Early and Late Human Capital Investments, Borrowing Constraints, and the Family*
(Preliminary and Incomplete)

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1 Introduction

Education and other human capital investments are central to both individual and economy-wide development. By limiting the incentives and capacity to invest in human capital, financing constraints and labor market uncertainty can play critical roles in determining aggregate productivity, national income distributions, social mobility, and economic growth and development (Becker 1975). The growing importance of parental income for child achievement and educational attainment (Belley and Lochner 2007, Duncan and Murnane 2011, Reardon 2011) raises serious questions about the ability (or willingness) of disadvantaged families to make efficient investments in their children. In this paper, we investigate the importance of family borrowing constraints in determining human capital investments in children at early and late ages. We also explore the extent to which different policies targeted to different ages can address this market failure, potentially improving economic efficiency and equity.

Despite considerable evidence that adolescent skill levels are important in determining subsequent schooling and lifetime earnings (see, e.g., Cameron and Heckman 1998, Keane and Wolpin 1997, 2001, and Carneiro and Heckman 2002), only recently has the literature begun to consider

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whether borrowing constraints inhibit early investments in young children.[1] Studies of consumption behavior often find stronger evidence of binding liquidity constraints for younger households (e.g. Meghir and Weber 1996, Alessie, Devereux, and Weber 1997, Stephens 2008).[2] Indirect evidence suggests that constraints at early ages may play a more important role in determining human capital investment than constraints at later ages. For example, most empirical studies find high lifetime returns for early childhood programs, especially for the most disadvantaged children (e.g., see Karoly et al. 1998, Blau and Currie 2006, or Cunha, et al. 2006, Heckman, et al. 2010). A few studies also find that family income received at early childhood ages has a greater impact on achievement when compared with income received at later ages (e.g. Duncan and Brooks-Gunn 1997, Duncan, et al. 1998, Levy and Duncan 1999, Caucutt and Lochner 2006).[3] In Section 2, we provide further evidence from the Children of the National Longitudinal Survey of Youth (CNLSY) that early income is more important for high school completion and college-going than income earned at later ages. More generally, recent studies show that exogenous increases in family income lead to improvements in early child development (e.g. Løken 2010, Løken, Mogstad and Wiswall 2010, Duncan, Morris and Rodrigues 2011, Milligan and Stabile 2011, and Dahl and Lochner, forthcoming).

Credit constraints are natural candidates to explain why most low-income children do not participate in quality preschool programs despite the high economic returns. First, while (generous) government student loan programs are available for college in the U.S. and other developed countries, neither governments nor private lenders typically offer loans to parents to help finance human capital investments in younger children. Second, even though elementary and secondary education is publicly provided, the quality of public schools available to poor families is often low, while high quality private schools and preschool programs are typically quite expensive. Kaushal,

[1] Cunha (2007) examines investment behavior over the lifecycle in the presence of uncertainty and self-imposed borrowing constraints as in Aiyagari (1994). Del Boca, Flinn, and Wiswall (2010) consider differential time and goods inputs throughout childhood in the absence of borrowing or saving.

[2] In related work, Attanasio, Goldberg and Kyriazidou (2008) conclude that many younger and middle-age American households are likely to be liquidity constrained based on differential car loan demand elasticities with respect to interest rates and loan maturity. In the presence of income uncertainty, Gourinchas and Parker (2002) empirically show that younger consumers behave as buffer-stock agents, saving small amounts (rather than borrowing) due to precautionary motives, while middle-age workers save primarily for lifecycle motives. The possibility of very low future income realizations causes young consumers to behave much like they are liquidity constrained.

[3] Carneiro and Heckman (2002) argue that early income should have a larger effect than later income due purely to discounting (e.g. \$1 at age 0 should have an effect that is $(1+r)^{10}$ larger than income at age 10, where r is the annual interest rate). Accounting for this, they estimate similar effects of 'early' and 'late' family income on college enrolment in the Children of the National Longitudinal Survey of Youth (CNLSY); however, they also control for age 12 achievement levels which may absorb much of the effect of earlier income. Caucutt and Lochner (2006) estimate that (discounted) income received at earlier ages has a larger impact on age 5-14 math and reading achievement in the CNLSY than (discounted) income received at later ages.

Magnuson, and Waldfogel (2011) find that families in the bottom family expenditure quintile spend 3% of their total expenditures on educational enrichment items, while families in the top quintile spend 9%. Parental spending on education-related items and activities rises with family expenditures. Parental time is also an important input in a young child's education that poor parents may be unable to afford.[4] Finally, most parents of young children are young themselves, in the early stages of their labor market careers and without a solid credit history. Even young college-educated parents may be constrained by mortgages, their own schooling loans, and other liabilities.

Investment in human capital is a multi-stage process that begins early in life.[5] As a result, investment in human capital is an intergenerational family problem.[6] In this paper, we develop a human capital-based theory of the family that incorporates the dynamic nature of investment in children, intergenerational transfers, and borrowing constraints faced by parents and college-age youth. Our theory recognizes that later investments build on earlier investments, that early childhood investments are made by young parents at the beginning of their careers, and that desired borrowing may differ substantially over the life-cycle of an individual.

In our framework, young parents make early investments in their children and provide them with consumption. These parents also make their own consumption choices and borrow or save to intertemporally allocate resources. Constraints on their borrowing may limit consumption and investments in young children. Once children become young adults, they make additional investments in themselves (e.g. college), using their own earnings, transfers from their parents, and student loans to cover their own costs and consumption. Again, choices may be constrained by imperfect credit markets. Older parents must decide how much to transfer to their college-age children and how much to borrow or save for their own current and future consumption. Once a child leaves the home to establish his own family, parents continue to work, save, and consume until retirement. This cycle repeats itself, as young adults grow into parenthood.

We show that the dynamic nature of human capital investment has important implications for the role of borrowing constraints and economic policies. Consistent with the analysis of Cunha and Heckman (2007), we show that *dynamic complementarity* in investment – the complementarity between early and late investments in human capital – plays a central role in determining the

[4]See Del Boca, Flinn, and Wiswall (2010)

[5]See, e.g., Becker (1975), Todd and Wolpin (2003, 2007), Cunha, et al. (2006), Cunha (2007), Cunha and Heckman (2007), Cunha, Heckman and Schennach (2010), Del Boca, Flinn, and Wiswall (2010).

[6]See, e.g., Becker and Tomes (1979, 1986), Loury (1981), Galor and Ziera (1993), Aiyagari, Greenwood and Seshadri (2002), Caucutt and Kumar (2003), and Restuccia and Urrutia (2004).

impacts of investment subsidies and borrowing constraints on investment over the lifecycle. When investments are sufficiently complementary, a policy that encourages investment at one stage of development will also tend to increase investment at other stages. By contrast, when investments are substitutable over time, a subsidy or loan increase at one stage of development tends to shift investment to that stage and away from others. Indirect evidence discussed in Cunha, et al. (2006) and estimates by Cunha, Heckman and Schennach (2010) suggest that investments are quite complementarity. Calibration of our intergenerational model produces a similar degree of dynamic complementarity.[7]

A large literature considers the impacts of college-age policies on schooling and labor market outcomes holding early investment and adolescent achievement levels fixed (e.g. Cameron and Heckman 1998, Keane and Wolpin 2001, Caucutt and Kumar 2003).[8] The degree of dynamic complementarity we calibrate suggests that these policies not only impact college-going, but they also have significant impacts on earlier investments in children. Our quantitative analysis suggests that ignoring those earlier investment responses may lead researchers to under-estimate the total human capital impacts of college-age investment subsidies by as much as 60%.

The timing of borrowing constraints is also important and interacts with dynamic complementarity in investment. As discussed above, early income appears to be more important than later income for educational attainment. Based on this feature of the CNLSY data, our calibration finds that borrowing constraints are more severe for young families with young children. Increasing borrowing opportunities or subsidizing investment for families with young children would lead to important (short-run) increases in human capital investments at early and college-going ages. By comparison, increasing borrowing limits for college students and older parents has very small effects. The latter is consistent with the findings of Keane and Wolpin (2001) and Johnson (2010). When compared with subsidies for early investments, subsidies for college also have weaker effects on human capital investment, since they come too late for many constrained families. Due to early borrowing constraints, many families cannot increase early investments to fully take advantage of later subsidies. Given strong complementarity of investments over time, additional later investments may not be worthwhile for these families. Thus, dynamic complementarity and early borrowing constraints interact in a way that limits the effectiveness of college-age policies

[7] Unlike Cunha, Heckman and Schennach (2010), we do not distinguish between cognitive and non-cognitive skills. To the extent that these skills are combined to create a composite productivity (i.e. human capital) level used in the labor market, we effectively identify the technology mapping early and late investments (in cognitive or non-cognitive skills) into this productivity measure.

[8] Traditional difference-in-difference or reduced form studies of the impact of tuition or aid levels on college-going also implicitly hold early investment behaviors fixed.

for disadvantaged families.

The intergenerational nature of human capital investment is also important. Keane and Wolpin (2001) and Johnson (2010) both emphasize the importance of differences in parental transfers by socioeconomic background in explaining differential schooling outcomes; however, parental transfers are exogenously determined and unaffected by policy and economic conditions in their models. By endogenizing parental transfers, we show how parents respond to different policies by adjusting transfers to their children. Furthermore, our dynastic approach to human capital investment enables us to study dynamic effects of lasting economic policies that are typically ignored by the literature.

First, we differentiate between the "current" and "future" effects of an income transfer or loan policy. For example, an income transfer to young borrowing constrained parents (that is expected to last for the forseeable future) affects child investment decisions in two distinct ways: (i) it directly encourages investment by increasing parental resources and transfers – the "current" effect, and (ii) it discourages investment by lowering its return, since children will also receive the transfer when they grow up – the "future" effect. The second effect implies that income transfers to young parents will have weaker effects on investment if the policy is permanent rather than a one-time giveaway to a single generation. Similar effects arise for policies to extend loans to young parents; however, both "current" and "future" effects of such policies tend to be weaker.

Second, our dynastic approach allows us to simulate the long-run effects of permanent policy changes in addition to the short-run effects typically measured in empirical studies. While short-run effects are based on the current distributions of wealth and human capital in the population, long-run effects take into account changes in these distributions over time. This turns out to be quite important when considering a policy to increase borrowing opportunities for young parents. In the short-run, such a policy has the expected effects of increasing borrowing and human capital investment as discussed earlier. In the long-run, increases in borrowing cause parents to accumulate more debt and transfer less resources to their children. Over time, this leads to higher debt levels, pushing families nearly as close to borrowing limits as initial generations were before the policy change. While early generations accumulate more human capital, later generations do not. By contrast, investment subsidies appear to have greater long-run impacts (relative to short-run impacts), because they encourage human capital investment without building debt levels.

This paper proceeds as follows. In Section 2, we use data from the CNLSY to study whether

the timing of when family income is earned is important for educational attainment. Our results suggest that income received at earlier ages of child development has greater impacts on high school completion and college-going than does income received at later ages. These findings are broadly consistent with binding borrowing constraints for many young families.

Section 3 develops a lifecycle model of human capital investment with borrowing constraints. Allowing for two periods of investment, we analytically study the impacts of relaxing borrowing constraints or providing financial transfers at different ages on investment behavior. This analysis establishes the importance of dynamic complementarity for the qualitative nature of investment responses.

Section 4 extends the model to an intergenerational setting, assuming that individuals live through six stages: early and late childhood, early and late parenthood, post-parenthood, and retirement. Parents and children are linked for the two periods of investment (early and late childhood/parenthood). Important analytical results on the role of early and late borrowing constraints are extended to a two-generation context with endogenous parental transfers. Then, we move to a fully dynastic overlapping-generations framework in which altruistic parents value the utility of their children. We calibrate this model in Section 5 using data from the CNLSY on family income at different stages of child development, educational attainment by children and their parents, and the earnings outcomes of children.[9] We assume a CES human capital production function without imposing any assumptions on the complementarity of investments across different stages of development. We allow for heterogeneity in ability and post-school shocks to labor market earnings. Borrowing constraints are assumed to depend on the discounted present value of lowest possible earnings levels.

