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Abstract

We propose a macroprudential theory of foreign reserve accumulation that can rationalize the secular trends in public and private international capital flows. In middle-income countries, the increase in international reserves has been associated with elevated private capital inflows, both in the aggregate and in the cross-section, and reserve holdings have been more prominent in economies with a more open capital account. We present an open economy model of financial crises that is consistent with these features. We show that the optimal reserve accumulation policy leans against the wind, raising gross private borrowing while improving the economy's net foreign asset position and reducing the exposure to financial crises.

Keywords: Macroprudential policy, international reserves, financial crises, gross capital flows

JEL Classifications: E58, F31, F32, F34

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1 Introduction

Central banks' holdings of international reserves have nearly quadrupled since the wave of financial globalization of the 1990s. Yet despite an extensive literature, accounting for this surge and the large variation in reserve holdings across countries has remained elusive. In this paper, we propose a simple theory of foreign reserve accumulation based on a macroprudential motive and show that it can quantitatively account for the buildup of international reserves while being consistent with salient cross-sectional patterns of capital flows.

Our theory is motivated by the intertwined relationship between foreign reserves and private external debt prevalent among middle-income countries. Four facts, which we document in Section 2, guide our analysis. First, on the aggregate, the secular increase in foreign reserves has occurred concomitantly with an increase in private external debt. Second, there is a positive association in the growth of reserves and debt, both over time and across countries. Third, reserves and private external debt accumulation appear to be procyclical for most countries. Fourth, reserve holdings are larger in economies with a more open capital account, a fact that resonates with the influential work by Ilzetzki, Reinhart and Rogoff (2019) linking the recent global accumulation of international reserves to the overall reduction in capital controls.

We argue that these facts point to a hypothesis linking international reserves to the government's prudential management of private capital flows. Few models of international capital flows, however, study explicitly the interaction between private and public capital flows. The literature has focused either on private or public flows, or considered a single borrowing agent without distinguishing between the two categories of flows.[1] A first contribution of our paper is to provide a framework that can quantitatively speak to the evolution of private and public capital flows within a unified setup.

The environment we consider is a two-sector small open economy model with incomplete markets and inefficient private borrowing. The framework builds on a workhorse model of sudden stops and capital flows in emerging markets (e.g., Mendoza (2002); Bianchi, 2011; Schmitt-Grohé and Uribe, 2017). A key feature of the model is that households' private borrowing is limited by an occasionally binding credit constraint that depends on income and links the borrowing capacity to the real exchange rate. In this setup, when an adverse shock hits and the economy is highly leveraged, households hit their credit constraint and become unable to smooth consumption. The contraction in spending leads to a depreciation of the real exchange rate, which further tightens the borrowing constraint and leads to a "sudden stop" in capital inflows. Our key departure from the literature is to allow for the accumulation of a non-state contingent asset, which we refer to

[1] A notable exception, which we discuss below, is Benigno, Fornaro and Wolf (2021).

as international reserves, and examine the implications for gross capital flows and optimal policy.

We show that while reserves provide a liquidity buffer to mitigate the contraction in consumption in a crisis, households do not internalize their general equilibrium benefits. Moreover, we demonstrate that the constrained-efficient allocation (i.e., the allocations that would prevail if the government were to make all financial decisions on behalf of private agents) can be implemented via a government reserve policy. When households deleverage, they fail to internalize how the contraction in their spending leads to a real exchange rate depreciation, further tightening economy-wide borrowing constraints. As a result, they do not face proper incentives to accumulate reserves in good times, when the credit constraint is not binding. A contribution of our paper is then to provide a theory of why it is the government rather than the private sector that must accumulate reserves.

By a Ricardian equivalence logic, households' gross borrowing increases in response to the government's accumulation of reserves (Barro, 1974). Ultimately, however, once the government accumulates a large enough stock of reserves, households become borrowing constrained and are unable to offset the government policy. Hence, the very same credit constraint that makes households overborrow in good times, relative to the constrained-efficient allocation, makes the reserve accumulation by the government effective. However, while gross debt increases under this government policy, the economy's net foreign asset position improves, leading to a reduction in the frequency and severity of sudden stops relative to the laissez-faire outcome.

A quantitative analysis of the model shows that the macroprudential motive for reserves can go a long way towards accounting for the intertwined relationship between private and public capital flows observed in the data. In particular, model simulations can account for the four aforementioned facts. The model generates the observed upward trend in reserves and debt, the positive association between yearly increases in reserves and debt in the cross-section, the procyclicality of debt and reserves over the business cycle, and the positive correlation between the degree of financial liberalization and reserves across countries.

Literature. Our paper is related to several areas of research. First, it belongs to a large literature seeking to explain the demand for international reserves. The idea of a precautionary motive for reserves has a long tradition in international macroeconomics (Kenen and Yudin, 1965; Heller, 1966, Clower and Lipsey, 1968; Clark, 1970; and Kelly, 1970). More recently, precautionary theories of reserves have focused on shocks to income or shocks to countries' access to credit markets, but in the context of models with a single decision maker controlling all external financial decisions. This literature has hence remained silent on the question of why it is the government that has to accumulate reserves.[2] Our paper tackles this question and underscores the presence of an externality by which private agents do not have incentives to accumulate reserves on their own.

Few papers model jointly private and public capital flows in quantitative settings. A notable exception is Benigno et al. (2021), who consider a model in which reserves held by the government are motivated by the presence of a learning-by-doing externality in the tradable sector. They show that in the absence of industrial policies, accumulating reserves is desirable to undervalue the real exchange rate and foster export-led growth. Our work is complementary in that it articulates a motive for reserve accumulation based on a macroprudential motive. Moreover, we examine optimal policy and show that the macroprudential motive can go a long way towards accounting for the observed levels of reserves and the interaction between private and public capital flows.

Our paper also relates to the literature that studies foreign exchange interventions in the presence of limits to international arbitrage. Examples include Cavallino (2019), who shows how foreign exchange interventions can deal with dynamic terms of trade externalities and capital account shocks, Amador, Bianchi, Bocola and Perri (2020), who show that reserve accumulation is needed to implement exchange rate policies at the zero lower bound, and Fanelli and Straub (2020), who characterize optimal policies when real exchange rate fluctuations lead to distributional consequences. While a common theme in these papers is that international intermediaries have limited leverage capacity, building on the work of Gabaix and Maggiori (2015), our focus is instead on frictions in domestic financial markets.[3] In addition, a key distinction of our paper is that we study the scope for reserve accumulation owing to financial stability, a motive notably raised by by Calvo (2006) and Obstfeld, Shambaugh and Taylor (2010). In this respect, our paper is complementary to Bocola and Lorenzoni (2020), who show that reserves can enhance the credibility of lender of last resort policies.

Our paper also relates to the literature on financial crises and macroprudential policy. This literature has shown how capital controls can correct pecuniary externalities that generate excessive systemic risk (e.g., Lorenzoni, 2008; Bianchi, 2011; Dávila and Korinek, 2017; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2018).[4] We contribute to this literature by showing how international reserves can serve as a macroprudential policy tool. Our results shed light on the observation by Ilzetzki, Reinhart and Rogoff (2019) of an increase in world reserves alongside an overall increase in capital mobility.

[2]See, for example, Durdu, Mendoza and Terrones (2009), Caballero and Panageas (2008), Jeanne and Rancière (2011), Bianchi, Hatchondo and Martinez (2018), Bianchi and Sosa-Padilla (2020), Aizenman and Lee (2007), Bacchetta, Benhima and Kalantzis (2013), Hur and Kondo (2016), and Jeanne and Sandri (2017).

[3]For a comprehensive review, see Maggiori (2021).

[4]The scope for prudential policies can also emerge from aggregate demand externalities (see e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016; Korinek and Simsek, 2016). See Bianchi and Lorenzoni (2021) for a review of the literature on prudential policies.

Our work is also related to a small set of papers that analyze the interaction between ex-ante and ex-post policies. The two most closely related are Benigno et al. (2013) and Schmitt-Grohé and Uribe (2021).[5] In Benigno et al. (2013), the government has access to a richer set of tax instruments, enabling it to relax borrowing constraints ex post, which results in more borrowing ex ante compared to the laissez-faire economy. In Schmitt-Grohé and Uribe (2021), the optimal government intervention induces more borrowing relative to competitive equilibria driven by self-fulfilling pessimistic beliefs. In contrast with these studies, our model distinguishes between private and official flows. While we also find that under the optimal intervention, households borrow more, the accumulation of reserves ultimately raises the net foreign asset position.

Finally, our paper is related to a large empirical literature on capital flows. Particularly relevant is the empirical work on the precautionary motive for reserves (e.g., Edwards, 1983; Frankel and Saravelos, 2012; Bussiere, Cheng, Chinn and Lisack, 2013; Calvo, Izquierdo and Loo-Kung, 2013). Our empirical and theoretical analysis emphasizes the interaction between private and public capital flows and the importance of considering gross positions, as stressed in Obstfeld (2012).

The paper is organized as follows. Section 2 outlines the motivating facts. Section 3 presents the model and the theoretical results. Section 4 contains a quantitative analysis, and Section 5 concludes.

2 Motivating facts: reserves, debt, and capital mobility

In this section, we present empirical evidence on international reserves and private external debt that illustrates the intertwined relationship between these two variables. We use data for middle-income countries from 1980 to 2015.[6] The data for private external debt are from the International Debt Statistics collected by the World Bank and measure private external debt as non-publicly guaranteed external debt.[7]

We summarize the evidence in four facts:

FACT 1: *Over the past three decades, there has been a concomitant substantial increase in private external debt and international reserves.* Figure 1 shows the evolution of the GDP weighted average

[5] See also Bianchi (2016), Bornstein and Lorenzoni (2018), and Jeanne and Korinek (2020).

[6] The complete list of countries, based on data availability and other considerations detailed in appendix A, is Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela

[7] An advantage of using data from the International Debt Statistics is that it allows us to differentiate PGD and non-PGD. This distinction is important, as some middle-income countries in our sample have large publicly owned companies that issue debt internationally.

of private external debt and reserves from 1980 to 2015.[8] Until 1990, both international reserves and private external debt were below 5 percent of total GDP for the average middle-income country. By 2015, reserves and private external debt reached, respectively, 16 percent of GDP and 12 percent of GDP. It is worth noting that the sharp rise in private external debt contrasts with the decline in publicly guaranteed external debt (PGD) in the countries in our sample. Over the same time period, PGD decreased from 27 percent of GDP in 1980 to 14 percent of GDP in 2015.

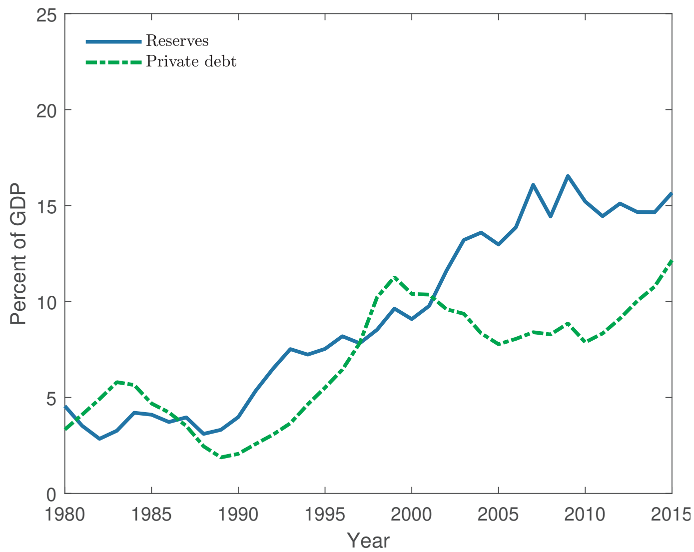

Figure 1: Evolution of reserves and private debt (GDP-weighted average)

FACT 2: *There is a positive association in the growth of reserves and debt, both over time and across countries.* To assess the relationship between growth in reserves and growth in private external debt at a more granular level, we estimate variants of the following panel regression:

$$\Delta a_{it} = \alpha_i + \alpha_t + \gamma x_{it} + \beta \Delta b_{it} + \varepsilon_{it}, \tag{1}$$

where Δa_{it} and Δb_{it} denote the yearly changes in the foreign reserves-to-GDP ratio and private external debt-to-GDP ratio for country i, and x_{it} denotes control variables. Our coefficient of interest is β.

Table 1 reports the estimation results, along with standard errors clustered at the country level. In all the regressions we include a constant and control for the current account-to-GDP ratio.[9] In addition, we consider the results with pooled ordinary least squares (OLS) regressions as well as country and time fixed effects.

[8] This trend also holds when we look at simple averages. Notice that Figure 1 excludes China.

[9] The estimates of the parameter of interest are still positive and statistically significant when we control for the net changes in the PGD-to-GDP ratio and real GDP growth.

In all cases, the coefficient on private external debt is positive and statistically significant at the 1 percent confidence level, indicating a robust statistical association between changes in external private debt and changes in reserves. The estimation with a country fixed effect only (column 2) wipes out all cross-sectional means. Consequently, our significant estimates confirm in this case that changes in private external debt and changes in reserves are positively correlated within country units. Similarly, the relationship is preserved when we allow for time fixed effects only (column 3), confirming that the positive association also holds in the cross-section. In other words, in a given year, countries that accumulate more private external debt also tend to accumulate more reserves.[10,11]

Table 1: Changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	0.261***	0.276***	0.254***	0.266***
	(0.0421)	(0.0420)	(0.0470)	(0.0460)
Current Account	0.175***	0.251***	0.162***	0.251***
	(0.0237)	(0.0294)	(0.0265)	(0.0356)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time

Note: Standard errors clustered at the country levels in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

FACT 3: *The accumulation of reserves and private external debt are procyclical for most countries.* Figure 2 displays correlations of real GDP growth with the growth rates of reserves and of private debt (panels a and b, respectively). In line with Bianchi et al. (2018), we find that reserves growth correlates positively with output growth for a wide majority of countries. In addition, we find a positive correlation between the growth of private external debt and output growth for most countries. As shown in the appendix, the positive association between output growth and reserves growth, and output growth and private debt growth is also apparent when using a pooled OLS estimation.

FACT 4: *Reserve holdings are larger in economies with a more open capital account.* Figure

[10] These findings complement Broner et al. (2013)'s results on the positive correlation between inflows of private debt and reserves over time, as well as Obstfeld, Shambaugh and Taylor (2010)'s results on the positive association between private *domestic* debt and reserves.

[11] The positive association between reserves and private external debt is also obtained with panel feasible generalized least squares (FGLS) estimation, as shown in the appendix.

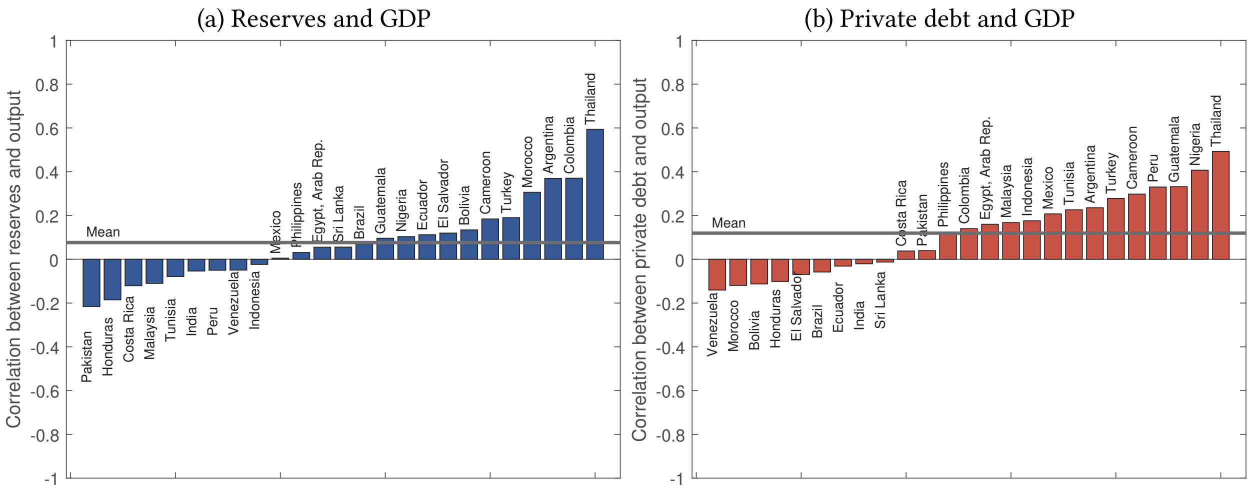

Figure 2: Time-series correlations

Note : Correlation between the growth rates of real GDP and growth rate of reserves (panel a), and growth rate of real GDP and growth rate of private debt (panel b)

3 shows a scatter plot of the Chinn and Ito (2008) index of capital account openness and the average ratio of reserves-to-GDP. It shows a positive correlation between reserves and capital account openness, in line with the evidence presented by Aizenman and Lee (2007) and Bussiere et al. (2013). In other words, emerging countries that impose significant controls on international private flows of capital tend to have relatively smaller ratios of reserves-to-GDP than countries with more liberalized capital accounts.[12]

To summarize the empirical evidence that motivates our theoretical analysis, the data shows that reserves and private external debt are deeply intertwined. The relationship is apparent from time-series and cross-sectional data. In particular, we highlight four facts: (i) there has been a substantial increase in private external debt and international reserves in the aggregate; (ii) there is a positive correlation between reserves and private external debt in the cross-section; (iii) reserve and private external debt accumulation are procyclical; (iv) reserve holdings are larger in economies with a more open capital account.

