Could Prometheus Be Bound Again?

A Contribution to the Convergence Controversy[1]

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Abstract

This paper presents a model of stochastic growth in which the probability of adverse shocks to production is inversely related to the aggregate stock of capital per capita. Postulating this endogenous relationship, justified by empirical evidence, the model yields long-run predictions consistent with the recent findings of cross-country club convergence and intra-distribution mobility.

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1 Introduction

This paper develops a model of growth with stochastic production. On the basis of observable empirical regularities, it postulates that the probability of adverse production shocks decreases as an economy advances to higher stages of development, proxied by the accumulated stock of capital. This is a departure from standard stochastic growth models, where the randomness in production is customarily taken as exogenous and invariant with respect to the level of accumulated capital. Endogenizing the probability distribution over the shocks permits the identification of a rich variety of dynamic equilibria, characterized by different shapes of the long-run, stationary distribution of capital per capita. This makes it possible to predict a number of observable empirical phenomena, such as 'club convergence', 'economic miracles', 'growth disasters' and 'reversals of fortune'.

The most recent empirical growth literature has debated the importance of the analysis of distribution dynamics for a full understanding of the mechanics of economic development. This view is strongly supported by Quah in numerous contributions (e.g. Quah [30], [31], [32], [33], [34]).[1] Empirical studies have shown consistent evidence of a cross-country income distribution displaying bimodality with a marked thinning in the middle.[2] Countries are clustered in two separate groups, the rich and the poor. However, there is a positive probability for an economy to move from one cluster to the other, that is, the bimodal distribution is ergodic. These results are interpreted as showing a lack of convergence among rich and poor countries, although it is possible to observe at times economic miracles, or previously poor economies that grow rapidly and move to join the rich club, as well as reversals of fortune, where the spells of fast growth are only temporary and are followed by abrupt halts and decumulation, and finally, economic disasters, i.e. previously rich economies

[1] The "classical" approach to the study of cross-sectional growth patterns originally analyzed the properties of the first and the second moment of the cross-country distribution, investigating the existence of the so-called β-convergence and σ-convergence (e.g. Barro and Sala-i-Martin [5], Mankiw, Romer and Weil [26]). Subsequently, much energy has been spent in a (perhaps at times overheated) dispute among those attacking the validity of this approach and those in its defense. The essence of the controversy is best captured in the contributions of Durlauf [18], Bernard and Jones [10], Sala-i-Martin [38], Quah [32] and Galor [20].

[2] In addition to Quah's contribution see also Ben-David [8], Benhabib and Gali [9] and Bianchi [13].

regressing to significantly lower levels of income.[3]

An important issue in this debate is related to the choice of the appropriate theoretical model which can predict club convergence and explain the occurrence of the above mentioned intra-distribution dynamics. Part of the research effort has been directed at analyzing whether such phenomena can be described in the world of diminishing marginal productivity and constant returns to scale of the neo-classical growth model. Galor [20] maintains that club convergence can still be predicted by such model once we 'augment' it by introducing individual heterogeneity, human capital accumulation, capital market imperfections, endogenous fertilities, etc.[4] Any one of these more sophisticated versions of the model can generate multiplicity of steady states: club convergence is generated as the result of history dependence. Nonetheless, two important caveats should be highlighted. First, the model predicts that "countries with similar structural characteristics converge to the same steady-state equilibrium if their initial per capita output levels are *similar* as well" (Galor [20], pag. 1058, emphasis is mine). However, two identical countries with very close initial conditions still converge to separate steady-state equilibria if they start on different sides of an unstable steady-state equilibrium.[5] Thus, empirically it may be difficult to predict the future development path of two identical countries from the observation of similar initial per capita output levels.[6] The second and perhaps more important criticism is that the standard deterministic growth model yields incorrect predictions on the intra-distribution dynamics: the kind of club convergence generated through multiplicity implies that a country will never leave its basin of attraction. Put differently, contrary to the above mentioned empirical findings, the model predicts a

[3]Examples of economic miracles might be Hong Kong, Singapore, South Korea and Taiwan, although recent events may suggest that for some of them the concept of reversals of fortune is a more apt description. As for economic disasters, oft cited examples are Argentina, one of the richest economies in the first part of this century but with a per capita income in 1988 only 42 percent of that of the United States, and Venezuela, the third richest country in 1960 with a 1988 income only 55 percent of that of the United States (Jones [25]).

[4]Examples of such models are Galor and Zeira [24], Azariadis and Drazen [3], Galor and Tsiddon [22], Barro and Becker [4], Galor and Weil [23]. A thourough survey of such models is in Azariadis [2].

[5]This arbitrariness is recognized by Galor himself ([20], pag. 1058, footnote 6).

[6]In fact, in the presentation of the results of this paper I will actually argue that relying on the similarity of initial conditions in per capita output is not only ambiguous but perhaps altogether misleading.

non-ergodic bimodal distribution.[7]

Given the necessity of investigating distributional properties and asking questions in probabilistic terms, could the stochastic version of the neo-classical growth model be a more appropriate theoretical framework?[8] In a growth model with stochastic production the long-run equilibrium is characterized by a stationary *distribution* of per capita income levels. Cross-sectional and time-series predictions compatible with the above mentioned stylized facts emerge naturally. However, a major caveat associated with standard stochastic growth models is that the randomness in production is exogenously imposed: the distributional patterns derived by the model are completely 'nature driven', and this makes it hard to extract explanatory power from the model.[9] Another problem is that by imposing exogenous randomness, one is lead to predict a similar pattern of uncertainty for all economies, regardless of the level of per capita output. This implies that output variability must be the same for both developing and developed economies. However, there is strong evidence of a much higher output variability in developing countries (see e.g. Acemoglu and Zilibotti [1] and Ramey and Ramey [35]).

The model presented in this paper refines the neoclassical, stochastic growth model by postulating that the likelihood of adverse shocks to production is higher when an economy is in early stages of development. There are plausible economic explanations to justify such a postulate. A developing economy is likely to suffer from the inexistence of markets to insure production risk. Lack of infrastructures and other institutional factors may hinder diversification (e.g. Chile and its former heavy dependence on copper production). Also, there may be a higher vulnerability to internationally driven shocks (foreign direct investment withdrawal, currency shocks, etc.). This endogenous relationship between the probability of an economic setback and aggre-

[7] This remark is also made by Benhabib and Gali [9].

[8] The origin of this alternative theoretical model is usually attributed to the contribution of Brock and Mirman [14]. See also Mirman [27], Mirman and Zilcha [28], Zilcha [40]. Also Wang [41], [42], Bertocchi [11] for the extension to the overlapping generations framework.

[9] In other words, the model predicts that the exact position within the stationary distribution is purely determined by 'luck'. It should be fair to note that the standard stochastic growth model was originally investigated simply to confirm the basic predictions of the deterministic version, that is to make sure that the deterministic equilibrium was robust to stochastic perturbations (see Brock and Mirman [14], pag. 480). Thus its importance is not diminished by the fact that it cannot answers more sophisticated questions.

gate capital per capita has actually been derived from first principles by Acemoglu and Zilibotti [1]. Their explanation is indeed based on market incompleteness and lack of diversification. While their model is important in that it shows a convincing mechanism generating such an inverse relationship, it does not focus on the issues of interest to this paper.[10] The postulated relationship in this paper is a reduced form representation of the Acemoglu and Zilibotti's first principles derivation.

In any event, there is empirical confirmation that an economic setback is more likely at lower levels of development. Quah [30] estimated a cross-country transition matrix of income per capita to analyze patterns of persistence and intra-distribution dynamics. Such a matrix (shown in Table 1 and extensively analyzed throughout the paper) indeed shows that the probability to drop in the income ranking is higher for countries with lower levels of income. This empirical evidence thus suggests (and the model would like to capture) an intrinsic higher frailty of countries at early stages of development.

How can this innovation in the standard stochastic growth model generate club convergence? At low stages of development, an economy is not only characterized by low capital but also by a low probability that that capital will be successful in production. This generates time persistence in the poverty region. An attempt to merely accelerate capital accumulation (e.g. policies increasing savings rate) may still be a failure. In turn, at high stages of development not only capital accumulation is high, but also the probability of success is high, generating time persistence in the developed region. 'Some' degree of fate–or, more interesting, policies affecting, for instance, the degree of diversification and of market incompleteness–are such that economies will transit from one region to the other, allowing for the observable intra-distribution mobility.[11]

[10] In fact their model would yield very strong predictions as far as the convergence debate goes: the stationary distribution in their model is unimodal (it is in fact a collapsed distribution on one steady-state value of capital per capita), and all economies are ultimately expected to converge with each other.

[11] The importance of 'luck' as a factor determining cross-country growth patterns, has been analyzed in Easterly et al. [19]. The authors actually argue that luck is perhaps the *main* explanation for the instances of economic miracles observed over time. In my model, luck is only a factor affecting the timing of intra-distribution mobility, since the postulated relationship between the probability of adverse shocks and aggregate capital still underlines the importance of economic, institutional factors responsible for its characteristics.

This paper contributes to the growth literature in different ways. First, the paper confirms the neoclassical model as an adequate framework of analysis for economic development. Second, it highlights the importance of uncertainty in economic development. It suggests that capital accumulation per se is a necessary but not a sufficient condition for permanent development, and that more attention should be devoted to policies specifically aimed at reducing the impact of adverse shocks. Third, it contributes to the convergence debate by identifying the conditions for the emergence of club convergence *without having to rely on multiplicity* of steady states. Moreover, the stationary distributions are ergodic, allowing for intra-distribution mobility.

Finally, the paper contributes to that stream of literature which offers, as an alternative explanation for these empirical findings, the possibility that equilibrium *indeterminacy* be a proper characterization of reality. Among others, still Benhabib and Gali [9] suggest that in the presence of technological complementarities, it is possible that more than one pair of expectations/outcomes are consistent with individuals' optimization. Failing to coordinate agents' expectations results in equilibrium indeterminacy. In fact, the endogenous relationship postulated in this paper is a type of technological complementarity and indeed equilibrium indeterminacy may emerge in the model. The paper shows under what conditions on the primitives this is likely to happen, but subsequently it shows that club convergence and intra-distribution mobility are identifiable even in the absence of indeterminacy.