Based on our calibrated model, Section 6 simulates the impacts of various policy changes, including increases in borrowing limits, investment subsidies, and income transfers. We consider both short- and long-run effects of policies. Section 7 concludes.

2 Effects of Early and Late Family Income on Educational Attainment

In this section, we estimate whether family income received when children are young is more important for final educational outcomes than income received when children are older. If intragenerational borrowing constraints are a problem for younger families and investments are

[9] We also use national measures of educational expenditures to determine the direct costs of schooling.

sufficiently complementary across ages, we would expect early income to be more important than later income (after properly discounting both). As noted by Carneiro and Heckman (2002), it is important to discount income by the prevailing interest rate, since later income is worth less in a present value sense. In the absence of constraints, only the discounted present value of lifetime family income should matter, not the timing of that income.

To explore this prediction of early borrowing constraints, we use data from the CNLSY, which follows the children born to all women in the NLSY. The mothers in our sample are original NLSY respondents from the random sample and were ages 14-22 in 1979 when the survey began. Importantly, the data contains measures of family income every year from 1979 to 1994 and biennially thereafter. We sum earned income from the mother and father each year as our measure of family income, discounting income back to age zero of the child using a 5% annual interest rate. Our measure of 'early' income averages family incomes over child ages zero to eleven, while our measure of 'late' income averages incomes over ages 12-23. These assumptions and age groups are consistent with those used later in our calibration.

We examine the effects of family income on the probability children of NLSY mothers have dropped out of high school, ever attended college by age 21 or by age 24, and completed college by age 24.[10] Since the NLSY mothers were ages 14-22 in 1979, many children in the survey are still young. Thus, our sample sizes are smaller when looking at college attendance or completion at age 24 compared with measures of high school dropout or college attendance at age 21.

The Children of the NLSY data also contain many child characteristics and measures of family background that may affect educational attainment. We consider year of birth, race/ethnicity, gender, whether the mother was a teenager when the child was born, maternal education (high school dropout, high school graduate, some college, college graduate), whether the mother was living in an intact family at age 14, whether the mother is foreign-born, and the mother's normed score on the Armed Forces Qualifying Test (AFQT) taken as part of the survey in 1980.[11]

We regress educational attainment indicators on early and late family income (measured in $10,000 year 2000 dollars) in Table 1. Estimates reported in Panel A only control for maternal education, while Panel B reports estimates controlling for other child and family background

[10] We categorize individuals with less than 12 years of completed schooling as high school dropouts, those with 13 or more years of completed schooling as having attended college, and those with 16 or more years of completed schooling as college graduates. If educational attainment is unavailable at age 21 (24), we use reported education at ages 22-24 (25-27).

[11] AFQT scores are a widely used measure of cognitive achievement in social science studies. They correlate strongly with educational attainment, earnings and many other socioeconomic measures.

characteristics.[12] The estimates are quite similar across the panels, suggesting that controlling for maternal education captures much of the important role played by family background. Most importantly, nearly all specifications suggest that early income is important. A $10,000 increase in discounted annual income at ages zero to eleven reduces the probability of high school dropout by about 4 percentage points, while it increases college attendance and completion rates by 4-6 percentage points. All of these estimates are statistically significant. The same increase in discounted annual income when the child is 12-23 has smaller (and generally statistically insignificant) effects on educational attainment. The differential effect of early and late income is most pronounced for high school dropout, where the difference is statistically significant at the 0.05 level; however, the difference is also statistically significant for college completion in both panels.

These results suggest that early income is important – more important than income received at later ages. Borrowing constraints are an obvious explanation for this pattern of results, although others are certainly possible. Building on these results, the rest of this paper more deeply examines the economic role of borrowing constraints for families and their implications for human capital investment.

3 A Lifecycle Model of Human Capital Investment and Borrowing Constraints

In this section, we develop a basic lifecycle human capital investment model to study 'early' and 'late' investments and their responses to changes in borrowing limits and financial transfers. We highlight the importance of dynamic complementarity in investments in determining the impacts of borrowing constraints and transfers on investment at different stages of development.

We assume that people live through six stages in their life. Human capital investment takes place in the first two periods (i.e. 'childhood'), followed by three periods of work and a period of retirement. Our main focus in this section is on the importance of multiple investment periods during childhood. The adult periods play a more prominent role in the following section when we consider an intergenerational framework of overlapping parents and children.

Let subscripts $j \in \{1, 2, 3, 4, 5, 6\}$ represent the six stages of the lifecycle, and denote an individual's ability to learn by θ. Investments in stages 1 and 2 are given by i_1 and i_2, respectively.

[12] Estimates in Panel A use data on 1,483 youth at age 21 and 828 at age 24, while those in Panel B use slightly fewer (1,422 and 802) due to missing observations on some of the child/family background variables.

Together, these investments produce period 3 human capital:

$$h_3 = f(i_1, i_2, \theta). \tag{1}$$

The human capital production function $f(\cdot)$ is strictly increasing in all of its arguments and strictly concave in both investment arguments. We further assume that f_{13} and f_{23} are non-negative, so that ability and investments are complements. To guarantee appropriate second order conditions hold in the decision problems described below, we assume the following:

Assumption 1. $f_{12} \geq max\left\{f_{22}\left(\frac{f_1}{f_2}\right), f_{11}\left(\frac{f_2}{f_1}\right)\right\}$ *and* $f_{12}^2 \leq f_{11}f_{22}$.

Most reasonable specifications for human capital production would have $f_{12} \geq 0$, satisfying the first part of this assumption. The second part limits the degree of complementarity in investments.

In our computational analysis below, we employ a modified-CES human capital production function of the form

$$f(i_1, i_2, \theta) = \theta(ai_1^b + (1-a)i_2^b)^{d/b}, \tag{2}$$

where $a \in (0,1)$, $b < 1$, and $d \in (0,1)$; however, our theoretical analysis does not rely on any particular functional form. Assumption 1 holds for this production function. We impose decreasing returns to scale (i.e. $d < 1$); otherwise unconstrained individuals would want to invest an infinite amount.[13]

Adult earnings depend on human capital acquired through childhood investments. Given our emphasis on childhood human capital investment (i.e. early childhood and schooling investments but not post-school training), we assume that human capital grows exogenously after childhood:

$$h_4 = \Gamma_4 h_3 \qquad and \qquad h_5 = \Gamma_5 h_4. \tag{3}$$

Adult earnings are given by

$$W(h_j) = wh_j, \text{ for } j \in \{3,4,5\}, \tag{4}$$

where $w > 0$ reflects the wage per unit of skill.

In this section, we assume that individuals receive exogenous financial transfers y_j during childhood. These may reflect transfers from parents or the government. For older children, they may also reflect modest earnings while in school. In the next section, we explicitly endogenize transfers to children (from parents) in an intergenerational framework.

[13] With constant (increasing returns) to scale, if any combination of first and second period investment produces a positive net value, then doubling those investments will double (more than double) the net value.

If individuals choose to save, they earn a gross return of R on savings. This interest rate also applies to any borrowing. Assets saved in period j are given by A_{j+1}, and total borrowing (negative A_{j+1}) may be limited by a restriction on debt carried over to the next period, $-L_j$. During retirement individuals live off of savings and do not work.

3.1 Human Capital Investment Behavior

Individuals choose investment $i = (i_1, i_2)$ and consumption $c = (c_1, , ..., c_6)$ to maximize their lifetime utility. For time discount factor $\beta \in (0, 1)$ and utility function $u(c)$ (a strictly increasing and strictly concave function), an individual of ability θ receiving transfers y_1 and y_2 solves the following problem:

$$U(y_1, y_2, \theta) = \max_{i,c} \sum_{j=1}^{6} \beta^{j-1} u(c_j) \tag{5}$$

subject to human capital accumulation and earnings functions (1), (3), and (4), budget constraints:

$$A_{j+1} = \begin{cases} RA_j + y_j - i_j - c_j & \text{for } j = 1, 2 \\ RA_j + W(h_j) - c_j & \text{for } j = 3, 4, 5, \end{cases} \tag{6}$$

borrowing constraints:

$$A_{j+1} \geq -L_j, \text{for } j = 1, ..., 4, \tag{7}$$

$A_1 = 0$, $c_6 = RA_6$, and non-negative consumption and investments: $c_j \geq 0 \ \forall j$, and $i_j \geq 0$ for $j = 1, 2$.

The first order conditions for consumption imply

$$u'(c_j) \geq \beta R u'(c_{j+1}) \quad \forall j,$$

where the inequality is strict if and only if the borrowing constraint for that period (L_j) binds. First order conditions for investment imply:

$$u'(c_1) = \beta^2 w \left[u'(c_3) + \beta\Gamma_4 u'(c_4) + \beta^2\Gamma_4\Gamma_5 u'(c_5)\right] f_1(i_1, i_2, \theta) \tag{8}$$

$$u'(c_2) = \beta w \left[u'(c_3) + \beta\Gamma_4 u'(c_4) + \beta^2\Gamma_4\Gamma_5 u'(c_5)\right] f_2(i_1, i_2, \theta). \tag{9}$$

Taking the ratio of these equations reveals that optimal investment equates the technical rate of substitution in the production of human capital with the marginal rate of substitution for consumption: $\frac{f_1(i_1,i_2,\theta)}{f_2(i_1,i_2,\theta)} = \frac{u'(c_1)}{\beta u'(c_2)} \geq R$.

Unconstrained optimal investments for an individual of ability θ, $i_1^u(\theta)$ and $i_2^u(\theta)$, satisfy $f_1(i_1^u(\theta), i_2^u(\theta), \theta) = \frac{R^2}{w\chi}$ and $f_2(i_1^u(\theta), i_2^u(\theta), \theta) = \frac{R}{w\chi}$, where $\chi = 1 + R^{-1}\Gamma_4 + R^{-2}\Gamma_4\Gamma_5$ is the

discounted present value of an additional unit of human capital. Importantly, unconstrained investments maximize the discounted present value of earnings net of discounted investment costs. Therefore, they are independent of the marginal utility of consumption, since individuals can optimally smooth across periods. This is the sense in which the timing of income is irrelevant for investment in the absence of borrowing constraints. This is not true when borrowing constraints bind as the following proposition shows.

Proposition 1. *Let i_1^* and i_2^* reflect optimal first and second period investment. Then, (i) $f_1(i_1^*, i_2^*, \theta) > f_1(i_1^u, i_2^u, \theta) = \frac{R^2}{w\chi}$ if and only if any borrowing constraint binds; (ii) $f_2(i_1^*, i_2^*, \theta) > f_2(i_1^u, i_2^u, \theta) = \frac{R}{w\chi}$ if and only if borrowing limits L_2, L_3 or L_4 bind; (iii) there is under-investment in at least one period and possibly both (i.e. $i_1^* < i_1^u$ and/or $i_2^* < i_2^u$) if and only if any borrowing constraint binds.*

Proof: See Appendix A.

Facing a binding constraint at any point, even later in life, implies under-investment in human capital during at least one period. When the first period borrowing constraint binds, $f_1(\cdot)/f_2(\cdot) > R$ so there is too little early investment relative to late investment. When only the second period (or later constraints) bind, both early and late investments tend to be too low even though $f_1(\cdot)/f_2(\cdot) = R$. We next explore how investments in both periods depend on borrowing limits and financial transfers.

For this analysis, it is useful to define the following function:

$$v_\tau(Y) = \max_{c_\tau,\ldots,c_6} \sum_{j=\tau}^{6} \beta^{j-\tau} u(c_j) \text{ subject to} \sum_{j=\tau}^{6} R^{\tau-j} c_j = Y. \tag{10}$$

This function reflects the maximized discounted lifetime utility for an unconstrained adult with Y in total resources as of period $\tau = 3, 4, 5$, and it is strictly increasing and strictly concave in Y given our assumptions on $u(\cdot)$. Thus, the remaining lifetime utility for an adult (unconstrained from that point on) with human capital h_3 and assets A_3 in period 3 is given by $v_3(RA_3 + wh_3\chi)$, since welfare is determined entirely by total lifetime wealth, which includes the value of current assets and the remaining value of human capital in the labor market.