These observations indicate positive associations, but they do not point to causality in *either* direction. We next propose a theory that sheds light on the interplay between private external debt and reserves and deliver dynamics consistent with the four aforementioned facts.

[12]In the appendix, we verify that positive correlation between reserves and capital account openness is present in a pooled OLS regression. The sign and significance of this association is not changed when we drop outliers from the panel.

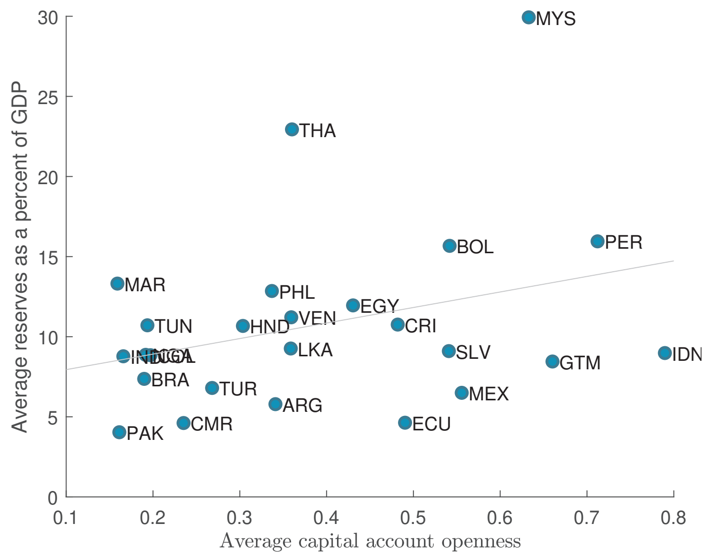

Figure 3: Average 1980–2015 reserves and Chinn and Ito (2008) capital account openness

3 Model

We consider a dynamic small open-economy model with tradable and non-tradable goods. The economy is populated by a continuum of identical households that borrow externally subject to an occasionally binding borrowing constraint. We describe first the households' problem, and then we analyze the competitive equilibrium and the role of international reserves.

3.1 Households' problem

Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada conditions; and consumption c_t is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta + 1)$ between tradable goods c_t^T and non-tradable goods c_t^N, given by

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \qquad \text{with} \quad \eta > -1, \omega \in (0,1).$$

In each period, households receive a random endowment of tradable goods y_t^T and a fixed endowment of non-tradable goods y^N. We use the tradable good as the numeraire.

Households can borrow (or save) using a one-period non-state-contingent bond b_{t+1} denominated in units of tradables paying an interest rate R_t, which is exogenously determined in international capital markets and may vary stochastically.[13] Their budget constraint, in units of tradables, is given by

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = y_t^T + p_t^N y^N - b_t - T_t, \tag{3}$$

where p_t^N is the price of non-tradable goods and T_t is a lump-sum tax. In addition, households face a credit constraint given by

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + p_t^N y^N \right). \tag{4}$$

This credit constraint captures in a parsimonious way the empirical fact that current income is critical in determining credit-market access (see e.g. Jappelli (1990); Lian and Ma, 2021), and it has been shown to be important for accounting for the dynamics of capital flows in emerging markets (e.g., Mendoza, 2002). Non-tradable goods enter the collateral constraint because while foreign creditors do not value these goods directly, they can seize them in the event of default and sell them in exchange for tradable goods on the domestic market.[14] We allow for shocks to the parameter κ_t, which we refer to as a financial shock. One interpretation of this shock is that it captures fluctuations in lenders' perceptions about households' ability to repay or in the country's institutional contract enforcement.

Households choose consumption and borrowing to maximize their utility (2) subject to their budget (3) and credit constraint (4), taking prices and taxes as given. Their optimality conditions are given by

$$p_t^N = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{\eta+1}, \tag{5}$$

$$\lambda_t = u_T(t), \tag{6}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t \quad \text{with } \mu_t = 0 \text{ if } b_{t+1}/R_t < \kappa_t \left(y_t^T + p_t^N y^N \right), \tag{7}$$

[13] Assuming no foreign inflation, it is equivalent to denominating the bonds in foreign currency, capturing the liability dollarization phenomenon.

[14] The credit constraint can be derived endogenously from a problem of limited enforcement under two assumptions. First, households can default at the end of the current period. Second, upon default, foreign creditors can seize a fraction κ_t of the current income, and households immediately regain access to credit markets. The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks (see Bianchi and Mendoza, 2018, for a derivation of a similar constraint).

where $u_T(t)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$ and μ_t denotes the non-negative Lagrange multiplier on the borrowing constraint. Condition (5) is a static optimality condition equating the marginal rate of substitution between tradable and non-tradable goods to their relative price. Condition (6) equates the marginal utility of tradable consumption to the shadow value of current wealth, and Condition (7) is the household's Euler equation for debt. When $\mu_t > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit and repaying next period.

3.2 Government

The government accumulates international reserves $A_{t+1} \geq 0$ and finances them with lump-sum taxes and existing holdings of reserves A_t.

$$\frac{A_{t+1}}{R_t} = T_t + A_t. \tag{8}$$

3.3 Competitive equilibrium

The market clearing condition for non-tradable goods is

$$c_t^N = y^N. \tag{9}$$

We can now define a competitive equilibrium for any government policies. Given initial conditions A_0, b_0, and government policies $\{T_t, A_{t+1}\}_{t\geq 0}$, a *competitive equilibrium* is defined as a stochastic sequence of prices $\{p_t^N\}_{t\geq 0}$ and households' policies $\{c_t^T, c_t^N, b_{t+1}\}_{t\geq 0}$ such that (i) households maximize their utility (2) subject to the sequence of budget constraints (3) and credit constraints (4), taking as given prices and government policies; (ii) the government budget constraint (8) is satisfied; and (iii) the market for non-tradable goods clears (9).

Combining the household's budget constraint (3) with the government's budget constraint (8) and the non-tradable goods market clearing (9), we obtain the economy's consolidated resource constraint for tradable goods:

$$c_t^T + \frac{A_{t+1} - b_{t+1}}{R_t} = y_t^T + A_t - b_t. \tag{10}$$

This constraint illustrates that from the perspective of the resource constraint in the small open economy, official reserves and household-issued bonds are perfect substitutes. Absent the credit constraint (4), Ricardian equivalence would hold and the amount of foreign reserves accumulated by the government would be completely irrelevant. However, as we argue below, the presence of the credit constraint (4) makes reserve accumulation both *relevant* and *desirable*.

Remark on households' accumulation of reserves. Notice that we have not explicitly considered the accumulation of the reserve asset by households, but this is without loss of generality. When the borrowing constraint is binding, the return on reserves R_t is lower than the shadow return on borrowing, and so households optimal choice of reserves is zero. When the borrowing constraint is not binding, households are indifferent between reserves and debt because both debt and reserves have the same maturity and deliver a risk-less return R_t in units of tradables.

3.4 Constrained efficiency

The competitive equilibrium is constrained inefficient due to a pecuniary externality. Households do not internalize that by borrowing more in the present and consuming less in the future, they put downward pressure on the future price of non-tradables and thereby contribute to tightening other agents' future credit constraint. Following Bianchi (2011), we consider the problem of a constrained social planner who directly chooses the economy's debt subject to the borrowing constraint and allows goods markets to clear competitively. In recursive form, the problem can be written as:

$$V(b, y^T, R, \kappa) = \max_{b', c^T} u(c(c^T, y^N)) + \beta \mathbb{E} V(b', y^{T\prime}, R', \kappa') \tag{11}$$

subject to

$$b + c^T = y^T + \frac{b'}{R}, \tag{12}$$

$$\frac{b'}{R} \leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N \right]. \tag{13}$$

where the substitution of the price of non-tradables expression (5) into (13) reflects the implementability constraints of the planner.

Using sequential notation, for convenience, the planner's Euler equation for debt is given by

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{14}$$

where λ_t and μ_t denote the Lagrange multipliers on (12) and (13). While equation (14) resembles the private Euler equation (7), a critical difference is that the shadow value of current wealth differs and is given by

$$\lambda_t = u_T(t) + \mu_t \Psi_t, \tag{15}$$

where Ψ_t denotes the equilibrium change in the collateral value associated to a marginal change in tradable consumption, defined as:

$$\Psi_t \equiv \kappa_t (p_t^N c_t^N)/(c_t^T)\,(1+\eta)\,. \tag{16}$$

The change in the value of collateral associated to a marginal change in tradable consumption is the product of three terms: the collateral parameter κ_t, the ratio of non-tradable to tradable expenditure, and the inverse of the elasticity of substitution.

The wedge between the planner's and the private marginal value of wealth captures how the planner internalizes that higher demand for consumption relaxes the economy's borrowing constraint. This wedge translates into an "overborrowing" externality whenever the credit constraint does not currently bind but is expected to bind with strictly positive probability in the next period.

3.5 Reserve accumulation

In this section, we demonstrate that the constrained-efficient allocations can be implemented using a policy for reserve accumulation. One potential advantage of the implementation with reserves relative to capital controls is the observation that leakages often undermine the effectiveness of the latter (Bengui and Bianchi, 2018). This may make reserve accumulation a more attractive policy to pursue in practice and can, in fact, rationalize why governments seldom resort to the use of capital controls (Fernandez, Rebucci and Uribe, 2015) and instead use reserves as a primary policy tool.

To establish our result, it is convenient to impose the following assumption.

Assumption 1. *Consumption is a Cobb-Douglas aggregator $c = (c^T)^\omega (c^N)^{1-\omega}$, and the credit constraint parameter satisfies $\kappa_t(1-\omega)/\omega < 1$.*

This assumption implies that $\Psi_t < 1$, for Ψ_t defined in (16), and guarantees that in any equilibrium, an increase in aggregate consumption by one unit does not relax the credit constraint by more than one unit. We return to the role played by this assumption later in this section, and emphasize that it is by no means necessary for our result to hold.

Our main normative result is summarized in the following proposition.

Proposition 1. *Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, b_{t+1}^\star, p_t^{N\star}\}_{t=0}^\infty$*. Then, given initial conditions* (b_0, A_0) *such that* $b_0^\star = b_0 - A_0$*, the competitive equilibrium features a tradable consumption allocation* $\{c_t^{T\star}\}_{t=0}^\infty$ *if the government follows the reserve policy* $\{A_{t+1}\}$ *given by*

$$\frac{A_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N\right) - \frac{b_{t+1}^\star}{R_t} \quad \textit{for all } t \geq 0. \tag{RP}$$

Proof. The proof is by construction. We will show that, given the sequence of prices $\{p_t^{N\star}\}_{t=0}^\infty$ and initial conditions, the sequence of consumption allocations $\{c_t^{T\star}, y^{N\star}\}_{t=0}^\infty$ satisfy the households' first-order conditions, which are both necessary and sufficient for optimality.

We start by guessing that given (RP), the households' credit constraint (4) holds with equality at all times:

$$\frac{b_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N\right). \tag{17}$$

Combining (17) with (RP), we obtain

$$b_{t+1}^\star = b_{t+1} - A_{t+1}. \tag{18}$$

Substituting (18) into the the tradable resource constraint (10) yields

$$c_t^T = y_t^T - (b_t - A_t) + \frac{b_{t+1}^\star}{R_t}. \tag{19}$$

Meanwhile, since $\{c_t^{T\star}, b_{t+1}^\star\}$ solve the constrained planning problem, we have

$$c_t^{T\star} = y_t^T - b_t^\star + \frac{b_{t+1}^\star}{R_t}. \tag{20}$$

Given the initial condition $b_0^\star = b_0 - A_0$, a comparison of (19) and (20) reveals that $c_t^T = c_t^{T\star}\ \forall t \geq 0$. That is, when households' borrowing policy satisfies (17) and reserves are set according to (RP), the constrained-efficient sequence of tradable consumption is consistent with the consolidated budget constraints of the household and the government. Notice that the non-negativity of A_{t+1} follows immediately from the reserve policy (RP) and the planner's credit constraint (13).

We are left to show that $c_t^T = c_t^{T\star},\ c_t^N = y^N$ satisfy the optimality conditions of the households. From conditions (14)-(15) characterizing the constrained-efficient allocation, we have

$$\mu_t^\star = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1) - \beta R_t \mathbb{E}_t \Psi_{t+1}^\star \mu_{t+1}^\star + \mu_t^\star \Psi_t^\star. \tag{21}$$

Rearranging the households' intertemporal Euler equation (7), we have that

$$\mu_t = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1). \tag{22}$$

Combining (21) and (22), we obtain

$$\mu_t = \beta R_t \mathbb{E}_t \Psi^\star_{t+1} \mu^\star_{t+1} + \mu^\star_t (1 - \Psi^\star_t) \geq 0, \tag{23}$$

where the the non-negativity of μ_t follows from $\Psi^\star_t = \kappa_t(1-\omega)/\omega < 1$, given Assumption 1, and the non-negativity of $\mu^\star_t$. Together, the conjecture (17) and the fact that $\mu_t \geq 0$ ensure that the households' intertemporal Euler equation and complementary slackness condition are satisfied. That is, condition (7) holds. Finally, notice that the households' intratemporal condition (5) follows directly from the definition of the constrained-efficient allocation, implying that $c^N_t = y^N$ is also optimal. ∎

The proposition establishes that under the reserve accumulation policy (RP), the competitive equilibrium achieves the same level of consumption as in the constrained-efficient allocation—and therefore delivers the same welfare. When the government accumulates reserves, households take on more debt to maintain the same level of consumption, until the credit constraint becomes binding. At that point, further increases in reserves generate a reduction in consumption and an increase in the net foreign asset position. The government then fine-tunes the amount of reserves to deliver the constrained-efficient net foreign asset position.[15]

The reserve policy effectively pushes private agents against their credit constraint whenever consumption in the laissez-faire economy would be above its level in the constrained-efficient allocation. Under the proposed reserve policy, the private credit constraint therefore holds with equality at all times. Notice that it strictly binds, however, only when there is a strictly positive probability of a binding credit constraint in the subsequent period under the constrained-efficient allocation. In states in which the credit constraint is not expected to bind next period, the constrained-efficient allocation can also be achieved by any alternative reserve policy satisfying $A_{t+1} \leq R_t \kappa_t \left(y^T_t + p^{N\star}_t y^N\right) - b^\star_{t+1}$ and such that the borrowing constraint does not hold with equality.[16] Intuitively, when the credit constraint does not bind, the anticipation that the constrained-

[15] We note that the proposition applies generally to an economy that is currently either a net foreign debtor or a net foreign creditor (but not if the economy is always a net creditor). What is important is that the economy faces at some point the possibility of a binding borrowing constraint. While this probability is higher when the country is a net debtor, a net creditor can also enter a crisis in the model after a sufficiently large negative shock.

