figure 5 shows how welfare losses over the simulated business cycle vary with uniform shifts away from the optimal interest rate policy in the benchmark model. The welfare losses are measured in consumption equivalent units relative to the social optimum simulated over the same 30 quarter cycle. In particular, we measure the percentage decrease in the optimal consumption from the social planner allocation necessary to give the consumer the same welfare as in the competitive equilibrium with private bonds and rating agencies.

Figure 5 also shows welfare losses from our benchmark model (without mispriced bonds) over the same 30 quarter cycle. As seen in the figure, depending on the interest rate policy, the 30-quarter cycle simulation of our benchmark economy may give higher welfare relative to the social planner. This is due to the fact that the simulated cycle has a much longer than average expansion phase, during which intermediaries in the competitive equilibrium invest in profitable risky projects.

To summarize Figure 5, in the presence of erroneous ratings of bonds, welfare losses are much larger than before. Equivalently, the welfare costs of relaxing the collateral constraint of intermediaries are quite large. We conclude that the collateral channel in our benchmark model is an important tool for restricting risk taking of financial intermediaries.

The bottom panel of Figure 5 shows the risk taking implications of interest rate variations in our benchmark model and the extension. The difference is remarkable. With erroneous ratings of bonds, lower policy rates unambiguously increase risk taking. The high-risk intermediaries are no longer constrained in their risk taking, even if they buy few government bonds in the primary market. In our model extension, the costs associated with excessive risk taking are borne by foreign investors and the government.

Our results suggest that lower than optimal policy rates can indeed contribute to more risk taking, provided there are some other sources of inefficiency in financial markets, which lead participants to misprice risk.[29]

[29] A binding capital adequacy requirement does not change these results, since this constraint mainly affects the balance sheet in the primary market.

5 Conclusion

The recent financial crisis has stirred interest in the relationship between lower than optimal interest rates and the risk taking behavior of financial institutions. We examine this relationship in a dynamic general equilibrium model that features deposit insurance, limited liability of financial intermediaries, as well as heterogeneity in the riskiness of intermediaries' portfolios.

There are two channels through which interest rate policy influences risk taking in our model. The *portfolio channel* illustrates the idea that lower than optimal policy rates reduce the returns to safe assets and lead intermediaries to shift investments towards riskier assets. In turn, given fewer bond purchases in the primary market, intermediaries have less collateral available for repo market transactions. Hence, *the collateral channel* constrains the ability of intermediaries to take on more risk through the repo market, after they receive further information regarding the riskiness of their projects. We calibrate our model to U.S. data, and show that, our decentralized economy with optimal interest rate policy has welfare that is very close, though below, the social optimum and features excessive risk taking. While both risk taking channels lead to important changes in the intermediaries' portfolios, we find that, for small variations around the optimal policy, the collateral channel dominates quantitatively. Thus, lower than optimal interest rates lead to less risk taking.

We also consider the implications of allowing for issuance of private bonds by intermediaries, which are misrepresented as being safe by rating agencies. In this version of the model, we find higher levels of excessive risk taking, and substantial welfare losses at low interest rates.

There are different potential extensions to our work. First, our paper focuses on a time consistent interest rate policy. It may be worthwhile to evaluate the consequences of a departure from this assumption. In addition, while we consider the implications of state independent capital requirements, we do not aim to characterize the features of an optimal capital regulation. The capital regulation in our model is successful in reducing risk taking, but also reduces welfare. A more flexible regulation has potential to reduce risk taking in the economy, while delivering higher welfare relative to an equilibrium with optimal interest rate policy alone. Thus, it seems worthwhile to consider the joint determination of optimal capital regulation and optimal interest rate policy.

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A Appendix, Not for Publication

A.1 Timing of Model Events

Let $s_t \in \{\overline{s}, \underline{s}\}$ be the aggregate shock at time t. Let $s^t = (s_1, s_2, ...s_t)$ be the history of the aggregate shock up to time period t. Note that $s^t = (s^{t-1}, s_t)$. The timing of the economy is as follows.

- Each nonfinancial firm enters period t with equity $M(s^{t-1})/\pi_m$, capital $k_m(s^{t-1})$ and labour $l_m(s^{t-1})$. Each financial intermediary enters period t with $\tilde{b}_j(s^{t-1})$ safe assets, $k_j(s^{t-1})$ risky assets, $d(s^{t-1})$ deposits, equity $z(s^{t-1})$ and labour $l(s^{t-1})$.
- At the beginning of period t, the aggregate shock s_t realizes and financial and nonfinancial firms find out their current productivity shocks: $q_j(s_t)$ for intermediaries of type $j \in \{h, l\}$ and $q_m(s_t)$ for nonfinancial firms. All firms produce output using the capital that has been allocated to production at the end of period $t-1$.
- Nonfinancial firms pay wage income $W_m(s^t) l_m(s^{t-1})$ and equity returns $R^m(s^t) k_m(s^{t-1})$.
- Financial intermediaries pay state contingent returns to labour and may declare bankruptcy, if they are unable to pay the return on deposits, $R^d(s^{t-1}) d(s^{t-1})$. Notice that bankruptcy occurs after the intermediaries know the riskiness of their projects j and the current aggregate shock s_t has realized. Bankrupt intermediaries are liquidated. Their equity holders receive no equity returns and the government steps in to guarantee the rate of return on deposits.
- The government uses lump-sum taxes or transfers $T(s^t)$ to cover expenses and to balance its budget.
- Household wealth, $w(s^t)$, is realized. Households use current wealth to purchase consumption and make investments that will pay off next period: $M(s^t), Z(s^t)$ and $D_h(s^t)$ and supply labour inelastically to financial intermediaries and nonfinancial firms.
- Each nonfinancial firms receives equity $M(s^t)/(1-\pi_m)$.

