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Abstract

Standard dynamic models of structural transformation, without knife-edge and counterfactual parameter values, preclude balanced growth path (BGP) analysis. This paper develops a dynamic equilibrium concept for a more general class of models — an alternative to a BGP, which we coin a Stable Transformation Path (STraP). The STraP characterizes the medium-term dynamics of the economy in a turnpike sense; it is the path toward which the economy (quickly) converges from an arbitrary initial capital stock. Calibrated simulations demonstrate that the relaxed parameter values that the STraP allows have important quantitative implications for structural transformation, investment, and growth. Indeed, analyzing the dynamics along the STraP, we show that the modern dynamic model of structural transformation makes progress over the Neoclassical growth model in matching key growth and capital accumulation patterns in cross-country data, including slow convergence.

Keywords: Growth, Investment Dynamics, Non-balanced Growth

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1 Introduction

Structural transformation — the reallocation process of economic activity across sectors, from agriculture to industry and ultimately services — is one of the most empirically salient and well-studied macroeconomic phenomena of economic development.[1] Understanding the processes of industrialization and de-industrialization in particular, as well as their relation to growth, is a chief goal. Investment, investment dynamics, and long-lasting nonhomotheticities are all important aspects of growth and industrialization, but analysis of dynamic structural transformation models with these features is difficult because generally they preclude standard balanced growth path (BGP) analysis.[2] Facing this limitation, researchers in existing work have focused on (i) knife-edge cases with BGPs, but where structural transformation is orthogonal and therefore irrelevant to aggregate growth, or (ii) static models.

In this paper, we develop a new dynamic equilibrium concept — an alternative to a BGP that characterizes the medium-run dynamics of transformation economies and enables analysis and consideration of the investment and growth implications of structural transformation. Specifically, we define a Stable Transformation Path (STraP) as a path from one asymptotic balanced growth path to another. Concretely, our application focuses on an economy that transitions from being initially agricultural to an eventual service economy, moving in and out of industry along the way. Such a path follows from the models that include the latest advances in the field.[3] The path is *stable* in the sense that, along dynamic equilibria, capital quickly converges to the STraP for different initial conditions. The STraP therefore has turnpike-like properties. The dynamics of capital along this convergence reflect standard Neoclassical convergence, whereas the dynamics of capital along the STraP reflect medium-run transformation dynamics toward an asymptotic BGP. Moreover, in addition to defining the STraP, we prove its existence and uniqueness in a general class of models, and address computational challenges by presenting a simple double-recursive shooting

[1] Herrendorf et al. (2014)'s handbook chapters provides an excellent review.

[2] The interactions between investment dynamics and structural change are the focus of recent contributions by García-Santana et al. (2016) and Herrendorf et al. (2018). Boppart (2014) and Comin et al. (2015) show the importance of persistent nonhomotheticities.

[3] See, instance, Boppart (2014), Comin et al. (2015), Duernecker et al. (2017a), Duernecker et al. (2017b), García-Santana et al. (2016), Herrendorf et al. (2018), Moro and Leon-Ledesma (2017), and Kehoe et al. (2018).

algorithm to solve for the STraP. Much like a BGP, the STraP itself can then be used to more easily compute transition paths from initial values off of the STraP.

We then calibrate and simulate a quantitative STraP for a typical model of structural change that starts with simple assumptions, i.e., differential productivity growth, constant elasticity of substitution (CES) sectoral aggregators, common Cobb-Douglas parameters across sectors, and constant intertemporal elasticity of substitution (CIES) preferences. The STraP concept allows us to move the model away from the knife-edge case of log intertemporal preferences and the counterfactual case of investment that only includes manufacturing value added (Herrendorf et al., 2013). Indeed, we can consider structural transformation within the investment sector, which has been recently shown to be empirically important (Herrendorf et al., 2018). This structural transformation leads to time-varying growth in the effective productivity of the investment sector and the relative price of investment, both of which can preclude BGPs.

The simulations show that the benchmark STraP is able to reproduce the salient features of structural transformation and secular growth patterns. The share of agriculture shows a prolonged decline, while services shows a prolonged growth. Interestingly, the simulations yield quantitatively important industrialization and de-industrialization — the hump shape in manufacturing that has eluded previous balanced growth models of structural transformation (Ngai and Pissarides, 2007; Kongsamut et al., 2001). More importantly, the structural transformation in the STraP yields time-varying aggregate productivity growth and a time-varying relative price of investment that affect the aggregate growth process. The model demonstrates a pronounced Baumol's disease slow down in aggregate growth of chain-weighted Gross Domestic Product (GDP), despite the investment rate increasing over time, and the investment rate increases, despite the interest rate declining with development.

In addition, we demonstrate that the model's STraP-enabled departures from earlier parameterizations are important for these implications. These departures not only change the quantitative features of the structural transformation, but they can also affect the qualitative growth patterns. For example, the simplifying assumption of log intertemporal preferences implies a declining investment rate rather than an increasing one, while the simplifying assumption of manufacturing-only investment implies an increasing growth rate rather than a decreasing growth rate.

Next, we introduce nonhomothetic CES preferences into the demand structure, which allows for long-lasting income effects. These preferences alone preclude a BGP, but they also are able to better match sustained structural transformation patterns. We calibrate these preferences following Comin et al. (2015) and analyze the STraP for the nonhomothetic case, which exhibits an even more pronounced pattern of industrialization and de-industrialization.

Finally, we examine the STraP's predictions for the overall investment and growth process relative to empirical patterns in the data. We show that the model predicts persistent, non-balanced patterns: a rising capital-output ratio, a falling relative price of investment, a falling interest rate, and falling growth rates over the course of development, all of which are inline with the patterns in the Penn World Tables' cross-country panel that we document. These patterns contrast starkly with the predictions of the Neoclassical growth model of Ramsey (1928), Cass (1965), and Koopmans (1965), endowed with the capital-to-output ratio observed in poor economies. The Neoclassical growth model implies counterfactually high initial growth rates and interest rates, but rapid convergence of the growth rate, interest rate, and capital-output ratio to the constant BGP, as well as a completely flat relative price of investment.

Thus, the STraP makes progress in addressing the well-known growth convergence puzzle and refocusing it by characterizing the medium-term dynamics of structural transformation. A poor economy, along a shared STraP with advanced economies but with trailing productivity, will grow faster than advanced economies even if each sector's productivity grows at the same rate. Importantly, these higher growth rates are not the product of transitionally low levels of capital. Instead, they are the result of structural transformation shifting resources from high-productivity growth to low-productivity growth sectors, matching the empirically observed, persistently declining pattern of growth rates of GDP over the course of development.

We view the STraP as a natural benchmark for studying sectoral and investment distortions in the macro development process for several reasons. The welfare theorems hold in the undistorted STraP model we present. As noted, the medium-term dynamics of the STraP track the patterns in the data. The stability and relatively quick convergence in our parameterized simulations, make the STraP a natural benchmark from which to evaluate growth trajectories. Since departures from the STraP that stem from initial conditions quickly disappear, more persistent departures may reflect underlying distortions. However, this very lack of a BGP is precisely what makes structural transformation informative about the overall growth process (see Buera and Kaboski, 2009). Thus, studying a richer environment of structural transformation without BGPs opens the door to normatively evaluating the sectoral composition of the economy and sectoral distortions — e.g., assessing the empirical evidence in Rodrik (2016) through the lens of dynamic theory. It also opens the door for sectoral distortions to have a more substantial role in affecting growth trajectories than models with BGPs.

The remainder of the paper is organized as follows. After reviewing the related literature, in Section 2 we present a benchmark structural transformation model. We define the STraP in Section 3 and prove its existence and uniqueness. In Section 4, we lay out the computational algorithm, simulate various STraPs, and show their relevance for understanding structural change and growth patterns. Section 5 concludes.

Related Literature The paper builds on and relates to an existing literature on structural transformation. There are some earlier analyses of non-stationary transformation paths from stable equilibria to asymptotic BGPs. Hansen and Prescott (2002) and Gollin et al. (2002) analyze transitions from stagnant or slow growing agricultural economies to modern growth. These papers provide a simple example of a STraP without recognizing it as a more general dynamic concept to distinguish short-run from medium-term dynamics. We show how to define the STraP in a general class of environments, where we show its existence and uniqueness.

Other papers have tried to reconcile structural transformation with perceived Kaldor (1957)'s stylized facts. Kongsamut et al. (2001) used Stone-Geary nonhomotheticities together with a knife-edge cross-restriction on the preferences and technology to yield a rising service share and declining agricultural share along a BGP. In contrast, Ngai and Pissarides (2007) used biased productivity growth and non-unitary elasticity of substitution across sectors to get structural transformation. They assumed a unitary intertemporal elasticity of substitution and investment that was exclusively manufacturing to yield constant growth in terms of the manufacturing good numeraire. As shown by Buera and Kaboski (2009), the assumptions in both studies effectively divorce growth from structural transformation, making the two phenomena orthogonal. Because we only require BGPs *asymptotically*, the STraP we develop allows for a rich, encompassing set of assumptions in models, including nonhomotheticities, imperfect substitutability of sectoral output in investment and consumption, and productivity growth in all sectors. It therefore reintegrates the twin macro development phenomena: structural transformation and growth. Moreover, empirically we show that Kaldor's stylized facts do not hold over the longer path of development.

The key role of investment in structural transformation has been examined in recent work by García-Santana et al. (2016) and Herrendorf et al. (2018). The former paper argues that the hump shape in the share of value added in the industrial sector can be explained by the combination of a hump shape in the investment rate and the fact that investment is relatively more intensive in industrial value added than consumption. They however assume a constant sectoral composition of investment. The latter paper shows that structural change occurs within the investment sector, and this is inconsistent with a balanced growth path. Acemoglu and Guerrieri (2008) and Ju et al. (2015) also argue that capital accumulation is central to the transformation process, but do not analyze the medium-term dynamics of the STraP. We solve a model with both the level and composition of investment endogenously time-varying, and both are ultimately important in yielding the hump shape of the industrial sector in the STraP.

Preferences incorporating nonhomotheticities with long-lasting income effects have been developed in recent years, in part to address the need for a persistent income effect in services emphasized by Buera and Kaboski (2009). Boppart (2014) applied "price-independent generalized linear" (PIGL) preferences into the structural change literature, and these preferences have been extended to three sectors by Alder et al. (2019). Comin et al. (2015) introduce nonhomothetic constant elasticity of substitution (CES) preferences as another alternative for long-lasting nonhomotheticities in an n-sector model that does not impose a correlation between income and price elasticities. We adopt these nonhomothetic CES preferences because they preserve the constant price elasticity, which allows us to have a clear nonhomothetic analog to the homothetic CES case that can be embedded into the standard CIES intertemporal preferences. Similar to this previous research, we find that nonhomotheticities help to better match empirical patterns, but we also show that departures from log intertemporal preferences are important.

This paper also relates to the normative literature on structural transformation. Wedge-based normative analyses of structural transformation, including work on the agricultural productivity gap (Gollin et al., 2014) and work on the distortions in command economies (Cheremukhin et al., 2017a,b), have focused exclusively on static distortions or distortions for a given level of capital. The STraP allows for a broadening of these analyses because economies converge to the STraP for different initial conditions. The STraP constitutes a benchmark dynamic model for interpreting the optimality in the aggregate level of capital (given technology levels), and one can therefore infer and interpret distortions away from this benchmark as reflecting dynamic intertemporal distortions. Such an analysis is precisely what our contemporaneous work Buera et al. (2019) undertakes.

Finally, in showing that competitive equilibria converge to the STraP for different initial conditions, our work relates to an early literature studying the turnpike properties of the Neoclassical growth model (see McKenzie, 1986, and references therein). While standard turnpike theorems state that dynamic equilibrium asymptotically approaches a stationary equilibrium, our numerical analyses show that along the transition to the asymptotic stationary equilibrium these trajectories first approach the STraP. Relatedly, when realistic calibrations are used, the convergence to the STraP is fast.[4]

2 Model

In this section, we start with a typical model of investment and structural transformation based on Ngai and Pissarides (2007) but then introduce an investment aggregator as in Herrendorf et al. (2018), which allows for a more generally time-varying relative price of investment.

[4]In turn, this result is reminiscent of the fast convergence of the Neoclassical growth model to the balance growth path (King and Rebelo, 1993).