The paper is organized as follows. In section 2 I lay out the details of the model. Section 3 analyzes the dynamic equilibrium and the potential emergence of equilibrium indeterminacy and its interpretation as a first result of permanent income disparity. The paper continues focusing on the dynamic in the case in which indeterminacy does not emerge. Section 4 describes the nature of the stationary equilibria of the economy, identifying potential causes for club convergence and the likelihood of economic miracles, disasters and reversals of fortune, in the absence of both indeterminacy and multiplicity of equilibria. Section 5 presents numerical simulations calibrated on Quah's empirical findings. The simulations help visualizing a stationary distribution and show how its shape is affected by the choice of parameters of the function representing the probability of an economic setback. Section 6 presents a summary and concluding remarks.

2 The economy

The model is based on the stochastic extension of the Diamond [16] overlapping generations economy with production. The economy is populated by cohorts of identical agents living for two periods. Population size is assumed to be constant and without loss of generality normalized to one.

Agents derive utility from consumption in both periods of a single, non-storable good, produced with a constant returns to scale production function. Production is subject in every period to an exogenous, multiplicative random shock, whose characteristics are fully described in the next section. There is full depreciation of capital.

2.1 Technology

The production technology is described by the general functional form $Y_t = F(K_t, L_t)\Theta_t$, where Y_t, K_t and L_t denote output, capital and labor, respectively, at time t. Θ_t is a random variable which can take two values, $0 < \Theta^L < \Theta^H = 1$. Therefore, in every period there is a positive probability the economy loses capital and drop down the income ranking. It is assumed for simplicity that $\Theta_t^L = \Theta^L$ and $\Theta_t^H = \Theta^H$, $\forall t$. The function $F(\cdot,\cdot)$ is assumed homogeneous of degree one in K_t and L_t. Therefore, putting all variables in per-capita terms, $y_t = f(k_t)\Theta_t$, where $f(\cdot)$ is the per-capita production function. In addition, $f \in C^2$, $f(0) = 0$, $f'(k) > 0$, $f''(k) < 0$, $\forall k \geq 0$. Finally, to ensure interior solutions, the Inada conditions, $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$, are satisfied.

I conjecture the existence of an economy-wide capital accumulation externality, such that as the economy reaches more advanced stages of development, proxied by its stock of accumulated capital, the likelihood of an economic setback decreases. In introduction I have provided economic arguments to justify this conjecture, but its plausibility seems indeed confirmed on empirical ground. Table 1 is taken from Quah [30], pag. 431. It describes a 23-year Markov chain transition matrix of per capita incomes for a sample of 118 countries. Each country's per capita income is calculated relative to the world average.

According to the table, an economy with income more than twice as high as the world average, has a 0.05 probability to drop down to lower levels. In my model this is equivalent to saying that the probability of Θ^L is 0.05 when an economy is at the highest stages of development. Continuing the analysis of the table, the probability to drop down for an economy with income between once and twice the world average is instead 0.24. Proceeding down the income ranking this probability increases until it reaches 0.76 for economies at the bottom rank. This is empirical evidence confirming the conjectured endogenous relationship between the probability of adverse shocks to production and the accumulated capital stock. Based on these empirical findings I postulate that

$$p_t = p(k_t), \tag{1}$$

where $p_t \equiv \Pr(\Theta_t = \Theta^H)$, and $p(0) \geq 0,\ p'(k) > 0$ and $p''(k) \lesseqqgtr 0$. Also, it is assumed that $\exists\ \bar{k} < \infty,\ s.t.\ p(\bar{k}) = 1,\ \forall k \geq \bar{k}$. This last assumption is discussed later in section 4.2.

In a recent contribution, Acemoglu and Zilibotti [1] derive an expression like (1) as a result of market incompleteness and lack of production differentiation. An important aspect of their model is that the authors generate this endogenous relationship from first principles. However, the price to pay to allow full tractability are quite substantial restrictions on the primitives.[12] Instead, by postulating such relationship, justified on empirical ground, I can maintain a higher degree of generality and work with very general expressions for technology, preferences and the function $p(k)$. This allows me to identify a broader class of dynamic equilibria, and highlight several instances of multiplicity and poverty traps, thus extending and complementing Acemoglu and Zilibotti's results.[13] Many comparisons with their paper will be drawn throughout the following sections. Thus in what follows, equation 1 could be seen as a reduced form representation of Acemoglu and Zilibotti's model.

[12] In their model intermediate goods are produced with linear technologies, the final good with a Cobb-Douglas, and preferences are logaritmic. Also, they construct the model so that the probability of success increases linearly in k.

[13] For example, as we will see in following sections, by not restricting preferences to be logaritmic, it is possible to identify potential equilibrium indeterminacy, shown to depend on the preferences' degree of risk aversion.

2.2 Preferences

Agents maximize a von Neumann-Morgenstern, time-separable, expected utility function over consumption in both periods, $U = v(c_t) + E_t u(c_{t+1})$. We assume that $v, u \in C^2$, $v'(\cdot)$, $u'(\cdot) > 0$, $v''(\cdot)$, $u''(\cdot) < 0$. Moreover, $\lim_{c_t \to 0} v'(c_t) = \infty$ and $\lim_{c_{t+1} \to 0} u'(c_{t+1}) = \infty$.

At the end of time t, young agents, who have worked and received wage income $\omega_t(\Theta_t)$, decide on the optimal amount of savings, s_t, necessary to finance consumption in period $t+1$.

The maximization problem faced by agents at time t is

$$\begin{aligned} &\underset{c_t, c_{t+1}}{\text{Max}}\ v(c_t) + E_t u(c_{t+1}) \qquad (2)\\ &c_t + s_t = \omega_t(\Theta_t),\\ &c_{t+1} = s_t \hat{r}_{t+1}(\Theta_{t+1}), \end{aligned}$$

where $\hat{r}_{t+1}$ is the expected rate of return on savings, whose randomness depends on the probability distribution governing the shock Θ_{t+1}. Note that at the time the problem is solved, the wage income, function of the random shock at time t, is known.

3 The dynamic equilibrium

Denote as $s_t^* = \arg\max\{v[\omega_t(\Theta_t) - s_t] + E_t u[s_t \hat{r}_{t+1}(\Theta_{t+1})]\}$. The optimal amount of savings at time t is the supply of capital at time $t+1$ available to firms.

Firms are assumed to be profit maximizers and in perfect competition among each other for the supply of capital s_t^* (and labor of generation $t+1$). Profit maximization and perfect competition ensure that factors will be priced according to their marginal contribution to production, i.e.

$$r_{t+1} = f'(k_{t+1})\Theta_{t+1}, \qquad (3)$$

$$\omega_{t+1} = [f(k_{t+1}) - k_{t+1} f'(k_{t+1})]\Theta_{t+1}, \qquad (4)$$

Agents of generation t solve the maximization problem in (2). The first order condition is

$$v'[\omega_t(\Theta_t) - s_t] = E_t\ u'[s_t \hat{r}_{t+1}(\Theta_{t+1})]\hat{r}_{t+1}(\Theta_{t+1}). \tag{5}$$

At the time the problem is solved agents need to form expectations regarding the future realization of the rate of return $r_{t+1}(\Theta_{t+1})$, which implies forming expectations on the probability distribution over the shock Θ_{t+1}. This is so because in this model the probability distribution over Θ is determined endogenously by the combined action of savers and firms through the effect of the capital accumulation externality. In other words, agents know that when they choose optimal savings, they are also affecting indirectly the determination of p_{t+1}.[14]

As is customary in this literature, we restrict our analysis to the class of rational, self-fulfilling expectations equilibria. In these type of equilibria agents make decisions based on expectations that will be confirmed ex-post correct, in every state of nature. Such an equilibrium (if it exists) is characterized as follows.[15] Given the wage income ω_t, recalling the expression for p in (1), substituting in (5) for $\hat{r}_{t+1}$ the expression for r_{t+1} in (3), k_{t+1} is a self-fulfilling expectations equilibrium if

$$\begin{aligned} v'[\omega_t(\Theta_t) - s_t] &= p_{t+1}u'[s_t f'(s_t)\Theta^H_{t+1}]f'(s_t)\Theta^H_{t+1} + && (6)\\ &+ (1-p_{t+1})u'[s_t f'(s_t)\Theta^L_{t+1}]f'(s_t)\Theta^L_{t+1}, &&\\ k_{t+1} &= s_t. && (7)\\ p_{t+1} &= p(k_{t+1}). && (8) \end{aligned}$$

The following proposition establishes sufficient conditions for the uniqueness of a self-fulfilling equilibrium.

Proposition 1 *Assume that the capital income, $sf'(s)\Theta$, is non-decreasing in s in all states of nature. In addition, assume that the ratio of the marginal utility in the "good" state of nature and the "bad" state of nature does not exceed Θ^L. More*

[14]In standard stochastic models of growth the random shock affecting production is either assumed to be drawn from an exogenous, fixed distribution of probabilities (see, for example, Brock and Mirman [14], Mirman and Zilcha [28], Zilcha [40], Wang [41], Bertocchi [11] or generated by an exogenous Markov process (see Duffie et al. [17], Wang [42]).

[15]The underlying proof strategy is based on Wang [41]. Wang has established the conditions for the existence and uniqueness of equilibria in a stochastic OLG model. However, my results differ since the probability distribution of the rate of return is determined endogenously.

precisely:

$$sf''(s) + f'(s) \geq 0 \;\; \forall s > 0, \tag{9}$$

$$\frac{u'(sf'(s)\Theta^H)}{u'(sf'(s)\Theta^L)} \leq \Theta^L. \tag{10}$$

Then, for any given $k_t = s_{t-1} > 0$*, there exists a unique* $k_{t+1} = s_t > 0$ *that is a self-fulfilling expectation.*

Proof. See Appendix.