- Financial intermediaries receive deposits $d\left(s^{t}\right)$ and equity $z\left(s^{t}\right)$.

- At the end of period t, financial intermediaries allocate the resources received from household into bonds and new risky projects. Subsequently, they find out the type of risky project $j \in \{h, l\}$ they invested in and trade repurchasing agreements on government bonds in the secondary bond market. The resulting investments into the risky projects pay returns at the beginning of period $t+1$, after shock s_{t+1} realizes.

A.2 Cost of Reallocation in the Social Planner Problem

We illustrate why the cost of issuing bonds in the competitive equilibrium equals the cost of reallocating resources in the social planner's problem.

Consider the following relationships between variables in the two frameworks (for clarity, we add subscript 'SP' or 'CE' to variables that exist in both): $C^{SP}\left(s^{t}\right) = C^{CE}\left(s^{t}\right)$, $k_{m}^{SP}\left(s^{t}\right) = k_{m}^{CE}\left(s^{t}\right) = \frac{M\left(s^{t}\right)}{\pi_{m}}$, $k_{b}^{SP}\left(s^{t}\right) = \frac{D_{h}\left(s^{t}\right)+Z\left(s^{t}\right)}{1-\pi_{m}}$, $k_{l}^{SP}\left(s^{t}\right) = k_{l}^{CE}\left(s^{t}\right)$, $k_{h}^{SP}\left(s^{t}\right) = k_{h}^{CE}\left(s^{t}\right)$. Then, the resource constraints in the two frameworks are identical. It remains to derive the expressions for $k_{l}^{SP}\left(s^{t}\right)$ and $k_{h}^{SP}\left(s^{t}\right)$ and show they include cost τ.

(i). First, we show that $k_{b}^{SP}\left(s^{t}\right) = k^{CE}\left(s^{t}\right) + \tau p\left(s^{t}\right) b\left(s^{t}\right)$. Using the deposit and equity market clearing conditions from the competitive equilibrium, we find (8):

$$\begin{aligned} D_{h}\left(s^{t-1}\right) + D_{g}\left(s^{t-1}\right) + Z\left(s^{t-1}\right) &= \left(1-\pi_{m}\right)\left[d\left(s^{t-1}\right) + z\left(s^{t-1}\right)\right] &(8) \\ D_{g}\left(s^{t-1}\right) = (1-\tau)\, p\left(s^{t-1}\right) B\left(s^{t-1}\right) &= (1-\tau)\, p\left(s^{t-1}\right) \cdot \left(1-\pi_{m}\right) b\left(s^{t-1}\right) &(9) \\ z\left(s^{t-1}\right) + d\left(s^{t-1}\right) &= k^{CE}\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right) &(10) \end{aligned}$$

Combine (8) with the expression for government deposits in (9) and the balance sheet of financial intermediaries in (10) to get: $k_{b}^{SP}\left(s^{t-1}\right) = \frac{D_{h}\left(s^{t-1}\right)+Z\left(s^{t-1}\right)}{1-\pi_{m}} = k^{CE}\left(s^{t-1}\right) + \tau p\left(s^{t-1}\right) b\left(s^{t-1}\right)$.

(ii). Next, we derive the expressions for $k_l^{SP}(s^t)$ and $k_h^{SP}(s^t)$.

$$\begin{aligned} k_h^{SP}\left(s^t\right) &= k_h^{CE}\left(s^t\right) \\ &= k^{CE}\left(s^t\right)+\tilde{p}\left(s^t\right)\tilde{b}_h\left(s^t\right)=k_b^{SP}\left(s^t\right)-\tau p\left(s^t\right)b\left(s^t\right)+\tilde{p}\left(s^t\right)\tilde{b}_h\left(s^t\right) \qquad (11) \\ k_l^{SP}\left(s^t\right) &= k_l^{CE}\left(s^t\right) \\ &= k^{CE}\left(s^t\right)+\tilde{p}\left(s^t\right)\tilde{b}_l\left(s^t\right)=k_b^{SP}\left(s^t\right)-\tau p\left(s^t\right)b\left(s^t\right)+\tilde{p}\left(s^t\right)\tilde{b}_l\left(s^t\right) \qquad (12) \end{aligned}$$

For equilibria with constrained repo markets, we have shown that $p(s^t) = \tilde{p}(s^t)$. In addition, when the high risk intermediary sells all bonds in the primary market (during booms), we have $\tilde{b}_h(s^t) = b(s^t)$ and $\tilde{b}_l(s^t) = -\frac{\pi_h}{\pi_l}\tilde{b}_h(s^t) = -\frac{\pi_h}{\pi_l}b(s^t)$. Then (11) and (12) become $k_l^{SP}(s^t) = k_b^{SP}(s^t) - \iota_n(s^t)\tau n(s^t) - \frac{\pi_h}{\pi_l}n(s^t)$ and $k_h^{SP}(s^t) = k_b^{SP}(s^t) - \iota_n(s^t)\tau n(s^t) + n(s^t)$, just like in the social planner's problem. Resources reallocated are $n(s^t) = p(s^t)\tilde{b}_h(s^t) = p(s^t)b(s^t) > 0$. Hence, reallocation costs in the two frameworks are identical: $\tau n(s^t) = \tau p(s^t)b(s^t)$. The planner reallocation during recessions is governed by the same relationships above, except that now $n(s^t) < 0$ and $\iota_n(s^t) = -1$. This means that resources $-n(s^t)$ are taken away from the high-risk technology and allocated to the low-risk technology, while the cost of reallocation is proportional, as before.

