A simple model of gross worker flows across labor market states

Marcelo Veracierto

Introduction and summary

While standard macroeconomic models of labor markets typically assume two labor market states (employment/nonemployment or employment/unemployment), a recent literature has extended those models to incorporate three labor market states: employment, unemployment, and nonparticipation (or out of the labor force). See, for example, Tripier (2004), Veracierto (2008), Christiano, Trabandt, and Walentin (2010), Haefke and Reiter (2011), and Shimer (2013). Most of these papers have focused on modeling the number of workers in each of these labor market states, but not the gross flows of workers across them. A notable exception is Krusell et al. (2012), which introduces search frictions (for the process through which workers meet job opportunities) into a real business cycle model with borrowing constraints in the household sector. Their model is rich enough to explicitly determine the gross flows of workers across the three labor market states, potentially providing a deeper understanding of what drives unemployment and other labor market shifts.

Interestingly, Krusell et al. (2012) find that under certain specifications of aggregate shocks, their model is able to broadly reproduce the cyclical behavior of gross worker flows across labor market states in the U.S. economy. While this is an important result, the economic mechanisms behind it are somewhat obscured by the real business cycle structure and the borrowing constraints. The purpose of this article is to strip the model in Krusell et al. (2012) down to its bare bones—that is, to develop (and analyze) a very simple version of it. The key difference between my model and theirs is that instead of embodying the search frictions in a real business cycle with borrowing constraints, I assume that technology and workers' preferences (for consumption and leisure) are linear. These assumptions allow for an analytical characterization of the model that makes the determination of gross worker flows transparent. Moreover, the simple structure of the model allows me to perfectly identify its shocks using U.S. data.

A key ingredient of any model useful for analyzing unemployment behavior is the presence of search frictions. While search frictions typically create bilateral bargaining situations between workers and employers, I (like Krusell et al., 2012) introduce them in such a way that wages are determined in perfectly competitive labor markets.[1] Moreover, the simple structure that is

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assumed allows for a single wage rate in the whole economy. Both features are obtained by assuming that all firms in the economy produce in a single geographical location where workers can move from one firm to another in a frictionless way.

Not all workers are present in the production location, though: Some of them are situated in a separate geographical location in which they are able to enjoy leisure. Search frictions are introduced by assuming that it is difficult to move from the leisure location to the production location. In particular, agents are assumed to be able to move from the leisure location to the production location with a fixed probability (interpreted as a job-finding rate). Also, workers who are present in the production location are forced to move to the leisure location with a fixed probability (interpreted as a job-separation rate). Aside from being subject to job-separation shocks that force individuals to move to the leisure location, workers always have the possibility of moving from the production location to the leisure location whenever they wish. Because workers are subject to idiosyncratic labor productivity shocks, they face nontrivial labor supply decisions. Also, because the shocks are idiosyncratic, workers end up moving in and out of employment in an unsynchronized way, generating gross flows across the different labor market states.

It turns out that calibrating the steady state (or long-run equilibrium) of the simple model's economy to average monthly U.S. gross worker flow rates requires that I introduce errors in the classification of agents' labor market states. Otherwise, the model would be inconsistent with those actual gross worker flows. The classification errors needed are somewhat more extreme than those indicated by the empirical evidence, but they are not vastly out of line. Once the model is calibrated, the equations describing the gross flows of workers are used to measure the reallocation shocks (that is, the job-finding and job-separation rates) and the aggregate productivity shocks that hit the economy. (In order to match the U.S. data, I find that the classification errors introduced into the model must also be allowed to vary over time.) A crucial test of the model is whether these measured aggregate shocks, classification errors, and gross worker flows are consistent with the optimal decisions of agents in the model. I find that they are. Thus, the model seems to provide a reasonable labor supply theory. However, the model does not provide a deep theory of labor market dynamics. The reason is that most of the success of the model at reproducing the labor market dynamics found in the U.S. economy relies on the realization of exogenous shocks: The optimal decisions of agents in the model economy play a minor role in generating endogenous fluctuations in the gross flows of workers across labor market states, as well as in employment and unemployment.

In the next section, I describe the simple model's economy in greater detail. In the subsequent section, I discuss how its classification of workers into the three labor market states approximates that of the U.S. Bureau of Labor Statistics (BLS). Next, I calibrate the model to U.S. data. Then, I test how well the model does in reproducing U.S. business cycle data. After that, I compare these results with those of my previous work (Veracierto, 2008) and offer goals for future research.

The model economy

The model economy is populated by an interval [0,1] of workers, who are distributed across two islands: a production island and a leisure island.[2] The production island has a representative firm that produces output with a linear production function that uses labor as the only input of production. The productivity of a worker is given by $p + z$, where p is an aggregate productivity level common to all workers and z is an idiosyncratic productivity level. Because the production island is assumed to be competitive, the wage rate that the worker receives is equal to $p + z$. While p is assumed to be constant, the idiosyncratic productivity level z evolves stochastically over time according to a Markov process. In particular, I will assume that with probability $1 - \gamma$, the next period's idiosyncratic productivity level z' will be equal to the current period's idiosyncratic productivity level z and that with probability γ, the next period's idiosyncratic productivity level z' will be drawn anew from a known distribution function F.

On the leisure island, workers do not produce but enjoy α units of leisure. Workers value consumption and leisure according to the following preferences:

$$\sum_{t=0}^{\infty} \beta^t \left[c_t + (1 - e_t)\alpha \right],$$

where $e_t \in \{0,1\}$ is an indicator function that is equal to 1 when the worker is employed and 0 otherwise. Observe that, given the linear preferences assumed, the worker cares about the present value of his consumption but not about its timing.

There are frictions to move from the leisure island to the production island. However, there are no frictions to move the other way.

The timeline within each time period is as follows: 1) Workers start the period distributed in some way across the two islands; 2) the idiosyncratic productivity shock z of each worker is realized; 3) workers who are initially located on the production island become exogenously relocated to the leisure island with probability σ; 4) each agent who is located on the leisure island at this point (including all those initially located on the leisure island and all those that were exogenously relocated from the production island) is exogenously relocated to the production island with probability λ; 5) agents located on the production island at this point (including all those that have just arrived from the leisure island) must decide whether to stay on the production island or move back to the leisure island; and 6) consumption and leisure are finally enjoyed. In what follows, I will refer to σ as the (exogenous) job-separation rate and to λ as the (exogenous) job-finding rate.

Next, I analyze the problem that workers face in this economy.

Workers' problem

Consider the decision problem of a worker situated on the production island during stage 5 of the timeline just described (this is the only situation in which a worker must make a decision). At this point, the worker already knows the realization of his idiosyncratic productivity for the current period (that is, z). Given this information, the worker must decide whether to stay on the production island and work or go back to the leisure island and enjoy leisure. Let his value of staying on the production island be $W(z)$ and his value of moving back to the leisure island be $L(z)$. Then, his optimal value $V(z)$ is given by

$$V(z) = \max\{W(z), L(z)\}.$$

The value of being on the production island is given by the following equation:

1) $$W(z) = p + z + \beta\sigma(1-\lambda)\begin{bmatrix}(1-\gamma)L(z) \\ +\gamma\int L(z')dF(z')\end{bmatrix} + \beta\left[1-\sigma(1-\lambda)\right]\begin{bmatrix}(1-\gamma)V(z) \\ +\gamma\int V(z')dF(z')\end{bmatrix}.$$

This equation states that if the worker stays on the production island, he receives the wage rate $p + z$ during the current period and starts the following period on the same island. Different things can happen from that point on. With probability $\sigma(1 - \lambda)$, the worker gets exogenously relocated to the leisure island and is not able to come back. In this case, with probability $1 - \gamma$, his idiosyncratic productivity level in the next period does not change and he obtains the value of being on the leisure island $L(z)$. However, with probability γ, he draws a new idiosyncratic productivity level and obtains the expected value $\int L(z')\, dF(z')$. The alternatives, which happen with probability $1 - \sigma(1 - \lambda)$, are either that the worker is not exogenously relocated to the leisure island or that if he is, he is able to come back to the production island. In either case, he starts stage 5 in the following period on the production island. Then, with probability $1 - \gamma$, he obtains the value $V(z)$, and with probability γ, he obtains the expected value $\int V(z')\, dF(z')$.