3.1 Equilibrium indeterminacy

The result of Proposition 1 highlights the potential emergence of coordination failure and consequent equilibrium indeterminacy. While condition (9) is also the one required in a model where the randomness is exogenous (see Wang [41]),[16] the additional condition (10) is needed because there is a 'strategic complementarity' between the private return on savings and the aggregate capital stock. If the marginal utility in the two states of nature is approximately equal (making their ratio close to one, hence larger than Θ^L), multiple levels of savings will result optimal. Given the assumption of strict concavity of the utility function, the pace at which the marginal utility decreases is determined by the degree of risk aversion. High degree of risk aversion means that the curvature of the utility function changes quickly. Therefore, condition (10) suggests that the likelihood of equilibrium indeterminacy is higher when agents are relatively risk tolerant. The intuition is that an agent perceives that if everybody else is saving a high amount, p^* will result high and consequently he should also choose to save a high amount. The opposite is instead true if he perceives that everybody else is saving a small amount. This can be seen as equivalent to facing a lottery on the realization of future income. When agents are risk tolerant (low risk aversion), they may be willing to 'bet' on what other agents will do. In such a case one cannot exclude the possibility that multiple pairs $\{s^*_{t,i}, p^*_i = p(s^*_{t,i})\}$ could solve the utility maximization problem. On the other hand, in a sufficiently risk averse

[16] It says that the elasticity of substitution (in absolute value) exceeds the labor's share of output. For example, a *CES* production function with elasticity of substitution greater or equal than one (in absolute value) satisfies this condition.

world, agents will not be inclined to bet on what everybody else's decision might be, and this will be more likely to result in a unique optimal decision level of savings.[17]

This example of coordination failure is peculiar to the model because of the postulated endogenous nature of aggregate risk in production. The predicted dynamics of *identical* economies subject to this problem is that one economy could build on a history of low levels of savings and remain trapped in such a scenario, while the other one with a history of high participation in the capital market could be expected to converge to a long-run equilibrium with high levels of capital accumulation. In addition, switches of beliefs over time may cause ranking reversal among identical economies, which identifies the patterns of intra-distribution mobility previously described.

A first plausible case of divergent dynamics among otherwise identical economies is therefore identified by the model and it is shown to depend on the degree of risk aversion exhibited by agents' preferences.

3.2 The dynamic evolution in the absence of indeterminacy

The existence of equilibrium indeterminacy in models of growth, and its recognition as a plausible explanation for club convergence, has been the focus of recent analysis by Benhabib and Gali [9]. While aware of the importance of this result, one should also highlight the difficulties associated with the econometric identification of equilibrium indeterminacy. In addition, it is easy to show that in the case, for example, of CRRA preferences, indeterminacy arises in this model if the coefficient of relative risk aversion is strictly less than one. Although there does not seem to be agreement on the exact empirical magnitude of such coefficient, this is customarily considered to be larger than one (e.g. Barsky et al. [6]).

Therefore, I will proceed throughout the rest of the paper assuming that conditions (9) and (10) hold. Even under this optimistic scenario, club convergence can still be generated.

Under conditions (9) and (10), for any $\omega_t(\Theta_t) > 0$, agents at time t choose a *unique* s_t^* such that the realization of outcome at time $t+1$ will be exactly

[17] In sum, this is analogous to stating the preference of a risk averse agent for a certainty equivalent level of income, rather than a lottery on it.

$$\begin{cases} y_{t+1} = f(s_t^*)\Theta_{t+1}^H & \text{with probability } p_{t+1}^*(s_t^*), \\ y_{t+1} = f(s_t^*)\Theta_{t+1}^L & \text{with probability } 1 - p_{t+1}^*(s_t^*) \end{cases}$$

Let us now define the dynamic equilibrium path for this economy.

Definition 1 *A dynamic equilibrium is a stochastic sequence* $\{k_t^*\}_{t=0}^{\infty}$ *with initial condition* $k_0 > 0$ *which depends on the history of the realizations of the random variable* $\Theta_t = \{\Theta_0, \Theta_1, \ldots, \Theta_t\}$ *such that the optimal plans of consumption and production are satisfied in every period.*

From Proposition 1, s_t is uniquely defined as a function of k_t and Θ_t. Hence, given the equilibrium condition in the capital market, equation (7), there exists a well defined, single valued function, Φ, such that

$$k_{t+1} = \Phi(k_t, \Theta_t). \tag{11}$$

The expression (11) is a first-order, stochastic, difference equation, fully characterizing the dynamic equilibrium of the economy.

Graphically, the dynamic evolution of (11) could be represented as in Figure 1.[18] In this picture $\Phi^H \equiv \Phi(k_t, \Theta^H)$ represents the dynamic evolution of the capital stock if the realization of the shock were Θ^H at any time t. Equivalently we can define $\Phi^L \equiv \Phi(k_t, \Theta^L)$. The values k_a and k_b are the fixed points corresponding to the two trajectories. The set $[k_a, k_b]$ is called a *stable set* for the capital stock.

3.3 Multiplicity of stable sets

It is well known that, even with strong restrictions on preferences and technology, in an overlapping generations model it is plausible to have scenarios where Φ^H and Φ^L show a multiplicity of fixed points, or cases with no fixed points at all. Figure 2 and 3 illustrate these two alternative cases. Perhaps also interesting is that in the presence of multiple stable sets, the dynamic system implies a set of initial conditions for which

[18] It can easily be shown that $\partial\Phi/\partial k_t > 0$, which assures that the phase diagram of k_{t+1} against k_t is upward sloping.

indeterminacy arises (a different kind of indeterminacy from the one highlighted in section 3.1). In Figure 2 this is true for any $k \in [k_c, k_d]$.[19] However, the model proceeds maintaining the focus on the case where Φ^H and Φ^L each have only one fixed point. Even in the case of a unique stable set, a broad class of distinct long-run equilibria will be identified.

4 Stationary equilibria

In order to obtain long-run predictions on the evolution of the economy, we need to analyze the steady-state conditions of the model. However, in a stochastic model we cannot define a steady state as a single value of the capital stock, but rather as a probability distribution over the capital stock that remains stationary over time: if we cannot predict whether in the long run the capital stock will assume a specific value, $\tilde{k}$, we can nevertheless state what is the probability that, in the long run, $k = \tilde{k}$.

Let us denote with μ_t the probability distribution over k_t, i.e. $\mu_t(B) = Pr\{k_t \in B\}$ for all $B \in R_+$.

Definition 2 *A probability distribution μ^* is a stationary probability distribution if it is the probability distribution over k_t, for all t. That is, $\mu_t \rightarrow \mu^*$ as $t \rightarrow \infty$.*

Thus, the stochastic analogue of a steady-state equilibrium in a deterministic framework is as follows.

Definition 3 *A stationary equilibrium for the stochastic process $k_{t+1} = \Phi(k_t, \Theta_t)$ is a stationary probability distribution over the capital stock k on R_+.*

Proposition 2 *If conditions in Proposition 1 hold and the stable set is defined over the interval $[k_a, k_b]$, then there is a unique stationary probability distribution over $[k_a, k_b]$.*

Proof. This is an application of Theorem 12.12 in Stokey and Lucas [39].

[19] Given the restrictions on preferences and technologies, global contraction, multiplicity (and the possible indeterminacy) cannot emerge in the set up chosen by Acemoglu and Zilibotti.

Referring to Figure 1 we see that for any initial condition $k_0 > 0$ the capital stock will fluctuate between the boundaries Φ^H and Φ^L, but once in $[k_a, k_b]$ it will stay there for ever. As $t \to \infty$, $Pr\{k \in [k_a, k_b]\} = 1$ and there is a unique stationary distribution μ^* measuring the probability that $k = \tilde{k}$, $\forall\, \tilde{k} \in [k_a, k_b]$.

4.1 Economic interpretation of the stable set

The features of the long-run equilibrium are determined by the characteristics of μ^*. The analysis of μ^* becomes especially interesting if we think of the stable set $[k_a, k_b]$ in the following fashion: the neighborhood of k_a can be considered as a *poverty region*, where the economy will stagnate if Θ^L recurs often. In contrast, the neighborhood of k_b can be considered as a *developed region* that is reached by the economy if the adverse shock does not occur over time.[20] Under this interpretation it should then be clear that the long run will be qualitatively very different depending on whether μ^* is characterized by a high concentration of mass on the upper or the lower end of the support.

In standard models of stochastic growth this would not be an interesting argument because the randomness in production, which ultimately determines the form of the stationary distribution, is exogenously imposed. But in this model the randomness is the result of an endogenous interaction between savers and firms. Thus, even in the case where an economy can converge to a unique stable set, substantially different long-run patterns can be observed. The identification of this class of dynamic equilibria is the focus of the following three sub-sections.

4.2 Liberating Prometheus forever

The first pattern to identify is the one where, in fact, the stable set degenerates into one point, k_b. Recall that, by assumption, there exists a value of k, defined as $\bar{k} < \infty$, such that $p(k) = 1$, for all $k \geq \bar{k}$. If an economy passes this threshold, the capital accumulation externality is strong enough to assure complete insulation of production from the occurrence of bad shocks. If this occurs, then k will converge with certainty

[20] Acemoglu and Zilibotti identify equilibrium points similar to k_a and k_b (in their notation, K^{qssb} and K^{qssg}) and they offer the same interpretation in terms of development regions.

to k_b and μ^* will collapse on the upper end of the stable set.

The following Proposition describes formally the condition for the identification of such long-run pattern.

Proposition 3 *Consider the case described in Figure 4. The stable set is defined as the interval* $[k_a, k_b]$*. If* $\overline{k} \leq k_b$*, then for any capital stock* $k_0 > 0$*, the economy will converge, with probability one, to a deterministic steady state equilibrium,* $k = k_b$*.*

Proof. When $\overline{k} \leq k_b$, then for any value of $k_0 > 0$ a finite number of favorable shocks is sufficient for the capital stock to enter the range of values $k \geq \overline{k}$. From that point on, k will evolve according to $k_{t+1} = \Phi(k_t, \Theta^H)$ and will converge in the long run to k_b. □

Proposition 3 describes a long-run equilibrium where the economy is perfectly insulated by exogenous shocks. The transition, however, could be considerably long and very painful: suppose that $p(k_a)$ is very low. Consequently, when k is close to k_a, there is a high probability to be hit by Θ^L, thus low probability to accumulate capital and therefore high likelihood that Θ^L will occur again. Time persistency will be observed when k is in a neighborhood of k_a. However, with probability one, sooner or later Θ^H will occur, capital will accumulate and the probability of Θ^H occurring again will increase, thus making more and more likely that the economy will approach the threshold $\bar{k}$.

