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Government Debt Management and Inflation with Real and Nominal Bonds*

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Abstract

In the wake of rising inflation in the aftermath of unprecedented debt financed stimulus packages, we ask: Can governments use real bonds (TIPS) as part of their debt portfolio to commit to stable inflation rates? We propose a novel framework of optimal debt management in the presence of sticky prices with a government that can issue nominal and real non state-contingent bonds. Nominal debt can be inflated away giving ex-ante flexibility, whereas real bonds are cheaper but constitute a real commitment ex-post. Under Full Commitment, the government chooses a leveraged portfolio of nominal liabilities and real assets to use inflation effectively to smooth fiscal policy. When the government cannot commit to future policies, it reduces borrowing costs ex ante using real debt strategically to mitigate incentives for the future government to monetize debt ex-post. Without commitment, the policies are quantitatively consistent with US data, suggesting that such a framework realistically captures the relevant constraints governments face.

Keywords: Optimal Fiscal Policy, Optimal Debt Management, TIPS, Incomplete Markets, Inflation, Inflation Risk Premia, Monetary Policy, Machine Learning, Time Inconsistency.

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1 Introduction

Inflation has returned.¹ Indeed, the annual inflation rate in the US edged up to a 40-year high of 8.6% for the 12 months ending in May 2022, with inflation expectations rising alongside. Similarly, after a decade that was dominated by central bankers' fear of deflation, inflation forecasts and long term Treasury yields have been widening recently as well. These concerns reflect not only the potential upward pressure on prices caused by supply and capacity shortages when demand recovers in the aftermath of the pandemic, but also the surge in government debt across the globe following fiscal stabilization programs and stimulus packages both around the Great Recession and the Pandemic. The 1.9 trillion dollar American Rescue Plan Act of 2021, further adds to US government debt, which is projected to reach around 200 percent of GDP in 2050, according to the CBO (as of March 2021). In situations with such unprecedented debt levels, governments and central banks may be tempted to restore budget balance by monetizing debt, thereby strengthening inflationary pressure.

In this paper, we examine how governments can optimally manage their debt portfolios in the presence of inflation concerns and high debt levels. Starting from the simple observation that real or indexed debt (TIPS) cannot be inflated away ex-post, we examine the government's optimal debt portfolios when it has access to both nominal and real non state-contingent bonds. We consider both governments that can commit to future policies under Full Commitment, and such that cannot and respond strategically to the actions of future governments under No Commitment, thereby solving for the Ramsey equilibrium and the optimal time-consistent policy, respectively.

First, we solve for the Ramsey equilibrium in a setting in which the government has to finance an exogenous stochastic expenditure stream either by levying distortionary labor taxes or by issuing real or nominal non state-contingent debt. We allow for multi-horizon debt and assess the implications of short versus long term debt for equilibrium quantities and debt portfolios. Inflation has real costs because of the presence of nominal rigidities through sticky prices and is affected by the monetary authority which sets the nominal short-term interest rate by responding to inflationary pressure following a Taylor rule. Therefore, our paper contributes to the literature started in the seminal work of [Lucas and Stokey \(1983\)](#) on optimal fiscal and monetary policy, and considers both long-term nominal and real debt with incomplete markets and in models with nominal rigidities, building on

¹*The Economist's* issue of December 12, 2020, was titled "will inflation return?".

to [Siu \(2004\)](#), [Schmitt-Grohe and Uribe \(2004\)](#), [Marcet, Oikonomou, and Scott \(2013\)](#) , and [Lustig, Sleet, and Yeltekin \(2008\)](#).

When the government cannot issue TIPS, the Ramsey planner faces a trade-off between responding to shocks using distortionary taxes versus inflation. On the one hand, by inflating away the nominal liability, the government can finance additional expenditures without increasing labor taxes. However, by raising expected inflation, the planner reduces the value of household savings and decreases the price of government nominal bonds. Therefore, both current and future prices of nominal bonds are lowered. The addition of inflation protected securities in the government debt portfolio affects this trade-off in two ways. On the one hand, inflation protected securities constitute a real commitment ex-post and cannot be inflated away as the planner needs to compensate real bond holders. On the other hand, higher inflation has smaller impact on the cost of current and future borrowing since inflation does not affect the price of real bonds.