The value of being on the leisure island is given by the following equation:

2) $$L(z) = \alpha + \beta(1-\lambda)\begin{bmatrix}(1-\gamma)L(z) \\ +\gamma\int L(z')dF(z')\end{bmatrix} + \beta\lambda\left[(1-\gamma)V(z)+\gamma\int V(z')dF(z')\right].$$

This equation states that if the worker is on the leisure island during the current period, he receives the value of leisure α during the current period and starts the following period on the same island. Different things can happen from then on. With probability $1 - \lambda$, the worker is not able to relocate to the production island and gets the expected value $(1 - \gamma)\, L(z) + \gamma \int L(z')\, dF$. However, with probability λ, the worker is able to relocate to the production island and obtains the expected value $(1 - \gamma)\, V(z) + \gamma \int V(z')\, dF$.

With equations 1 and 2, I can show straightforwardly that $W'(z) > 0$ and that $L'(z) > 0$ (that is, both value functions are strictly increasing in z). Thus, the optimal decision for the worker is characterized by a threshold idiosyncratic productivity level z^* that satisfies that

$$W(z^*) = L(z^*),$$

$$W(z) < L(z), \text{ for } z < z^*,$$

$$W(z) > L(z), \text{ for } z > z^*.$$

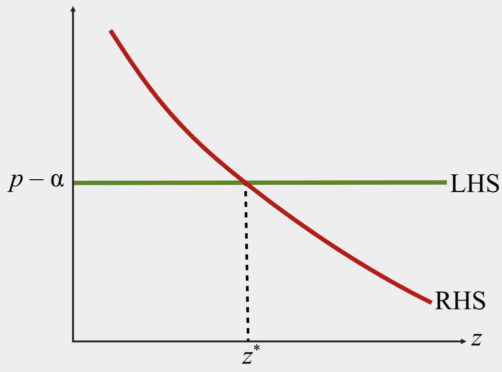
That is, at the threshold level z^*, the worker is indifferent between staying on the production island and going back to the leisure island. For values of z lower than the threshold, the worker prefers to go back to the leisure island; and for values of z higher than the threshold, the worker prefers to stay on the production island.

Evaluating the condition $W(z^*) = L(z^*)$ using equations 1 and 2, I get that

3) $$p + z^* = \alpha - \beta\gamma(1-\sigma)(1-\lambda) \times \int_{z^*}^{\infty}\left[W(z') - L(z')\right]dF(z').$$

FIGURE 1

Optimal threshold for idiosyncratic productivity



Note: See the text for further details.

Hence, I get the familiar condition that the reservation wage rate $p + z^*$ (that is, the lowest wage rate acceptable to the worker) is less than the value of leisure α because of the possibility that z may improve over time (the value of this possibility is called its "option value").

Evaluating equations 1 and 2 for $z \geq z^*$ and differentiating with respect to z, I get that

$$W'(z) - L'(z) = \frac{1}{1 - \beta(1 - \gamma)(1 - \lambda)(1 - \sigma)},$$

for $z \geq z^*$. Integrating by parts in equation 3 thus results in the following:

$$4)\quad p - \alpha = -z^* - \frac{\beta\gamma(1-\sigma)(1-\lambda)}{1 - \beta(1-\gamma)(1-\sigma)(1-\lambda)} \times \int_{z^*}^{\infty} \left[1 - F(z')\right] dz',$$

which implicitly gives the idiosyncratic productivity threshold z^* as a function of the fundamental parameters of the model.

Figure 1 provides a graphic representation of the determination of z^*. It shows the left-hand side (LHS) of equation 4 as a horizontal line, independent of z. The right-hand side (RHS) of equation 4 is depicted as a decreasing function.[3] The intersection of both lines determines the idiosyncratic productivity threshold z^*.

When either the aggregate productivity level p increases or the value of leisure α decreases, the RHS of equation 4 is not affected but the LHS of that equation shifts up, lowering the value of z^*. This is quite intuitive, since in both cases the value of being employed increases relative to the value of enjoying leisure, inducing the worker to become less picky and accept employment with lower values of idiosyncratic productivity.

When either the probability of being exogenously relocated to the leisure island σ or the probability of being exogenously relocated to the production island λ goes up, the LHS of equation 4 is not affected but the RHS of that equation shifts up. Similar effects take place when the probability of obtaining a new draw for the idiosyncratic productivity level γ decreases. In all three cases, the idiosyncratic productivity threshold z^* increases.

To get intuition about these effects, recall from equation 3 that the reservation wage rate $p + z^*$ is less than the value of leisure α because of the option value of z improving over time. In all three cases, the option value of waiting for an improvement in the idiosyncratic productivity level happens to decrease, making the worker less tolerant of low realizations of the idiosyncratic productivity level. When the probability of being exogenously relocated to the leisure island σ increases, the option value of waiting decreases because it is more likely that the worker will find himself on the leisure island at the time that the idiosyncratic productivity level z improves. When the probability of being exogenously relocated to the production island λ increases, the option value of waiting decreases because it becomes easier to get back to the production island in the future (once an improvement in z takes place). When the probability of obtaining a new draw for the idiosyncratic productivity level γ decreases, the option value of waiting decreases because productivity improvements become less frequent.

Measuring labor market states

Given the idiosyncratic productivity threshold z^* described in the previous subsection, it is straightforward to describe the evolution of aggregate employment in the economy.

Let E_{-1} be the total number of workers that have been employed in the previous period. Then, the total number of workers that are employed during the current period is given by

$$5)\quad E = (1 - \sigma + \sigma\lambda)(1 - \gamma)E_{-1} + (1 - \sigma + \sigma\lambda)\gamma \times [1 - F(z^*)]E_{-1} + \lambda(1 - \gamma)[1 - E_{-1} - F(z^*)] + \lambda\gamma[1 - F(z^*)](1 - E_{-1}).$$

The first term is the total number of workers employed in the previous period that continued with the same idiosyncratic productivity level during the current period and that either were not exogenously relocated to the leisure island or, if they were, were able to come back to the production island within the same period.

The second term is the total number of workers employed in the previous period that received a new idiosyncratic productivity above the threshold $z^*$and that either were not exogenously relocated to the leisure island or, if they were, were able to come back to the production island within the same period. The third term is all those workers not employed in the previous period, even though they had an idiosyncratic productivity level higher than the threshold z^*, who did not receive a new productivity draw during the current period and were relocated to the production island. The last term is all those workers not employed in the previous period who received a new idiosyncratic productivity level above the threshold z^* during the current period and were relocated to the production island.

Defining employment and nonemployment is quite natural in this two-island model; however, dividing nonemployed workers into the unemployed and nonparticipants is less obvious. Hereon I will follow Krusell et al. (2012) and perform the classification by surveying workers at the end of the period with the following questionnaire:

1) Are you employed?

2) If you are not employed, do you wish you had been employed?

If the worker answers yes to the first question, he is classified as employed. If he answers no to the first question but yes to the second question, he is classified as unemployed. If he answers no to both the first and second questions, he is classified as not in the labor force (nonparticipant).

The total number of nonparticipants in the economy will then be given by

6) $N = F(z^*)$;

that is, it is the total number of workers with idiosyncratic productivity levels below the threshold z^*. Irrespective of whether these agents had the opportunity of becoming employed or not, they end the period being nonemployed and answering that they do not wish to be employed. Observe that absent changes in total productivity and other parameter values, the total number of nonparticipants in the economy is constant over time.

