



Federal Reserve Bank of Chicago

## **Micro Data and Macro Technology**

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## Abstract

We study the implications of microeconomic heterogeneity for aggregate technology, showing that the aggregate elasticity of substitution between capital and labor can be expressed as a simple function of plant level structural parameters and sufficient statistics for plant heterogeneity. This allows for a new approach to estimating the aggregate elasticity using microeconomic data and allows us to examine how the aggregate elasticity varies over time or across countries. We then use plant level data from the Census of Manufactures to construct an aggregate elasticity of substitution for the manufacturing sector, and estimate an aggregate elasticity of approximately 0.72 in 1987. We find that the aggregate elasticity has risen over time in the US and is higher in less developed countries. These differences are quantitatively important; our estimates imply that a change in the interest rate has a 50 percent larger impact on India than the US. Finally, we measure the bias of aggregate technical change using our estimates of the aggregate elasticity, and find that the bias of technical change has increased in recent years.

KEYWORDS: Elasticity of Substitution, Aggregation, Bias of Technical Change.

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# 1 Introduction

At the foundation of much of macroeconomics is the shape of aggregate production technology. Since [Hicks \(1932\)](#), economists have recognized that the aggregate elasticity of substitution determines how income is divided between labor and capital. The elasticity is crucial to understand how these factor shares and aggregate output respond to corporate tax changes or international interest rate differentials, and how international trade affects factor intensities and welfare ([Hall and Jorgenson \(1967\)](#), [Mankiw \(1995\)](#), [Dornbusch et al. \(1980\)](#)). [Hicks \(1932\)](#) also emphasized that innovation is often not neutral. The value of the elasticity is crucial in measuring biased innovation, and determines how changes in aggregate factor quantities affect biased innovation ([Acemoglu \(2002\)](#), [Acemoglu \(2010\)](#)).

Aggregate technology, however, is composed of a multitude of micro production units. The empirical microeconomic literature has repeatedly found vast heterogeneity across these units in size, productivity, and capital intensity; see ([Bartlesman and Doms \(2000\)](#), [Syverson \(2011\)](#)). Moreover, aggregate and industry composition responds to regulation, trade barriers, and structural change, and varies considerably across countries.<sup>1</sup> The central question of this paper is to characterize how this microeconomic heterogeneity impacts aggregate technology.

We first show that the aggregate elasticity of substitution is a convex combination of

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<sup>1</sup>For examples of these, see [Olley and Pakes \(1996\)](#) on deregulation, [Holmes and Stevens \(2010\)](#) on trade, [Buera and Kaboski \(2012\)](#) on structural change, and [Bartlesman et al. \(2009\)](#) on cross-country differences.

the micro elasticity of substitution and the elasticity of demand. The macro and micro elasticities of substitution differ because the macro elasticity also includes how consumers respond to changes in relative prices; this response is larger when demand is more elastic. Factor price movements lead to greater changes in relative prices when plants produce at more varied capital intensities. We show that a cost-weighted variance of capital shares is sufficient to capture all of the micro heterogeneity.

We use this decomposition to make two contributions. First, we provide a new estimation strategy for the aggregate elasticity by recovering its components from cross-sectional microdata. The aggregate elasticity of substitution we estimate is local; it depends on the composition of firms at a point in time. Because we do not assume a stable functional form for the aggregate production function, in contrast to most of the empirical literature, our analysis is robust to large structural changes that might alter aggregate production technology over time. Second, we are able to examine both how aggregate technology varies across countries and time, and why it varies.<sup>2</sup> For example, policies that lead to a higher variance of capital shares will raise the aggregate elasticity.

We use the US Census of Manufactures to estimate the elasticity of substitution for the manufacturing sector. Across industries, we find an average plant-level elasticity of substitution of roughly one-half, and a scale elasticity of roughly two.<sup>3</sup> Given the variation

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<sup>2</sup>The aggregate time series lacks the power to examine movements in the aggregate elasticity across time.

<sup>3</sup>To account for the fact that firms can also substitute towards materials when relative factor prices change, the scale elasticity is a combination of the elasticity of demand and the elasticity of substitution between materials and a capital-labor aggregate.

in capital shares, the aggregate elasticity in 1987 was 0.72, so that roughly one-quarter of substitution between capital and labor comes from the shift in composition.<sup>4</sup> We find an aggregate elasticity of 0.79 for the US in 1997; the increase over our 1987 estimate is mostly attributable to compositional changes.

We then use our methodology to estimate how the shape of aggregate elasticity varies across countries using manufacturing censuses from Chile, Colombia, and India. We find an average manufacturing sector elasticity of 0.82 for Chile, 0.86 for Colombia, and 1.11 for India, with most of the increase for Chile and India driven by larger variation in capital shares. This implies, for example, that a rise in the wage decreases the labor share in India while increasing the labor share in the US. These differences are also quantitatively important; the response of output per worker to a change in the interest rate is over fifty percent larger in India than in the US, as is the welfare cost of capital taxation.

Movements in the aggregate labor share can come from changes in factor prices or the bias of aggregate technical change. Our approach leads to a natural way to separate the contribution of each of these factors, because we place no assumptions on the behavior of technical change over time, unlike estimates of the elasticity using the aggregate time series. Technical change is biased towards capital, with an average rate of biased technical change of 2.20 percent. The rate of biased technical change has not been constant; the rate of

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<sup>4</sup>Our estimate is a long run elasticity of substitution between capital and labor, owing to a proper interpretation of our estimates of firm level parameters. This elasticity is then an upper bound on the short run elasticity of substitution, which might be more relevant for interpreting fluctuations at business cycle frequencies.

biased technical change is 3.70 percent from 2000–2011, compared to only 0.30 percent from 1970–1979.

Our work is related to two separate literatures. First, [Houthakker \(1955\)](#) demonstrated theoretically how different the micro and macro elasticities of substitution could be. In his model, an economy composed of micro units with Leontieff production functions has an aggregate Cobb-Douglas production function. However, [Levhari \(1968\)](#) showed that this result relies critically on the assumption of Pareto distributed factor-augmenting productivities.<sup>5</sup> More recently, [Jones \(2005\)](#), [Lagos \(2006\)](#), and [Luttmer \(2012\)](#) all used distributional assumptions to derive an aggregate Cobb-Douglas production function.<sup>6</sup> Rather than assuming a particular distribution, we measure it directly using plant level data. Our work is closest to [Sato \(1967\)](#), who showed that in an economy with two goods the aggregate elasticity of substitution is always between the micro elasticity of substitution and the elasticity of demand.

Second, our approach complements an empirical literature that has used aggregate time series data to estimate an aggregate production function. Because of the impossibility theorem of [Diamond et al. \(1978\)](#), identification requires strong assumptions. The literature using aggregate data assumes both a stable functional form for the aggregate production

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<sup>5</sup>Independent Pareto distributions and a cost cutoff imply that plants' cost shares of capital are uncorrelated with size, which does not match the US micro data. Rather, larger plants tend to have higher capital shares.

<sup>6</sup>[Lagos \(2006\)](#) used similar assumptions as [Houthakker \(1955\)](#) in a model with labor search. [Jones \(2005\)](#) assumed independent Pareto distributions of factor augmenting productivities in microfounding a Cobb-Douglas production function, while [Luttmer \(2012\)](#) provided an alternative microfoundation.

function and substantial restrictions on the bias of technical change.<sup>7</sup> In contrast, our identifying assumptions are related to our estimation of micro parameters in the cross-section. The recent time series literature ([Antras \(2004\)](#), [Klump et al. \(2007\)](#)) finds estimates from 0.5 to 0.9; our 1987 estimate of 0.72 for manufacturing lies within their range.

The rest of the paper is organized as follows. In [Section 2](#), we present our theoretical analysis of the aggregation problem, while in [Section 3](#) we estimate the US aggregate elasticity. In [Section 4](#), we examine cross country differences in the aggregate elasticity. [Section 5](#) examines the robustness of our estimates. In [Section 6](#), we examine the bias of technical change given our estimates of the aggregate elasticity of substitution. Finally, in [Section 7](#) we conclude.

## 2 Theory

In this section, we derive the aggregate elasticity of substitution from the actions of heterogeneous firms. Throughout, we maintain the following assumptions in order to greatly simplify our characterization of the aggregate elasticity:

**Assumption 1** *(1) All firms produce using constant returns to scale technology; (2) Factor markets are competitive; (3) All firms face isoelastic demand curves, and firms within an*

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<sup>7</sup>For example, [Berndt \(1976\)](#) found a unitary elasticity of substitution in the US time series assuming neutral technical change, while [Antras \(2004\)](#) and [Klump et al. \(2007\)](#) subsequently found an elasticity of substitution significantly less than unity once they allow for specific forms of non-neutral productivity growth. Even with such assumptions, [Leon-Ledesma et al. \(2010\)](#) showed that it is difficult to identify the macro elasticity of substitution without using restrictions from the entire system of equations of the representative firm's maximization problem, relying more heavily on a stable aggregate production function.

industry face a common demand elasticity; (4) All firms maximize profits.

## 2.1 Industry Elasticity of Substitution

For simplicity, first consider an industry composed of firms whose production functions share a common, constant elasticity of substitution between capital and labor,  $\sigma_i = \sigma$ . A firm produces output  $Y_i$  from capital  $K_i$  and labor  $L_i$  using the following CES production function:

$$Y_i = \left[ (A_i K_i)^{\frac{\sigma-1}{\sigma}} + (B_i L_i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

Productivity differences between firms are factor augmenting:  $A_i$  is the firm's capital augmenting productivity and  $B_i$  its labor augmenting productivity.