[16] This can be seen by noting that if $\mu^\star_{t+1} = 0$ in all future states and $\mu^\star_t = 0$, (23) implies that $\mu_t = 0$, implying that the credit constraint is slack in the competitive equilibrium. In our quantitative analysis, the indeterminacy of the reserve policy arises only in 2.4% of the simulations.

efficient consumption allocation will prevail in the future leads households to pick the constrained-efficient consumption even without a government intervention.

In Appendix C, we also provide a dual result, by which the optimal accumulation of reserves that maximizes welfare in the competitive equilibrium yields the constrained-efficient allocation. In line with this dual result, in the remainder of the paper, we occasionally refer to the implementation of the constrained-efficient allocation via reserve policy as the "optimal reserve policy" outcome.

Reserve depletion and liquidity value. As the expression (RP) indicates, when the credit constraint holds with equality in the constrained-efficient allocation, the government depletes its stock of reserves, setting $A_{t+1} = 0$. This result illustrates the liquidity value of reserves for the economy. The government accumulates reserves in good times to be used during crisis times. By rebating reserves to households during a crisis, it stabilizes consumption, raises the price of non-tradables and reduces the amount of deleveraging. Because households do not internalize how a reserve buffer would generate positive general equilibrium effects during crises, it is the government that must accumulate the reserves.

To illustrate the importance of depleting reserves during a crisis, consider an alternative policy by which the government keeps a fraction ϕ of reserves: $A_{t+1} = \phi A_t$. Substituting this reserve policy into the economy's resource constraint (10) when the credit constraint (13) is binding yields a level of tradable consumption of

$$c_t^T = \frac{(1+\kappa_t)\, y_t^T + \left[1 - \frac{\phi}{R_t}\right] A_t - b_t}{1 - \kappa_t \frac{1-\omega}{\omega}}.$$

Hence, maximizing current consumption—the planner's effective objective when the credit constraint binds—requires a full reserve depletion (i.e., setting $\phi = 0$). The above expression also clarifies why private households undervalue reserves in a crisis. While from an individual perspective, a unit of reserves provides enough resource to consume one additional unit of tradable goods, in equilibrium a unit of reserves raises tradable consumption by $1/[1 - \kappa_t(1-\omega)/\omega] > 1$.

The sharp reduction in reserves when the planner's credit constraint binds is consistent with the evidence that central banks use a large portion of reserves during crises (see, e.g., Broner et al., 2013). In many cases, however, reserves are not entirely depleted (Aizenman and Sun, 2012). A potential explanation for why central banks may choose to keep a positive level of reserves during crises is that policy makers may fear that losing large amounts of international reserves would send a bad signal to market participants. An alternative explanation is pursued by Bocola and Lorenzoni (2020). In their model, holding reserves allows the government to convey to market participants that it has the necessary fiscal resources to intervene if a bad equilibrium occurs. Thus, the sole availability of reserves can implement the good equilibrium without reserves actually being used. By contrast, it is essential in our model that the government actually uses the reserves during crises.

3.6 Discussion

In this section, we briefly analyze extensions and variations of the basic model and discuss the extent to which the main theoretical result continues to hold.

Assumption 1 and equilibrium multiplicity. Assumption 1 is sufficient for our reserve implementation to work, but it is by no means necessary. In Appendix B, we prove a more general version of Proposition 1, in which we relax Assumption 1—departing in particular from Cobb-Douglas preferences—and show that the reserve accumulation policy (RP) still implements the constrained-efficient allocation under an alternative weaker condition. As we show in the appendix, a necessary condition is

$$\mu_t^\star(\Psi_t^\star - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^\star \Psi_{t+1}^\star \tag{24}$$

where stars refer to variables evaluated at the constrained-efficient allocations.

The implications of a violation of condition (24) can be more easily understood by assuming that $\mu_{t+1}^\star = 0$ in all successor states. In this case, when $\Psi_t^\star > 1$ and $\mu_t^\star > 0$, the planner is borrowing constrained but chooses a level of consumption which is higher than the unrestricted one (i.e., the level that would prevail at date t absent the date t credit constraint). This occurs because a low elasticity of substitution or a high κ_t (leading to a value of $\Psi_t > 1$) generates a non-convexity in the planner's problem such that that the planner may be forced to choose between very low levels of consumption or very high ones. As can be seen from (23), such an allocation cannot be implemented with reserves because it would imply a negative Lagrange multiplier on the borrowing constraint for the household.[17] Intuitively, households would never choose an allocation such that $u_T(t) < \beta R_t \mathbb{E}_t u_T(t+1)$. Achieving this allocation would require a subsidy on borrowing in these states, and reserve accumulation alone would not be enough. Even though our implementation result would not hold in this case, reserves would remain an effective

[17] If the credit constraint is expected to bind in the following period (i.e., $\mu_{t+1}^\star$ in some state at date $t+1$), the necessary condition is weaker. Intuitively, even though $\Psi_t^\star > 1$ indicates that a collective increase in borrowing is feasible, the planner may still choose a level of borrowing below the unconstrained level because it internalizes that more borrowing would tighten the constraint in the next period. As a result, given (23), we still have a positive Lagrange multiplier for households evaluated at the constrained-efficient allocations.

tool to reduce overborrowing ex ante.

Assumption 1 is related to the condition for self-fulfilling financial crises identified in Schmitt-Grohé and Uribe (2021). As they explain, equilibrium multiplicity may occur in this model because even though for an individual agent, an increase in debt tightens the borrowing constraint, in equilibrium, an increase in aggregate debt may actually raise the borrowing capacity more than one-for-one and relax the borrowing constraint. Assumption 1 is sufficient to ensure that when aggregate borrowing increases by one unit, the borrowing capacity does not increase by more than one unit. As Schmitt-Grohé and Uribe show, this rules out multiple equilibria by which sharp drops in consumption can become self-fulfilling.

It is worth highlighting that our implementation result would still hold under multiplicity of equilibria, as long as (24) holds. However, the reserve policy would not be able to *uniquely* implement the constrained-efficient allocation. To understand why, consider a situation in which, for a given level of debt, reserves, and shocks, the economy features multiple equilibrium levels of current consumption. A planner that can directly choose the level of borrowing would choose the high consumption equilibrium, but it may not be able to implement it using reserve accumulation. If agents were to coordinate on the bad equilibrium, the government would deplete its reserves to raise consumption and support the real exchange rate, but this would not be sufficient to increase the borrowing capacity to reach the good equilibrium unless all households were to further increase their consumption simultaneously. Nevertheless, the macroprudential role for reserves remains intact. In fact, the possibility of being trapped in a bad equilibrium could give rise to an even more significant role for reserves ex ante.[18]

Credit constraint. In the model, government reserves do not relax households' private borrowing capacity. We can show, however, that under alternative formulations of (17) by which government holdings of reserves raise the borrowing capacity (e.g., because of future lump-sum transfers), our main result of Proposition 1 continues to hold. Intuitively, as long as reserve accumulation does not raise households' borrowing capacity one-for-one, it is always possible for the government to reduce the economy's net borrowing by pushing agents eventually against their credit constraint.

In addition, the borrowing constraint (17) could also admit after-tax income as a source of collateral. In this case, reserve accumulation tightens the constraint, and we can show that lower levels of reserves implement the constrained-efficient allocations.

[18]The parameterization we use for our quantitative analysis violates Assumption 1 but delivers a unique equilibrium, as in Bianchi (2011). Appendix E provides details on how we check numerically for the presence of multiplicity.

Financing of reserves. The main result also holds when the government finances the reserves with domestic debt, in addition to taxes. By offering a high-interest rate on domestic bonds, the government can induce households to postpone consumption forward. In this case, the rate on government bonds is given by $u_T(t)/\beta u_T(t+1)$ and households would cease to be against their borrowing constraint. We note that to replicate the constrained-efficient allocations, it is essential here that foreign investors do not trade the bonds issued by the government. If foreign investors (with limited pockets) had access to domestic bonds, they would earn a rent at the expense of the small open economy, generating an extra cost from interventions see e.g., (Amador et al., 2020; Fanelli and Straub, 2020) and implying that capital controls would dominate foreign reserve intervention.[19]

3.6.1 Implementation via a feedback rule

Proposition 1 describes a state contingent reserve accumulation policy that implements the constrained-efficient allocations. In this section, we show that the constrained-efficient allocation can also be implemented using a simple *feedback rule* that directly specifies reserve policy as a function of the private sector's borrowing choice. Feedback rules are common in the study of monetary policy (i.e., the "Taylor rule") and under some circumstances can achieve the same outcomes as the state-contingent optimal policy (see e.g. Woodford, 2007). It turns out that a similar equivalence applies in our model.

Proposition 2. *Suppose Assumption 1 holds and initial conditions (b_0, A_0) are such that $b_0^\star = b_0 - A_0$. Then, the constrained-efficient allocation is achieved if the reserves are set according to the feedback rule given by*

$$\mathcal{A}_{t+1}(b_{t+1}) = b_{t+1} - b_{t+1}^\star. \tag{25}$$

Proof. See Appendix B.2. ∎

The feedback rule provides a simple, yet clear policy insight: the government should save an amount equal to the gap between the private sector's borrowing b_{t+1} and the constrained-efficient level of borrowing $b_{t+1}^\star$.

To understand the mechanics of the feedback rule and the strategic interactions between the government and households, consider a scenario in which the constrained-efficient allocation is

[19] In terms of the government problem in C, the difference is that the resource constraint now features a loss, which results from debt having a higher interest rate than reserves. Analyzing the trade-off in this case between the externality and the losses is an interesting area for future research. Davis, Devereux and Yu (2021) consider the role of domestic financial intermediaries where this loss is not present. Another friction in government financing could come from distortionary taxation, as explored in Kim and Zhang (2020).

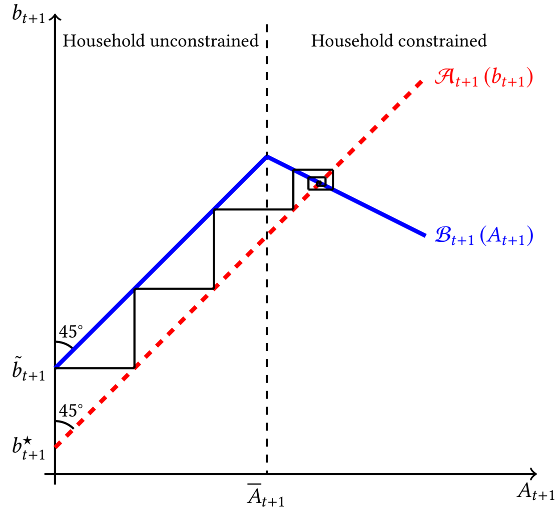

Figure 4: Illustration of the implementation of constrained-efficient allocation via feedback rule for reserves when the borrowing constraint is slack for the planner and $c^{\star}_{T} < \tilde{c}_t$.

implemented from date $t+1$ onward and let us focus on the outcome at date t. Denote by $\tilde{c}^T_t$ the level of tradable consumption the household would choose at date t in the absence of any reserve intervention, and by $\tilde{b}_{t+1}$ the associated level of private borrowing. Meanwhile, denote by $c^{T\star}_t$ and $b^{\star}_{t+1}$ the constrained-efficient tradable consumption and net borrowing. Given the overborrowing result, we have $\tilde{c}^T_t \geq c^{T\star}_t$ and $\tilde{b}_{t+1} \geq b^{\star}_{t+1}$.

Let us now examine how households respond to the purchases of reserves A_{t+1} at date t. To stay on their Euler equation (7), households adjust their borrowing according to

$$\mathcal{B}_{t+1}(A_{t+1}) = \begin{cases} A_{t+1} + \tilde{b}_{t+1} & \text{for } A_{t+1} < \bar{A}_{t+1} \\ R_t \kappa_t \frac{\frac{1}{\omega} y^T_t - \frac{1-\omega}{\omega}\left(b_t + \frac{A_{t+1}}{R_t}\right)}{1-\kappa_t \frac{1-\omega}{\omega}} & \text{for } A_{t+1} \geq \bar{A}_{t+1}, \end{cases}$$

where $\bar{A}_{t+1} \equiv R_t \kappa_t \left(y^T_t + \frac{1-\omega}{\omega}\tilde{c}^T_t\right) - \tilde{b}_{t+1}$ denotes the threshold of reserve purchases at which point the households' borrowing constraint becomes binding. For $A_{t+1} < \bar{A}_{t+1}$, households react to the lump-sum tax (expected to be offset by a positive future transfer) by a one-to-one increase in debt, following a Ricardian equivalence type of logic. For $A_{t+1} > \bar{A}_{t+1}$, the private debt level required to offset the tax is so large that it violates the household's credit constraint. In fact, above the threshold, more reserves contract the borrowing capacity of the economy and lead to *less* private debt rather than more private debt.

What is the level of reserves that the government needs to accumulate to implement a consumption of $c^{T\star}_t$ and a net borrowing of $b^{\star}_{t+1}$? Figure 4 illustrates how the interaction between the government's policy and the households' response determines the necessary level of reserves. The solid blue line represents the private sector's best response $\mathcal{B}_{t+1}(A_{t+1})$, and the dashed red line represents the government's policy $\mathcal{A}_{t+1}(b_{t+1})$ described by (25). Notice that the fact that households borrow more in response to the accumulation of reserves leads the government to accumulate even more reserves. Equilibrium is reached when both the private sector and the government play their best responses, i.e., when $\mathcal{A}_{t+1}(b_{t+1})$ and $\mathcal{B}_{t+1}(A_{t+1})$ intersect. At that point, official reserves are positive and private indebtedness has increased, but the economy's net foreign asset position has improved relative to the level that would have resulted from households' borrowing choice absent any reserve intervention at date t.[20]

4 Quantitative analysis

In this section, we present a quantitative analysis.[21] We organize the results as follows. First, we describe the calibration. Second, we present the policy functions to illustrate the workings of the model. Third, we show that the model is able to account for the four empirical facts presented in Section 2. Fourth, we present a simple rule for reserve accumulation. Finally, we present various extensions and perform sensitivity analysis.

4.1 Calibration

A period in the model represents a year. The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2,\ 1/(1+\eta) = 0.83$. The value for the interest rate is set to 4%, also standard in the literature.[22] For the calibration of the remaining parameters, we use data from Mexico, a common choice in studies of reserve accumulation (e.g., Bianchi et al., 2018) during the period 1980-2015.

To estimate the tradable endowment stochastic process, we use the value added series in the primary and industrial (net of construction) sectors.[23] We assume a first-order autoregressive process for the cyclical component: $\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y$ with $\varepsilon_t^y \sim N(0, \sigma_y)$, and estimate values

[20] In a state in which the borrowing constraint binds under the constrained-efficient allocation, we have $\bar{A}_{t+1} = 0$ and hence the households' best response only features a decreasing segment. In that case, the two best responses intersect at $A_{t+1} = 0$.

[21] The competitive equilibrium is solved numerically using time iteration and the optimal policy problem is solved with value function iteration, as in Bianchi (2011).

[22] In our baseline calibration, we assume a constant interest rate. We consider a stochastic interest rate in Section 4.5.

[23] We use value added data in local currency from Mexico's National Institute of Statistics and Geography (INEGI) for 1980-2015, deflated by sector specific prices.

of $\rho^y = 0.24$ and $\sigma_y = 0.034$. The value of ω is set to match the share of tradable GDP in the data, which is 33%.[24]

We assume that the process for the financial shock κ_t follows a first-order autoregressive process in logs: $\log(\kappa_t) = (1-\rho^\kappa)\log(\bar{\kappa}) + \rho^\kappa \log(\kappa_{t-1}) + \varepsilon_t^\kappa$ with $\varepsilon_t^\kappa \sim N(0, \sigma_\kappa)$. Unlike income, the financial shock is not directly observable. To discipline the process for κ_t, we exploit the fact that the credit constraint holds with equality under the optimal reserve intervention, and follow the approach proposed by Jermann and Quadrini (2012). Namely, taking (13) with equality, we back out a time series for κ_t using the observed sequence of output and debt. Since before the 1994 Tequila crisis, Mexico had very low levels of reserves, we take 1995-2015 as the reference period. We then estimate the aforementioned AR(1) process and obtain $\rho^\kappa = 0.82$, $\bar{\kappa} = 0.46$ and $\sigma_\kappa = 0.11$.