The total number of unemployed workers is then given by

7) $U = 1 - E - F(z^*)$;

that is, it is the total number of nonemployed workers that have a productivity level above the threshold z^*. Observe that U changes over time because E (the total number of employed workers) does. In the long run, as employment converges to a constant level, unemployment will also converge. In particular, by setting $E = E_{-1}$ in equation 5, I get that the long-run employment level is equal to

$$E = \frac{\lambda\left[1-F\left(z^*\right)\right]}{1-(1-\sigma)(1-\lambda)\left(1-\gamma F\left(z^*\right)\right)}.$$

From equation 7, I get that the long-run unemployment level is then equal to

$$U = 1-\frac{\lambda\left[1-F\left(z^*\right)\right]}{1-(1-\sigma)(1-\lambda)\left(1-\gamma F\left(z^*\right)\right)}-F\left(z^*\right).$$

Measuring gross flows of workers across labor market states

Given the classification of workers into employment (E), unemployment (U), or nonparticipation (N) described in the previous subsection, I can now define the flow rates of workers across the different labor market states. The flow rates of workers are as follows:

8) $f_{EN} = \gamma F(z^*)$,

9) $f_{UN} = \gamma F(z^*)$,

10) $f_{UE} = \lambda(1 - f_{UN})$,

11) $f_{NE} = \lambda\gamma[1 - F(z^*)]$,

12) $f_{EU} = \sigma(1 - \lambda)(1 - f_{EN})$,

13) $f_{NU} = (1 - \lambda)\gamma[1 - F(z^*)]$,

where f_{ij} is the flow rate from labor market state i (E, U, or N) to labor market state j (E, U, or N). The flow rate f_{EN} is given by the probability that an employed worker receives a new idiosyncratic productivity level times the probability that this level is below the threshold z^*. The flow rate f_{UN} is similarly given by the probability that an unemployed worker receives a new idiosyncratic productivity level times the probability that this level is below the threshold z^*. The flow rate f_{UE} is given by the probability that an unemployed worker does not transition into nonparticipation times the probability of moving to the production island. The flow rate f_{NE} is given by the product of the probability that a nonparticipant draws a new idiosyncratic productivity level, the probability that this productivity level is above the threshold z^*, and the probability that the worker moves to the production island. The flow rate f_{EU} is given by the product of the probability that an employed worker does not transition into nonparticipation, the probability that he is exogenously relocated to the leisure island, and the probability that he is not able to make it back to the production island within the same period. The flow rate f_{NU} is given by the product of the probability that a nonparticipant worker draws a new idiosyncratic productivity level, the probability that this productivity level is above the threshold z^*, and the probability that the worker is not able to move to the production island.

Observe that absent changes in total productivity and other parameter values, the flow rates described by equations 8–13 are constant over time.

Using the BLS classification of labor market states

The U.S. Bureau of Labor Statistics classifies people into employment, unemployment, and nonparticipation by essentially asking the following two questions:

1) Are you employed?

2) If you are not employed, did you search for a job in the past four weeks?

If the person answers yes to the first question, he is classified as employed. If he answers no to the first question but yes to the second one, he is classified as unemployed. If he answers no to both questions, he is classified as being a nonparticipant. In the model of this article as well as that of Krusell et al. (2012), there is no search activity. As a consequence, there is no distinction between unemployment and nonparticipation in the BLS sense.

This article, as well as Krusell et al. (2012), works around this difficulty by substituting the second BLS question with the following one: If you are not employed, do you wish you had been employed? As shown in the previous section, this rephrasing led to a very clear classification between unemployment and nonparticipation in the model. But how well does the model's classification approximate the BLS's classification? Krusell et al. (2012) argue that it does this very well.

To see why this is the case, consider changing the relocation from the leisure island to the production island from being exogenous to being endogenous (for stage 4 in the timeline described near the beginning of the previous section). In particular, assume that agents now have to pay an infinitesimal cost in order to make such a transition with probability λ. If they choose not to pay that infinitesimal cost, then the agents remain on the leisure island.

Consider now the same productivity threshold z^* as in the original equilibrium, which determined who would work and who would relocate to the leisure island (at stage 5 of the timeline). Because idiosyncratic productivity shocks are realized at the beginning of the period (during stage 1 of the timeline), it is clear that agents located on the leisure island at stage 4 of the timeline with a $z < z^*$ will not want to pay the infinitesimal search cost (because they would not want to stay on the production island anyway). Because these agents will end the period not employed and will not have not searched, they will be correctly classified as nonparticipants.

In turn, someone located on the leisure island at stage 4 of the timeline with a $z > z^*$ will be willing to pay the infinitesimal search cost in order to move to the production island with probability λ. However, some of these agents will not be lucky enough to move to the production island. These agents will end the period not employed but will have searched. Hence, they will be correctly classified as unemployed.

Finally, those agents who were employed in the previous period and who get to stage 5 of the timeline while located on the production island with a $z < z^*$ will decide to move to the leisure island. Because these workers will end the period nonemployed and will not have searched, they will be correctly classified as nonparticipants. Thus, it's clear that if an infinitesimal search cost is introduced into the model, the Krusell et al. (2012) classification of labor market states, which I use for my model, coincides with the BLS classification.

The particular timeline assumed plays a critical role in obtaining this equivalence. To see why, consider changing it slightly. In particular, instead of assuming that idiosyncratic productivity shocks are realized at the beginning of the period, assume that they are realized after the search decisions are made.[4] Assuming the infinitesimal search cost would now produce a stark result: Nobody in the economy would be classified as a nonparticipant by the BLS. To see why, observe that all the agents would be willing to pay the infinitesimal search cost to see if they are lucky enough to draw an idiosyncratic productivity shock $z > z^*$. If they aren't, they would move to the leisure island and remain nonemployed. However, because they searched, all of these agents will be classified as unemployed. Thus, with this slight change in the timeline, one can see that the Krusell et al. (2012) classification would be widely different from the BLS classification.

TABLE 1

U.S. monthly transition rates

From	To: E	U	N
E	0.9559	0.0140	0.0271
U	0.2503	0.5290	0.2207
N	0.0472	0.0278	0.9250

Notes: This table presents the average monthly transition rates between the three labor market states—employment (*E*), unemployment (*U*), and nonparticipation (*N*)—reported by the BLS in the CPS between April 1992 and October 2014. See the text for further details.

Source: Author's calculations based on data from the U.S. Bureau of Labor Statistics (BLS), *Current Population Survey* (CPS), from Haver Analytics.

I conclude that both the infinitesimal search cost and the particular timeline assumed are necessary for using the model to analyze U.S. data in a meaningful way.

Calibration of the model

In principle, data on transition rates between employment, unemployment, and nonparticipation could be used to determine the four "parameters" that appear in equations 8–13: γ, λ, σ, and $F(z^*)$. To this end, I present in table 1 the average monthly transition rates between the three labor market states reported by the BLS in its *Current Population Survey* (CPS) between April 1992 and October 2014. It turns out that these average transition rates are not in line with the simple model considered so far because they are inconsistent with two key testable implications of the model.

The first testable implication of the model is obtained from equations 8 and 9:

$$14)\quad f_{EN} = f_{UN};$$

that is, the transition rate from employment to nonparticipation is exactly the same as the transition rate from unemployment to nonparticipation. However, table 1 indicates that in the data, f_{UN} is about eight times larger than f_{EN}: 22.07 percent versus 2.71 percent. Krusell et al. (2012) faced similar difficulties in generating a large f_{UN} relative to f_{EN}, but here the difficulty indicated by equation 14 is even more striking because of the particular stochastic process for idiosyncratic productivity assumed.

The second testable implication of the model is obtained from equations 10, 11, and 13. From equations 11 and 13, I have that

$$f_{NU} = \left(\frac{1-\lambda}{\lambda}\right) f_{NE}.$$

Using equation 10, I then have that

$$15)\quad f_{NU} = \left(\frac{1-f_{UN}}{f_{UE}} - 1\right) f_{NE}.$$

Using the values for f_{UN}, f_{UE}, and f_{NE} in table 1, I get from equation 15 an implied value for f_{NU} of 9.97 percent. However, table 1 shows an empirical value of 2.78 percent for this transition rate.

Given that the results of the simple model do not align with the empirical data thus far, I will follow Krusell et al. (2012) and introduce classification error as in Poterba and Summers (1986).[5] In particular, I will introduce two probabilities, ψ_{UN} and ψ_{NU}, which represent the probability of classifying as a nonparticipant someone who actually is unemployed and the probability of classifying as unemployed someone who actually is a nonparticipant, respectively. Employment is assumed to be measured without error.