A.3 Computation of Equilibrium

We compute a recursive formulation of the model, where the state variables at each time period are the aggregate state, s_t, and the household wealth, w_t. Our strategy is to solve for consumption and capital stocks as functions of the state variables using a collocation method with linear spline functions.

We separate the household problem into two parts: a portfolio choice problem and an intertemporal problem. The household's portfolio choice allocates resources to the nonfinancial and financial sectors to equate expected returns of investing in these sectors. Then, given the overall resources allocated to the financial sector, the split between equity and deposits is determined to equalize expected returns from the two types of investments (for details, see Carroll (2011), Section 7 on multiple control variables).

There are two main challenges when solving the financial sector problem. First, some financial intermediaries may be constrained in their secondary market trades and, second, financial intermediaries may go bankrupt when the aggregate state is realized. We consider all the possible combinations in sequence and verify which is an equilibrium. For example, an ex-ante assumption that we may make is that when the bad aggregate state occurs, high risk intermediaries are constrained in their secondary market trade and go bankrupt, while the low risk intermediaries are unconstraint and do not go bankrupt. After solving the financial intermediaries' problems, we check whether the ex-post outcome is consistent with the assumed ex-ante behavior.

A.4 Sketch of Proof for Proposition 1

To simplify notation in our derivations, we use subscripts as a short hand notation for the entire history, s^{t-1}. For example, $\tilde{b}_{j,t-1} \equiv \tilde{b}_j\left(s^{t-1}\right)$ and $b_{t-1} \equiv b\left(s^{t-1}\right)$.

Deriving the relationship between bond prices and the return to deposits in our model involves studying three possible outcomes on the secondary bond market. Transactions of bonds either satisfy: (i) $\tilde{b}_{j,t-1} < b_{t-1}$ for both $j \in \{h, l\}$ or (ii) $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$ or (iii) $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$. Here, we sketch the proof of Proposition 1 for case (ii). The proof is obtained in an analogous fashion for cases (i) and (iii) and is omitted here for brevity.[30]

In case (ii), the high-risk intermediary increases the amount of resources allocated to risky investments by selling all bond holdings in the secondary bond market.

[30] The full derivation is available upon request from the authors.

Step 1: Some Key Relationships

In finding and characterizing the equilibrium, it is useful to define the share of resources a financial intermediary retains for risky investment in the primary market, call it x_{t-1}. Then,

$$\begin{aligned} k_{t-1} &= x_{t-1}\left(z_{t-1}+d_{t-1}\right) &(13)\\ b_{t-1} &= \frac{1-x_{t-1}}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right) &(14) \end{aligned}$$

where the second equation was obtained from equation (1).

For the case presented here, high-risk intermediaries use all their bonds as collateral in the secondary market, while low-risk intermediaries give resources against this collateral. We have:

$$\begin{aligned} \tilde{b}_{h,t-1} &= b_{t-1}=\frac{1-x_{t-1}}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right) &(15)\\ \tilde{b}_{l,t-1} &= -\frac{\pi_h}{\pi_l}b_{t-1}=-\frac{\pi_h}{\pi_l}\frac{1-x_{t-1}}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right) &(16) \end{aligned}$$

Lastly, using equations $(13)-(16)$, the resources allocated to risky investments by high-risk and low-risk intermediaries after the secondary market trades are given by (17) and (18).

$$k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{h,t-1}=\left[x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right) \tag{17}$$

$$k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{l,t-1}=\left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right) \tag{18}$$

Step 2: Equilibrium Conditions for the Financial Sector

In what follows, we make use of the equilibrium result $l_{t-1} = 1$.

We rewrite the secondary market problem given in $(P2)$ as below:

$$\max_{\tilde{b}_{j,t-1}} \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \left(\begin{array}{l} q_{j,t}\left[\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta}+(1-\delta)\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)\right] \\ +\left(b_{t-1}-\tilde{b}_{j,t-1}\right)-R^d_{t-1}d_{t-1}-W_{j,t} \end{array} \right)$$

where $\tilde{b}_{j,t-1} \in \left[-\frac{k_{t-1}}{\tilde{p}_{t-1}}, b_{t-1}\right]$ and $1_{j,t}$ is an indicator function given by $1_{j,t} \equiv \begin{cases} 1 \text{ if } V_{j,t} > 0 \\ 0 \text{ otherwise} \end{cases}$.

The first order conditions with respect to bond trades, $\tilde{b}_{h,t-1}$ and $\tilde{b}_{l,t-1}$, are given by:[31]

$$\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \left\{ q_{j,t}\tilde{p}_{t-1}\left[\theta\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta-1}+1-\delta\right]-1\right\} - \mu_{j,t-1} = 0 \tag{19}$$

where $\mu_{j,t-1}$ for $j \in \{h,l\}$ are the Lagrange multipliers on the constraints $\tilde{b}_{j,t-1} \leq b_{t-1}$ and they satisfy the complimentary slackness conditions: $\mu_{j,t-1} \geq 0$, $\mu_{j,t-1}\left(b_{t-1}-\tilde{b}_{j,t-1}\right) = 0$.

