Supplementary Materials for "Macroeconomic Implications of Agglomeration"*

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Abstract

These supplementary materials provide details on (1) our growth model, (2) our data, (3) deriving the moment conditions underlying our estimation, (4) measuring the impact of agglomeration on per capita consumption growth (5) solving the model and comparing it with the data, (6) standard errors of our estimates, and (6) Monte Carlo analysis of our estimation strategy.

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*Any views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

A First Order Conditions, Prices and Growth

This section describes details about the growth model that were omitted from the main text. These include (1) the first order conditions of the planning problem, (2) how to map the Lagrange multipliers for this problem into competitive equilibrium prices, (3) how the planner's first order conditions relate to those of the agents' in the competitive equilibrium, (4) the stationarity inducing transformation of the growing economy, and (5) how to calculate the increase in steady state consumption and housing required to compensate households for not having the growth due to local agglomeration.

A.1 The Model Without Growth

The competitive equilibrium can be found as the solution to an optimization problem with side conditions. Idiosyncratic technology z_t evolves as a stationary discrete Markov chain. Let $q_t(z^t)$ denote the time t distribution of cities across productivity histories z^t and $Q(z, z')$ denote the probability that $z_{t+1} = z'$ conditional on $z_t = z$. The planner's problem is given by:

$$\max_{\substack{\{C_t, K_{bt+1}, K_{st+1}, y(z^t), l_b(z^t), l_h(z^t), \\ k_b(z^t), k_s(z^t), k_{ft+1}(z^t), n(z^t), h(z^t)\}_{t=0}^{\infty}}} \left[\sum_{t=0}^{\infty} \beta^t \ln C_t + \psi \sum_{t=0}^{\infty} \beta^t \sum q_t(z^t) n(z^t) \ln \frac{h(z^t)}{n(z^t)} \right]$$

subject to

$$\begin{aligned} C_t &+ P_{bt}\left[K_{bt+1} - (1-\kappa_b)K_{bt}\right] \\ &+ P_{st}\left[K_{st+1} - (1-\kappa_s)K_{st}\right] \\ &+ P_{ft}\sum q_t(z^t)\left[k_{ft+1}(z^t) - (1-\kappa_f)k_{ft}(z^{t-1})\right] \\ &\leq \left[\sum q_t(z^t) y(z^t)^{\eta}\right]^{\frac{1}{\eta}} \end{aligned} \tag{1}$$

$$y(z^t) \leq A_t^{(1-\alpha)\phi} z_t^{(1-\alpha)\phi} \left[\frac{\tilde{y}(z^t)}{\tilde{l}_b(z^t)}\right]^{\frac{\lambda-1}{\lambda}} l_b(z^t)^{1-\phi} k_b(z^t)^{\alpha\phi} n(z^t)^{(1-\alpha)\phi}, \ \forall z^t \tag{2}$$

$$h(z^t) \leq l_h(z^t)^{1-\omega} k_s(z^t)^{\omega}, \ \forall z^t \tag{3}$$

$$l_h(z^t) + l_b(z^t) \leq k_{ft}(z^{t-1})^{\zeta}, \ \forall z^t \tag{4}$$

$$\sum q_t(z^t) k_b(z^t) \leq K_{bt} \tag{5}$$

$$\sum q_t(z^t) k_s(z^t) \leq K_{st} \tag{6}$$

$$\sum q_t(z^t) n(z^t) \leq 1 \tag{7}$$

and $K_{b0}, K_{s0}, k_f(z_0), \{A_t, P_{bt}, P_{st}, P_{ft}, z^t\}_{t=0}^{\infty}, \tilde{y}(z^t)$ and $\tilde{l}_b(z^t)$ given. Competitive equilibrium allocations are obtained as a solution to this optimization problem such that $y(z^t) = \tilde{y}(z^t)$ and $l_b(z^t) = \tilde{l}_b(z^t)$. Prices corresponding to an equilibrium are easy to obtain from the constraint's Lagrange multipliers. We now show how to do this and how to relate the planners first order conditions to those of individual agents in the competitive equilibrium.

Let the Lagrange multipliers for the above constraints be $\beta^t\pi_t$, $\beta^t\pi_t q_t(z^t)p_y(z^t)$, $\beta^t\pi_t q_t(z^t)r_h(z^t)$, $\beta^t\pi_t q_t(z^t)r_l(z^t)$, $\beta^t\pi_t r_{bt}$, $\beta^t\pi_t r_{st}$ and $\beta^t\pi_t\theta_t$. Then the first order conditions for C_t, K_{bt+1}, K_{st+1}, $k_{ft+1}(z^t)$, $y(z^t)$, $l_b(z^t)$, $k_b(z^t)$, $n(z^t)$, $h(z^t)$, $l_h(z^t)$, and $k_s(z^t)$ are

$$
\begin{aligned}
\frac{1}{C_t} &= \pi_t && (8)\\
\pi_t P_{bt} &= \beta\pi_{t+1}\left[r_{bt+1} + P_{bt+1}(1-\kappa_b)\right] && (9)\\
\pi_t P_{st} &= \beta\pi_{t+1}\left[r_{st+1} + P_{st+1}(1-\kappa_s)\right] && (10)\\
\pi_t P_{ft} &= \beta\pi_{t+1}\sum_{z_{t+1}}\left[r_{ft+1} + P_{ft+1}(1-\kappa_f)\right]Q(z_t, z_{t+1}) && (11)\\
p_y(z^t) &= Y_t^{1-\eta}y(z^t)^{\eta-1} && (12)\\
r_l(z^t) &= p_y(z^t)\,(1-\phi)\,z_t l_b(z^t)^{-\phi}k_b(z^t)^{\alpha\phi}n(z^t)^{(1-\alpha)\phi} && (13)\\
r_b(z^t) &= p_y(z^t)\alpha\phi z_t l_b(z^t)^{1-\phi}k_b(z^t)^{\alpha\phi-1}n(z^t)^{(1-\alpha)\phi} && (14)\\
\pi_t\theta_t &= \psi\ln\frac{h(z^t)}{n(z^t)} - \psi + \pi_t w(z^t) && (15)\\
\frac{\psi}{h(z^t)} &= \pi_t r_h(z^t) && (16)\\
r_l(z^t) &= (1-\omega)\,r_h(z^t)l_h(z^t)^{-\omega}k_h(z^t)^{\omega} && (17)\\
r_s(z^t) &= \omega r_h(z^t)l_h(z^t)^{1-\omega}k_s(z^t)^{\omega-1} && (18)
\end{aligned}
$$

where

$$
\begin{aligned}
w(z^t) &= p_y(z^t)(1-\alpha)\phi z_t l_b(z^t)^{1-\phi}k_b(z^t)^{\alpha\phi}n(z^t)^{(1-\alpha)\phi-1}, && (19)\\
Y_t &= \left[\sum q(z^t)y(z^t)^{\eta}\right]^{\frac{1}{\eta}} && (20)\\
Q(z_t, z_{t+1}) &= \frac{q_{t+1}(z^t, z_{t+1})}{q_t(z^t)} && (21)\\
r_f(z^t) &= r_l(z^t)\zeta k_{ft}(z^{t-1})^{\zeta-1}. && (22)
\end{aligned}
$$

In the competitive equilibrium $\{P_{xt},\ x = b, s, f\}$, p_y, $\{r_x,\ x = b, s, f, l, h\}$, and w correspond to investment prices, intermediate good prices, rental rates and wages. Under this this interpretation we can relate the planner's first order conditions to those of the individual agents described in Section 2.2 and referred to in Sections 3.1 and 3.2. Equations (9)-(11), after substituting for π using (8), correspond to the representative household's first order conditions for capital accumulation. Equation (12) corresponds to the first order condition of the final good producer for intermediate input demand. Equations (13),(14) and (19) correspond to the intermediate good producers' first order conditions for finished land, business capital and labor. Equation (16), after substituting for π using (8), corresponds to the household's first order condition for housing in each location. Equations (17) and (18) correspond to the first order conditions of housing service providers for finished land use and rental of residential structures. Equation (22) corresponds to the first order condition of finished land service providers (landlords) for infrastructure use.

A.2 The Model With Growth

With growth, that is under the assumptions

$$\begin{aligned} P_{xt} &= \gamma_x^{-t},\ x=s,b,f; \\ A_t &= \gamma_a^t; \\ N_t &= \gamma_n^t, \end{aligned}$$

we need to obtain the mapping from the growing economy to a stationary planning problem. The mapping is driven by the balanced growth expressions derived in the main text.

The model's quantities are transformed as follows:

$$\begin{array}{ccc} g_c^t \bar{C}_t = \frac{C_t}{N_t} & g_x^t \bar{K}_{xt} = \frac{K_{xt}}{N_t}, x=b,s & g_c^t \frac{\bar{y}(z^t)}{\bar{n}(z^t)} = \frac{y(z^t)}{n(z^t)} \\ g_l^t \frac{\bar{l}_{bt}(z^t)}{\bar{n}(z^t)} = \frac{l_{bt}(z^t)}{n(z^t)} & g_x^t \frac{\bar{k}_{xt}(z^t)}{\bar{n}(z^t)} = \frac{k_{xt}(z^t)}{n(z^t)}, x=b,s & \gamma_x^{-t} \bar{P}_{xt} = P_{xt}, x=b,s,f \\ g_l^t \bar{l}_{ht}(z^t) = l_{ht}(z^t) & g_f^t \frac{\bar{k}_{ft}(z^{t-1})}{\bar{n}(z^t)} = \frac{k_{ft}(z^{t-1})}{n(z^t)} & \bar{n}(z^t) = \frac{n(z^t)}{N_t} \\ & g_h^t \bar{h}(z^t) = h(z^t) & \end{array}$$

All of the growth rates in these expressions are derived in the main text except for g_h. This growth rate equals $\left[\gamma_n^{\zeta-1} g_f^{\zeta}\right]^{1-\omega} g_s^{\omega}$.

The multipliers and prices are transformed as follows:

$$\begin{aligned} \bar{r}_{xt} &= \gamma_x^t r_{xt}, x=b,s,f \\ \bar{r}_{lt} &= g_{p_l}^{-t} r_{lt} \\ \bar{r}_{ht} &= g_{p_h}^{-t} r_{hlt} \\ \bar{\theta}_t &= g_c^{-t} \theta_t \\ \bar{\pi}_t &= g_c^t \gamma_n^t \pi_t \\ \bar{w}_t(z^t) &= g_c^{-t} w(z^t) \end{aligned}$$

Replacing the growing variables in the original planning problem using these transformations one can show the planning problem reduces to

$$\max_{\substack{\{\bar{C}_t, \bar{K}_{bt+1}, \bar{K}_{st+1}, \\ \bar{y}(z^t), \bar{l}_b(z^t), \bar{l}_h(z^t), \\ \bar{n}(z^t), \bar{k}_b(z^t), \bar{k}_s(z^t), \\ \bar{k}_f(z^t), \bar{h}(z^t)\}_{t=0}^{\infty}}} \left[\sum_{t=0}^{\infty} \beta^t \ln \bar{C}_t + \sum_{t=0}^{\infty} \beta^t \sum q_t(z^t) \bar{n}(z^t) \psi \ln \frac{\bar{h}(z^t)}{\bar{n}(z^t)} + \sum_{t=0}^{\infty} \beta^t \sum q_t(z^t) \bar{n}(z^t) \psi \ln g_h^t \right]$$

subject to

$$
\begin{aligned}
\bar{C}_t &+ \bar{P}_{bt}\left[\gamma_n g_b \bar{K}_{bt+1} - (1-\kappa_b)\bar{K}_{bt}\right] \\
&+ \bar{P}_{st}\left[\gamma_n g_s \bar{K}_{st+1} - (1-\kappa_b)\bar{K}_{st}\right] \\
&+ \bar{P}_{ft}\sum q_t(z^t)\left[\gamma_n g_f \bar{k}_{ft+1}(z^t) - (1-\kappa_f)\bar{k}_{ft}(z^t)\right] \\
&\leq \left[\sum q_t(z^t)\left(\bar{y}(z^t)\right)^{\eta}\right]^{\frac{1}{\eta}} \\
\bar{y}(z^t) &\leq z_t\left[\frac{\tilde{y}(z^t)}{\tilde{l}_b(z^t)}\right]^{\frac{\lambda-1}{\lambda}}\left[\bar{l}_b(z^t)\right]^{1-\phi}\left[\bar{k}_b(z^t)\right]^{\alpha\phi}\left[\bar{n}(z^t)\right]^{(1-\alpha)\phi},\ \forall z^t \\
\bar{h}(z^t) &\leq \left[\bar{l}_h(z^t)\right]^{1-\omega}\left[\bar{k}_s(z^t)\right]^{\omega},\ \forall z^t \\
\bar{l}_h(z^t)+\bar{l}_b(z^t) &\leq \left[\bar{k}_{ft}(z^t)\right]^{\zeta}\gamma_n^{1-\zeta},\ \forall z^t \\
\sum q_t(z^t)\bar{k}_b(z^t) &\leq \bar{K}_{bt} \\
\sum q_t(z^t)\bar{k}_s(z^t) &\leq \bar{K}_{st} \\
\sum q_t(z^t)\bar{n}(z^t) &\leq 1
\end{aligned}
$$

and $\bar{K}_{b0}, \bar{K}_{s0}, \bar{k}_f(z_0), \left\{\bar{P}_{bt}, \bar{P}_{st}, \bar{P}_{ft}, z^t\right\}_{t=0}^{\infty}, \tilde{y}(z^t)$ and $\tilde{l}_b(z^t)$ given. Competitive equilibrium allocations for the growing economy are obtained in two steps. First we find the solution to the transformed problem such that $y(z^t) = \tilde{y}(z^t)$ and $l_b(z^t) = \tilde{l}_b(z^t)$. The second step translates the stationary allocations and prices to their growing counterparts using the transformations described above.