When classification error is introduced, I must make a distinction between *true* unemployment and nonparticipation, U and N, and *measured* unemployment and nonparticipation, $\bar{U}$ and $\bar{N}$. (The true labor market states do not account for any classification error, whereas the measured labor market states do.) In particular, by a law of large numbers, I have the following relations:

$$16)\quad \bar{U} = U(1-\psi_{UN}) + N\psi_{NU},$$

$$17)\quad \bar{N} = N(1-\psi_{NU}) + U\psi_{UN}.$$

Equation 16 states that measured unemployment $\bar{U}$ is constituted by a fraction $1-\psi_{UN}$ of unemployed workers U that do not get misclassified and a fraction ψ_{NU} of nonparticipants N that get misclassified as unemployed. Equation 17 states that measured nonparticipation $\bar{N}$ is constituted by a fraction $1-\psi_{NU}$ of nonparticipants N that do not get misclassified and a fraction ψ_{UN} of unemployed workers U that get misclassified as nonparticipants.

With classification error introduced into the model, the measured transition rates become the following:

$$18)\quad f_{E\bar{N}} = f_{EN}(1-\psi_{NU}) + f_{EU}\psi_{UN},$$

$$19)\quad f_{E\bar{U}} = f_{EU}(1-\psi_{UN}) + f_{EN}\psi_{NU},$$

$$20)\quad f_{\bar{U}E} = f_{UE}\frac{U(1-\psi_{UN})}{\bar{U}} + f_{NE}\frac{N\psi_{NU}}{\bar{U}},$$

$$21)\quad f_{\bar{N}E} = f_{NE}\frac{N(1-\psi_{NU})}{\bar{N}} + f_{UE}\frac{U\psi_{UN}}{\bar{N}},$$

TABLE 2

Model's true transition rates

From	To: E	U	N
E	0.9588	0.0193	0.0219
U	0.2503	0.7278	0.0219
N	0.0068	0.0198	0.9734

Notes: This table presents the monthly transition rates between the three true (without classification error) labor market states—employment (*E*), unemployment (*U*), and nonparticipation (*N*)—in the model economy. See the text for further details.

$$22)\ f_{\bar{U}\bar{N}} = \left[f_{UU}\psi_{UN} + f_{UN}\left(1-\psi_{NU}\right)\right]\frac{U\left(1-\psi_{UN}\right)}{\bar{U}} + \left[f_{NN}\left(1-\psi_{NU}\right) + f_{NU}\psi_{UN}\right]\frac{N\psi_{NU}}{\bar{U}},$$

$$23)\ f_{\bar{N}\bar{U}} = \left[f_{NN}\psi_{NU} + f_{NU}\left(1-\psi_{UN}\right)\right]\frac{N\left(1-\psi_{NU}\right)}{\bar{N}} + \left[f_{UU}\left(1-\psi_{UN}\right) + f_{UN}\psi_{NU}\right]\frac{U\psi_{UN}}{\bar{N}}.$$

Equation 18 states that the probability that an employed worker makes a transition to measured nonparticipation $f_{E\bar{N}}$ is given by the probability of transitioning to nonparticipation f_{EN} times the probability of not being misclassified $1 - \psi_{NU}$, plus the probability of transitioning to unemployment f_{EU} times the probability of being misclassified as a nonparticipant ψ_{UN}. Equation 19 is similar to equation 18, but it is for the transition rate from employment to measured unemployment $f_{E\bar{U}}$. Equation 20 states that the probability that a worker measured as an unemployed worker makes a transition to employment $f_{\bar{U}E}$ is given by the probability that the worker is truly unemployed $\frac{U\left(1-\psi_{UN}\right)}{\bar{U}}$ times the probability of transitioning into employment f_{UE}, plus the probability that the worker is actually a nonparticipant $\frac{N\psi_{NU}}{\bar{U}}$ times the probability of transitioning into employment f_{NE}. Equation 21 is similar to equation 20, but it is for the transition rate from measured nonparticipation to employment $f_{\bar{N}E}$. Equation 22 states that the probability of transitioning from measured unemployment to measured nonparticipation $f_{\bar{U}\bar{N}}$ is given by the sum of two terms. The first term is that with probability $\frac{U\left(1-\psi_{UN}\right)}{\bar{U}}$, the worker is truly unemployed—in which case he makes a transition to measured nonparticipation if he remains unemployed and this lack of change in labor market status is mismeasured, $f_{UU}\psi_{UN}$, or if he transitions to nonparticipation and this change is not mismeasured, $f_{UN}(1 - \psi_{NU})$. The second term is that with probability $\frac{N\psi_{NU}}{\bar{U}}$, the worker is truly a nonparticipant—in which case he makes a transition to measured nonparticipation if he remains a nonparticipant and this lack of change in labor market status is not mismeasured, $f_{NN}(1 - \psi_{NU})$, or if he transitions into unemployment and this change is mismeasured, $f_{NU}\psi_{UN}$. Equation 23 is similar to equation 22, but it is for the transition rate from measured nonparticipation to measured unemployment.

Because I am interested in reproducing the transition rates in table 1, which are monthly averages over a long time period, I will impose the following steady-state conditions:

$$24)\ (f_{UE} + f_{UN})U = f_{EU}E + f_{NU}N,$$

$$25)\ (f_{NE} + f_{NU})N = f_{EN}E + f_{UN}U,$$

$$26)\ U + N + E = 1.$$

Equation 24 states that the total flows out of unemployment must be equal to the total flows into unemployment. Equation 25 states that the total flows out of nonparticipation must be equal to the total flows into nonparticipation. Finally, equation 26 states that the sum of all workers across the three labor market states must add up to the total population.

The system of nonlinear equations 14–26 could in principle be solved for the 13 unknowns $\bar{U}$, $\bar{N}$, U, N, E, f_{EN}, f_{EU}, f_{UN}, f_{UE}, f_{NE}, f_{NU}, ψ_{UN}, and ψ_{NU} with the target values for $f_{E\bar{N}}$, $f_{E\bar{U}}$, $f_{\bar{U}E}$, $f_{\bar{N}E}$, $f_{\bar{U}\bar{N}}$, and $f_{\bar{N}\bar{U}}$ being taken from table 1. However, performing an exhaustive computer analysis indicates that such a solution does not exist. Instead, the best approximate solution is obtained by setting ψ_{NU} to 0 (representing a corner solution), ψ_{UN} to 0.2733, and the true transition probabilities f_{EN}, f_{EU}, f_{UN}, f_{UE}, f_{NE}, and f_{NU} to the values given in table 2. Based on equations 14–26, these transition probabilities and misclassification probabilities imply the measured transition probabilities that are given in table 3. Many of the values in table 3 do exactly match those in table 1, and those that do not are not that far apart. In fact, only $f_{\bar{N}E}$ and $f_{\bar{N}\bar{U}}$ miss their target values—and not by much. Also, while the classification errors $\psi_{NU} = 0$ and $\psi_{UN} = 0.2733$ are more

TABLE 3

Model's measured transition rates

From	To		
	E	$\bar{U}$	$\bar{N}$
E	0.9559	0.0140	0.0271
$\bar{U}$	0.2503	0.5290	0.2207
$\bar{N}$	0.0163	0.0344	0.9493

Notes: This table presents the monthly transition rates between the three measured (including classification error) labor market states—employment (E), unemployment ($\bar{U}$), and nonparticipation ($\bar{N}$)—in the model economy. See the text for further details.

extreme than those reported by Poterba and Summers (1986) (their reported values are $\psi_{NU} = 0.0064$ and $\psi_{UN} = 0.1146$), they are not completely out of line.