The complementarity of investment across periods plays a central role in determining individual responses to borrowing constraints. In particular, the following complementarity condition is important for a number of results:

Condition 1. $\frac{f_{12}}{f_1 f_2} > -\frac{v_3''(-RL_2+wh_3\chi)}{v_3'(-RL_2+wh_3\chi)} w\chi$.

This condition simplifies nicely with the CES production function given in equation (2) and a value function of the form $v(x) = \eta \frac{x^{1-\sigma}}{1-\sigma}$. In this case, Condition 1 cannot hold if $d \leq b$, but this only rules out very strong substitution between early and late investments, since $d > 0$.[14] For the more relevant case of $d > b$, Condition 1 simplifies to

$$\underbrace{\frac{1}{1-b}}_{\text{elast. of sub.}} < \underbrace{\frac{1}{\sigma}}_{\text{CIES}} \cdot \underbrace{\left(1 - \frac{RL_2}{w\chi h_3}\right)}_{1\text{-}\frac{maximum\ debt}{lifetime\ income}} \cdot \left(\frac{d-b}{d(1-b)}\right).$$

As the elasticity of substitution between early and late investments decreases (i.e. investments become more complementary) or the consumption intertemporal elasticity of substitution (CIES) increases (i.e. individuals become less concerned about maintaining smooth consumption profiles), this inequality is more likely to hold. For $b \leq 0$ (complementarity at least as strong as implied by a Cobb-Douglas production function) and $L_2 = 0$, this condition is satisfied whenever the CIES is greater than the elasticity of substitution between early and late investments. More generally, when individual preferences for smooth consumption are strong, Condition 1 requires strong complementarity between early and late investments.

We analyze the effects of borrowing constraints on young and old children when individuals are unconstrained as adults.[15] We begin by studying the effects of relaxing constraints on older children on investment at young and old ages.

Proposition 2. *Assume that borrowing constraints bind during late childhood (i.e. $A_3 = -L_2$) but that there are no future borrowing constraints.*

(i) If early constraints do not bind (i.e. $A_2 > -L_1$), then: $\frac{\partial i_1}{\partial L_2} > 0$, $\frac{\partial i_2}{\partial L_2} \in (0,1)$, and $\frac{\partial h_3}{\partial L_2} > 0$.

(ii) If early constraints also bind (i.e. $A_2 = -L_1$), then: $\frac{\partial i_1}{\partial L_2} > 0$ if Condition 1 holds; $\frac{\partial i_2}{\partial L_2} \in (0,1)$; and $\frac{\partial h_3}{\partial L_2} > 0$.

Proof: See Appendix A.

An individual that is constrained as an old child responds to an increase in borrowing opportunities that period by unambiguously increasing late investments. However, late investment increases less than borrowing, since the individual consumes some of the additional resources. If individuals are unconstrained during early childhood, then relaxing the late borrowing constraint always increases early investments along with late investments. However, if the early constraint

[14] When $d \leq b$, $f_{12} \leq 0$.

[15] Similar results hold throughout this section if the person is borrowing constrained as an adult using a modified Condition 1 that incorporates those future constraints in $v_3(\cdot)$.

also binds, then early investment only increases with sufficient dynamic complementarity. If early and late investments are sufficiently substitutable, early investment may fall (e.g. if investments are perfect substitutes). Regardless of whether early investment rises or falls, individuals acquire more total human capital, h_3, when the borrowing constraint on older children is relaxed.

Now, consider relaxing the constraint on young children when children are constrained in both investment periods. Relaxing the early constraint effectively moves resources from late to early childhood.

Proposition 3. *Assume that borrowing constraints bind during early and late childhood (i.e. $A_2 = -L_1$ and $A_3 = -L_2$), but there are no future borrowing constraints. If Condition 1 does not hold, then $\frac{\partial i_1}{\partial L_1} > 0$ and $\frac{\partial i_2}{\partial L_1} < 0$.*

Proof: See Appendix A.

Sufficient substitutability in investments over time implies that shifting resources from late to early childhood by relaxing the early borrowing constraint causes investment to shift toward the early period and away from the later period. By contrast, with strong complementarity, investment will tend to move in the same direction in both periods. In most cases, investments are likely to increase; however, it is possible that investments may actually *decline* in both periods.[16]

As an alternative to increasing borrowing opportunities for young children, consider increasing financial transfers. The following proposition compares the effects of increasing early transfers on human capital investment with the impacts of an increase in early borrowing limits.

Proposition 4. *Assume that borrowing constraints bind during early and late childhood (i.e. $A_2 = -L_1$ and $A_3 = -L_2$), but there are no future borrowing constraints.*[17] *Then,*

(i) if Condition 1 holds, then $\frac{\partial i_1}{\partial y_1} > 0$ and $\frac{\partial i_2}{\partial y_1} > 0$; $\frac{\partial i_1}{\partial y_1} > \frac{\partial i_1}{\partial L_1}$ and $\frac{\partial i_2}{\partial y_1} > \frac{\partial i_2}{\partial L_1}$;

(ii) if Condition 1 does not hold, then $0 < \frac{\partial i_1}{\partial y_1} < \frac{\partial i_1}{\partial L_1}$ and $\frac{\partial i_2}{\partial L_1} < \frac{\partial i_2}{\partial y_1} < 0$.

Proof: See Appendix A.

If investments are sufficiently complementary, a pure income transfer to young children increases early and late investments; otherwise, it increases early investment but reduces late investments. Loans and transfers have the same impact on the marginal cost of early investment;

[16] One can show that if $\frac{\partial i_1}{\partial L_1} < 0$, then $\frac{\partial i_2}{\partial L_1} < 0$. Intuitively, if late investment is very productive, then relaxing the early borrowing constraint can 'starve' investment at later ages. With sufficient complementarity, individuals may choose to reduce early investment as well. In an online appendix, we discuss this seemingly perverse case for a Leontiff human capital production function. This strange scenario does not seem to be empirically relevant – early investment rises given the parameters calibrated in Section 5.1.

[17] If constraints do bind in the future, similar results hold using a complementarity condition that is modified to incorporate the future constraints.

however, the marginal cost of late investment increases more with an early loan since it must be repaid. The latter causes late investment to increase by less (or fall by more) in response to an increase in the loan limit. When investments are sufficiently complementary, early and late investment both increase more with a transfer; otherwise, a loan increases early investment and decreases late investment more than does a transfer.

Finally, we show how policies targeted to adult ages can affect investments at earlier ages due to the forward-looking nature of investment decisions. These results are important below in our intergenerational framework when we consider the impacts of permanent policy changes that affect the constraints faced by young parents. These policies have two distinct effects on child investment behavior in equilibrium. First, they affect young children via their impacts on parental transfers, y_1 in the current context. Second, forward-looking children adjust their investment behavior in response to changes in future opportunities when they become adults.

Transfers and loans for constrained young adults affect the marginal returns to early and late investment. As seen in equations (8) and (9), the marginal returns are increasing in the marginal utility of consumption at adult ages and decreasing in consumption levels. Consider a financial transfer y_3 to constrained adults in period 3 (modifying the budget constraint in that period accordingly). By increasing adult consumption, this income transfer reduces the marginal return on investments (both early and late). An equivalent increase in borrowing opportunities L_3 raises period 3 consumption but lowers subsequent consumption as the loan is repaid. As a result, it implies weaker (or even positive) effects on investment. These results are summarized in the next proposition. We concentrate on the case where investments are complementary, since that is most empirically relevant.

Proposition 5. *Suppose $f_{12} > 0$. Assume that the borrowing constraint binds for young children (i.e. $A_2 = -L_1$) and there are no borrowing constraints after period 3. Let y_3 reflect an income transfer received in period 3. Then $\frac{\partial i_1}{\partial y_3} < \frac{\partial i_1}{\partial L_3}$ and $\frac{\partial i_2}{\partial y_3} < \frac{\partial i_2}{\partial L_3}$. As a consequence, if $A_4 > -L_3$, then $\frac{\partial i_1}{\partial y_3} < 0$ and $\frac{\partial i_2}{\partial y_3} < 0$.*

Proof: See Appendix A.

First, consider the case when the individual is constrained while a young child, but not constrained in young adulthood. This means that increasing the borrowing limit in the third period will have no effect on optimal investments. An income transfer in young adulthood reduces the marginal benefit of earlier investments, which due to the early constraint are measured in terms

of the marginal utility of consumption in future periods. Therefore, investment in both periods falls. When the person is also constrained in young adulthood, things are a bit more complicated. Even if he is not constrained in the investment periods, this later constraint has a negative effect on investment in at least one period (Proposition 1). Offering an income transfer mitigates this effect, increasing investment towards the unconstrained choice. On the other hand, if the person is also constrained during the investment periods, this income transfer has the additional effect of decreasing the marginal benefit of investment, which in turn decreases optimal investment. Together, this implies an ambiguous effect on investment of an income transfer to a constrained young adult. The effect of a loan relative to a transfer in young adulthood on early investment is always greater. This is because a loan in young adulthood reduces the marginal benefit of investment received in that period, but increases the marginal benefit of investment received in the next period, when the loan is repaid. Therefore, the potentially negative effect of a loan is less than that of a transfer.

Suppose that an individual is constrained when a young child. Altogether, the above results imply that if an individual receives a transfer *while investing* in human capital, he will increase both early and late investments. We call this the "current effect" of a transfer. If instead, he expects to receive a transfer *while earning the returns* on his investments, we call the effect that this has on investments the "future effect" of a transfer. Current increases in borrowing opportunities may increase human capital investments, but the increases will be less than when an equivalent transfer is offered. The expectation of improved future borrowing opportunities might reduce human capital investments, but it will do so less than a future transfer does. While both loans and transfers increase consumption during the period offered, the loan also reduces future consumption. This key difference implies more muted responses to loan policies. Considered together, these "current" and "future" effects of policy highlight the importance of quantitative work in an intergenerational context when children are affected both directly and indirectly by policies targeted at adults. Children are directly impacted, since they will become adults themselves one day. They are indirectly impacted through changes in transfers from their parents as we discuss further below. Since the "current" and "future" effects tend to push investment in opposite directions, it is unclear *ex ante*, whether transfers and loan policies will increase or decrease investment if the policies are operative over the entire lifecycle. Furthermore, it is unclear which type of permanent policy change, increasing loans or transfers to young adults/parents, will have a greater impact on human capital investments.

These results have assumed that transfers are exogenous. Next, we consider an intergenerational framework in which y_1 and y_2 are endogenously determined parental transfers.

4 An Intergenerational Framework

In this section, we explicitly incorporate intergenerational relationships in which parents endogenously transfer resources to their children. Doing so allows us to investigate policies that affect a parent and child jointly. Moving to an intergenerational setting does not alter the main insights about investment highlighted in the lifecycle model of the previous section. Relaxing constraints on old children has the same qualitative effect on investments and human capital, while allowing constrained parents of old children to borrow more is akin to giving old children an income transfer. Relaxing constraints on parents of young children is qualitatively similar to relaxing constraints on young children.

Parents typically respond to increases in their own borrowing capacity by increasing transfers to their children that period; however, they may reduce transfers in later periods when loans must be repaid. Parents respond to improvements in their older children's borrowing opportunities by reducing their transfers that period; however, they may increase or decrease earlier transfers depending on the complementarity of investments (and whether early investments increase or decrease).