2.1 Environment

Consider a standard continuous time intertemporal problem of a representative household with constant intertemporal elasticity preferences over a consumption aggregate $C(t)$. The household exogenously provides labor which earns a wage $w(t)$, and owns capital, $K(t)$, which earns a rental rate, $R(t)$. Capital depreciates at a rate $\delta \in [0,1]$, but can be accumulated through investment, $X(t)$. A bond, $B(t)$, which is priced in units of consumption and pays off in units of consumption, is in zero net supply, but prices the (consumption-based) interest rate, $r(t)$. The household's problem is therefore:

$$\max_{C(t),X(t),K(t),B(t)} \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} \frac{C(t)^{1-\theta}}{1-\theta} \tag{1}$$

s.t.

$$P_c(t)\,C(t) + P_x(t)\,X(t) + P_c(t)\,\dot{B}(t) = W(t)\,L + R(t)\,K(t) + r(t)\,P_c(t)\,B(t) \tag{2}$$

and

$$\dot{K}(t) = X(t) - \delta K(t)\,. \tag{3}$$

Note that consumption and investment have distinct, time-varying prices, $P_c(t)$ and $P_x(t)$, respectively. To this investment problem, we add structural transformation, which can impact the price of investment relative to consumption. Specifically, we assume the household also faces an intratemporal problem of choosing consumption of value added from agriculture, $C_a(t)$, manufacturing, $C_m(t)$, and services, $C_s(t)$, to produce the consumption aggregate:

$$C(t) = \left[\sum_{j=a,m,s} \omega_{cj}^{\frac{1}{\sigma_c}} C_j(t)^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}},$$

where we normalize the CES weights, $\sum_{j=a,m,s} \omega_{cj} = 1$. Consistent with standard structural change patterns, we further assume that sectors are gross complements, i.e., $\sigma_c < 1$.

A competitive firm uses a similar CES aggregator to take value added in agriculture, $X_a(t)$, manufacturing, $X_m(t)$, and services $X_s(t)$ and produce the final investment aggregate, $X(t)$:

$$X(t) = A_x(t)\left[\sum_{j=a,m,s} \omega_{xj}^{\frac{1}{\sigma_x}} X_j(t)^{\frac{\sigma_x - 1}{\sigma_x}}\right]^{\frac{\sigma_x}{\sigma_x - 1}}. \tag{4}$$

Note that the weights again sum to one, $\sum_{j=a,m,s} \omega_{xj} = 1$, but they are specific to the investment sector. Note also that the investment aggregator also differs from the intratemporal utility function in that it experiences sector-neutral technological change through $A_{x,t}$, which we assume occurs at a constant rate:

$$\dot{A}_x(t) = \gamma_x A_x(t), \tag{5}$$

with $\gamma_x > 0$.

Finally, we note that the elasticity of substitution in the investment aggregator can potentially differ from that in the consumption aggregator. However, consistent with standard structural change patterns, we again assume that sectors are gross complements in investment, i.e., $\sigma_x < 1$.

A competitive representative firm in each sector $j \in \{a, m, s\}$ produces value added using common Cobb-Douglas technologies, except for a factor-neutral productivity parameters, A_j, which vary by sector:

$$C_j(t) + X_j(t) = A_j(t) K_j(t)^{\alpha} L_j(t)^{1-\alpha}. \tag{6}$$

These productivities grow at constant rates:

$$\dot{A}_j(t) = \gamma_j A_j(t), \tag{7}$$

where the growth rates are also sector-specific and ordered as follows: $\gamma_a > \gamma_m > \gamma_s > 0$.

Finally, feasibility requires that the labor and capital used by each sector be less than the aggregate supply:

$$\sum_{j=a,m,s} L_j(t) \leq L \tag{8}$$

and

$$\sum_{j=a,m,s} K_j\left(t\right) \leq K\left(t\right). \tag{9}$$

2.2 Equilibrium

Analysis of the equilibrium conditions of the model gives intuition for (i) the important roles for both the effective productivity of the investment sector and the relative price of investment and (ii) how structural change leads to its growth rate varying over time and precludes an *aggregate balanced growth path* (aggregate BGP) for general parameter values.[5] This result is a simple three-sector extension of the results in Herrendorf et al. (2018).

We start with the the Euler equation for the households dynamic problem:

$$\theta\frac{\dot{C}\left(t\right)}{C\left(t\right)} = r\left(t\right) - \rho = \frac{R\left(t\right)}{P_x\left(t\right)} - \delta - \rho + \left(\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right), \tag{10}$$

which is the standard single-sector Euler equation except for two differences on the right-hand side: the interest rate involves (i) the growth rate of relative price of investment, $\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}$, and (ii) the rental rate of capital *in terms of investment.* We will highlight the importance of these differences.

The second dynamic equation is the law of motion for capital:

$$\frac{\dot{K}\left(t\right)}{K\left(t\right)} = \frac{X\left(t\right)}{K\left(t\right)} - \delta = \frac{P\left(t\right)Y\left(t\right)}{P_x\left(t\right)K\left(t\right)} - \frac{P_c\left(t\right)C\left(t\right)}{P_x\left(t\right)K\left(t\right)} - \delta.$$

In the first equation one can see that constant growth in capital requires real investment and capital to grow at a constant rate. Using the definition of total output, $P(t)Y(t) = P_c(t)C(t) + P_x(t)X(t)$ and substituting in for $X(t)$, one can see in the second equation that this implies constant growth in output and consumption expen-

[5]Following Ngai and Pissarides (2007), Herrendorf et al. (2018) define an aggregate BGP as an an equilibrium path along which aggregate variables (expressed in a common unit) grow at constant, though potentially different, rates. This latter characteristic allows for structural change.

ditures *when translated into units of the investment good*.[6] Moreover, it is not real consumption that grows at a constant rate, but consumption *expenditures* (in units of investment).

Defining $\tilde{C}(t) \equiv P_c(t) C(t) / P_x(t)$, the Euler equation becomes

$$\theta \frac{\dot{\tilde{C}}(t)}{\tilde{C}(t)} = \frac{R_{t+1}}{P_{x,t+1}} - \delta - \rho + (1-\theta)\left[\frac{\dot{P}_x}{P_x} - \frac{\dot{P}_c}{P_c}\right].$$

Herrendorf et al. (2018) show that for our preferences, an assumption of log intertemporal preferences (i.e., $\theta = 1$) and a constant productivity growth rate in investment are necessary and sufficient for such a balanced growth path. One can see that $\theta = 1$ eliminates the problematic role of non-constant growth in the relative price of investment, since the household does not respond to it. As we will show, constant growth in productivity in investment production leads the rental rate of capital in units of investment to also be constant, but structural change in investment precludes this constant productivity growth in our model.

To study the dynamics of the relative price of investment, we start by solving for the prices of value added. The cost-minimizing competitive price for value added in sector j is

$$P_j(t) = \frac{1}{A_j(t)} \left(\frac{R(t)}{\alpha}\right)^{\alpha} \left(\frac{W(t)}{1-\alpha}\right)^{1-\alpha} \tag{11}$$

Hence, given the common Cobb-Douglas parameter for all sectors, relative prices become the inverse of relative productivities:

$$\frac{P_j(t)}{P_{j'}(t)} = \frac{A_{j'}(t)}{A_j(t)}. \tag{12}$$

Hence, given $\gamma_a > \gamma_m > \gamma_s > 0$, prices move differentially; relative to manufacturing, the price of services rises and the price of agriculture falls. These feed into the price indexes for consumption and investment, which follow from cost-minimization:

[6] It is the fact that investment enters the law of motion in terms of real units of capital, that other variables must have constant growth in units of investment. This is the reason that Ngai and Pissarides (2007) and Herrendorf et al. (2018) choose investment as a numeraire. We do not choose a numeraire at this point in order to make the role of the relative price of investment more explicit.

$$P_c(t) = \left[\sum_{j=1}^{J} \omega_{cj} P_j(t)^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}}, \tag{13}$$

and

$$P_x(t) = \frac{1}{A_x(t)} \left[\sum_{j=1}^{J} \omega_{xj} P_j(t)^{1-\sigma_x} \right]^{\frac{1}{1-\sigma_x}}. \tag{14}$$

The relative price of investment is then

$$\frac{P_x(t)}{P_c(t)} = \frac{1}{A_x(t)} \frac{\left[\sum_{j=1}^{J} \omega_{xj} P_j(t)^{1-\sigma_x} \right]^{\frac{1}{1-\sigma_x}}}{\left[\sum_{j=1}^{J} \omega_{cj} P_j(t)^{1-\sigma_c} \right]^{\frac{1}{1-\sigma_c}}}. \tag{15}$$

The structural transformation model provides a theory for an endogenously time-varying relative price of investment. This relative price can trend for three reasons: (i) technical progress in the investment aggregator, $A_x(t)$; (ii) the CES weights differing across consumption and investment; and (iii) the elasticities differing across consumption and investment. The last two will typically lead to different and changing compositions of agriculture, manufacturing, and service value added across investment and consumption, and so differential rates of price changes in value-added will lead to differential rates of change of the relative price. Our model allows for all three of these, and we allow for the case when $\theta \neq 1$, when the changing relative price of investment does not allow for a balanced growth path. Note that even if the investment aggregator had no structural transformation because it only included manufacturing (i.e., $\omega_{xm} = 1$), a BGP would not exist in this case.[7]

We now turn to the second important feature of the modified Euler equation: the rental rate of capital in units of the investment good, $\frac{R(t)}{P_x(t)}$.

Since all value-added production shares the same Cobb-Douglas parameter, it all uses the same capital-labor ratio, $K_j(t)/L_j(t) = \alpha/(1-\alpha) W(t)/R(t) = K(t)/L(t)$. One can solve for an aggregate production function for the investment sector in terms of the total capital and labor embodied in the value-added aggregated into investment:

[7] Ngai and Pissarides (2007) achieve a BGP for the special case where $\omega_{xm} = 1$ and $\theta = 1$.

$$X(t) = \mathcal{A}_x(t)\, K_x(t)^{\alpha}\, L_x(t)^{1-\alpha},$$

where

$$\mathcal{A}_x(t) = A_x(t)\left[\sum_{j=a,m,s} \omega_{xj} A_j(t)^{\sigma_x - 1}\right]^{\frac{1}{\sigma_x - 1}} \tag{16}$$

Herrendorf et al. (2018) refer to $\mathcal{A}_x(t)$ as *effective* productivity because it includes both the direct productivity of the aggregator, and the productivities in producing the different sector j value-added components from labor.[8] From this, one can easily calculate the rental rate of capital in units of the investment good:

$$\frac{R(t)}{P_x(t)} = \alpha \mathcal{A}_x(t)\left(\frac{K(t)}{L}\right)^{\alpha-1} \tag{17}$$

Examining (17), since $K(t)$ must grow at a constant rate on a BGP, $\mathcal{A}_x(t)$ must also grow at a constant rate on a BGP. However, Equation (16) shows that this effective productivity of investment, like the price of investment, is subject to the changing composition of value added, and so it will not grow at a constant rate. Thus, the presence of structural change in the investment aggregator leads to the lack of a BGP, even in the case of log intertemporal preferences.

One can also analogously solve for an aggregate production function for the consumption sector:

$$C(t) = \mathcal{A}_c(t)\, K_c(t)^{\alpha}\, L_c(t)^{1-\alpha},$$

where

$$\mathcal{A}_c(t) = \left[\sum_{j=a,m,s} \omega_{cj} A_j(t)^{\sigma_c - 1}\right]^{\frac{1}{\sigma_c - 1}}. \tag{18}$$

Finally, we note that the wage is the value of the marginal product of labor:

$$W(t) = (1-\alpha)\, P_x(t)\, \mathcal{A}_x(t)\left(\frac{K(t)}{L}\right)^{\alpha}. \tag{19}$$

[8] Herrendorf et al. (2018) also refer to this as a "pseudo" aggregate production function because it holds in equilibrium rather than as a primitive.

Definition 1. *Given an initial state consisting of* $K(0)$, $A_x(0)$, *and* $\{A_j(0)\}_{j=a,m,s}$, *a* ***competitive equilibrium*** *for the model is:*

- *an allocation,* $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$; *and*
- *prices,* $P_c(t)$, $P_x(t)$, $W(t)$, $R(t)$, $r(t)$ *and* $\{P_j(t)\}_{j=a,m,s}$;

for $t \geq 0$ *that solve:*

- $B(t) = 0$;
- *equations (2)-(11), (13), (14), and (16)-(19); and*
- *the transversality condition,* $\lim_{t\to\infty} e^{-\rho t} C(t)^{-\theta} K(t) = 0$.