Given the transition rates estimated in table 2, I can use equations 8 and 10–12 to back up the values for $F(z^*)$, λ, γ, and σ consistent with them. In particular, from equation 10, I have that

$$\lambda = \frac{f_{UE}}{1-f_{UN}} = \frac{0.2503}{1-0.0219} = 0.2558.$$

In turn, from equation 11, I have that

$$f_{NE} = \lambda\gamma - \lambda\gamma F(z^*) = \lambda\gamma - \lambda f_{EN}.$$

Hence,

$$\gamma = \frac{f_{NE} + \lambda f_{EN}}{\lambda} = \frac{0.0068 + 0.2558 \times 0.0219}{0.2558} = 0.0484.$$

From equation 8, I then have that

$$27)\ F\left(z^*\right) = \frac{f_{EN}}{\gamma} = \frac{0.0219}{0.0484} = 0.4513.$$

In addition, from equation 12, I get that

$$\sigma = \frac{f_{EU}}{\left(1-\lambda\right)\left(1-f_{EN}\right)} = \frac{0.0193}{\left(1-0.2558\right)\left(1-0.0219\right)} = 0.0265.$$

In order to calibrate the difference between aggregate productivity and the value of leisure ($p - \alpha$) that appears in equation 4, I must take a stance on the shape of the distribution function of idiosyncratic productivity levels F. For the sake of convenience, I will assume that it is exponential:

$$28)\ F(z) = 1 - e^{-\phi z}.$$

Equation 4 allows for a normalization. I will therefore normalize the threshold value z^* to 1 and find the parameter value ϕ that satisfies equation 27; that is,

$$F(z^*) = 1 - e^{-\phi z^*} = 1 - e^{-\phi} = 0.4513.$$

From this, I get $\phi = 0.6003$.

Observe that

$$\int_{z^*}^{\infty}\left[1-F\left(z\right)\right]dz = \int_{z^*}^{\infty} e^{-\phi z}\,dz = -\frac{1}{\phi}e^{-\phi z}\Big]_{z^*}^{\infty} = \frac{1}{\phi}e^{-\phi z^*}.$$

Substituting this expression in equation 4 yields the following equation:

$$29)\ p - \alpha = -z^* - \frac{\beta\gamma\left(1-\sigma\right)\left(1-\lambda\right)}{1-\beta\left(1-\gamma\right)\left(1-\sigma\right)\left(1-\lambda\right)}\frac{1}{\phi}e^{-\phi z^*}.$$

Using the values calibrated in this section, the normalization $z^* = 1$, and a discount rate $\beta = 0.9967$ (which implies an annual interest rate of 4 percent), I get a value of $p - \alpha = -1.1021$.

Observe that aggregate labor productivity is given by the following:

$$\frac{Y}{E} = p + \frac{1}{1-F\left(z^*\right)}\int_{z^*}^{\infty} zF'\left(z\right)dz.$$

That is, it is given by the aggregate labor productivity level common to all workers p plus the average idiosyncratic productivity z of employed workers. Using the functional form for the distribution function F in equation 28 and integrating by parts, I find that aggregate labor productivity is given as follows:

$$30)\ \frac{Y}{E} = p + z^* + \frac{1}{\phi}.$$

In order to determine the value of leisure α, I follow Shimer (2005) and assume that it is equal to 40 percent of aggregate labor productivity Y/E. That is,

$$\alpha = 0.40\left(p + z^* + \frac{1}{\phi}\right).$$

Because it has already been determined that $p - \alpha = -1.1021$, it follows that

$$\begin{aligned} p + 1.1021 &= 0.40\left(p + z^* + \frac{1}{\phi}\right) \\ &= 0.40\left(p + 1 + \frac{1}{0.6003}\right), \end{aligned}$$

where I have used the values for z^* and ϕ already determined. It follows that $p = -0.0596$ and $\alpha = 1.0425$. The resulting value for aggregate productivity Y/E in equation 30 is 2.6063.

Business cycles

In this section, I test how well the model does in reproducing U.S. business cycle data. To this end, I will follow Krusell et al. (2012) and allow the probability of being relocated to the production island λ, the probability of being relocated to the leisure island σ, and aggregate productivity p to fluctuate over time. All other parameters, including those describing the stochastic process for idiosyncratic productivity levels, will be assumed to be constant.

The plan is to use monthly data on measured gross flow rates to *infer* true gross flow rates in the model. These true gross flow rates will then be used to construct monthly time series for the probability of being relocated to the production island λ, the probability of being relocated to the leisure island σ, and the idiosyncratic productivity threshold z^*. Given that I have an empirical time series for aggregate labor productivity Y/E, I will then use equation 30 to construct a time series for the aggregate productivity p. A key test of the model will be to compare this time series with the time series for p that is obtained from equation 29, using the already determined time series for z^*, λ, and σ. This is a key test because equation 29 reflects the optimal decision of agents in the model economy.

In order to obtain empirical counterparts for true gross flow rates, I will use equations 18, 19, 20, and 22. The reason for using these equations is that these are the four equations among equations 18–23 that happen to hold with equality in the calibration performed in the previous section. Because the classification error ψ_{NU} that best described the data was equal to 0 (that is, was a corner solution), I will impose that it is 0 over the business cycle fluctuations. In contrast, I will allow the classification error ψ_{UN} not only to be positive but to fluctuate over the business cycle.[6]

Under the assumption that the classification error ψ_{NU} is equal to 0, which implies that $\bar{U} = U(1-\psi_{UN})$, equations 18, 19, 20, and 22 become the following:

$$f_{E\bar{N}} = f_{EN} + f_{EU}\psi_{UN},$$
$$f_{E\bar{U}} = f_{EU}(1-\psi_{UN}),$$
$$f_{\bar{U}E} = f_{UE},$$
$$f_{\bar{U}\bar{N}} = (1-f_{UE}-f_{UN})\psi_{UN} + f_{UN},$$

where, from equation 14,

$$f_{UN} = f_{EN}.$$

This system of equations turns out to have the following solution:

$$31)\ \psi_{UN} = \frac{f_{\bar{U}\bar{N}} - f_{E\bar{N}}}{1-f_{\bar{U}E}-f_{E\bar{N}}-f_{E\bar{U}}},$$

$$32)\ f_{EU} = \frac{f_{E\bar{U}}}{1-\psi_{UN}},$$

$$33)\ f_{EN} = f_{E\bar{N}} + f_{E\bar{U}} - f_{EU},$$

$$34)\ f_{UE} = f_{\bar{U}E},$$

$$35)\ f_{UN} = f_{EN}.$$

With monthly CPS data for $f_{\bar{U}\bar{N}}$, $f_{E\bar{N}}$, $f_{E\bar{U}}$, and $f_{\bar{U}E}$, I can then construct monthly time series for ψ_{UN}, f_{EU}, f_{EN}, f_{UE}, and f_{UN} using equations 31–35. Panels A–D of figure 2 show the time series for these true transition rates as well as the corresponding measured transition rates (which coincide, by construction, with U.S. data). On the one hand, one can see from panels A and B that f_{EU} is higher than $f_{E\bar{U}}$, and that f_{EN} is lower than $f_{E\bar{N}}$ (which was expected from tables 2 and 3) but that the true transition rates track the measured transition rates quite closely. On the other hand, panel C shows that f_{UE} and $f_{\bar{U}E}$ coincide (as indicated by equation 34). A large discrepancy shows up in panel D, where one can see that f_{UN} is not only much smaller than $f_{\bar{U}\bar{N}}$ (as was expected from tables 2 and 3) but that f_{UN} hardly fluctuates at all. In fact, panel E shows that most of the fluctuations in $f_{\bar{U}\bar{N}}$ are accounted for by fluctuations in the classification error ψ_{UN}.[7]

Given the constructed monthly time series for f_{UN} and f_{UE}, I can obtain the probability of being relocated to the production island λ from equation 10 as follows:

$$\lambda = \frac{f_{UE}}{1-f_{UN}}.$$

In turn, I can obtain the probability of being relocated to the leisure island σ from equation 12 and the constructed time series for f_{EU}, f_{EN}, and λ as follows:

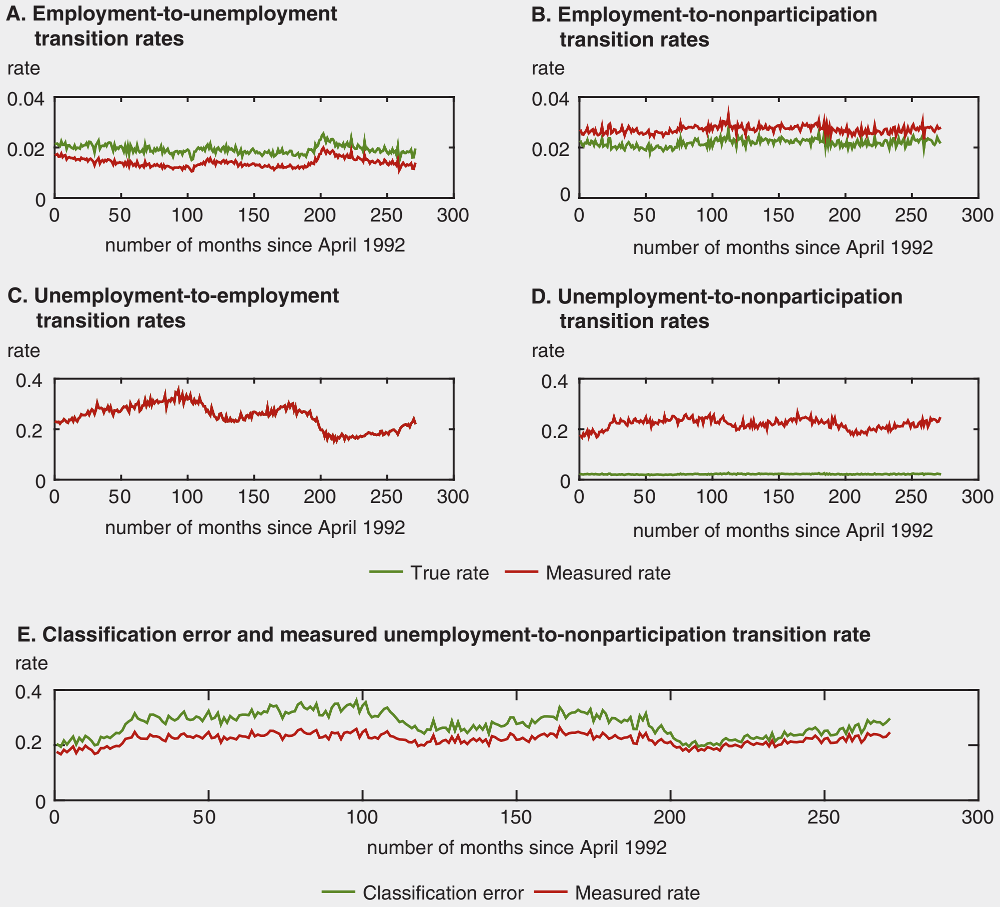
$$\sigma = \frac{f_{EU}}{(1-\lambda)(1-f_{EN})}.$$

From the constructed monthly time series for f_{EN} and the calibrated value for γ in the previous section, I can measure the time series for $F(z^*)$ from equation 8 as

$$F(z^*) = \frac{f_{EN}}{\gamma}.$$

FIGURE 2

True versus measured transition rates



Notes: Panels A–D present the monthly true and measured transition rates between the three labor market states—employment, unemployment, and nonparticipation—in the model economy. True transition rates do not account for classification error, while measured transition rates do. (The measured rates coincide, by construction, with U.S. data.) Panel E shows the measured unemployment-to-nonparticipation transition rate and the unemployment-to-nonparticipation classification error. See the text for further details on the panels.

Source: Author's calculations based on data from the U.S. Bureau of Labor Statistics, *Current Population Survey*, from Haver Analytics.

Using this time series, equation 28, and the calibrated value for ϕ from the previous section, I can then construct a monthly time series for z^* from the following equation:

$$z^* = -\frac{1}{\phi}\ln\left[1 - F\left(z^*\right)\right].$$

Given this constructed time series for z^* and an empirical time series for aggregate labor productivity Y/E, I can then use equation 30 to construct a time series for p. For an empirical time series for aggregate output Y, I use the forecasting firm Macroeconomic Advisers' monthly real gross domestic product (GDP) series, which is a monthly indicator of real aggregate output that is conceptually consistent with GDP in the U.S. Bureau of Economic Analysis's national income and product accounts (NIPAs).[8] For aggregate employment E, I use employment of the civilian noninstitutional population aged 16 years and over from the CPS provided by the BLS. The aggregate labor productivity Y/E obtained from dividing both time series happens to grow over time. Because aggregate labor productivity is constant in the model economy, I detrend the data using a linear regression. The deviations from trend thus obtained are then used to construct a time series for Y/E with an average value of 2.6063, which was the value for Y/E implied by the calibration of the previous section.

TABLE 4

Stochastic properties of shocks

	λ	σ	p
Standard deviation	0.0490	0.0024	0.0914
Autocorrelation	0.97	0.68	0.28
		Correlation matrix	
λ	1.00	0.40	0.26
σ		1.00	0.34
p			1.00

Note: See the text for further details.
Sources: Author's calculations based on data from Macroeconomic Advisers and the U.S. Bureau of Labor Statistics, *Current Population Survey*, all from Haver Analytics.

Table 4 reports summary statistics for the joint stochastic behavior of the time series for λ, σ, and p obtained through the equations discussed in this section. One can see that the probability of being exogenously relocated to the production island λ is highly persistent, the probability of being exogenously relocated to the leisure island σ is less so, and aggregate productivity p is much less persistent than usually assumed (see the second row of table 4 reporting autocorrelation statistics). All three shocks are pairwise weakly positively correlated (see the final three rows of table 4). This contrasts with Krusell et al. (2012), who assumed perfect correlations between the shocks. In particular, their "good-times/bad-times" assumption implies that $\rho(\lambda, \sigma) = -1$, $\rho(\lambda, p) = 1$, and $\rho(\sigma, p) = -1$.

Panel A of table 5 reports business cycle statistics for U.S. data. The labels u and $lfpr$ denote the unemployment rate and labor force participation rate, respectively. All statistics correspond to monthly time series. Before any statistics were computed, the data were logged and applied a Hodrick–Prescott filter with smoothing parameter of 10^5 in order to obtain their cyclical components (that is, their deviations from a slow-moving trend that reflect their fluctuations at business cycle frequencies). The labels $\sigma(x_t)$, $\rho(x_t, Y_t)$, and $\rho(x_t, x_{t-1})$ denote the standard deviation of variable x_t, the contemporaneous correlation of the variable x_t with output Y_t, and the serial autocorrelation of the variable x_t, respectively. I note that compared with output, employment is somewhat less variable, the unemployment rate is much more variable, and the labor force participation rate is much less variable (see the first column of table 5, panel A). Employment is strongly procyclical (that is, rising when economic times are good and falling when they are bad), the unemployment rate is strongly countercyclical (that is, falling when economic times are good and rising when they are bad), and the labor force participation rate is roughly acyclical (that is, moving independently of the overall state of the economy) (see the second column of table 5, panel A). All of these variables are significantly persistent, as shown by the serial autocorrelation statistics (see the third column of table 5, panel A). Reviewing the statistics for the transition rates, I note that all of them are highly volatile. The $f_{\bar{U}E}$ transition rate is strongly procyclical and persistent, while the $f_{E\bar{U}}$ and $f_{\bar{N}\bar{U}}$ transition rates are countercyclical and somewhat persistent. All other transition rates display weak cyclical patterns and have little persistence.

Panel B of table 5 reports similar statistics for artificial data generated using the following procedure. Given the monthly time series for z^*, λ, and σ constructed earlier, equations 8–13 were used to construct monthly time series for the true transition rates $f_{EN}, f_{UN}, f_{UE}, f_{NE}, f_{EU}$, and f_{NU}. Given these time series, monthly paths for true employment E, true unemployment U, and true nonparticipation N were constructed using the following equations:

$$U_t = (1 - f_{UE,t-1} - f_{UN,t-1})U_{t-1} + f_{EU,t-1}E_{t-1} + f_{NU,t-1}N_{t-1},$$

$$N_t = (1 - f_{NE,t-1} - f_{NU,t-1})N_{t-1} + f_{EN,t-1}E_{t-1} + f_{UN,t-1}U_{t-1},$$

$$E_t = 1 - U_t - N_t.$$

Given the time series for these variables and for the classification error ψ_{UN} obtained earlier, paths for measured unemployment $\bar{U}$ and measured nonparticipation $\bar{N}$ were obtained from equations 16 and 17. Given all these series, paths for the measured transition rates $f_{\bar{U}\bar{N}}$, $f_{\bar{N}\bar{U}}$, $f_{\bar{N}E}$, $f_{E\bar{N}}$, $f_{E\bar{U}}$, and $f_{\bar{U}E}$ were constructed using equations 18–23. In turn, given the time series for z^* and p constructed earlier, aggregate labor productivity Y/E was obtained from equation 30. Output Y was then obtained by multiplying aggregate labor productivity Y/E by employment E. Finally, the unemployment rate was calculated as $u = \bar{U}/\left(E + \bar{U}\right)$ and the labor force participation rate as $lfpr = E + \bar{U}$.