We find that in equilibrium, the Ramsey planner uses both types of bonds and that the optimal government portfolio prescribes a substantial role to real bonds. We derive analytical results showing that the use of inflation allows to implement real and nominal price differences that help to complete the markets and that the investment position in real and nominal bonds depends on the type of shock considered. In the quantitative model we consider an economy with exogenous government expenditure shocks and find that the optimal policy prescribes the allocation to nominal bonds in good times and reallocation to real bonds in bad times. By doing this the planner uses inflation to reduce the nominal liability and at the same time issues real bonds, whose price does not decrease as much in the presence of rising inflation expectations. Quantitatively, in our baseline calibration, inflation is more volatile but on average lower than in the model with only nominal bonds. This implies a welfare gain of 0.223%, which is achieved through better management of inflation risk and bond prices.

We find that inflation response is shaped by (i) the outstanding nominal debt and (ii) the maturity of debt. When the outstanding nominal debt is high, it becomes more tempting to use inflation as the same inflation rate allows to alleviate a larger debt burden, while creating the same misallocation cost due to nominal rigidities. We find that higher nominal debt leads to high inflation, which is optimal as long as the government reallocates to real bonds once the rising inflation begins to affect nominal bond prices. Longer debt maturity, on the other hand, is related to lower inflation rates as a longer planning horizon allows to spread inflation costs across multiple periods. We find that longer maturity implies

inflation that is less volatile but more responsive to expenditure shocks, which, overall, improves household welfare.

Critically, we find that the commitment friction drives the difference between the observed debt portfolios in the data and the optimal allocations under Full Commitment. The optimal policy without commitment is strategically biased, designed not only to smooth fiscal policy but also to best respond to the future government in order to reduce borrowing costs. A hedging portfolio with levered positions constitutes an expensive financial choice ex-ante and exacerbates the dilemma posed by the lack of commitment ex-post. Future governments have incentives to monetize debt ex-post to which households respond by raising the current government’s borrowing costs ex ante. In this situation, the current government finds it optimal to borrow using real debt so as to lower borrowing costs and mitigating future governments’ incentive to inflate nominal debt away. Notably, the tension is resolved by an optimal debt management policy that match the data.

The nonlinear nature of the equilibrium inflation response in our model requires an accurate global solution. We solve the optimal policy under Full Commitment using an algorithm similar in spirit to the Parameterized Expectations Algorithm ([den Haan and Marcet, 1990](#)). This is computationally challenging in our environment, as the complexity of solving Ramsey problems with multiple maturities increases in the length of the longest maturity and the state space is highly multicollinear. In this paper we exploit a machine learning algorithm based on artificial neural networks to tackle these problems, as proposed in [Valaitis and Villa \(2021\)](#). As mentioned, we build on a version of the parameterized expectations algorithm ([den Haan and Marcet, 1990](#)) and use neural networks to project expected value terms on the state space. A detailed description of the solution algorithm under Full Commitment can be found in [Appendix A.1](#). We solve the optimal policy with No Commitment with a different methodology, adopting an algorithm similar in spirit to the one introduced by [Clymo and Lanteri \(2020\)](#). A detailed description of the solution algorithm under No Commitment can be found in [Appendix A.2](#).

2 Related Literature

The paper builds on the papers studying the Ramsey problem with non-state contingent government debt ([Aiyagari, Marcet, Sargent, and Sappala, 2002](#); [Angeletos, 2002](#); [Buerra and Nicolini, 2004](#); [Faraglia, Marcet, Oikonomou, and Scott, 2019](#); [Bhandari, Evans, Golosov, and Sargent, 2019](#)). [Aiyagari, Marcet, Sargent, and Sappala \(2002\)](#) show that

when the government can only issue real bonds of one period maturity, the Ramsey planner achieves the complete markets outcome in the long-run by accumulating assets and using government savings to smooth tax distortions. [Angeletos \(2002\)](#) shows that complete markets outcome can be achieved if the number of maturities available is weakly greater than the number of states, while [Buerra and Nicolini \(2004\)](#) argue quantitatively that this requires unrealistically large long and short positions and rebalancing of government debt. [Bhandari, Evans, Golosov, and Sargent \(2019\)](#) study optimal maturity structure in a model with Epstein-Zin preferences and show that such extreme positions are optimal because of counterfactual asset pricing implications. With Epstein-Zin preferences the optimal policy implies moderate portfolio positions with little rebalancing. [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) remove the assumption that government buys back the whole debt in every period and, instead, consider another extreme where bonds cannot be repurchased before the maturity. They show that under this assumption the optimal debt positions are closer to the data and government borrows in both types of bonds. Debt in long bonds is used to smooth taxes over states and short bonds are used to smooth taxes over time.