4.1 Two-Generation Problem

We begin with a two-generation problem in which the parent makes transfers to her child in periods 3 and 4 of her life (periods 1 and 2 of the child's life). (In the next subsection, we consider a dynastic framework in which children have children of their own.) Children make their decisions based on transfers y_1 and y_2 given to them by their parents. Parents receive utility from transferring resources to their children. Denote the value of transferring y_1 and y_2 to a child of ability θ by $\tilde{U}(y_1, y_2, \theta)$. Parents choose their own consumption, savings/borrowing, and transfers to their children subject to any borrowing constraints. A young parent with assets A_3, human capital h_3, and a child with ability θ solves the following problem:

$$V(h_3, A_3, \theta) = \max_{c_3, c_4, c_5, y_1, y_2} \sum_{j=3}^{6} \beta^{j-3} u(c_j) + \tilde{U}(y_1, y_2, \theta) \tag{11}$$

subject to equations (3)-(4), budget constraints:

$$A_{j+1} = \begin{cases} RA_j + W(h_j) - y_{j-2} - c_j & \text{for } j = 3, 4 \\ RA_j + W(h_j) - c_j & \text{for } j = 5, \end{cases} \tag{12}$$

borrowing constraints:

$$A_{j+1} \geq -L_j, \text{for } j = 3, 4, \tag{13}$$

$c_6 = RA_6$, and non-negative consumption and transfers: $c_j \geq 0$ for $j = 3, ..., 6$, and $y_j \geq 0$ for $j = 1, 2$.

We assume that $\tilde{U}(\cdot)$ reflects parental altruism (as in Becker-Barro preferences) for the child's utility over those transfers, so $\tilde{U}(\cdot) = \rho U(\cdot)$ where $U(\cdot)$ is defined in equation (5). In this case, the parent's and the child's problem clearly become interdependent. Parents' choices depend on their children's ability and investment choices, and children's choices depend on the transfer functions of their parents.[18] Our analysis here assumes that $L_1 = 0$, so that young children cannot borrow; however, we continue to allow for the fact that older children may be able to borrow themselves. We summarize key analytical results below and provide detailed propositions in Appendix B.

When old children are allowed to borrow more, their investment responses are qualitatively similar to the case with fixed parental transfers (i.e. Proposition 2). However, parents endogenously respond to their old children's improved borrowing conditions by lowering transfers to them, partially offsetting the benefits of increased credit. Interestingly, when investment is sufficiently complementary, parents increase transfers to their children at young ages to help finance desired increases in early investment. If investments are sufficiently substitutable over time, children shift investment from the earlier to the later period, and parents reduce transfers at both ages.

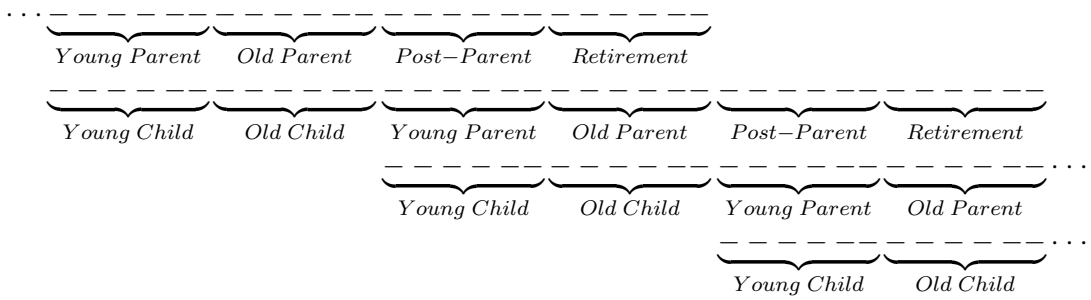
Now, consider the effects of relaxing borrowing constraints on parents. We begin with the constraint on young parents. Analogous to Proposition 3, allowing young parents to borrow more leads to increased early investments and a decrease in late investments if investments are sufficiently substitutable. Strong complementarity will tend to produce increases in investment at both stages of development.[19] Parental transfers to young children increase if early investments rise, while transfers to older children fall if late investments fall.

Lastly, we look at the effects of loosening the borrowing constraint on older parents. When the old parent can borrow more, he passes some of the extra resources onto his old child. Be-

[18] Whether the parents determine the level of transfers and investments, or merely the level of transfers and allow the child to determine the level of investments can be important in some contexts. Brown, et al. (2011) develop a multi-period family model in which strategic behavior can arise. In their framework, children have an incentive to under-invest in period one to extract more from the parent in period two. Parents respond by tying transfers to educational spending. We simply assume parents make investment and consumption choices for young children, thus strategic behavior is not a problem at this stage. Because the link between parents and children is severed after children become parents themselves, we avoid strategic behavior in the second period as well.

[19] For similar reasons to those discussed for Proposition 3, it is still possible for investment to fall in both periods as a result of the shift of resources from the second to the first period.

Diagram 1: Generations of a Dynasty



cause the link between parent and child is severed after this period (by assumption), an increase in the parent's borrowing limit effectively results in a larger transfer to the old child. The old child takes this additional income and increases late investment and consumption. If investments are complementary enough, early investment increases; otherwise, it falls. Human capital always increases. The parent responds to increases (decreases) in early investment by increasing (decreasing) transfers that period, although by less than the change in investment.

4.2 A Dynastic Framework

To understand the long-run effects of policy, we need to track the evolution of assets and human capital over time. This requires an intergenerational dynastic framework. Assuming that children become parents themselves, valuing their own children's utility, and that those children have children of their own, the problem becomes dynastic. Parents transfer resources to their children, who grow up, become parents of their own children, and transfer resources to them in an analogous fashion. In a dynastic framework, permanent policy changes not only affect human capital investments via their impact on opportunities today but also through their impacts on opportunities in the future. As demonstrated in Propositions 4 and 5, the current and future impacts of new loans or transfers can move investment in opposite directions requiring quantitative work.

As above, we assume that people live through six stages in their life. We now refer to these stages as young and old childhood, young and old parenthood, post-parenthood, and retirement. The lifecycle of different generations in a dynasty is given by Diagram 1.

We assume that young children cannot borrow or save themselves (i.e. $A_2 = 0$). We further assume that young parents make investment and consumption decisions for their young children.

Although old children make investment decisions, we assume that it is their last period of financial interaction with their parents, so there is no scope for strategic behavior. It is, therefore possible to write the entire family problem from the point of view of parents. We assume that the ability of a child depends on that of the parent following a simple Markov process. Once a child is born, the parent's ability becomes irrelevant. However, the child's ability will affect parental decisions, because it affects the child's ability to accumulate human capital, and it affects the future ability of the grandchild. Therefore, the value function for a young parent with a young child depends on the child's ability in addition to the parent's own human capital and assets.

To make the problem more realistic and suitable for quantitative analysis, we modify the family problem in three important ways: We introduce earnings shocks, allow borrowing constraints to depend on human capital levels, and incorporate public spending on education and investment subsidies. Due to the nature of assumed earnings shocks, it is useful to allow for human capital-specific growth rates, $\Gamma_4(h_3)$ and $\Gamma_5(h_4)$, as discussed further below.

To account for variation in earnings within education classes, we introduce period j-specific earnings shocks ϵ_j for young and old parents, so

$$W(h_j, \epsilon_j) = wh_j + \epsilon_j, \quad \text{for } j = 3, 4, \tag{14}$$

$$W(h_5) = wh_5. \tag{15}$$

These shocks are distributed such that earnings are always non-negative. To simplify computation, we abstract from shocks in period 5, when parents and children are no longer economically linked.

We allow borrowing constraints to depend on the future human capital of an individual to account for the possibility that more educated persons are able to borrow more. This is both theoretically and empirically attractive for reasons discussed in Lochner and Monge (2011).[20] We assume that borrowing limits depend on the lowest possible discounted value of future earnings, since that determines the amount a person can credibly commit to re-pay under any circumstances (Ayagari 1994). Letting $\underline{\epsilon}_j = min\{\epsilon_j\}$ represent the lowest possible earnings shock in period j,

[20] Lochner and Monge (2011) argue that more skilled individuals can commit to re-pay higher debts, explaining why private lenders offer them more credit. Furthermore, the federal student loan system explicitly links loan amounts to post-secondary enrollment and the level of schooling attended.

we assume the following limits on borrowing:

$$\begin{aligned}
L_2(h_3) &= \gamma[R^{-1}(wh_3 + \underline{\epsilon}_3) + R^{-2}(w\Gamma_4(h_3)h_3 + \underline{\epsilon}_4) + R^{-3}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3], \\
L_3(h_3) &= \gamma[R^{-1}(\Gamma_4(h_3)h_3 + \underline{\epsilon}_4) + R^{-2}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3], \\
L_4(h_3) &= \gamma R^{-1}w\Gamma_4(h_3)\Gamma_5(h_3 + \Gamma_4(h_3))h_3,
\end{aligned}$$

where $\gamma \in [0, 1]$. Intuitively, the parameter γ reflects the efficiency of credit markets, since γ near zero implies that no borrowing is allowed while γ near one implies that individuals can borrow fully against guaranteed future earnings.[21]

We incorporate freely provided public investment in each period of childhood, denoting these public investments p_1 and p_2. Thus, total investment in period j is given by $p_j + i_j$. We further assume that private spending on investment in each period is subsidized at rates s_1 and s_2. Below, we consider the effects of policies that adjust these publicly provided investment levels and subsidy rates.

Letting prime superscripts denote the child's variables, the problem facing a young parent with a young child is given by:

$$V_3(h_3, \epsilon_3, A_3, \theta') = \max_{c_3, A_4, c_1', i_1'} \{u(c_3) + \rho u(c_1') + \beta E_{\epsilon_4} V_4(h_4, \epsilon_4, A_4, h_2', \theta')\}$$

subject to

$$\begin{aligned}
A_4 &= RA_3 + W(h_3, \epsilon_3) - c_3 - i_1'(1 - s_1) - c_1', \\
A_4 &\geq -L_3(h_3), \\
h_2' &= p_1 + i_1', \\
h_4 &= \Gamma_4(h_3)h_3,
\end{aligned}$$

$c_3 \geq 0$, $c_1' \geq 0$ and $i_1' \geq 0$. Since young children are not allowed to borrow on their own, the only constraint on borrowing represents that imposed on young parents. The expectation of V_4 is taken over the earnings shock the young parent will receive as an old parent.

The problem facing an old parent with an old child is given by:

$$V_4(h_4, \epsilon_4, A_4, h_2', \theta') = \max_{c_4, A_5, c_2', i_2', A_3'} \{u(c_4) + \beta V_5(h_5, A_5) + \rho \left[u(c_2') + \beta E_{\theta'', \epsilon_3'} \left(V_3(h_3', \epsilon_3', A_3', \theta'' | \theta')\right)\right]\}$$

[21] Of course, γ could vary across stages of the lifecycle. We do not pursue this extension, because we do not expect to be able to separately calibrate three different γ values given our data.

subject to

$$\begin{aligned}
A_3' + A_5 &= RA_4 + W(h_4, \epsilon_4) + W_2 - c_4 - c_2' - i_2'(1 - s_2), \\
A_3' &\geq W_2 - c_2' - i_2'(1 - s_2), \\
A_5 &\geq -L_4(h_4), \\
A_3' &\geq -L_2(h_3'), \\
h_3' &= f(h_2', p_2 + i_2', \theta'), \\
h_5 &= \Gamma_5(h_4)h_4,
\end{aligned}$$

$c_4 \geq 0$, $c_2' \geq 0$ and $i_2' \geq 0$. The second constraint ensures that parental transfers are non-negative. That is, parents cannot borrow against their offspring's earnings. Both the old parent and the old child face constraints on their borrowing as shown in the third and fourth constraints. The expectation of V_3 is taken over the earnings shock the old child will receive as a young parent and the ability level of the future grandchild conditional on the ability of the child, θ'.

The problem facing a post-parent with no child at home is a standard lifecycle consumption/savings problem:

$$V_5(h_5, A_5) = \max_{A_6} \{u(RA_5 + W(h_5) - A_6) + \beta u(RA_6)\}.$$

This is easily solved analytically (given our assumed utility function below) and incorporated into the old parent's problem.