Let the Lagrange multipliers for the above constraints be $\beta^t\bar{\pi}_t$, $\beta^t\bar{\pi}_t q_t(z^t)p_y(z^t)$, $\beta^t\pi_t q_t(z^t)\bar{r}_h(z^t)$, $\beta^t\pi_t q_t(z^t)\bar{r}_l(z^t)$, $\beta^t\pi_t\bar{r}_{bt}$, $\beta^t\pi_t\bar{r}_{st}$,and $\beta^t\pi_t\bar{\theta}_t$ Then the first order conditions for $\bar{C}_t$, $\bar{K}_{bt+1}$, $\bar{K}_{st+1}$, $\bar{k}_{ft+1}(z^t)$, $\bar{y}(z^t)$, $\bar{l}_b(z^t)$, $\bar{k}_{bt}(z^t)$, $\bar{n}(z^t)$,$\bar{h}(z^t)$,$\bar{l}_h(z^t)$ and $\bar{k}_s(z^t)$ are

$$\begin{aligned}
\frac{1}{\bar{C}_t} &= \bar{\pi}_t \\
\gamma_n g_b \bar{\pi}_t \bar{P}_{bt} &= \beta \bar{\pi}_{t+1} \left[\bar{r}_{bt+1} + \bar{P}_{bt+1}(1-\kappa_b) \right] \\
\gamma_n g_s \bar{\pi}_t \bar{P}_{st} &= \beta \bar{\pi}_{t+1} \left[\bar{r}_{st+1} + \bar{P}_{st+1}(1-\kappa_s) \right] \\
\gamma_n g_f \bar{\pi}_t \bar{P}_{ft} &= \beta \bar{\pi}_{t+1} \sum_{z_{t+1}} \left[r_{ft+1} + P_{ft+1}(1-\kappa_f) \right] Q(z_t, z_{t+1}) \\
p_y(z^t) &= \bar{Y}_t^{1-\eta} \bar{y}(z^t)^{\eta-1} \\
\bar{r}_l(z^t) &= p_y(z^t)\,(1-\phi)\, z_t^{(1-\alpha)\phi} \left[\frac{\tilde{y}(z^t)}{\tilde{l}_b(z^t)} \right]^{\frac{\lambda-1}{\lambda}} \bar{l}_b(z^t)^{-\phi} \bar{k}_b(z^t)^{\alpha\phi} \bar{n}(z^t)^{(1-\alpha)\phi} \\
\bar{r}_b(z^t) &= p_y(z^t) \alpha\phi z_t^{(1-\alpha)\phi} \left[\frac{\tilde{y}(z^t)}{\tilde{l}_b(z^t)} \right]^{\frac{\lambda-1}{\lambda}} \bar{l}_b(z^t)^{-\phi} \bar{k}_b(z^t)^{\alpha\phi-1} \bar{n}(z^t)^{(1-\alpha)\phi} \\
\bar{\pi}_t \bar{\theta}_t &= \psi \ln \frac{\bar{h}(z^t)}{\bar{n}(z^t)} - \psi + \psi \ln g_h^t + \bar{\pi}_t \bar{w}(z^t) \\
\bar{n}(z^t) \frac{\psi}{\bar{h}(z^t)} &= \pi_t \bar{r}_h(z^t) \\
\bar{r}_l(z^t) &= (1-\omega)\, \bar{r}_h(z^t) \bar{l}_h(z^t)^{-\omega} \bar{k}_h(z^t)^{\omega} \\
\bar{r}_s(z^t) &= \omega \bar{r}_h(z^t) \bar{l}_h(z^t)^{1-\omega} \bar{k}_s(z^t)^{\omega-1}
\end{aligned}$$

where

$$\bar{w}(z^t) = p_y(z^t)(1-\alpha)\phi z_t^{(1-\alpha)\phi} \left[\frac{\tilde{y}(z^t)}{\tilde{l}_b(z^t)} \right]^{\frac{\lambda-1}{\lambda}} \bar{l}_b(z^t)^{1-\phi} \bar{k}_b(z^t)^{\alpha\phi} \bar{n}(z^t)^{(1-\alpha)\phi-1}$$

and

$$\bar{Y}_t = \left[\sum q(z^t) \bar{y}(z^t)^{\eta} \right]^{\frac{1}{\eta}}$$

It is straightforward to established that the first order conditions for the untransformed economy correspond to these first order conditions once the stationary variables are replaced with their growing counterparts using the transformations given above.

A.3 Compensation for lost growth

Here we derive the formulas used to evaluate the level increase in consumption and housing required to compensate households for giving up the growth due to local agglomeration.

$$\begin{aligned}
g_h &= g_l^{1-\omega} g_s^{\omega} \\
&= \left[\gamma_n^{\zeta-1} g_f^{\zeta} \right]^{1-\omega} g_s^{\omega} \\
&= \left[\gamma_n^{\zeta-1} \left(\frac{g_c}{g_{p_f}} \right)^{\zeta} \right]^{1-\omega} \left(\frac{g_c}{g_{p_s}} \right)^{\omega}
\end{aligned}$$

Therefore the growth rate of per capita housing absent agglomeration is

$$g_h^* = \left[\hat{\gamma}_n^{\zeta-1}\left(\frac{g_c^*}{\hat{g}_{p_f}}\right)^{\zeta}\right]^{1-\omega}\left(\frac{g_c^*}{\hat{g}_{p_s}}\right)^{\omega}$$

Notice that the utility of the representative household is

$$\left[\sum_{t=0}^{\infty}\beta^t \ln \bar{C}_t + \sum_{t=0}^{\infty}\beta^t \ln g_c^t + \sum_{t=0}^{\infty}\beta^t \sum q_t(z^t)\bar{n}(z^t)\psi \ln \frac{\bar{h}(z^t)}{\bar{n}(z^t)} + \sum_{t=0}^{\infty}\beta^t \sum q_t(z^t)\bar{n}(z^t)\psi \ln g_h^t\right]$$
$$= \frac{\ln \bar{C}}{1-\beta} + \frac{\beta \ln g_c}{(1-\beta)^2} + \frac{\ln \frac{\bar{h}}{\bar{n}}}{1-\beta} + \psi\frac{\beta \ln g_h}{(1-\beta)^2}$$

We seek μ so that utility with and without agglomeration is equated. That is

$$(1+\psi)\frac{\ln \mu}{1-\beta} + \frac{\ln \bar{C}}{1-\beta} + \frac{\beta \ln g_c^*}{(1-\beta)^2} + \psi\frac{\ln \frac{\bar{h}}{\bar{n}}}{1-\beta} + \psi\frac{\beta \ln g_h^*}{(1-\beta)^2}$$
$$= \frac{\ln \bar{C}}{1-\beta} + \frac{\beta \ln \hat{g}_c}{(1-\beta)^2} + \psi\frac{\ln \frac{\bar{h}}{\bar{n}}}{1-\beta} + \psi\frac{\beta \ln \hat{g}_h}{(1-\beta)^2}$$

Solving for μ,

$$\mu = \left(\frac{\hat{g}_c}{g_c^*}\left(\frac{\hat{g}_h}{g_h^*}\right)^{\psi}\right)^{\frac{\beta}{(1-\beta)(1+\psi)}}.$$

B Data

This section provides a detailed description of how we construct empirical counterparts to model variables from various data sources and how we merge our different data sources.

B.1 MSA-Level Panel Data, 1978-2009

B.1.1 CPS Data (Wages and Hours Worked by Skill)

The March CPS data are available for download at *http://cps.ipums.org/cps/* as part of the Integrated Public Use Microdata Series (IPUMS-CPS) project at the University of Minnesota Population Center.

We download the March CPS data from 1979 through 2009. We chose 1979 as our starting year because the number of metropolitan areas we can identify in the CPS and then match to data on housing rents drops off rapidly prior to 1979. The CPS wage and employment questions refer to the "previous calendar year." Therefore, data for any given year's CPS is treated as data appropriate for the previous calendar year. For example, variables generated from the March 2005 CPS would be treated as data for the year 2004.

In each year of our data, we use the following criteria to restrict the sample (with IPUMS-CPS variables in italics)

- Respondent lives in a household, not in group quarters or vacant units ($gq = 1$)
- Is aged 20 to 65 ($age \geq 20$ and $age \leq 65$)
- Wage and salary income in the previous calendar year is identified and is nonzero ($incwage > 0$ and $incwage < 999998$)
- Weeks worked in the previous calendar year is identified and is between 1 and 52 ($wkswork1 \geq 1$ and $weekswork1 \leq 52$)
- Hours worked in a typical week in the previous year (if the respondent worked) is identified and is between 1 and 99 ($uhrswork \geq 1$ and $uhrswork \leq 99$)
- Educational attainment is recorded ($educ \geq 2$ and $educ \leq 115$)
- Has an identified metro area of residence (*metarea* non missing)[1]

For each MSA, we use the CPS data to create the following three variables:

1. Ratio of labor input of high skill to labor input of low skill, $m = n_e/n_u$
2. Ratio of total wages paid to total wages paid to low skill workers, s
3. Average weekly wage of high skill workers, w_e.

We use the *educ* categorical variable to label respondents as either "low" or "high" skill workers. High skill workers are assumed to have completed 1+ years of college ($educ \geq 80$ and $educ \leq 115$). Everyone else in the sample is assumed to be a low skill worker.

n_e is created as the total of weeks worked the previous calendar year ($wkswork1$) multiplied by the number of hours per week the respondent usually worked ($uhrswork$) for high skill workers. n_u is the same, but for low skill workers. For each respondent, we weigh the product of $wkswork1$ and $uhrswork$ using the IPUMS-CPS sampling person weights, $perwt$.

s is computed as

$$\frac{w_e n_e + w_u n_i}{w_u n_u} = \frac{\sum_{j \epsilon MSA_i} perwt_j \cdot wages_j}{\sum_{j \epsilon MSA_i} perwt_j \cdot wages_j \cdot 1\{unskilled_j\}}$$

for respondent j in MSA i, i.e. as the sum of all low- and high- skill workers' pre-tax wage and salary income for the previous calendar year ($incwage$) divided by the sum of all low skill

[1] According to notes from the IPUMS-CPS, the metro area of residence was not collected from respondents, but added by the Census Bureau. The metro areas of residence is based on FIPS codes used in the 1990 census.

workers' pre-tax wage and salary income for the previous calendar year. We weigh pre-tax wage and salary income for all persons using the IPUMS-CPS sampling person weights.

w_e is created as the sum of all high skill workers' pre-tax wage and salary income for the previous calendar year (created as an input into s) divided by n_e.

B.1.2 BEA Data (Output Prices)

We assume that the price of output varies across MSAs because that industry composition varies across MSAs, and the price index for industry output varies across industries.

Chain-type price indexes for industry output are available over the 1947-2009 period in the Annual Industry Accounts, *http://www.bea.gov/industry/index.htm#annual*. To construct a price index for output produced by MSA, we merge this information with MSA-level data on earnings by industry that is available in Tables CA05 and CA05N of the Regional Economic Accounts, *http://www.bea.gov/regional/reis/*. Earnings is inclusive of wage and salary disbursements, supplements to wages and salaries, and proprietors' income.

In the remainder of this section, section B.1.2, the notation will differ from that used in the paper.

Denote $g_{t,j}$ as the growth rate of the price of industry output j from periods t to $t+1$ and g_t^i as the growth rate of the price of all output produced in MSA i between years t and $t+1$. Assuming output from $j = 1, \ldots, N$ industries is produced in MSA i in year t, we set the growth rate of the price of output produced in MSA i between years t and $t+1$ as

$$g_t^i = \sum_{j=1}^{N} \omega_{t,j}^i \; g_{t,j} \; . \tag{23}$$

The weight on each industry, $\omega_{t,j}^i$, is the share of our estimate of total value of the MSA i attributable to value add of industry j in year t:

$$\omega_{t,j}^i = \frac{\mu_j \; \epsilon_{t,j}^i}{\sum_{k=1}^{N} \mu_j \; \epsilon_{t,k}^i} \; , \tag{24}$$

where $\epsilon_{t,j}^i$ stands for total earnings of employees in industry j in MSA i during year t and μ_j is a time- and MSA- invariant "markup" that scales earnings of industry j to value add from industry j (described next).[2] For each MSA, we construct a price index for output, normalized to 1.0 in the year 1969, that is consistent with the sequence of time-series estimates of g_t^i.

Before describing how we compute μ_j, we note two details about the earnings and industry data. First, on a somewhat infrequent basis, Tables CA05 and CA05N do not report estimates of earnings for a given industry in an MSA in a given year. In these cases, we

[2] The markup is allowed to change in 1997, which industry classifications change from SIC-based to NAICS-based.

set earnings for this industry-MSA-year cell to zero.[3] Also, some of the industry-MSA-year employment estimates are marked with code E. According to the BEA web site, these estimates "constitute the major portion of the true estimate." In these cases, we assume that the reported estimate is equal to the actual estimate.