I see many similarities between panels A and B of table 5. To some extent this is not surprising because the monthly time series for p, z^*, λ, and σ were constructed in such a way that this would be the case. Indeed, the values of Y/E, $f_{E\bar{U}}$, $f_{E\bar{N}}$, $f_{\bar{U}E}$, and $f_{\bar{U}\bar{N}}$ must necessarily be identical in both cases (because these variables have been used as targets in the construction of the shocks).[9] Interestingly, similarities are also apparent in the rest of the variables. While the transition rate $f_{\bar{N}\bar{U}}$ and the unemployment rate are right on target, the transition rate $f_{\bar{N}E}$, output Y, employment E, and the labor force participation rate $lfpr$ are more volatile in the artificial

TABLE 5

Monthly business cycle statistics

	A. U.S. data			B. Artificial data		
	$\sigma(x_t)$ (*percent*)	$\rho(x_t, Y_t)$	$\rho(x_t, x_{t-1})$	$\sigma(x_t)$ (*percent*)	$\rho(x_t, Y_t)$	$\rho(x_t, x_{t-1})$
Y	1.12	1.00	0.88	1.47	1.00	0.92
E	0.95	0.74	0.98	1.23	0.85	0.97
u	10.79	–0.84	0.98	10.70	–0.75	0.96
$lfpr$	0.28	0.22	0.76	0.75	0.30	0.91
Y/E	0.77	0.55	0.69	0.77	0.55	0.69
$f_{E\bar{U}}$	7.68	–0.70	0.64	7.68	–0.63	0.64
$f_{E\bar{N}}$	4.30	0.16	0.07	4.30	0.14	0.07
$f_{\bar{U}E}$	7.96	0.77	0.81	7.96	0.66	0.81
$f_{\bar{U}\bar{N}}$	5.75	0.53	0.52	5.75	0.46	0.52
$f_{\bar{N}E}$	4.79	0.43	0.30	7.96	0.58	0.61
$f_{\bar{N}\bar{U}}$	7.65	–0.68	0.66	6.93	–0.62	0.90

Notes: All statistics correspond to monthly time series and are for the period April 1992–October 2014. See the text for further details.
Sources: Author's calculations based on data from Macroeconomic Advisers and the U.S. Bureau of Labor Statistics, *Current Population Survey*, all from Haver Analytics.

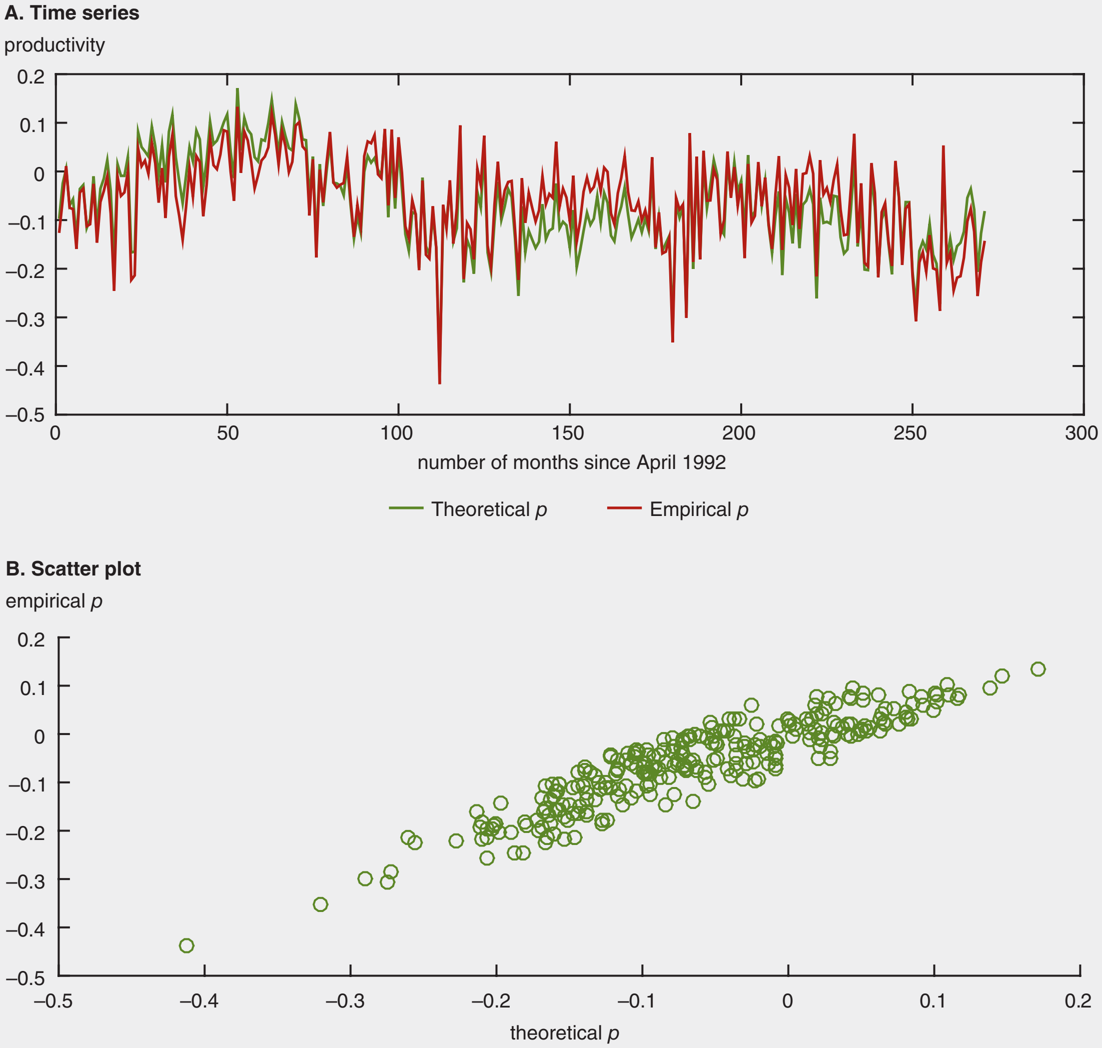
data than in the U.S. economy. However, the differences are not large and the correlations with output and serial autocorrelation statistics are quite similar in both cases.

While this is all quite satisfactory, it does not represent a test of the model yet. The reason is that the relationship between the productivity threshold z^* and the shocks p, λ, and σ that has been used so far has been determined by the data (and by part of the model structure); but it is not clear that such a relationship is completely consistent with the model economy. To fully test the empirical plausibility of the model, I must use equation 29, which represents the optimal decision of agents.[10] The way that I implement such a test is to plug into equation 29 the time series for z^*, λ, and σ constructed earlier in this section and to solve for the theoretical aggregate productivity level p implied by the equation.[11] The resulting time series is then compared with the empirical time series for p constructed earlier in this section. The comparison is displayed in figure 3, panel A. The result is striking. Not only do both time series display similar properties, but they align on top of each other quite well. This is better seen in figure 3, panel B, which displays a scatter plot of both the theoretical and empirical aggregate productivity levels. While the points are not perfectly aligned along the 45-degree line (which should be the case if both time series were identical), they are not far from it. I interpret this as a surprising success of the labor supply theory embodied in the model.