5 An Empirically Based Quantitative Analysis

In our computational analysis, we assume that there are a finite number of investment and ability levels, but a continuum of asset levels. The finite investment and ability grids imply a finite number of human capital levels.

We assume a CES human capital production function, as in equation (2), and a CIES utility function, given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \sigma \geq 0.$$

5.1 Calibration

We use data from the March Current Population Surveys (CPS), National Longitudinal Survey of Youth 1979 Cohort (NLSY79), and Children of the NLSY79 to calibrate our model to the U.S. economy.[22] The six model periods are mapped into ages 0-11, 12-23, 24-35, 36-47, 48-59, and 60-

[22] In this analysis, we use NLSY79 and Children of the NLSY79 collected through 2010 and CPS data from 2006.

71. We consider four values of i_2 associated with different observed schooling levels: high school dropouts (less than 12 years of completed schooling), high school graduates (exactly 12 years of completed schooling), some college (13-15 years of completed schooling), and college graduates (16 or more years of completed schooling). An annual interest rate of $r = 0.05$ is assumed throughout, so $R = (1+r)^{12} = 1.7959$. We assume $\beta = R^{-1}$. All earnings are in 2008 dollars (deflated by the CPI-U). We normalize $w = 1$, so human capital is measured in 2008 dollars per year. Finally, we choose the preference parameter $\sigma = 2$, which implies an intertemporal elasticity of substitution for consumption of 0.5. This is consistent with estimates in the literature (Browning, Hansen and Heckman 1999).

We assume that income shocks are *iid* log normally distributed.

Assumption 2. $\epsilon_j \sim logN(m, s^2)$, *for* $j = 3, 4$.

We also assume a two-state Markov process for ability.

Assumption 3. $\theta \in \{\theta_1, \theta_2\}$ *with* $Pr(\theta_j = \theta'_j) = \pi_j$ *for* $j = 1, 2$.

Along with using data to guide our choice for the investment grids, the following parameters must be determined empirically: potential earnings in school (W_2), post-school income shock distributions (m, s), human capital growth rates (Γ_4, Γ_5), the human capital production function, (a, b, d), the Markov process for ability $(\theta_1, \theta_2, \pi_1, \pi_2)$, parental altruism towards children (ρ), and the debt constraints (γ). We first discuss parameters that are chosen to match data outside of the model and then sketch the calibration process for all remaining parameters.

Second Period Earnings and Investment Costs

We first directly estimate potential earnings for ages 12-23, W_2, using the Children of NLSY79. We also estimate foregone earnings from these data, which are combined with direct educational expenditures by schooling level (from the Digest of Education Statistics 2008) to determine second period investment amounts, i_2.

Using the random sample of Children of the NLSY79, we estimate the discounted present value of average earnings for high school dropouts over ages 16-23.[23] Dividing the average annual discounted income over this period by 12 yields an annualized potential income measure of $W_2 = 11,187$. This also reflects the total amount of foregone earnings for individuals in our highest schooling category: college completion. Foregone earnings for 'high school graduates' (those with

[23] A discount rate of $r = 0.05$ was used to discount earnings to age 18. All dollar amounts are adjusted to year 2008 dollars using the CPI-U.

'some college') are given by the discounted present value of earnings for dropouts over ages 16-18 (16-20), dividing by 12 to annualize the amounts. We assume no foregone earnings for high school dropouts, since individuals cannot typically work before age 16.

We distinguish between total investment amounts and the amount privately paid by individuals themselves, since education is heavily subsidized in the U.S. Total investment amounts include foregone earnings and total public and private education expenditures. Consider first the investments made by old children ages 12-23. To calculate expenditures associated with grades 6-12, we use the average expenditure per pupil for all public elementary and secondary schools from 1990-91 to 1994-95, \$8,552. For the schooling category 'some college', we add two years of current-fund expenditures per student at all post-secondary institutions (\$25,902 per year) to the costs of high school. For 'college graduates', we add five years of current-fund expenditures per student at four-year post-secondary institutions (\$32,712 per year) to the costs of high school.[24] Combining foregone earnings with direct expenditures and dividing by 12 to annualize the amounts, we obtain total investment amounts ($p_2 + i_2$) of \$3,563, \$5,912, \$13,369, and \$29,805 for high school dropouts, high school graduates, some college, and college graduates, respectively.

Foregone earnings are borne by individuals, but we assume that primary and secondary schooling is otherwise publicly provided at no private cost. Since dropping out of high school entails no foregone earnings or other private costs, we set $p_2 = 3,563$. This amount is subtracted from total investment amounts to obtain private i_2 investments (inclusive of marginal subsidies) of \$0, \$2,260, \$9,374, and \$25,082 for high school dropouts, high school graduates, some college, and college graduates, respectively. High school graduates only pay foregone earnings (roughly two-fifths of their total investment), while college students pay both foregone earnings and a share of direct costs, which are heavily subsidized. Dividing revenue from tuition and fees by total revenue for all degree-granting post-secondary institutions in 1995-96 suggests that student payments account for only 28% of college revenues. Altogether, individuals pay roughly 40% of their total marginal costs of finishing high school and about 55-60% of the total marginal costs of college. Striking a balance between these figures, we set the second period marginal investment subsidy rate to $s_2 = 0.5$.

Since there are no forgone earnings for young children, we take the annualized value of \$3,563

[24] All schooling expenditure figures are taken from the Digest of Education Statistics (2008) and are adjusted to year 2008 dollars using the CPI-U. Primary and secondary expenditures are based on averages over the 1990-91 to 1994-95 period (Table 181). Post-secondary expenditures are based on all degree-granting institutions in 1995-96 (Table 360).

as the minimum period one investment.[25] Assuming this level of investment is completely subsidized for young children, we set $p_1 = 3,563$ and consider a grid for period one private investments i_1 ranging from zero to \$15,000, an amount very few parent wish to invest.[26] We set $s_1 = 0$, since private investments by parents in their young children are not typically subsidized in the U.S.

Earnings Growth Rates

We set $\Gamma_4(h_3)$ and $\Gamma_5(h_4)$ to match growth in average income levels, $E[W(h_4, \epsilon_4)]/E[W(h_3, \epsilon_3)] = 1.478$ and $E[W(h_5)]/E[W(h_4, \epsilon_4)] = 1.077$, using data from the NLSY79 and 2006 March CPS, respectively.[27] This approach assumes that individuals face the same expected growth in earnings regardless of their human capital level. Because earnings shocks ϵ_3 and ϵ_4 are non-negative, different human capital growth rates are needed for each level of human capital to produce the same growth rates in expected earnings.

Calibrating other Parameters Using Simulated Method of Moments

A number of remaining parameters need to be calibrated by simulating the model and comparing the resulting allocations with those observed in the data. In particular, we need to determine parameters of the earnings shock distribution (m, s), the human capital production function (a, b, d), parental altruism towards their children (ρ), the ability distribution $(\theta_1, \theta_2, \pi_1, \pi_2)$, and the debt constraint parameter (γ). We use a simulated method of moments procedure, which chooses parameters to best fit moments for educational and earnings dynamics using data from Children of the NLSY79. This step requires fully solving the dynastic fixed point problem of Section 4.2 in steady state, simulating a number of conditional moment conditions, and comparing those moments with their empirical counterparts. In particular, we fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 and 36-47 in the NLSY79, (iii) child schooling levels conditional on parental income and maternal schooling, and (iv) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and parental income levels (when the child is ages 0-11). Appendix C provides greater detail on the calibration.

[25] This corresponds to the sum of average annual expenditures per pupil of \$8,552 for grades 1–5 divided by 12 (to annualize the amount).

[26] Our grid for early investments includes seven points from 0 to 15,000, with equally spaced increments of 2,500. Modest changes in the number of grid points or the upper limit produces very similar results.

[27] In both cases, we use data for men deflated to year 2008 dollars. We discount within period earnings to ages 30, 42, and 54 using a 5% interest rate. We drop observations for respondents with annual earnings less than \$200 or greater than \$275,000 or those with less than 9 years of completed schooling.

Table 2 shows the distribution of educational attainment for our NLSY calibration sample along with the calibrated steady state distribution produced by our model. While the model matches high school completion and college attendance rates quite well, it under-predicts college completion rates.

To help identify the earnings shock distribution and human capital levels, we match the mean and standard deviation of earnings from the random sample of men in the NLSY79 ages 24-35 and 36-47 (discounted to ages 30 and 42, respectively, using a 5% interest rate). Table 3 reports these statistics in the NLSY79 and the calibrated steady state for our model.

Child educational attainment (i_2' in our model) should depend on early investments, child ability levels, second period parental income, and parental assets. Not all of these are observed in the NLSY data; however, we could simulate our model to fit $Pr(i_2'|I_3, I_4, i_2)$ where I_j reflects total parental earnings in period $j = 1, 2$ of the child's life. In practice, we condition on three categories of parental incomes I_3 and I_4: bottom quartile, second quartile, and top half of the age-specific income distribution. This approach reflects the fact that parental income (when children are young and old) should affect investments in older children. Furthermore, parental education should be related to parental ability, which affects the child's ability and therefore investments in children. The extent to which parental income and education affects child education levels depends on the unknown parameters in complex ways. Given our other moments, these moments are most useful in identifying parameters affecting the extent of credit constraints, the complementarity of early and late investments, the intergenerational correlation in ability, and parental altruism. In the absence of binding credit constraints, the timing of parental income should have relatively minor effects on family investment decisions and children's educational outcomes. Timing becomes more important as early credit constraints become more stringent and investments become more complementary. Finally, the intergenerational correlation in educational investments will clearly depend on the correlation in abilities and parental altruism.

Table 4 reports the education distribution for our model and the NLSY79 data by parental education, while Table 5 shows the education distribution by parental income when the child is young and old.[28] Educational attainment is strongly increasing in parental education and income. The model slightly over-predicts the importance of parental education for high school completion, while it under-predicts its importance for college attendance and completion. The intergenerational correlation in i_2 investment amounts is 0.33 for our model and 0.32 in the NLSY

[28]See Appendix C for the full set of moments (i.e. child education by parental education and income at young and old ages) used in calibration along with their steady state counterparts.

data.[29]

A few interesting patterns emerge from Table 5 when we simultaneously condition on both early and late parental income. Considering college completion, late income is relatively unimportant conditional on early income, while the reverse is not true. Early income is quite important for college even after conditioning on later income. These patterns are clearly evident in both the NLSY data and our model. When looking at high school completion, the model suggests that both early and late income are important (conditional on the other), while the NLSY suggests a more modest role for late parental income. Overall, the model replicates key features of the relationship between child educational attainment and parental education and income. As noted earlier, these relationships are central for identifying the human capital production technology and borrowing limits.

Wages during early adulthood depend on human capital levels, h_3'. Conditional on schooling i_2', human capital and wages should be generally increasing in early childhood investments and the child's raw ability. Thus, we also fit $E(W_3'|I_3, i_2, i_2')$, using wage income for youth ages 24-35 as our measure of W_3 and three income categories for I_3 as described above.[30] These moments are helpful in identifying parameters of the human capital production function, including the distribution of ability and its intergenerational correlation. Appendix C reports these moments in the data and our calibrated steady state.

Our calibrated parameter values are reported in Table 6. A value of $b = -1.1$ implies an elasticity of substitution between early investments and late investments of 0.48 – strong dynamic complementarity similar to that estimated by Cunha, Heckman and Schennach (2010). Interestingly, the model implies a similar weight on early and late investments, with a near one-half. Values for θ suggest that high ability individuals are roughly 2.5 times as productive as their low ability counterparts. Our calibration suggests modest intergenerational persistence in ability: high ability parents have a high ability child 76% of the time, while low ability parents only have

[29] Assuming that high school dropouts have 10 years of schooling, high school graduates 12 years, some college 14 years, and college graduates 17 years, the model produces an intergenerational correlation for years of schooling of 0.40 compared to 0.34 in our NLSY sample.