What I have just described is actually the scenario identified in Acemoglu and Zilibotti. Passing $\bar{k}$ corresponds in their model to supply capital to the entire set of projects, thus covering all possible states of nature. Prometheus will eventually be unbound.[21]

However, in principle there does not seem to be a particular reason to believe that an economy can ever grow enough to reach the threshold $\bar{k}$. Whether $\bar{k} \lesseqgtr k_b$ is a condition depending on the primitives of the economy, and we may not have sufficient 'a prioris' to claim that the scenario depicted in Proposition 3 holds unconditionally. In fact, if $\bar{k} \leq k_b$ were a general condition, we should expect all economies to be

[21] Although he will not be unbound by chance, since–as just mentioned–the probability that k grows larger than $\bar{k}$ is equal to one. Of course, chance matters for the actual timing of the transition process.

joining the club of the most developed countries, and once there, to exhibit a very low degree of output variability. But this is a too strong implication in the light of the club convergence patterns observed in reality.

The long-run equilibrium described in Proposition 3 should therefore be considered only as a *special case* of a more general set of long-run equilibria, distinct from each other by the shape of the stationary distribution μ^*. According to the definition of a stationary equilibrium, the economy is in fact in a stable, long-run equilibrium even if μ^* is not collapsed on the upper bound of the support.[22]

How can we characterize the long run in this more general case? If $\bar{k} > k_b$, then even a long, repeated sequence of lucky draws will not allow the economy to become totally insulated from production risk. The intuition is that, depending on the specific parameterization of preferences and technology (recall we are working purposefully with the most general functional forms restrictions), agents may never optimally save enough to allow an accumulated capital stock larger than $\bar{k}$. This more general scenario can be formally described as follows.

Proposition 4 *If $\overline{k} > k_b$ then for any value of $k_0 > 0$, the long-run equilibrium is characterized by perpetual fluctuation in the stable set $[k_a, k_b]$.*

Proof. The proof follows from a brief inspection of Figure 5.

Thus, under condition in Proposition 4, Prometheus is destined to be bound for eternity. However, the characteristics of the long-run equilibrium (i.e., keeping the metaphore, Prometheus' well being) will be very different depending on the shape of the resulting stationary distribution μ^*.

4.3 Convergence to a unique neighborhood

Suppose, for example, that $p(k_a)$ is already substantially high and then it grows higher as k goes to k_b. In this case μ^* will be characterized by a high concentration of mass toward the upper bound of the support. See Figure 6a.

[22] The choice for the title of this paper, in direct reference to Acemoglu and Zilibotti's, should now be clear.

If we look at a time-series prediction, an economy under this scenario will spend most of its time in the developed region. The transition path is very similar to the one highlighted in the previous sub-section. The main difference is that this equilibrium configuration is compatible with temporary, but not the least painful, reversals of fortune.

In a cross-section interpretation, this result suggests convergence of countries to a developed region and decreasing variability as capital increases, exactly as in the case described in the previous section.

The opposite scenario could be generated if $p(k_a)$ is low and it is still low at $p(k_b)$. In this case μ^* will be characterized by a concentration of mass toward the lower bound of the support. See Figure 6b.

In a time-series interpretation, an economy should be observed, most of the time, in the poverty region, although temporary spurts of growth may still be observed.

Cross-sectionally, this equilibrium pattern suggests a result of convergence to low levels of income and the identification of a poverty trap (although, again, it is at least temporarily possible to escape from the trap before Prometheus is bound again).

4.4 Club convergence

Finally, suppose that $p(k_a)$ is low but $p(k_b)$ is high. Under such a scenario, and repeating the steps described above, the model implies time persistency when k is *either* in the neighborhood of k_a or k_b. Thus, both k_a and k_b represent poles of attraction for the capital stock in the long run. As Figure 7 shows, a "twin-peaks" stationary distribution characterizes the long-run equilibrium.

In a cross-section interpretation, this is a legitimate identification of club convergence. It accounts for the observation of *identical* economies, some clustered in a poverty region and others clustered in the developed region. It is important to highlight that such pattern emerges under the conditions for a unique stable set and therefore a unique ergodic distribution. Consequently, as anticipated in introduction, we do not need to invoke multiplicity of steady-state equilibria. In fact, we can now move one step further in the interpretation of the results and argue that the empirical prediction associated with multiple steady states (close initial conditions in per capita output levels imply convergence to the same steady state) is not only ambiguous but actually misleading. As shown here, two identical economies, even with exactly the same initial level of income per capita and both with the same stationary distribution featuring club convergence, can still be observed to have permanently divergent capital levels.[23]

5 Numerical simulation

This section presents numerical simulations where specific functional forms were chosen for the primitives of the economy. The simulation exercise helps visualizing the stationary distribution of capital per capita, previously analyzed theoretically. In addition, it shows how the shape of the stationary distribution is affected by a change in the characteristics of the $p(k)$ function, thus adding normative value to the analysis. This section also presents several robustness checks of the main results..

The utility function is assumed to be log-linear, $U = \beta log(c_t) + E[(1 - \beta)log(c_{t+1})]$ and the production function is a Cobb-Douglas, $f(k_t) = k_t^{\alpha}\Theta_t$. This choice rules out equilibrium indeterminacy and assures that the economy converges to a unique stable set.

Given this specific functions, the law of motion for the capital stock is $k_{t+1} = (1-\beta)(1-\alpha)k_t^{\alpha}\Theta_t$. The lower and upper ends of the stable set are $k_a = [(1-\beta)(1-\alpha)\Theta^L]^{\frac{1}{1-\alpha}}$ and $k_b = [(1-\beta)(1-\alpha)\Theta^H]^{\frac{1}{1-\alpha}}$.

The model was calibrated around Quah's transition matrix, shown in Table 1. If x is world average income per-capita, the width of the support of the ergodic distribution in the Table is equal to $2x/0.25x = 8$. Setting $\Theta^H = 1$, I selected Θ^L accordingly, so that $k_b/k_a = 8$.

For every exercise I have run numerical simulations consisting of up to 100,000 iterations. For each iteration a random realization was drawn from the uniform distribution over $[0, 1]$. Starting from an assigned level of k, already in the stable set, the simulation describes the random process of capital accumulation. In every

[23] More precisely, the two economies are clearly indistiguishable in a distributional sense. However, if one takes a snapshot at any point in time or observes any finite interval of time (which is what we do empirically), the economies may very well be observed having highly different levels of capital per capita (one attracted in the poverty region while the other attracted in the developed region).

iteration the shock Θ would take value Θ^L if the generated random number were greater than the value of p, or value 1 otherwise. The value assigned to p in a specific iteration depended on the chosen expression for $p(k)$. For every simulation I have then drawn the corresponding stationary distribution, approximated by a frequency histogram. In order to maintain the parallel with Quah's empirical exercise, the histograms were drawn choosing a 5 states grid.

In the first part of the exercise $p(k)$ was assumed to be a linear, increasing function. Its slope and intercept were set so that the function would fit closely the values of the off-diagonal elements of Quah's transition matrix. Specifically, in Table 1 the probability of an economic setback for countries at the bottom of the ranking is 0.76, while it is 0.05 for countries in the top cluster. Consequently, I set $p(k_a) = 0.24$ and $p(k_b) = 0.95$. This yielded the function $p_t = 0.14 + 8.99 \cdot k_t$.

Setting $\beta = 0.4$ and $\alpha = 0.5$ in the utility and production functions respectively, I ran a first simulation. The top part of Figure 8 shows the resulting stationary distribution, exibiting bimodality with a clear thinning in the middle. Table 2 reports in the first row the numerical values for the simulated ergodic distribution and in the second row the corresponding values from Quah's empirical findings (the bottom row in Table 1).

As a first robustness check, I ran multiple independent simulations. The bottom part of Figure 8 shows the contour plot of the corresponding tri-dimensional frequency histogram, confirming the result of clustering and thinning in the middle. As an alternative robustness check, I also ran simulations letting α vary in a plausible sub-range in [0.4, 0.75], and β in [0.3, 0.7]. Table 3 and 4 reports the corresponding ergodic distributions, showing a substantially unchanged result. It may be worthwhile noticing that changes in β, i.e. changes in agents preferences over savings for future consumption, does not seem to have any effect on the shape of the distribution. This may hint that indeed, as suggested in introduction, traditional policies aimed at stimulating economic growth through an increase in savings rate may not be effective unless there is also an improvement in insulation from production shocks.

In a second exercise, I ran simulations keeping the intercept of the $p(k)$ function constant and varying the slope. A specific parameterization of the function $p(k)$ captures the degree of market incompleteness and product differentiation present in the economy. In the model, a steeper $p(k)$ function reflects a superior insulation from adverse shocks. As the stationary distributions in Figure 9 show, a relatively flat $p(k)$ function generates unimodal convergence to the poverty region, while the opposite is true with a relatively steep function. This exercise confirms the impact that policies aimed at improving the insulation of the country from the occurrence of adverse shocks would have on the pattern of long-run capital accumulation.

Additional robustness checks were done by running simulations assuming a continuous *Beta* distribution for $\Theta \in [\Theta^L, \Theta^H]$, rather than a binomial.[24] An appropriate change of the parameters of a *Beta* distribution shifts probability mass from the lower to the upper extreme of the support. The conjectured endogenous relationship between the probability of an adverse shock and the level of capital accumulation can be translated in a continuous framework by assuming the *mean* of the distribution as an increasing function of k. Therefore, in the simulation Θ would be drawn from a different distribution, depending on the value of k in the stable set. Table 5 reports the resulting ergodic distribution, which is still bimodal.

As a final robustness check I have run simulations keeping $p(k)$ constant. The reason for this exercise was to verify whether one could actually obtain a multimodal ergodic distribution within the standard stochastic growth model.