Each firm faces an isoelastic demand curve with firm specific demand level  $D_i$  and common elasticity of demand  $\varepsilon > 1$ :<sup>8</sup>

$$Y_i = D_i P_i^{-\varepsilon} \quad (2)$$

We define the industry elasticity of substitution,  $\sigma_n^N$ , to be the response of the industry

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<sup>8</sup>Demand would be of this form if consumers have Dixit-Stiglitz preferences with an industry aggregate  $Y_n = \left( \sum_{i \in I_n} \tilde{D}_i^{\frac{1}{\varepsilon}} Y_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$ . In this case  $D_i = Y_n P_n^\varepsilon \tilde{D}_i$ , where  $P \equiv \left( \sum_{i \in I_n} \tilde{D}_i P_i^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$  is the ideal price index.



capital-labor ratio,  $K_n/L_n$ , to a change in relative factor prices,  $w/r$ :

$$\sigma_n^N \equiv -\frac{d \ln K_n/L_n}{d \ln r/w} \quad (3)$$

The industry capital labor ratio is the sum of each firm's capital-labor ratio weighted by the firm's share of industry labor:

$$\frac{K_n}{L_n} = \sum_{i \in I_n} \frac{K_i}{L_i} \frac{L_i}{L_n} \quad (4)$$

To find the industry elasticity of substitution, we can simply differentiate the right hand side of [equation \(4\)](#). After some manipulation (see [Appendix A](#) for details), we can show that the industry elasticity of substitution is a convex combination of the elasticity of substitution and elasticity of demand:

$$\begin{aligned} \sigma_n^N &= (1 - \chi)\sigma + \chi\varepsilon \\ \chi &\equiv \sum_{i \in I_n} \frac{(\alpha_i - \alpha_n)^2}{(1 - \alpha_n)\alpha_n} \frac{c_i}{c_n} \end{aligned} \quad (5)$$

where  $c_i$  denotes the total cost of production for firm  $i$  and  $c_n = \sum_{i \in I_n} c_i$  total industry cost,  $\alpha_i \equiv \frac{rK_i}{rK_i + wL_i}$ , the firm cost share of capital, and  $\alpha_n$  the industry cost share of capital.

The first term is a substitution effect that captures the change in factor intensity holding firm size fixed.  $\sigma$  measures how much individual firms will substitute across factors.

The second term is a scale effect that captures how firm size changes with relative factor prices. By Shephard's Lemma, a firm's cost share of capital  $\alpha_i$  measures how relative factor prices affect its marginal cost,  $\lambda_i \equiv [(r/A_i)^{1-\sigma} + (w/B_i)^{1-\sigma}]^{\frac{1}{1-\sigma}}$ :

$$\frac{\partial \ln(\lambda_i/\lambda_j)}{\partial \ln(r/w)} = \alpha_i - \alpha_j \quad (6)$$

High capital share firms have a relative cost advantage when wages rise, so their scale increases:

$$-\frac{\partial \ln(c_i/c_j)}{\partial \ln(r/w)} = (\varepsilon - 1) [\alpha_i - \alpha_j] \quad (7)$$

Consumers substitute more towards low cost firms when demand is more elastic, increasing the scale effect.

$\chi$  is the cost-weighted variance of capital shares normalized to range from zero to one.<sup>9</sup> When each firm produces at the same capital intensity, each firm's marginal cost responds to relative factor prices in the same way, so that there is no change in relative prices. Thus  $\chi$  is zero, and all substitution is within plants. In contrast, if all firms use either capital only or labor only technologies, all factor substitution is across firms and  $\chi$  is one. When the variance of cost shares is low, the substitution effect dominates the scale effect.

We can easily modify the formulae above to allow firms to have heterogenous constant

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<sup>9</sup>A simple proof:  $\sum_{i \in I_n} (\alpha_i - \alpha_n)^2 \frac{c_i}{c_n} = \sum_{i \in I_n} \alpha_i^2 \frac{c_i}{c_n} - \alpha_n^2 \leq \sum_{i \in I_n} \alpha_i \frac{c_i}{c_n} - \alpha_n^2 = \alpha_n - \alpha_n^2 = \alpha_n(1 - \alpha_n)$ . It follows that  $\chi = 1$  if and only if each firm uses only capital or only labor (i.e., for each  $i$ ,  $\alpha_i \in \{0, 1\}$ ).

returns to scale production functions. In this case, each firm's elasticity of substitution is defined locally as  $\sigma_i \equiv -\frac{d \ln K_i/L_i}{d \ln r/w}$ . **Proposition 1** then expresses the industry elasticity of substitution  $\sigma_n^N$ . The proofs of all propositions are contained in [Appendix A](#).

**Proposition 1** *Under Assumption 1, an industry composed of firms with local elasticities of substitution  $\sigma_i$  and demand elasticities  $\varepsilon$  has an industry elasticity of substitution  $\sigma_n^N = -\frac{\partial \ln(K_n/L_n)}{\partial \ln(r/w)}$ .*

$$\begin{aligned}\sigma_n^N &= (1 - \chi)\bar{\sigma}_n + \chi\varepsilon \\ \chi &\equiv \sum_{i \in I_n} \frac{(\alpha_i - \alpha_n)^2 c_i}{(1 - \alpha_n)\alpha_n c_n} \\ \bar{\sigma}_n &\equiv \sum_{i \in I_n} \frac{c_i \alpha_i (1 - \alpha_i)}{\sum_{i \in I_n} c_i \alpha_i (1 - \alpha_i)} \sigma_i\end{aligned}\tag{8}$$

This is the same formula as [equation \(5\)](#), except we replace the firm elasticity of substitution with a weighted average of the firms' local elasticities,  $\bar{\sigma}_n$ , where firms that are larger and have less extreme factor shares are weighted more heavily.

We now extend this benchmark case to multiple industries in [Section 2.2](#) and additional inputs such as materials in [Section 2.3](#).

## 2.2 Aggregating across Industries

To aggregate to the manufacturing sector as a whole, we assume that demand has a nested structure with a constant elasticity at each level of aggregation. Such a structure is consistent

with a representative consumer whose preferences exhibit constant elasticities of substitution  $\varepsilon_n$  within an industry and  $\eta$  across industries:

$$\left[ \sum_{n \in N} \tilde{D}_n \left( \sum_{i \in I_n} \tilde{D}_i^{\frac{1}{\varepsilon_n}} Y_i^{\frac{\varepsilon_n - 1}{\varepsilon_n}} \right)^{\frac{\varepsilon_n}{\varepsilon_n - 1} \frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}} \quad (9)$$

The innermost sum is over  $I_n$ , the set of all producers  $i$  in industry  $n$ , while the outer sum is over  $N$ , the set of all industries  $n$ .

The analysis is the same at the industry level, as each of the firms in the industry faces isoelastic demand. The aggregate elasticity has a parallel structure; the aggregate elasticity of substitution is a convex combination of the industry elasticity of demand  $\eta$  and a weighted average  $\bar{\sigma}^N$  over the industry elasticities of substitution  $\sigma_n^N$ . The weight between these two parameters depends on the cost weighted variance in capital cost shares across industries, rather than across firms. **Corollary 1** states this formally:

**Corollary 1** *The aggregate elasticity of substitution  $\sigma^{agg} = -\frac{d \ln(K/L)}{d \ln(r/w)}$  is:*

$$\sigma^{agg} = (1 - \chi^{agg}) \bar{\sigma}^N + \chi^{agg} \eta \quad (10)$$

$$\chi^{agg} = \sum_{n \in N} \frac{c_n (\alpha_n - \alpha)^2}{c \alpha (1 - \alpha)}$$

$$\bar{\sigma}^N = \sum_{n \in N} \frac{c_n \alpha_n (1 - \alpha_n)}{\sum_{n \in N} c_n \alpha_n (1 - \alpha_n)} \sigma_n^N \quad (11)$$

where  $\{\sigma_n^N\}_{n \in N}$  are the industry elasticities of substitution defined in **Proposition 1**.

## 2.3 Materials

So far, we have examined firms that only use capital and labor as inputs. We allow materials to enter production through a nested CES gross output firm production function between materials  $M_i$  and a CES capital-labor aggregate:

$$G_i(K, L, M) = \left( \left[ (A_i K)^{\frac{\sigma_n - 1}{\sigma_n}} + (B_i L)^{\frac{\sigma_n - 1}{\sigma_n}} \right]^{\frac{\sigma_n}{\sigma_n - 1} \frac{\zeta - 1}{\zeta}} + (C_i M)^{\frac{\zeta - 1}{\zeta}} \right)^{\frac{\zeta}{\zeta - 1}} \quad (12)$$

Here,  $\zeta$  is the elasticity of substitution between materials and the capital-labor aggregate. Implicit in [equation \(12\)](#) is that materials is separable from a capital-labor aggregate and that each firm uses the same bundle of materials.<sup>10</sup>

The industry elasticity of substitution can be computed in the same way, as summarized in [Proposition 2](#). As before,  $\alpha_i = \frac{rK_i}{rK_i + wL_i}$  is firm  $i$ 's capital share of non materials cost and  $c_i = rK_i + wL_i$  firm  $i$ 's total expenditure on capital and labor. A new term is  $s_i^M = \frac{qM_i}{rK_i + wL_i + qM_i}$ , firm  $i$ 's materials share of total cost, where  $q$  is the price of the materials bundle.

We assume that the materials bundle has the same factor content as total output.