[30] Given enough data, we could fit $E(W_3'|I_3, I_4, i_2, i_2')$. However, conditioning on I_4 probably adds little additional identifying variation, since only i_1' and θ' affect expected wages conditional on i_2' and these are largely determined by I_3 and i_2. Note that we use weekly earnings for our measure of W_3 (due to data availability and the desire to best capture differences in human capital), while we use the distribution of annual earnings for men in helping identify earnings growth and the distribution of shocks (as described above). Since the units for these are quite different, we fit the ratio of $E(W_3'|I_3, i_2, i_2')$ for each category of (I_3, i_2, i_2') relative to the corresponding average for a baseline group of high school graduates with high school graduate mothers whose early parental income is in the lowest quartile. Wages for all relevant ages are discounted to age 30 using an interest rate of 5%. For youth in the high school dropout and graduate categories, we use wages measured over ages 22-35 rather than 24-35 to expand our sample and increase precision.

a high ability child 41% of the time. The calibrated value of $\rho = 0.67$ is far from 'pure altruism', but it still implies that considerable value is placed on children and grandchildren. Finally, our calibrated value for γ implies that individuals can only borrow up to 45% of their minimal discounted lifetime earnings at any age. Thus, credit limits are far more stringent than the 'present value' limit of Aiyagari (1994).

5.2 Additional Features of the Baseline Steady State

Tables 4 and 5 show the relationship between children's educational attainment and parental education and income in our baseline steady state. Table 7 shows how average early (i_1) and late (i_2) private investment amounts vary with parental education in our benchmark steady state. On average, parents annually invest \$2,121 in their young children and \$7,227 in their older children. (Because of the 50% marginal subsidy on late investments, private investment expenditures for older children are only \$3,864.) Private investments in young (old) children are roughly five (four) times as great for the children of college graduates compared to high school dropouts. Kaushal, Magnuson, and Waldfogel (2011) find that high school dropout parents spend \$825 per child, annually on educational enrichment, while parents that graduated from college spend \$4,671. Aggregating early and late private investment expenditures (i.e. $\frac{i_1+(1-s_2)i_2}{2}$), we obtain very comparable measures: \$1,115 and \$4,873, for high school dropout and college graduate parents, respectively.[31]

Our calibrated steady state suggests that roughly half of all young parents and 12% of all old parents are borrowing at their limit, while no older youth are borrowing constrained. The share of young parents that are borrowing constrained is greater for those who attended (60%) or completed (68%) college relative to those who only finished high school (38%) or who dropped out (51%). Interestingly, this relationship is non-monotonic: high school graduates are less likely to be constrained than high school drop outs and college attendees and graduates. These patterns are consistent with a relatively high demand for credit among young high school dropouts that experience a bad income shock. For them, a low earnings shock is quite costly given already low expected income levels. More educated young parents tend to be constrained for other reasons. First, many already have considerable debt from their own education. Second, more educated parents desire more credit to fund higher levels of investment in their children, since their children are very likely to be of high ability.

[31] See Table 3 of the online appendix from Kaushal, Magnuson, and Waldfogel (2011). Amounts reported here exclude enrichment spending allocated to parents.

The share of old parents that are borrowing constrained is monotonically increasing in educational attainment. Adverse income shocks at older ages increase the demand for credit less than at younger ages (especially for the least educated), since expected income levels are higher (due to lifecycle wage growth) and retirement is closer. Even though none of the older youth are borrowing constrained, the possibility of facing future borrowing constraints once they become parents can affect their current human capital investment behavior as we discuss further below.

In our calibrated steady state, roughly 5% of all older parents are 'transfer' constrained, transferring zero to their old children. Unlike borrowing constraints, the least educated parents are most likely to be transfer constrained (15% of high school dropouts make zero transfers to their older children while all college graduates make a positive transfer). This relationship is driven by the fact that less-educated parents are more likely to have low income relative to what their children can expect to earn. Most high school dropouts are of low ability; however, 40% of those with low ability will have a high ability child. Many of these parents apparently would like to receive income transfers from their children.

6 Policy Analysis

We next simulate a series of different policies to emphasize important economic forces affecting investment in human capital. In particular, we focus on policies that shed light on the interaction of borrowing constraints and investments at different ages. Intergenerational linkages through endogenous parental transfers play a key role in our analysis. First, we consider different loan policies to determine the importance of borrowing constraints at different stages of child development. We differentiate between the short- and long-run effects of increased borrowing, where the latter accounts for changes in human capital and asset distributions through intergenerational linkages. Second, we study fiscally equivalent early and late investment subsidy policies. Comparing these policies demonstrates the strong interaction between dynamic complementarity and early borrowing constraints. We also discuss the quantitative importance of incorporating early investment responses to policies that target subsidies at later ages. Lastly, we compare the effects of income transfers with those of loans. Here, we distinguish between "current" and "future" effects of these policies on investment. As noted earlier, policies that change the budget/borrowing constraints for parents not only affect children through parental transfer decisions ("current" effects); they also affect investment decisions by changing the returns to investment when children become parents themselves ("future" effects). This highlights the importance of

considering the full effects of lasting policy changes in a dynamic intergenerational environment.

6.1 Increasing Borrowing Limits

Given the level of complementarity that we find between early and late investments and the fact that borrowing constraints bind for many young parents in our baseline steady state, our analytical results suggest that relaxing early borrowing constraints should lead to increases in investment during both early and late childhood. To investigate this quantitatively, we simulate the 'short-run' and 'long-run' responses to a permanent $2,500 increase in the borrowing limit for all young parents (leaving all other borrowing limits unchanged). The effects this has on early and late investments in children and on their average post-school wages are reported in Table 8. By 'short-run', we refer to responses by the first generation to be fully affected by the policy change. By 'long-run', we refer to decisions in the new steady state many generations later. The former shows how families respond to the policy, given the distribution of assets and human capital in the baseline steady state, while the latter takes into account the fact that parental asset and human capital distributions change over time in response to expanded borrowing opportunities.

Focusing first on short-run impacts, Table 8 reveals that relaxing borrowing constraints on young parents would lead to sizeable increases in both early and late investments in children. Interestingly, increases in early investments would be greatest among children from more educated households. This partly reflects the fact that college educated parents are the most likely to be borrowing constrained. It also reflects the fact that more educated parents want additional credit to bolster investment, while constrained high school dropouts appear to desire additional credit primarily to help smooth consumption. Parents with a college degree would increase early investments in their children by 19% on average, while there would be no early investment response among parents that dropped out of high school. Despite the lack of an early investment response among less educated parents, their children are more likely to now invest in getting a high school education (with negligible average impacts on their final human capital and wage outcomes). Older children are willing to take on more debt to invest in their education, because they know they will be able to borrow more when they become young parents themselves (when they are likely to be constrained). This underlines an important point: Even if a person is not currently borrowing up to his debt limit, his decisions are affected by the possibility of future binding constraints. Among children whose parents attended or completed college, effects on high school completion are small (almost all already complete high school) while there are sizeable increases in the probability of finishing college. The combined effects of increased early and late investment

on average wage levels upon labor market entry are as high as 3.6% for the children of college graduates. Average short-run increases in wages among all young workers are about 1.5%.

The right half of Table 8 reports the long-run changes in investment and wages in the new steady state (many generations later). These changes incorporate the fact that many older children borrow more and find themselves in greater debt when they become young parents. While constraints on young parents are less likely to bind, more older parents (especially those with lower education levels) become borrowing and transfer constrained (see Table 9). Asset distributions at all ages generally shift left. Despite the fact that constrained persons with any given level of assets and human capital are likely to invest more in their children (this is precisely what the short-run effects demonstrate), the long-run shifts in asset distributions lead to lower overall early investment levels. This is most pronounced for children from the least educated families; although, the 16% drop in average i_1 among these families is only about $100 given their low initial investment levels. Due to dynamic complementarity, these drops in early investment are accompanied by reductions in college completion rates among children of less-educated parents; however, high school completion rates actually increase slightly. By contrast, long-run responses by children of college graduates are more positive for both early and late investments.

These results suggest that relaxing borrowing constraints on young parents can be a double-edged sword in terms of investment in human capital. In the short-run, there are obvious gains in human capital investment among constrained families. Although the increased borrowing opportunities do not directly benefit unconstrained parents, they benefit their children and future generations who may become constrained. Parents take some of the 'family' gains by transferring less to their children. While this is good in terms of 'family' or 'dynastic' welfare, it can saddle future generations with more debt. This debt gets passed on across generations through smaller financial transfers and, in some cases, less human capital investment. In the long-run, asset distributions shift left and investment declines slightly. In general, these forces appear to be most pronounced at the bottom of the education distribution. While one may not typically be concerned about outcomes many generations into the future, we observe long-run-like investment responses for second- and third-generations affected by the policy. These results, therefore, underscore the importance of considering long-run policy impacts along with more immediate effects on current generations. They also highlight the fact that some policies may have important indirect effects on asset accumulation if future generations are affected: a policy may cause current generations to respond even if they themselves are not directly affected by the policy.

Because old children are not borrowing constrained in our baseline steady state, relaxing their borrowing limits has no effect on investment behavior.[32] Yet, this does not mean that investment decisions for old children are optimal (even conditional on early investment choices), since many of these children will face binding constraints as young and old parents. Still, allowing them to borrow more as old children does nothing to alleviate these future constraints.

Relaxing constraints on older parents has fairly small effects. While this enables parents to smooth their consumption and transfer more wealth to their children, the magnitude of new transfers is small and has little impact on children's investment behavior. In the long-term, the increase in parental transfers prevents the type of leftward shift in the asset distribution observed with increased borrowing for young parents. As a result, the long-run effects of increasing borrowing opportunities for older parents are positive and larger than the short-run effects, although they are still quite small. In the long-run, increasing loan limits for older parents by \$2,500 increases average early investment by 1.5% and average late investment by 2.3%.

6.2 Subsidizing Education

We next consider the effects of increasing subsidy rates for early and late human capital investments. This analysis highlights the implications of dynamic complementarity in investments and borrowing constraints when considering policies targeted to different stages of development.

In comparing the effects of subsidies to early and late investments, we increase s_1 and s_2 so that total expenditures on all education subsidies increase by roughly the same amount. Given the complementarity of early and late human capital investments, subsidizing investments at one age will tend to increase investments at all ages. Because $s_2 > 0$ in the baseline economy, the total cost of subsidizing early investment includes both the direct cost associated with raising s_1 and the indirect cost associated with any increase in subsidized late investments. Since early investments are not subsidized in the baseline economy, an increase in s_2 only entails direct costs for additional outlays on second period investment.[33]

Table 10 shows the short- and long-run effects of subsidizing early and late investments on average investments, the percent who graduate from high school and college, and average entry wage outcomes. The first row reports the effects of subsidizing early human capital investment at a rate of 10%. The per capita total cost of this policy is about \$900, with roughly two-

[32] This result is roughly consistent with with findings of Keane and Wolpin (2001) and Johnson (2010).

[33] The total per capita cost of increasing s_1 from zero to s_1' is given by $s_1'\bar{i}_1(s_1', s_2) + s_2[\bar{i}_2(s_1', s_2) - \bar{i}_2(0, s_2)]$, where $\bar{i}_j(s_1, s_2)$ reflects average investment in period j under subsidy policy (s_1, s_2). The total cost of increasing subsidies to late investment is $s_2'\bar{i}_2(0, s_2') - s_2\bar{i}_2(0, s_2)$.

thirds of this coming from the increased costs associated with subsidies for late investments. Not surprisingly, there are large increases in early investments in both the short- and the long-run (28% and 34%, respectively). Because investments are so complementary, this policy also increases late investments by roughly 12% in the short-run and 17% in the long-run. Most of the changes in the education distribution come from increases at the upper end. The percent who graduate college increases 30% in the short-run and 40% in the long-run. Changes in high school completion rates are negligible. Average post-school wages increase 3.1% in the short-run and 3.9% in the long-run. Unlike modest increases in borrowing limits, increased subsidy rates do not produce leftward shifts in asset distributions, since they enable families to invest more in their children without spending much more out-of-pocket.