Second, the definition of industries in the Regional Accounts is not consistent across years. Table CA05 reports employment based on SIC-industry classifications over the 1969-2000 period and CA05N reports employment based on NAICS industry classifications spanning the years 2001-2006.

We map SIC and NAICS industry employment from Tables CA05 and CA05N to prices from the Annual Industry Accounts according to the tables shown later in this section. These tables list all the categories of nonfarm private employment. The sum of the earnings estimates in each of these categories is considered as total nonfarm earnings, and is used to compute the denominator of equation (24).

In all cases except one, there is an exact correspondence of earnings estimates from Tables CA05 and CA05N to prices from the Annual Industry Accounts. For the SIC category of "Transportation and public utilities," line 500 of Table CA05, there is no clean analogous price index in the Annual Industry Accounts. Instead, the Annual Industry Accounts includes separate price indexes for "Transportation and warehousing" and "Utilities." In Table CA05, we therefore separate earnings of the single Transportation and public utilities into earnings in two categories: Earnings from utilities ("electric, gas, and sanitary services", line 570) and earnings from transportation and public utilities less earnings from utilities (i.e. line 500 less line 570).

Finally, we need to compute a markup that maps earnings to value add. For each industry, we compute the markup μ_j as the product of two estimated values. The first is the fraction of earnings, by industry, not attributable to proprietor's income. We compute this in order to remove an estimate of proprietors' income from reported earnings by industry by MSA. For each of the SIC industry classifications covering the 47-97 period, we compute this fraction using data on the components of value-add by industry, available in the file "GDPbyInd_VA_SIC" which is available at *http://www.bea.gov/industry/io_histannual.htm*. Similar data are not available for NAICS, so we map our SIC-based estimates to NAICS industries for the 97-09 period. Taking the construction industry as an example, in 1947 reported compensation of employees in this industry (in millions) is \$6266 and reported proprietors' income is \$2123. We compute the fraction of earnings not attributable to proprietor's income in this year as $0.747 = 6266/(6266+2123)$. We repeat this process each year over the 47-97 period, and compute the average over all years for the construction sector as 0.788. Thus, for the construction sector, in each MSA in each year we scale reported earnings of construction sector employees by 0.788 to remove an estimate of proprietor's income.

In the second step, we scale the estimate of compensation of employees less proprietors' income to value added. For SIC industries over the 47-97 period, we use data from the "GDP-

[3] The three reasons that are listed for omission are (a) avoid disclosure of confidential information (code D), (b) earnings are less than \$50,000 (code L), or (c) data not available for this year (code N). These omissions occur in approximately six percent of industry-MSA-year cells from 1969 to the mid-1990s and about thirteen percent of cells after the mid-1990s.

byInd_VA_SIC" file and for NAICS industries over the 98-09 period we use similar data from the "GDPbyInd_VA_NAICS" file available at *http://www.bea.gov/industry/gdpbyind_data.htm* to make this computation. Again, using the example of the construction industry to illustrate how this process works, according to the GDPbyInd_VA_SIC file, in 1947 the reported value-added of the industry (in millions) is $9057 and compensation of employees is $6266, and thus the ratio of value-add to compensation of employees in that year is 1.445 = 9057/6266. Averaged over all years in the 47-97 period, the ratio of value-add to compensation of employees in the construction industry is 1.432. We use a similar procedure to compute the mapping of compensation of employees to value add using a similar procedure for the NAICS industries over the 98-09 period.

Summarizing our procedure for the construction sector: We set value added from the construction sector in MSA i in any year t over 47-97 equal to total earnings of employees (from table CA05) in that year in that MSA multiplied by μ_j for construction, which we compute as 1.128 = 0.788*1.432. We repeat this for every SIC industry (47-97) and every NAICS industry (98-09) for every MSA in every year. In the tables below, we list our estimates of the two components of μ_j in the right-most columns.

Data for Earnings, $w^i_{t,j}$ Regional Accounts Table CA05, 1969-2000		Data for Growth in Prices, $g^p_{t,j}$ Industry Accounts, 1969-2001		$\mu_j = a * b$	
				a	b
Line	Label	Line	Label		
100	Agricultural services, forestry fishing and other	3	Agriculture, forestry, fishing and hunting	0.300	4.858
200	Mining	6	Mining	0.895	3.092
300	Construction	11	Construction	0.788	1.432
400	Manufacturing	12	Manufacturing	0.979	1.454
500[c]	Transportation and public utilities less electric, gas, and sanitary services	36	Transportation and warehousing	0.932	1.981
570	Electric, gas, and sanitary services	10	Utilities	0.925	3.197
610	Wholesale trade	34	Wholesale trade	0.899	1.873
620	Retail trade	35	Retail trade	0.806	1.721
700	Finance, insurance and real estate	50	Finance, insurance, real estate, rental and leasing	0.856	4.784
800	Services	59	Professional and business services	0.742	1.557
900	Government and government enterprises	82	Government	1.000	1.236

a. Adjustment to remove proprietor's income from earnings. b. Mapping of wage compensation to value added. c. See text for details.

Data for Earnings, $w^i_{t,j}$ Regional Accounts Table CA05N, 2001-2005		Data for Growth in Prices, $g^p_{t,j}$ Industry Accounts, 2001-2006		$\mu_j = a * b$	
				a	b
Line	Label	Line	Label		
100	Forestry, fishing, related activities and other	5	Forestry, fishing and related activities	0.300	3.441
200	Mining	6	Mining	0.895	3.397
300	Utilities	10	Utilities	0.925	3.737
400	Construction	11	Construction	0.788	1.520
500	Manufacturing	12	Manufacturing	0.979	1.659
600	Wholesale trade	34	Wholesale trade	0.899	1.889
700	Retail trade	35	Retail trade	0.806	1.729
800	Transportation and warehousing	36	Transportation and warehousing	0.932	1.541
900	Information	45	Information	0.742	2.203
1000	Finance and insurance	51	Finance and insurance	0.856	1.888
1100	Real estate and rental and leasing	56	Real estate and rental and leasing	0.856	15.856
1200	Professional, scientific and technical services	60	Professional, scientific and technical services	0.742	1.534
1300	Management of companies and enterprises	64	Management of companies and enterprises	0.742	1.174
1400	Administrative and waste services	65	Administrative and waste management services	0.742	1.329
1500	Educational services	69	Educational services	0.742	1.132
1600	Health care and social assistance	70	Health care and social assistance	0.742	1.218
1700	Arts, entertainment and recreation	75	Arts, entertainment and recreation	0.742	1.705
1800	Accommodation and food services	78	Accommodation and food services	0.742	1.625
1900	Other services except public administration	81	Other services except government	0.742	1.546
2000	Government and government enterprises	82	Government	1.000	1.236

a. Adjustment to remove proprietor's income from earnings. b. Mapping of wage compensation to value added.

B.1.3 BLS Data and 1990 Decennial Census of Housing (Housing Rents)

We create annual estimates over the 1978-2009 period of the average rents paid for certain types of rental units, by MSA, using a two-step procedure.

In the first step, we estimate the average rents paid for certain types of rental housing units in 1990 using household-level data from the 1990 Decennial Census of Housing (DCH). These data are available for download at *http://usa.ipums.org/usa/* as part of the Integrated Public Use Microdata Series (IPUMS-USA) project at the University of Minnesota Population Center. We use data from the 1990 DCH reports data by metropolitan area for more metropolitan areas than the 2000 DCH.

With IPUMS-USA variables in italics, we restrict the 1990 DCH sample to renter non-farm households in 2-19 unit residences in a building built between 1940 and 1986 and living in an identifiable MSA ($ownershg = 2$, $farm \neq 1$, $unitsstr \in \{5, 8\}$, $builtyr \in \{3, 7\}$, and $metarea > 0$) who live in households and do not live in group quarters ($gq \in \{3, 4, 6\}$) and where the reported monthly gross rent of the house (rent inclusive of utilities) is nonzero

($rentgrs > 0$). Conditional on these restrictions, we compute the weighted average value of units by MSA using the sampling weight variable $hhwt$. These calculations yield estimates of the average rental price of housing for 272 metro areas as identified in the 1990 DCH. We exclude single-family rented units, rented high-rise units (> 20 units), and units in very old (built before 1940) or very new (built after 1986) apartment buildings to attempt to keep the average characteristics of rental units roughly constant across metropolitan areas without resorting to hedonic regressions.

In the second step, we extrapolate the annual rental price of housing in each metro area forward from 1990 to 2009 and backwards from 1990 to 1978 using annual MSA-specific constant-quality price indexes for the price per unit of shelter. These price indexes for shelter are published by the Bureau of Labor Statistics (BLS) as part of computations for the Consumer Price Index, and are available at *http://www.bls.gov*. The BLS reports rental price indexes for 27 MSAs, but the indexes of three of these MSAs (Phoenix, AZ, Washington, DC, and Tampa Bay, FL) do not have data available prior to 1985 and we exclude these from our sample. The CPS does not have data on Anchorage and Honolulu back to 1978, explaining our sample of 22 MSAs.

In 1983, the BEA changed its procedure for measuring the price of owner-occupied rent, which accounts for about 73 percent of all spending on shelter. After 1983 the BEA began measuring the price of owner-occupied rent using the "rental equivalence" approach, whereas in earlier years the BLS used the "asset price method."[4] To eliminate this nontrivial inconsistency in the data, we replace the reported values of the shelter indexes from 1978-1982 with predicted values, essentially predicting what the BLS would have reported if the owner-occupied data had been collected using the "rental equivalence approach." Specifically, we regress the log BLS shelter indexes on MSA dummies and the log BLS tenant-rent indexes over the 1983-2009 period. The R^2 of the regression is 0.99 and the coefficient on log tenant rents is 1.055. Based on the regression results, and the values of the log tenant-rent indexes in the 1978-1982 period, we predict the log MSA shelter indexes from 1978-1982.

B.1.4 Merging the MSA-Level Data, 1978-2009

We merge the CPS data on wages and employment (section B.1.1) with the BEA data on output prices (B.1.2) and the annual data we construct on housing rents (B.1.3). The data are merged by MSA and by year. After all data are merged, we are left with a balanced panel of 22 MSAs. Note that the MSA definitions may not be completely consistent across data sources. In the BEA data, MSAs definitions are given by the list in the December, 2009 report of the Office of Management and Budget (OMB).[5] The MSA definitions in the CPS data are consistent with the definitions as of the 1990 Census. The MSA definitions in the BLS rental data change over time – in each year, they are consistent with the OMB definition of that year, but as OMB definitions change, the definitions of the MSA change. Since the MSAs in our sample constitute most of the largest concentrations of populations in

[4]See *http://www.bls.gov/cpi/cpifact6.htm* for details.

[5]For a complete list of the counties comprising each MSA, go to *http://www.census.gov/population/www/metroareas/metrodef.html*.

the U.S. changes or inconsistencies in in MSA definitions over our sample period are likely to be inconsequential to our results.

In every MSA and date in our sample, the minimum number of respondents from the CPS is never less than 200; it is typically about 250 until 1999 and then jumps to about 450 after 2000. The median number of respondents is about 540 until about 2000, at which point the median jumps to about 1,000. The maximum number of respondents is always above 3,000 and is typically about 4,000.

B.2 Aggregate Data

B.2.1 Data used for Depreciation Rate of Residential Structures

One of our moment conditions involving the depreciation rate on residential structures, κ_S, is

$$E\left[\kappa_s - \frac{P_{st}D_{st}}{P_{st}K_{st}}\right] \;=\; 0 \;,$$

where $P_{st}D_{st}$ is nominal value of aggregate depreciation on structures in year t and $P_{st}K_{st}$ is the nominal value of the aggregate stock of structures in year t. Our data on $P_{st}D_{st}$ are from line 7 (Residential Fixed Assets) of the BEA Fixed Assets Table 1.3, Current Cost Depreciation of Fixed Assets and Consumer Durable Goods.[6] Our data on $P_{st}K_{st}$ are from line 7 of the BEA Fixed Assets Table 1.1, Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods. The capital stocks reported Fixed Assets Table 1.1 are year-end values. To adjust for this, we set K_{st} as the once-lagged reported year-end value, that is we set K_{st} for the year 2000 as the year-end reported value for 1999.

B.2.2 Data used for Depreciation Rate of Infrastructure Capital

One of our moment conditions involving the depreciation rate on residential structures, κ_F, is

$$E\left[\kappa_f - \frac{P_{ft}D_{ft}}{P_{ft}K_{ft}}\right] \;=\; 0 \;,$$

where $P_{ft}D_{ft}$ is nominal value of aggregate depreciation on infrastructure capital in year t and $P_{ft}K_{ft}$ is the nominal value of the aggregate stock of infrastructure in year t.