The remaining parameter is the discount factor. We calibrate β so that the average NFA in the economy without government intervention matches the average of Mexicos's NFA position.[25] This calibration yields $\beta = 0.94$. A summary of parameter values is provided in Table 2.

Table 2: Parameter Values

	Value	Source/Targets
Interest Rate	$r = 0.04$	Standard value
Risk Aversion	$\sigma = 2$	Standard value
Elasticity of Substitution	$1/(1+\eta) = 0.83$	Standard value
Weight on Tradables in CES	$\omega = 0.33$	Share of T, output = 33%
Stochastic structure y^T	$\rho^y = 0.24, \sigma_y = 0.034$	See text
Stochastic structure κ	$\rho^\kappa = 0.82, \bar{\kappa} = 0.46, \sigma_\kappa = 0.11$	See text
Discount Factor	$\beta = 0.94$	Average NFA $= -37.0\%$

4.2 Reserves and gross debt

We start by describing the workings of the model through an analysis of the policy functions for reserve accumulation and debt. We highlight how the reserve intervention differs markedly from those based on a Pigouvian tax intervention.

[24] In a non-stochastic version of the model with a level of net foreign asset position equal to $\overline{NFA}$ and tradable and non-tradable output normalized to one, the relative share of non-tradable to tradable output is given by the value of ω such that $1/[1 + \frac{1-\omega}{\omega}(1 + r\,\overline{NFA})] = 33\%$. Given the mean value of the NFA to be calibrated below, this yields $\omega = 0.325$

[25] Although gross positions have increased quite substantially over time, the average NFA is about the same in the periods 1980-1994 and 1995-2015.

Policy function for reserves. Figure 5 presents the optimal reserve accumulation policy as a function of the shocks the economy faces and the current value of debt. In panel (a), the amount of reserves is shown as a fraction of the tradable endowment, for a value of κ one standard deviation below the mean and for two possible values of beginning-of-period debt. In panel (b), the amount of reserves is shown as a function of the financial shock, for a value of y^T one standard deviation below the mean, again for two possible values of debt. In both cases, reserves are reported as a function of average GDP.

Figure 5 shows that the government finds it optimal to hold more reserves in good times, that is, when income is high or when financial conditions are less stringent. The intuition for these results is that when the amount that households can borrow rises (because of either higher y^T or higher κ), the government needs to accumulate more reserves to close the gap between the net amount of borrowing desired by the planner and the borrowing capacity of households. Similarly, when beginning-of-period debt is lower, households are further away from their credit constraint—they want to borrow less and they have more spare borrowing capacity—and the government accumulates more reserves.

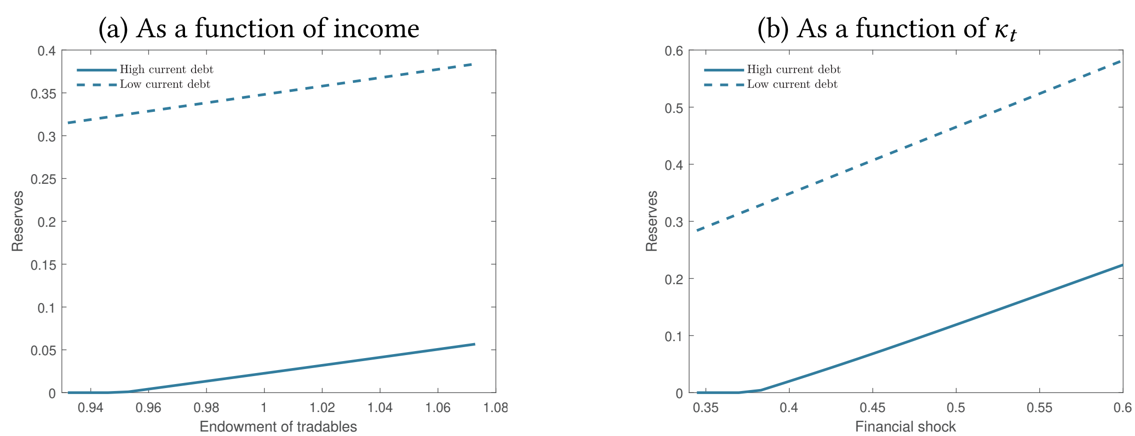

Figure 5: Policy function for reserves

Note: In the left (right) panel, the financial (output) shock is set to one standard deviation below the mean value.

Comparison with taxes on debt. An important fact that motivated our analysis was that countries that rely less on capital controls appear to hold larger amounts of reserves (fact 4). In our model, reserves and taxes on debt are substitute policies: a government that uses capital controls has no need for reserve accumulation and conversely, a government that accumulates reserves does not need to impose capital controls. It is interesting, however, to contrast the properties of the optimal reserve intervention policy with those of the optimal capital control policy. Figure 6 again displays policy functions for reserves, but this time together with policy functions for the optimal tax on debt, following the optimal borrowing tax formula of Bianchi (2011).

A common feature of the two policies is that they are passive when the constraint is already binding (both taxes on debt and reserve holdings are zero in this case). However, despite both responding to a macroprudential motive when the credit constraint is not binding, they differ markedly in terms of their cyclical properties. While reserves tend to *increase* with output, the tax on debt tends to *decrease* with output. The reason for this different cyclical pattern is as follows. When output is low, agents have stronger incentives to borrow, leading to a higher probability of a binding borrowing constraint in the future—hence, calling for a higher tax on debt. By contrast, as we explained above, when output is low, there is a smaller excess borrowing capacity, which calls for a smaller amount of reserves. A similar contrast applies with respect to the financial shock (see panel (b) of Figure 6).

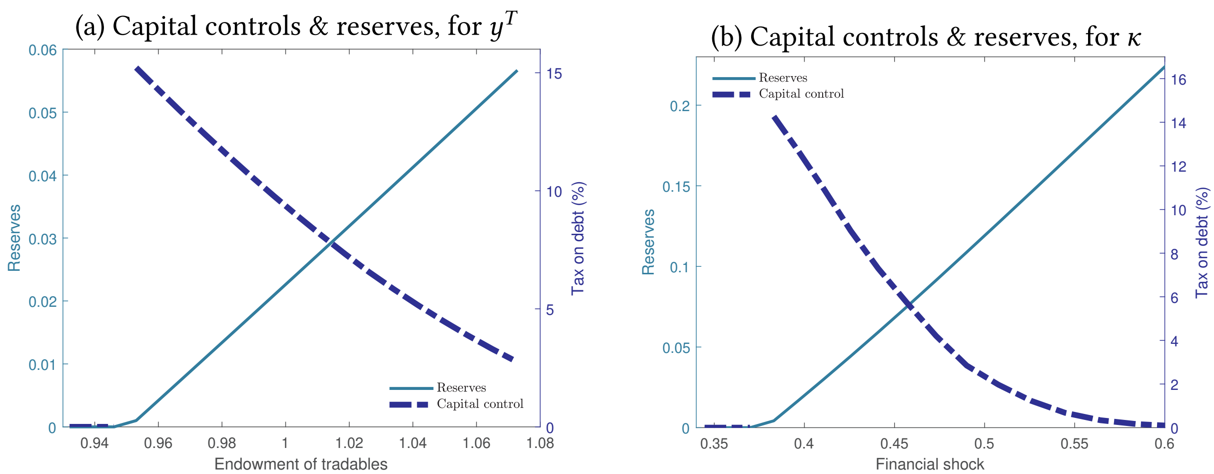

Figure 6: Reserve accumulation vs. capital controls

Note: In the left (right) panel, the financial (output) shock is set to one standard deviation below the mean value.

Policy functions for gross private debt. We now show how the profile of private debt depends critically on the government intervention. Figure 7 shows the law of motion for b' and its ergodic distribution for three economies: (i) laissez-faire, (ii) constrained-efficient and (iii) foreign reserve intervention.[26] Panel (a) shows that when current debt is high enough, the borrowing constraint binds and all three economies have the same end-of-period debt. For low debt levels, however, private debt choices differ: the constrained-efficient economy is the one in which the least amount of private debt is accumulated, followed by the laissez-faire economy and the economy with foreign

[26]By "constrained-efficient," we mean the allocation described in Section 3.4, while by "foreign reserve intervention," we mean the implementation of the constrained-efficient allocation presented in Section 3.5.

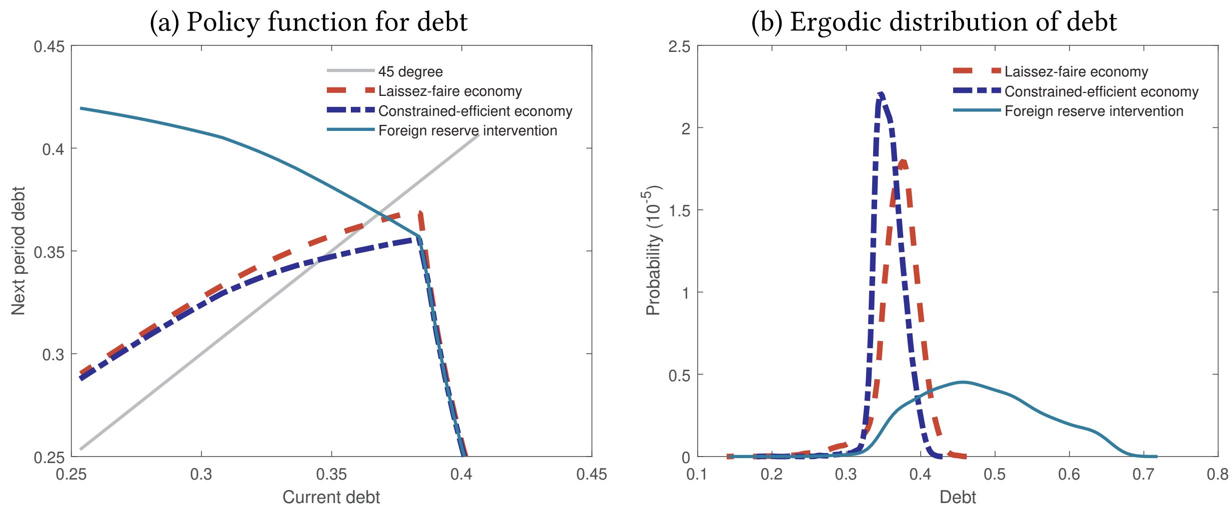

Figure 7: Debt: policy functions and ergodic distributions

Note:In the left panel, the financial and output shocks are set to one standard deviation below their respective mean values.

reserve intervention. In line with these results, panel (b) shows that the ergodic distribution of gross private indebtedness is located further to the right in the economy with foreign reserve intervention than in the other two economies.

A finding that stands out is that gross private indebtedness is *higher* under the foreign reserve intervention than in the laissez-faire economy.[27] This result emerges even though the laissez-faire economy features overborrowing relative to the constrained-efficient allocation. In other words, the laissez-faire economy displays a lower NFA position than the economy with the optimal reserve intervention but larger gross debt positions. This "underborrowing" result is thus different from the one described by Benigno et al. (2013). In that paper, the laissez-faire economy also issues too little debt; but critically, it has a higher NFA position relative to an economy in which the government has access to ex post policies.

Figure 8 further shows how the optimal reserve intervention changes the cyclical properties of private borrowing: the debt policy functions with respect to income and financial conditions are shown in panels (a) and (b), respectively. When income is low, borrowing is increasing in income for both the laissez-faire economy and the economy with the optimal reserve intervention. The reason is that when income is low, the borrowing constraint is binding and higher income helps relax it. When income is high, however, the two economies differ in the cyclical properties of borrowing: while borrowing is countercyclical under the laissez-faire, it is procyclical under the optimal reserve intervention. Under the laissez-faire, when the credit constraint does not bind,

[27]In the state space, this occurs technically for all values of debt except those at which the borrowing constraint is binding under the laissez-faire but not in the constrained-efficient allocation.

the economy borrows less when income is high, following a permanent income logic. Under the optimal foreign reserve intervention, in contrast, since the excess borrowing capacity is procyclical in the constrained-efficient allocations, the government accumulates more reserves when output is high, inducing households to take on more debt. On the other hand, panel (b) shows that private borrowing is procyclical with respect to financial conditions in both economies.

Our finding that optimal foreign reserve interventions may lead to higher private indebtedness has implications for empirical studies on credit booms and financial crises. In particular, it stresses the importance of taking official reserve dynamics into consideration when determining the role of private credit in predicting financial crises. In our model, the optimal foreign reserve intervention results in higher private indebtedness, yet a lower exposure to financial crises.

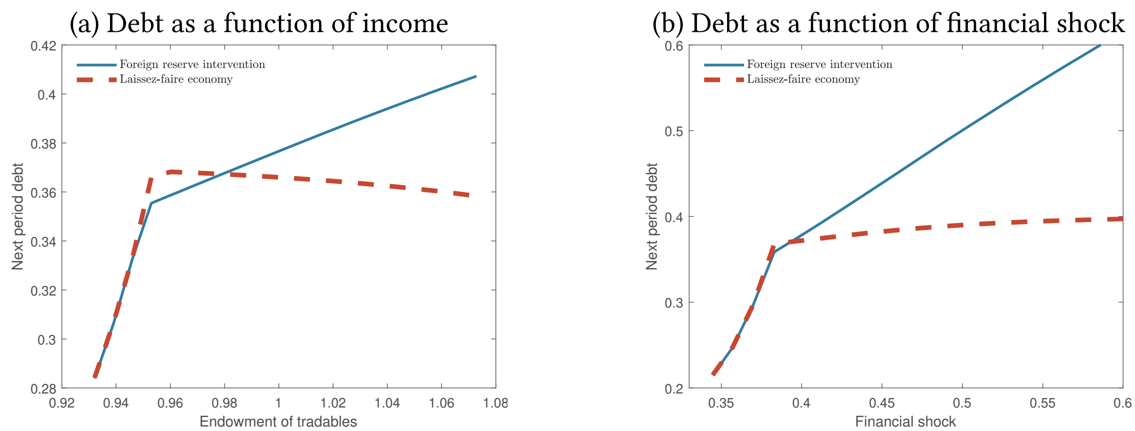

Figure 8: Equilibrium policy function for debt

Note: In the left (right) panel, the financial (output) shock is set to one standard deviation below the mean value.

Long-run moments. Table 3 displays average debt and reserves (as percent of output), together with the probabilities of crisis, according to the ergodic distribution for the different versions of the model. The amount of overborrowing in the laissez-faire economy relative to the constrained-efficient allocations is about 1% of GDP, but the amount of debt under the optimal intervention is about 11% of GDP higher than under the laissez-faire. Meanwhile, the optimal average level of reserves is 12% of output. This value is in the range of the recently observed level of reserves, which is around 15% of GDP for Mexico.[28]

[28] As we explained in Section 3, when the credit constraint is currently slack and its probability of being binding in the following period is zero, the reserve policy is indeterminate. While we assume that reserves follows the policy (RP) in our quantitative analysis, assuming that reserves are zero in these circumstances would only change the average level of reserves from 12.2% to 12.0%. The results differ very little because the indeterminacy arises only 2.4% of the time in the simulations.

It is worth emphasizing that even though the presence of overborrowing is key to justify the reserve accumulation policy, the scope of the intervention needed is not determined by the difference between the constrained-efficient level of borrowing and the level of borrowing in the laissez-faire. Rather, as indicated by (RP), the amount of intervention is determined by the difference between the constrained-efficient level of borrowing and *the economy's borrowing capacity*. This difference can actually be quite large, especially in states in which financial conditions are loose.[29]

Finally, the optimal intervention is quite successful at reducing the exposure to financial crises. We compute the probability of a financial crisis, defined as an episode where the current account increases by more than two standard deviations above its mean, following the empirical literature. In the laissez-faire economy, the probability of a crisis is 1.8%, which is in the range of the estimated frequency of financial crises for emerging markets (e.g., Calvo et al., 2006). In the simulations, these events are always characterized by binding credit constraints. The probability that the credit constraint binds is 2.5%, implying that nearly 70% of the times that a shock triggers a binding credit constraint, there is a sharp reversal of capital flows. Following the optimal reserve accumulation policy reduces this probability of financial crises to 0.4%.[30] While the optimal reserve intervention does not fully eliminate the occurrence of crises, it substantially reduces their frequency. As we show in Section 4.4, it also reduces their severity.