Notice that for the case we are analyzing here, $\mu_{l,t-1} = 0$ and $\mu_{h,t-1} \geq 0$. Using this, along with the expressions in (17) and (18), we can rewrite equation (19) for $j \in \{h,l\}$ as (20) and (21) below:

$$\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta = \frac{\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}\tilde{p}_{t-1}} \tag{20}$$

$$\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta \geq \frac{\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t}{\tilde{p}_{t-1}\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}} \tag{21}$$

Notice that equation (20) can be equivalently written as:

$$\left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right](z_{t-1}+d_{t-1}) = \left[\frac{1}{\theta}\left(\frac{\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}\tilde{p}_{t-1}}-1+\delta\right)\right]^{\frac{1}{\theta-1}} \tag{22}$$

[31] In equilibrium, the constraint $-\frac{k_{t-1}}{\tilde{p}_{t-1}} \leq \tilde{b}_{j,t-1}$ does not bind as returns to capital invested in risky projects would become infinite.

Using equations $(13)-(18)$ we rewrite the primary market problem $(P1)$ as below:

$$\max_{\substack{x_{t-1}\in[0,1] \\ d_{t-1}\geq 0}} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda_t V_{j,t}$$

subject to :

$$V_{l,t} = \max\left\{\begin{array}{l} q_{l,t}\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta} \\ +q_{l,t}(1-\delta)\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1}) \\ +\frac{1}{\pi_l}\frac{(1-x_{t-1})}{p_{t-1}}(z_{t-1}+d_{t-1})-R^d_{t-1}d_{t-1}-W_{l,t},\ 0 \end{array}\right\}$$

$$V_{h,t} = \max\left\{\begin{array}{l} q_{h,t}\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta} \\ +q_{h,t}(1-\delta)\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})-R^d_{t-1}d_{t-1}-W_{h,t},\ 0 \end{array}\right\}$$

$$z_{t-1}-\eta x_{t-1}(z_{t-1}+d_{t-1})\geq 0$$

Let ζ_{t-1} be the Lagrange multiplier on the capital regulation constraint. The first order conditions with respect to x_{t-1} and d_{t-1} are given by (23) and (24), respectively.[32]

$$\begin{aligned} &\frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}}\lambda_t 1_{l,t} \qquad (23) \\ =\ &\left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_l\sum_{s^t|s^{t-1}}1_{l,t}\lambda_t q_{l,t} \\ &+\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h\sum_{s^t|s^{t-1}}1_{h,t}\lambda_t q_{h,t} \\ &-\zeta_{t-1}\eta \end{aligned}$$

[32] In order to obtain equation (24), we derive the first order condition with respect to deposits and simplify it by using the expression in (23).

$$
\begin{aligned}
& R^d_{t-1} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \qquad (24)\\
= & \left\{\theta\left[\left(x_{t-1} - \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1} + 1-\delta\right\}\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t} \\
& + \left\{\theta\left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1} + 1-\delta\right\}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} - \zeta_{t-1}\eta
\end{aligned}
$$

Step 3: Bond Prices

Using (22) , we rewrite the equilibrium condition for the choice of x_{t-1}, equation (23) , as below:

$$
\begin{aligned}
& \left(\frac{1}{p_{t-1}} - \frac{\pi_l}{\tilde{p}_{t-1}}\left(1+\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\right)\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t \\
= & \left\{\theta\left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1} + 1-\delta\right\}\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} \\
& -\zeta_{t-1}\eta
\end{aligned}
$$

Using $\pi_l + \pi_h = 1$, we can simplify the left hand side of the above equation and write it equivalently as:

$$
\left(1-\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\cdot \Xi - \zeta_{t-1}\eta = 0 \qquad (25)
$$

$\Xi \equiv \left\{\theta\left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1} + 1-\delta\right\}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} + \frac{\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\tilde{p}_{t-1}}$.
Notice that $\Xi > 0$, unless all financial intermediaries go broke. Then, equation (25) implies that, in the absence of capital regulation or if the capital regulation does not bind (i.e. $\eta = 0$ or $\zeta_{t-1} = 0$), the primary and secondary market bond prices are equated, $\tilde{p}_{t-1} = p_{t-1}$. However, if $\eta > 0$ and capital regulation binds $\zeta_{t-1} > 0$, then equation (25) implies that $\tilde{p}_{t-1} < p_{t-1}$.

Step 4: Primary Market Bond Price and Return to Deposits

We combine equations (23) and (24) to eliminate the term $\zeta_{t-1}\eta$. We find:

$$\frac{1}{p_{t-1}}\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t - R^d_{t-1}\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \tag{26}$$
$$= \left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\pi_h\frac{\tilde{p}_{t-1}}{p_{t-1}}\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}$$
$$-\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1}+1-\delta\right\}\frac{\tilde{p}_{t-1}}{p_{t-1}}\pi_h\sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}$$

Using (20) and (21), equation (26) becomes $R^d_{t-1}\geq\frac{1}{p_{t-1}}$. This completes the proof of Proposition 1 for the case in which the high-risk intermediary sells all bonds in the secondary bond market. The other cases are derived analogously, but are omitted here to keep the exposition short.