While my analysis thus far has demonstrated that the optimal decisions of agents (summarized by the idiosyncratic productivity threshold z^*) are consistent with empirical observations, it is natural to wonder about the role that the endogenous fluctuations in z^* play in business cycle dynamics. I evaluate this role by comparing two scenarios, whose results are reported in table 6. Panel A of table 6—which I label "Variable z^*"—reproduces the statistics of panel B of table 5 (the business cycle statistics of the benchmark economy). Panel B of table 6—which I label "Constant z^*"—reports business cycle statistics under the assumption that the productivity threshold z^* is constant at its steady-state value while all exogenous shocks (including the classification error) remain the same as in the benchmark case. One can see that the constant z^* significantly reduces the fluctuations in $f_{E\bar{N}}$ and increases its persistence (see the first and third columns of both panels A and B of table 6); however, the behavior of all other gross worker flow rates is largely unchanged. The constant z^* hardly affects the behavior of employment or unemployment: Their standard deviations and serial autocorrelations remain largely the same (see the first and third columns of both panels A and B of table 6). However, fluctuations in labor force participation are considerably dampened. This is not surprising because equation 6 indicates that true nonparticipation N is constant whenever z^* is. What is largely affected by the constant z^* is aggregate labor productivity Y/E, which becomes about four times more variable and completely loses its persistence (see the first and third columns of both panels A and B of table 6). Because aggregate output Y is given by aggregate employment E (which is hardly affected by the constant z^*) times aggregate labor productivity Y/E, Y also becomes more volatile and less persistent.

FIGURE 3

Theoretical versus empirical aggregate productivity level *p*



Notes: Panel B displays a scatter plot of both the theoretical and empirical aggregate productivity levels. See the text for further details on how to interpret both panels.
Sources: Author's calculations based on data from Macroeconomic Advisers and the U.S. Bureau of Labor Statistics, *Current Population Survey*, all from Haver Analytics.

I conclude that while the model described in this article captures salient features of labor market dynamics, it does so by relying almost completely on exogenous shocks. The only endogenous margin in the model, the choice of the productivity threshold z^*, provides little insight into such dynamics. Conditional on observed labor market dynamics, the main role of the endogenous fluctuations in z^* is to generate empirically relevant fluctuations in aggregate output and labor productivity.

Discussion

The results in this article are widely different from those in Veracierto (2008). In that paper, I studied a business cycle model with three labor market states but found that the model generated counterfactual business cycle dynamics. In particular, I found that unemployment was procyclical and that labor force participation was highly volatile (in fact, as much as employment). The intuition for why I got such a result is straightforward. In that model, being out of the labor force provided agents more leisure than being unemployed. Therefore, when a bad aggregate shock hit the economy that made leisure more attractive than working, workers made transitions from employment to nonparticipation instead of making transitions from employment to unemployment (as the data largely indicate).

TABLE 6

Variable versus constant z^*

	A. Variable z^*			B. Constant z^*		
	$\sigma(x_t)$ (percent)	$\rho(x_t, Y_t)$	$\rho(x_t, x_{t-1})$	$\sigma(x_t)$ (percent)	$\rho(x_t, Y_t)$	$\rho(x_t, x_{t-1})$
Y	1.47	1.00	0.92	3.16	1.00	0.09
E	1.23	0.85	0.97	1.29	0.30	0.98
u	10.70	–0.75	0.96	10.60	–0.34	0.96
$lfpr$	0.75	0.30	0.91	0.23	–0.07	0.57
Y/E	0.77	0.55	0.69	3.03	0.92	0.01
$f_{E\bar{U}}$	7.68	–0.63	0.64	7.67	–0.21	0.64
$f_{E\bar{N}}$	4.30	0.14	0.07	1.87	0.27	0.34
$f_{\bar{U}E}$	7.96	0.66	0.81	7.98	0.28	0.81
$f_{\bar{U}\bar{N}}$	5.75	0.46	0.52	5.78	0.36	0.52
$f_{\bar{N}E}$	7.96	0.58	0.61	7.85	0.32	0.69
$f_{\bar{N}\bar{U}}$	6.93	–0.62	0.90	6.02	–0.26	0.94

Notes: All statistics correspond to monthly time series and are for the period April 1992–October 2014. See the text for further details.
Sources: Author's calculations based on data from Macroeconomic Advisers and the U.S. Bureau of Labor Statistics, *Current Population Survey*, all from Haver Analytics.

As a consequence, fluctuations in labor force participation ended up mirroring fluctuations in employment, while unemployment became procyclical. Moreover, when the negative shock reversed, there was a surge of unemployment because agents needed to search in order to become employed. This reinforced the procyclicality of unemployment.

In principle, the model in this article would be subject to the same difficulties. To see why, suppose that a negative aggregate productivity shock hits the economy that lowers p. Because this makes working less attractive than enjoying leisure, the threshold productivity level z^* will increase. As a consequence, fewer people will choose to work (if given the opportunity); and of those not working, fewer will say that they would like to work. Thus, when the negative aggregate productivity shock hits the economy, employment and unemployment will decrease and nonparticipation will increase. When the aggregate productivity shock reverses, more of those on the production island will decide to work. And of those not making it to the production island, more of them will report that they would like to be employed. Thus, employment and unemployment will increase and nonparticipation will decrease. It is clear that with aggregate productivity shocks alone, the model will tend to generate procyclical unemployment and variations in labor force participation that mirror those in employment. That is, the model would display the same counterfactual behavior found in Veracierto (2008).

The reason why the model does not experience these difficulties is because there are exogenous variations in the job-separation rate σ and the job-finding rate λ. While in Veracierto (2008) these rates varied endogenously in response to an aggregate productivity shock, here they were chosen to fluctuate as much as needed to reproduce U.S. observations. In fact, the previous section showed that most of the success of the model at reproducing labor market dynamics relied on the exogenous variations in job-separation and job-finding rates, with little role for the endogenous decisions. The challenge for future researchers will be to develop models that generate exactly those same variations, but endogenously. This promises to be an exciting area of research.

Conclusion

In this article, I develop and analyze a simple model of the gross flows of workers across labor market states that is based on a model by Krusell et al. (2012). The simplicity of the model allows for analytical derivations that make the determination of these flows transparent. Moreover, this same simplicity allows me to perfectly identify the shocks that drive labor market fluctuations in the model by using U.S. data. I find that if errors in the classification of agents' labor market states are introduced and allowed to vary over time, the model has the ability to generate business cycle dynamics similar to those observed in the U.S. data. However, the labor market dynamics generated by the model are essentially driven by exogenous factors; the endogenous labor supply decisions embodied in the model barely affect them. The challenge for the future will be to develop models that reproduce actual labor market dynamics like my model did—but through endogenous factors. Such models may help further our understanding of what may be driving unemployment and other shifts in the labor market.

NOTES

[1]A perfectly competitive labor market is a market comprising many well-informed buyers (firms) and sellers (individuals) of labor that take the wage rate as given. Competitive wage determination in models with search frictions was first introduced by Phelps (1970) and, more systematically, by Lucas and Prescott (1974).

[2]For the rest of the article, I will follow the Lucas and Prescott (1974) tradition of referring to geographically distinct locations as "islands."

[3]The slope of the right-hand side of equation 4 is equal to

$$-1+\frac{\beta\gamma(1-\sigma)(1-\lambda)}{1-\beta(1-\gamma)(1-\lambda)(1-\sigma)}\left[1-F\left(z^{*}\right)\right]<0.$$

[4]This alternative timeline seems quite natural. In fact, standard search models assume that agents search without knowing the wage offer that they will receive (for example, McCall, 1970).

[5]Krusell et al. (2012) introduce classification error in an appendix to show that it can improve certain failures of their benchmark calibration.

[6]Here I depart from Krusell et al. (2012) because in an appendix, they only consider the case of constant classification error.

[7]It is difficult to take a stance on the plausibility of the classification error ψ_{UN} fluctuating over the business cycle by this magnitude.

[8]In fact, the quarterly growth rate of the Macroeconomic Advisers' GDP time series closely resembles the growth rate of real GDP in the NIPAs.

[9]In fact, only the standard deviations and serial autocorrelations of these variables must be the same in both cases. Their correlations with output will generally differ because output has not been used as a target in the construction of the shocks.

[10]Equation 29 has been used to calibrate the model, but it has not been used so far to analyze business cycle fluctuations.

[11]This "theoretical" aggregate productivity level should be interpreted as the aggregate productivity level p that is needed to reconcile the model with the data.

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