The paper is most closely related to the literature studying the optimal mix of monetary and fiscal policy with non-state contingent nominal debt ([Chari and Kehoe, 1999](#); [Siu, 2004](#); [Schmitt-Grohe and Uribe, 2004](#); [Lustig, Sleet, and Yeltekin, 2008](#); [Marcet, Oikonomou, and Scott, 2013](#); [Leeper and Zhou, Forthcoming](#)). As known since [Lucas and Stokey \(1983\)](#), the Ramsey planner seeks to manage government debt in order to smooth distortionary taxes over time and across states. [Chari and Kehoe \(1999\)](#) show that such smoothing of tax distortions can be achieved with inflation surprises when the Ramsey planner has control over the monetary policy. [Chari and Kehoe \(1999\)](#)'s conclusion is achieved in a model without nominal rigidities, which means that inflation is no real cost. [Siu \(2004\)](#) and [Schmitt-Grohe and Uribe \(2004\)](#) contemporaneously consider an optimal fiscal and monetary policy mix when planner faces a trade-off between distortionary taxes and inflation in the presence of nominal rigidities. In such a setting optimal policy prescribes a very limited role for inflation even when nominal rigidities are small. [Lustig, Sleet, and Yeltekin \(2008\)](#) show that inflation's role is larger when the government can issue bonds with long maturities. The idea is that large inflation implies a higher interest rate on new debt and long maturity allows the government to postpone such costly increase. Such idea is reaffirmed in [Marcet, Oikonomou, and Scott \(2013\)](#). In addition, [Leeper and Zhou \(Forthcoming\)](#) show that importance of inflation also depends on the starting level of government debt and [Siu \(2004\)](#) shows that the role of inflation in optimal policy increases with the size

of government expenditure shocks. [Marcet, Oikonomou, and Scott \(2013\)](#) show that the optimal use of inflation depends on the independence of the monetary authority and, when it is independent, on the values of Taylor rule coefficients. Overall, the Ramsey planner is more likely to inflate the debt when the monetary authority is independent. Another paper closely related to ours is [Equiza-Goni, Faraglia, and Oikonomou \(2020\)](#), which studies the role of inflation-indexed debt when the planner issues long-term debt that is nominal and short-term debt that is inflation-indexed. In this paper we study the trade-off between nominal and real bonds with the same maturity.

Lastly, we contribute to the literature on optimal fiscal policy under no commitment. In analyzing the allocations under No Commitment, we follow an approach by [Klein, Krusell, and Rios-Rull \(2008\)](#) and solve for optimal time-consistent policies. Our paper is related to [Debortoli, Nunes, and Yared \(2017\)](#) who show how commitment friction changes the implications for the optimal maturity structure. Similar to them, we find that under No Commitment leveraged position of nominal and real bonds create incentives for the future governments to monetize debt, which increases the borrowing costs for the current government. The need to reduce the borrowing costs is quantitatively important and the optimal portfolio allocation features positive amounts of both real and nominal bonds.

Solving the Ramsey problem with multiple maturities is computationally challenging because the number of state variables increases in the length of the largest maturity and the state space is highly multicollinear. In this paper we exploit the neural networks approach to tackle these problems, as proposed in [Valaitis and Villa \(2021\)](#).