**Proposition 2** *The industry elasticity of substitution is:*

$$\sigma_n^N = (1 - \chi_n) \bar{\sigma}_n + \chi_n [(1 - \bar{s}_n^M) \varepsilon_n + \bar{s}_n^M \zeta] + \alpha (\zeta - \varepsilon_n) \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

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<sup>10</sup>The assumption that each firm uses the same bundle of inputs can be relaxed, but we exclude it because we do not have data on factor content of materials at the firm level.

where  $\chi_n = \sum_{i \in I_n} \frac{(\alpha_i - \alpha_n)^2}{\alpha_n(1 - \alpha_n)} \frac{c_i}{c_n}$  and  $\bar{s}_n^M = \sum_{i \in I_n} \frac{(\alpha_i - \alpha_n) \alpha_i \frac{c_i}{c_n}}{\sum_{i \in I_n} (\alpha_i - \alpha_n) \alpha_i \frac{c_i}{c_n}} s_i^M$

There are two differences relative to [Proposition 1](#). First, the demand elasticity is replaced by a convex combination of the elasticity of demand,  $\varepsilon$ , and the elasticity of substitution between materials and the capital-labor aggregate,  $\zeta$ . The scale effect rises with a higher materials capital-labor elasticity because firms now have an additional margin of substitution in materials.  $\bar{s}_n^M$ , a weighted average of materials shares, determines the relative importance of these two elasticities. When materials shares are high, capital and labor are a small share of overall cost. Thus, substitution between materials and the capital-labor aggregate is more important than substitution across goods. For example, when the materials share approaches one, changes in capital and labor have minimal effects on firm marginal costs.

The second term determines the shift in composition due to changes in the price of materials. When the price of materials falls, plants that use materials more intensively will tend to expand. If these plants also happen to be more capital intensive, this raises the industry capital labor ratio. Thus this term is a (cost-weighted) covariance between firm capital shares and materials shares. This is multiplied by  $\alpha$ , which reflects the factor content of the materials bundle.<sup>11</sup>

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<sup>11</sup> $\alpha$  represents the contribution from the change in relative materials price with changes in relative factor prices of capital and labor. Since the materials bundle has the same factor content as total output,  $\frac{d \ln q/w}{d \ln r/w} = \alpha$ .

### 3 US Aggregate Elasticity of Substitution

The methodology developed in the previous section shows how to estimate the aggregate elasticity from micro parameters and sufficient statistics for micro heterogeneity. We now use plant level data concerning US manufacturing plants to estimate all of the micro components of the aggregate elasticity. We then put all of these components together to estimate the US aggregate elasticity of substitution and examine its behavior over time.

#### 3.1 Data

We use the US Census of Manufactures and Annual Survey of Manufactures for microdata on manufacturing plants. The Census of Manufactures is a census of all manufacturing plants conducted every five years. It contains more than 180,000 plants per year.<sup>12</sup> The Annual Survey of Manufactures (ASM) tracks about 50,000 plants over five year panel rotations between Census years, and includes the largest plants with certainty. It has the capital and investment history required to construct perpetual inventory measures of capital. In this study, we primarily use factor shares measured at the plant level. Labor costs are the total salaries and wages at the plant level, supplemented with benefits data for the ASM plant subsamples. For the Census samples, we measure capital by the end year book value of

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<sup>12</sup>We exclude small Administrative Record plants with fewer than five employees, for whom the Census only tracks payroll and employment. This omission is in line with the rest of the literature using manufacturing Census data.

capital, deflated using a current cost to historic cost deflator. For the ASM subsamples, we create perpetual inventory measures of capital, accounting for retirement data when possible as in Caballero et al. (1995). Capital costs are these measures of capital stocks multiplied by the appropriate rental rate, using rental rates based upon an external real rate of return of 3.5 percent as in Harper et al. (1989).

## 3.2 Components of the Aggregate Elasticity

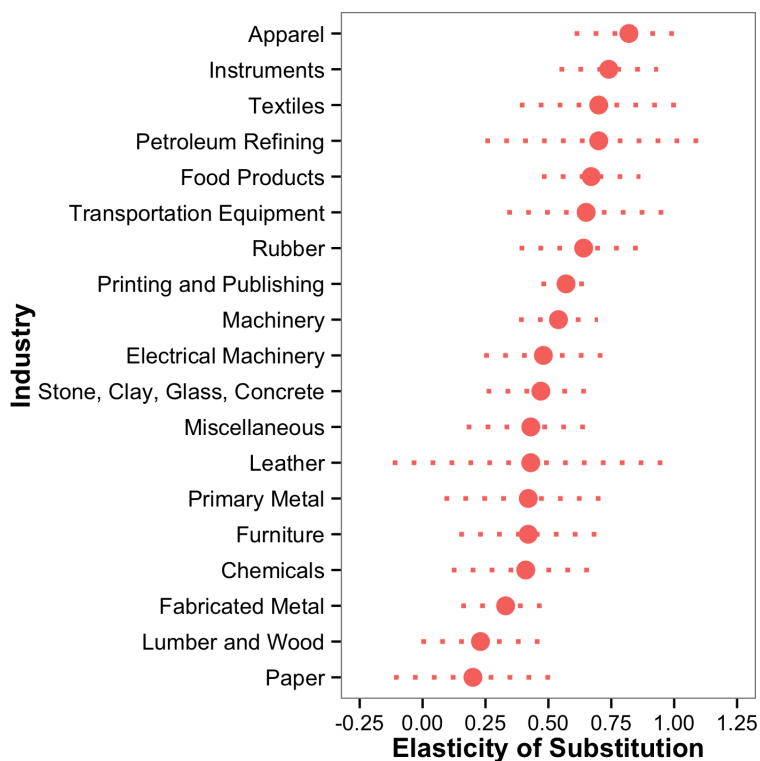
### 3.2.1 Plant Level Production Parameters

The micro elasticity of substitution anchors the substitution effect. Given cost minimization, the relationship between relative factor costs of capital and labor  $rK_i/wL_i$  and relative factor prices  $w/r$  identifies this elasticity. We use the estimates of Raval (2012) that exploit persistent wage differences across local areas in the US in order to identify the long run micro elasticity of substitution between capital and labor. The wage is a residual MSA average after controlling for observed individual heterogeneity based on the Population Censuses. All regressions control for industry, as well as plant age and multi-unit status.

Raval (2012) estimates a plant level elasticity of substitution close to one-half for overall manufacturing in both 1987 and 1997. We allow plant elasticities of substitution and demand to vary at the industry level, defining industries at the two digit SIC level. Figure 1 displays the estimates by industry along with the 95 percent confidence interval. Most of the estimates range between 0.4 and 0.7.



**Figure 1** Plant Elasticity of Substitution by Industry, 1987



**Note:** For each industry, this graph plots the plant level elasticity of substitution between capital and labor as estimated in [Raval \(2012\)](#), together with the 95 percent confidence interval for each estimate.

Plants have an additional avenue to substitute towards materials that depends upon the elasticity of substitution between materials and the capital-labor aggregate  $\zeta$ . Given cost minimization, this elasticity is identified by how relative factor costs of the capital-labor aggregate and materials vary with changes in relative factor prices. We use the same

cross-area variation in the wage as before to identify this elasticity.<sup>13 14</sup>

**Table I** contains these elasticities for 1987 and 1997. These estimates are on average slightly lower than one; in our subsequent empirical work, we use the 1987 MSA level estimate of 0.90.

**Table I** Elasticities of Substitution between Materials and the Capital-Labor Aggregate for the Manufacturing Sector

	No State Effects	State Effects
1987	.90 (.06)	1.01 (.10)
1997	.67 (.04)	.81 (.06)
N	≈ 140,000	

**Note:** Standard errors are in parentheses. All regressions include industry fixed effects, age fixed effects and a multi-unit status indicator, and have standard errors clustered at the two digit industry-MSA level. The wages used are the average log wage for the MSA, where the wage is computed as wage and salary income over total number of hours worked adjusted for differences in education, experience, race, occupation, and industry. Wages are adjusted so that the resulting elasticity is divided by the plant labor share to identify the materials–capital labor aggregate elasticity.

### 3.2.2 Demand Parameters

Within industries, the scale effect relies upon consumer substitution across plants when relative prices change, and thus upon the demand elasticity. We derive the elasticity of

<sup>13</sup>Given cost minimization,  $\zeta$  is identified by the elasticity of firm capital-labor aggregate cost over firm materials cost  $(rK_i + wL_i)/q_i M_i$  with changes in the marginal cost of the capital labor aggregate relative to the materials price  $\lambda_i/q_i$ . We can decompose this elasticity into how relative firm factor costs vary with wage changes and how relative firm marginal costs vary with wage changes:

$$-\frac{d \log[(rK_i + wL_i)/q_i M_i]}{d \log(\lambda_i/q_i)} = -\frac{d \log[(rK_i + wL_i)/q_i M_i]}{d \log w} \left[ \frac{d \log(\lambda_i/q_i)}{d \log w} \right]^{-1}$$

If the materials price does not vary with local changes in the wage, the elasticity of relative firm marginal costs with the wage is the plant labor share of non-materials cost. In this case, we estimate  $\zeta$  above through a regression of the log plant capital-labor aggregate cost to materials cost on the plant level wage multiplied by the labor share.

<sup>14</sup>**Atalay (2012)** pursues a complementary approach using differences in materials prices across plants and finds preliminary estimates within the range of **Table I**.

demand using the implications of profit maximization; optimal price setting behavior implies that the markup over marginal cost is equal to  $\frac{\varepsilon}{\varepsilon-1}$ . We thus invert the average markup across plants in an industry (using the revenue to cost ratio for the markup) to obtain the elasticity of demand. [Figure 2](#) displays the elasticity of demand across all industries in 1987. The elasticity of demand varies between three to five at the industry level.