Table 3: Long-run moments

	Laissez-faire	Constrained-efficient	Optimal Reserves
Debt	37.2	35.9	48.1
Reserves	-	-	12.2
Crisis probability	1.8	0.4	0.4

4.3 Accounting for the stylized facts

We now assess the model's ability to account for the facts 1–4 outlined in Section 2. To do so, we simulate the model to generate artificial data comparable with the data used in our empirical analysis of Section 2.

[29] On the other hand, the difference in the unconditional level of borrowing between the constrained-efficient economy and the laissez-faire is only about 1% of GDP, consistent with Bianchi (2011).

[30] The probability of a binding credit constraint 0.7% in the constrained efficient economy.

Fact 1. First, we examine whether the model can account for the recent increase in reserves, while being consistent with the simultaneous rise in private external debt observed in the data. We use our calibration for Mexico. Starting in 2000, we fix the initial gross positions from the data and feed the observed path for income. A simulation of the model also requires a path for the financial shock, which is not observable. Given our interest in the joint trend between reserves and debt, we feed the sequence of financial shocks that deliver the sequence of debt observed in the data. As we show in Figure F.1, the model estimates a secular increase in κ, which is consistent with the rise in overall capital mobility. [31]

Panel (a) of Figure 9 shows that this exercise makes the model predict a significant increase in reserves, of about 10% of GDP, consistent with the increase observed in the data. While the model predicts more volatility in the path of reserves than there is in the data, the model's ability to account for the magnitude of the overall increase is quite remarkable. The model is therefore able to jointly explain the increase in debt and reserves. Notably, while the debt path was targeted in our simulation (see panel (b)), the path of reserves was not.[32]

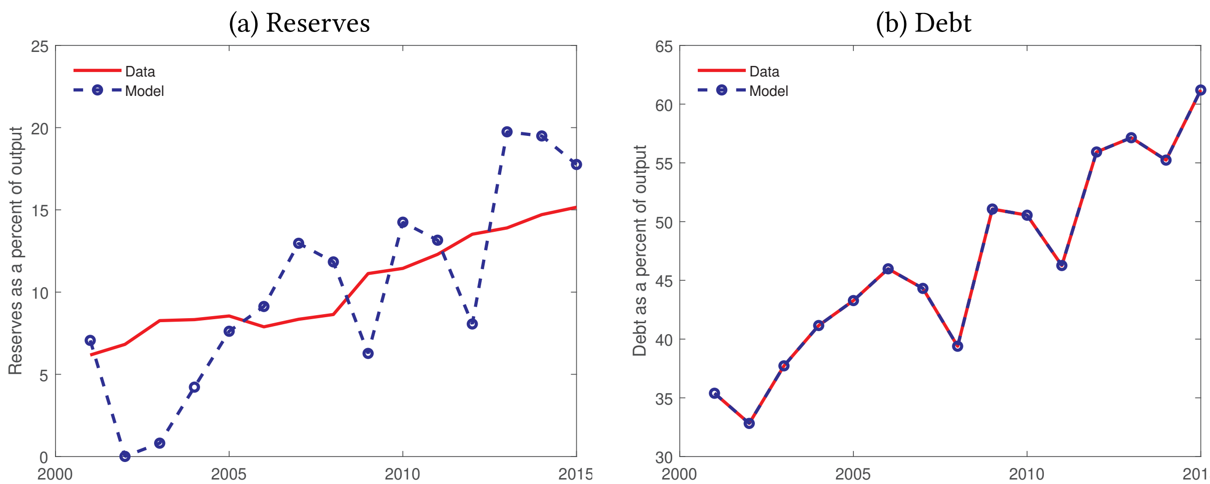

Figure 9: Evolution of reserves and debt, 2001–2015: data and model

Note: Model simulation obtained by feeding observed income shocks and calibrating financial shocks to match sequence of NFA (excluding reserves) observed in the data.

Fact 2. Second, we argue that our model is also consistent with the positive cross-sectional association between changes in reserves and changes in private external debt observed in the

[31]The sequence of debt in the data is constructed, analogously to the model, as the sum of the NFA plus reserves.

[32] We note that the financial shocks that are reverse-engineered in the procedure are consistent with the log-normal process estimated in the calibration section. In particular, the financial shock is within the 95% confidence interval generated by the estimated process in all but one year from 2001 to 2015 (see Appendix F for details).

data. To examine this fact through the lens of our model, we proceed in a way analogous to the way we proceeded in the data: (i) we construct 25 samples of simulations of 10035 years each, and focus on the last 35 years; (ii) we use this simulated data to construct model counter parts of the changes in reserves-to-output ratio, changes in the external debt-to-output ratio, and the current account-to-otuput ratio.; (iii) we estimate the effect of increases in debt on increases in reserves on a series panel regressions on the simulated data.

Table 4 reports panel regressions on the simulated data where the dependent variable is the net changes in foreign reserves-to-output ratios, and the controls the external private debt-to-output ratios and the current account-to-output ratio. Columns 1 to 4 are therefore the model counterparts of columns 1 to 4 on Table 1 respectively. As in the data, all the regressions include a constant and all standard errors are clustered at the country (simulated panel) level. Column 1 is a pooled OLS regression, column 2 includes country fixed effects only, column 3 includes time fixed effects only, and column 4 includes both time and country fixed effects. The table shows a positive and significant relation between reserves and private external debt in all specifications, consistent with fact 2. This association is holds both within country panels and across countries.[33] Of course, reserves in the data are also driven by factors other than the macroprudential motive studied in the paper. Hence, the estimated parameters in the data in Table 1 are lower than their model counterparts in Table 4.

Table 4: Changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	1.107^{***}	1.108^{***}	1.104^{***}	1.105^{***}
	(0.0210)	(0.0211)	(0.0220)	(0.0221)
Current Account	1.043^{***}	1.050^{***}	1.044^{***}	1.052^{***}
	(0.199)	(0.203)	(0.201)	(0.206)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country levels in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Results based on 25 samples of simulations of 35 years each, at their respective ergodic distribution, with each simulation representing a country year.

[33] In the appendix we show that we also obtain positive and significant relation where we apply panel FGLS to the simulated data.

Fact 3. Third, we show that our model generates time-series correlations between the changes in reserves, private external debt, and output. For each of our 10,000 samples, we compute the time-series of first differences of reserves, private debt and output.[34] We then calculate the correlation between the reserves and output series, and between the private debt and output series. Finally, we sort these correlations from the lowest to the highest.

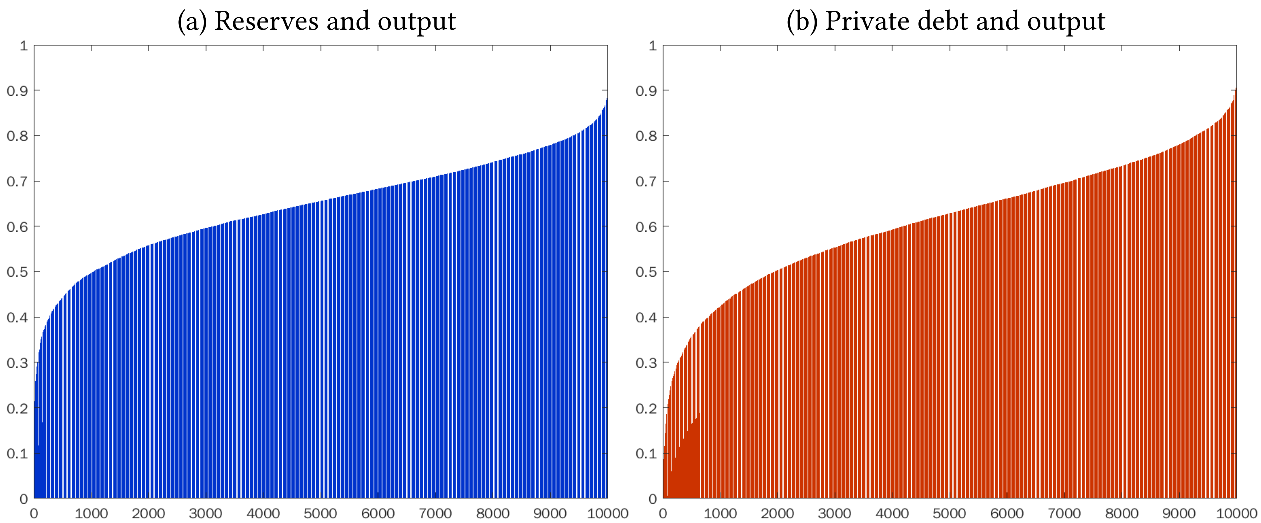

Figure 10: Correlations between changes in reserves and output (panel (a)), and changes in private debt and output (panel (b))

Note: Based on 10,000 samples of simulations of 30 years each, with each vertical bar measuring the correlation between two variables in a given sample.

Figure 10 displays these results: panel (a) shows the correlation between the reserve and output series; and panel (b) shows the correlation between the private debt and output series. Like in the data, both reserves and private debt are procyclical.

Fact 4. Fourth, we address the correlation between the accumulation of international reserves and capital account openness. So far, we have only considered implementations with either taxes or reserves. However, we can extend our analysis to address fact 4. We postulate that in the background there is a maximum tax rate $\bar{\tau}$ on borrowings that governments can or are willing to impose, either because of a fear of leakages or other unintended consequences. We assume that this maximum tax rate is heterogeneous across countries and draw for each of 10,000 simulations a different $\bar{\tau}$ from a uniform distribution between zero and τ^{max}, where we take τ^{max} to be the largest tax necessary to implement the constrained-efficient allocation in the ergodic distribution.

[34] We use the log of the private debt and output series, but not of the reserve series since there are several occurrences of zero reserves in the samples.

For each of these samples, we consider a government using a mix of capital controls and reserves. We assume that the government implements the constrained-efficient allocation using taxes on borrowing if the optimal tax rate is below the drawn maximum tax rate $\bar{\tau}$ while if the maximum tax is binding, the government sets the maximum tax rate and resorts to reserve accumulation to close the gap.

Figure 11 plots the average tax rate and reserves for each sample over 30 years. We find that in samples where average reserves are high, taxes on private borrowing are low. This negative correlation between reserves and traditional capital control policy is consistent with our fact 4.

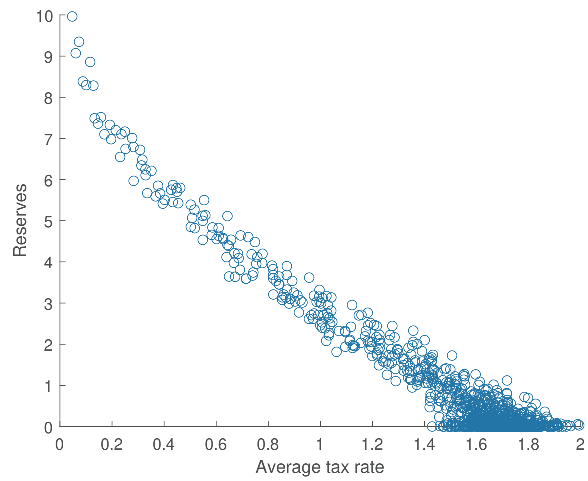

Figure 11: Average reserve and taxes in simulated economies

Note: We simulate 10,000 samples for 30 years each, with each dot representing a sample. The x-axis measures the average tax rate over the 30 periods. The y-axis measures the average level of reserves as a percent of output over 30 periods.

4.4 A simple rule for reserve accumulation

Motivated by practical policy considerations, we analyze a version of the model in which the government follows a simple policy rule for the accumulation of international reserves.[35]

The simple rule we propose approximates the optimal reserve accumulation by a linear function of state variables, including the NFA position:

$$A_{t+1}^{SR} = \max\{\beta_0 + \beta_1 y_t^T + \beta_2 \kappa_t + \beta_3 (A_t - b_t), 0\},$$

[35]Different from the feedback rule (25), the simple rule we consider does not require the government to know the constrained-efficient level of borrowing. It is possible to map this simple rule into a feedback rule where we replace the constrained-efficient level of borrowing in (25) by a target that depends on observables.

where the $\{\beta_i\}_{i=0}^{3}$ are constant parameters. We estimate these coefficients by maximizing the unconditional welfare gains from moving from the laissez-faire economy to the economy with the optimal simple rule.[36]

The results from the optimization yields the following coefficients:

$$\beta_0 = -0.34, \ \beta_1 = 0.45, \ \beta_2 = 0.68, \ \beta_3 = 0.29.$$

The estimated rule implies a reserve accumulation policy that is increasing in the current NFA position, income and the financial shock. The rule hence inherits the same basic qualitative properties as the optimal state-contingent policy, but differs in that it is a simple linear rule.

The financial stability gains from adopting the simple rule can be illustrated by conducting the following event analysis. First, we simulate the laissez-faire economy for a large number of periods, identify all the financial crisis episodes, and construct 7-year window events centered around the financial crisis episodes. Second, we take the average of key variables across the window period for the laissez-faire economy. Third, we feed the initial state and shock sequence for each event from the laissez-faire economy to the policy functions of the economies with the simple rule and with the optimal state-contingent reserve policy, again taking the average of key variables.

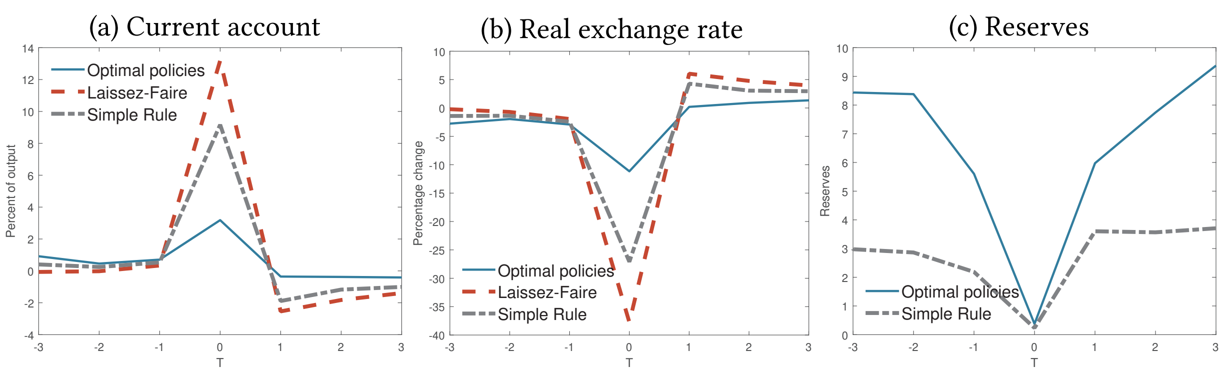

Figure 12: Financial crisis event analysis under optimal policies, laissez-faire, and a simple rule

The dynamics of the event analysis are shown in Figure 12. The path for the current account

[36]Numerically, we proceed by first running an OLS regression of the optimal level of reserves at the ergodic distribution on the exogenous state variables and the current NFA position in the economy with the optimal reserve intervention. We restrict the sample to observations where next period's level of international reserves is strictly positive ($A_{t+1} > 0$). Then, we construct a grid for each of the four parameters $\{\beta_i\}_{i=0}^{3}$ centered around the OLS estimates. Given three values for each parameter and a total four parameters, we have eighty-one possible combinations. We select the combination that gives the highest welfare gains. We repeat the process by centering the new grids on this combination. We iterate until we cannot increase the welfare gains by selecting any other point in the grid different from our initial guess.

and the real exchange rate are shown in panels (a) and (b), respectively, comparing outcomes in the laissez-faire economy, the economy with the optimal state-contingent reserve policy and the economy with the simple rule. The figure shows how, in a crisis, the laissez-faire economy experiences a large current account reversal of about 13% of GDP while the real exchange rate depreciates by close to 40%. These magnitudes are in line with empirical regularities of sudden stops (see, e.g., Calvo et al., 2006). The optimal reserve policy is successful at mitigating the severity of sudden stops as reflected in a more modest current account reversal of about 3% of GDP and a real exchange rate depreciation of 10%. The optimal use of reserves, therefore, reduces capital outflows by 10% of GDP and reduces the exchange rate depreciation by 30 percentage points.[37] The simple rule also delivers significant gains, reducing the current account reversal by about 5% of GDP and the real exchange rate depreciation by about 10 percentage points. In terms of welfare, we find that the simple policy rule delivers 12% of the total welfare gains achieved by optimal state-contingent reserve policy.