The simulations were run both with Θ drawn from a binomial distribution and from a continuous *Beta* distribution. Table 6 reports the ergodic distributions for the binomial case, for $p = (0.2, 0.4, 0.5, 0.6, 0.8)$. Regardless of the value taken by p, the distributions never come close to approximate one with twin peaks and a marked thinning in the middle, such as the one observed empirically. Unimodality is also clearly displayed in the case in which Θ was drawn from a *Beta* distribution, as it is reported in Table 7.

6 Conclusions

The main question asked in economic development is whether poor countries catch up with the rich. Empirical evidence suggests a negative answer, displaying a bimodal, ergodic cross-country distribution of income per capita. The poor on average stay

[24] The Beta distribution has a density function $b(x, v, w) = \frac{x^{v-1}(1-x)^{w-1}}{\int_0^1 x^{v-1}(1-x)^{w-1}dx}$, for $0 \leq x \leq 1$. The mean of the distribution is $Mean(x) = \frac{v}{v+w}$.

poor, but it is still possible to observe intra-distribution mobility. How does growth theory explain this empirical evidence? The standard neoclassical model, in its augmented versions, do predict club convergence, but cannot explain economic miracles, reversal of fortunes and growth disasters, due to the non-ergodic properties of the predicted stationary distribution.

In the standard stochastic version of the neoclassical model, every distributional characteristic is fully explained by nature, leaving little room for economics. In this paper I have proposed a refinement of the neoclassical, stochastic growth model, in which the likelihood of adverse shocks to production is postulated to be higher when an economy is in early stages of development. The assumption is justified economically arguing that a developing country may suffer from lack of diversification and missing markets and institutions, which may leave the economy overly exposed to the occurrence of adverse shocks. The empirical evidence from Quah [30]'s transition matrix shown in Table 1 indeed confirms an intrinsic higher fragility of less developed countries.

The paper contributes to the convergence debate by identifying the conditions for the emergence of club convergence, without relying on multiplicity of steady states. In addition, the predicted stationary distribution is ergodic, allowing for intra-distribution mobility. The paper also identifies conditions for equilibrium indeterminacy, considered as a possible alternative explanation for club convergence and intra-distribution mobility.

In sum, this approach confirms the neoclassical model as an adequate framework of analysis for economic development but it highlights uncertainty in production as a fundamental second dimension. Capital accumulation per se is a necessary but not a sufficient condition for *permanent* development. In fact, the paper suggests that the very *definition* of a developing country should be based on a metric that incorporates risk considerations.

Appendix

Proof of Proposition 1

Let us write equation (6) from section 3:

$$v'(\omega_t(\Theta_t) - s_t) = p_{t+1}(s_t)u'(s_t f'(s_t)\Theta^H_{t+1})f'(s_t)\Theta^H_{t+1} \quad (12)$$
$$+ \quad (1 - p_{t+1}(s_t))u'(s_t f'(s_t)\Theta^L_{t+1})f'(s_t)\Theta^L_{t+1}. \quad (13)$$

Existence and uniqueness of a self-fulfilling equilibrium requires that the left-hand side of (12), (LHS), and the right-hand side, (RHS), have only one intersection.

Given the concavity of preferences, LHS is strictly increasing in s_t over $[0, \omega_t]$, with $LHS(0) > 0$ and $LHS(\omega_t) = +\infty$.

Let us now focus on RHS. Note that $kf'(k) < f(k)$ in all states of nature. Thus it follows that $\lim_{k\to 0} kf'(k)\Theta = 0$. Therefore, $RHS(0) = +\infty$ and $RHS(\omega_t) > 0$, which assures the existence of at least one intersection between (LHS) and (RHS).

In order to obtain uniqueness it is sufficient to establish conditions under which (RHS) is strictly decreasing in s_t over $[0, \omega_t]$.

Differentiating (RHS) in s_t, (RHS) is strictly decreasing if

$$\begin{aligned}
&p'(\cdot)u'(\cdot)f'(\cdot)\Theta^H_{t+1} + \\
&p(\cdot)[u''(\cdot)(s_t f''(\cdot) + f'(\cdot))\Theta^H_{t+1} f'(\cdot)\Theta^H_{t+1} + u'(\cdot)f''(\cdot)\Theta^H_{t+1}] + \\
&-p'(\cdot)u'(\cdot)f'(\cdot)\Theta^L_{t+1} + \\
&(1 - p(\cdot))[u''(\cdot)(s_t f''(\cdot) + f'(\cdot)\Theta^L_{t+1})f'(\cdot)\Theta^L_{t+1} + u'(\cdot)f''(\cdot)\Theta^L_{t+1}] < 0.
\end{aligned}$$

This expression is strictly less than zero if conditions (9) and (10) in Proposition 1 hold. Under these conditions there is only one intersection between LHS and RHS over $[0, \omega_t]$. $\square$

Table 1

Real GDP per capita relative to world average

23-year transition (1962-1985). Quah [30]

	0-1/4	**1/4-1/2**	**1/2-1**	**1-2**	**>2**
0-1/4	0.76	0.12	0.12	0.0	0.0
1/4-1/2	0.52	0.31	0.10	0.07	0.0
1/2-1	0.09	0.20	0.46	0.26	0.0
1-2	0.0	0.0	0.24	0.53	0.24
>2	0.0	0.0	0.0	0.05	0.95
Ergodic distribution	0.20	0.09	0.13	0.12	0.47

Results of the simulations (Table 2-7)

Table 2

Binomial Θ. $\alpha = 0.5, \beta = 0.4$.

$p_t = 0.14 + 8.99k_t$

	0-1/4	1/4-1/2	1/2-1	1-2	>2
Simulated ergodic distribution	0.32	0.11	0.06	0.05	0.46
Quah's ergodic distribution	0.20	0.09	0.13	0.12	0.47

Table 3

Simulated ergodic distribution.

Binomial Θ. $p_t = 0.14 + 8.99k_t$

Varying α (β=0.4).

	0-1/4	1/4-1/2	1/2-1	1-2	>2
$\alpha = 0.4$	0.26	0.03	0.04	0.03	0.62
$\alpha = 0.5$	0.32	0.11	0.06	0.05	0.46
$\alpha = 0.6$	0.37	0.14	0.08	0.05	0.36
$\alpha = 0.7$	0.48	0.18	0.08	0.05	0.21
$\alpha = 0.75$	0.49	0.20	0.09	0.05	0.15

Table 4

Simulated ergodic distribution.

Binomial Θ. $p_t = 0.14 + 8.99k_t$

Varying β (α=0.5).

	0-1/4	1/4-1/2	1/2-1	1-2	>2
$\beta = 0.3$	0.31	0.11	0.06	0.04	0.47
$\beta = 0.4$	0.32	0.11	0.06	0.05	0.46
$\beta = 0.5$	0.30	0.11	0.05	0.05	0.49
$\beta = 0.6$	0.31	0.11	0.06	0.05	0.48
$\beta = 0.7$	0.31	0.11	0.06	0.05	0.48

Table 5

Beta distributed Θ. $Mean(\Theta)$ increasing in k.

$\alpha = 0.5, \beta = 0.4.$

	0-1/4	1/4-1/2	1/2-1	1-2	>2
Simulated ergodic distribution	0.37	0.06	0.03	0.06	0.48

Stable set: $k \in [0.01125, 0.09]$.

For $k \in [0.01125, 0.27], \Theta$ drawn from $Beta(2, 6)$.

For $k \in [0.027, 0.57], \Theta$ drawn from $Beta(6, 6)$.

For $k \in [0.057, 0.09], \Theta$ drawn from $Beta(6, 2)$.

Table 6

Simulated ergodic distribution.

Binomial Θ. Constant p. $\alpha = 0.5, \beta = 0.4.$

	0-1/4	1/4-1/2	1/2-1	1-2	>2
$p = 0.2$	0.76	0.16	0.05	0.02	0.02
$p = 0.4$	0.49	0.23	0.14	0.08	0.04
$p = 0.5$	0.35	0.25	0.17	0.13	0.10
$p = 0.6$	0.23	0.24	0.19	0.16	0.18
$p = 0.8$	0.05	0.15	0.15	0.14	0.50

Table 7

Simulated ergodic distribution.

Beta distributed Θ. Constant $Mean(\Theta)$. $\alpha = 0.5, \beta = 0.4.$

	0-1/4	1/4-1/2	1/2-1	1-2	>2
$Beta(2, 6)$	0.78	0.22	0.08	0.00	0.00
$Beta(6, 6)$	0.001	0.64	0.35	0.001	0.00
$Beta(6, 2)$	0.00	0.01	0.29	0.65	0.05