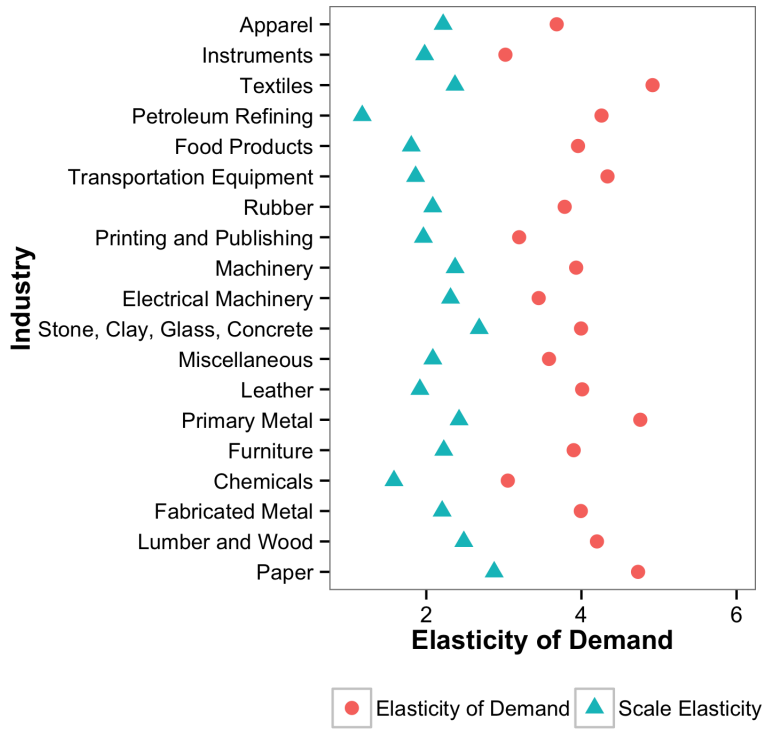
The overall scale elasticity  $\varepsilon_n(1 - \bar{s}_n^M) + \bar{s}_n^M\zeta$  is smaller than the demand elasticity, however, because it is a weighted average of the demand elasticity and .90, our estimate of the materials-capital labor aggregate elasticity  $\zeta$ .  $\bar{s}_n^M$  is an average of materials shares, which are high in manufacturing; the average across industries in 1987 is .59. A high materials share lowers the scale elasticity and hence the scale effect. [Figure 2](#) contains the scale elasticity; it is close to two for most industries with an average of 2.12.<sup>15</sup> Given our estimates of the micro elasticity of substitution, we can roughly bound the aggregate elasticity of substitution between one-half and two.

Across industries, the magnitude of the scale effect depends upon the cross industry elasticity of demand  $\eta$ . We estimate this elasticity using panel data on quantity and price for all four digit manufacturing industries from 1962-2005. Since least squares estimates conflate demand and supply, we instrument for price using the average real cost per unit produced, which is the appropriate measure of industry productivity in our model. In addition, we control for industry and year effects and industry trends.

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<sup>15</sup>This, along with other averages across industries, is a weighted average where the weight on industry  $n$  is  $\frac{c_n \alpha_n (1 - \alpha_n)}{\sum_{n \in N} c_n \alpha_n (1 - \alpha_n)}$  as in [equation \(11\)](#).

**Figure 2** Elasticity of Demand and Scale Elasticity by Industry, 1987



**Note:** For each industry, this graph plots both the elasticity of demand estimated from the revenue to total cost ratio, and the scale elasticity  $\varepsilon_n(1 - \bar{s}_n^M) + \bar{s}_n^M \zeta$ .

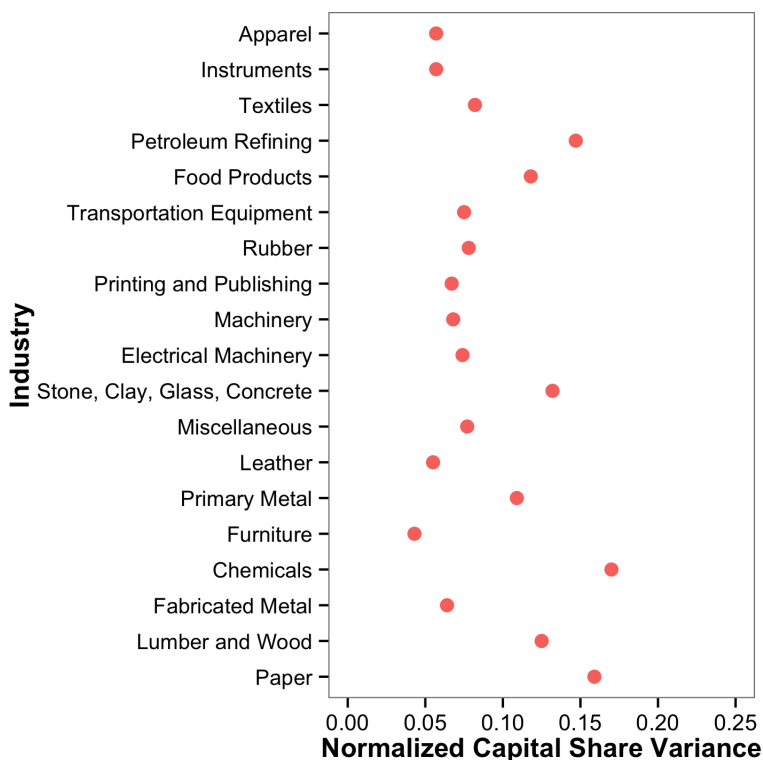
The IV estimate is 1.49 and above the least squares estimate of .62; both have a standard error of .02. As we would expect, the cross industry demand elasticity  $\eta$  is much lower than the firm demand elasticity; varieties in the same industries are better substitutes than varieties in other industries.

### 3.2.3 Micro Heterogeneity

The normalized variance of the capital shares of cost determines the relative importance of the substitution and scale effects. In [Figure 3](#), we depict these variances for each industry in

1987.<sup>16</sup> The average variance across industries is 0.10, while the variance is less than 0.2 for all industries. Given this level of variation in capital shares, the substitution effect accounts for most of overall labor-capital substitution for the US.

**Figure 3** Normalized Variance of Capital Shares by Industry, 1987



**Note:** For each industry, this graph displays the normalized variance of capital shares  $\chi_n$ .

Finally, we also have to consider the weighted covariance between materials shares and

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<sup>16</sup>We compute the normalized variance of capital shares using the ASM subsample with perpetual inventory capital, because we later examine its change over time and the overall Census does not have capital data until 1987.

capital shares, which if positive would tend to lower the scale effect:

$$\sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n(1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

These covariances are small, ranging from -0.05 to 0.05 in 1987, so this component does not play a major role.

### 3.3 Estimates

We can now combine the substitution and scale effects to estimate the industry and manufacturing sector level elasticities of substitution. In [Figure 4](#), we depict the plant level and industry level elasticities of substitution. Because the normalized variance of capital shares tends to be small, the substitution effect dominates and the industry level elasticity of substitution is only moderately higher than the plant level elasticity. The average industry elasticity is 0.66, 27 percent higher than the average plant elasticity of 0.52.

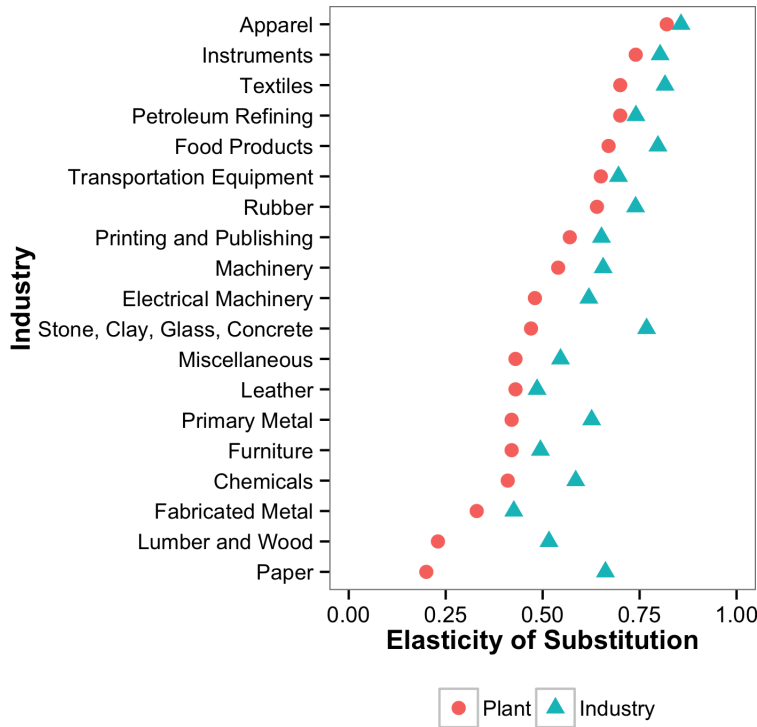
A cross-industry normalized variance of capital shares of .05 implies an overall manufacturing level elasticity of substitution of 0.72.<sup>17</sup> Thus, the overall manufacturing sector elasticity is 38 percent higher than the average plant-level elasticity of substitution.

Our methodology now allows us to examine the stability of the aggregate elasticity of substitution over time by changing all of the individual components. Using this approach,

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<sup>17</sup>Standard errors depend upon the correlation between industry plant elasticity estimates. Assuming zero correlation, the standard error is 0.03, while assuming perfect correlation the standard error is 0.11.

**Figure 4** Plant and Industry Level Elasticity of Substitution, 1987



**Note:** The figure displays the plant level elasticity and industry level elasticity of substitution for each industry.

the 1997 manufacturing level elasticity is 0.79, higher than the 1987 manufacturing-level elasticity of 0.72. Most of the rise from 1987 to 1997 is due to compositional changes.