Panel (c) compares the path for reserves under the optimal policy with its counterpart under the simple rule. As it turns out, the simple rule prescribes less reserve accumulation than the fully optimal policy. The inability to conduct a perfectly state-contingent policy leads the government to err on the side of lower reserves. The intuition for this result is that too large reserve accumulation relative to the optimal may have the effect of excessively tightening households' borrowing constraint. In both cases, reserves fall to a value close to zero around crises, in line with our result on reserve depletion following from Proposition 1 and the empirical evidence that reserves fall sharply during crises (Broner et al., 2013).[38]

4.5 Sensitivity and Model Extensions

Production economy. We consider an extension of the model with production. This extension is important in light of the findings that endogenous production may call for ex-post stabilization policies and affect the efficient amount of borrowing (Benigno et al., 2013).

As we describe in more detail in Appendix D, a key feature that distinguishes this production economy from our baseline endowment economy is a reallocation of labor across sectors occurring in tandem with movements in the real exchange rate, in line with the empirical regularity documented, for instance, by Benigno, Converse and Fornaro (2015). During booms, labor moves

[37] We note that while the model is purely real, it would be relatively straightforward to extend it to a monetary model where the government defends the *nominal* exchange rate. For example, if the government followed an inflation targeting policy, it would prevent a large nominal exchange depreciation to keep inflation on target during a sudden stop.

[38] The reason why reserves do not exactly fall to zero in the Figure is that for some shock sequences that lead to crises the laissez-faire economy, the ex ante reserve accumulation succeeds at averting a crisis altogether.

to the non-tradable sector as the real exchange rate appreciates, while during crises, labor moves to the tradable sector as the real exchange rate depreciates. From a normative standpoint, the scope for labor reallocation also opens the door to welfare-improving ex-post stabilization policy interventions, such as sector specific taxes on labor, as in Benigno et al. (2013). Such interventions make crises less severe and increase borrowing ex ante by reducing the need for precautionary savings. However, we show in Appendix D, that irrespective of whether the planner has access to an ex-post instrument, such as a labor tax in the non-tradable sector, our main result that the constrained-efficient allocation can be implemented with reserves remains intact. What is key for our results is that households continue to face a low private shadow cost of debt relative to the social one. Hence, reserve accumulation remains desirable as a tool to increase the NFA position, irrespective of whether the planner has access to an ex-post instrument.

A quantitative analysis of the production economy model with an ex-post stabilization policy shows that the scope for reserve accumulation remains broadly in line with those of our baseline endowment economy model.[39] Implementing the constrained-efficient allocation with reserves and a labor tax in the non-tradable sector results in a reduction in the frequency and severity of financial crises comparable to those obtained in the endowment economy model, as indicated in Table 5. The average level of reserves, standing at 8.7% of output, is also close to the baseline value.

Liquidity role of reserves. We argued in Section 3.5 that in a crisis, it is optimal for the government to fully deplete its reserves. To further highlight the liquidity role of reserves in mitigating the severity of financial crises, we consider a scenario in which the government suboptimally keeps a fraction of reserves. More precisely, we construct an event analysis, as in Section 4.4, but assume that at the time of the crisis, the government unexpectedly deviates from the optimal reserve policy for one period and keeps 25% of its current level of international reserves. Figure 13 compares the dynamics of crises under this suboptimal policy with that associated with the optimal intervention. The experiment points to a strong liquidity role of reserves, as the depletion of an additional 1 percentage point (of GDP) of reserves reduces the current account reversal by over 3 percentage points (of GDP) and the real exchange rate depreciation by nearly 10 percentage points.

Interest rate shocks Finally, we consider shocks to the risk-free interest rate R_t to capture fluctuations in US monetary policy. Naturally, when interest rates are low, the model predicts an

[39] Our calibration assumes that the productivity of tradables follows a log-normal AR(1) of the form: $\ln A_t^T = \rho^A \ln A_{t-1}^T + \varepsilon_t^A$ with $\varepsilon_t^A \sim N(0, \sigma_A)$. The persistence parameter $\rho^A = 0.24$ is equal to the persistence to the endowment of tradables in the original model. The volatility $\sigma_A = 0.017$ is chosen to ensure that the standard deviation of tradable output as a share of output at the ergodic distribution coincides with its counterpart in the endowment economy.

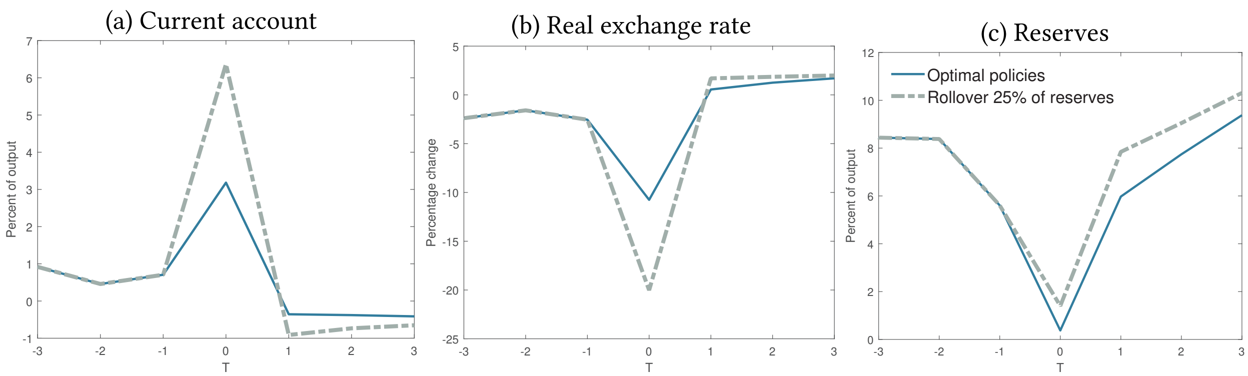

Figure 13: Financial crises event analysis when reserves are not fully depleted.

increase in consumption and a reduction in the desired NFA position. However, the effects on gross positions are more subtle. Reserves, in particular, respond to the gap between the economy's borrowing capacity and the constrained-efficient level of borrowing, as revealed by (RP). Keeping all other parameters at their calibrated values and using the estimated process from Bianchi, Liu and Mendoza (2016), we obtain quantitative results very similar to those obtained in our baseline model with a constant risk-free rate (see Table 5). In particular, the average level of reserves is unchanged at 12.2% of output and the reduction in the frequency and severity of financial crises remain comparable.

Table 5: Long-run moments in extended models.

	Baseline		Production economy		Stochastic R	
	Laissez-faire	Optimal	Laissez-faire	Optimal	Laissez-faire	Optimal
Reserves-GDP	-	12.2	-	8.7	-	12.2
Crisis probability	1.8	0.4	1.9	0.3	1.8	0.3
CA reversal	13.2	2.8	6.6	2.6	13.0	2.6

5 Conclusions

This paper has articulated a macroprudential theory of foreign reserve accumulation in which reserves provide a liquidity value not internalized by private agents. The model is consistent with key aspects of the interaction between private and public capital flows observed in the data. It can account for the increase in reserves and private debt over the last twenty years, the positive association between these variables in the cross-section and the time-series, and the observation that countries with more open financial accounts accumulate more international reserves.

There are several interesting avenues for future research. One would be to apply and further investigate the lessons of our theory for the use of reserve accumulation in models of financial crises that combine aggregate demand externalities and pecuniary externalities. In addition, the theory can also be extended by considering foreign intermediaries with limited capital. This would generate an additional cost from reserve accumulation which the government would balance against the financial stability benefits uncovered in this paper.

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Online Appendix to "A Macroprudential Theory of Foreign Reserve Accumulation"

A Appendix to Empirical Analysis

A.1 Data Sources

The sources for the data used in the paper are as follows:

Gross Domestic Product (GDP): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)"*, GDP current USD in current USD converted from domestic currency using the period-average exchange rate, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

International reserves: *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, "FX reserves" in millions of current USD. Does not include gold holdings, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Net Foreign Assets (NFA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, NFA in millions of current USD, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Private debt: *World Bank 2018. International Debt Statistics*, External debt stocks, private nonguaranteed (PNG) (DOD, current USD) from 1980 to 2015.

Public debt: *World Bank 2018. International Debt Statistics*, External debt stocks, public and publicly guaranteed (PPG) (DOD, current USD) from 1980 to 2015.

Real GDP growth: *IMF 2018. The World Economic Outlook (WEO) database*, Gross domestic product, constant prices in Percent change, from 1980 to 2015.

Capital Account openness: *Chinn, Menzie, and Ito (2006).* Index of capital account openness. Annualized using data from 1980Q1-2015Q4.

Current account (CA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, CA in millions of current USD, for Mexico from 1970 to 2015.

Tradable share of GDP: *National Institute of Statistics and Geography (INEGI))* We first compute the annual sectoral value added in the primary and secondary sector as a share of total value added from 1980 to 2015. Since the secondary sector includes construction we compute the average share in the desegregated series, available from 1993-2015. We estimate the share of the construction sector in total value added, 8% with a standard deviation of 0.5%. We subtract this average share from our first measure of tradable value added.

A.2 Sample of Countries

The sample of countries correspond to “Middle-Income Countries”. We arrive to our sample as follows. We start by considering the universe of all countries included in the International Debt Statistics dataset and exclude those listed as “Advanced economies” by the IMF and ”Low income countries” by the World Bank. To have a balanced panel from 1980-2015 we keep only countries that have positive values of private debt in the International Debt Statistics and that do not have missing values for the Chinn-Ito index of capital account openness or in the WEO and Lane and Milesi-Ferretti databases. The requirement of a balanced panel subtracts 40 countries from the sample. Finally, we also exclude countries that record a net foreign assets positions above or below 150% GDP at any point from 1980-2015: Mauritius, Cote d’Ivoire, Jamaica, Papua New Guinea, and Paraguay.

The final list includes the following 25 countries: Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela.

A.3 Robustness Results

This section presents additional regression results regarding empirical Fact 2 from Section 2. The econometric model is once again:

$$\Delta a_{it} = \alpha_i + \alpha_t + \gamma x_{it} + \beta \Delta b_{it} + \varepsilon_{it}, \tag{A.1}$$

where Δa_{it} and Δb_{it} denote the yearly changes in the foreign reserves-to-GDP ratio and private external debt-to-GDP ratio for country i. This time, however, the parameter of interest, β, is estimated using panel Feasible Generalized Least Squares (FGLS). The regression results are presented in Table 6. As before, all the regressions include a constant and control for the current account-to-GDP ratio. The regressions of columns 2 and 6 also control for the net changes in the PGD-to-GDP ratio and real GDP growth.

Moreover, to obtain accurate standard errors, we conduct preliminary tests on the error terms, ε_{it}. We find that error structure is heteroskedastic, serially autocorrelated within panels, and contemporaneously correlated across panels. Our parameter estimates in Table 6 are therefore robust to heteroskedasticity and cross-sectional correlation and assume a panel-specific autoregressive process of order one for the error terms. The coefficient of interest is again positive and statistically significant at the 1 percent confidence level, confirming a robust statistical association between changes in private external debt and changes in reserves.

We next verify that this positive association also holds in the simulated data. As in Section , we use the calibrated model to construct 25 samples of simulations of 10035 years each, and focus on the last 35 years. We use the model counterparts of the changes in reserves-to-output

Table 6: Net changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1)	(2)	(3)	(4)
	Reserves	Reserves	Reserves	Reserves
Private External	0.253^{***}	0.268^{***}	0.250^{***}	0.249^{***}
Debt	(0.0150)	(0.0143)	(0.0150)	(0.0146)
Current Account	0.170^{***}	0.246^{***}	0.159^{***}	0.244^{***}
	(0.0119)	(0.0130)	(0.0124)	(0.0138)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Time+Country

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

ratio, changes in the external debt-to-output ratio, and the current account-to-output ratio in FGLS panel regressions. The parameter estimates are shown in Table 7. The estimates and their respective standard errors are computed assuming the same error structure as in the empirical data. We obtain positive and significant parameters for all model specifications.

Table 7: Net changes in Reserves-to-output Ratios on changes in Private External Debt-to-output Ratios in the simulated data

	(1)	(2)	(3)	(4)
	Reserves	Reserves	Reserves	Reserves
Private External	1.107^{***}	1.107^{***}	1.108^{***}	1.108^{***}
Debt	(0.00514)	(0.00514)	(0.00586)	(0.00586)
Current Account	1.114^{***}	1.114^{***}	1.115^{***}	1.115^{***}
	(0.0229)	(0.0229)	(0.0254)	(0.0254)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country levels in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: Based on 25 samples of simulations of 35 years each, at their respective ergodic distribution, with each simulation representing a country year.

In table 8 we verify the robustness of Fact 3 and Fact 4 using a pooled OLS regression approach with clustered standard errors at the country level. In column 1 (column 2) we regress the growth rate in foreign reserves (private debt) on a constant ant the real GDP growth rate. Once again we find that reserves and debt accumulation are procyclical and that this relation is statistically significant. In column 3 we regress the ratio of reserves-to-GDP on a constant and the the Chinn and Ito (2008) index of capital account openness for our panel of countries. The positive association between reserves and capital account openness holds. To make sure that this result is not driven by outliers, in column 4 we drop from the sample all the observations from Thailand and Malaysia, and verify that the relation is still positive and significant.

Table 8: Robustness of Fact 3 and Fact 4

	Fact 3		Fact 4	
	(1)	(2)	(3)	(4)
	Growth in Reserves	Growth in Private External Debt	Reserves	Reserves
Real GDP Growth	1.098^{**}	0.907^{***}		
	(0.474)	(0.265)		
Index of Capital Account Openness			4.925^{*}	7.776^{***}
			(2.991)	(1.921)
Observations	865	865	875	805
Countries	25	25	25	23
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country levels in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B Proofs

In this section, we present the proofs not presented in the main text.

B.1 Implementation for more general parameterizations

We provide here a proposition analogous to Proposition 1, but without using Cobb-Douglas preferences and Assumption 1.

Proposition B1. *Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, b_{t+1}^\star, p_t^{N\star}\}$. *Assume that the solution satisfies the following condition*

$$\mu_t^\star(\Psi_t^\star - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^\star \Psi_{t+1}^\star \tag{B.1}$$

for all t *where* Ψ_t *is defined in* (16) *and* $\mu_t^\star$ *is given by* (21). *Then, given initial conditions* (b_0, A_0) *such that* $b_0^\star = b_0 - A_0$, *the decentralized equilibrium with a consumption allocation* $\{c_t^{T\star}\}$ *can be implemented if the government follows the reserve policy* $\{A_{t+1}\}$ *given by* (RP).