We next consider the effects of increasing the subsidy to late investments from 50% to 53% at a cost of roughly $900 in the new steady state. We begin by discussing the effects of this policy when parents are aware of the higher subsidy rate when their children are young (row two of Table 10). Thus, both early and late investments may respond. We then discuss the short-term impacts on families who are unaware of the policy when making early investments in their children, so only late investments respond (row three of Table 10). This effectively measures the short-run effects for families with older children when the policy is first announced and introduced.

The second row of Table 10 shows the effects of increasing s_2 on families who are aware of the program when their children are young. Although this policy costs the same as a 10% subsidy to early investment, it has much weaker effects on human capital accumulation. Early investments increase by only 4% and 7% in the short- and long-run, respectively, compared with 30-40% for the early investment subsidy. Perhaps more surprisingly, increases in average late investments are quite similar to those for an increase in early investment subsidies.[34] While late subsidies have weaker impacts on college completion (compared to early subsidies), they appear to increase high school graduation rates more. Altogether, these investment responses imply a much smaller increase (1.4% in the short-run and 1.8% in the long-run) in average entry wage rates relative to a policy that subsidizes early investment.

These results underscore the important interaction between credit constraints and the dynamic complementarity of early and late investments in human capital. The fact that many young

[34]To put these numbers in context, our s_2 increase of 0.03 is roughly equivalent to a $1,300 reduction in annual tuition for the first two years of college. Our simulations suggest that this increases college attendance (i.e. some college or more) by 5-6 percentage points (depending on whether early investments are allowed to adjust). This is well within the range of estimated impacts of tuition and financial aid on college attendance in the U.S. Kane (2006) and Deming and Dynarski (2009) provide recent surveys of this literature, which concludes that a $1,000 reduction in tuition leads to a 3-5 percentage point increase in college attendance.

parents are credit constrained means that they cannot easily finance additional early investments in response to policies targeted to later ages. While unconstrained families increase both early and late investments in response to an increase in s_2, constrained young parents are limited in how much they can increase investments in their young children. Complementarity implies that if children do not receive adequate early investments, it may not be worth it for parents to make later investments, even if they are heavily subsidized. By contrast, early investment subsidies enable families to increase investments in their young children without having to sacrifice current consumption or borrow more. Those early investments can then be matched with later investments, when constraints are not as binding.

Row three of Table 10 reports the effects of an increase in s_2 that is announced after early investments have already been made. Looking at the short-term effects of this policy, we see more modest effects on late investment and human capital accumulation, because early investment is held fixed. Overall, average late investment increases about 8.7%, a little more than half the effect observed when early investment is also able to adjust. This, coupled with no change in early investment, produces a much smaller increase in wages (0.6% vs. 1.4% when early investment adjusts). Interestingly, increases in high school completion rates are quite similar whether or not early investment is able to adjust; yet, effects on college completion are negligible when early investment cannot respond, compared to a 23% increase when it can. This is because in order for college to be productive, considerable early investment must be made. This is less true for high school.

These results demonstrate the importance of considering the interaction of early and late investments when studying human capital investment decisions. Assuming that early investments and skill levels are fixed when analyzing policies that affect high school or college attendance decisions is not innocuous. Due to dynamic complementarity in investment, failing to account for adjustments in early investment not only neglects those changes, but it also leads one to underestimate the policy's true impact on late investments. Together, these imply substantial underestimation of policy effects on human capital and wages (except, of course, for those families with older children at the time of the policy change). In our framework, failure to account for early investment responses would cause the researcher to underestimate the full impact of post-secondary subsidies on wages by almost 60%.[35]

[35] It is worth noting that these concerns not only apply to structural models of schooling decisions (e.g. Cameron and Heckman 1998, Keane and Wolpin 2001), but they also apply to more standard regression or differences-in-differences estimates of the effect of tuition or financial aid changes on college attendance. These strategies may identify the very short-run effects on older cohorts of college-age children when the policy is implemented, but they

6.3 Income Transfers (currently being updated)

7 Conclusion

Empirically, we find that family income received at earlier ages of child development improves educational outcomes more than income received at later ages. Our estimates suggest that a $10,000 increase in discounted annual income from birth to age 11 would reduce the probability of high school dropout by 4 percentage points and increase college attendance and completion by 4-6 percentage points. The same increase in income over ages 12-23 has much smaller and statistically insignificant effects. The timing of family income is important, consistent with early borrowing constraints.

Our theoretical analysis of borrowing constraints on investment at different ages establishes the central role played by dynamic complementarity. When investments are sufficiently complementary over the lifecycle, policies that encourage investment in one period tend to raise investment in other periods as well. Our calibration identifies a strong degree of dynamic complementarity, and our quantitative analysis suggests that investments do move together in response to policy changes. Our quantitative analysis yields a number of other important general insights.

We find that many young and old parents are borrowing constrained, especially those with higher education who took out loans to finance their own education and who tend to have high ability children. However, like Keane and Wolpin (2001) and Johnson (2010), our model suggests that there would be little impact on human capital investment ('early' or 'late') from relaxing borrowing constraints on college-age youth or their parents. At least in the short-run, relaxing constraints on young parents would substantially increase both 'early' investments in young children and 'late' investments in older children (e.g. high school completion and college). For example, we find that a modest increase in the borrowing limit faced by young parents would increase early investment by 11% and college graduation rates by 10%. Interestingly, the effects are greater for families with more educated parents, since these families are constrained and want more credit for investment in their children. Less-educated parents want more credit primarily for current consumption.

We also consider the long-run impacts of permanently relaxing borrowing constraints, allowing the distribution of assets and human capital to change in response. Here, the results are quite different. Since relaxing the borrowing constraint for young parents causes families to accumulate more debt over time, future generations find themselves constrained to nearly the same extent

are unlikely to identify the medium-term effects on younger or future cohorts.

that initial generations were before the constraint was relaxed. On average, this shift in assets results in negligible long-run effects of relaxing the constraint on average human capital levels. Modest increases can be a double-edged sword, increasing human capital in the short-run but lowering family assets in the long-run.[36]

We explore the impact of subsidies for 'early' vs. 'late' investment. Two interesting lessons emerge from this. First, subsidies for investment at either stage raise investments at both stages, calling into question traditional analyses of college-age policies that ignore the response of early investment; this omission would cause one to under-estimate the final impact on post-school wages by nearly 60%. Second, subsidies for early investment produce much greater short- and long-run gains in human capital than (fiscally equivalent) subsidies for late investment. Dynamic complementarity implies that families that are constrained when their children are young do not fully capitalize on subsidies at later ages, because it is too costly to adjust early investments. Those that receive inadequate early investments do not find it worthwhile to make additional later investments (especially college) even if it is heavily subsidized. By contrast, early investment subsidies enable families to increase investments in their young children without sacrificing current consumption or borrowing more. Those investments can then be matched with later investments when constraints are less severe.

Lastly, we show that it is important to take into account intergenerational effects when evaluating policy. A one-shot policy that gives transfers to young parents increases human capital investments more than an equivalent loan to young parents. However, if the policy is permanently put in place, the loan increases human capital investments more. Transfers today decrease the cost of investment, but transfers tomorrow decrease the benefit of investments. In our framework, this latter effect is quantitatively important.

Many simplifying assumptions have been made in order to make our intergenerational problem tractable. Future work should attempt to incorporate a richer structure for family size, marriage/divorce behavior, and labor supply decisions. Shrinking periods to one or two years would certainly enrich the nature of human capital production and other important lifecycle issues. General equilibrium concerns also deserve attention. While improvements along these lines should add credibility to any policy analysis, our main conclusions are quite general.

[36] Of course, welfare of the dynasty is improved by relaxing the constraint; however, initial generations capture most of this gain.

Appendix A Proofs of Propositions 1-5

Coming soon...

Appendix B Details and Results for Two-Generation Model

This Appendix provides key results for the two-generation model. Those results are proven in an online appendix.

Proposition 6. *Assume that $\tilde{U}(\cdot) = \rho U(\cdot)$, the old child is borrowing constrained (i.e. $A_3 = -L_2$) while a child, but not as an adult. Then,*

(i) $\frac{\partial i_2}{\partial L_2} \in (0, 1)$*;*

(ii) if Condition 1 holds, then $\frac{\partial i_1}{\partial L_2} > 0$*;*

(iii) $\frac{\partial h_3}{\partial L_2} > 0$*;*

(iv) $0 > \frac{\partial y_2}{\partial L_2} > \frac{\partial i_2}{\partial L_2} - 1$*;*

(v) if $\frac{\partial i_1}{\partial L_2} > 0$*, then* $\frac{\partial i_1}{\partial L_2} > \frac{\partial y_1}{\partial L_2} > 0$*, else* $\frac{\partial i_1}{\partial L_2} \leq \frac{\partial y_1}{\partial L_2} \leq 0$*.*

Proof: See online appendix.

Proposition 7. *Assume that $\tilde{U}(\cdot) = \rho U(\cdot)$, the old child is borrowing constrained (i.e. $A_3 = -L_2$) while a child, but not as an adult. Then, if the young parent is borrowing constrained,*

(i) if Condition 1 does not hold, then $\frac{\partial i_1}{\partial L_3} > 0$ *and* $\frac{\partial i_2}{\partial L_3} < 0$*;*

(ii) if $\frac{\partial i_1}{\partial L_3} < 0$*, then* $\frac{\partial i_2}{\partial L_3} < 0$*;*

(iii) if $\frac{\partial i_1}{\partial L_3} > 0$*, then* $\frac{\partial y_1}{\partial L_3} > 0$*;*

(iv) if $\frac{\partial i_2}{\partial L_3} < 0$*, then* $\frac{\partial y_2}{\partial L_3} < 0$*.*

Proof: See online appendix.

Proposition 8. *Assume that $\tilde{U}(\cdot) = \rho U(\cdot)$, the old child is borrowing constrained (i.e. $A_3 = -L_2$) while a child, but not as an adult. Then, if the parent is borrowing constrained when old,*

(i)$\frac{\partial i_2}{\partial L_4} \in (0, 1)$*;*

(ii) if Condition 1 holds, then $\frac{\partial i_1}{\partial L_4} > 0$*, else* $\frac{\partial i_1}{\partial L_4} < 0$*;*

(iii) $\frac{\partial h_3}{\partial L_4} > 0$*;*

(iv) $\frac{\partial i_2}{\partial L_4} < \frac{\partial y_2}{\partial L_4} < 1$*;*

(v) if $\frac{\partial i_1}{\partial L_4} > 0$ *then* $\frac{\partial i_1}{\partial L_4} > \frac{\partial y_1}{\partial L_4} > 0$*, else* $\frac{\partial i_1}{\partial L_4} \leq \frac{\partial y_1}{\partial L_4} \leq 0$*.*

Proof: See online appendix.

Appendix C Details on Calibration

We calibrate parameters of the earnings shock distribution (m, s), the human capital production function (a, b, c), parental altruism towards their children (ρ), the ability distribution $(\theta_1, \theta_2, \pi_1, \pi_2)$, and the debt constraint parameter γ by simulating the model in steady state to best fit a number of moments in the NLSY79 and Children of the NLSY79 data. In particular, we fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 and 36-47 in the NLSY79, (iii) child schooling levels conditional on parental income and maternal schooling, and (iv) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and parental income levels (when the child is ages 0-11).

When classifying individuals by education (either mother or child), we categorize them by highest grade completed (completing less than 12 years of school, 12 years of school, 13-15 years, or 16 or more years).

We minimize $ERR = \sum_{j=1}^{4} W_j ERR_j$, where each ERR_j represents the error associated with one of the four sets of moments we fit and W_j is the weight placed on that set of moments. We briefly describe each of these moments.