Figure 1

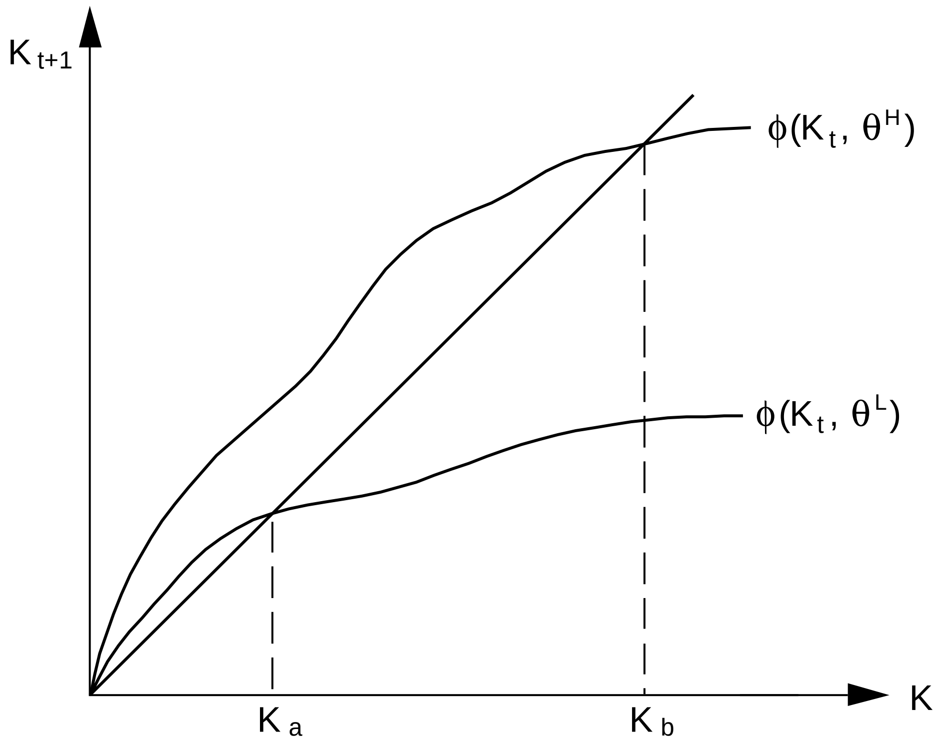
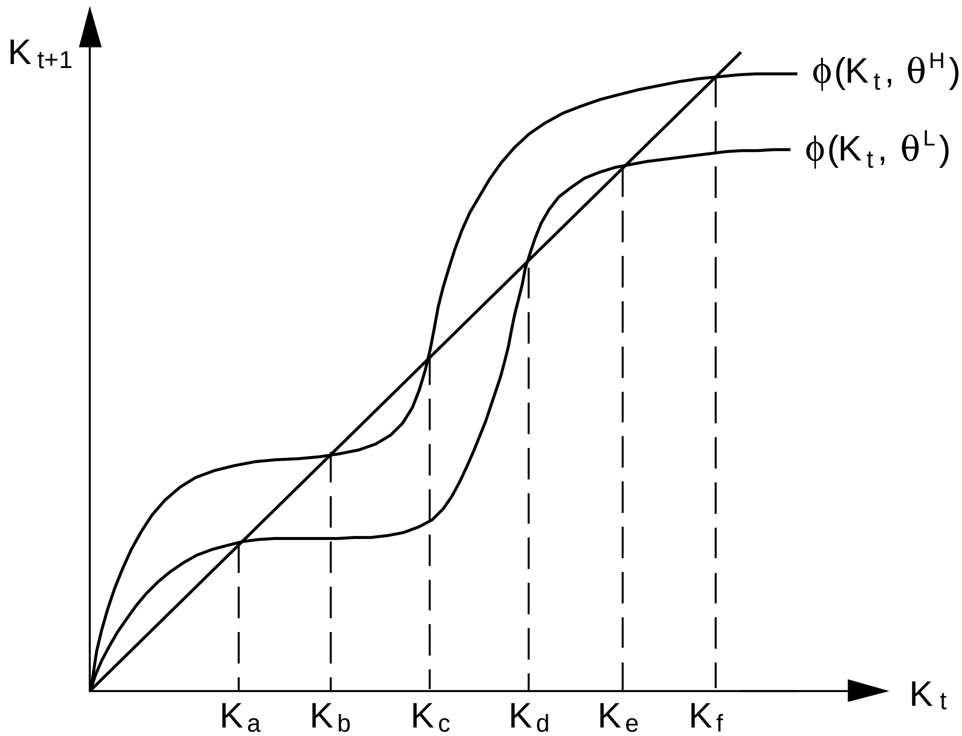


Figure 2



$[K_a, K_b]$: Stable set
$[K_e, K_f]$: Stable set
$K_0 \in [K_c, K_d]$: Indeterminacy

Figure 3

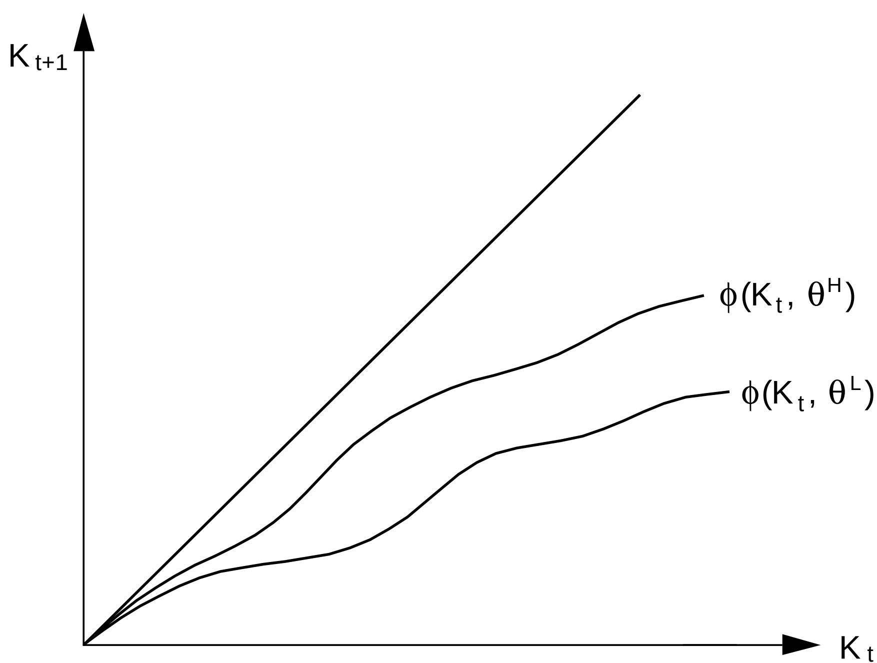
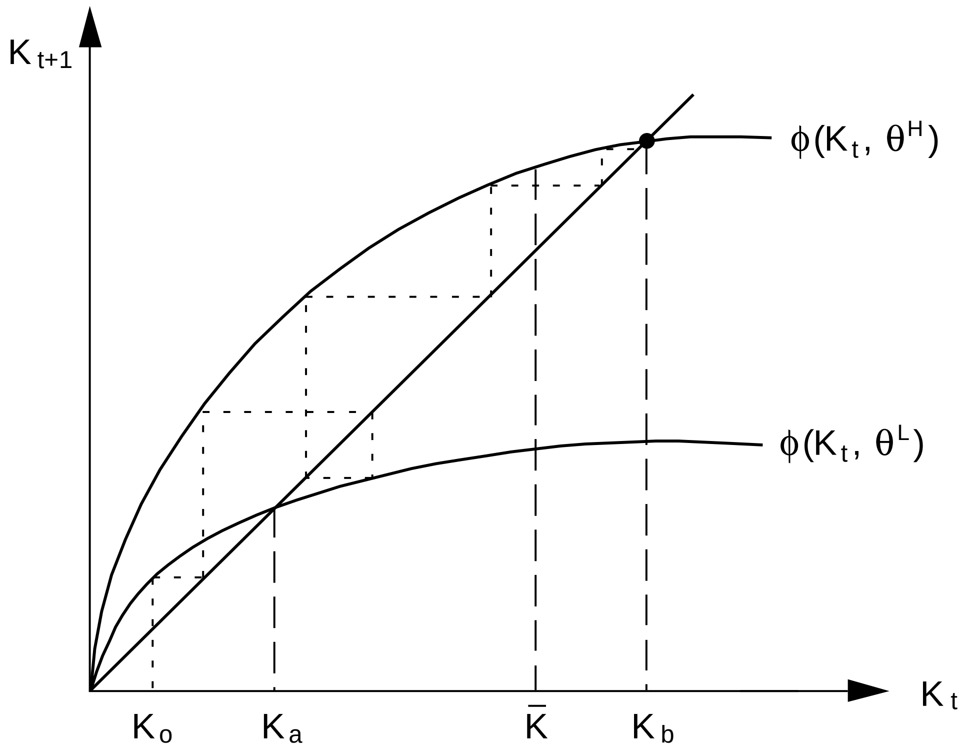
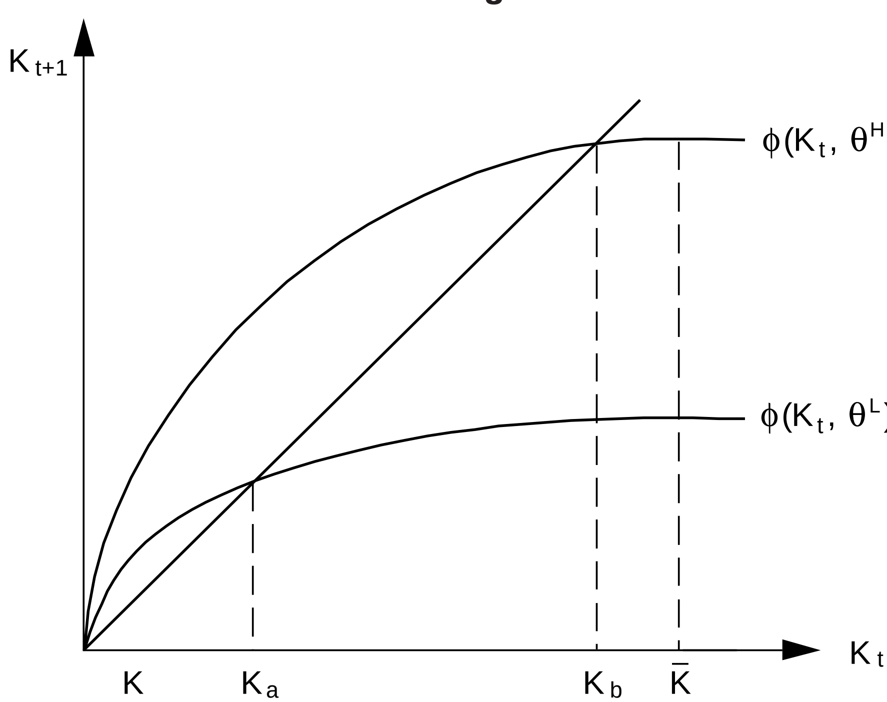


