Not a stock answer

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"Estimating default correlations using a reduced-form model" (*Risk*, January 2005) contains default correlations for thirteen selected firms. This note converts the default correlations to the asset correlations useful in a credit portfolio model. The values of these asset correlations are distinctly less than the stock return correlations often employed in such models. The difference appears to reflect a difference in the firms being observed. Defaulting firms, such as observed by authors Jarrow and van Deventer, are more likely to be relevant for credit portfolio loss models.

To make the conversion from default correlation to asset correlation we begin by simplifying the notation for the default indicators. Default indicators are Bernoulli random variables:

(1) $$\begin{aligned} D_i &= 1 \textbf{ if firm } i \textbf{ defaults in the next year,} \\ &= 0 \textbf{ otherwise} \end{aligned} .$$

Expectation and variance are therefore:

(2) $$\mathbf{E}[D_i] = \mathbf{Pr}[D_i = 1] = PD_i; \quad \mathbf{Var}[D_i] = PD_i(1 - PD_i) .$$

We denote by $PD_{i,j}$ the probability that in the next year firms i and j both default. The correlation between two indicators can then be stated:

(3) $$\mathbf{Corr}[D_i, D_j] = \frac{\mathbf{Cov}[D_i, D_j]}{\sqrt{\mathbf{Var}[D_i]\,\mathbf{Var}[D_j]}} = \frac{PD_{i,j} - PD_i PD_j}{\sqrt{PD_i(1 - PD_i)PD_j(1 - PD_j)}} .$$

Inverting this relation produces the probability that firms i and j both default as a function of the quantities that appear in the lower part of Jarrow and van Deventer's Table B, "Correlations in events of default":

(4) $$PD_{i,j} = Corr[D_i, D_j]\sqrt{PD_i(1 - PD_i)PD_j(1 - PD_j)} + PD_i PD_j .$$

In a CreditMetrics-style model, the default of a firm is triggered when a standard normal asset variable drops below a specified threshold. The asset variables of a pair of firms are connected by an asset correlation, ρ. The probability that both firms default in the same year equals the integral of the joint density over the relevant region:

(5) $$PD_{i,j} = \int_{-\infty}^{\Phi^{-1}(PD_j)} \int_{-\infty}^{\Phi^{-1}(PD_i)} \phi(x, y, \rho)\, dx\, dy ,$$

where $\phi(\cdot)$ symbolizes the bivariate normal density and $\Phi^{-1}(\cdot)$ symbolizes the inverse of the standard normal cumulative distribution function. A unique value of ρ brings Equation (5) into equality with Equation (4), and quantifies the asset correlation implied by the published default correlations.

Table A shows for selected firms the asset correlations implied by Jarrow and van Deventer's default correlations. For the same firms, Table B shows the correlation between monthly stock price returns from January 1990 through 2004.

Table A

Asset correlations implied by default correlations

	Volkswagen	AMR	Citigroup
Volkswagen	100%	6.2%	3.8%
AMR	6.2%	100%	18.2%
Citigroup	3.8%	18.2%	100%

Table B

Correlations of monthly stock price returns

	Volkswagen	AMR	Citigroup
Volkswagen	100%	35.7%	44.5%
AMR	35.7%	100%	43.1%
Citigroup	44.5%	43.1%	100%

The asset correlations are notably less than the stock correlations. Of the full matrix of correlations, average asset correlation equals 8.1%, much less than average stock correlation of 42.4%. Each asset correlation is less than one-half its associated stock correlation.

The reason for the difference between the values may not be immediately obvious. Every correlation is based on the histories of two firms, and the periods of observation are roughly similar for the two correlation estimates. Further, every correlation depends in some way on the monthly stock price, though the asset correlations in Table A employ additional information.

Importantly, stock correlations take no special notice of default. They measure the relation between firms as they have existed historically. The analyzed firms do not default in the sample period, and their stock prices rise and fall along with other stocks in concert with broad macroeconomic trends.

By contrast, Jarrow and van Deventer take an extra step, the reduced-form model, that ties their estimates to default rates. As the authors state, default is usually an idiosyncratic event. By calibrating Equation (3) to default data, they allow for these idiosyncrasies. As a result, the asset correlations inferred here have much lower values than correlations observed between stocks.

If a credit portfolio model uses stock correlations, it assumes that a correlation observed between historical stock returns will remain fixed even if one or both of the firms deteriorate toward default. This assumption is convenient, but it has never been shown to reflect the facts. The evidence presented here suggests the assumption is false. The correlation between solvent firms may simply be greater than the correlation between firms that default, because firms default for mostly idiosyncratic reasons. Correlations estimated from stock prices may apply to solvent firms, but correlations calibrated to default data arguably give a better idea of the correlation relevant for credit loss.

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