ERR_1 is the sum of squared differences between the model's steady state education probabilities and the corresponding sample proportions based on the random sample of all mothers in the NLSY79 (sample size of 2,478). See Table 2 in the paper for these moments in the data and our calibrated steady state.

ERR_2 reflects differences between moments associated with the model's steady state earnings distribution and their corresponding sample moments in the NLSY79 data. Let $E(W_j)$, $SD(W_j)$, and $SK(W_j)$ reflect the mean, standard deviation, and skewness for steady state wages in period $j = 3, 4$ for the model. For corresponding sample moments in the data ($\hat{E}(W_j)$, $\widehat{SD}(W_j)$, $\widehat{SK}(W_j)$ for $j = 3, 4$) we use annual earnings averaged over ages 24-35 and 36-47 (discounted at annual rate $r = 0.05$ to ages 30 and 42) for the random sample of men in the NLSY79. We then compute

$$ERR_2 = \frac{1}{N_3 + N_4} \sum_{j=3}^{4} N_j \left[\frac{E(W_3) - \hat{E}(W_3)}{\hat{E}(W_3)} \right]^2 + \frac{1}{N_3 + N_4} \sum_{j=3}^{4} N_j \left[\frac{SD(W_j) - \widehat{SD}(W_j)}{\widehat{SD}(W_j)} \right]^2 .$$

Here, $N_3 = 2,696$ and $N_4 = 2,399$ reflect the number of observations used in each age-specific calculation.

ERR_3 is a weighted sum of squared differences between the model's steady state child education probabilities (conditional on parental income in periods 3 and 4 and parental schooling) and the corresponding sample proportions from the Children of the NLSY. We separate our sample in the model and data depending on whether parental income (maternal plus paternal earnings) that period is in quartile 1, quartile 2, or above the median.[37] We use the maternal education

[37] In calculating empirical income cutoffs for the first quartile and median, we use the distribution of average

categories discussed earlier. To determine child education probabilities, we use highest grade completed at age 21 to assign high school dropout and completion status, and age 24 to assign college attendance and completion status. We calculate

$$ERR_3 = \frac{1}{N}\sum_{j=1}^{4}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{4} N_{jklm}[P(e^c = j|I_3^p = k, I_4^p = l, e^p = m) - \hat{P}(e^c = j|I_3^p = k, I_4^p = l, e^p = m)]^2,$$

where $P(e^c = j|I_3^p = k, I_4^p = l, e^p = m)$ is the steady state probability a child chooses education category $e^c = j$ conditional on family income categories $I_3^p = k$ and $I_4^p = l$, and maternal education in category $e^p = m$. $\hat{P}(e^c = j|\cdot)$ reflects the corresponding conditional sample moment in the full sample of Children of the NLSY. N_{jklm} is the number of observations used in calculating each conditional moment in the data and $N = \sum_{j,k,l,m} N_{jklm}$.[38]

ERR_4 reflects the extent to which the model fits period 3 average wages of children conditional on their own education, paternal education, and parental income when they were young. We classify parental income and education as we did for ERR_3 (in the model and data). We use average child weekly wages over ages 24-35 (all discounted to age 30 using $r = 0.05$) for children of the NLSY.[39] Because we consider weekly wages for children (rather than annual income) to better reflect human capital levels at younger ages, we scale all average wage measures by those for children with a high school degree, whose mothers had a high school degree, and whose parental income was in the lowest quartile. We compute

$$ERR_4 = \sum_{k=1}^{4}\sum_{l=1}^{3}\sum_{m=1}^{4} N_{klm}\left[\frac{E(w_3|e^c = k, I_3^p = l, e^p = m) - \hat{E}(w_3|e^c = k, I_3^p = l, e^p = m)}{\hat{E}(w_3|e^c = 2, I_3^p = 1, e^p = 2)}\right]^2,$$

where $E(w_3|e^c = k, I_3^p = l, e^p = m)$ is the average steady state period-three wage w_3 for a child conditional on own education category $e^c = k$, early parental income category $I_3^p = l$, and maternal education category $e^p = m$. $\hat{E}(w_3|\cdot)$ reflects the corresponding conditional sample moment in the full sample of Children of the NLSY. N_{klm} is the number of observations used in calculating each conditional moment in the data and $N = \sum_{k,l,m} N_{klm} = 3,049$.

All of our ERR_j errors should be of similar magnitudes given the scaling of various moments. Reflecting sample sizes used to compute each error, we use weights $W_1 = 0.24$, $W_2 = 0.33$, $W_3 = 0.10$, and $W_4 = 0.33$. We generally fit all sets of moments well. Our calibration yields $ERR = 0.033$, with $ERR_1 = 0.003$, $ERR_2 = 0.07$, $ERR_3 = 0.02$, and $ERR_4 = 0.04$.

family income over maternal ages 24-35 and 36-47 (discounted at annual rate $r = 0.05$ to ages 30 and 42) based on all mothers in the random sample of the NLSY79. We use family income averaged over child ages 0-11 and 12-23 for Children of the NLSY to categorize children by parental income in periods 3 and 4.

[38] N_{jklm} depends on the child education category, since we use different ages to determine high school dropout and graduate vs. some college and college completion. Altogether, our sample includes 3,928 individuals ages 21+ and 2,966 individuals ages 24+.

[39] We drop observations with weekly wages less than \$40 or greater than \$2,500. To calculate more precise wage measures for high school dropouts and graduates, we also include weekly wage measures at ages 22-23 in computing average wages.

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Table 1: Effects of Early and Late Family Income (in $10,000s PDV as of birth year) on Child Educational Attainment

	Sample Size	Early Income	Later Income	Equal Effects (p-value)
A. Controls only for Maternal Education				
HS Dropout	1,483	**-0.042**	-0.001	0.003
(Ages 21-24)		(0.007)	(0.008)	
Attended Any College	1,483	**0.044**	**0.019**	0.096
(Ages 21-24)		(0.008)	(0.009)	
Attended Any College	828	**0.056**	0.012	0.333
(Ages 24-27)		(0.012)	(0.013)	
Graduated College	828	**0.051**	0.015	0.039
(Ages 24-27)		(0.009)	(0.010)	
B. Control for Maternal Education and Child/Family Background				
HS Dropout	1,422	**-0.041**	-0.001	0.006
(Ages 21-24)		(0.008)	(0.009)	
Attended Any College	1,422	**0.037**	0.018	0.211
(Ages 21-24)		(0.008)	(0.009)	
Attended Any College	802	**0.052**	**0.026**	0.272
(Ages 24-27)		(0.013)	(0.013)	
Graduated College	802	**0.047**	0.012	0.048
(Ages 24-27)		(0.010)	(0.010)	

Notes: All specifications control for indicators for maternal education categories (high school dropout, high school gradaute, some college, college graduate). Additional controls for 'child/family background' include year of birth, black, hispanic, male, teen mother, maternal AFQT (normalized and adjusted for age at time of test), foreign born mother, and mother from intact family (age 14). Early income reflects average discounted family income over child ages 0-11; late income reflects average discounted family income over ages 12-23. A discount rate of 5% is used to discount income to age 0. Bold = sig. at 0.05 level.

Table 2: Calibrated Education Distribution

Education	NLSY Data	Model
High school dropout	.18	.21
High school graduate	.40	.36
Some college	.23	.29
College graduate and beyond	.19	.13

Table 3: Calibrated Annual Earnings Distributions for Men Ages 24-35 and 36-47

Earnings Statistic	NLSY Data	Model
Mean (ages 24-35)	40,868	43,889
Standard deviation (ages 24-35)	23,108	26,362
Mean (ages 36-47)	60,392	64,826
Standard deviation (ages 36-47)	41,416	29,444

Table 4: Educational Attainment by Parental Education (Baseline)

	Model			NLSY Data		
Parental Education	High School Graduate or More	Some College or More	College Graduate	High School Graduate or More	Some College or More	College Graduate
High School Dropout	0.54	0.18	0.03	0.61	0.23	0.05
High School Graduate	0.77	0.39	0.08	0.78	0.42	0.14
Some College	0.90	0.55	0.20	0.84	0.53	0.21
College Graduate	0.99	0.66	0.30	0.93	0.76	0.46

Table 5: Educational Attainment by Parental Income (Baseline)

Parental Income Quartile:		Model		NLSY Data	
Early Ages	Late Ages	High School Graduate or More	College Graduate	High School Graduate or More	College Graduate
1	Any	0.58	0.01	0.65	0.07
2	Any	0.75	0.03	0.79	0.16
3 or 4	Any	0.91	0.24	0.89	0.29
Any	1	0.56	0.06	0.66	0.06
Any	2	0.72	0.11	0.77	0.14
Any	3 or 4	0.93	0.18	0.87	0.29
1	1	0.46	0.01	0.64	0.06
2	1	0.56	0.03	0.73	0.10
3 or 4	1	0.71	0.16	0.90	0.08
1	2	0.56	0.01	0.69	0.08
2	2	0.71	0.03	0.80	0.17
3 or 4	2	0.85	0.24	0.84	0.18
1	3 or 4	0.84	0.01	0.69	0.14
2	3 or 4	0.90	0.03	0.82	0.17
3 or 4	3 or 4	0.96	0.27	0.90	0.34

Table 6: Calibrated Parameter Values

Parameter	Value
a	0.45
b	-1.10
d	0.77
θ_1	7.82
θ_2	20.00
π_1	0.59
π_2	0.76
m	9.94
s	0.74
ρ	0.67
γ	0.45

Table 7: Average Baseline Investment Amounts by Parental Education

Parental Education	Average i_1	Average i_2
All Levels	2,121	7,227
High School Dropout	686	3,088
High School Graduate	1,711	5,989
Some College	2,970	9,473
College Graduate	3,643	12,205

Table 8: Effects of Increasing Young Parent's Borrowing Limit by $2,500

	Short-Run Effects (% Change)				Long-Run Effects (% Change)			
Parental Education	Avg. i_1	HS+	College Grad.	Avg. W_3	Avg. i_1	HS+	College Grad.	Avg. W_3
All Levels	11.0	2.8	10.0	1.5	-0.4	1.7	-1.9	0.0
HS Dropout	0.0	5.2	0.0	0.1	-16.4	2.6	-14.2	-0.8
HS Graduate	9.7	3.9	12.7	1.4	-5.2	1.5	-3.1	-0.6
Some College	9.3	1.7	4.5	1.6	-2.1	0.9	-8.2	-0.4
College Graduate	18.8	0.4	17.6	3.6	10.5	0.3	8.1	2.0

Table 9: Effects of Increasing Young Parent's Borrowing Limit by $2,500 on Fraction of Constrained Young and Old Parents, and Old Children

	Fraction of Young Parents Constrained		Fraction of Old Parents Constrained		Fraction of Old Parents with $y_2 = 0$	
Parental Education	Baseline SS	New SS	Baseline SS	New SS	Baseline SS	New SS
All Levels	0.51	0.46	0.12	0.18	0.05	0.12
High School Dropout	0.51	0.39	0.04	0.11	0.15	0.29
High School Graduate	0.38	0.35	0.04	0.10	0.04	0.14
Some College	0.60	0.56	0.14	0.22	0.01	0.03
College Graduate	0.68	0.66	0.41	0.46	0.00	0.00

Table 10: Effects of Early and Late Investment Subsidies

	Short-Run Effects (% Change)					Long-Run Effects (% Change)				
Policy	Avg. i_1	Avg. i_2	HS+	College Grad.	Avg. W_3	Avg. i_1	Avg. i_2	HS+	College Grad.	Avg. W_3
Announced early										
$s_1 = .10$	28.1	12.1	0.0	30.5	3.1	34.0	17.0	1.3	39.6	3.9
$s_2 = .53$	4.0	15.5	10.1	22.9	1.4	7.2	18.1	10.9	26.8	1.8
Announced late										
$s_2 = .53$	0.0	8.7	10.0	1.4	0.6	7.2	18.1	10.9	26.8	1.8