- We compute the nominal value of infrastructure capital, $P_{ft}K_{ft}$, as the sum of the nominal stocks of (a) federal non-defense and state and local government highways and streets, (b) federal non-defense and state and local "other structures" (pre-1996) and Transportation and Power structures (post-1997) (c) state and local sewer systems structures, (d) state and local water supply facilities, and (d) privately owned power

[6] These tables are available at *http://www.bea.gov/national/FA2004/SelectTable.asp*.

and communication, transportation, and "other" structures. The data for the nominal stocks of federal and state and local infrastructure capital are in the BEA Fixed Asset Tables 7.1a (lines 38, 40, 49, 51, 52, and 53 covering the period 1925-1996) and 7.1b (lines 41, 42, 43, 56, 57, 58, 59, and 60 covering 1997-2009). The data for the nominal stocks of private infrastructure capital are in the BEA Fixed Asset Table 2.1, lines 50, 63, and 67.[7]

- We compute the nominal value of depreciation of infrastructure capital, $P_{ft}D_{ft}$, analogously. The data for the nominal depreciation of infrastructure capital for the federal and state and local government are in BEA Fixed Asset Tables 7.3a (lines 38, 40, 49, 51, 52, and 53) and 7.3b (lines 41, 42, 43, 56, 57, 58, 59, and 60). Depreciation for privately owned infrastructure capital is reported in BEA Fixed Asset Table 2.4, lines 50, 63, and 67.

As mentioned earlier, the BEA reports the capital stocks data at year-end. To adjust for this we define K_{ft} as the lag of reported year-end values.

B.2.3 Data used for Depreciation Rate of Business Capital

One of our moment conditions involving the depreciation rate on capital used in production, κ_B, is

$$E\left[\kappa_b - \frac{P_{bt}D_{bt}}{P_{bt}K_{bt}}\right] \quad = \quad 0 \ ,$$

where $P_{bt}D_{bt}$ is the nominal value of aggregate depreciation on capital used in production in year t and $P_{bt}K_{bt}$ is the nominal aggregate stock of capita used in production in year t.

- We compute $P_{bt}D_{bt}$ as nominal depreciation of all fixed assets and consumer durable goods (line 1) less nominal depreciation of private residential structures (line 7) of the BEA Fixed Assets Table 1.3, all less nominal depreciation of infrastructure capital, defined in section B.2.2.
- We compute $P_{bt}K_{bt}$ as the nominal stock of all fixed assets and consumer durable goods (line 1) less the nominal stock of private residential structures (line 7) of the BEA Fixed Assets Table 1.1, all less nominal depreciation of infrastructure capital as defined in section B.2.2.

As mentioned earlier, the BEA reports the capital stocks data at year-end, and we adjust for this by setting $P_{bt}K_{bt}$ equal to the lag of reported year-end values.

[7] The BEA notes that "other" government structures consist "primarily of electric and gas facilities, transit systems, and airfields" whereas "other" private structures include structures pertaining to "water supply, sewage and waste disposal, public safety, highway and street, and conservation and development."

B.2.4 Data used for the Growth Rate of the Price of Housing Structures, Infrastructure Capital, and Business Capital

Our moment conditions involving the trend growth rate of the aggregate real price of housing structures g_{PS} and the trend growth rate of the aggregate real price of business capital g_{p_b} are

$$\begin{aligned} E\left\{\left(\ln P_{st} - \ln\left(g_{p_s}\right)t\right)t\right\} &= 0 \\ E\left\{\left(\ln P_{ft} - \ln\left(g_{p_f}\right)t\right)t\right\} &= 0 \\ E\left\{\left(\ln P_{bt} - \ln\left(g_{p_b}\right)t\right)t\right\} &= 0 \end{aligned}$$

- P_{st} is the real price for housing structures, defined as the nominal price index for structures divided by the price index of consumption. The nominal price index for structures is computed as the nominal stock of housing structures, line 7 of BEA Fixed Asset Table 1.1, divided by the chain-type quantity index for residential structures, line 7 of BEA Fixed Asset Table 1.2.

- P_{ft} is the real price for infrastructure capital, defined as the nominal price index for infrastructure capital divided by the price index of consumption. The nominal price index for infrastructure capital is computed by chain-weighting the price indexes of each of the components of infrastructure capital described in section B.2.2: federal non-defense and state and local government highways and streets and other (pre-1996) or transportation and power (post-1997), state and local sewer systems structures, state and local water supply facilities, state and local transportation structures and power structures, and privately owned power and communication structures, transportation structures, and other structures. The price indexes for each of the components is computed as the ratio of the nominal stock (Fixed Asset Tables 7.1a, 7.1b, and 2.1) to the chain-type quantity indexes (Fixed Asset Tables 7.2a, 7.2b, and 2.2).

- P_{bt} is the real price for business capital, defined as the nominal price index for business capital divided by the price index of consumption. We compute the nominal price index for business capital by chain-weighting the price index for (a) all fixed assets and consumer durable goods less (b) the price index for housing structures less (c) the price index for infrastructure capital.

  - The nominal price index for all fixed assets and consumer durable goods is computed by dividing the nominal stock of all fixed assets and consumer durable goods, line 1 of BEA Fixed Asset Table 1.1, by the chain-type quantity index for all fixed assets and consumer durable goods, line 1 of BEA Fixed Asset Table 1.2.
  - The price indexes for housing structures and infrastructure capital are defined above.

- We discuss how we create the price index for consumption in section B.2.9.

B.2.5 Data used for Growth Rate of Housing, Structures, and Land Prices

Our moment condition involving the average of rental prices across MSAs, the aggregate growth rate of the real price of land rents g_{p_l} and the real price of housing structures g_{p_s}, and the share of housing rents attributable to housing structures ω is

$$E\{(\ln E_t r_{hit} - [(1-\omega)\ln(g_{p_l}) + \omega \ln(g_{p_s})]\, t)\, t\} \;=\; 0$$

We compute $E_t r_{hit}$ in each period as the average level of real rental prices in each MSA. In each MSA, the real rental price is computed as the nominal rental price divided by the price index for consumption. We discuss how we create the price index for consumption in section B.2.9.

B.2.6 Data used for Structures' Share of Housing Rents

One of our moment conditions for structures' share of housing rents, ω, the growth rate of the price of land rents, g_{p_l}, the growth rate of the price of housing structures, g_{p_s}, and the depreciation rate on housing structures, κ_s, is:

$$E\left(\frac{\sum p_{lit} l_{hit}}{\sum (P_{st} k_{sit} + p_{lit} l_{hit})}\left[\frac{\omega}{1-\omega}\frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_s} + \kappa_s - 1} + 1\right] - 1\right) \;=\; 0,$$

We set $\sum p_{lit} l_{hit}$ as the market value aggregate value of finished land in residential use, taken from a study by Davis and Heathcote (2007), and available at *http://www.lincolninst.edu/subcenters/land-values/price-and-quantity.asp*. We compute the annual data as the average of the reported quarterly data. We set $\sum (P_{st} k_{sit} + p_{lit} l_{hit})$ as the market value of housing (land and structures), taken from the same Davis and Heathcote (2007) study. Again, we set annual values as the average of the reported quarterly values.

B.2.7 Parameters of the Production Function Related to Capital and Land Shares of Production

Two of our moment conditions related to capital's and finished land's share of production, α and ϕ are

$$E\left(\frac{\sum p_{lit} l_{bit}}{\sum (P_{bt} k_{bit} + p_{lit} l_{bit})}\left[\frac{\alpha\phi}{1-\phi}\frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_b} + \kappa_b - 1} + 1\right] - 1\right) \;=\; 0 \qquad (25)$$

$$E\left(\frac{\sum w_{it} n_{it}}{\sum [w_{it} n_{it} + r_{Lit} l_{bit} + r_{bt} k_{bit}]} - \phi(1-\alpha)\right) \;=\; 0 \qquad (26)$$

Data for equation (25)

We set $\sum p_{lit} l_{bit}$ equal to the aggregate value of finished land used in production, computed as the sum of

1. The value of land used for nonresidential purposes by Nonfarm Nonfinancial Corporate Businesses. These data come from Table B.102 of the Flow of Funds Accounts of the United States (FFA). We set the value of land equal to the value of real estate owned by this sector (line 3) less the replacement cost of structures owned by this sector (lines 33 and 34). We set the annual as the average of the reported quarterly observations.

2. The value of land used for nonresidential purposes by Nonfarm Nonfinancial Noncorporate Businesses. These data come from Table B.103 of the FFA. We set the value of land equal to the value of nonresidential real estate owned by this sector (line 5) less the replacement cost of nonresidential structures owned by this sector (line 33). We set the annual as the average of the reported quarterly observations.

3. The current cost of privately owned infrastructure capital (power and communication structures, transportation structures, and other structures) as described in section B.2.2.

4. The value of land used for nonresidential purposes by financial corporations. We compute this as $\mathcal{R}$ (to be defined later) times the Current-Cost Net Stock of Private Nonresidential Fixed Assets Owned by Financial Corporations, line 25 of BEA Fixed Asset Table 4.1. We set the annual as the average of the current and lagged year-end observations.

5. The value of land used for nonresidential purposes by Nonprofit Organizations. We compute this as $\mathcal{R}$ times the sum of equipment and software owned by nonprofit organizations (line 6 of B.100 of the FFA) and the replacement cost of nonresidential structures owned by nonprofit organizations (line 46 of B.100 of the FFA). We set the annual as the average of the reported quarterly observations.

6. The value of land used for nonresidential purposes by the Government. We compute this as $\mathcal{R}$ times the following: The Current-Cost Net Stock of Government Fixed Assets, line 8 of BEA Fixed Asset Table 1.1, less the current cost of infrastructure capital owned by the federal and state and local government, as defined in section B.2.2. We set the annual as the average of the current and lagged year-end observations.

We set $\sum (P_{bt}k_{bit} + p_{lit}l_{bit})$ equal to the aggregate value of all capital and finished land used in production, computed as the sum of

a. The total market value of tangible assets owned by Nonfarm Nonfinancial Corporate Businesses less the replacement cost of residential structures owned by Nonfarm Nonfinancial Businesses, line 2 less line 33 of Table B. 102 of the FFA.[8] We set the annual as the average of the reported quarterly observations.

b. The total market value of tangible assets less the market value of residential real estate owned by Nonfarm Nonfinancial Noncorporate Businesses, line 2 less line 4 of Table B. 103 of the FFA. We set the annual as the average of the reported quarterly observations.

[8] Residential structures are typically a very small fraction of total tangible assets: In 2009, they accounted for 1.4 percent of value.

c. The value of land used for nonresidential purposes by financial corporations computed in step 4 above; plus the Current-Cost Net Stock of Private Nonresidential Fixed Assets owned by Financial Corporations, line 25 of BEA Fixed Asset Table 4.1. We set the annual as the average of the current and lagged year-end observations.

d. The value of land used for nonresidential purposes by Nonprofit Organizations computed in step 5 above; plus equipment and software owned by nonprofit organizations (line 6 of B.100 of the FFA); plus the replacement cost of nonresidential structures owned by nonprofit organizations (line 46 of B.100 of the FFA). We set the annual as the average of the reported quarterly observations.

e. The value of land used for nonresidential purposes by the Government computed in step 6 above; plus the Current-Cost Net Stock of Government Fixed Assets, line 8 of BEA Fixed Asset Table 1.1; less the current cost of infrastructure capital owned by the federal and state and local governments, as defined in section B.2.2. We set the annual as the average of the current and lagged year-end observations.

f. The Current-Cost Net Stock of Consumer Durable Goods, line 13 of BEA Fixed Asset Table 1.1. We set the annual as the average of the current and lagged year-end observations.

We define $\mathcal{R}$ as the value of all land used for nonresidential purposes by businesses (the sum of items 1-3 above) divided by the value of all tangible assets less land used for nonresidential purposes by businesses (the sum of a and b above less the sum of items 1-3 above).

Also note that (as mentioned previously), when we use data from the BEA Fixed Asset Tables, we compute current-year values as the average of the reported current- and previous-year values. We do this because the BEA reports values at year-end; this adjustment aligns the timing of the BEA data with that of the FFA data.

Data for equation (26)

We compute

$$\frac{\sum w_{it} n_{it}}{\sum \left[w_{it} n_{it} + r_{lit} l_{bit} + r_{bt} k_{bit} \right]}$$

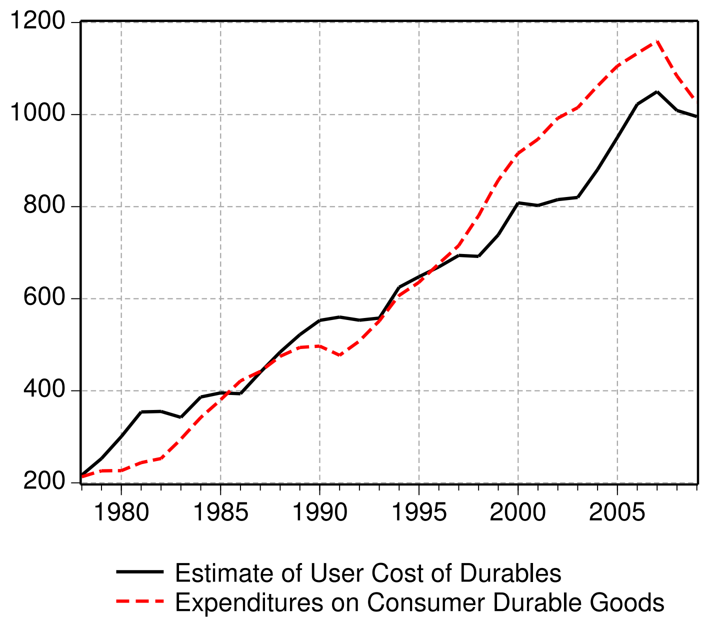
as follows. We set the numerator equal to "unambiguous labor income." We set the denominator equal to total gross domestic income plus an estimate of the nominal service flow from the stock of durable goods less the reported consumption of housing services less an estimate of "ambiguous income" (i.e. income that is not either unambiguous capital or unambiguous labor income).