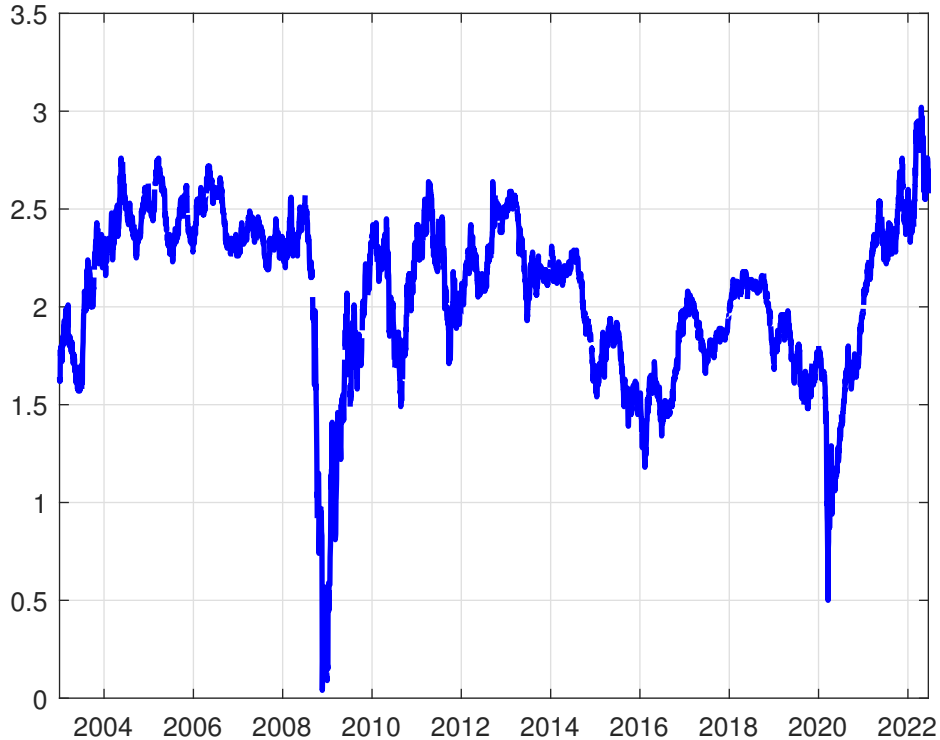
The paper is organized as follows. Section 3 presents stylized facts that strengthen the motivation to investigate the proposed question. Section 4 describes our model and characterizes the optimal policy under Full Commitment. Section 4 also includes a stylized two-period version of the model designed to convey intuition. Section 4.8 augments the baseline model of section 4 with generic long-term bonds. Section 5 presents our quantitative analysis. Section 6 describes and characterizes the optimal time-consistent policy (without Commitment). Section 7 concludes.

3 Stylized Facts

We begin by presenting some stylized facts that motivate our analysis. We focus on the evolution of inflation, government debt and real bonds.

Figure 1 illustrates the evolution of inflation expectations, as captured by the ten-year break-even inflation. The break-even inflation rate stabilizes at a level of about 2.5% from 2004 through 2007. In 2008, the break-even inflation rate sharply fell. After having reached almost a value of zero during the pandemic, inflation expectations recently spiked up sharply above pre-crisis level but remained fairly volatile.

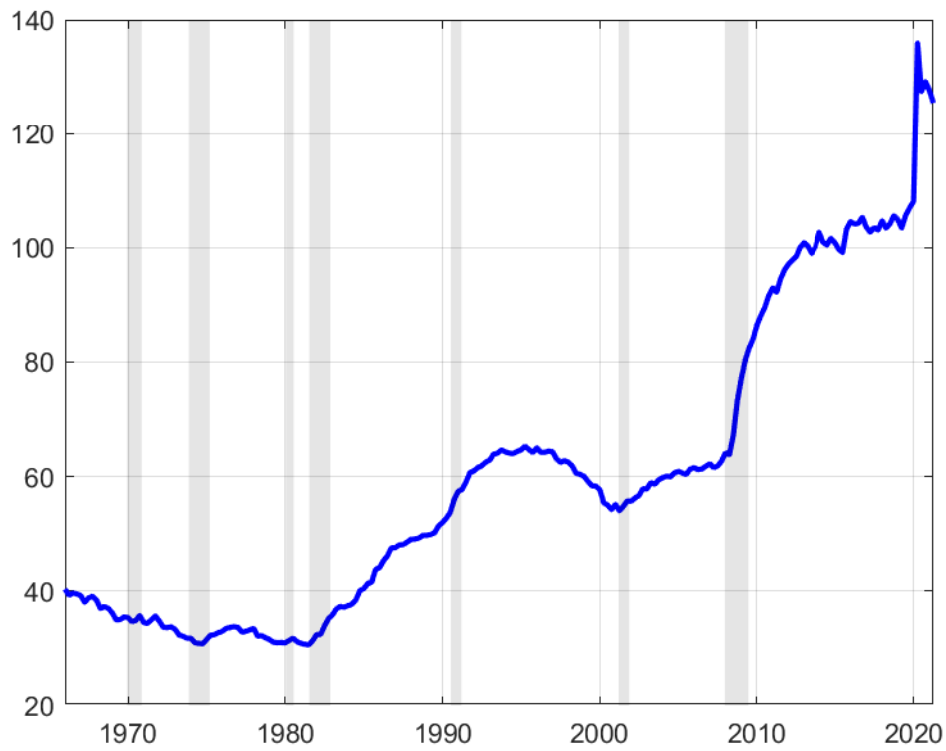
Figure 1: 10 YEAR BREAK-EVEN inflation [%]