A.5 Model Extension: Private Bonds

In the model extension discussed in Section 4.2, the financial intermediaries' first stage problem changes to:

$$\max\sum_{j\in\{h,l\}}\pi_j\sum_{s^t|s^{t-1}}\left\{\pi_F\lambda^F\left(s^t\right)V_j^F\left(s^t\right)+(1-\pi_F)\left[\lambda^{NF}\left(s^t\right)V_j^{NF}\left(s^t\right)\right]\right\}$$

subject to:

$$z\left(s^{t-1}\right)+d\left(s^{t-1}\right)=k\left(s^{t-1}\right)+p\left(s^{t-1}\right)b\left(s^{t-1}\right)$$
$$z\left(s^{t-1}\right)/k\left(s^{t-1}\right)\geq\eta$$

where $V_j^{NF}\left(s^t\right)$ and $V_j^F\left(s^t\right)$ are defined below

$$V_j^{NF}\left(s^t\right) = \max\left\{\begin{array}{l} q_j\left(s_t\right)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right]^{\theta}\left[l\left(s^{t-1}\right)\right]^{1-\theta-\alpha} \\ +q_j\left(s_t\right)\left(1-\delta\right)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right) - \tilde{b}_j\left(s^{t-1}\right)\right] - R^d\left(s^{t-1}\right)d\left(s^{t-1}\right) - W_j\left(s^t\right)l\left(s^{t-1}\right), 0 \end{array}\right\}$$

$$V_j^{F}\left(s^t\right) = \max\left\{\begin{array}{l} q_j\left(s_t\right)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)a_j\left(s^{t-1}\right)\right]^{\theta}\left[l\left(s^{t-1}\right)\right]^{1-\theta-\alpha} \\ +q_j\left(s_t\right)\left(1-\delta\right)\left[k\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)\tilde{b}_j\left(s^{t-1}\right) + \tilde{p}\left(s^{t-1}\right)a_j\left(s^{t-1}\right)\right] \\ +\left[b\left(s^{t-1}\right) - \tilde{b}_j\left(s^{t-1}\right)\right] - \left(1+\xi_j\left(s^{t-1}\right)\right)a_j\left(s^{t-1}\right) \\ -R^d\left(s^{t-1}\right)d\left(s^{t-1}\right) - W_j\left(s^t\right)l\left(s^{t-1}\right), 0 \end{array}\right\}$$

and where the real cost of stamping bonds $\xi_j\left(s^{t-1}\right)$ is defined as:

$$\xi_j\left(s^{t-1}\right) = \left\{\begin{array}{c} 0.01 \quad \text{if} \quad a_j\left(s^{t-1}\right) > 0 \\ 0 \quad \text{if} \quad a_j\left(s^{t-1}\right) \leq 0 \end{array}\right\}$$

Since these are assumed to be the real costs, we need to subtract $(1-\pi_m)\sum_{j=l,h}\pi_j\xi_j\left(s^{t-1}\right)a_j\left(s^{t-1}\right)$ from the total output. Also, in order for the private bond market to clear we must have: $\sum_{j=l,h}\pi_j a_j\left(s^{t-1}\right) + a_F\left(s^{t-1}\right) = 0$, where $a_F\left(s^{t-1}\right) < 0$ is the quantity of private bonds purchased by foreign investors. In exchange, foreign investors give $\tilde{p}\left(s^{t-1}\right)a_F\left(s^{t-1}\right)$ in real capital to domestic financial intermediaries. After the production takes place, foreigners receive $a_F\left(s^{t-1}\right)$ in return, if the aggregate state is good, and only $0.9 \times a_F\left(s^{t-1}\right)$, if the aggregate state is bad.[33]

[33] The values of π_F, ξ and the recovery rate of 90 percent are arbitrary and were chosen for illustration purposes.

B Tables

Table 1: Calibrated Parameters

Parameter/Value	Moment
$\beta = \left(\frac{1}{1.04}\right)^{1/4}$	Real interest rate of 4 percent
$\theta = 0.29$	Capital income share[1]
$\tau = 0.008\%$	Brokerage fees for the issuance of U.S. T-bills[2]
$\Phi = \begin{bmatrix} 0.9447 & 0.0553 \\ 0.20 & 0.80 \end{bmatrix}$	Average length of expansions/contractions of business sector
$\pi_l = 0.85$, $\pi_h = 1 - \pi_l = 0.15$	Sensitivity analysis

[1]This is the average share for the corporate nonfinancial sector from 1948 to 2009. [2]Stigum (1983, 1990) reports values between 0.0013% and 0.008%. We use the upper bound.

Table 2: Estimated Parameters

Parameter	Value
The following parameters are determined jointly to match the moments in Table 3	
Share of nonfinancial firms	$\pi_m = 0.6949$
Depreciation rate	$\delta = 0.0264$
Fixed factor income share	$\alpha = 0.00070317$
Productivity parameters	
nonfinancial firms	$q_m(\overline{s}) = 0.9617$
	$q_m(\underline{s}) = 0.9281$
low-risk financial firms	$q_l(\overline{s}) = 0.9381$
	$q_l(\underline{s}) = 0.9344$
high-risk financial firms	$q_h(\overline{s}) = 1$ (normalization)
	$q_h(\underline{s}) = 0.6785$

Table 3: Comparison of Data and Model Moments

Moment	Data in %	Model in %
Coefficient of variation of output[1]	3.75	3.94
Coefficient of variation of household net worth[2]	8.17	9.11
Average maximum decline in output during contractions[3]	6.48	6.98
Average deposits over total household financial assets[2]	17.2	26.0
Recovery rate in case of bankruptcy[4]	42.0	28.4
Mean output share of nonfinancial sector[5]	66.9	71.3
Average capital depreciation rate in economy	2.5	2.5
Equity to asset ratio of the financial sector[2,6]	7.6	5.2

[1]Output is measured as the value added for the business sector from 1987Q1 to 2010Q2. This is also the reference period for the other moments, unless otherwise stated. [2]From the U.S. Flow of Funds accounts. [3]This is the absolute decline taking the growth trend into account. [4]As reported in Acharya, Bharath, and Srinivasan (2003). [5]We identify the nonfinancial sector with the corporate nonfinancial sector. [6]When calculating the equity to asset ratio, we exclude mutual funds.