Table II breaks down the estimates as we change each the component parts from the 1987 value to the 1997 value. We report the elasticity after each component change, considering seven component changes in turn when moving from 1987 to 1997.<sup>18</sup> The bottom value in

<sup>18</sup>In turn, we change the cross industry weights used to construct the average industry elasticity, the plant elasticities of substitution, the demand elasticities, the within industry capital share variance, the materials shares, the covariance between materials shares and capital shares, and the cross industry capital share variance. All reported effects are from this order of changes, as the decomposition is not invariant to the order that the changes are made.

the table contains the 1997 aggregate elasticity of substitution.

Together, the change in materials shares and the change in the within industry capital share variance drive about 80 percent of the difference between 1987 and 1997, with about an equal effect from each change. First, the average within industry capital share variance rises from 0.10 in 1987 to 0.12 in 1997. Second, lower weighted average materials shares increase the average plant scale elasticity from 2.12 in 1987 to 2.23 in 1997. By contrast, changes in estimated plant level elasticities of substitution and elasticities of demand have only small effects on the aggregate elasticity. Thus, compositional changes explain most of the increase in the aggregate elasticity from 1987 to 1997.

**Table II** Decomposition of Change in Manufacturing Level Elasticity of Substitution from 1987 to 1997

Component Change	Aggregate Elasticity
1987 Value	.72
1997 Industry Composition	.72
1997 Plant Elasticities of Substitution	.72
1997 Demand Elasticities	.72
1997 Within Industry Capital Share Variance	.75
1997 Materials Shares	.78
1997 Materials-Capital Share Covariance	.79
1997 Cross Industry Capital Share Variance	.79
1997 Value	.79

**Note:** This table records the manufacturing level elasticity of substitution as components of the manufacturing level elasticity of substitution are changed from their 1987 value to their 1997 value.



## 4 Cross–Country Elasticities

We now use our methodology to examine how the aggregate elasticity of substitution varies across countries. There are a number of reasons to think that less developed countries have greater microeconomic heterogeneity, which would translate into higher aggregate elasticities of substitution. Researchers have generally found that productivity is more dispersed in less developed countries, and that resources are allocated less efficiently (Hsieh and Klenow (2009), Bartlesman et al. (2009)).

Another reason for greater heterogeneity is that less developed countries typically have much lower relative wages. If new technologies are discovered in the developed world and are biased towards capital, as in a model like Acemoglu and Zilibotti (2001), they may not be as cost-effective in poorer countries. These poor countries may then operate a mix of old and new technologies.

We look at three countries other than the US– Chile, Colombia, and India– and find that the aggregate elasticity of substitution is higher in these countries because of greater microeconomic heterogeneity.

### 4.1 Data

We obtain plant-level data for Chile, Colombia, and India from national plant level manufacturing censuses. The Chilean data spans from 1979–1996 with about 5,000 plants per year,

the Colombia data from 1981–1991 with about 7,000 plants per year, and the Indian data from 2000–2003 with about 30,000 plants per year. The Chilean and Colombian data cover all manufacturing plants with at least 10 employees, while the Indian data are a sample of all plants with at least 10 employees (20 if without power), with plants with at least 100 workers sampled with certainty. We define industries at a similar level to two digit US SIC; for Chile and Colombia this at the three digit ISIC level, and for India at the two digit NIC level.

Capital costs are the most involved variable to construct. For each country, we construct capital rental rates based upon private sector lending rates reported in the IMF Financial Statistics, depreciation rates and a measure of the inflation rate of capital assets. For Chile, we use the capital services constructed by [Greenstreet \(2007\)](#), while for Colombia we broadly follow the perpetual inventory procedure of [Tybout and Roberts \(1996\)](#). Because the Indian data is not panel, we use book values of capital for each type of capital. Total capital services are then the sum of the individual capital stocks weighted by the respective average rental rates plus rental payments.

## 4.2 Estimates

To estimate the aggregate elasticity of substitution for these countries, we allow the composition of plants to change across countries and fix production and demand elasticities at their US 1987 values for matching industries. We then compute the average aggregate elas-

ticity for 1986–1996 for Chile, 1981–1991 for Colombia, and 2000–2003 for India.<sup>19</sup> **Table III** reports these estimates. We find an aggregate manufacturing elasticity of 0.81 in Chile, 0.85 in Colombia, and 1.11 in India, all of which are higher than the US 1987 value of 0.72.

**Table III** Estimates of the Cross Country Aggregate Elasticity of Substitution

	US	Chile	Colombia	India
Aggregate Elasticity	.72	.81	.85	1.11
Average Industry Elasticity	.66	.76	.83	1.05
Average Plant Elasticity	.52	.51	.55	.53
Time Period	1987	1986–1996	1981–1991	2000–2003

What causes the cross country differences in the aggregate elasticity of substitution? **Table IV** examines the contribution to the aggregate elasticity difference from each component change in sequence as in **Table II**. For India and Chile, a larger within industry capital share variance explains a lion’s share of the aggregate elasticity difference. The Chilean normalized variance is 0.15, 50 percent higher than the US value, and the Indian variance .30, three times the US value. For both Chile and India, this explains roughly 70 percent of the difference with the US.

The difference for Colombia is due to a number of factors. Both a different industrial composition and an increase in within industry capital share variance are roughly equally important. The most important determinant is a rise in scale elasticities from lower materials shares, which contributes about 48 percent of the change.

<sup>19</sup>For Chile, we exclude the initial years to avoid the Chilean financial crisis of the early 1980s.

**Table IV** Decomposition of Cross Country Differences in Manufacturing Level Elasticity of Substitution

	Chile	Colombia	India
Industry Composition	18.9%	28.2%	3.7%
Within-Industry Capital Share Variance	71.6%	27.8%	73.4%
Scale Elasticities	-5.2%	47.6%	14.8%
Materials-Capital Share Covariance	-29.6%	-14.3%	-1.4%
Cross-Industry Capital Share Variance	44.3%	10.6%	9.5%
Time Period	1986-1996	1981-1991	2000-2003

**Note:** This table records the fraction of the overall difference in the elasticity of substitution as components of the manufacturing level elasticity of substitution are changed one by one from their 1987 US value to their values for each country.

### 4.3 Effects of Policy Changes to Capital Rental Rate

We now show that these cross-country differences in elasticities can imply large differences in outcomes. In particular, we examine two potential policy changes that would affect the capital rental rate. The first policy change lowers foreign interest rates to the US real interest rate, while the second policy change lowers corporate taxes. Our approach allows us to examine the effect of each policy change without assuming that each country shares the same aggregate technology.

The capital rental rate is composed of the real interest rate  $R$ , depreciation rate  $\delta$ , and effective corporate tax rate  $\tau$ :

$$r = \frac{R + \delta}{1 - \tau}$$

For the real interest rate, we use the nominal private sector lending rate for each country

from the IMF International Financial Statistics adjusted for inflation by the change in the GDP deflator and then averaged from 1992-2011.<sup>20</sup> Table V contains these estimates. The US real interest rate is 4.15 percent, close to the 3.5 percent real interest rate we use when constructing capital rental rates. The real interest rate is higher in all three other countries, with an interest rate differential of 1.8 percentage points for Chile, 1.9 percentage points for India, and 5.3 percentage points for Colombia. The Colombian interest rate differential is particularly high, probably reflecting uncertainty given its civil war during the sample period.

For corporate tax rates, we use the 1 year effective tax rate collected by Djankov et al. (2010).<sup>21</sup> Table V contains these tax rates: the US effective tax rate is 18.2 percent. Colombia and India have slightly higher tax rates and Chile a slightly lower tax rate. Finally, we set the depreciation rate to 9.46 percent based upon US manufacturing data. All three countries have higher capital rental rates than the US; the Chilean capital rental rate is 9 percent higher than the US rate, the Indian rate 17 percent higher, and the Colombian rate 51 percent higher.

The elasticity of output per worker in manufacturing with changes in relative factor prices

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<sup>20</sup>We employ a discrete time correction as some countries have high inflation rates, so  $R = \frac{i_t - \pi_t}{1 + \pi_t}$  for lending rate  $i_t$  and inflation rate  $\pi_t$ .

<sup>21</sup>Djankov et al. (2010) derive effective tax rates for fiscal year 2004 by asking a major accounting firm to calculate the tax rate for the same fictitious corporation in 85 countries.

**Table V** Cross Country Differences in Real Interest Rates, Effective Corporate Taxes, and Capital Rental Rates

	US	Chile	Colombia	India
Real Interest Rate, 1992–2011 Average	4.2%	5.9%	9.5%	6.0%
Effective Corporate Tax Rate	18.2%	15.1%	24.3%	20.3%
Capital Rental Rate	16.6%	18.1%	25.0%	19.4%

**Note:** This table records the average real interest rate from 1992–2011 using the private sector lending rate from the IMF International Financial Statistics adjusted for inflation using the change in the GDP deflator, the 1 year effective corporate tax rate from Djankov et al. (2010), and the capital rental rate given the average real interest rate, corporate tax rate, and a depreciation rate of 9.46 percent.

is:

$$\frac{d \ln(Y/L)}{d \ln(r/w)} = \alpha \sigma$$

where  $\alpha$  is the manufacturing capital share. First, we examine the effect on output per worker of moving from the country’s real interest rate to the US interest rate. Second, we simulate a fall in the effective corporate tax rate to zero. <sup>22</sup> Table VI displays the change in output per worker from each policy change, as well as the change in output per worker if we used the US elasticity for all countries.