Notes: The figure show the US 10-year break-even inflation rate. Break-even inflation is the difference between 10-year nominal and inflation-indexed bond yields. Source: St. Louis Fred database.

Figure 2 depicts the evolution of government debt as measured by the debt-to-gdp ratio. The evolution of government debt exhibits long swings, and hovered between around forty and sixty percent of GDP before the financial crisis. In response to fiscal stimulus packages around the financial crisis and then the pandemic, it has recently reached World War II levels for the first time. Moreover, according to the CBO, under current policies it is projected to reach two hundred percent of GDP by around 2050.

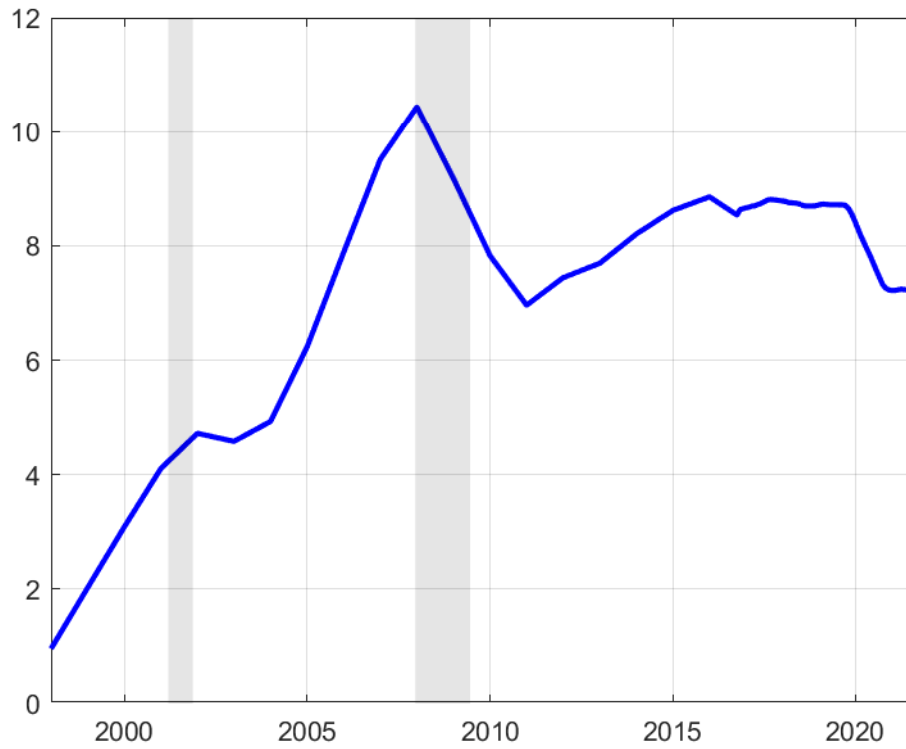
Figure 2: US DEBT TO GDP RATIO [%]



Notes: The figure shows US total public debt to GDP ratio. Data is quarterly and seasonally adjusted. Source: St. Louis Fred database.

Figure 3 plots the evolution of real debt as a fraction of total US government debt. That fraction has grown since the inception of the market for inflation-protected bonds (TIPS) and has stabilized around a modest eight percent in the last ten years.

Figure 3: SHARE OF US REAL DEBT [%]



Notes: The figure shows the share of US inflation-protected securities (TIPS) to US total public debt. Source: US department of Treasury. Treasury data can be found at the following link: <https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding>.

We now turn to a general equilibrium model that informs us about the optimal composition of government debt portfolios in the presence of a high fiscal burden and inflation pressure.