- We set unambiguous labor income equal to line 2 of Table 1.10 of the National Income and Product Accounts (NIPA), "Compensation of employees, paid." This table is available at *http://www.bea.gov/national/nipaweb/SelectTable.asp?Selected=N*.

- We set gross domestic income equal to line 1 of NIPA Table 1.10, "Gross domestic income."

- We estimate the nominal service flow from the stock of durable goods as the sum of nominal depreciation on the stock of durable goods, line 13 of BEA Fixed Asset Table 1.3, plus the rate of interest on a 5-year Treasury bond times the nominal stock of durable goods. We take the nominal rate of interest on a 5-year Treasury bond from the web site of the Federal Reserve Board, *http://www.federalreserve.gov/releases/h15/data.htm*. We set the nominal stock of durable goods as the average of the current- and previous- year reported (year-end) values of the stock, as reported in line 13 of BEA Fixed Asset Table 1.1.

  The graph below compares our estimate of the user cost of durables to expenditures on consumer durable goods as reported in the NIPA over our sample period, 1978-2009. Both data series are in billions of dollars. Broadly speaking, the levels and growth rates of the two series are similar.



- We set consumption of housing services equal to line 50 of NIPA Table 2.4.5., "Household consumption expenditures (for services): Housing."

- As in Cooley and Prescott (1995), we use the following data from NIPA Table 1.10 to determine ambiguous income

  Line 9, Taxes on production and imports

  − line 10, Subsidies

  + line 15, Proprietors' income with inventory valuation and CCA

  + line 22, Current surplus of government enterprises

  + line 26, Statistical discrepancy

B.2.8 Data used for Infrastructure Share of Finished Land

One of our moment conditions for infrastructure's share of finished land rents, ζ, the growth rates of the price of finished land and infrastructure capital, g_{p_l} and g_{p_f}, and the depreciation rate on infrastructure capital κ_f, is

$$E\left(\frac{\sum P_{ft}k_{fit}}{\sum\left(p_{lit}l_{bit}+p_{lit}l_{hit}\right)}-\frac{R/g_{p_l}-(1-\kappa_f)^{\zeta}}{R/g_{p_f}-(1-\kappa_f)}\zeta\right) \;=\; 0$$

We set $\sum p_{ft}k_{fit}$ equal to the aggregate value of infrastructure capital, measured as defined in section B.2.2. We set $\sum\left(p_{lit}l_{bit}+p_{lit}l_{hit}\right)$ as the sum of the aggregate value of finished land used for business purposes, $\sum p_{lit}l_{bit}$, measured as defined in B.2.7 - data for equation (26), and the aggregate value of finished land used in housing, $\sum p_{lit}l_{hit}$, measured as defined in B.2.6.

B.2.9 Data for the growth of aggregate per-capita real consumption

Our moment condition for growth in aggregate real per-capita consumption, g_c, is

$$E\{(\ln C_t-\ln(g_c)\,t)\,t\} \;=\; 0\;.$$

We compute C_t as nominal aggregate consumption, divided by the appropriate price index, and divided again by the population.

We define the population as the civilian non-institutional population ages 16 and older. These data are available from the Bureau of Labor Statistics. We replace the reported population with a predicted value based on a regression of the BLS data on a 4th order polynomial in year (R^2 of 0.998). This smoothes a few odd peaks in the reported BLS series.

We define nominal aggregate consumption as

- Total consumption as reported by the NIPA, line 1 of NIPA Table 2.4.5,
- Less expenditures on durable consumption goods, line 3 of NIPA Table 2.4.5,
- Less the consumption of housing services, line 50 of NIPA Table 2.4.5,
- Plus government consumption expenditures, line 3 of NIPA Table 3.9.5,
- Plus an estimate of the nominal service flow from the stock of durable goods. This estimate is described in detail in the previous section.

We compute the price index for this definition of consumption by chain-weighting the appropriate price indexes. For total consumption, expenditures on durable goods, and the consumption of housing services, the price indexes are available in NIPA Table 2.4.4. The price index for government consumption expenditures is available in NIPA Table 3.9.4. Finally, we set the price index for the service flow from durable goods equal to an estimate of the price index for the stock of durable goods. We estimate this as the average of the current and previous year values of the price index, which is computed as the reported nominal year-end stock of consumer durable goods (line 13 of BEA Fixed Asset Table 1.1) divided by the reported quantity index for this stock (line 13 of BEA Fixed Asset Table 1.2).

B.3 Other Data

In addition to the lagged endogenous panel variables in the system, we use two other MSA-level variables as instruments in our GMM analysis: Per-capita personal income, as measured by the BEA in Table CA1-3 of its Local Area Personal Income and Employment Tables, and repeat-sales price indexes for existing homes as produced by the Federal Housing Finance Agency. In our GMM analysis, we log and demean both variables. For the purposes of comparing simulated model output on employment and average wages across all 366 MSAs in the United States, we use data from the Local Area Personal Income and Employment Tables of the BEA. By MSA, wage and salary employment is reported on line 7020 of Table AMSA04 (Personal income and its components) and we compute average wage as the sum of "Wage and salary disbursements" (line 50) and "Supplements to wages and salaries" (line 60) divided by wage and salary employment. For the 22 MSAs in our sample, we compare the BEA-based average wage and total employment measures to estimates from the CPS, generated as total hours worked for all respondents (for employment) and average wage per hour for all hours worked by all workers. After removing year effects and taking logs, the correlation of the average wage estimates is 0.76 and for the employment estimates is 0.99.

C Derivation of Aggregate Moment Conditions

In the main text we assumed variables were de-meaned when stating moment conditions used to calculate trend rates of growth. In practice we incorporate the estimates of the means in our calculations. The moment conditions stated here incorporate this estimation of the means.

We use the following moment conditions to identify $\kappa_b, \kappa_s, \kappa_f$:

$$\begin{aligned} E\left\{\kappa_b - \frac{D_{bt}}{K_{bt}}\right\} &= 0 \\ E\left\{\kappa_s - \frac{D_{st}}{K_{st}}\right\} &= 0 \\ E\left\{\kappa_f - \frac{D_{bt}}{K_{ft}}\right\} &= 0 \end{aligned}$$

where D_{Xt} is nominal depreciation of capital of type $X = B, S, F$.

Along the balanced growth path (without aggregate uncertainty, but *with* idiosyncratic uncertainty) the household's Euler equation for *finished land* holds for each city:

$$p_{lit} = E_{t|i}\left\{\frac{1}{R}\left[r_{lit+1} + (1-\kappa_f)^{\zeta} p_{lit+1}\right]\right\} \tag{27}$$

where $E_{t|i}$ denotes expectation at t conditional on i, r_{li} is the rental price of finished land in city i and p_{li} is the capital price of finished land in city i. This equation is derived as follows. Finished land k_{lit} is a Cobb-Douglas aggregate of raw land L_i (normalized to one in the main text) and infrastructure capital k_{fit}, such that

$$k_{lit} = l_i^{1-\zeta} k_{fit}^{\zeta}$$

Raw land does not depreciate but infrastructure capital depreciates at rate κ_f such that in the absence of any investment

$$\frac{k_{lit+1}}{k_{lit}} = (1 - \kappa_f)$$

This implies that in the absence of any investment in infrastructure capital, finished land essentially depreciates. To see this, write:

$$\begin{aligned} \frac{k_{lit+1}}{k_{lit}} &= \left(\frac{k_{fit+1}}{k_{fit}}\right)^{\zeta} \\ &= (1 - \kappa_f)^{\zeta} \end{aligned}$$

Consider the household raising its holdings of finished land in city i by k_{lit} units this period, and next period you rents the land and then resell it after it depreciates. The no arbitrage condition for this transaction is equation (27).

Equation (27) implies that along a balanced growth path the average rental price of land and the average price of land grow at the same rate, g_{p_l}. We do not measure rents from land, but we do measure rents from housing, which includes land and structures. We assume housing services are derived from structures and finished land as follows:

$$h_{it} = k_{sit}^{\omega} l_{hit}^{1-\omega}$$

From profit maximization of housing service providers, at each date and in each city

$$r_{hit} = \omega^{-\omega}(1-\omega)^{\omega-1} r_{lit}^{1-\omega} r_{st}^{\omega} \tag{28}$$

where r_h denotes the rent on services from houses and r_s denotes the rent on housing structures. Therefore along a balanced growth path

$$\begin{aligned} r_{hit} &= g_{p_l}^{(1-\omega)t} g_{p_s}^{\omega t} r_{hi0} \\ \ln E_t r_{hit} &= \ln r_{hi0} + \left[(1-\omega)\ln(g_{p_l}) + \omega \ln(g_{p_s})\right] t \end{aligned}$$

since rent on housing structures and land follow the same trends as their respective asset prices.

Along a balanced growth path, for $x = b, s, f, l$

$$\begin{aligned} P_{xt} &= P_{x0} g_{p_x}^{t} \\ \ln P_{xt} &= \ln P_{x0} + \ln(g_{p_x}) t \end{aligned}$$

where P_{xt} is the real price of the indicated type of capital (in the case of land this is the average price). We identify $g_{P_B}, g_{P_S}, g_{P_F}$ and g_{P_L} using the following moment conditions:

$$\begin{aligned} E\left\{\ln P_{xt} - \ln P_{x0} - \ln(g_{p_x})t\right\} &= 0,\ x = b, s, f \\ E\left\{(\ln P_{bt} - \ln P_{x0} - \ln(g_{p_x})t) \cdot t\right\} &= 0,\ x = b, s, f \\ E\left\{\ln E_t r_{hit} - \ln E r_{hi0} - \left[(1-\omega)\ln(g_{p_l}) + \omega \ln(g_{p_s})\right] t\right\} &= 0 \\ E\left\{(\ln E_t r_{hit} - \ln E_t r_{hi0} - \left[(1-\omega)\ln(g_{p_l}) + \omega \ln(g_{p_s})\right] t) \cdot t\right\} &= 0 \end{aligned}$$

The moment conditions for identifying g_{p_l}(the final two conditions) are based on the fact that $r_{lit} = g_{p_l}^t r_{li0}$ implies $E_t r_{lit} = g_{p_l}^t E_0 r_{li0}$, where E_t denotes expectation at t over i.

We now use equation (27) evaluated along the balanced growth path to relate prices of land to rent from land. Analogous relationships hold for the other forms of capital. We will use these relationships to formulate moment conditions to identify ω, α, ϕ and ζ. Notice that along a balanced growth path

$$\begin{aligned} E_t p_{lit+1} &= g_{p_l} E_t p_{lit} \\ E_t r_{lit+1} &= g_{p_l} E_t r_{lit} \end{aligned}$$

Since $E_t E_{t|i} x_t = E_t x_t$, it follows from (27) that

$$E_t p_{lit} = \frac{g_{p_l}}{R - (1-\kappa_f)^{\zeta} g_{p_l}} E_t r_{lit}.$$

Similar conditions hold for the other types of capital.

We identify ζ, the share of development capital in the production of finished land as follows. In every city

$$\zeta = \frac{r_{fit} k_{fit}}{r_{lit} l_{bit} + r_{lit} l_{hit}},$$

so that

$$\zeta = \frac{E_t r_{fit} k_{fit}}{E_t r_{lit} l_{bit} + E_t r_{lit} l_{hit}}.$$

Using the relationship between values and incomes,

$$\frac{E_t P_{ft} k_{fit}}{E_t p_{lit} l_{bit} + E_t p_{lit} l_{hit}} = \frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_f} - (1-\kappa_f)} \frac{E_t r_{ft} k_{fit}}{\left[E_t r_{lit} l_{bit} + E_t r_{lit} l_{hit}\right]}$$

Therefore we identify ζ using

$$E\left\{\frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_f} - (1-\kappa_f)}\zeta - \frac{\sum P_{ft} k_{fit}}{\sum (p_{lit} l_{bit} + p_{lit} l_{hit})}\right\} = 0$$

To identify ω first relate the ratio of capital to land income in the housing sector:

$$\frac{r_{st} k_{sit}}{r_{lit} l_{hit}} = \frac{\omega}{1-\omega}$$

Use this and the relationship between value and income ratios to obtain a relationship between the share of land in house values and ω:

$$\begin{aligned}\frac{E_t p_{lit} l_{hit}}{E_t p_{st} k_{sit} + E_t p_{lit} l_{hit}} &= \frac{1}{\frac{E_t P_{st} k_{sit}}{E_t p_{lit} l_{hit}} + 1} \\ &= \frac{1}{\frac{\omega}{1-\omega}\frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_s} + \kappa_s - 1} + 1}\end{aligned}$$

The moment condition identifying ω is then

$$E\left\{\frac{\sum p_{lit} l_{hit}}{\sum (P_{st} k_{sit} + p_{lit} l_{hit})}\left[\frac{\omega}{1-\omega}\frac{R/g_{p_l} - (1-\kappa_f)^{\zeta}}{R/g_{p_s} + \kappa_s - 1} + 1\right] - 1\right\} = 0,$$