Although straightforward to define, an equilibrium is neither conceptually nor computationally straightforward to solve, since the model lacks a BGP toward which an economy quickly converges.

3 The STraP

In this section, we introduce the new concept of a stable growth path, the STraP, define it formally, and show its existence and uniqueness.

We start with some intuitive motivation for the STraP. First, the value-added technologies in the model have diminishing returns to capital, which imply Neoclassical convergence dynamics with a speed of convergence that is rapid given the elasticity of output with respect to capital (α), the productivity growth rate, and depreciation. This can be easily seen in the asymptotic limit, where the model implies in the long run a single-sector service economy BGP with purely Neoclassical features. However, in the medium run — the time period of structural transformation — the target capital stock toward which Neoclassical dynamics aims to converge is moving because structural transformation causes time-varying growth rates of effective investment productivity and relative price of capital that influence the return to investing. This medium-run path has turnpike-like characteristics in that regardless of initial values, the economy converges first toward this path, which we call the STraP.

How might one conceptualize and characterize this path? Given that the Neoclassical convergence dynamics are rapid, one could simply start from an arbitrary initial capital stock at time 0 and let the economy converge to it. The STraP itself, however, is driven by productivity dynamics, not initial capital conditions. If we wanted to eliminate the role of initial conditions and know the STraP at time 0, we could construct the "what if" exercise of assuming the productivity path had started at some earlier time ($\tau << 0$) such that the economy's capital stock had already converged to the STraP by time 0. If we wanted to trace the STraP for all periods of time, we could go to the hypothetical extreme of assuming that the productivity process had existed eternally. However, to do this, we note that just as the economy ultimately converges to a single-sector service economy governed by the productivity growth rate of services as $t \to \infty$, the hypothetical "eternal" productivity process implies a single-sector agricultural economy governed by the productivity growth rate of agriculture value added, as a limiting case as $t \to -\infty$. Although this case is hypothetical — the productivity process may not have been valid for time periods before t=0 — consideration of this hypothetical limit helps to solve for the stable medium-run dynamics.

We therefore utilize the two limiting asymptotic BGPs. In the case of $t \to \infty$ the growth rates of effective investment productivity, $\mathcal{A}_x(t)$, converges to a constant $\gamma_x + \gamma_s$, while in the case of $t \to -\infty$ it converges to an analogous $\gamma_x + \gamma_a$. In both cases, the composition of investment and consumption are identical to each other, so the growth rate of the relative price of investment converges to $-\gamma_x$, and, similarly, in both cases the normalizing factor is $\mathcal{A}_x(t)^{1/(1-\alpha)}$. We use lowercase to indicate normalized variables and, therefore, define the limiting normalized capital stocks for these two economies as:

$$\bar{k}_{-\infty} \equiv \left[\frac{\alpha}{\delta + \rho + \left(1 + \theta\frac{\alpha}{1-\alpha}\right)(\gamma_x + \gamma_a) - (1-\theta)\gamma_a}\right]^{1/(1-\alpha)}$$

and

$$\bar{k}_{\infty} \equiv \left[\frac{\alpha}{\delta + \rho + \left(1 + \theta\frac{\alpha}{1-\alpha}\right)(\gamma_x + \gamma_s) - (1-\theta)\gamma_s}\right]^{1/(1-\alpha)}.$$

3.1 Defining the STraP

We can now define the STraP as the time path of objects connecting the asymptotic agricultural BGP to the asymptotic service BGP.

Definition 2. *Given initial productivities, $A_x(t)$, and $\{A_j(t)\}_{j=a,m,s}$, the* ***Stable Transformation Path (STraP)*** *is:*

- *an allocation, $C(t)$, $K(t)$, $X(t)$, $\{C_j(t), X_j(t), K_j(t), L_j(t)\}_{j=a,m,s}$; and*
- *prices, $P_c(t)$, $P_x(t)$, $W(t)$, $R(t)$, $r(t)$ and $\{P_j(t)\}_{j=a,m,s}$;*

defined $\forall t \in \mathbb{R}$ that solves:

- *$B(t) = 0$;*
- *equations (2)-(11),(13), (14), and (16)-(19); and*
- *asymptotic conditions, $\lim_{t\to\infty} \frac{K(t)}{\mathcal{A}_x(t)^{1/(1-\alpha)}} = \bar{k}_\infty$, and $\lim_{t\to-\infty} \frac{K(t)}{\mathcal{A}_x(t)^{1/(1-\alpha)}} = \bar{k}_{-\infty}$.*

Comparing the STraP to the definition of an equilibrium at the end of the previous section, we see two main differences. First, whereas an equilibrium is only defined forward from an initial value, $t = 0$, the STraP is defined for all real numbers. Second, an equilibrium is solved for a specific initial value and asymptotic boundary condition (the transversality condition) of the capital stock, while the STraP uses two asymptotic boundary conditions.

The fact that the initial value of capital, $K(0)$, is not an arbitrary initial condition in the STraP implies that, given the productivity process, the STraP passes through *a particular value of* $K(0)$, call it $K(0)_{STraP}$. That is, the STraP gives a particular time path of capital that is stable for the productivity process, whereas an equilibrium can be defined for any positive value of $K(0)$. The STraP path for capital is stable in that for $K(0) \neq K(0)_{STraP}$, the dynamic equations in the equilibrium will lead capital to converge to the STraP level of capital over time via standard Neoclassical convergence dynamics. In simulation, these Neoclassical convergence dynamics are quick, as they are in the single-sector model. In the next subsection, we formally define a more general version of the STraP and prove its existence and uniqueness.

3.2 Existence and Uniqueness of the STraP

We start by presenting a more general class of growth models for which we will prove existence and uniqueness of the STraP. We consider models in which the First and Second Welfare Theorems hold, so that competitive equilibria coincide with solutions to the planner's problem. In particular, we suppose that starting at an arbitrary time, $-\tau$, the planner's intertemporal problem can be written

$$\max_{c(t),k(t)} \int_{t=-\tau}^{\infty} e^{-\rho t} \mathcal{A}_u(t) u(c(t),t) dt, \tag{20}$$

where

$$\dot{k}(t) = f(k(t),t) - (\delta + \gamma_k(t))\, k(t) - c(t). \tag{21}$$

To ensure everything is well behaved, we assume that $\gamma_u(t), \gamma_k(t)$ are continuously differentiable; u is three times continuously differentiable, strictly concave for each t, and $\lim_{c\to 0} u'(c,t) = \infty$ for all t; the function $f(k,t)$ is twice continuously differentiable and satisfies the Inada conditions for each t; and $\gamma_k(t) > 0$ for all t.

For the concept of the STraP to make sense, we need that this problem converges to a standard optimal growth problem in the limits as $t \to \pm\infty$. This means the growth rates, the production function, and the utility function all need to converge. Defining $\gamma_u(t) \equiv \frac{\dot{\mathcal{A}}_u(t)}{\mathcal{A}_u(t)}$, we assume

$$\lim_{t\to\infty} \gamma_u(t) = \gamma_{+u} > 0, \lim_{t\to-\infty} \gamma_u(t) = \gamma_{-u} > 0,$$

$$\lim_{t\to\infty} \gamma_k(t) = \gamma_{+k} > 0, \lim_{t\to-\infty} \gamma_k(t) = \gamma_{-k} > 0.$$

For the production functions, we assume that there exist functions $f_+(k), f_-(k)$ such that

$$\lim_{t\to\infty} f(k,t) = f_+(k), \lim_{t\to-\infty} f(k,t) = f_-(k),$$

$$\lim_{t\to\infty} \frac{\partial f(k,t)}{\partial k} = f'_+(k), \lim_{t\to-\infty} \frac{\partial f(k,t)}{\partial k} = f'_-(k)$$

uniformly on $k \in [\varepsilon, \bar{k}]$ for all $\varepsilon, \bar{k} > 0$. Similarly, we assume there exist functions $u_+(c)$ and $u_-(c)$ such that

$$\lim_{t\to\infty} u(c,t) = u_+(c), \lim_{t\to-\infty} u(c,t) = u_-(c),$$

$$\lim_{t\to\infty} \theta(c,t) = \theta_+(c), \lim_{t\to-\infty} \theta(c,t) = \theta_-(c)$$

uniformly on $c \in [\varepsilon, \bar{C}]$ for all $\varepsilon, \bar{C} > 0$, where $\theta(c,t) \equiv -\frac{\frac{\partial^2 u(c,t)}{\partial c^2} c}{\frac{\partial u(c,t)}{\partial c}}$. To these assumptions, we add the assumption that $\gamma_{+u}, \gamma_{-u} < \rho$ so that utility is well defined shooting forward and in both asymptotic balanced growth paths.

Our model in Section 2 is a special case of this more general setting. Once normalized, consumption and investment correspond to those in the model. Similarly, $u(c,t) = \frac{c^{1-\theta}}{1-\theta}$, $f(k,t) = k^\alpha$, $\gamma_k(t) = \frac{1}{1-\alpha}\frac{\dot{\mathcal{A}}_x(t)}{\mathcal{A}_x(t)}$, $\mathcal{A}_u(t) = \left(\mathcal{A}_c(t)\mathcal{A}_x(t)^{\frac{\alpha}{1-\alpha}}\right)^{1-\theta}$, which all fit the model assumptions. It is also straightforward to show that the Welfare Theorems hold in the model of Section 2 using standard techniques as in Acemoglu (2008), for example.[9]

Similarly, we can define the asymptotic conditions more generally. Denote k_∞, the asymptotic steady state level of capital, as the steady state level of capital for the problem

$$\max_{c(t),k(t)} \int_0^\infty e^{(-\rho+\gamma_{+u})t} u_+(c(t))dt,$$

where

$$\dot{k}(t) = f_+(k(t)) - (\delta + \gamma_{+k})\, k(t) - c(t),$$

and $k_{-\infty}$ as the steady state levels of capital for the problem

$$\max_{c(t),k(t)} \int_0^\infty e^{(-\rho+\gamma_{-u})t} u_-(c(t))dt,$$

where

$$\dot{k}(t) = f_-(k(t)) - (\delta + \gamma_{-k})\, k(t) - c(t).$$

Naturally, there are also corresponding steady state levels of consumption, c_∞ and $c_{-\infty}$.

[9]In the online appendix, we formally show the above mapping, as well as the mapping for the nonhomothetic CES case in Section 4.4.

For the sake of completeness and ease of stating the theorem, we now restate the definition of the STraP in this more general set up.

Definition 3. *Given time-τ productivities, $\mathcal{A}_u(\tau)$ and $\gamma_k(\tau)$, the* ***Stable Transformation Path (STraP)*** *is an allocation, $c(t)$ and $k(t)$ defined $\forall t \in \mathbb{R}$, that solves the maximization problem in equations (20) and (21) and satisfies the boundary conditions:*

- $\lim_{t\to\infty} k(t) = k_\infty$*; and*
- $\lim_{t\to-\infty} k(t) = k_{-\infty}$.

Given the model and definitions, we make an additional assumption.

Assumption 1. *There exists a function $h : \mathbb{R} \to (0,1)$ such that:*

- *h is strictly increasing and invertible,*
- *Both h and h^{-1} are twice continously differentiable,*
- $\lim_{t\to\pm\infty} h'(t)$ *exists,*
- $\lim_{t\to\pm\infty} \frac{\dot{\gamma}_k(t)}{h'(t)} = \lim_{t\to\pm\infty} \frac{\dot{\gamma}_u(t)}{h'(t)} = 0,$
- $\lim_{t\to\pm\infty} \frac{\frac{\partial f(k,t)}{\partial t}}{h'(t)} = \lim_{t\to\pm\infty} \frac{\frac{\partial^2 f(k,t)}{\partial k \partial t}}{h'(t)} = 0$ *uniformly on* $k \in [\underline{\varepsilon}, \bar{k}]$,
- $\lim_{t\to\pm\infty} \frac{\frac{\partial \theta(c,t)}{\partial t}}{h'(t)} = 0$ *uniformly on* $c \in [\varepsilon, \bar{C}]$ *for all* $\varepsilon > 0$,
- $\lim_{t\to\infty} \frac{h''(t)}{h'(t)} = a_+ \in (-\infty, 0), \lim_{t\to-\infty} \frac{h''(t)}{h'(t)} = a_- \in (0, \infty).$

We construct an $h(t)$ in the online appendix that satisfies Assumption 1 for any monotonic, twice-differentiable $\gamma_u(t)$ and $\gamma_k(t)$, including the model in Section 2. The theorem can now be stated quite simply.