4 Model

In this section, we describe an infinite-horizon model with non state-contingent nominal and real bonds. The key friction in this environment is the lack of state-contingent bonds. That is, the value of outstanding debt at time t is independent from the realization of the shock at time t but, instead, measurable with respect to $t - 1$. If state-contingent bonds were available, i.e. bonds' markets were complete, the trade-off between nominal and real

bonds would not be meaningful. We introduce the reader to our framework with short-term one-year bonds. We then augment the model with long-term nominal and real bonds before presenting the quantitative results.

4.1 Environment

We consider a stochastic production economy populated by a continuum of identical households, a continuum of identical firms, a central bank and a government. Time is discrete and infinite, $t = 0, 1, 2, \dots$

Preferences. Households rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t)], \quad (1)$$

where $\beta \in (0, 1)$ is the discount factor, $u(\cdot)$ and $v(\cdot)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. A continuum of perfectly competitive intermediate firms, indexed by $i \in [0, 1]$, produces output through a linear production function $F(h_i)$, where hours worked is the only input. Intermediate goods are sold at a price $P_{i,t}$ to the final good producer. Aggregate output is given by $Y_t = A \cdot h_t$.

Resource. The resource constraint of the economy is given by

$$c_t + \Phi_t + g_t = Y_t, \quad (2)$$

where $h_t = 1 - l_t$ is labor, and g_t is an exogenous stochastic stream of government expenditures. Furthermore, we assume each firm can set prices $P_{i,t}$ incurring the following convex quadratic reduced-form adjustment cost

$$\Phi_t = \frac{\varphi}{2} \cdot (\pi_t - \pi)^2,$$

where $\pi_t \equiv P_{i,t}/P_{i,t-1}$ denotes inflation, and π is the inflation target of the central bank.

Shocks. We assume that g_t follows a discrete Markov process with transition probability matrix P_g . We denote by $g^t \equiv \{g_0, g_1, \dots, g_t\}$ a history of realizations of government spend-

ing. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t , but it is understood that c_t , and l_t are measurable with respect to g^t .

Households demand consumption goods, supply labor and trade: (i) claims S_t to the aggregate firm's dividend d_t , (ii) nominal and (iii) real non-contingent government bonds denoted as B_t and b_t , respectively. To simplify notation, we avoid explicitly denoting bonds as functions of histories g^{t-1} , but it is understood that B_t , and b_t are measurable with respect to g^{t-1} . The household budget constraint reads

$$c_t + Q_t B_{t+1} + q_t b_{t+1} + p_t S_{t+1} = (1 - \tau_t) w_t h_t + \frac{B_t}{\pi_t} + b_t + (p_t + d_t) S_t, \quad (3)$$

where Q_t is the price of nominal bonds, q_t is the price of real bonds, π_t denotes inflation, and p_t is the price of the firm's claim to dividend.² In equilibrium $S_t = 1$, since all households are identical.

4.2 Household and Firm Optimality

Households maximize utility (1) subject to their budget constraint (3). The intratemporal labor-consumption margin and the Euler equations for all savings instruments are

$$(1 - \tau_t) \cdot u_c(c_t) \cdot w_t = v_l(l_t), \quad (4)$$

$$u_c(c_t) \cdot Q_t = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot \pi_{t+1}^{-1}, \quad (5)$$

$$u_c(c_t) \cdot q_t = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot 1, \quad (6)$$

$$u_c(c_t) \cdot p_t = \beta \mathbb{E}_t u_c(c_{t+1}) \cdot [p_{t+1} + d_{t+1}]. \quad (7)$$

Intermediate firms set prices $P_{i,t}$ and hire labor to maximize expected net present value of dividends

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{u(c_t)}{u(c_0)} \cdot [P_{i,t} Y_{i,t} - P_t w_t h_{i,t} - P_t \Phi_t],$$

where the demand for the intermediate good is given by static profit maximization of the final good producer $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{1}{\nu}} Y_t$. In a symmetric equilibrium ($P_{i,t} = P_t$), the

²Notice that we did not allow households to trade risk-free bonds among themselves, since they are identical. In equilibrium these bonds would be in zero-net supply, rendering these bonds immaterial for equilibrium allocations.

intermediate firm's profit maximization problem yields the New-Keynesian Phillips Curve

$$Y_t \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A} \right) - \Phi_\pi(\pi_t)\pi_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \Phi_\pi(\pi_{t+1})\pi_{t+1} \right] = 0. \quad (8)$$

4.3 Government

The government needs to finance spending g_t using labor income taxes and bonds, subject to the following budget constraint:

$$q_t b_{t+1} + Q_t B_{t+1} + \tau_t w_t h_t = g_t + b_t + \frac{B_t}{\pi_t}. \quad (9)$$

At date t , the government chooses current tax rate τ_t , and current bonds b_{t+1} and B_{t+1} , which are measurable with respect to g^t .