Now consider the identification of α and ϕ. For this we make use of the following relationships implied by the intermediate good producer's production function:

$$\begin{aligned}\phi(1-\alpha) &= \frac{E_t w_{it} n_{it}}{E_t w_{it} n_{it} + E_t r_{lit} l_{bit} + E_t r_{bt} k_{bit}} \\ \frac{\alpha\phi}{1-\phi} &= \frac{E_t r_{bt} k_{bit}}{E_t r_{lit} l_{bit}}\end{aligned} \qquad (29)$$

We use the last equality to relate the ratio of the value of land to the value of tangible assets in the business sector to α and ϕ :

$$\begin{aligned}\frac{E_t p_{lit} l_{bit}}{E_t P_{bt} k_{bit} + E_t p_{lit} l_{bit}} &= \frac{1}{\frac{E_t P_{bt} k_{bit}}{E_t p_{lit} l_{bit}} + 1} \\ &= \frac{1}{\frac{\alpha\phi}{1-\phi}\frac{R/g_{p_l} - 1}{R/g_{p_b} + \kappa_b - 1} + 1}\end{aligned}$$

Using the last equality and (29) we arrive at the moment conditions used to identify α and ϕ:

$$\begin{aligned}E\left\{\frac{\sum p_{lit} l_{bit}}{\sum [P_{bt} k_{bit} + p_{lit} l_{bit}]}\left[\frac{\alpha\phi}{1-\phi}\frac{R/g_{p_l} - 1}{R/g_{p_b} + \kappa_b - 1} + 1\right] - 1\right\} = 0 \\ E\left\{\frac{\sum w_{it} n_{it}}{\sum [w_{it} n_{it} + r_{lit} l_{bit} + r_{bt} k_{bit}]} - \phi(1-\alpha)\right\} = 0\end{aligned}$$

We also need to estimate g_c, the gross growth rate of per capita consumption. Along a balanced growth path

$$\begin{aligned}C_t &= C_0 {g_c}^t \\ \ln C_t &= \ln C_0 + \ln(g_c)t\end{aligned}$$

where C_t is per capita consumption. Therefore we identify g_c using the following two moment conditions:

$$\begin{aligned}E\{\ln C_t - \ln C_0 - \ln(g_c)t\} &= 0 \\ E\{(\ln C_t - \ln C_0 - \ln(g_c)t) \cdot t\} &= 0\end{aligned}$$

D Measuring the Effect of Agglomeration on Per Capita Consumption Growth

The balanced growth path of our model has

$$g_c = \gamma_a^{\frac{(1-\alpha)\delta}{(1-\alpha)\delta+(\zeta-1)(\delta-1)}} \gamma_n^{\frac{(1-\zeta)(\delta-1)}{(1-\alpha)\delta+(\zeta-1)(\delta-1)}} \gamma_b^{\frac{\alpha\delta}{(1-\alpha)\delta+(\zeta-1)(\delta-1)}} \gamma_f^{\zeta\frac{1-\delta}{(1-\alpha)\delta+(\zeta-1)(\delta-1)}}$$

Use the balanced growth equation to express γ_a as

$$\hat{\gamma}_a = \hat{g}_c^{\frac{(1-\alpha)\delta+(\zeta-1)(\delta-1)}{(1-\alpha)\delta}} \hat{\gamma}_n^{\frac{(1-\zeta)(1-\delta)}{(1-\alpha)\delta}} \hat{g}_{p_b}^{\frac{\alpha}{(1-\alpha)}} \hat{g}_{p_f}^{\zeta\frac{1-\delta}{(1-\alpha)\delta}}$$

$$\begin{aligned}
g_c^* &= \hat{\gamma}_a^{\frac{(1-\alpha)\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} \gamma_n^{\frac{(1-\zeta)(\phi-1)}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} g_{p_b}^{\frac{-\alpha\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} g_{p_f}^{-\zeta\frac{1-\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} \\
&= \left[\hat{g}_c^{\frac{(1-\alpha)\delta+(\zeta-1)(\delta-1)}{(1-\alpha)\delta}} \hat{\gamma}_m^{\frac{(1-\zeta)(1-\delta)}{(1-\alpha)\delta}} \hat{g}_{p_b}^{\frac{\alpha}{(1-\alpha)}} \hat{g}_{p_f}^{\zeta\frac{1-\delta}{(1-\alpha)\delta}}\right]^{\frac{(1-\alpha)\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} \\
&\quad \times \gamma_n^{\frac{(1-\zeta)(\phi-1)}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} g_{p_b}^{\frac{-\alpha\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} g_{p_f}^{-\zeta\frac{1-\phi}{(1-\alpha)\phi+(\zeta-1)(\phi-1)}} \\
&= \hat{g}_c^{\frac{\phi[(1-\alpha)\delta+(\zeta-1)(\delta-1)]}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}} \hat{\gamma}_n^{\frac{(\phi-\delta)(1-\zeta)}{\delta[(1-\alpha)\phi+(1-\zeta)(1-\phi)]}} \hat{g}_{p_f}^{\frac{\zeta(\phi-\delta)}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}}
\end{aligned}$$

Therefore

$$\begin{aligned}
\Lambda &= \frac{\hat{g}_c - g_c^*}{g_c^* - 1} \\
&= \frac{\hat{g}_c - \hat{g}_c^{\frac{\phi[(1-\alpha)\delta+(\zeta-1)(\delta-1)]}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}} \hat{\gamma}_n^{\frac{(\phi-\delta)(1-\zeta)}{\delta[(1-\alpha)\phi+(1-\zeta)(1-\phi)]}} \hat{g}_{p_f}^{\frac{\zeta(\phi-\delta)}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}}}{\hat{g}_c^{\frac{\phi[(1-\alpha)\delta+(\zeta-1)(\delta-1)]}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}} \hat{\gamma}_N^{\frac{(\phi-\delta)(1-\zeta)}{\delta[(1-\alpha)\phi+(1-\zeta)(1-\phi)]}} \hat{g}_{p_f}^{\frac{\zeta(\phi-\delta)}{\delta[(1-\alpha)\phi+(\zeta-1)(\phi-1)]}} - 1}
\end{aligned}$$

E Solving the Model and Comparing it to Data

This section describes how we solve our model with $\omega = 0$ and $\xi = 1$. The notation is somewhat different from the main text of the paper, but is internally consistent.

In this model there is a representative household with a large number of members who share consumption risk perfectly. Each period the household allocates its workers and capital across locations, chooses how much infrastructure capital to build in each location for the next period, and decides much business capital it wants to allocate across locations in the next period. We assume these decisions are made after the household observes total factor productivity in each location in that period. The household takes all prices and the distribution of total factor productivity as given. That is, it behaves competitively and does not take into account the effect of its actions on the density of production in each location. In each location there are developers and producers. The developers rent local infrastructure capital from the household and combine it with raw land to produce developed land which they then rent to local producers. Producers rent capital and labor from the household and developed land to produce the city-specific intermediate good. There is also a final good producer who combines city-specific intermediate goods to produce the final good. Developers and goods producers maximize profits taking all prices and total factor productivity as given.

The competitive equilibrium for this economy can be found as the solution to an optimization problem with side conditions. Notation is borrowed from the main text except that here we assume the support of the distribution of technology is discrete. Let $m(z^t)$ denote the distribution of cities across idiosyncratic productivity histories z^t. The household perfectly insures itself against consumption risk, so we write the optimization problem as

$$\max_{\{C_t,K_{t+1},y(z^t),l(z^t),k(z^t),n(z^t),h(z^t),x(z^t),d(z^t)\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \sum m(z^t)n(z^t) \left[\ln C_t + \psi \ln h(z^t)\right]$$

subject to

$$\begin{aligned}
C_t + K_{t+1} - (1-\kappa)K_t + X_t &\leq \left[\sum m(z^t)y(z^t)^{\eta}\right]^{\frac{1}{\eta}} \\
y(z^t) &\leq z_t^{(1-\alpha)\phi} \left[\frac{\hat{y}(z^t)}{\hat{l}(z^t)}\right]^{\frac{\lambda-1}{\lambda}} l(z^t)^{1-\phi}k(z^t)^{\alpha\phi}n(z^t)^{(1-\alpha)\phi} \\
\sum m(z^t)k(z^t) &\leq K_t \\
n(z^t)h(z^t) + l(z^t) &\leq d(z^{t-1})^{\varsigma}, \ \forall z^t \\
\sum m(z^t)n(z^t) &\leq 1 \\
d(z^t) &\leq (1-\zeta)d(s^{t-1}) + x(z^t), \ \forall z^t \\
\sum m(z^t)x(z^t) &\leq X_t \\
& \quad K_0, d(z_0)\,\forall\, z_0, \hat{y}(z^t) \text{ and } \hat{l}(z^t) \text{ given.}
\end{aligned}$$

The un-indexed summations are over productivity histories z^t. The competitive equilibrium corresponds to a solution to this optimization problem such that $y(z^t) = \hat{y}(z^t)$ and $l(z^t) = \hat{l}(z^t)$. Note that for simplicity, and in contrast to the main text, we have assumed that housing services are derived from developed land only.

E.1 Steady State with One City

This section derives the steady state with one city which we use for starting values in the general case. The first order condition and constraints for land development, evaluated in steady state, are

$$\begin{aligned}
1 &= \frac{1}{R}\left[r_d + (1-\zeta)\right] \\
x &= \zeta d
\end{aligned}$$

where r_d is the rental rate for infrastructure capital. Then

$$\begin{aligned}
1 &= \frac{1}{R}\left[r_l \varsigma d^{\varsigma-1} + (1-\zeta)\right] \\
r_d^* &= R - 1 + \zeta \\
R &= 1/\beta \\
r_l \varsigma d^{\varsigma-1} &= r_d^*
\end{aligned}$$

where r_l is rent on finished land.

Output in the city is

$$y = z^{(1-\alpha)\phi}\left[\frac{y}{l}\right]^{\frac{\lambda-1}{\lambda}} l^{1-\phi} k^{\alpha\phi} n^{(1-\alpha)\phi}$$

Let $\delta = \lambda\phi$. Solving for output:

$$y = z^{\delta(1-\alpha)} l^{1-\delta} k^{\delta\alpha} n^{\delta(1-\alpha)} \tag{30}$$

Aggregate output is

$$Y = y,$$

output prices are

$$q = 1$$

The labor supply constraint implies

$$n = 1$$

and the aggregate resource constraint is

$$C + \kappa K + \zeta D = Y.$$

where the total stock of developed land is equal to the developed land in the city, $D = d$. The equilibrium rent on business capital r_k^* is found using the business capital accumulation and is given by

$$r_k^* = 1/\beta - 1 + \kappa$$

Rental demand for capital, developed land demand, labor demand, housing, land development, labor allocation first order necessary conditions (FONCS) and the land constraint are:

$$\begin{aligned}
k &= \alpha\phi y r_k^{*-1} \\
r_l &= y\,(1-\phi)\,l^{-1} \\
w &= (1-\alpha)\phi y n^{-1} \\
\psi C &= r_l h \\
r_l \varsigma d^{\varsigma-1} &= r_d^* \\
\psi \ln h &= \frac{1}{C}\theta + \frac{1}{C}\left[\psi C - w\right] \\
nh + l &= d^{\varsigma}.
\end{aligned}$$

Here w denotes the wage. From the infrastructure FONC

$$d^{\varsigma} = \frac{r_d^* d}{\varsigma r_l}$$

Since $n = 1$ the land constraint can be written

$$l = \frac{r_d^* d}{\varsigma r_l} - h$$

Using this, the land demand FONC and the housing demand FONC, we have

$$y\,(1-\phi) = \frac{r_d^* d}{\varsigma} - \psi C. \tag{31}$$

Substitute from the capital rental and land demand FONCS into 30 to express y as function of r_l

$$\begin{aligned} y &= z^{\delta(1-\alpha)} \left[\frac{(1-\phi)\,y}{r_l}\right]^{1-\delta} \left[\frac{\alpha\phi}{r_k^*} y\right]^{\delta\alpha} \\ &= \left[z^{\delta(1-\alpha)} \left[1-\phi\right]^{1-\delta} \left[\frac{\alpha\phi}{r_k^*}\right]^{\delta\alpha}\right]^{\frac{1}{\delta(1-\alpha)}} r_l \end{aligned}$$

Use the definition of r_d to solve for d as function of r_l

$$d = \left[\frac{\varsigma}{r_d^*}\right]^{\frac{1}{1-\varsigma}} r_l$$

Solve for k as function of r_l using the capital rental FONC

$$k = \frac{\alpha\phi}{r_k^*} \left[z^{\delta(1-\alpha)} \left[1-\phi\right]^{1-\delta} \left[\frac{\alpha\phi}{r_k^*}\right]^{\delta\alpha}\right]^{\frac{1}{\delta(1-\alpha)}} r_l$$

Solve for C as function of y and d using 31

$$C = \frac{r_d^*}{\psi\varsigma} d - \frac{1-\phi}{\psi} y.$$

With this last expression we solve the aggregate resource constraint for r_l using the expressions for y and d as functions of r_l from above. First substitute in for the right hand side of the aggregate resource constraint:

$$\begin{aligned} y &= C + \kappa k + \zeta d \\ &= \frac{r_d^*}{\psi\varsigma} d - \frac{1-\phi}{\psi} y + \kappa \frac{\alpha\phi}{r_k^*} y + \zeta d \end{aligned}$$

Substituting for y and d in the last equation and solving for r_l we arrive at

$$r_l^* = \left[\frac{\left[\frac{r_d^*}{\varsigma\psi} + \zeta\right] \left[\frac{\varsigma}{r_d^*}\right]^{\frac{1}{1-\varsigma}}}{\left[1 + \frac{1-\phi}{\psi} - \kappa\frac{\alpha\phi}{r_k^*}\right] \left[z^{\delta(1-\alpha)} \left[1-\phi\right]^{1-\delta} \left[\frac{\alpha\phi}{r_k^*}\right]^{\delta\alpha}\right]^{\frac{1}{\delta(1-\alpha)}}} \right]^{\frac{1}{\frac{\delta-1}{\delta(1-\alpha)} - \frac{1}{1-\varsigma}}}.$$

With r_l^* in hand we can solve for C^*, k^*, d^*, y^* using expressions derived above. In addition, from the housing FONC

$$h^* = \psi C^* / r_l^*$$

and from the land constraint

$$l^* = r_d^* d^* / r_l^* - h^*.$$

Finally the labor demand and labor allocation FONCS yield

$$\begin{aligned} w^* &= y^*(1-\alpha)\phi \\ \theta^* &= C^*\psi \ln h^* - \psi C^* + w^* \end{aligned}$$

The foregoing demonstrates that, for admissible parameters, a non-stochastic steady state exists and is unique with one level of productivity.