Theorem 1. *If Assumption 1 holds, then the STraP exists and is unique.*

We provide a brief overview of the proof here and leave the formal proof to online appendix A. Given the setup, the Hamiltonian conditions and transversality condition, which k_∞ satisfies, are sufficient to yield a unique path forward from any $k(t) > 0$. We denote the unique optimal consumption level by $c(k, t)$. This simplifies the system to a 1-dimensional non-autonomous system in $k(t)$. Proving existence and uniqueness of the STraP therefore comes down to proving that from time τ there exists one unique path that has $k(t) \to k_{-\infty}$ as $t \to -\infty$.

We apply an existing theorem (Theorem 4.7.5 in Hubbard and West, 1991) for the existence and uniqueness of time paths in an antifunnel, which requires several conditions: 1) a single differential equation, 2) a Lipschitz condition within the antifunnel, 3) narrowing upper and lower fences that define the antifunnel; and 4) a condition on the derivative of the right-hand side of the differential equation that bounds it away from $-\infty$ in a particular sense.[10] Verifying these conditions requires characterizing $c(k, t)$ to some extent. The function $h(t)$ in Assumption 1 is used to transform the original non-autonomous 2-dimensional system into a more easily analyzed 3-dimensional autonomous system by including time as a variable and reparameterizing it onto the compact interval $[0, 1]$ interval. This requires that the system is well behaved in the limit as $t \to \pm\infty$, which the conditions in Assumption 1 ensure.

That reparametrization allows us to show that $c(k, t)$ gets arbitrarily close to the consumption function in the negative asymptotic growth problem. With that, we can construct the upper and lower narrowing fences that define the antifunnel and verify that the other conditions hold.

4 STraP Implications for Medium-Term Structural Change and Growth

In this section, we: (i) provide an algorithm to solve for the STraP, (ii) calibrate the model and show the relevance of the relaxed parameterizations it enables for structural change and growth dynamics, (iii) extend the model to nonhomothetic CES, and (iv) show the STraP's implications for aggregate development patterns. The medium-

[10]Heuristically, a fence is a one-way gate for a dynamic path. A lower fence pushes solutions up; an upper fence pushes them down. An antifunnel is a region defined by an upper and a lower fence, as lower and upper limits, respectively, which a path, starting from the outside, cannot enter. Narrowing is the property that the size of this region shrinks to 0. For a deeper explanation of antifunnels and fences, see Hubbard and West (1991). Chapter 1 gives a nice intuitive introduction.

term STraP dynamics allow us to better understand important development patterns in the data, including slow convergence, which constitutes a well-known puzzle for the Neoclassical growth model.

4.1 Computing the STraP

Although Theorem 1 ensures that a unique STraP exists, an issue of practical relevance is how to solve for the STraP. For its computation, we return to the more specific model in Section 2 and move to discrete time. We maintain the same growth notation, but we use the discrete analogs, e.g., $A_{x,t+1}/A_{x,t} = 1 + \gamma_{x,t}$, and the discount rate, ρ, is replaced by the discount factor, β.[11] The computational algorithm we present is a double-recursive shooting algorithm. We recursively shoot both forwards and backwards. Again, we normalize values by the effective investment productivity, $\mathcal{A}_{x,t}^{1/(1-\alpha)}$, and continue denoting normalized variables using lowercase letters. In an inner loop, we shoot forward, solving for a time 0 value of consumption expenditures that asymptotically leads to the services BGP. In an outer loop, we shoot backwards and solve for a time 0 value of capital that asymptotically leads to the agriculture BGP. In practice, it is quite difficult to shoot toward an asymptotic BGP, which requires more precision at the initial value of consumption expenditure, $\tilde{c}_0$, than is computationally practical. Instead, we find a reasonably precise initial $\tilde{c}_0$, but we allow for small adjustments midpath that keep the overall path following the ideal path with a high level of precision. A useful analogy is a hypothetical launch of a rocket toward a planet in another solar system, where over time small deviations from the ideal launch angle could compound and require small retro-rocket adjustments to keep the rocket on target.

To make our computation, we assign initial values for $A_{x,0}$ and $A_{j,0}$ for $j = a, m, s$,

[11] In discrete time, the analogous Euler equation and law of motion for capital are

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta} = \beta\left[1 - \delta + \frac{R_{t+1}}{P_{x,t+1}}\right]\left[\frac{P_{x,t+1}/P_{c,t+1}}{P_{x,t}/P_{c,t}}\right]^{1-\theta}.$$

and

$$\frac{K_{t+1}}{K_t} = \frac{X_t}{K_t} + (1 - \delta),$$

respectively.

and, for the sake of convenience, we now declare investment as the numeraire. Given these, the steps of the algorithm are as follows:

1. **Define initial bounds for k_0.** We solve for k_0 using the bisection method, which requires an upper and lower bound. Clearly, $\bar{k}_\infty$ and $\bar{k}_{-\infty}$ are candidate upper and lower bounds, but one can use the fact that the relative price of investment is increasing to bound it more tightly. One can solve for a pseudo-BGP level of capital, which is the normalized level of capital that would result if the effective productivities $\mathcal{A}_{x,t}$ and $\mathcal{A}_{c,t}$ grew at their initial rates perpetually:

$$k_{pseudoBGP} = \left[\frac{\alpha\beta\mathcal{A}_{x,0}}{\left(1 + \frac{\mathcal{A}_{x,1}}{\mathcal{A}_{x,0}}\right)^{1+\frac{\alpha\theta}{(1-\alpha)}} \left(1 + \frac{\mathcal{A}_{c,1}}{\mathcal{A}_{c,0}}\right)^{\theta/(1-\alpha)} - \beta\left(1-\delta\right)} \right]^{1/(1-\alpha)}$$

   A lower bound that is sufficiently low can then be chosen. In our simulations we chose this as $0.9 * k_{pseudoBGP}$.

2. **Choose a trial value for k_0.** Using the bisection method, we choose the midpoint of the two bounds.

3. **Define initial bounds for $\tilde{c}_0$.** Again, we use the bisection method and choose upper and lower bounds. Consumption expenditures are naturally bounded between 0 and y_0, but we choose tighter bounds: $1.2 * \tilde{c}_{pseudoBGP}$ and 10 percent of initial output, $0.1 * k_0^\alpha$.

4. **Choose a trial value for $\tilde{c}_0$.** We choose the midpoint between the upper and lower bound.

5. **Shoot forward toward $\bar{k}_\infty$.** In principle, we would consider it a successful shot if we get sufficiently close to a stable value of capital close to the target, i.e., k_t within a tolerance of $\bar{k}_\infty$ and k_{t+1}/k_t within a tolerance of one. In this case, we would skip to the backward shooting in Step 7. However, in practice, we also stop the shooting attempt at any point in which either $\tilde{c}_{t+1} < \tilde{c}_t$ or $k_{t+1} < k_t$. (Define this point of divergence as t^*.) If the former, assign the $\tilde{c}_0$ as the new lower bound; if the latter assign $\tilde{c}_0$ as the new upper bound. Check if the bounds on $\tilde{c}_0$ are sufficiently tight. If not, return to Step 4.

6. **Update t_0 to shoot recursively.** Regardless of the precision of $\tilde{c}_0$, we have found a point of divergence, t^*. Our recursive approach is to back up from this point to some $t_n < t^*$. Practically, we choose t_n as the nearest period to $0.95 * t^*$. We then return to Step 3 at this new t_0. (Note that one need not shoot forward until convergence to the asymptotic BGP, i.e., within a tolerance of $\bar{k}_\infty$, but one can stop shooting whenever the desired simulation period is finished.)

7. **Shoot backward (i.e., $t \to -\infty$) from k_0 and $\tilde{c}_0$ toward $\bar{k}_{-\infty}$.** Here we iterate backwards using the Euler equation and the laws of motion for capital and technology. Again, we consider it a successful shot if we get sufficiently close to a stable value of capital close to the target, i.e., k_t within a tolerance of $\bar{k}_{-\infty}$ and k_t/k_{t-1} within a tolerance of one. In this case, we are finished. However, we also stop the shooting attempt if capital diverges too strongly at any point. If capital becomes too large (we choose $k_{t-1} > \bar{k}_\infty$), we update the upper bound on k_0. If capital gets too small (we choose $k_{t-1} < 0.01$), we update the lower bound. We then return to Step 2.

The stopping procedure in the last step further illustrates the fact that capital is only backwards stable along the STraP.

4.2 Calibration

To illustrate and compute the STraP, we start with a benchmark model that stick closely with the existing literature but adds in the new variations that the STraP allows for. Namely, we assign standard values for depreciation and the capital elasticity parameter, $\delta = 0.05$ and $\alpha = 0.33$. We choose an intertemporal elasticity parameter of $\theta = 2$, which diverges from the log utility in the Ngai and Pissarides (2007) and Herrendorf et al. (2018) but is a more common value in the broader macro literature.[12] We calibrate the rest of the parameters to match the available time series for the United States.[13] The relative sectoral productivity growth rates are calibrated

[12] Havránek (2015)'s meta-analysis of the 2,735 estimates of the elasticity of intertemporal substitution in consumption from 169 published studies covering 104 countries produces a mean of 0.5, which implies $\theta = 2$.

[13] We combine the historical GDP by Industry data 1947–1997 together with the 1997–2018 to yield sectoral prices and value-added. We use aggregate real (chain-weighted) GDP, real personal

using time series data on relative prices according to equations (12) and (15). The absolute growth rates are scaled to match growth in income per capita. This yields values of $\gamma_a = 0.050$, $\gamma_m = 0.021$, $\gamma_s = 0.012$, and $\gamma_x = 0.0026$. Absent nonhometheticities, Leontieff substitution between sectors provides a best fit to long run data (Buera and Kaboski, 2009). We therefore assign a common elasticity of substitution for consumption and investment that approaches Leontieff, $\sigma_c = \sigma_x = 0.01$. We then calibrate the aggregator weights to match the average shares over the time series, where we use the input-output tables to yield the sectoral composition of investment. These values are $\omega_{c,a} = 0.013$, $\omega_{x,a} = 0.015$, $\omega_{c,m} = 0.231$, $\omega_{x,m} = 0.502$, $\omega_{c,s} = 0.756$, and $\omega_{x,s} = 0.483$.[14] Finally, we choose $\beta = 0.99$, to match the average interest rate. Gomme et al. (2011) calculate the after tax return to business capital, which averages 6.1 percent between 1950-2000. This yields $\beta = 0.99$.

4.3 Implications for structural transformation, growth, and investment

In this section, we show that the calibrated benchmark STraP combines interesting implications for structural transformation, growth, and investment patterns. Simulations of the key growth variables for the benchmark economy are displayed in Figure 1.

The top panel of Figure 1 shows the structural transformation patterns over time by tracking the current-value, value-added share of each sector over time. The patterns of the benchmark economy replicate the qualitative empirical patterns of most countries, including a sharp decline of the share of agriculture that asymptotes toward zero, a hump shape in manufacturing's share (with a peak under 50 percent of the economy), and an increase and late acceleration of the share of services that eventually constitutes the majority of the economy. Using industry as a gauge, one can

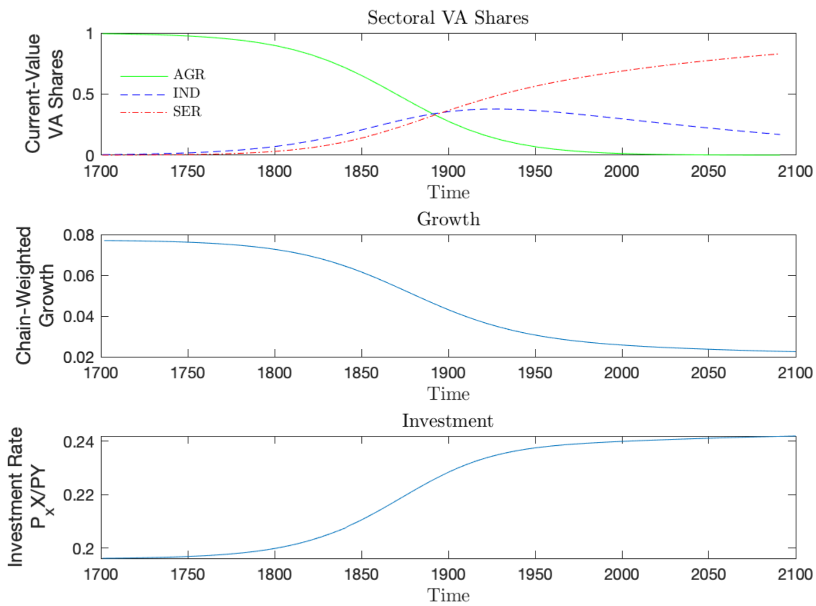
consumption expenditures (PCE), real private investment from Table 1.1.6, and the prices of PCE and real private investment from Table 1.1.4.