Figure 4



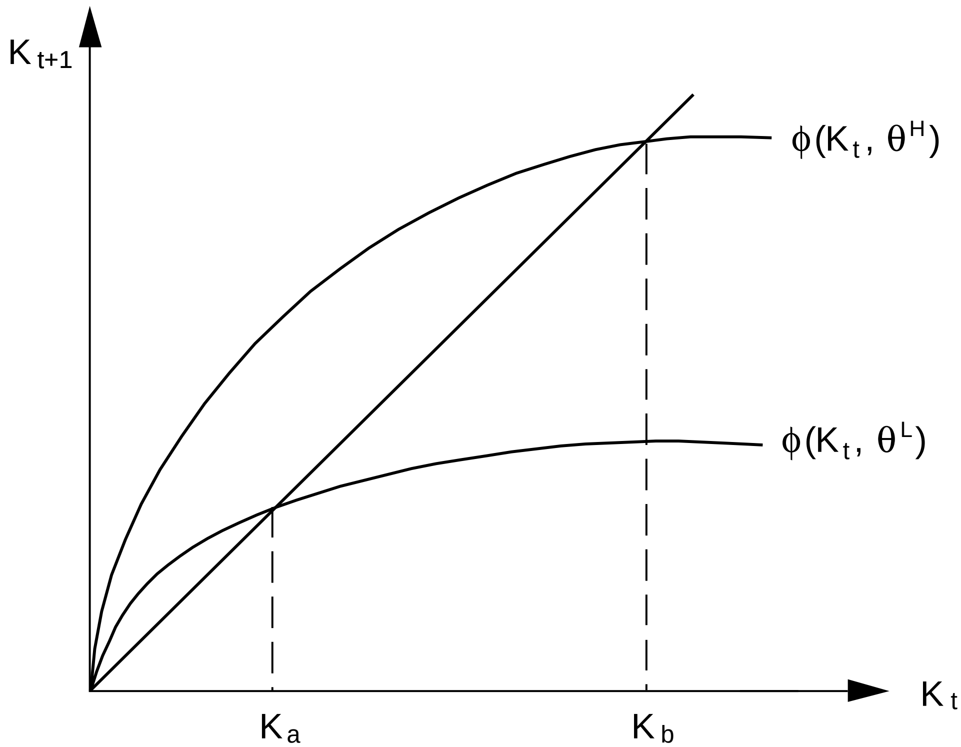
$\bar{K} \leq K_b$: Convergence to K_b

Figure 5



$\bar{K} > K_b$: Permanent fluctuation in $[K_a, K_b]$

Figure 6a



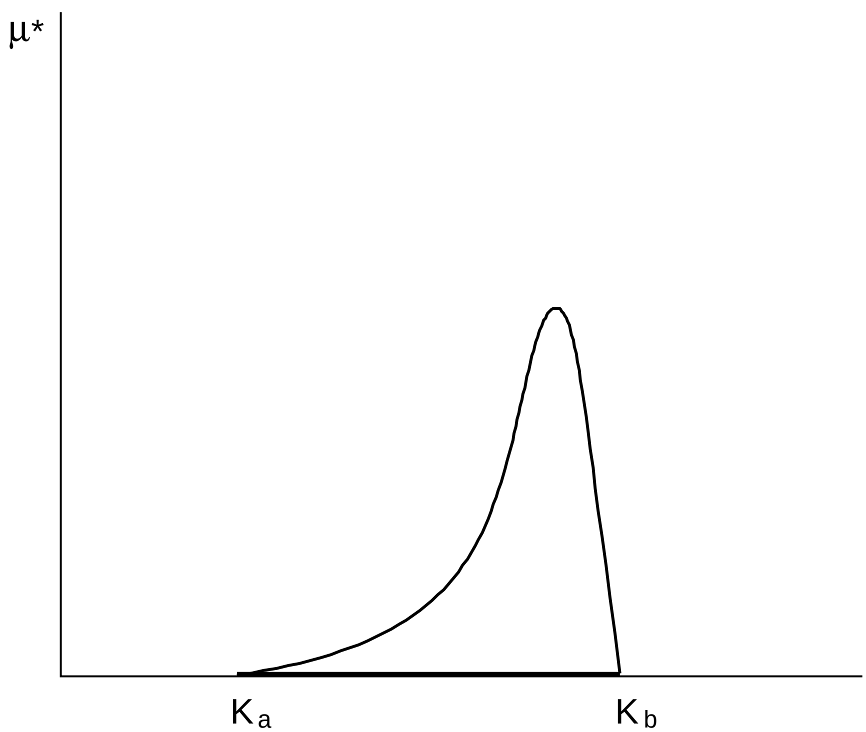


Figure 6b

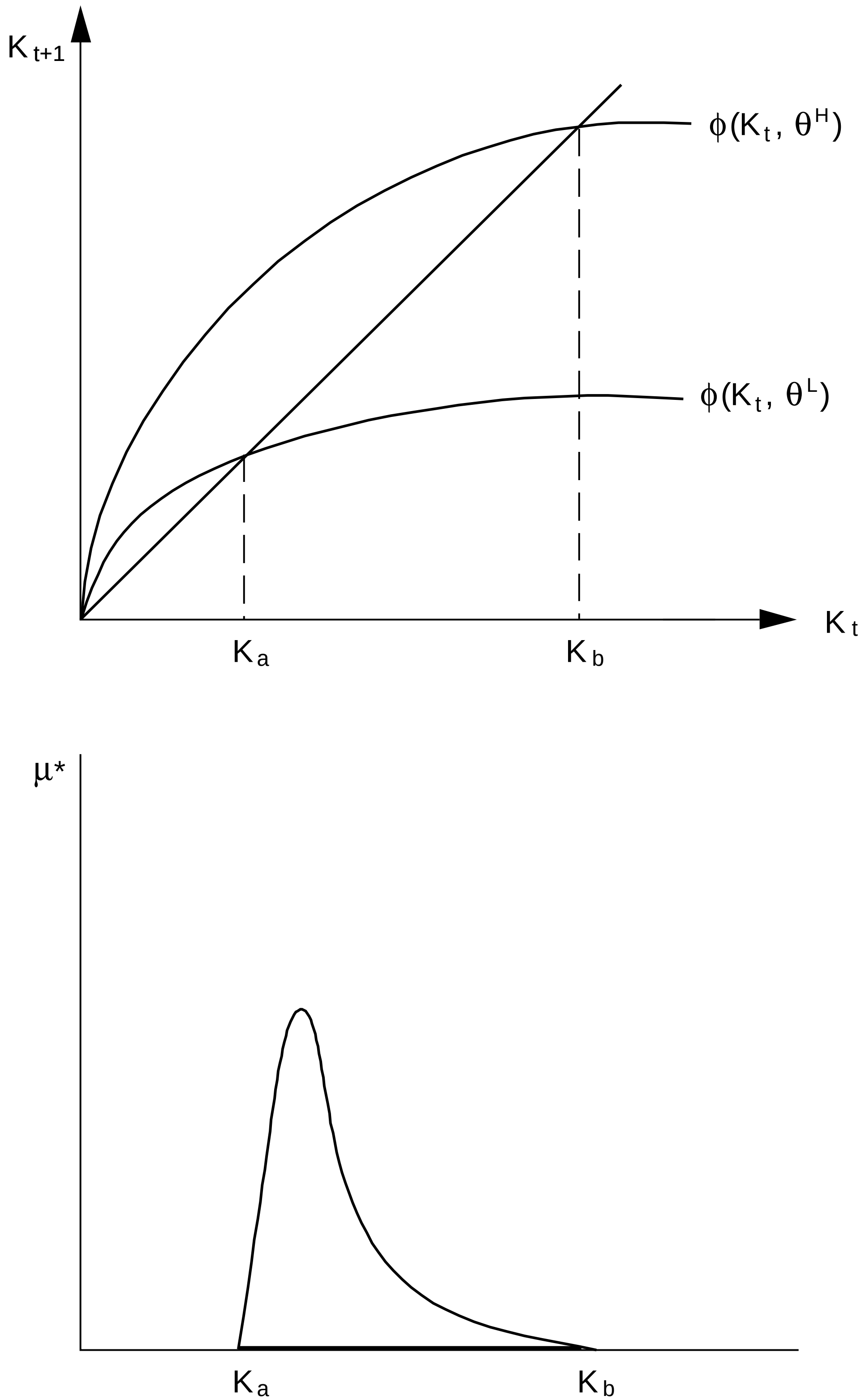


Figure 7

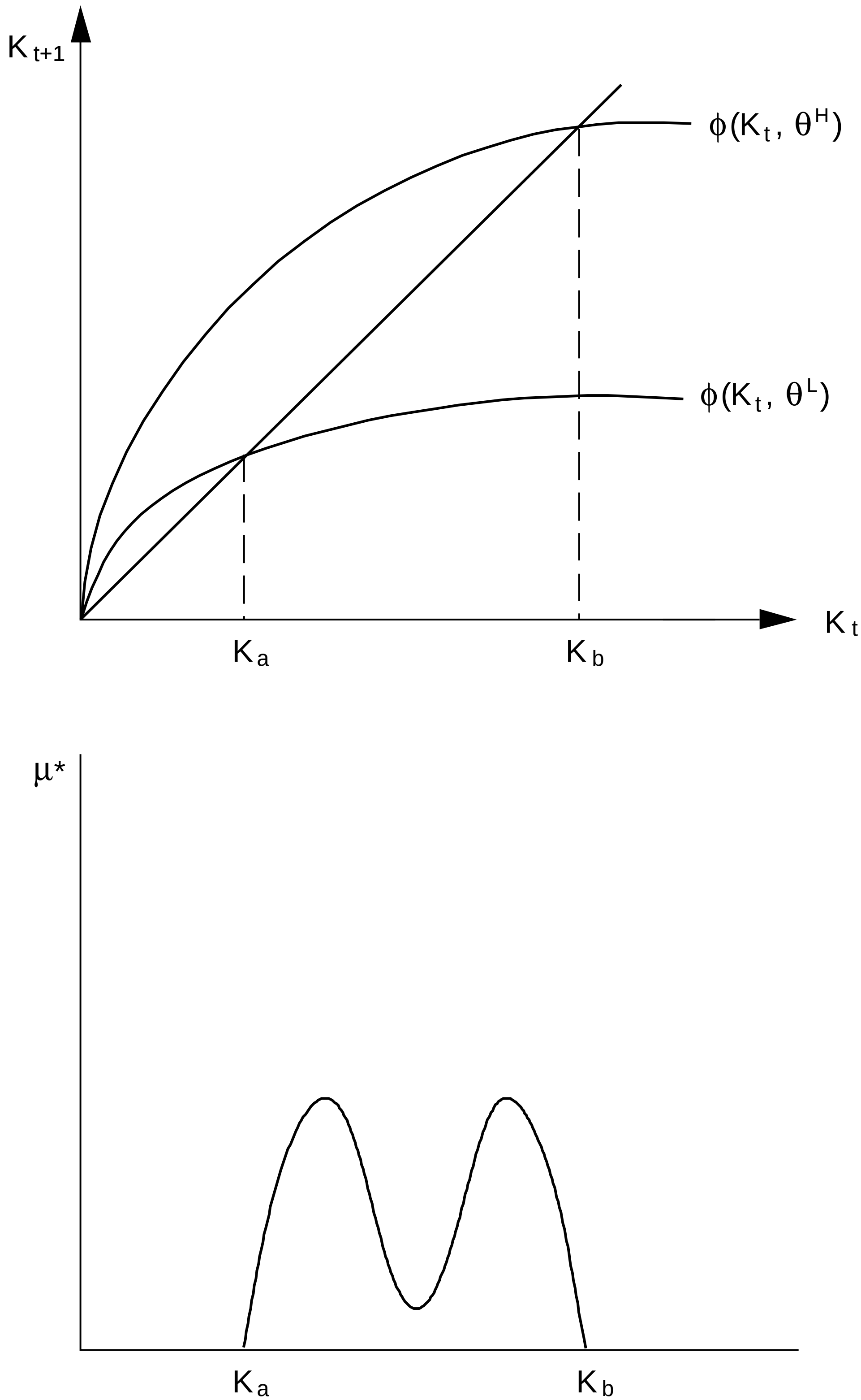
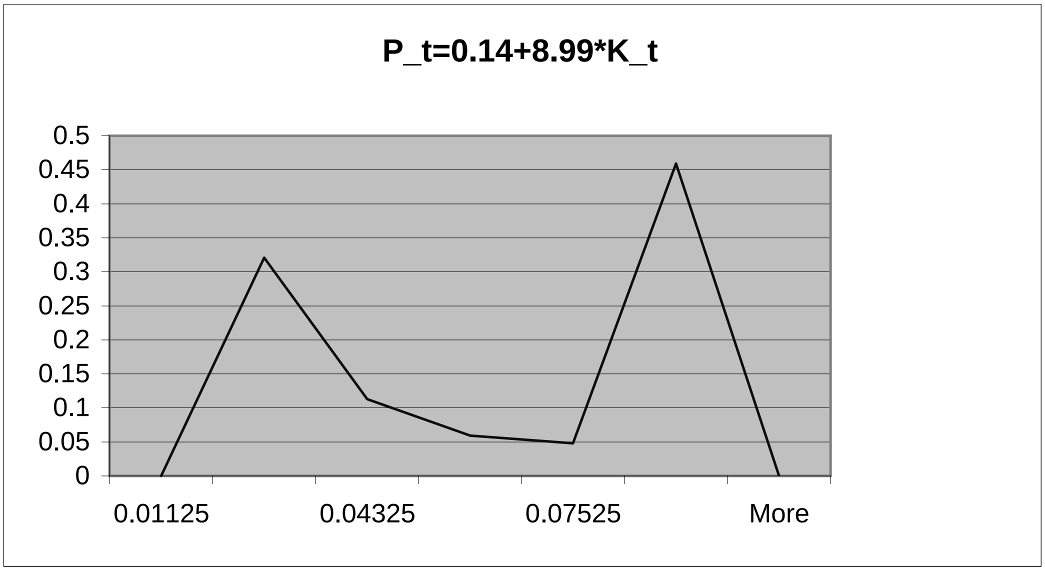


Figure 8


P_t=0.14+8.99*K_t
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.01125
0.04325
0.07525
More