Proof. The proof follows the same steps as in Proposition 1, but uses (B.1) to show that given (23), we have that $\mu_t \geq 0$ is satisfied in the decentralized equilibrium. Notice that when Assumption 1 holds, condition (B.1) is trivially satisfied. ∎

B.2 Proof of Proposition 2

Proof. By construction, the feedback rule (25) guarantees that the economy's net foreign asset (NFA) position always coincides with its counterpart in the constrained efficient allocation $A_{t+1} - b_{t+1} = -b_{t+1}^\star$. Substituting this equality into economy's resource constraint (10) shows that the constrained efficient level of consumption is also achieved. Following the same steps as in the proof of Proposition 1, conditions (22), (C.16) and (23) must hold. When $\mu_t^\star > 0$ or $\mu_{t+1}^\star > 0$ in at least one successor state, (23) indicates that $\mu_t > 0$, so that $b_{t+1}/R_t = \kappa_t(y_t^T + p_t^{N\star} y^N)$ and $A_{t+1} = R_t\kappa_t(y_t^T + p_t^{N\star} y^N) - b_{t+1}^\star \geq 0$. Meanwhile, when $\mu_t^\star = 0$ and $\mu_{t+1}^\star = 0$ in every successor state, (23) indicates that $\mu_t = 0$, so that $b_{t+1} = b_{t+1}^\star$ is optimal and $A_{t+1} = 0$. Either way, the private borrowing choice is optimal and reserves are non-negative. ∎

C Optimal reserve accumulation policy

We consider the problem of the government that chooses a state-contingent sequence $\{A_{t+1}\}_{t=0}^{\infty}$ to maximize welfare in the competitive equilibrium. Let us use $U(c^T, c^N)$ to denote $u(c(c^T, c^N))$ The problem consists of solving:

Problem 1 (Optimal Policy).

$$\max_{b_{t+1}, A_{t+1}, c_t^T, \mu_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, y^N) \tag{C.1}$$

subject to

$$b_t + c_t^T = y^T + A_t + \frac{b_{t+1} - A_{t+1}}{R_t}, \qquad (\hat{\lambda}_t) \tag{C.2}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \qquad (\hat{\mu}_t) \tag{C.3}$$

$$A_{t+1} \geq 0, \qquad (\hat{\zeta}_t) \tag{C.4}$$

$$u_T(c_t^T, y^N) = \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) + \mu_t, \qquad (\hat{\xi}_t) \tag{C.5}$$

$$0 = \mu_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \qquad (\hat{\chi}_t) \tag{C.6}$$

$$\mu_t \geq 0. \qquad (\hat{\nu}_t) \tag{C.7}$$

where $\hat{\lambda}_t, \hat{\mu}_t, \hat{\xi}_t, \hat{\zeta}_t, \hat{\chi}_t$ and $\hat{\nu}_t$ are the multipliers associated with the constraints (C.2)-(C.7).

The government's first-order conditions for b_{t+1}, A_{t+1}, c_t^T and μ_t are given by

$$b_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t} + \hat{\chi}_t \mu_t = \beta \mathbb{E}_t \hat{\lambda}_{t+1} + \frac{\hat{\mu}_t}{R_t}, \tag{C.8}$$

$$A_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t} = \beta \mathbb{E}_t \hat{\lambda}_{t+1} + \hat{\zeta}_t, \tag{C.9}$$

$$c_t^T: \quad \hat{\lambda}_t = u_T(t) + \hat{\xi}_{t-1} R u_{TT}(c_t^T, y^N) - \hat{\xi}_t u_{TT}(c_t^T, y^N) + \Psi_t(\hat{\mu}_t - \hat{\chi}_t \mu_t), \tag{C.10}$$

$$\mu_t: \quad \hat{\xi}_t = \hat{\chi}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right] + \hat{\nu}_t, \tag{C.11}$$

and complementary slackness conditions

$$0 = \hat{\mu}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \tag{C.12}$$

$$0 = \hat{\zeta}_t A_{t+1}. \tag{C.13}$$

Combining conditions (C.9) and (C.10) reveals that the government faces different marginal benefit from accumulating reserves. In a state in which (C.6)-(C.7) are not binding—as will be indeed the case at the optimum—and assuming that the constraint for reserves and borrowing are currently slack, we have that

$$\begin{aligned} u_T(t) = \beta R_t \mathbb{E}_t [u_T(t+1) + \hat{\mu}_{t+1} \Psi_{t+1} - \\ \hat{\xi}_{t-1} R u_{TT}(c_t^T, y^N) + \hat{\xi}_t (R u_{TT}(c_{t+1}^T, y^N) - u_{TT}(c_t^T, y^N)) + \hat{\xi}_{t+1} u_{TT}(c_{t+1}^T, y^N)] \end{aligned} \tag{C.14}$$

There are two new terms in the planner's Euler equation for reserves relative to the household version. First, it is the pecuniary externality, captured by the $\hat{\mu}_{t+1}\Psi_{t+1}$ term, which reflects that the planner internalizes that having more reserves in a future state with a binding borrowing constraint has positive general equilibrium effects. Second, there is an incentive term that captures how households respond to government policy. Chiefly important for this effect is that the planner is subject to households' borrowing Euler equation as an implementability constraint and that because of the overborrowing externality, the Lagrange multiplier is non-negative. When the government accumulates reserves, this lowers at the margin current consumption and increases future consumption. These effects tighten today's implementability constraint and relax next period implementability constraint, as reflected in the two components of the "incentive term."

As it turns out, at the optimum ξ_t becomes zero. Intuitively, once the level of reserves is large enough, the borrowing constraint becomes binding, and thus households cannot offset the government policy. When optimizing, the government exactly fine-tunes the accumulation of reserves so that the borrowing constraint becomes binding exactly at the level of tradable consumption that corresponds to the constrained-efficient allocation. This result is a corollary of the proposition below.

Proposition C2. *Suppose Assumption 1 holds. Then, the solution to the optimal reserve accumulation policy presented in Problem 1 achieves the same utility as the constrained-efficient allocations. Moreover, the optimal policy is time consistent.*

Proof. We guess and verify that (C.5)-(C.7) are slack, and so $\hat{\xi}_t = \hat{\chi}_t = \hat{\nu}_t = 0$. Using this conjecture, and combining (C.8) and (C.10), we arrive at

$$u_T(c_t^T, y^N) = \beta R_t \mathbb{E}_t [u_T(c_{t+1}^T, y^N) + \hat{\mu}_{t+1} \Psi_{t+1}] + \hat{\mu}_t (1 - \Psi_t). \tag{C.15}$$

By Assumption 1, $\Psi_t < 1$ and since $\hat{\mu}_{t+1} \geq 0$ and $\Psi_{t+1} \geq 0$, we have that:

$$u_T(c_t^T, y^N) - \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) \geq \hat{\mu}_{t+1} \Psi_{t+1}. \tag{C.16}$$

Setting $\mu_t = \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) - u_T(c_t^T, y^N))$ and using (C.16), it follows that μ and $\hat{\mu}$ have the same sign. Hence (C.7) is satisfied. Moreover, from (C.12), it follows that (C.6) is satisfied as well as conjectured.

Finally, (C.8) and (C.9) imply that (C.4) binds if and only if (C.3) binds, so that (C.3) and (C.4) can be combined to deliver

$$\frac{b_{t+1} - A_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right). \tag{C.17}$$

Using $b_{t+1} - A_{t+1} = b_{t+1}^{\star}$, we can see that (C.17) is equivalent to the borrowing constraint in the constrained-efficient problem (13). Therefore, Problem 1 reduces to the same constrained-efficient planning problem of Section 3.4. Hence, it follows that the optimal reserve policy achieves the constrained-efficient allocations and is time consistent. ∎

D Reserve accumulation in production economy

This appendix provides additional details on the production economy model described in Section 4.5 and shows that the reserve implementation of the constrained efficient allocation outlined in Proposition 1 equally applies to that framework, irrespective of whether an ex post instrument is available.

We assume that households are endowed with a fixed amount of hours $\bar{h}$, , and do not value leisure. They receive a competitive wage w_t for their labor, as well as profits from firms in the tradable and nontradable sectors, π_t^T and π_t^N. The household's budget and credit constraint are given by:

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = w_t \bar{h} + \pi_t^T + \pi_t^N - b_t - T_t, \tag{D.1}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(w_t \bar{h} + \pi_t^T + \pi_t^N \right), \tag{D.2}$$

The tradable and nontradable goods are produced by competitive firms that maximize profits and solve:

$$\max_{h_t^T} z_t^T (h_t^T)^\alpha - w_t h^T \tag{D.3}$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^\alpha - (1 + \tau_t^N) w_t h^N + T_t^N, \tag{D.4}$$

where z_t^T is a stochastic productivity shock, z^N and α are constant parameters, and τ_t^N is a labor tax in the non-tradable sector, to be rebated lump-sum via a transfer T_t^N to non-tradable goods-producing firms.

The competitive equilibrium is given by sequences of consumption, labor, wages and prices of non-tradables such that all optimality conditions are satisfied and market clearing holds for all goods and labor:

$$c_t^T - \frac{b_{t+1}}{R_t} = z_t^T (h_t^T)^\alpha - b_t, \tag{D.5}$$

$$c_t^N = z^N (h_t^N)^\alpha, \tag{D.6}$$

$$\bar{h} = h_t^N + h_t^T. \tag{D.7}$$

We consider two constellations in turn. First, in Section D.1, we consider a constellation, similar to that considered in our baseline model, where the planner chooses directly the level of non-state contingent debt every period, but lets markets for labor and goods clear competitively. Then, in Section D.2, we consider an alternative constellation, where, similar to Benigno et al. (2013), the planner has in addition also access to a distortionary tax on labor.

In either case, households' optimality conditions are still given by (5), (6) and (7). The tradable and non-tradable goods firm's optimality conditions are respectively given by

$$w_t = z_t^T \alpha \left(h_t^T\right)^{\alpha-1}, \tag{D.8}$$

$$(1+\tau_t^N) w_t = p_t^N z^N \alpha \left(h_t^N\right)^{\alpha-1}. \tag{D.9}$$

Combining (D.8) and (D.9), we obtain

$$1+\tau_t^N = \frac{z^N \left(h_t^N\right)^{\alpha-1}}{z_t^T \left(h_t^T\right)^{\alpha-1}} p_t^N. \tag{D.10}$$

In what follows, we present the planning problems with one and two instruments in Sections D.1 and D.2, respectively. In each case, we then re-state our main normative result that the constrained efficient allocation can be implemented via reserves for this production economy and show that the proof works similarly to that of Proposition 1.

D.1 Without availability of ex post instrument

In recursive form, the planner's problem can be written as:

$$V(b, y^T, R, \kappa) = \max_{c^T, c^N, h^T, h^T, b'} u(c^T, c^N) + \beta \mathbb{E} V(b', y^{T\prime}, R', \kappa') \tag{D.11}$$

subject to

$$c^T - \frac{b'}{R} = z^T \left(h^T\right)^{\alpha} - b, \tag{D.12}$$

$$c^N = z^N \left(h^N\right)^{\alpha} \tag{D.13}$$

$$\bar{h} = h^T + h^N \tag{D.14}$$

$$\frac{b'}{R} \leq \kappa \left[z^T \left(h^T\right)^{\alpha} + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} z^N \left(h^N\right)^{\alpha} \right], \tag{D.15}$$

$$0 = \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} - \frac{z^T}{z^N} \left(\frac{h^T}{h^N}\right)^{\alpha-1}. \tag{D.16}$$

where (D.12) is the resource constraint for tradable goods, (D.13) is the resource constraint for non-tradable goods, (D.14) is the time constraint for labor, (D.15) is the credit constraint, and (D.16) is an implementability constraint associated firms' and households' optimal intratemporal choices when no labor tax is available.

Using sequential notation for convenience, the planner's Euler equation for debt is still given by (14), while its remaining optimality conditions for c_t^T, c_t^N, h_t^T and h_t^N are given by

$$\lambda_t = u_T(t) + (\mu_t\kappa_t + \nu_t)(1+\eta)\frac{p_t^N c_t^N}{c_t^T} \tag{D.17}$$

$$\delta_t = u_N(t) - (\mu_t\kappa_t + \nu_t)(1+\eta)p_t^N \tag{D.18}$$

$$\chi_t = (\lambda_t + \mu_t\kappa_t)\, z_t^T \alpha \left(h_t^T\right)^{\alpha-1} - \nu_t\,(\alpha-1)\, p_t^N \frac{1}{h_t^T} \tag{D.19}$$

$$\chi_t = \left(\delta_t + \mu_t\kappa_t p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha-1} + \nu_t\,(\alpha-1)\, p_t^N \frac{1}{h_t^N} \tag{D.20}$$

where λ_t, δ_t, χ_t, μ_t and ν_t respectively denote the multipliers on constraints (D.12), (D.13), (D.14), (D.15) and (D.16).

Combining (D.17), (D.18), (D.19) and (D.20) to eliminate δ_t, χ_t, and ν_t leads to an expression for the planner's marginal utility of tradable wealth given by

$$\lambda_t = u_T(t) + \mu_t^\star \widetilde{\Psi}_t, \tag{D.21}$$

for $\widetilde{\Psi}_t \equiv \Psi_t \Upsilon_t$, where Ψ_t was defined in (16) and

$$\Upsilon_t \equiv \frac{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{c_t^N h_t^T}}{\frac{1-\alpha}{\alpha}\frac{\bar{h}}{c_t^N h_t^T} + (1+\eta)\left[c_t^T/(c_t^T + p_t^N c_t^N)\right]^{-1}}, \tag{D.22}$$

Therefore, a wedge between the planner's and private shadow value of wealth similar to the one present in our baseline endowment economy model is apparent from (D.21). Note that relative to the endowment economy model, the wedge includes an attenuation term $0 < \Upsilon_t < 1$ reflecting the fact that the labor reallocation from the tradable to the non-tradable sector, and the associated rise in non-tradable consumption, mutes some of the increase in the price of non-tradable brought about by a marginally higher tradable goods consumption.

Proposition D3. *Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^\star, p_t^{N\star}\}_{t=0}^\infty$. Then, given initial conditions (b_0, A_0) such that $b_0^\star = b_0 - A_0$, the decentralized equilibrium features allocations $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^\infty$ if the government follows the reserve policy $\{A_{t+1}\}$ given by*

$$\frac{A_{t+1}}{R_t} = \kappa_t\left[z_t^T\left(h_t^{T\star}\right)^\alpha + p_t^{N\star} z_t^N \left(h_t^{N\star}\right)^\alpha\right] - \frac{b_{t+1}^\star}{R_t} \quad \textit{for all } t \geq 0. \tag{D.23}$$

Proof. The proof follows the same steps as the proof of Proposition 1. We will show that, given the sequence of prices $\{p_t^{N\star}\}_{t=0}^\infty$ and initial conditions, the sequence of allocations $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^\infty$ satisfy the households' and firms' fist-order conditions, which are both necessary and sufficient for optimality.

First, we note that from the firms' first-order conditions (D.8) and (D.9), the labor allocation must satisfy

$$p_t^{N\star} = \frac{z_t^T}{z^N}\left(\frac{h_t^T}{\bar{h} - h_t^T}\right)^{\alpha-1}. \tag{D.24}$$

It follows that $h_t^T = h_t^{T\star}$ and $h_t^N = h_t^{N\star}$. Next, we guess that given (D.23), the households' credit constraint (D.2) holds with equality at all times:

$$\frac{b_{t+1}}{R_t} = \kappa_t\left[z_t^T\left(h_t^{T\star}\right)^\alpha + p_t^{N\star} z_t^N\left(h_t^{N\star}\right)^\alpha\right]. \tag{D.25}$$

Combining (D.25) with (D.23), we obtain

$$b_{t+1}^\star = b_{t+1} - A_{t+1}. \tag{D.26}$$

Substituting (D.26) into the tradable resource constraint (D.5) yields

$$c_t^T = z_t^T\left(h_t^T\right)^\alpha - (b_t - A_t) + \frac{b_{t+1}^\star}{R_t}. \tag{D.27}$$

Meanwhile, since $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^\star\}$ solve the constrained planning problem, we have

$$c_t^{T\star} = z_t^T\left(h_t^T\right)^\alpha - b_t^\star + \frac{b_{t+1}^\star}{R_t}. \tag{D.28}$$

Given the initial condition $b_0^\star = b_0 - A_0$, a comparison of (D.27) and (D.28) reveals that $c_t^T = c_t^{T\star}$. That is, when households' borrowing policy satisfies (D.25) and reserves are set according to (D.23), the constrained-efficient sequence of tradable consumption is achieved. Further, the non-negativity of A_{t+1} again follows immediately from the reserve policy (D.23) and the planner's credit constraint (D.15).