[14] We solve for relative weights for $j = a, m$ using the relationship

$$\left(\frac{P_{j,t}}{P_{m,t}}\right)^{\sigma_c} = \frac{\omega_{c,j}}{\omega_{c,m}}\frac{C_{m,t}}{C_{j,t}},$$

and the normalization that weights sum to one to get the actual weights.

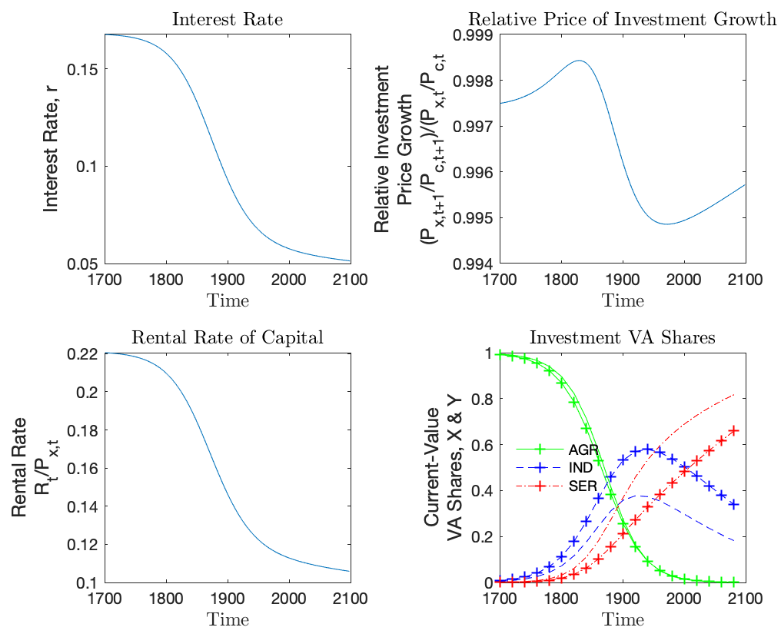
Figure 1: Structural Transformation, Growth, and Investment



see that the dynamics of structural transformation are of comparable orders of magnitude to the historical data (see Buera and Kaboski (2012) and the original sources therein). In 1870, when data are first available, the share is roughly 0.28 in the model, somewhat higher than the 0.24 in the data. By the middle of the twentieth century, that share has risen considerably. The peak in the model is 0.38 in 1927, and the peak in the data is 0.40 in 1941. The growth to the peak is therefore 10 percentage points in the data, and 16 percentage points in the model. By 2000, however, the share has fallen to 0.30 in the model and to 0.19 in the data. The decline of 8 percentage points in the model is therefore also smaller than the observed decline of 18 percentage points over this period. Nevertheless, the predicted hump shape in industry is quantitatively important and not something that the Ngai and Pissarides (2007) model accomplished.

The middle panel of Figure 1 presents the benchmark economy's growth rate of real GDP, where real aggregates are constructed using a chain-weighted index, as done in national income data. We see that the real growth rate varies along the STraP, and indeed structural transformation has implications for growth. As the economy moves from the fastest total factor productivity (TFP) growth sector (agriculture) to the

Figure 2: Understanding Growth Dynamics



slowest (services), Baumol (1967)'s disease is at work, and the real growth rate slows from 4.3 to 2.6 percent over the twentieth century.[15]

The bottom panel of Figure 1 shows that the growth rate is determined not only by the sectoral TFP patterns that drive Baumol's disease, but also by the dynamics of capital that motivate the dynamic model. The investment rate increases with structural transformation — about 22 to 24 percent over the twentieth century, for example. This acts to partially counteract Baumol's disease. Interestingly, the growth of investment coincides with an increase in the value-added share of manufacturing, consistent with García-Santana et al. (2016). However, the subsequent decline in the share of manufacturing occurs despite maintaining the high rate of investment.

Figure 2 explores the factors behind the investment dynamics in more depth. The top left panel plots the interest rate over time, which is crucial to growth in the Euler equation, equation (10). It falls over time, with a substantial drop from 17 to 5 percent. This interest rate drop can be decomposed into the (gross) growth rate of the relative price of investment and the rental rate in terms of the investment good, per

[15]If we use investment as the numeraire in constructing real growth, as in Ngai and Pissarides (2007), the growth rate shows a similar fall, but from about 4.7 to 3.0 percent.

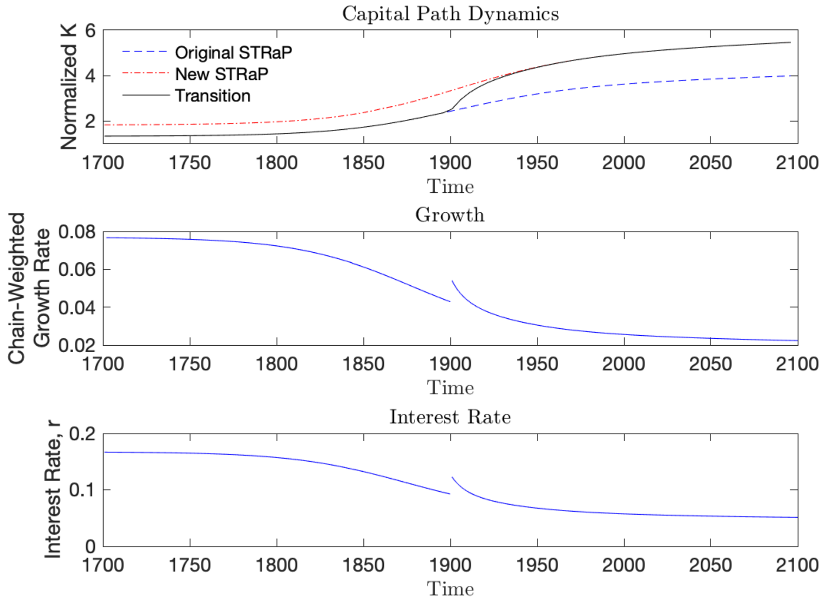
equation (10). The top right panel shows that the growth rate of the relative price of investment is below one, but it is relatively stable with a slight hump-shaped pattern. The lower left panel shows that the drop in the interest rate is driven primarily by the decline in the rental rate of capital in terms of manufactured goods. Recall that the price of the manufactured good is a function of both the productivity growth rate in the investment aggregator, and the changing composition of investment caused by structural transformation within the investment. The bottom right panel shows this sectoral composition of investment (the lines with the plus symbols) compared to the overall sectoral composition of value added (the lines without the plus symbols). Investment undergoes the same qualitative transformation as the economy overall, but is always more intensive in manufacturing, and eventually services. This changing composition gives insight into the de-industrialization process: the rate of investment stays high during the period when the share of manufacturing is declining, but given the high productivity growth in manufacturing, the composition of investment shifts away from manufacturing. The composition of investment also contributes to changes in the growth rate of the relative price of investment. Given the productivity growth in the investment aggregator, the relative price of investment is always falling, but it falls slower in the middle period when agriculture — the sector with the fastest TFP growth — is relatively unimportant in investment, but still vital to consumption.

4.3.1 Capital Dynamics and Stability

The STraP leads to time-varying dynamics in capital, even appropriately normalized capital (i.e., $K_t/\mathcal{A}_{x,t}^{1/(1-\alpha)}$), and we have emphasized that this time path is stable. Figure 3 illustrates both of these features of the STraP. The top panel shows the time path for the normalized capital stock in the top panel for two different STraPs. Both STraPs follow the same productivity growth processes, but the higher path has a 10 percent higher *level* of productivity at all points in time. Over time, capital increases relative to normalized investment productivity. (Here we have normalized by the higher productivity process.) Hence, the STraP predicts a lengthy capital deepening phenomenon over the development process.

The black line in the top panel of Figure 3 captures a transition from one STraP to the other. That is, the black line is a simulated STraP in which all productivity levels,

Figure 3: Transitional Dynamics



instantly and unexpectedly, jump at year 1900 from the lower to higher productivity process (with the same constant growth rate thereafter). This simulates a capital stock that is now below the stable level of capital for the higher productivity STraP.

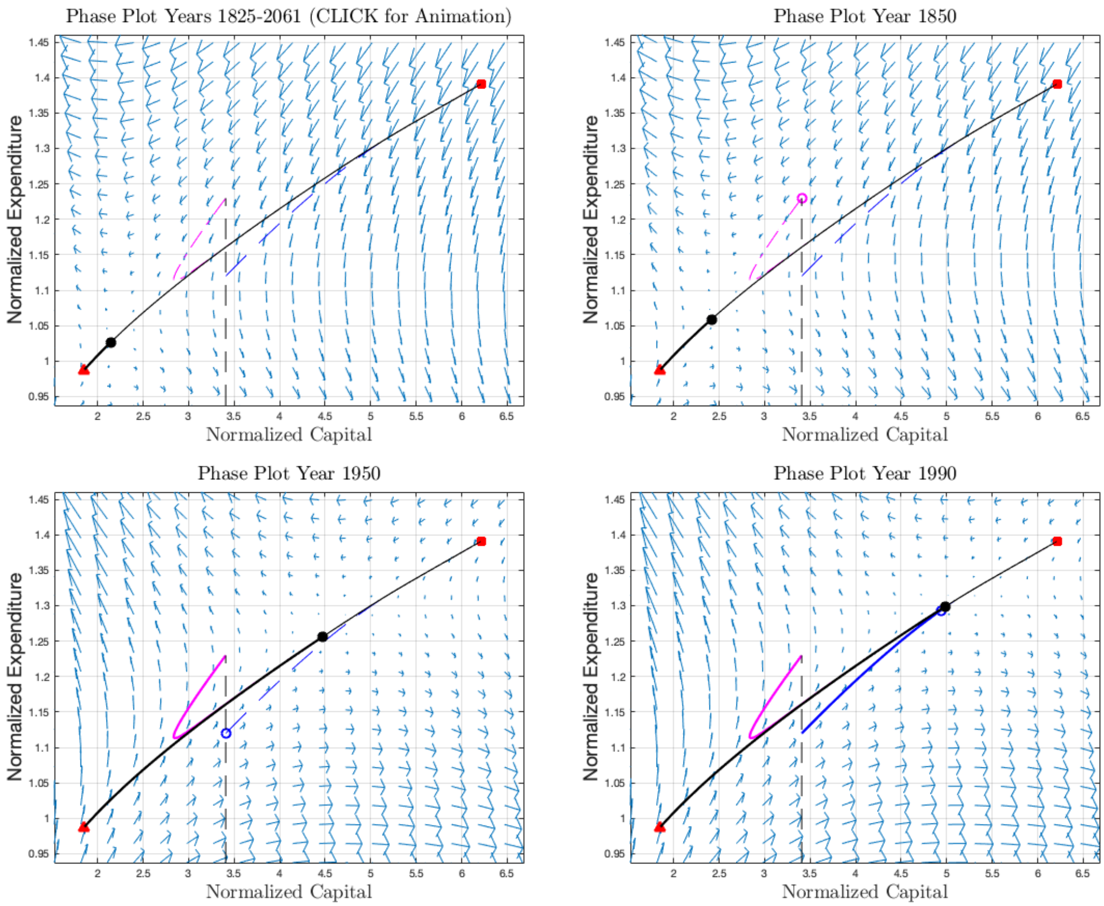
We describe the STraP as stable because capital quickly converges from the lower productivity to the higher productivity STraP. At the time of this jump, there is also a jump in the growth rate (see the middle panel of Figure 3), as the marginal product of capital increases and capital is accumulated at a higher rate. Thus, the mechanics are akin to Neoclassical dynamics toward a BGP, except that the path for stable (normalized) capital is itself time-varying. This example illustrates not only the stability of the STraP but also the speed of convergence. The half-life in this simulated case is just eight years. This stands in contrast to the relatively slow capital-deepening dynamics along the STraP. Thus, Neoclassical forces lead to rapid convergence of capital, whereas structural transformation itself leads to slower time-varying capital deepening.