If all interest rates fall to the US interest rate, output per worker increases by 4.9 percent in Chile, 10.7 percent in Colombia, and 7.8 percent in India. These effects are exacerbated

<sup>22</sup>Two notes: First, because the elasticity that we estimate is local, counterfactual predictions for non-local changes in rental rates involves extrapolating from our local elasticity. Second, we hold the wage fixed in these experiments; the change in wage induced from changes in rental rates depends upon labor supply as well as demand. Our estimates are a lower bound on the full effects of removing interest rate differentials, while the wage change for the capital tax change depends upon whether the revenue loss is compensated for by other tax changes.

because the aggregate elasticity of substitution is larger. For example, the impact of the interest rate differential in India is more than fifty percent larger than it would be if India had the same elasticity of substitution as the US. The story is similar with corporate tax rates. Reducing the corporate tax rate to zero would raise output per worker by 4.1 percent for the US, 6.5 percent for Chile, 9.0 percent for Colombia, and 13.6 percent for India; with the US aggregate elasticity, this change becomes 5.8 percent for Chile, 7.7 percent for Colombia, and 8.8 percent for India.<sup>23</sup> Differences in aggregate technology across countries have large effects on the outcome of these policy changes.

**Table VI** Change in Output per Worker from Policy Changes Affecting the Capital Rental Rate

Policy Change	US	Chile	Colombia	India
Equalize to US Interest Rate	0%	4.9%	10.7%	7.8%
Equalize to US Interest Rate, using US Elasticities	0%	4.3%	9.1%	5.0%
Zero Corporate Tax	4.1%	6.5%	9.0%	13.6%
Zero Corporate Tax, using US Elasticities	4.1%	5.8%	7.7%	8.8%

**Note:** This table records the change in output per worker from two policy experiments- equalizing all interest rates to the US interest rate and setting the corporate tax rate to zero. For each experiment, we examine the change in output per worker using the country elasticity and using the US 1987 elasticity.

<sup>23</sup>We cannot interpret the change in output per worker from these policy changes as a gain in welfare because that comparison contrasts two different steady states. However, [Chamley \(1981\)](#) found in a full general equilibrium model that the welfare cost of capital taxation is proportional to the elasticity of substitution.

## 5 Robustness

In this section, we show how our estimates include extensive margin effects and how to incorporate misallocation frictions. We also examine the robustness of our micro elasticities of substitution and demand.

### 5.1 Extensive Margin

So far, our estimate of the aggregate elasticity of substitution has assumed that the distribution of firms remains the same. What about the extensive margin? Would a higher wage cause entering firms to choose more capital intensive technologies? If so, the aggregate elasticity of substitution should include that shift.

It turns out that our estimate already includes the extensive margin. Our estimate of the plant level elasticity of substitution uses differences in wages across US locations. Because these differences are persistent over time, the distribution of plants in each location should have adjusted to the wage differences. Since we are comparing the distribution of capital-labor ratios across locations, we are picking up changes in both the intensive and extensive margins.

We can see this most easily through the lens of the putty-clay model.<sup>24</sup> In the putty-clay model, firms initially face a menu of fixed-proportions technologies (putty), but once they

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<sup>24</sup>The model of [Houthakker \(1955\)](#) is also a model of extensive margin.



choose, they cannot switch. The short run elasticity of substitution is just the plant intensive margin – the elasticity of clay. However, the long run elasticity would include a shift to an alternative technology, and would pick up the elasticity of putty.<sup>25</sup> Our estimate picks up the elasticity of putty, which is the appropriate elasticity to use when measuring the long run aggregate elasticity of substitution.

## 5.2 Misallocation

The section above found higher aggregate elasticities in Chile, Colombia, and India precisely because those countries possess greater micro heterogeneity in factor shares. However, the recent misallocation literature has attributed some of this micro heterogeneity to distortions; see (Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2009)).

We can easily modify our theoretical framework to include distortions that come in the form of plant specific factor prices, as in Hsieh and Klenow (2009). In this case, our formulae for the industry and aggregate elasticity remain the same, but all of the sufficient statistics from micro data are now measured using the plant specific factor prices.<sup>26</sup> Thus, we can allow misallocation frictions so long as the factor costs in our data include these plant specific

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<sup>25</sup>Formally, assume plant  $i$  faces a menu of short-run CRS production functions, indexed by  $j \in J_i$ . If  $\Lambda^j(r, w)$  is the unit cost associated with production function  $j$ , then the long run production function has the long run unit cost function  $\Lambda^{LR}$ :

$$\Lambda^{LR}(r, w) = \min_{j \in J_i} \Lambda^j(r, w)$$

<sup>26</sup>For example, given plant specific frictions  $\tau_K$  and  $\tau_L$  on capital and labor, the correct capital share is  $\alpha_i \equiv \frac{(r\tau_K)K_i}{(r\tau_K)K_i + (w\tau_L)L_i}$ , and plant cost  $c_i = (r\tau_K)K_i + (w\tau_L)L_i$ .

factor prices.

### 5.3 Plant Level Elasticity of Substitution

We estimated the plant level elasticity of substitution using cross-sectional wage differences across US locations. The natural question is whether these wage differences are exogenous to plant non-neutral productivity differences. To account for such endogeneity problems, we use the [Bartik \(1991\)](#) instrument for labor demand, which varies labor demand through the differential impact of national level industry shocks across locations with a different initial industrial composition.<sup>27</sup> We restrict the instrument to non-manufacturing industries only. [Table VII](#) contains estimates of the elasticity of substitution using these instruments.<sup>28</sup> All of the estimates are close to one-half, ranging from .47 to .55, and are similar to the previous least squares estimates.

### 5.4 Elasticity of Demand

Our demand elasticity estimates rely on optimal price setting; we examine their robustness through econometric demand elasticity estimates. For a set of several homogenous products, the US Census of Manufactures collects both price and physical quantity data. We can thus

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<sup>27</sup>Formally, it is the interaction between initial MSA employment shares of four digit SIC industries and the 10 year national employment growth rate of these industries.

<sup>28</sup>We use wages from establishment data for the same Census year to match the instrument timing. While these wages do not control for differences in individual worker characteristics, the instrument should be orthogonal to the measurement error in wages. Because the SIC industry definitions changed from 1972 SIC basis to 1987 SIC basis in 1987, for 1987 we use the 1976-1986 instrument.

**Table VII** Elasticities of Substitution between Labor and Capital for Manufacturing using Labor Demand Instruments for Wages

	No State Effects	State Effects
1987	.49 (.05)	.55 (.07)
1997	.52 (.08)	.47 (.10)
N	$\approx 140,000$	

**Note:** Standard errors are in parentheses. All regressions include industry fixed effects and have robust standard errors. Labor demand instruments are based on the interaction between 10 year lag industry composition of employment in the MSA and nationwide changes in labor demand at the industry level, where industries are defined at the 4 digit SIC level and exclude manufacturing. Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level. For 1987, we match an instrument using 1976-1986 changes because the SIC industry definitions change from 1972 basis to 1987 basis in 1987. Controls are age fixed effects and a multi unit status indicator.

estimate the elasticity of demand instrumenting for price using average cost in an approach similar to [Foster et al. \(2008\)](#). For these products, the average industry elasticity is 0.52 using the industry estimates from our IV regressions, 0.54 using the estimates in [Foster et al. \(2008\)](#), and 0.54 using the industry estimates from price-cost margins. Thus, we find similar aggregate substitution elasticities across various alternative estimates of the demand elasticity.

## 6 Bias of Technical Change

The aggregate elasticity dictates how movements in factor prices affect the aggregate capital share. Thus, our estimates of this elasticity allow us to examine why the aggregate capital share moves over time. The residual change after accounting for factor price movements is

then ascribed to directed technical change.<sup>29</sup> Since our estimated elasticity of substitution is less than one, technical change that augments labor increases the capital share.

Figure 6 depicts the manufacturing aggregate capital share since 1970, measured either using estimates from the Bureau of Labor Statistics (BLS) or from income and compensation data from the National Income and Product Accounts (NIPA).<sup>30</sup> Using either measure, the aggregate capital share has risen quite considerably in the past forty years. The largest rise has been since 2000; in the BLS data the capital share has increased from roughly 0.35 to 0.45 in the 2000s.

Because our elasticity estimates are based upon the cross-section, we have not placed any assumptions on the behavior of technical change. We can thus examine whether the rate of directed technical change has varied over time. Table VIII measures the rate of directed technical change by decade using the BLS data and our 1987 estimate of the aggregate elasticity of 0.72. The average rate of biased technical change is 2.20 percent per year. Directed technical change is labor augmenting in every decade; however, it is over three percentage points larger in the 2000s than in the 1970s.

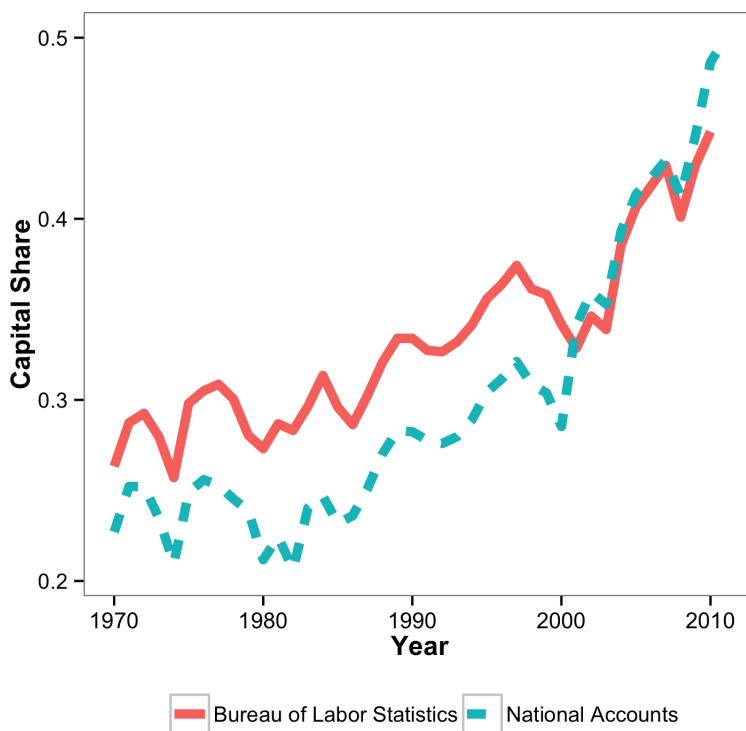
What causes the bias of technical change in the first place? One possibility is that plants adopt increasingly capital intensive technologies over time. However, biased technical change at the aggregate level does not necessarily imply changes in plant level technology; growth in

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<sup>29</sup>Formally, we define biased technical change as  $\phi^{agg}$ , where  $d \ln \frac{rK}{wL} = (1 - \sigma^{agg})d \ln \frac{r}{w} + \phi^{agg}$ .

