Technical Report for "Estimating the trend rate of economic growth using the CFNAI"

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The DF-CFNAI is estimated according to the following dynamic factor model, eqs. (1-3), where X_t is a vector of the 85 demeaned and standardized CFNAI monthly data series, F_t represents the monthly DF-CFNAI, and F_t^3 is its three-month moving average. We append to this model a nowcasting equation, eqs. (4-6), for annualized quarterly real GDP growth, Y_t.

$$
\begin{aligned}
X_t &= \Gamma F_t + \epsilon_t && (1)\\
F_t &= \beta_1 F_{t-1} + \beta_2 F_{t-2} + \beta_3 F_{t-3} + \beta_4 F_{t-4} + \nu_t && (2)\\
F_t^3 &= \frac{F_t + F_{t-1} + F_{t-2}}{3} && (3)\\
Y_t &= T_t^3 + \gamma_0 F_t^3 + \gamma_1 F_{t-1}^3 + \gamma_2 F_{t-2}^3 + \gamma_3 F_{t-3}^3 + \gamma_4 F_{t-4}^3 + \gamma_5 F_{t-5}^3 + \upsilon_t && (4)\\
T_t &= \alpha + T_{t-1} + \eta_t && (5)\\
T_t^3 &= \frac{T_t + T_{t-1} + T_{t-2}}{3} && (6)
\end{aligned}
$$

We assume that $\epsilon_t \sim N(0, H)$, where H is a diagonal matrix. The OLS variant of the DF-CFNAI parameterizes the variance-covariance matrix H as $\sigma^2 * I$ where I is the 85x85 identity matrix. The HR variant instead assumes a heteroskedastic representation where the diagonal elements of H are equal to σ_i^2. In addition to allowing for heteroskedasticity, the AR variant allows ϵ_i to be serially correlated up to first order.[1] By assumption, ϵ_t and ν_t are uncorrelated.

Our nowcast is based on a trend-cycle decomposition for quarterly annualized real GDP growth, Y_t, where the cyclical dynamics of real GDP growth are assumed to be captured by current and past values of the three-month moving average of the DF-CFNAI. We only observe Y_t in the third month of each quarter, so that eq. (4) strictly relates each quarterly realization of real GDP growth to its corresponding end-of-quarter trend value, T_t^3. We assume that $\upsilon_t \sim N(0, V)$ and $\eta_t \sim N(0, W)$ and are uncorrelated with each other, ϵ_t, and ν_t.[2]

[1] We choose the degree of serial correlation for each of the 85 data series prior to estimation of the DF-CFNAI according to the Hannan-Quinn Information Criterion.

[2] We experimented with allowing ν_t and υ_t to be correlated as in Morley et al. (2003), but doing so did not appreciably alter our results.

The state-space representation for the DF-CFNAI and nowcast is given by the following measurement and state equations

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Gamma & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_0 & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_t \\ F_{t-1} \\ F_{t-2} \\ F_{t-3} \\ F_t^3 \\ F_{t-1}^3 \\ F_{t-2}^3 \\ F_{t-3}^3 \\ F_{t-4}^3 \\ F_{t-5}^3 \\ T_t \\ T_{t-1} \\ T_{t-2} \\ T_t^3 \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ \upsilon_t \end{bmatrix}$$

$$\begin{bmatrix} F_t \\ F_{t-1} \\ F_{t-2} \\ F_{t-3} \\ F_t^3 \\ F_{t-1}^3 \\ F_{t-2}^3 \\ F_{t-3}^3 \\ F_{t-4}^3 \\ F_{t-5}^3 \\ T_t \\ T_{t-1} \\ T_{t-2} \\ T_t^3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha \\ 0 \\ 0 \\ \frac{\alpha}{3} \end{bmatrix} + \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\beta_1+1}{3} & \frac{\beta_2+1}{3} & \frac{\beta_3}{3} & \frac{\beta_4}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{t-1} \\ F_{t-2} \\ F_{t-3} \\ F_{t-4} \\ F_{t-1}^3 \\ F_{t-2}^3 \\ F_{t-3}^3 \\ F_{t-4}^3 \\ F_{t-5}^3 \\ F_{t-6}^3 \\ T_{t-1} \\ T_{t-2} \\ T_{t-3} \\ T_{t-1}^3 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{3} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \nu_t \\ \eta_t \end{bmatrix}.$$

The number of lags of F_t in eq. (2) and F_t^3 in eq. (4) were chosen to minimize the root-mean squared error of the nowcast of real GDP growth from the first quarter of 1967 through the fourth quarter of 2012.

Notice that the trend component of quarterly real GDP growth, T_t^3, evolves as a "time-varying mean" and that the assumption of a unit root in eq. (5) introduces a nonstationary element into the state equation. Written in this way, we assume that changes in the monthly mean of real GDP growth, T_t, have a permanent component (α) and a transitory component (η_t). Our quarterly time-varying mean, T_t^3, is then a weighted average of these two components. Identification of the transitory component of T_t^3 is made possible by the fact that it is a weighted average of the η_t's that span months within the quarter, whereas υ_t is assumed to be serially uncorrelated.

The model is estimated using the expectations-maximization (EM) algorithm. The M-step in the algorithm consists of ordinary least squares estimation of the model's parameters, i.e. $\Theta = \{\Gamma, \beta, \gamma, \alpha, H, Q, V, W\}$. To initialize the algorithm, we use the time series for the CFNAI as the first estimate of F_t to run the linear regressions implied by eqs. (1-2) to obtain Γ and β. The variance-covariance matrix H then follows from the first linear regression, while we follow Doz et al. (2012) in fixing the scale of the latent factor by setting $Q = I$.

For the initial nowcast, we assume that $E_t(T_t^3)$ is the constant in the linear regression of quarterly annualized real GDP growth on current and previous values of the CFNAI-MA3 implied by eq. (4). From this regression, we obtain our initial estimates of γ and W. We then initialize α at zero and calibrate V according to the median unbiased estimation procedure described in Stock and Watson (1998) applied to a "local-level" unobserved components model for real GDP growth. At subsequent iterations, α and V are then estimated by the linear regression implied by eq. (5) using estimates of T_t.

The E-step in the EM algorithm consists of using the Kalman filter and smoother to obtain new estimates of F_t and T_t given our initial estimates of the model's parameters and the data series X_t and Y_t. The Kalman filter requires that we specify the initial values for the mean and variance of F_t and T_t. To do so, we follow the procedure described in Harvey (1989), setting $F_0 = 0$ and $T_0^3 = Y_0$ in the first quarter of 1965 and giving them both a large variance. The impact of this initialization dies out slowly over time; and for this reason, we do not consider estimates in the two year period prior to 1967.

This process is then repeated until the likelihood function computed in the E-step becomes stable, using the estimates of F_t and T_t from the E-step in the next M-step and taking into account the additional uncertainty associated with using generated regressors in the linear regressions.[3] The algorithm requires only a few iterations, as it begins its search for a local maximum in a neighborhood of the parameter space associated with the initial consistent estimates of the parameters identified with the CFNAI and the median unbiased estimate of the variance of the time-varying mean.

[3]The likelihood function is similar to that derived in the appendix to Brave and Butters (2012). However, allowing for serially correlated ϵ_i requires a slight alteration as discussed in Jungbacker et al. (2011). Our stability criterion for the likelihood function where k references the iteration is $|\log L(k) - \log L(k-1)/((\log L(k) + \log L(k-1))/2)| < 10^{-4}$.

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