Figure 3 also illustrates an important aspect of backwards shooting given the model. Although the backward shooting assumes that the productivity process is constant going backwards, this process need not have any historical basis (as shown by the dashed red line in the top panel of Figure 3); it simply helps solve for the STraP from time 0 onward. One can therefore solve for the STraP going forward without any knowledge of past productivities.

Figure 4 illustrates the multidimensional aspect of convergence, by plotting the dynamic paths in the normalized capital-normalized consumption expenditure space. The top left panel of the figure adds the remaining dimension of time, which controls productivity— this is shown with an animation of the movement of the economy (the black dot) along the STraP (black line), from the agricultural BGP (the red triangle) through the transformation to the services BGP (the red square). (The animation starts when clicked on.) The vector field of time-varying arrows is a phase diagram showing the systems instantaneous trajectories for arbitrary expenditure-capital combinations. To further illustrate convergence properties, we start two economies from the identical normalized capital stock, but at two different points in time. The upper pink trajectory is an economy that starts at an earlier time with a higher-than-STraP level of capital given its productivity at that time, while the lower blue trajectory starts at a later date with a lower-than-STraP level of capital. The animation shows the rapid convergence to the STraP over time for both initial values. Note that the distances of the convergence paths in normalized expenditure-capital space are *not* reflective of the time required to converge. (Indeed, for more extreme levels of productivity — either extremely early or extremely late time periods — the convergence paths could go directly toward the BGPs.)

The other panels of Figure 4 illustrate snapshots from this animation. The different panels are discrete jumps in time, which serves to emphasize how the vector field — and the point toward which an economy moves — varies with time. The top right panel illustrates this starting point of the upper pink trajectory in the year 1850. Given high levels of capital, initial expenditures also exceed those of the STraP economy as the open pink circle indicates. The higher expenditures of the pink economy drive down the (normalized) capital stock over time. The bottom left panel brings the economy forward to the year 1950, where the full, completed convergence path of the pink economy can be seen. Interestingly, although convergence of the pink economy is from above, it involves a period of both decreasing and increasing normalized capital stocks. The same panel also shows the starting point of the blue dashed trajectory. Given the later date, the same capital stock is now to the left of the

Figure 4: STraP Convergence in the Capital-Expenditure Space.

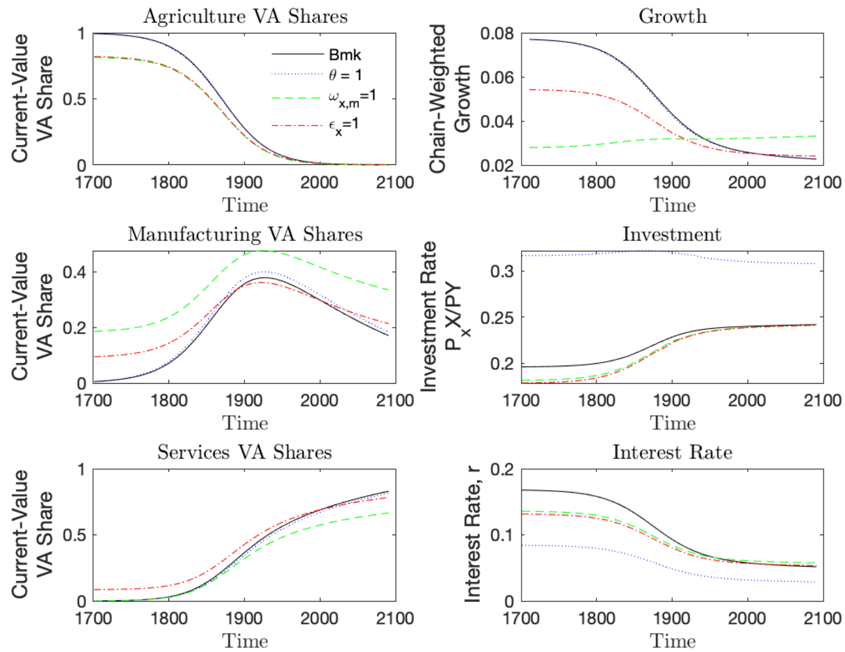


STraP and the vector field has rotated. Hence, the economy chooses a lower-than-STraP level of expenditures and will accumulate capital. The bottom right panel shows the economy in the year 1990, where both economies have converged to the STraP (here we show the instance where the blue hollow dot has almost converged so that it can be compared with the black dot).

4.3.2 Comparative Statics and Variant Models

Our benchmark model relaxed assumptions that were necessary for a BGP but no longer necessary in the STraP. In this section, we examine the importance of these assumptions by simulating alternative models with different parameter choices. That is, we perform simulated comparative statics changing one assumption at a time.

Figure 5: Alternative Models



Ultimately, not only do these parameters matter quantitatively for structural transformation and interest rates, but they also influence the qualitative patterns of growth and investment.

The alternative models we present are as follows. The first two adopt the assumptions in Ngai and Pissarides (2007), one at a time. The first alternative is their assumption of $\theta = 1$, i.e., log intertemporal preferences (relative to our benchmark of $\theta = 2$). The second alternative is their assumption that investment consists only of manufacturing value added, i.e., $\omega_{x,m} = 1$, (relative to the benchmark where it is heavy in manufacturing but still a mix that undergoes structural transformation). Finally, we consider an alternative in which the elasticity of substitution in the investment sector is unitary. This captures the idea of an investment sector with a mixed but stable composition, as in García-Santana et al. (2016), and contrasts it with an investment sector that undergoes structural transformation itself.

Figure 5 plots these alternatives versus the benchmark to demonstrate their impact on sectoral shares (left panels), growth (top right), investment (middle right), and interest rates (bottom right).

The impact of the first alternative, log preferences (the dotted blue lines), shows up most strikingly in the investment rate and interest rate. The level of the investment rate starts out much higher than in the benchmark (roughly, 0.31 versus 0.20) and falls slightly over time rather than rising as in the benchmark (and other alternatives). On the other hand, the interest rate starts substantially lower (8 percent vs. 17 percent) and declines over time as in the benchmark, though by substantially less (5 percentage points relative to 11 percentage points). Focusing on the structural transformation patterns, one can see the impact of the higher investment rate; the peak in the manufacturing hump is slightly higher, since investment is relatively intensive in manufacturing value-added.

The second alternative, only manufacturing in investment (the dashed green line), leads to different sectoral distributions, growth rates, and interest rates. The fact that all investment comes from the manufacturing sector changes the sectoral distributions. Manufacturing naturally constitutes a higher share of output, but the impact can be most easily seen in the asymptotic sectoral compositions; agriculture is less than one initially, while the economy never fully becomes exclusively services as it grows. Compared with the growth rate in the benchmark economy, the growth rate is lower and varies less over time.[16] Moreover, it rises with structural transformation rather than declining steeply as the economy leaves agriculture. Finally, we see that the interest rate starts lower (roughly 14.5 percent versus 17 percent), but it is relatively stable and indeed 1 percentage point higher by the end of the sample than the benchmark rate, which falls much more. (The impact on the investment rate is more subtle. It starts half a percentage point lower than the benchmark rate but rises a bit faster and is comparable to the benchmark rate by the end of the sample.)

The third alternative, a unitary elasticity of substitution in investment (the dash-dotted red line) tends to dampen the patterns relative to our benchmark. The decline in agriculture, hump shape in manufacturing, and increase in services are all less pronounced that in the benchmark simulations. Looking at the panels on the right, we see that the decline in growth is also less pronounced, as is the decline in the interest rate. The intuition is clear: structural transformation is weaker because it

[16]It is not absolutely constant as in Ngai and Pissarides (2007) because we report a chain-weighted growth rate rather than using manufacturing as the numeraire.

is only occurring within the consumption sector. However, the investment rate rises somewhat more without structural transformation as agriculture is not as important to investment early on, so the relative price of investment does not fall as rapidly.

4.4 Nonhomothetic Preferences

So far, we have relied on a biased productivity growth and inelastic substitution across sectors to drive structural change. The combination of rising income and nonhomothetic preferences is another key explanation for structural change (e.g., Kongsamut et al., 2001). We utilize nonhomothetic CES preferences as in Comin et al. (2015). These preferences exhibit persistent nonhomotheticities, which have been shown to be quantitatively important to account for structural transformation (Boppart, 2014; Comin et al., 2015; Alder et al., 2019). Nonhomothetic CES preferences implicitly define $C(t)$ as the solution to

$$1 = \sum_{j=a,m,s} \tilde{\omega}_{cj}^{\frac{1}{\tilde{\sigma}_c}} \left[\frac{C_j\left(t\right)}{C\left(t\right)^{\epsilon_j}} \right]^{\frac{\tilde{\sigma}_c - 1}{\tilde{\sigma}_c}}, \tag{22}$$

with $\tilde{\omega}_{cj}$, $\tilde{\sigma}_c$, and $\epsilon_j > 0$.[17] We further assume $\tilde{\sigma}_c < 1$. Here we have chosen our notation to emphasize the analogous roles of $\tilde{\omega}_{cj}$ and $\tilde{\sigma}_c$ with their counterparts ω_{cj} and σ_c in the earlier CES aggregators. The intratemporal first-order condition from the household's problem illustrates these analogs as well as the unique characteristics and parameter dependence of these preferences:

$$\frac{C_j\left(t\right)}{C_i\left(t\right)} = \frac{\tilde{\omega}_j}{\tilde{\omega}_i} \left(\frac{P_j\left(t\right)}{P_i\left(t\right)} \right)^{-\tilde{\sigma}_c} C\left(t\right)^{(\epsilon_j - \epsilon_i)(1-\tilde{\sigma}_c)}. \tag{23}$$

When $\epsilon_j = \epsilon_i$, the expression is exactly analogous to that in the CES case, with $\tilde{\omega}_{cj}$ and $\tilde{\sigma}_c$ determining the weights and constant elasticity of substitution, respectively. However, when $\epsilon_j > \epsilon_i$, consumption of value added from sector j will rise relative to that from sector i. Thus the ϵ_i parameters govern the income elasticity. It is the relative value of these parameters that determine demand patterns, s we follow Comin et al. (2015) and normalize one of them, $\epsilon_m = 1$.

[17] Note that $\epsilon_j > 0$ ensures that utility $C(t)$ is increasing in $C_i(t)$. This can be readily verified using the implicit function theorem in equation (22).

An important observation is that changing the demand structure from homothetic to nonhomothetic CES does not alter the aggregate production function representation of the economy, since the aggregation result is based on the investment side of the economy and value-added technologies (which we do not modify in this section).

The household's dynamic problem yields the following modified Euler equation:

$$\theta\frac{\dot{C}\left(t\right)}{C\left(t\right)}=r\left(t\right)-\rho-\frac{\dot{\bar{\epsilon}}\left(t\right)}{\bar{\epsilon}\left(t\right)}=\frac{R\left(t\right)}{P_{x}\left(t\right)}-\delta+\left(\frac{\dot{P}_{x}}{P_{x}}-\frac{\dot{P}_{c}}{P_{c}}\right)-\frac{\dot{\bar{\epsilon}}\left(t\right)}{\bar{\epsilon}\left(t\right)},\tag{24}$$

where we define $\bar{\epsilon}_t \equiv \sum_i \epsilon_i \frac{P_i(t)C_i(t)}{\sum_j P_j(t)C_j(t)}$ as the consumption share-weighted elasticity. Comparing the above Euler equation (24) with the benchmark Euler equation (10), we see that on the right-hand side the growth rate of $\bar{\epsilon}\left(t\right)$ is subtracted in equation (24).[18] If the income effects reinforce the relative price effects, as is empirically the case (Comin et al., 2015), consumption shares shift toward luxury goods and $\bar{\epsilon}\left(t\right)$ rises over time. Hence, during structural transformation (i.e., except in the asymptotic limits) nonhomotheticities depress consumption growth.