E.2 General Steady State Solution

The solution strategy is to fix C, K, D, θ, solve for all other endogenous variables conditional on these variables, and then find a fixed point in C, K, D, θ that satisfies market clearing.

Using the transition equation for exogenous productivity it is straightforward to compute the steady state distribution of cities by exogenous productivity. The mass of cities of type i is m_i and aggregate output is

$$Y = \left[\sum m_i {y_i}^{\eta}\right]^{\frac{1}{\eta}}. \tag{32}$$

Output prices are

$$q_i = Y^{1-\eta} y_i^{\eta-1}.$$

The aggregate resource constraint is

$$C + \kappa K + \zeta D = Y.$$

So that

$$q_i = \left[C + \kappa K + \zeta D\right]^{1-\eta} y_i^{\eta-1}.$$

Except where it helps the exposition we drop the i subscript hereon. The FONC for infrastructure capital accumulation is

$$1 = \beta E\left[r_l \varsigma d'^{\varsigma-1} + 1 - \zeta\right].$$

From the housing FONC and land constraint

$$r_l = \frac{\psi C n}{d^\varsigma - l}.$$

Therefore the FONC for infrastructure capital can be written

$$1 = \beta E\left[\frac{\psi C n'}{d'^\varsigma - l'}\varsigma d'^{\varsigma-1} + 1 - \zeta\right].$$

Equilibrium r_k is given by

$$r_k^* = 1/\beta - 1 + \kappa.$$

From the FONCS and constraints:

$$
\begin{aligned}
n_i h_i + l_i &= d_i^\varsigma & (33)\\
\psi C &= r_l h_i & (34)\\
\psi C \ln h_i &= \theta + [\psi C - w_i] & (35)\\
w_i &= q_i (1-\alpha)\phi y_i n_i^{-1} & (36)\\
r_{li} &= q_i\,(1-\phi)\,y_i l_i^{-1} & (37)\\
q_i &= [C + \kappa K + \zeta D]^{1-\eta}\,y_i^{\eta-1} & (38)\\
r_k^* &= q_i \alpha\phi y_i k_i^{-1} & (39)\\
y_i &= z_i^{\delta(1-\alpha)} l_i^{1-\delta} k_i^{\delta\alpha} n_i^{\delta(1-\alpha)} & (40)\\
1 &= \beta E\left[\frac{\psi C}{h_i'}\varsigma d_i'^{\varsigma-1} + 1 - \zeta\right] & (41)\\
k_i &= q_i \alpha\phi y_i r_k^{*-1}. & (42)
\end{aligned}
$$

Here we have used the i subscript to be clear on which variables are location specific and which are common to all locations. Combine all but 38 and 40, 41, 42 to form

$$
\begin{aligned}
\frac{\psi C}{r_k^* h} &= \frac{(1-\phi)}{\alpha\phi}\frac{k}{l} & (43)\\
\frac{(1-\alpha)k}{\alpha n} &= \frac{\theta + \psi C - \psi C \ln h}{r_k^*} & (44)\\
nh + l &= d^\varsigma. & (45)
\end{aligned}
$$

It follows that

$$\frac{l\psi C}{r_k^*(d^\varsigma - l)} = \frac{(1-\phi)}{\alpha\phi}\frac{k}{n}. \quad (46)$$

Using 43, we can rewrite the infrastructure FONC

$$1 = \beta E\left[\frac{(1-\phi)\,r_k^*}{\alpha\phi}\frac{k}{l}\varsigma d'^{\varsigma-1} + 1\right].$$

Furthermore, combining 44 and 46:

$$\frac{l\psi C}{r_k^*(d^\varsigma - l)} = \frac{(1-\phi)}{(1-\alpha)\phi}\frac{\theta + \psi C - \psi C \ln h}{r_k^*}.$$

Solving this last equation for h :

$$\begin{aligned} h &= \exp\left(\frac{\theta}{\psi C} + 1 - \frac{(1-\alpha)\phi l}{(d^\varsigma - l)(1-\phi)}\right) \\ &= \hat{h}(d, l; C, \theta). \end{aligned}$$

Using 43

$$\begin{aligned} k &= \frac{\alpha\phi\psi C}{(1-\phi)\, r_k^* \hat{h}(d, l; C, \theta)} l \\ &= \hat{k}(d, l; C, \theta). \end{aligned}$$

Using 33

$$\begin{aligned} n &= \frac{d^\varsigma - l}{\hat{h}(d, l; C, \theta)} \\ &= \hat{n}(d, l; C, \theta). \end{aligned}$$

Using 38, 39 and 40 and substituting in the expressions for k and n we solve for l_j for each d_i' using

$$r_k^* = \alpha\phi\,[C + \kappa K + \zeta D]^{1-\eta}\, z_j^{\delta(1-\alpha)\eta} l_j^{(1-\delta)\eta} \hat{k}(d_i', l_j; C, \theta)^{\delta\alpha\eta - 1} \hat{n}(d_i', l_j; C, \theta)^{\delta(1-\alpha)\eta}$$

The 'prime' superscript denotes choice of the indicated variable made for the following period when the current state is given by the subscript. For each i this yields

$$l_{ji} = \tilde{l}(C, K, D, \theta, z_j, d_i') \tag{47}$$

$$n_{ji} = \tilde{n}(C, K, D, \theta, z_j, d_i') \tag{48}$$

$$k_{ji} = \tilde{k}(C, K, D, \theta, z_j, d_i'). \tag{49}$$

Here we use the subscript convention that the variable is chosen contemporaneous with the technology state corresponding to the first subscript and state corresponding to the second subscript in the period before.

From the land infrastructure FONC, for each i

$$\begin{aligned} 1 &= \beta \sum \pi_{ij} \left[\frac{(1-\phi)\, r_k^*}{\alpha\phi} \frac{\tilde{k}(C, K, D, \theta, z_j, d_i')}{\tilde{l}(C, K, D, \theta, z_j, d_i')} \varsigma d_i'^{\varsigma - 1} + 1 - \zeta\right] \\ &= \beta \sum \pi_{ij} \left[f(C, K, D, \theta, z_j, d_i') + 1 - \zeta\right]. \end{aligned} \tag{50}$$

Here the π_{ij} denote the probability of transitioning from state i to state j. Because π_{ij} depends on z_i, solving 50 yields

$$d_i' = d(C, K, D, \theta, z_i).$$

Using 47, 48 and 49 we may now obtain

$$\begin{aligned} l_{ij} &= l(C, K, D, \theta, z_i, z_j) \\ n_{ij} &= n(C, K, D, \theta, z_i, z_j) \\ k_{ij} &= k(C, K, D, \theta, z_i, z_j). \end{aligned}$$

The switch in the order of the subscripts is by design. These expressions can be substituted into 40 to construct

$$y_{ij} = y(C, K, \theta, D, z_i, z_j)$$

By the definition of developed land investment

$$\begin{aligned} x_{ij} &= d_i' - (1-\zeta)d_j \\ &= x(C, K, D, \theta, z_i, z_j). \end{aligned}$$

In addition

$$\begin{aligned} Y &= C + \kappa K + \zeta D \\ &= Y(C, K, X) \end{aligned}$$

We solve for C, K, D, θ using

$$\begin{aligned} Y(C, K, D, \theta) &= \left[\sum m_{ij} y(C, K, D, \theta, z_i, z_j)^{\eta}\right]^{\frac{1}{\eta}} \\ K &= \sum m_{ij} k(C, K, D, \theta, z_i, z_j) \\ 1 &= \sum m_{ij} n(C, K, D, \theta, z_i, z_j) \\ X &= \sum m_{ij} x(C, K, D, \theta, z_i, z_j) \end{aligned}$$

where m_{ij} denotes the steady state mass of cities that have z_i today and z_j yesterday. Notice that the city level variables are entirely determined by lagged and current technology. In particular for each possible pair of technology states there is a unique value of infrastructure capital chosen for the next period. Consequently the number of infrastructure capital states corresponds to the number of possible pairs of technology states.

We use the GAUSS non-linear equation solver `eqSolve` to solve this system of four equations. To evaluate these equations we need to solve 50. We accomplish this using a version of `eqSolve` that we have modified to exploit the sparseness of the transition matrix formed with the π_{ij} (see below).

For this we need to find m_{ij}, the steady state distribution of (z_t, z_{t-1}). For simplicity, consider the case of three z states. The results described below are based on a grid for z_t with 75 points so there are 5,625 (z_t, z_{t-1}) states. We need the transition probabilities for

(z_t, z_{t-1}) to (z_{t+1}, z_t). The state is summarized by

$$\begin{bmatrix} z_1 & z_1 \\ z_1 & z_2 \\ z_1 & z_3 \\ z_2 & z_1 \\ z_2 & z_2 \\ z_2 & z_3 \\ z_3 & z_1 \\ z_3 & z_2 \\ z_3 & z_3 \end{bmatrix}$$

We obtain the π_{ij} (defined above) from the underlying transition matrix for z_t. Then the matrix of transition probabilities is

$$\begin{bmatrix} \pi_{11} & 0 & 0 & \pi_{12} & 0 & 0 & \pi_{13} & 0 & 0 \\ \pi_{11} & 0 & 0 & \pi_{12} & 0 & 0 & \pi_{13} & 0 & 0 \\ \pi_{11} & 0 & 0 & \pi_{12} & 0 & 0 & \pi_{13} & 0 & 0 \\ 0 & \pi_{21} & 0 & 0 & \pi_{22} & 0 & 0 & \pi_{23} & 0 \\ 0 & \pi_{21} & 0 & 0 & \pi_{22} & 0 & 0 & \pi_{23} & 0 \\ 0 & \pi_{21} & 0 & 0 & \pi_{22} & 0 & 0 & \pi_{23} & 0 \\ 0 & 0 & \pi_{31} & 0 & 0 & \pi_{32} & 0 & 0 & \pi_{33} \\ 0 & 0 & \pi_{31} & 0 & 0 & \pi_{32} & 0 & 0 & \pi_{33} \\ 0 & 0 & \pi_{31} & 0 & 0 & \pi_{32} & 0 & 0 & \pi_{33} \end{bmatrix}$$

From this matrix we can calculate the steady state m_{ij}. Obtaining the steady state and evaluating conditional expectations is greatly accelerated by exploiting the sparseness of this matrix.

E.3 Approximating the Technology Process

We consider an underlying technology process

$$\ln z_t = \max \{\gamma_z + \ln z_{t-1} + \varepsilon_t, \ln z_{\min}\} \tag{51}$$

where ε_t is iid normally distributed with mean zero and variance σ_ε^2. This is isomorphic to the process considered by Gabaix (2000). As long as $g < 0$ for fixed $z_{\min}$ this process has an invariant distribution in z_t, which is convenient for solving our model. The tail of this distrubtion is exponential, that is it has the property that

$$\Pr [z_t > b] = \frac{a}{b^\vartheta}$$

for some $a > 0$ and $\vartheta > 0$. Zipf's law is $\vartheta = 1$ for z_t population. We can always find a γ_z to match an admissible ϑ. The parameter ϑ can be estimated in our data by regressing the log rank z_t on $\ln z_t$. The coefficient on $\ln z_t$ is a consistent estimate of ϑ. Technology is well approximated by an exponential distribution with $\vartheta = 2.5$.

We approximate 51 with a discrete Markov chain. The grid is chosen to be equally spaced in logs. The elements of the transition equation are conditional probabilities of transiting from a given grid point to intervals around all possible grid points, where the intervals are equally spaced from the mid-points between grid points. These probabilities are calculated using 51. In practice we assume a wide domain for the grid that is 200 times the standard deviation of the innovation. This ensures that mass does not accumulate on the largest grid points. We solve for the γ_z which yields $\vartheta = 2.5$ in the steady state, excluding a small number of the smallest and largest grid points. We exclude some grid points to focus on the region of the state space where the approximation is best. This strategy yields a remarkably good approximation to an exponential distribution, except in the extreme tails. In fact the overall approximation resembles empirical plots of log city rank by population versus log population, with a different slope of course. We set $z_{\min} = 1 - 1/\zeta$. This sets the mean of the distribution to approximately 1.

We select σ_ε to match our estimate of the variance of employment growth. This yields $\sigma_\varepsilon = .005$.