Table 4: Welfare and Risk Taking Results Relative to the Social Planner[1]

Experiment	LTCE[2] in %	Risk taking[3] in %
Equilibrium without repo market	−0.8754	33.1
Optimal interest rate policy	−0.0431	23.6
Optimal policy −0.1 percentage points	−0.0433	21.1
Optimal policy +0.1 percentage points	−0.0436	26.2
Optimal interest rate policy and capital regulation	−0.0444	0.3

[1]The statistics are computed using 5000-period simulations of the model economy and the planner's problem. [2]Lifetime Consumption Equivalents (LTCE) is the percentage decrease in the optimal consumption from the social planner problem needed to generate the same welfare as the competitive equilibrium with a given interest rate policy. [3]Risk taking is the percentage deviation in the amount of resources invested in the high-risk projects in the competitive equilibrium relative to the social planner's choice. The numbers reported here are averages over expansions and contractions in our calibrated model. A positive number indicates too much risk taking relative to the social planner, on average.

Table 5: SENSITIVITY ANALYSIS FOR FRACTION OF HIGH RISK INTERMEDIARIES
WELFARE AND RISK TAKING RESULTS RELATIVE TO THE SOCIAL PLANNER[1]

	LTCE[1] in %			Risk taking[2] in %		
Experiment / π_h value	0.13	**0.15**	0.17	0.13	**0.15**	0.17
Equilibrium without repo market	−0.7814	**−0.8754**	−0.9624	37.4	**33.1**	29.1
Optimal interest rate policy	−0.0439	**−0.0431**	−0.0397	20.4	**23.6**	21.6
Optimal policy −0.1 percentage points	−0.0536	**−0.0433**	−0.0474	9.4	**21.1**	13.3
Optimal policy +0.1 percentage points	−0.4403	**−0.0436**	−0.0428	89.4	**26.2**	30.6

[1]The statistics are computed using 5000-period simulations of the model economy and the planner's problem. [2,3]See notes to Table 4.