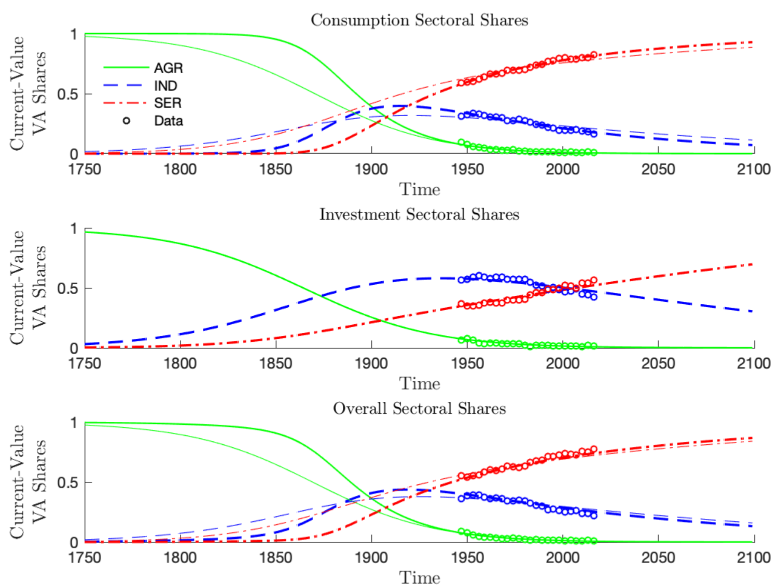
For the simulations, we again move to discrete time and continue to use US technology parameters and technology processes. As with the earlier model, we calibrate the nonhomothetic simulation using the best fit parameters values estimated for the US over the period 1947–2017. After normalizing $\epsilon_m = 1$, we estimate $\epsilon_a = 0.43$ and $\epsilon_s = 1.64$. Hence, relative to manufacturing, agriculture is a necessity and services are a luxury. The incorporation of nonhomothetic income effects lowers the need for low substitutability to do all of the work in structural transformation. Hence, we estimate a somewhat higher value of the substitution parameters, $\tilde{\sigma}_c = 0.30$. For the weights in consumption, we again normalize the manufacturing weight to one, $\tilde{\omega}_{c,m} = 1$, which yields $\tilde{\omega}_{c,a} = 0.024$ and $\tilde{\omega}_{c,s} = 8.85$. Online appendix D contains further details.

Figure 6 shows the fit of the simulation by plotting the sectoral share of consumption,

[18] The discrete time analog to the Euler equation in normalized expenditures, which we use for simulation, is

$$\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{\theta}=\beta(1+r_{t+1})\frac{\bar{\epsilon}_t}{\bar{\epsilon}_{t+1}}=\beta\frac{\bar{\epsilon}_t}{\bar{\epsilon}_{t+1}}\left(1-\delta+\frac{R_{t+1}}{P_{x,t+1}}\right)\left(\frac{\frac{P_{c,t}}{P_{x,t}}}{\frac{P_{c,t+1}}{P_{x,t+1}}}\right)^{1-\theta}.$$

Figure 6: US data 1947–2017, Homothethic and Nonhomothetic Benchmarks



Note: Thick lines correspond to NHCES benchmark. Thinner lines, to CES.

investment and value-added over the period 1947–2017 against the calibrated STraP simulations. The thick lines indicate the NHCES simulation, while the thinner lines are the original CES simulation. While both models fit the data well, the NHCES is somewhat better at capturing the steep growth in services and decline in manufacturing. We can also see the impact of nonhomothetic preferences in the out-of-sample simulation: Relative to the homothetic model, the addition of nonhomothetic preferences delays structural transformation in consumption and leads to the hump shape in manufacturing being quicker and more pronounced.

4.5 STraP and the Development Path

We now shift our focus away from the US to the STraP in relation to empirical growth and development patterns. We use Penn World Tables (PWT) 9.1 cross-country panel (Feenstra et al., 2015) to establish these empirical patterns and compare the model's predictions with respect to real (purchasing power parity, PPP) expenditure income per capita for real capital-output ratios, current-value investment rates, relative price of investment, (consumption-based) interest rates, and real (within-country) growth rates per capita.[19]

Relative to the US data, the PWT data differ in two important ways that are relevant to the STraP. First, growth rates in real income per capita are somewhat slower than in the US BEA data. Second, the capital-output ratio is substantially higher in the PWT data. To account for this, we scale all primitive productivity growth rates down by a common factor. To account for the higher capital-output ratio, we adjust the discount factor up (and no longer target the interest rate).[20] For calibration, we focus on comparable countries, and include all country-year observations within the US real income per capita range for the period 1950–2000, which is approximately 14,600-46,500 US dollars.[21] We ensure that average growth and the average capital-output ratio in our simulations for the years 1950–2000 match the corresponding moments in the PWT data (an average annual growth of 1.55 percent and a capital-output ratio of 4.54).[22,23]

We present the results over a wide range of development that spans well beyond the calibrated range, from a log real income per capita of 7 (roughly $1100) up to the US income per capita in 2000, 10.75 (roughly $46,500).[24] With 126 countries and over 4000 observations, the data themselves are dense and have wide variance. To make the data clearer, we use three lines to characterize them: nonparametric fits (using 100 income bins) of the 25th and 75th percentile of the data at each income level and a linear fit.[25] For comparison's sake, we include a simulated, calibrated Cass-Koopmans-

[19]Given the importance of capital dynamics, we include only country-year observations for which arbitrarily initialized capital stocks are no longer relevant, as discussed in Inklaar et al. (2019). Such indicators are not in the publicly available PWT 9.1 data series, but were provided upon request. See online appendix C for details on the construction of the sample and all data variables.

[20]The absolute scale of income matters in the case of nonhomothetic preferences. For our NHCES comparisons, we therefore change growth rates by pivoting around the midpoint of the simulated sample.

[21]All dollars are in real international PPP dollars, which equals 2011 US dollars.

[22]A third difference between the model and the data is the presence of cyclical fluctuations in the data. Given these fluctuations and the fact that we are more interested in medium-term growth patterns, we construct annual growth rates using ten-year growth averages.

[23]With nonhomothetic preferences, the absolute scale of income matters for calibrated parameters. For this reason, while we adjust the growth rates, we effectively pivot around the midpoint of the calibrated years in the sample by rescaling the level of income in the series so that the midpoint sectoral allocations are unchanged.

[24]Although limited data are available above and below these ranges, the set of countries thins out quickly, and patterns can be easily driven by country-specific effects and the changing sample.

[25]In all the figures that we present, the coefficient in the linear fit is significant at the 5 percent level.

Ramsey growth model, the benchmark one-sector Neoclassical growth model. We start the Neoclassical simulation off at log real income per capita of 7 with the average capital-output ratio in the data bin at that level of income.[26]

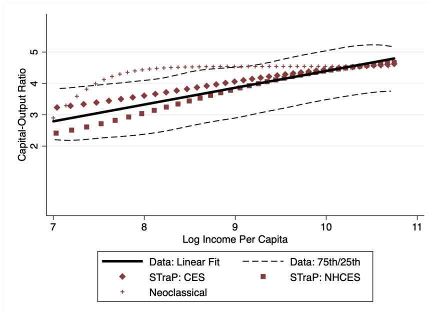
In the figures that follow, we stress that everything is out-of-sample except for the *average* capital-output ratios and annual growth rates at very high incomes (log incomes above 9.59) and the initial (i.e., log income of 7) capital-output ratio in the Neoclassical model. Recalling our discussion of the role of the backwards STraP, we emphasize that these moments are not even necessary implications of the models themselves, which need only be forward looking. Instead, looking back on lower incomes is a test of whether stability of the productivity process and the structural transformation forces modeled can add insight into broader development patterns.

We start by examining the patterns and determinants of the capital-output ratio, since, at the aggregate level, structural transformation provides a theory for a time-varying capital-output ratio over the medium-term, and the STraP characterizes these dynamics. Figure 7 presents these results. The top left panel plots the capital-output ratio over development. The black lines show the data, which have wide variance but clearly trends up and in a economically important way. In the linear fit, the capital-output ratio rises from roughly 2.8 to 4.8. The capital-output ratios in the STraPs for the CES (diamonds) and NHCES (squares) structural transformation models also trend up in fairly linear fashion. The NHCES trend is somewhat stronger than the fit, while the CES trend is somewhat weaker, but both stay within the 75/25 bands of the data. In contrast, the Neoclassical growth model ('+' symbols) displays its well-known short-lived dynamics, rising quickly and then remaining flat. The key point here is over the broad range of development, the assumption of a constant, balanced growth capital-output ratio does not hold in the data. The structural transformation models yield persistent medium-run capital accumulation dynamics that (i) go beyond the rapid convergence dynamics in the Neoclassical growth model that stem from initial levels of capital, and (ii) are consistent with the overall idea of a rising capital-output ratio over development.

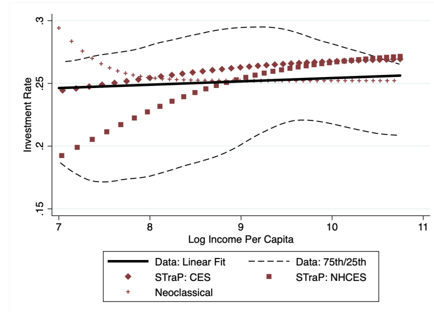
The remaining panels of Figure 7 explore the determinants of these capital accu-

[26]The calibrated Neoclassical growth model uses the same Cobb-Douglas parameter and depreciation rate, and productivity growth and the discount factor are again chosen to match the identical calibration targets.

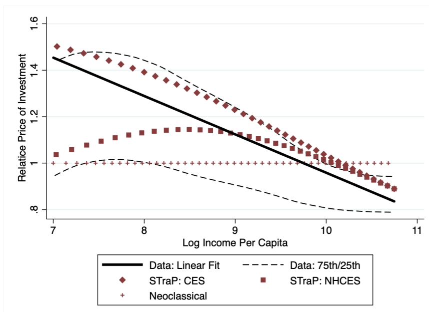
Figure 7: Capital Accumulation and its Determinants over Development.



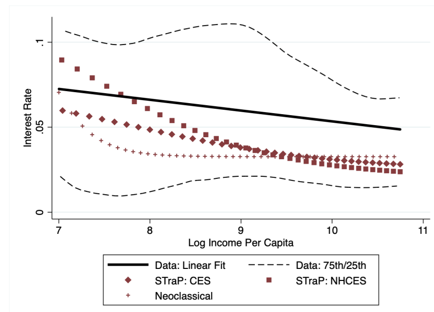
(a) Capital-Output Ratio.



(b) Investment Rate.



(c) Relative Price of Investment.

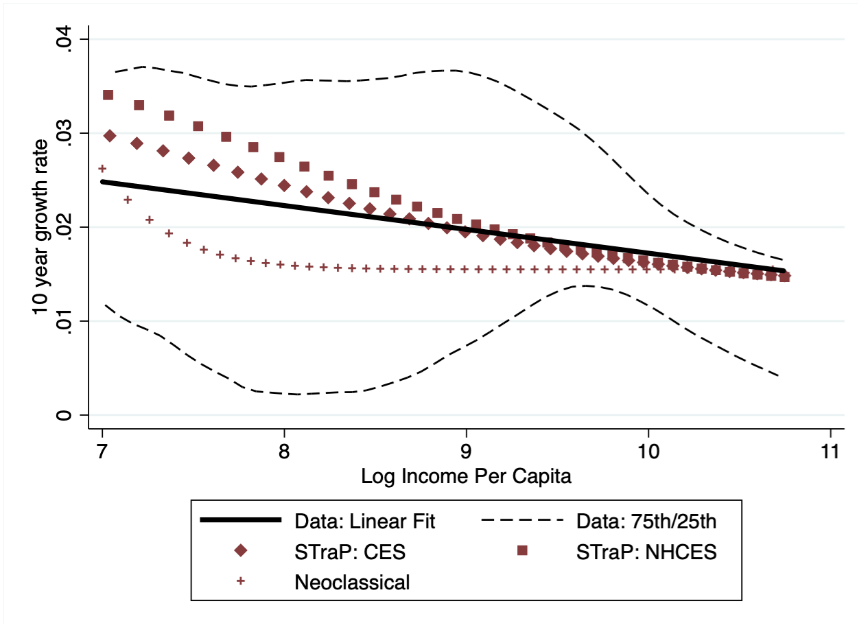


(d) Interest Rate.

mulation dynamics. The top right panel shows the (current value) investment rate. The linear fit of the data shows a very mild, though statistically significant, increase. The relative flatness of this pattern was emphasized by Hsieh and Klenow (2007), while the significance of the increase has been noted by Inklaar et al. (2019). Both STraPs shows an increase in the investment rate, with the increase stronger for the NHCES model. Both are somewhat stronger than in the data, but they stay in the 75/25 bands of the data. Again, the Neoclassical growth model, in contrast, has a high initial investment rate and a rapid stabilization consistent with the well-known short-run dynamics.