We are left to show that $c_t^T = c_t^{T\star}$, $c_t^N = c_t^{N\star}$ satisfy the optimality conditions of the households. From conditions (14) and (D.21) characterizing the constrained-efficient allocation, we obtain (21) with $\widetilde{\Psi}$ instead of Ψ. Rearranging the households' intertemporal Euler equation (7), we have (22). Combining (21) and (22), we obtain (23), again with $\widetilde{\Psi}$ instead of Ψ. The non-negativity of μ_t follows from $\widetilde{\Psi}_t = \Upsilon_t\kappa_t(1-\omega)/\omega < 1$, given Assumption 1 and $\Upsilon_t^\star < 1$, and the non-negativity of $\mu_t^\star$. Together, the conjecture (D.25) and the fact that $\mu_t \geq 0$ ensure that the households' optimality (7) condition is satisfied. Finally, notice that the intratemporal condition (5) follows directly from the definition of the constrained-efficient allocation, implying that $c_t^N = c_t^{N\star}$ is also optimal for the household. ∎

D.2 With availability of ex post instrument

In this case, the planner's problem is given by:

$$V(b, y^T, R, \kappa) = \max_{c^T, c^N, h^T, h^T, b'} u(c^T, c^N) + \beta \mathbb{E} V(b', y^{T'}, R', \kappa') \tag{D.29}$$

subject to

$$c^T - \frac{b'}{R} = z^T \left(h^T\right)^{\alpha} - b, \tag{D.30}$$

$$c^N = z^N \left(h^N\right)^{\alpha} \tag{D.31}$$

$$\bar{h} = h^T + h^N \tag{D.32}$$

$$\frac{b'}{R} \leq \kappa \left[z^T \left(h^T\right)^{\alpha} + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} z^N \left(h^N\right)^{\alpha} \right], \tag{D.33}$$

where the only difference with (D.11)-(D.16) is that the implementability constraint (D.16) associated with the intratemporal allocation of labor accross sector can now be dropped due to the availability of a labor tax in the non-tradable sector.

Again using sequential notation for convenience, the planner's Euler equation for debt is still given by (14), while its remaining optimality conditions for c_t^T, c_t^N, h_t^T and h_t^N are given by

$$\lambda_t = u_T(t) + \mu_t \kappa_t (1+\eta) \frac{p_t^N c_t^N}{c_t^T} \tag{D.34}$$

$$\delta_t = u_N(t) - \mu_t \kappa_t (1+\eta) p_t^N \tag{D.35}$$

$$\chi_t = \left(\lambda_t + \mu_t \kappa_t\right) z_t^T \alpha \left(h_t^T\right)^{\alpha-1} \tag{D.36}$$

$$\chi_t = \left(\delta_t + \mu_t \kappa_t p_t^N\right) z^N \alpha \left(h_t^N\right)^{\alpha-1} \tag{D.37}$$

where λ_t, δ_t, χ_t and μ_t again respectively denote the multipliers on constraints (D.30), (D.31), (D.32) and (D.33).

Combining (D.34)-(D.36) and (D.37), together with (D.10) leads to an expression for the optimal labor tax in the non-tradable sector:

$$\tau_t^N = \mu_t \kappa_t c_t^N (1+\eta) p_t^N \frac{\left(c_t^T\right)^{-1} + \left(c_t^N\right)^{-1} \frac{z^N \left(h_t^N\right)^{\alpha-1}}{z_t^T \left(h_t^T\right)^{\alpha-1}}}{u_T + \mu_t \kappa_t} \geq 0. \tag{D.38}$$

This optimal tax is positive whenever the credit constraint binds for the planner (i.e., whenever $\mu_t > 0$), indicating that the planner optimally redirects production away from the non-tradable sector so as to support the price of non-tradable goods and relax the credit constraint at the margin.

Proposition D4. *Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}, b_{t+1}^{\star}, p_t^{N\star}, \tau_t^{N\star}\}_{t=0}^{\infty}$*. Then, given initial conditions* (b_0, A_0) *such that* $b_0^\star = b_0 - A_0$*, the decentralized equilibrium features allocations* $\{c_t^{T\star}, c_t^{N\star}, h_t^{T\star}, h_t^{N\star}\}_{t=0}^{\infty}$ *if the government follows the reserve policy* $\{A_{t+1}\}$ *given by*

$$\frac{A_{t+1}}{R_t} = \kappa_t \left[z_t^T \left(h_t^{T\star} \right)^{\alpha} + p_t^{N\star} z_t^N \left(h_t^{N\star} \right)^{\alpha} \right] - \frac{b_{t+1}^{\star}}{R_t} \quad \textit{for all } t \geq 0. \tag{D.39}$$

Proof. The proof follows the same steps as those of Proposition D3, but adds the sequence of labor taxes $\{\tau_t^{N\star}\}_{t=0}^{\infty}$ to the set of variables conditional upon which private agents make their decisions. Accordingly, we invoke the firms' optimality conditions (D.8) and (D.9), together with the optimal tax expression (D.38), to show that the labor allocation satisfies $h_t^T = h_t^{T\star}$ and $h_t^N = h_t^{N\star}$. Furthermore, (D.34) is used instead of (D.21) to obtain (21). Combining (21) and (22), we obtain (23), again concluding that $\mu_t \geq 0$. The remaining steps are analogous those followed in the proof of Proposition D3. ∎

E Checking for Multiple Equilibria

Open economy models with a collateral constraint like the one we study may feature multiple equilibria, as formally established by Schmitt-Grohé and Uribe (2021). They provide necessary and sufficient conditions under which a non-stochastic version of the model will feature equilibrium multiplicity. Our parameterization does not fall within the conditions that would allow us to determine unambiguously that there is unique or multiple equilibria. Because of this, we provide a numerical algorithm designed to check whether our equilibrium is unique. For our calibration, we find that there are no other equilibria.

We begin from the solution to the competitive equilibrium absent any intervention, which we solve using time iteration, following Bianchi (2011).[40] We denote the equilibrium law of motion for debt as $\mathcal{B}(b, s)$. We construct a grid for possible values of debt $B_N = [b_1 .... b_{max}]$. For the upper bound of the grid, we take a value arbitrarily close to the natural debt limit. The natural debt limit can be obtained as the fixed point of the following problem:

$$
\begin{aligned}
b_{\max}(s) &= \max_{b'} y^T + \frac{b'}{R} && \text{(E.1)}\\
&\quad \text{subject to} \\
\frac{b'}{R} &\leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{y^T + \frac{b'}{R} - b_{\max}}{y^N} \right)^{1+\eta} y^N \right]
\end{aligned}
$$

Considering that this condition must be satisfied for every possible shock, we obtain $b_{max} = y_{min}(1 + \kappa)$.

For every point in the grid for initial debt and shocks, we then check whether the following conditions are satisfied for every $b' \in B_N$ other than $\mathcal{B}(b, s)$.

$$
\begin{aligned}
u_T(y^T - b + b'/R_t, y^N) &= \beta R_t E u_T(c_T(\mathcal{B}(b', s), s'), y^N) + \mu && \text{(E.2)}\\
b' &\leq \kappa \left[\frac{1-\omega}{\omega} \left(\frac{y^T - b + b'/R_t}{y^T} \right)^{\eta+1} y^N + y^T \right] && \text{(E.3)}\\
\mu &\geq 0 && \text{(E.4)}
\end{aligned}
$$

If there is another $b' \in B_N$ that satisfies these conditions, then there is multiple equilibria.

Following this procedure, we do not find any state with more than one solution to the system of equations. To illustrate this result, we turn to Figure E.1. In the two panels of the figure, the red line computes the excess borrowing capacity, that is the difference between the debt limit and

[40] We use 100 equally spaced points in the grid for bonds between 0 and the natural debt limit, 20 values for the tradable endowment shock, and 17 values for the financial shock. We interpolate linearly for values of next-period bonds not in the grid. To solve the constrained efficient economy, we use Lagrange multiplier iteration. Specifically, we compute the planner's λ_t to solve for the optimal policies, and update our policies using equation (15).

the issued level of debt (E.4), for each value on the grid. The blue line uses (E.3) to calculate the Lagrange multiplier associated with the collateral constraint (μ) consistent with each potential debt level. In an equilibrium with a binding collateral constraint the excess borrowing capacity is zero and the Lagrange multiplier associated with the collateral constraint (μ) is positive. Conversely, in an equilibrium with a non-binding collateral constraint the excess borrowing capacity is strictly positive and the Lagrange multiplier associated with the collateral constraint is zero.

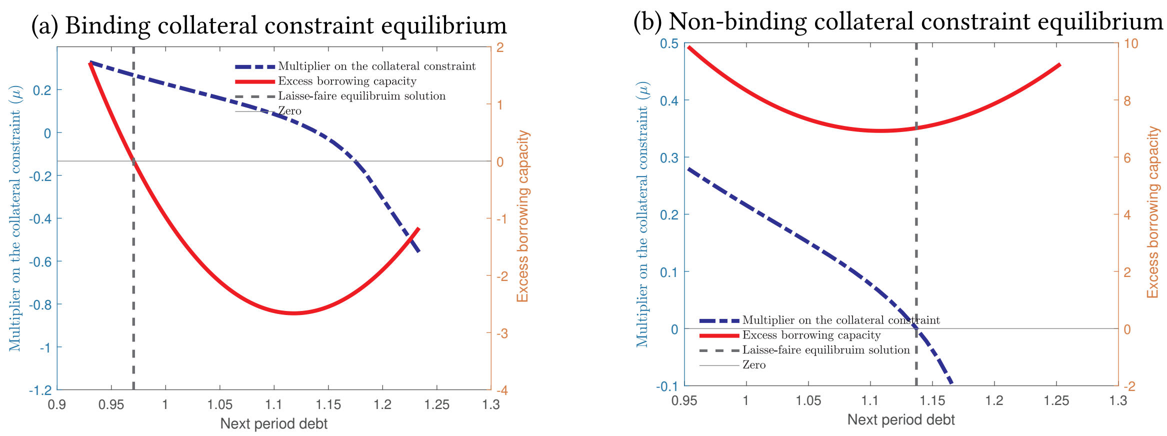

Figure E.1: Verifying Uniqueness

Note: Solutions to the optimality conditions for the states with the highest values of the financial shock where the collateral constraint binds and an endowment of tradables two standard deviations below its mean

Panel A, plots the solutions to equations (E.3) and (E.4) for the state with an endowment of tradables two standard deviations below its mean and the highest value of the parameter κ such that the collateral constraint still binds for high enough levels of initial debt. As we can see in panel A, it is possible that a higher level of next period debt that the one that solves the competitive equilibrium will also will also satisfy (E.4) with equality (zero excess borrowing capacity). However, at those higher values of debt, the Lagrange multiplier associated with the constraint is not positive. Similarly, a lower level of end of period debt, will be consistent with a positive excess borrowing capacity, but at those values the Lagrange multiplier μ is strictly positive.

Panel B plots instead an equilibrium at the same exogenous state as panel A but where the initial level of debt is smaller and the competitive equilibrium no longer exhibits a binding collateral constraint. As one can see, it is not possible to find equilibrium with a binding collateral constraints. Solutions with a b Alternative levels of debt are also not a solution with a binding constraint, since the excess borrowing capacity is never zero in this case.

So we have established that under our parameterization, we can only find one equilibrium. The only caveat of this procedure is that we take as given a continuation equilibrium from $t > 0$. However, given a continuation equilibrium for $t > 0$, the procedure can exhaustively determine whether there is multiplicity or not at $t = 0$ for every point in the grid.

F Financial shocks

In the dynamic exercise presented in Figure 9 we feed to the regulated economy a sequence of financial shocks (κ_t) that is consistent with our assumption that Mexico was following optimal reserve accumulation policies during those years. As explained in section 4.1, the financial shock follows a first-order autoregressive process of the form:

$$\log(\kappa_t) = (1 - \rho^{\kappa})\log(\bar{\kappa}) + \rho^{\kappa}\log(\kappa_{t-1}) + \varepsilon_t^{\kappa}$$

Figure F.1 plots this sequence of kappa shocks (panel a) as well as the innovations of this autoregressive process (panel b). We also plot the 95% confidence intervals associated with each observation.

(a) Financial shock (b) Innovations in the financial shock

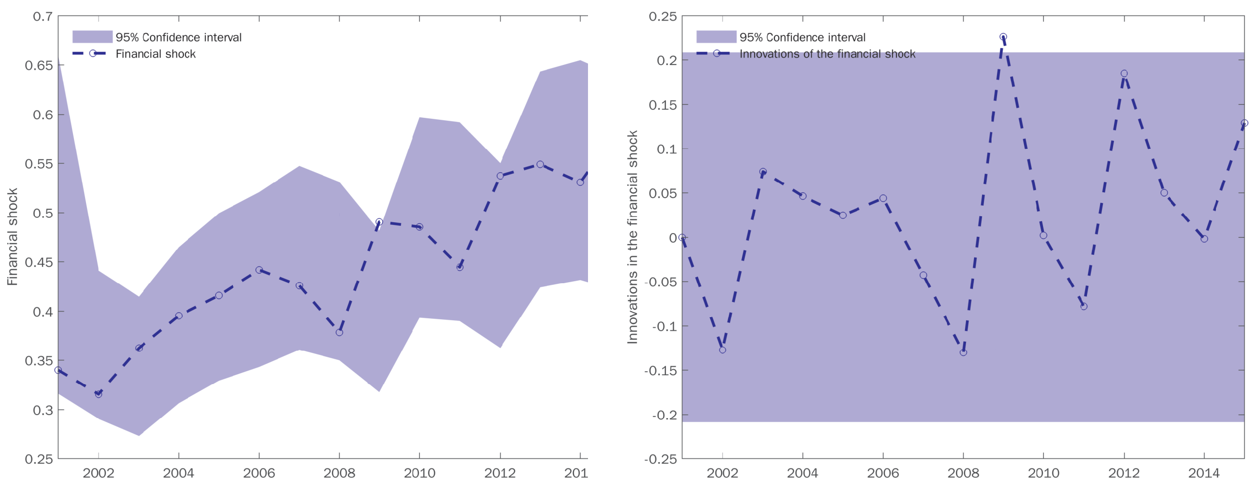

Figure F.1: Financial shock and innovations of the financial shock in the dynamic exercise

Note: For the first observation of the financial the bounds of the 95% confidence intervals are computed using the unconditional mean and standard deviation of the auto-regressive process.

G Additional Tables and Figures

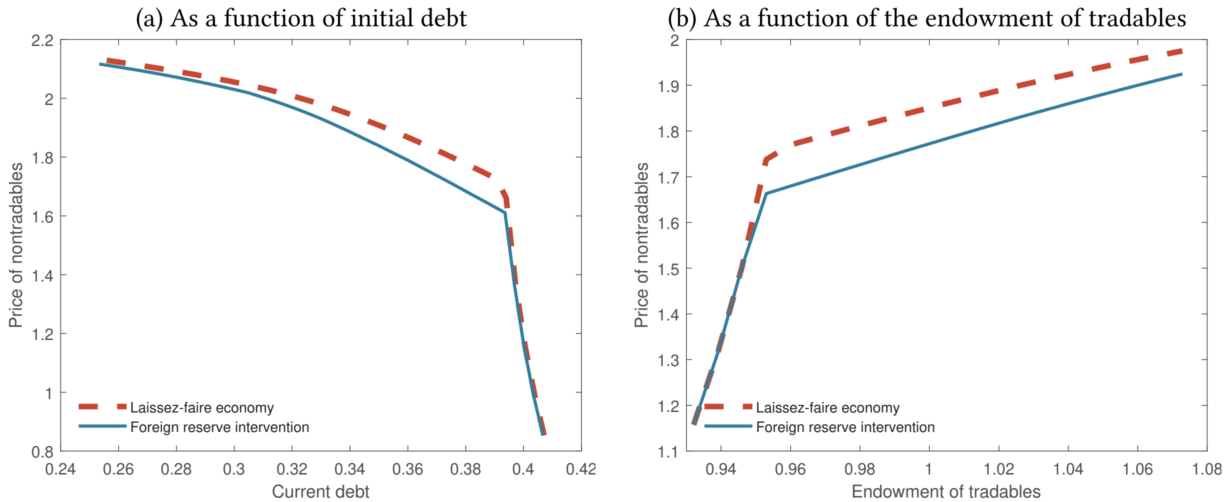

Figure G.1: Price of nontradables