E.4 Estimating Statistics to Compare to Model

We study log growth rates in the model. The empirical counterpart to the log growth rate of variable x_{it} is $\hat{x}_{it} - \hat{x}_{it-1}$. The empirical statistics take into account measurement error in employment and in wages. We have two sources of data for these series, the BEA and the BLS. We assume classical measurement error, that the measurement error in the BEA is orthogonal to that in the BLS and that measurement error in wages and employment is correlated if the measures are from the same source (BEA or BLS).

To be specific, denote two measures of variable $\hat{x}_t$ as $\tilde{x}_{1t} = \hat{x}_t + u^x_{1t}$ and $\tilde{x}_{2t} = \hat{x}_t + u^x_{2t}$ where u^x_i is classical measurement error associated with measures $\tilde{x}_{it}, i = 1, 2$. We have dropped the dependence of each variable on its city of origin for simplicity. It follows that consistent estimates of $var(\Delta\hat{x}_t)$ and $cov(\Delta\hat{x}_t, \Delta\hat{y}_t)$ are $cov(\Delta\tilde{x}_{1t}, \Delta\tilde{x}_{2t})$ and $cov(\Delta\tilde{x}_{1t}\Delta\tilde{y}_{2t})$ from which we can derive all statistics involving employment and wages. Here Δ is the first difference operator. With only one source for each of the remaining variables we must assume they are measured without error. Statistics are based on our sample of 22 cities (results are similar for a broader sample of cities).

F Standard Errors

We estimate Λ in three steps. To begin, we collect the expressions in the moment conditions described in the last sub-section into a vector-valued function $\Psi_1(X_t, \theta_1)$, so that

$$E\Psi_1(X_t, \theta_1) = 0. \tag{52}$$

Here, X_t is a vector of the aggregate variables included in these moment conditions, and θ_1 is a parameter vector given by:

$$\theta_1 \equiv \left[\kappa_b, \kappa_b, \kappa_b, g_{p_l}, g_{p_b}, g_{p_s}, g_{p_f}, \gamma_n, g_c, \alpha, \phi, \omega, \zeta\right]'.$$

Because this system of moment conditions is exactly identified, the dimensions of Ψ_1 and θ_1 are equal. The first step is to estimate equation (34) by GMM, in which we use a Newey-West weight matrix with a lag length of 2.

In the second step we estimate δ and ξ using the moment conditions in equation (26) in the main text. This estimation requires that we plug in the estimates of ω and α from the first step into (26). To account for the sampling variation associated with these two plug-in parameters, we adjust the weight matrix using the methods described in Newey and McFadden (1994). Specifically, write the moment condition in (26) as

$$E\Psi_2\left(X_{it},\theta_2\right)=0, \tag{53}$$

in which X_{it} is the vector of panel data in (26) and $\theta_2=\{\delta,\xi,\omega,\alpha\}$. Next, let $\psi_\omega\left(X_t\right)$ and $\psi_\alpha\left(X_t\right)$ be the influence functions associated with ω and α. To express the optimal weight matrix for the GMM estimation based on the moment condition in (53), we define

$$\tilde{\Psi}_2\left(X_{it},\theta_2\right)\equiv\Psi_2\left(X_{it},\theta_2\right)-\sqrt{N}\left(\frac{\partial\Psi_2\left(X_{it},\theta_2\right)}{\partial\omega}\psi_\omega\left(X_t\right)+\frac{\partial\Psi_2\left(X_{it},\theta_2\right)}{\partial\alpha}\psi_\alpha\left(X_t\right)\right). \tag{54}$$

The $\sqrt{N}$ term appears in this expression to account for the fact that the parameters ω and α are estimated with only T observations, instead of with NT observations. Finally, the optimal weight matrix is given by

$$\mathbf{\Omega}\equiv E\left[\tilde{\Psi}_2\left(X_{it},\delta,\xi,\omega,\alpha\right)\tilde{\Psi}_2\left(X_{it},\delta,\xi,\omega,\alpha\right)'\right].$$

The rest of the GMM estimation proceeds by averaging the moment conditions over both i and t, and by clustering the weight matrix at the city level.

The third step is to substitute the point estimates for g_c, g_{p_f}, δ, α, ϕ, and ζ into equation (17) in the main text to obtain Λ. To calculate the sampling variance of Λ, we need the joint covariance matrix of these six parameters, which we calculate by stacking the parameters' influence functions as shown by Erickson and Whited (2002). As in (54), we multiply the influence functions for the parameters estimated with time-series data by $\sqrt{N}$. After this calculation, a standard application of the delta method gives the variance of Λ.

G Monte Carlo Study

We perform a Monte Carlo study of our estimator using simulated data whose distribution closely approximates that of our own data set, which consists of some variables that vary in only the time dimension and others that vary in both the time and city dimension. We first consider the times series variables:

$$\boldsymbol{x}_t\equiv\left(\begin{array}{c}\frac{D_{bt}}{K_{bt}},\frac{D_{st}}{K_{st}},\frac{D_{ft}}{K_{ft}},Er_{hit},p_{bt},p_{st},p_{ft},c_t,\\ \frac{\sum p_{lit}l_{hit}}{\sum(p_{st}k_{sit}+p_{lit}l_{hit})},\frac{\sum p_{lit}l_{bit}}{\sum(p_{bt}k_{bit}+p_{lit}l_{bit})},\frac{\sum w_{it}n_{it}}{\sum(w_{it}n_{it}+r_{lit}l_{bit}+r_{bt}k_{bit})},\frac{\sum p_{ft}k_{fit}}{\sum(p_{lit}l_{bit}+p_{lit}l_{hit})}\end{array}\right). \tag{55}$$

As a first step, we use our actual data to estimate a time-trend regression for $\boldsymbol{x}_t$:

$$\boldsymbol{x}_t=\boldsymbol{a}+\boldsymbol{b}t+\boldsymbol{u}_{xt}, \tag{56}$$

in which $\boldsymbol{b}$ is a vector of time-trends for the individual elements of $\boldsymbol{x}_t$, $\boldsymbol{a}$ is a vector of intercepts, and $\boldsymbol{u}_{xt}$ is a vector of disturbances with covariance matrix Σ_{ux}. With the estimates of $(\boldsymbol{a}, \boldsymbol{b}, \Sigma_{ux})$, we simulate each variable as follows. We generate a matrix of normal disturbances of length 132 and width equal to the dimension of $\boldsymbol{x}_t$. These disturbances are serially uncorrelated, but are contemporaneously correlated with a covariance matrix of Σ_u. We then generate $\boldsymbol{x}_t$ using (56). Finally, we keep the last 32 observations, where 32 is the time-span of our actual data set. This procedure give us time series variables with the same first and second moments as those in our actual data.

Next we describe our panel variables $\Delta\hat{\boldsymbol{y}}_{it} = (\Delta\hat{w}_{eit}, \Delta\hat{r}_{hit}, \Delta\hat{p}_{yit}, \Delta\hat{s}_{it}, \Delta\hat{m}_{it}, \Delta\hat{v}_{it})$, in which $\Delta\hat{v}_{it}$ is a vector containing our two additional instrumental variables, house prices, and per capita income. We simulate directly in first-differenced, hatted form. We first calculate the means and covariances of these variables in our actual data. We also calculate the first-order serial correlations from OLS estimates of a simple $AR(1)$ model:

$$\Delta\hat{\boldsymbol{y}}_{it} = A\Delta\hat{\boldsymbol{y}}_{it-1} + \boldsymbol{u}_{yit}, \tag{57}$$

in which A is a diagonal matrix of autoregressive coefficients. We denote the estimated covariance matrix of the residuals as Σ_y.

Next, we use these estimates to create simulated panel variables. First we generate a matrix of normal disturbances, $\tilde{\boldsymbol{u}}_{yit}$ of length 132 and width equal to the dimension of $\Delta\hat{\boldsymbol{y}}_{it}$ times 22, which is the number of cities in our panel. We then update the variables $(\Delta\hat{r}_{hit}, \Delta\hat{p}_{yit}, \Delta\hat{s}_{it}, \Delta\hat{m}_{it}, \Delta\hat{v}_{it})$ in each of these cities using (57). Finally, we construct $\Delta\hat{w}_{eit}$ using

$$\Delta\hat{w}_{eit} = \frac{1}{1-\omega}\frac{\delta-1}{\delta(1-\alpha)}\Delta\hat{r}_{hit} + \frac{1}{\delta(1-\alpha)}\Delta\hat{q}_{it} + \frac{1-\xi}{\xi}\Delta\hat{\chi}_{it} + (\xi-1)\Delta\hat{m}_{it} + \varepsilon_{it}, \tag{58}$$

in which ε_{it} is constructed as $\varepsilon^*_{it} + a'\tilde{\boldsymbol{u}}_{yit}$, in which a is vector of coefficients corresponding to $(\Delta\hat{r}_{hit}, \Delta\hat{p}_{yit}, \Delta\hat{s}_{it}, \Delta\hat{m}_{it})$, and in which ε^*_{it} an *i.i.d.* normal variable. Thus, the error term in (58) shares common contemporaneous variation with $(\Delta\hat{r}_{hit}, \Delta\hat{p}_{yit}, \Delta\hat{s}_{it}, \Delta\hat{m}_{it})$, as our model predicts. We set the variance of ε^*_{it} so that the variance of $\Delta\hat{w}_{eit}$ in our simulated data equals the variance of $\Delta\hat{w}_{eit}$ in our real data. We set the correlation parameters, a, so that the covariance between $\Delta\hat{w}_{eit}$ and $(\Delta\hat{r}_{hit}, \Delta\hat{p}_{yit}, \Delta\hat{s}_{it}, \Delta\hat{m}_{it})$ in our simulated data approximates this covariance in our actual data.

Finally, we note that all of the panel variables are the residuals from regressing the raw variables on time dummies. They are therefore by construction orthogonal to any time-series variables, so we set the covariances between the time-series and panel variables equal to zero.

We repeat this procedure 10,000 times (thus generating 10,000 data sets), where we set the true value values of the coefficients equal to our estimates from Table 3. Specifically, $\delta = 1.04$, $\xi = 0.54$, and the time series coefficients all equal their estimated values. We use these true values to evaluate $\Lambda = 0.102$.

We estimate the model using twice lagged values of $(\Delta\hat{w}_{eit}, \Delta\hat{r}_{hit}, \Delta\hat{q}_{it}, \Delta\hat{\chi}_{it}, \Delta\hat{m}_{it}, \Delta\hat{v}_{it})$. We average our GMM moment conditions in both the cross-sectional and time-series dimensions. We calculate standard errors as we do for Table ???, with a Newey-West correction for the first-stage time-series estimation, and clustering at the city level for the second stage panel estimation.

We report the results from this simulation in the following table.

	δ	ξ	Λ
Coefficients			
Average Coefficient	1.0414	0.5493	0.1021
Average Bias	0.0014	0.0093	0.0013
Mean Absolute Deviation	0.0366	0.0319	0.0600
RMSE	0.0475	0.0505	0.0803
Test statistics			
Upside null rejection frequency (2.5%)	0.0300	0.0468	0.0083
Downside null rejection frequency (2.5%)	0.0823	0.0542	0.0907
J-test 5% rejection rate	0.0102		
$m2$-test 5% rejection rate	0.0968		
$m3$-test 5% rejection rate	0.0000		

The top panel of this table reports the average estimated coefficient over the 10,000 trials, as well as the average bias, mean absolute deviation, and root mean square error (RMSE). We see that despite the small sample size, the our two-step GMM estimator produces nearly unbiased coefficient estimates. The mean absolute deviations and RMSEs are low for δ and ξ, but somewhat larger for Λ. This result makes sense inasmuch as Λ is estimated from both time-series and panel data, and the time-series data contains much less variation identifying information.

The bottom panel of this table reports the two tail probabilities from nominal 5% tests that the coefficient estimates equal their true values. "Upside" refers to the right tail, and "downside" refers to the left tail. In general, these tests are slightly oversized, which arises because the standard errors are "too small." However, the overrejection is not symmetric. For δ and Λ the probability of rejecting the null on the upside is much smaller that the probability of rejecting the null on the downside. For δ the right tail is approximately correctly sized, and for Λ the right tail is undersized, which means that there is a negligible probability of rejecting the null in favor of the alternative of $\Lambda > 0$. Of course, the right tails are the ones that matter for our application because our theory implies that both δ and Λ are positive. Our test rejection results are therefore comforting in that they imply that our significant coefficients are not an artifact of a test over-rejection in small samples.

Finally, we report the rejection rates for our three diagnostic tests. We find that the J-test and the $m3$-test underreject but that the $m2$-test overrejects slightly. The first two results imply that the J-test and the $m3$-test are unlikely to be useful specification tests. In contrast, the third result implies that the insignificant $m2$ statistic we find in our estimation is not likely to be an artifact of an undersized test.

References

Davis, M. and J. Heathcote (2007). The price and quantity of residential land in the United States. *Journal of Monetary Economics 54*(8), 2595–2620.

Erickson, T. and T. Whited (2002). Two-step gmm estimation of the errors-in-variables model using high-order moments. *Econometric Theory 18*(3), 776–799.

Newey, W. and D. McFadden (1994). Large sample estimation and hypothesis testing. In R. Engle and D. McFadden (Eds.), *The Handbook of Econometrics, Vol. 4*. North-Holland.