The bottom left panel shows the relative price of investment over development. The marked decline in the data, from over 1.4 to less than 0.9, is another pattern emphasized by Hsieh and Klenow (2007) as important for understanding development: rich countries get more real investment bang for their current-value investment rate buck. Here the structural tranformation in the STraPs lead to variation over development that is consistent with a declining relative price of investment. The CES STraP shows

Figure 8: 10-Year Growth Over Development.



a remarkably similar decline as the linear fit in the data (although the level — calibrated to the US in 2000 — is somewhat higher). The NHCES STraP shows a rise and fall of the relative price of investment. This is the result of the investment sector being CES and experiencing an earlier onset of structural transformation (recall 6), which slows effective productivity growth in investment, while the NHCES preferences delay structural transformation in consumption, so that no such slowdown in productivity occurs in the consumption sector. (Introducing a NHCES aggregator into the investment sector as well could presumably eliminate this.) Again, in contrast, the Neoclassical growth model delivers a flat prediction for relative prices.

The bottom right panel plots interest rates over development.[27] The data show a mild decline, with the linear fit dropping from roughly 7 percent to 5 percent. The STraPs and the Neoclassical model simulation all show a somewhat stronger decline in interest rates, with the NHCES STra'sP particularly strong. The difference between the models is in the timing, however. The STraPs for the transformation models show prolonged declines over development, whereas the decline in the Neoclassical growth model transition is immediate, intense, and short-lived.

[27]We measure interest rates in the PWT data to be consistent with our consumption-based definition of interest rates.

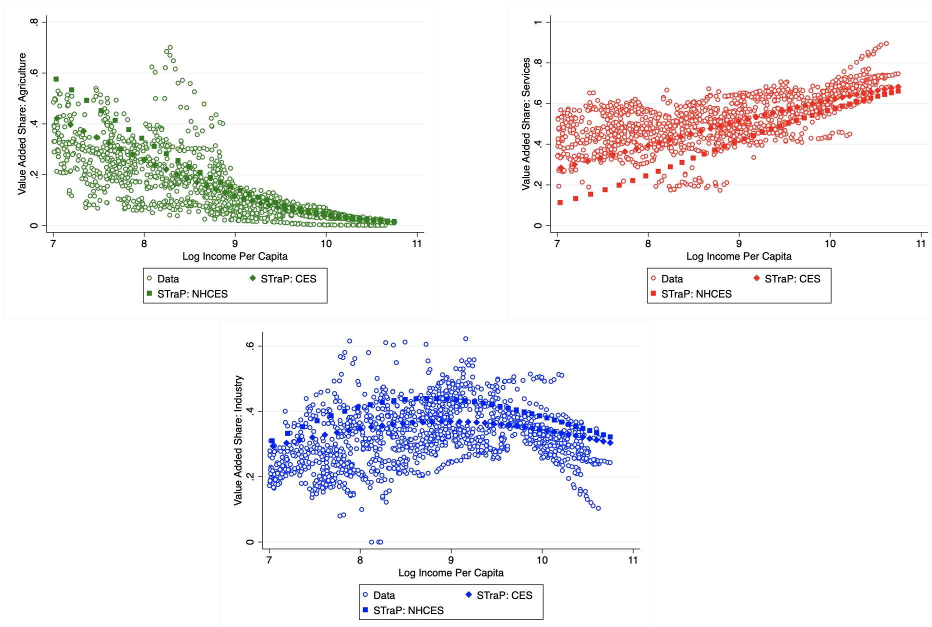
We also look at the models' implications for (ten-year) average annual growth (and implicitly convergence) over development. Again, the ten-year growth rate is used because our focus is on medium-term growth dynamics. Figure 8 shows what is really the key punch line of this analysis. In the data, ten-year annual growth rates, on average, fall with development from 2.7 percent to under 1.5 percent. The Neoclassical growth model displays a version of the well-known convergence puzzle.[28] Growth declines very quickly over development, as the economy rapidly converges to its BGP. In contrast, the STraPs of the transformation models converge only asymptotically to a BGP, and they exhibit slowly and persistently declining growth rates. Of course, these declining growth rates do not come from transitional capital dynamics — K/Y is growing stably throughout — but from the structural transformation itself as the economy shifts resources toward more slowly growing sectors.

Our final analysis is to examine whether the structural transformations that underlie the aggregate behavior in the model also align with the data. Sectoral data are not available in the PWT data, but we turn to the Groningen Growth and Development Centre (GGDC) 10-Sector Database (Timmer et al., 2015), which provides data on sectoral shares for 39 countries of varying levels of development over the years 1950–2010. Given the smaller sample, we can plot the CES and NHCES against the actual data. (We omit the Neoclassical model because it has no sectoral implications.)

Figure 9 shows the fit of the CES and NHCES simulations relative to the GGDC data (empty circles) for agriculture (top left panel), industry (lower panel), and services (top right panel). Despite the fact that, viewed together, the figures make a sad face, both models actually follow the overall patterns of structural change in the data quite well. Again, we emphasize that these patterns are both out of sample and also not a necessary implication of the forward-looking STraP itself. In sum, the structural transformation mechanisms driving the growth dynamics in the model have supporting evidence in the data as well.

[28] The more extreme version of the convergence puzzle in Barro and Sala-i Martin (1992) assumes that all countries have the same technology (at a point in time). We combine time and cross-sectional variation and allow poorer countries at any point in time to be further behind in the productivity process.

Figure 9: Structural Transformation over Development.



5 Conclusion

We have developed a new dynamic concept to characterize growth models with asymptotic BGPs, but non-trivial medium-run dynamics like structural change models. We have proven its existence and uniqueness for a general class of growth models, and we have presented an algorithm for computing it.

The STraP allows us to study a broader class of structural change models and enables us to link two important sets of development patterns: those of structural transformation with those of aggregate growth. The STraP is therefore valuable in that it allows us to characterize the medium-term dynamics of capital accumulation and growth. The model and its predictions, even out-of-sample predictions, make progress in matching and understanding important and well-known empirical patterns of development.

We believe that the STraP concept can be useful for future research in development economics, macroeconomics, and international economics. In development economics, for instance, our concurrent research, Buera et al. (2019), is using the STraP as a benchmark for normative wedge analysis of structural transformation. In macroeconomics, the STraP can be helpful for disentangling business cycle frequency from medium-term transformation dynamics. In international economics, the STraP is useful for work on premature de-industrialization in the global economy. These are examples of the potentially wide applicability of the concept.

Link to Online Appendix (Click Here)

References

Acemoglu, D. (2008): *Introduction to Modern Economic Growth*, Princeton University Press.

Acemoglu, D. and V. Guerrieri (2008): "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116, 467–498.

Alder, S., T. Boppart, and A. Müller (2019): "A theory of structural change that can fit the data," CEPR Discussion Papers 13469, C.E.P.R. Discussion Papers.

Barro, R. J. and X. Sala-i Martin (1992): "Convergence," *Journal of Political Economy*, 100, 223–251.

Baumol, W. J. (1967): "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, 57, 415–426.

Boppart, T. (2014): "Structural Change and the Kaldor Facts in a Growth Model With Relative Price Effects and Non-Gorman Preferences," *Econometrica*, 82, 2167–2196.

Buera, F. J. and J. P. Kaboski (2009): "Can Traditional Theories of Structural Change Fit The Data?" *Journal of the European Economic Association*, 7, 469–477.

——— (2012): "The Rise of the Service Economy," *American Economic Review*, 102, 2540–69.

Buera, F. J., J. P. Kaboski, M. Mestieri, and D. G. O'Connor (2019): "The Natural Rate of Structural Change," Unpublished Manuscript.

Cass, D. (1965): "Optimum Growth in an Aggregative Model of Capital Accumulation," *The Review of Economic Studies*, 32, 233–240.

Cheremukhin, A., M. Golosov, S. Guriev, and A. Tsyvinski (2017a): "IThe Industrialization and Economic Development of Russia through the Lens of a Neoclassical Growth Model," *Review of Economic Studies*, 84, 613–649.

——— (2017b): "The Economy of People's Republic of China from 1953," Unpublished Manuscript.

Comin, D. A., D. Lashkari, and M. Mestieri (2015): "Structural Change with Long-run Income and Price Effects," Working Paper 21595, National Bureau of Economic Research.

Duernecker, G., B. Herrendorf, and A. Valentinyi (2017a): "Quantity Measurement and Balanced Growth in Multi-Sector Growth Models," CEPR Discussion Papers 12300, C.E.P.R. Discussion Papers.

——— (2017b): "Structural Change within the Service Sector and the Future of Baumol's Disease," Mimeo.

Feenstra, R. C., R. Inklaar, and M. P. Timmer (2015): "The Next Generation of the Penn World Table," *American Economic Review*, 105, 3150–82.

García-Santana, M., J. Pijoan-Mas, and L. Villacorta (2016): "DP11636 Investment Demand and Structural Change," .

Gollin, D., D. Lagakos, and M. E. Waugh (2014): "The Agricultural Productivity Gap," *The Quarterly Journal of Economics*, 129, 939–993.

Gollin, D., S. Parente, and R. Rogerson (2002): "The Role of Agriculture in Development," *American Economic Review*, 92, 160–164.

Gomme, P., B. Ravikumar, and P. Rupert (2011): "The Return to Capital and the Business Cycle," *Review of Economic Dynamics*, 14, 26–278.

Hansen, G. C. and E. D. Prescott (2002): "From Malthus to Solow," *American Economic Review*, 92, 1205–1217.

Havránek, T. (2015): "Measuring intertemporal substitution: The importance of method choices and selective reporting," *Journal of the European Economic Association*, 13, 1180–1204.

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–89.

——— (2014): "Growth and Structural Transformation," in *Handbook of Economic Growth*, Elsevier, vol. 2 of *Handbook of Economic Growth*, chap. 6, 855–941.

——— (2018): "Structural Change in Investment and Consumption: A Unified Approach," Tech. rep., National Bureau of Economic Research.

Hsieh, C.-T. and P. J. Klenow (2007): "Relative Prices and Relative Prosperity," *American Economic Review*, 97, 562–585.

Hubbard, J. and B. West (1991): *Differential equations: a dynamical systems approach. Part I: ordinary differential equations*, Springer-Verlag.

Inklaar, R., P. Woltjer, and D. Gallardo Albarrán (2019): "The Composition of Capital and Cross-country Productivity Comparisons," *International Productivity Monitor*, 34–52.

Ju, J., J. Y. Lin, and Y. Wang (2015): "Endowment structures, industrial dynamics, and economic growth," *Journal of Monetary Economics*, 76, 244–263.

Kaldor, N. (1957): "A Model of Economic Growth," *The Economic Journal*, 67, 591–624.

Kehoe, T. J., K. J. Ruhl, and J. B. Steinberg (2018): "Global Imbalances and Structural Change in the United States," *Journal of Political Economy*, 126, 761–796.

King, R. G. and S. T. Rebelo (1993): "Transitional Dynamics and Economic Growth in the Neoclassical Model," *American Economic Review*, 83, 908–931.

Kongsamut, P., S. Rebelo, and D. Xie (2001): "Beyond Balanced Growth," *Review of Economic Studies*, 68, 869–82.

Koopmans, T. C. (1965): "On the Concept of Optimal Economic Growth," in *The Econometric Approach to Development Planning*, Rand McNally, Pontifical Academy of Science, 225–287.

McKenzie, L. W. (1986): "Optimal economic growth, turnpike theorems and comparative dynamics," Elsevier, vol. 3 of *Handbook of Mathematical Economics*, 1281 – 1355.

Moro, A. and M. Leon-Ledesma (2017): "The Rise of Services and Balanced Growth in Theory and Data," Discussion Papers 1714, Center for Macroeconomics.

Ngai, L. R. and C. A. Pissarides (2007): "Structural Change in a Multisector Model of Growth," *American Economic Review*, 97, 429–443.

Ramsey, F. P. (1928): "A Mathematical Theory of Saving," *The Economic Journal*, 38, 543–559.

Rodrik, D. (2016): " Premature deindustrialization," *Journal of Economic Growth*, 21, 31–33.

Timmer, M., G. de Vries, and K. de Vries (2015): *Patterns of structural Change in Developing Countries*, Routledge, 65–83, Routledge Handbooks.