C Figures

Figure 1: Portfolio Investments in the Primary Market and After Repo Trades

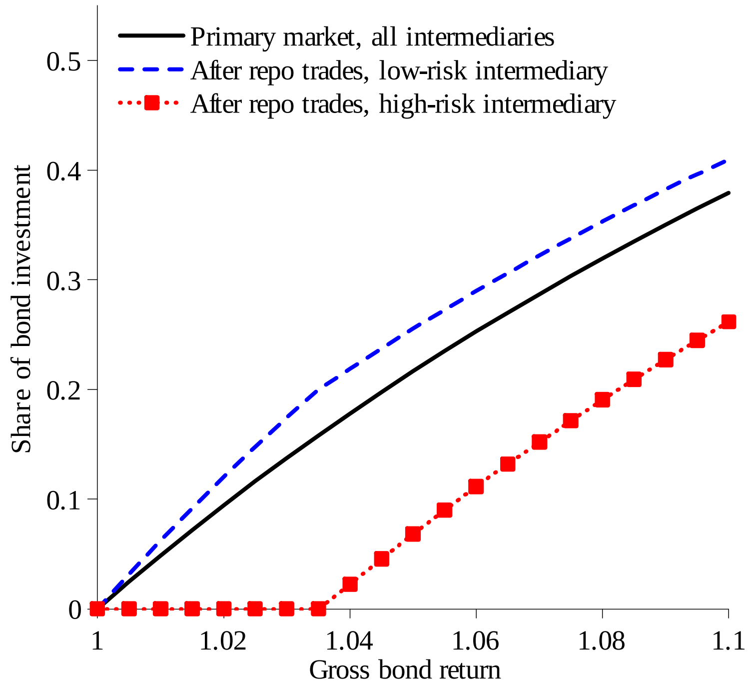

Figure 2: Simulation Results for Benchmark Model

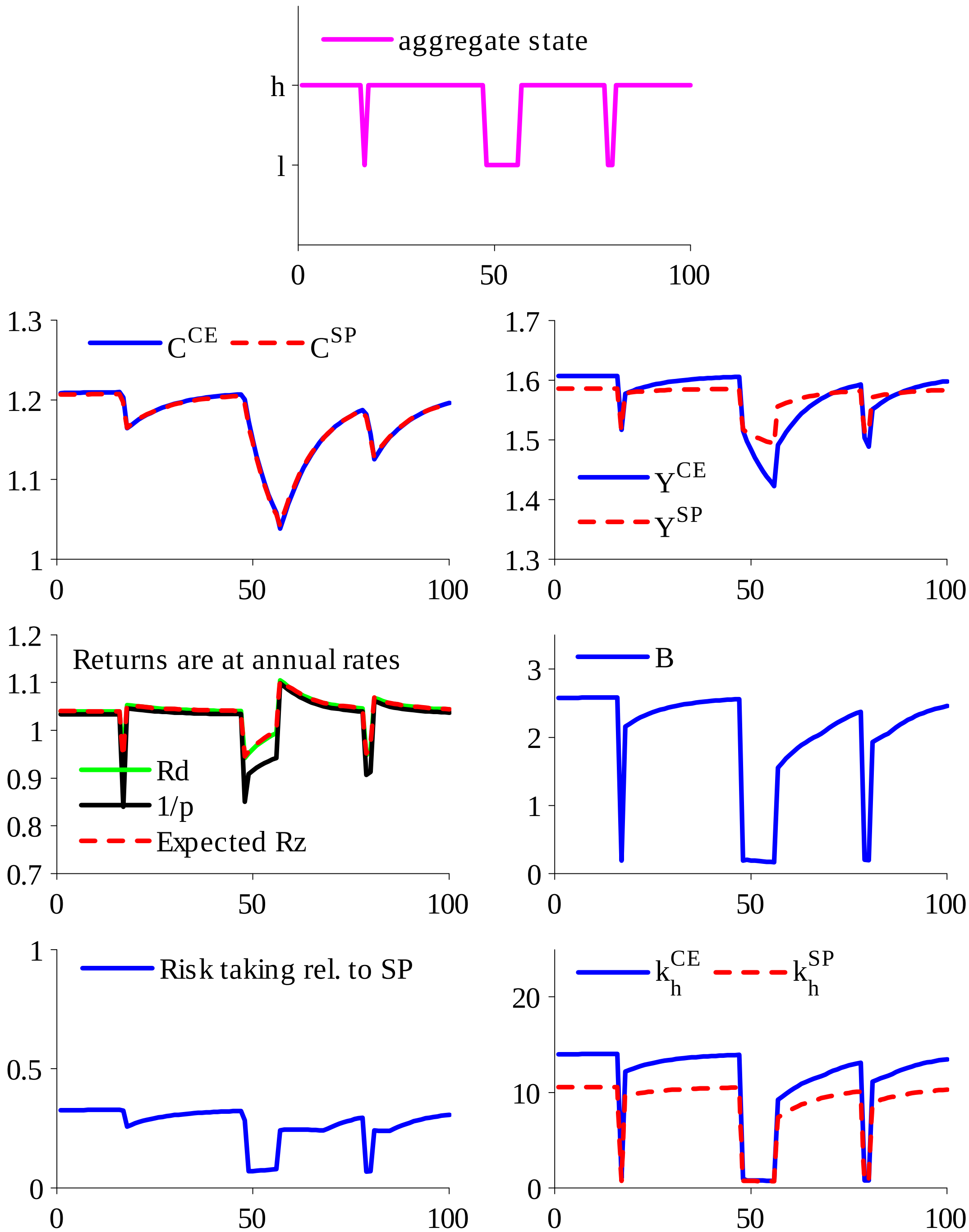

Figure 3: Benchmark Model: Welfare and Risk Taking Relative to the Social Planner; Results From 5000-Period Simulations