<sup>30</sup>Using the NIPA data, we measure labor income as total compensation plus two-thirds of proprietor's income and capital income as value added minus labor income.

**Figure 5** Manufacturing Capital Share over Time



**Note:** The red, solid line is the capital share for manufacturing based on the BLS Multifactor Productivity series. The blue, dashed line is the capital share for manufacturing calculated using data from NIPA – labor income is compensation plus two-thirds of proprietor’s income and capital income value added minus labor income.

**Table VIII** Annual Rate of Biased Technical Change by Decade

Time Period	1970-1979	1980-1989	1990-1999	2000-2011	1970-2011
Annual Rate of Bias	0.30%	2.90%	1.60%	3.70%	2.20%

**Note:** Each annual rate of biased technical change is the growth in the capital cost to labor cost ratio for manufacturing after deducting the effects of factor prices using the 1987 aggregate elasticity of 0.72. We use rental rates based on NIPA deflators for equipment and structures, accounting for the growth in equipment through a Tornqvist index. Wages are total compensation over total number of employees for manufacturing from NIPA, corrected for labor quality using indices available in [Fernald \(2012\)](#) based on the method of [Aaronson and Sullivan \(2001\)](#).

high capital share plants can cause aggregate biased technical change. We are now working on using the microdata for a decomposition of the sources of aggregate bias between biased

technical change within plants and compositional change across plants.

## 7 Conclusion

This paper has made two main contributions. First, we have developed a new approach to estimate the aggregate elasticity of substitution between capital and labor based on micro structural parameters and sufficient statistics for micro heterogeneity. Using our methodology and data on plants in the US manufacturing sector, we estimated the elasticity of substitution between capital and labor to be roughly 0.72 in 1987.

Second, our approach allowed us to estimate how the aggregate elasticity varies across time and countries, and to understand the underlying reasons for such variation. In particular, a higher capital share variance implies a higher aggregate elasticity of substitution. We then estimated an elasticity of 0.79 for the US in 1987, higher than our 1987 estimate, with the difference mostly due to changes in the distribution of plants. All three of the developing countries we examined had higher elasticities than the US, with an average manufacturing sector elasticity of 0.82 for Chile, 0.86 for Colombia, and 1.11 for India. The major reason for the difference for Chile and India was due to a higher capital share variance.

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# Appendix

## A Proofs of Propositions

### A.1 Industry Elasticity of Substitution

Below we derive a formula for the elasticity of substitution given in [equation \(1\)](#).

We first assume that (i) each firm produces with a constant returns to scale production function, (ii) each firm minimizes cost, and (iii) face competitive factor markets. We later specialize to the case in which firm's maximize profit and face isoelastic demand curves.

**Claim 1** *Cost minimization, constant returns to scale, and competitive factor markets imply*

$$\bar{\sigma}_n^N = (1 - \chi) \bar{\sigma}_n + \sum_{i \in I_n} \sum_{j \in I_n} \frac{d \ln(Y_j/Y_i)}{d \ln(r/w)} \frac{L_j}{L_n} \frac{K_i}{K_n}$$

**Proof.** First, constant returns to scale along with Shephard's lemma imply that  $\frac{d \ln(\lambda_i/w)}{d \ln(r/w)} = \alpha_i$  so that

$$\frac{d \ln(\lambda_i/\lambda_j)}{d \ln(r/w)} = \alpha_i - \alpha_j \quad (13)$$

Second, note that

$$\frac{d \ln \left[ \left( \frac{r}{w} \frac{K_i}{L_i} + 1 \right) / \left( \frac{r}{w} \frac{K_j}{L_j} + 1 \right) \right]}{d \ln(r/w)} = \alpha_i (1 - \sigma_i) - \alpha_j (1 - \sigma_j) \quad (14)$$

Third, we can write the total cost to firm  $i$  as  $\lambda_i Y_i = r K_i + w L_i$ . Dividing by the analogous expression for  $j$ , taking logs and differentiating gives

$$\frac{d \ln(Y_i/Y_j)}{d \ln(r/w)} + \frac{d \ln(\lambda_i/\lambda_j)}{d \ln(r/w)} = \frac{d \ln(L_i/L_j)}{d \ln(r/w)} + \frac{d \ln \left[ \left( \frac{r}{w} \frac{K_i}{L_i} + 1 \right) / \left( \frac{r}{w} \frac{K_j}{L_j} + 1 \right) \right]}{d \ln(r/w)}$$

Solving for  $\frac{d \ln(L_i/L_j)}{d \ln(r/w)}$  and using [equation \(13\)](#) and [equation \(14\)](#) gives

$$\frac{d \ln(L_i/L_j)}{d \ln(r/w)} = \frac{d \ln(Y_i/Y_j)}{d \ln(r/w)} + [\alpha_i \sigma_i - \alpha_j \sigma_j] \quad (15)$$

With this, the change in firm  $i$ 's share of total labor can be written as

$$\begin{aligned} d \ln \left( \frac{L_i}{L_n} \right) &= \sum_{j \in I_n} \frac{L_j}{L_n} d \ln \left( \frac{L_i}{L_j} \right) \\ &= \sum_{j \in I_n} \frac{L_j}{L_n} \left\{ \frac{d \ln (Y_i/Y_j)}{d \ln (r/w)} + (\alpha_i \sigma_i - \alpha_j \sigma_j) \right\} \end{aligned}$$

To compute the industry elasticity, we will use the following expression:

$$\sum_{i \in I_n} \sum_{j \in I_n} [\alpha_i \sigma_i - \alpha_j \sigma_j] \frac{L_j}{L_n} \frac{K_i}{K_n} = \sum_{i \in I_n} \alpha_i \sigma_i \frac{K_i}{K_n} - \sum_{j \in I_n} \alpha_j \sigma_j \frac{L_j}{L_n} = \sum_{i \in I_n} \alpha_i \sigma_i \left( \frac{K_i}{K_n} - \frac{L_i}{L_n} \right)$$

The industry elasticity of substitution  $\sigma_n^N$  is then:

$$\begin{aligned} \sigma_n^N &= \sum_{i \in I_n} \sigma_i \frac{K_i}{K_n} - \sum_{i \in I_n} \frac{d \ln (L_i/L_n)}{d \ln (r/w)} \frac{K_i}{K_n} \\ &= \sum_{i \in I_n} \sigma_i \frac{K_i}{K_n} - \sum_{i \in I_n} \alpha_i \sigma_i \left( \frac{K_i}{K_n} - \frac{L_i}{L_n} \right) - \sum_{i \in I_n} \sum_{j \in I_n} \frac{d \ln (Y_i/Y_j)}{d \ln (r/w)} \frac{L_j}{L_n} \frac{K_i}{K_n} \end{aligned}$$

Combining the first two terms give

$$\begin{aligned} \sum_{i \in I_n} \left[ \sigma_i \frac{K_i}{K_n} - \alpha_i \sigma_i \left( \frac{K_i}{K_n} - \frac{L_i}{L_n} \right) \right] &= \sum_{i \in I_n} \left[ \frac{\alpha_i}{\alpha_n} - \alpha_i \left( \frac{\alpha_i}{\alpha_n} - \frac{1 - \alpha_i}{1 - \alpha_n} \right) \right] \sigma_i \frac{c_i}{c_n} \\ &= \sum_{i \in I_n} \frac{\alpha_i (1 - \alpha_i)}{\alpha_n (1 - \alpha_n)} \sigma_i \frac{c_i}{c_n} \\ &= \bar{\sigma}_n \sum_{i \in I_n} \frac{\alpha_i (1 - \alpha_i)}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n} \\ &= (1 - \chi) \bar{\sigma}_n \end{aligned}$$

where  $\bar{\sigma}_n = \sum_{i \in I_n} \frac{\alpha_i (1 - \alpha_i) c_i}{\sum_{j \in I_n} \alpha_j (1 - \alpha_j) c_j} \sigma_i$ . ■

**Claim 2** *If all firms maximize profit and face isoelastic demand with common demand elasticity  $\varepsilon$ , then  $\sum_{i \in I_n} \sum_{j \in I_n} \frac{d \ln (Y_j/Y_i)}{d \ln (r/w)} \frac{L_j}{L_n} \frac{K_i}{K_n} = \chi \varepsilon$ .*

**Proof.** With isoelastic demand, the profit maximizing price is  $P_i = \frac{\varepsilon}{\varepsilon - 1} \lambda_i$ , and the profit maximizing quantity is  $Y_i = D_i \left( \frac{\varepsilon}{\varepsilon - 1} \right)^{-\varepsilon} \lambda_i^{-\varepsilon}$ . We therefore have

$$\frac{d \ln (Y_j/Y_i)}{d \ln (r/w)} = \varepsilon \frac{d \ln (\lambda_i/\lambda_j)}{d \ln (r/w)} = \varepsilon \left( \frac{d \ln (\lambda_i/w)}{d \ln (r/w)} - \frac{d \ln (\lambda_j/w)}{d \ln (r/w)} \right) = \varepsilon (\alpha_i - \alpha_j)$$

This gives

$$\begin{aligned}
\sum_{i \in I_n} \sum_{j \in I_n} \frac{d \ln (Y_j / Y_i)}{d \ln (r / w)} \frac{L_j}{L_n} \frac{K_i}{K_n} &= \varepsilon \sum_{i \in I_n} \sum_{j \in I_n} (\alpha_i - \alpha_j) \frac{L_j}{L_n} \frac{K_i}{K_n} \\
&= \varepsilon \sum_{i \in I_n} \alpha_i \left( \frac{K_i}{K_n} - \frac{L_i}{L_n} \right) \\
&= \varepsilon \sum_{i \in I_n} \alpha_i \left( \frac{\alpha_i}{\alpha_n} - \frac{1 - \alpha_i}{1 - \alpha_n} \right) \frac{c_i}{c_n} \\
&= \varepsilon \chi
\end{aligned}$$

■

## A.2 Industry Elasticity of Substitution with Materials

We now derive the industry elasticity of substitution given that materials enter the production function as in [equation \(12\)](#).