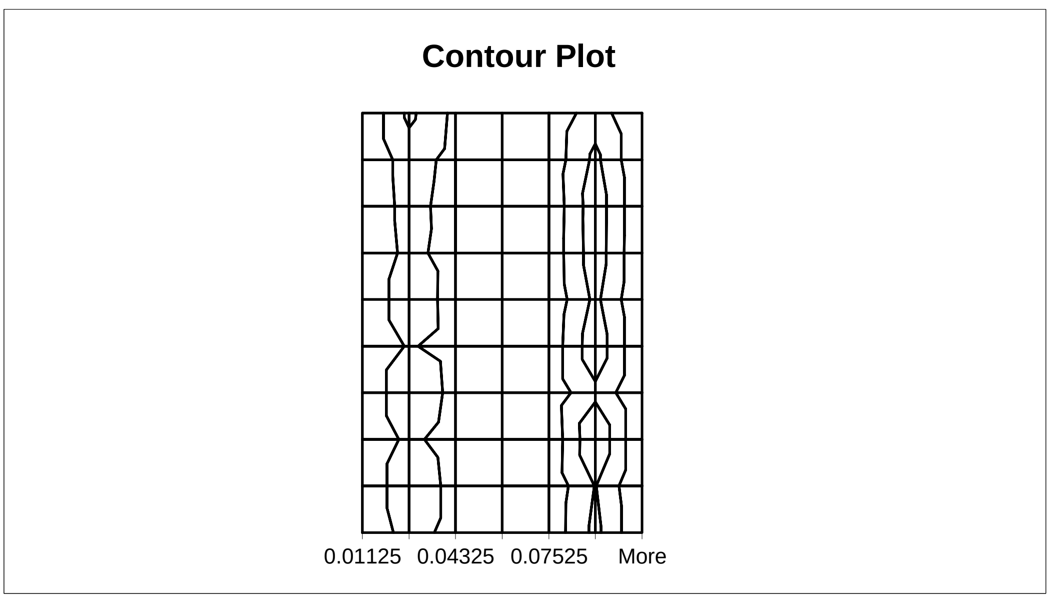
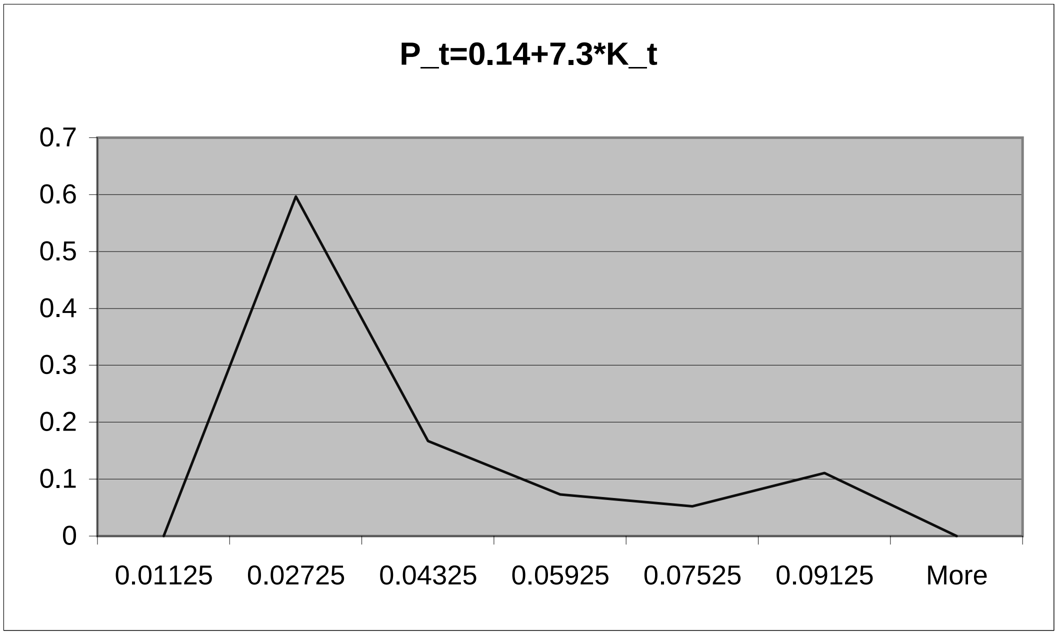
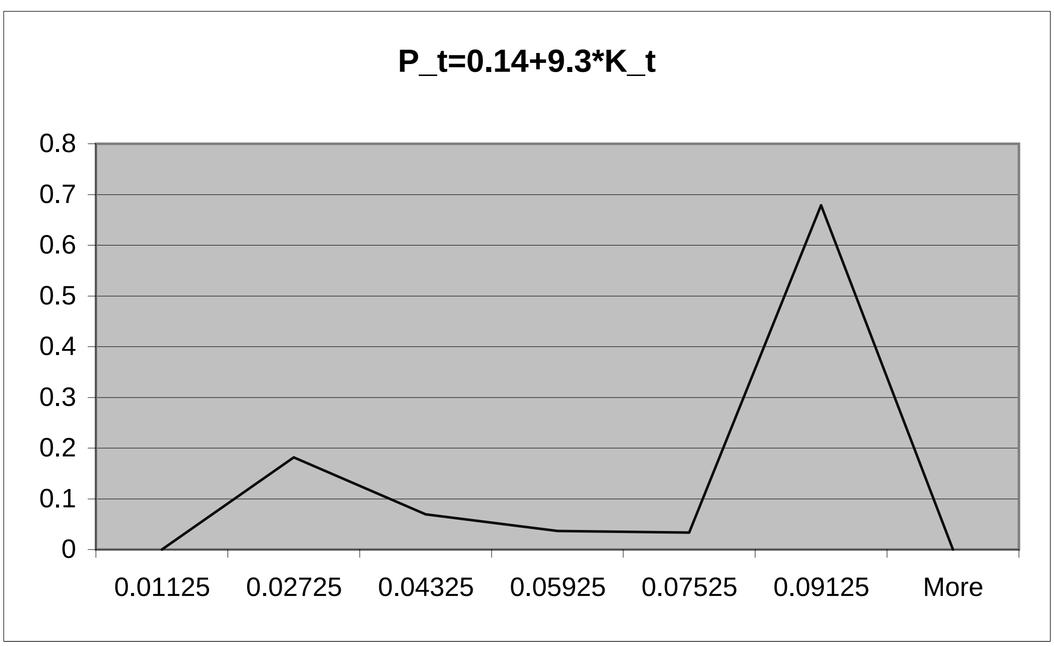

Contour Plot
0.01125
0.04325
0.07525
More

Figure 9


P_t=0.14+7.3*K_t
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.01125
0.02725
0.04325
0.05925
0.07525
0.09125
More


P_t=0.14+9.3*K_t
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.01125
0.02725
0.04325
0.05925
0.07525
0.09125
More

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