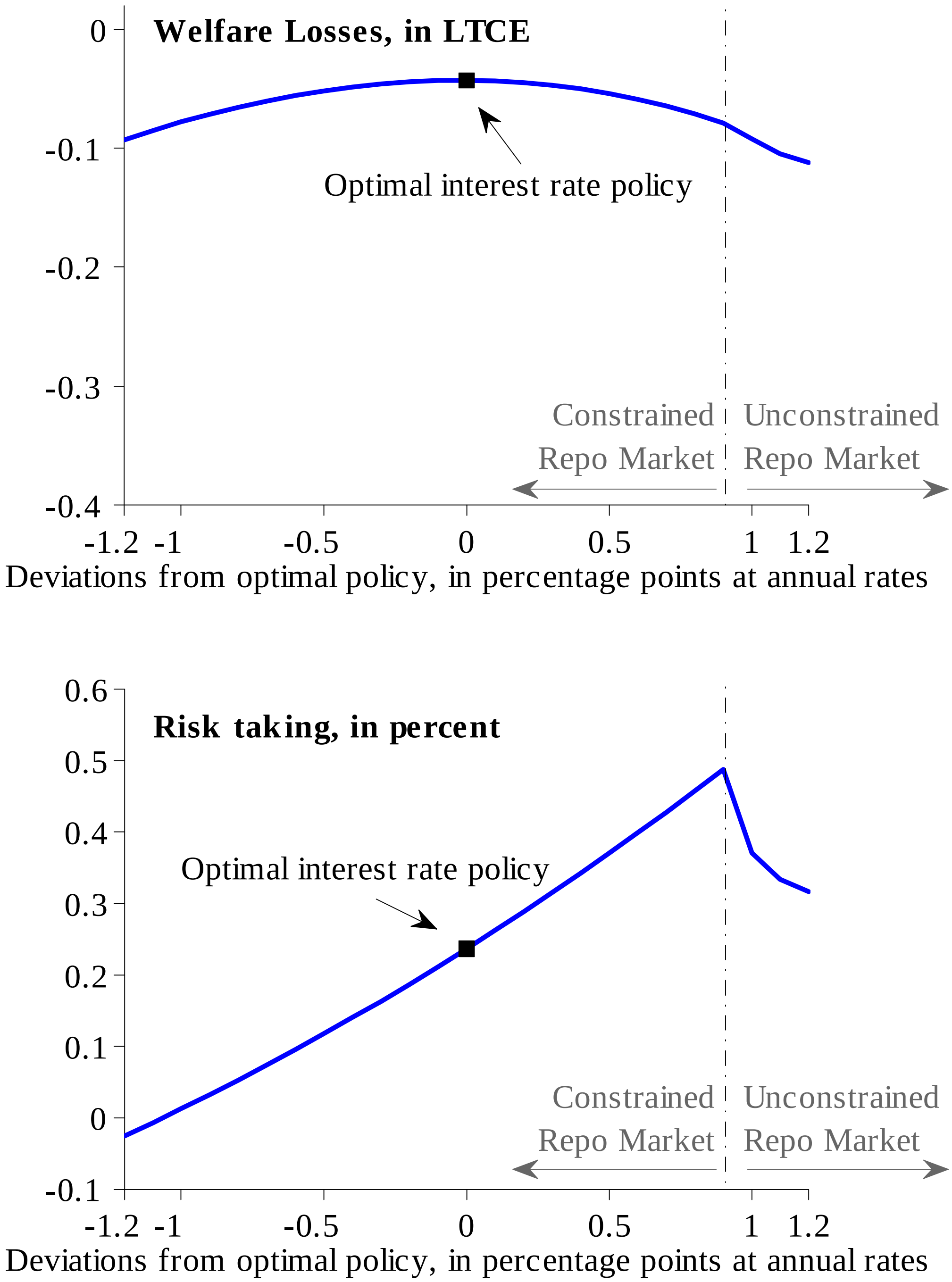

Figure 4: Simulation Results for Model Extension

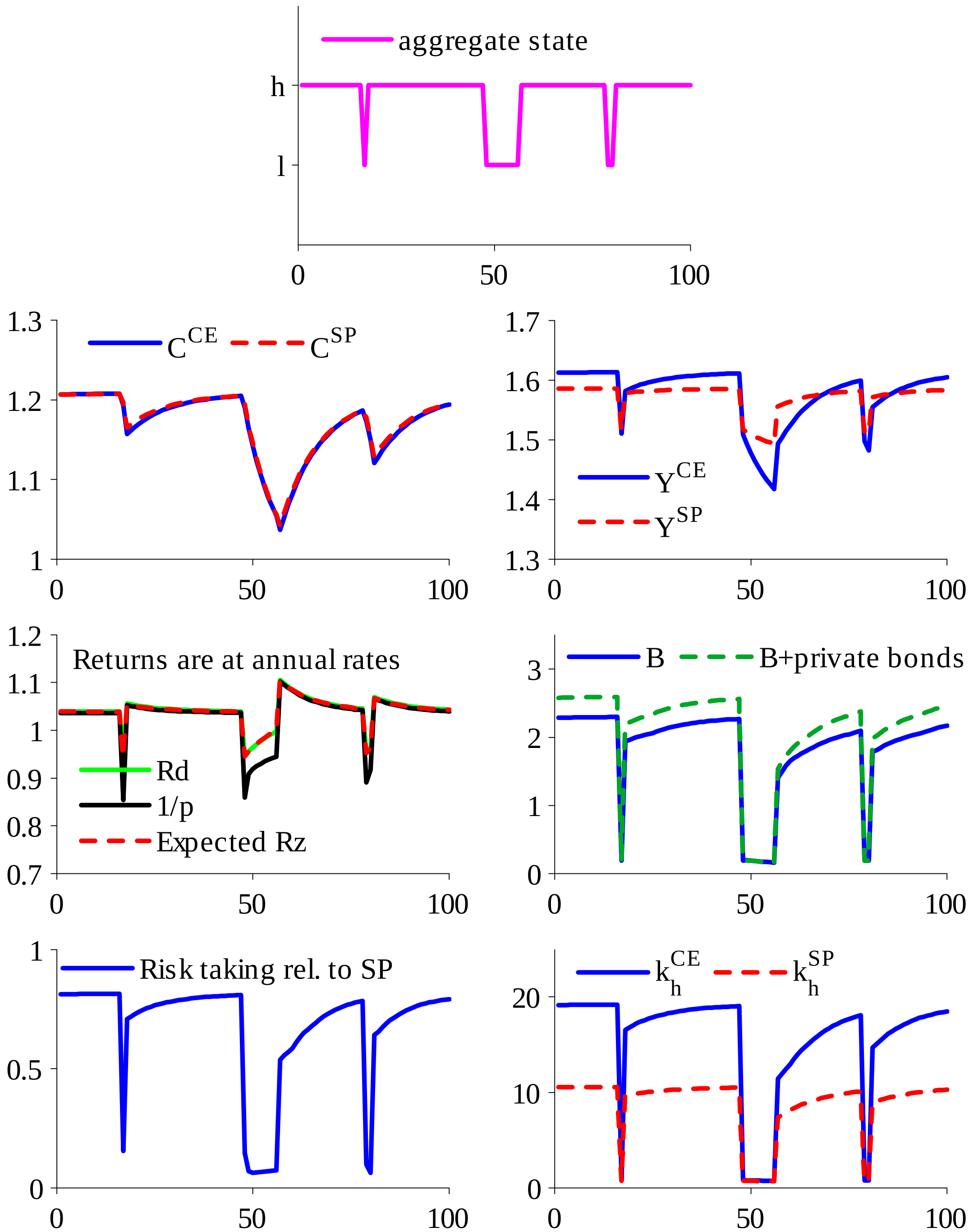

Figure 5: Welfare and Risk Taking Relative to the Social Planner; Results From a 30-quarter Business Cycle Simulation