**Claim 3** *Profit minimization, constant returns to scale, and competitive factor markets imply*

$$\sigma_n^N = (1 - \chi_n) \sigma_n + \chi_n [\bar{s}_i^M \zeta + (1 - \bar{s}_i^M) \varepsilon] + \frac{d \ln q / w}{d \ln r / w} (\zeta - \varepsilon) \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

**Proof.**

The industry elasticity of substitution is

$$\begin{aligned}
\sigma_n^N - 1 &= -\frac{d \ln \frac{r K_n}{w L_n}}{d \ln r / w} = -\frac{d \ln \sum_{i \in I_n} \frac{\alpha_i}{1 - \alpha_n} \frac{c_i}{c_n}}{d \ln r / w} = -\sum_{i \in I_n} \frac{\frac{\alpha_i}{1 - \alpha_n} \frac{c_i}{c_n}}{\sum_{j \in I_n} \frac{\alpha_j}{1 - \alpha_n} \frac{c_j}{c_n}} \left[ \frac{d \ln \frac{\alpha_i}{1 - \alpha_n}}{d \ln r / w} + \frac{d \ln c_i / c_n}{d \ln r / w} \right] \\
&= -\sum_{i \in I_n} \frac{\alpha_i}{\alpha_n} \frac{c_i}{c_n} \left[ \frac{d \ln \frac{\alpha_i}{1 - \alpha_n}}{d \ln r / w} + \frac{d \ln c_i / c_n}{d \ln r / w} \right]
\end{aligned}$$

Letting  $Z_i$  be firm  $i$ 's total expenditure,  $r K_i + w L_i + q M_i$ , the two terms in the brackets can be written as

$$\begin{aligned}
\frac{d \ln c_i / c_n}{d \ln r / w} &= \sum_{j \in I_n} \frac{c_j}{c_n} \frac{d \ln c_i / c_j}{d \ln r / w} = \sum_{j \in I_n} \frac{c_j}{c_n} \frac{d \ln \frac{(1 - s_i^M) Z_i}{(1 - s_j^M) Z_j}}{d \ln r / w} \\
\frac{d \ln \alpha_i / (1 - \alpha_n)}{d \ln r / w} &= \frac{d \ln \frac{r K_i}{r K_i + w L_i}}{d \ln r / w} - \frac{d \ln \frac{w L_n}{r K_n + w L_n}}{d \ln r / w} = (1 - \alpha_i) (1 - \sigma_n) + \alpha_n (1 - \sigma_n^N)
\end{aligned}$$

Plugging these in and rearranging gives

$$\begin{aligned}
\sigma_n^N - 1 &= \sum_{i \in I_n} \frac{\alpha_i}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n} \left[ (1 - \alpha_i) (\sigma_n - 1) - \sum_{j \in I_n} \frac{c_j}{c_n} \frac{d \ln \frac{(1 - s_i^M) Z_i}{(1 - s_j^M) Z_j}}{d \ln r/w} \right] \\
&= (1 - \chi_n) (\sigma_n - 1) - \sum_{i \in I_n} \frac{\alpha_i}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n} \sum_{j \in I_n} \frac{c_j}{c_n} \frac{d \ln \frac{(1 - s_i^M) Z_i}{(1 - s_j^M) Z_j}}{d \ln r/w}
\end{aligned}$$

The last equality uses  $1 - \chi_n = \sum_{i \in I_n} \frac{\alpha_i (1 - \alpha_i)}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n}$ . To get at the second term, we now solve for

$$\frac{d \ln \frac{(1 - s_i^M) Z_i}{(1 - s_j^M) Z_j}}{d \ln r/w}.$$

The change in the non-materials share of cost is

$$\begin{aligned}
\frac{d \ln (1 - s_i^M)}{d \ln r/w} &= - \frac{s_i^M}{1 - s_i^M} \frac{d s_i^M}{d \ln r/w} \\
&= - \frac{s_i^M}{1 - s_i^M} (1 - \zeta) \left( \frac{d \ln q/w}{d \ln r/w} - \frac{d \ln \phi_i/w}{d \ln r/w} \right) \\
&= - \frac{s_i^M}{1 - s_i^M} (1 - \zeta) \left( \frac{d \ln q/w}{d \ln r/w} - s_i^K - s_i^M \frac{d \ln q/w}{d \ln r/w} \right) \\
&= s_i^M (1 - \zeta) \left[ \alpha_i - \frac{d \ln q/w}{d \ln r/w} \right]
\end{aligned}$$

To find  $\frac{d \ln Z_i/Z_j}{d \ln r/w}$ , we let  $\phi_i$  be firm  $i$ 's marginal cost, so that shepherd's lemma implies  $\frac{d \ln \phi_i/w}{d \ln r/w} = s_i^K + s_i^M \frac{d \ln q/w}{d \ln r/w}$ . This yields

$$\begin{aligned}
\frac{d \ln Z_i/Z_j}{d \ln r/w} &= (1 - \varepsilon) \frac{d \ln \phi_i/\phi_j}{d \ln r/w} \\
&= (1 - \varepsilon) \left\{ s_i^K + s_i^M \frac{d \ln q/w}{d \ln r/w} - s_j^K - s_j^M \frac{d \ln q/w}{d \ln r/w} \right\} \\
&= (1 - \varepsilon) \left\{ (1 - s_i^M) \alpha_i + s_i^M \frac{d \ln q/w}{d \ln r/w} - (1 - s_j^M) \alpha_j + s_j^M \frac{d \ln q/w}{d \ln r/w} \right\} \\
&= (1 - \varepsilon) \left\{ (1 - s_i^M) \left( \alpha_i - \frac{d \ln q/w}{d \ln r/w} \right) - (1 - s_j^M) \left( \alpha_j - \frac{d \ln q/w}{d \ln r/w} \right) \right\}
\end{aligned}$$

After some manipulation, the scale effect can be written as

$$\begin{aligned} & \sum_{i \in I_n} \frac{\alpha_i}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n} \sum_{j \in I_n} \frac{c_j}{c_n} \frac{d \ln \frac{(1-s_i^M)Z_i}{(1-s_j^M)Z_j}}{d \ln r/w} = \\ & \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} \left\{ [s_i^M (\zeta - 1) + (1 - s_i^M) (\varepsilon - 1)] \left( \frac{d \ln q/w}{d \ln r/w} - \alpha_i \right) \right\} \frac{c_i}{c_n} = \\ & \left\{ \begin{aligned} & - \sum_{i \in I_n} \frac{(\alpha_i - \alpha_n) \alpha_i}{\alpha_n (1 - \alpha_n)} \{ [s_i^M (\zeta - 1) + (1 - s_i^M) (\varepsilon - 1)] \} \frac{c_i}{c_n} \\ & + \frac{d \ln q/w}{d \ln r/w} \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} \{ [s_i^M (\zeta - 1) + (1 - s_i^M) (\varepsilon - 1)] \} \frac{c_i}{c_n} \end{aligned} \right\} \end{aligned}$$

Letting  $\bar{s}_n^M = \frac{\sum_{i \in I_n} (\alpha_i - \alpha_n) \alpha_i s_i^M}{\sum_{i \in I_n} (\alpha_i - \alpha_n) \alpha_i}$ , the first term is

$$\sum_{i \in I_n} \frac{(\alpha_i - \alpha_n) \alpha_i}{\alpha_n (1 - \alpha_n)} \frac{c_i}{c_n} [\bar{s}_n^M (\zeta - 1) + (1 - \bar{s}_n^M) (\varepsilon - 1)] = \chi_n [\bar{s}_n^M (\zeta - 1) + (1 - \bar{s}_n^M) (\varepsilon - 1)]$$

The second term is term  $\frac{d \ln q/w}{d \ln r/w}$  multiplied by

$$\sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} [s_i^M (\zeta - 1) + (1 - s_i^M) (\varepsilon - 1)] \frac{c_i}{c_n} = (\zeta - \varepsilon) \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

Putting these pieces together, we have

$$\sigma_n^N - 1 = (1 - \chi_n) (\sigma_n - 1) + \chi_n [\bar{s}_n^M (\zeta - 1) + (1 - \bar{s}_n^M) (\varepsilon - 1)] + \frac{d \ln q/w}{d \ln r/w} (\zeta - \varepsilon) \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

Or more simply

$$\sigma_n^N = (1 - \chi_n) \sigma_n + \chi_n [\bar{s}_n^M \zeta + (1 - \bar{s}_n^M) \varepsilon] + \frac{d \ln q/w}{d \ln r/w} (\zeta - \varepsilon) \sum_{i \in I_n} \frac{\alpha_i - \alpha_n}{\alpha_n (1 - \alpha_n)} s_i^M \frac{c_i}{c_n}$$

■

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