Given initial conditions b_{-1}, B_{-1} , the benevolent government chooses stochastic sequences of current tax rates τ_t and bonds B_t, b_t to maximize the households utility (1).

4.4 Central Bank

We assume the central bank seeks to achieve an inflation target π by setting the nominal rate according to the following Taylor Rule:

$$Q_t^{-1} = \frac{1}{\beta} \pi \left(\frac{\pi_t}{\pi} \right)^{\phi_\pi}. \quad (10)$$

We would like to emphasize that we are particularly interested in answering our question from the perspective of a government that is separate from the monetary authority. That is: we analyze the problem of a government that takes the behavior of the monetary authority as given and utilizes fiscal policy tools to hedge against unexpected government expenditures.³

4.5 Implementability Constraint

We now derive the implementability constraint of the government problem and follow [Lucas and Stokey \(1983\)](#) by taking the primal approach to the characterization of competitive equilibria since this allows us to abstract away from bond prices and taxes.

³From a theoretical perspective, consolidating the monetary and fiscal authorities could be interesting. In practice, that would require to remove the Taylor Rule and study optimal fiscal and monetary policy jointly.

The government budget constraint (9) can be combined with the private sector's first order conditions (4)-(6), to obtain a single implementability constraint for $t = 0, 1, \dots$ that reads:

$$\left(\frac{B_t}{\pi_t} + b_t \right) = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot \left(\frac{B_{t+1}}{\pi_{t+1}} + b_{t+1} \right) \right], \quad (11)$$

where $s_t \equiv \left(1 - \frac{v_l(l_t)}{u_c(c_t)} w_t \right) \cdot w_t \cdot h_t - g_t$ denotes the government's surplus and wage w_t can be obtained from the New-Keynesian Phillips Curve (8). Moreover, we substitute leisure and labor $l_t = 1 - h_t$ everywhere using the resource constraint (2). The implementability constraint (11) prices the government's liabilities $\frac{B_t}{\pi_t} + b_t$ as an expected net present value of surpluses. We assume that there exist debts limits to prevent Ponzi schemes:

$$B_t \in [\underline{B}, \overline{B}], \quad b_t \in [\underline{b}, \overline{b}].$$

In our calibration, we let the bounds $(\underline{B}, \underline{b})$ be sufficiently low and $(\overline{B}, \overline{b})$ be sufficiently high so that they never bind in equilibrium. Forward substitution into equation (11) combined with a transversality condition implies the following implementability condition:

$$\frac{B_t}{\pi_t} + b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right].$$

4.6 Optimal Policy with Full Commitment

We now consider optimal debt management and fiscal policy under the assumption that the government has Full Commitment. The government chooses stochastic sequences of allocations and prices $\{c(g^t), w(g^t), \pi(g^t)\}_{t=0}^{\infty}$, and stochastic sequences of nominal and real non state-contingent bonds $\{B(g^{t-1}), b(g^{t-1})\}_{t=0}^{\infty}$ to maximize the household's utility (1) subject to the implementability constraint (11), with multiplier μ_t , the New-Keynesian Phillips Curve (8), with multiplier λ^π , the Taylor Rule (10), with multiplier λ^T , the bounds $(\overline{B}, \underline{B}, \overline{b}, \underline{b})$, with multipliers $(\overline{\Lambda}, \underline{\Lambda}, \overline{\lambda}, \underline{\lambda})$.

The first order conditions with respect to nominal bonds B_t and real bonds b_t are

$$\mu_t \cdot \mathbb{E}_t \left[\pi_{t+1}^{-1} \cdot u_c(c_{t+1}) \right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+1}) \cdot \pi_{t+1}^{-1} \right] + \beta^{-1} (\overline{\Lambda}_t - \underline{\Lambda}_t), \quad (12)$$

$$\mu_t \cdot \mathbb{E}_t \left[u_c(c_{t+1}) \right] = \mathbb{E}_t \left[\mu_{t+1} \cdot u_c(c_{t+1}) \right] + \beta^{-1} (\overline{\lambda}_t - \underline{\lambda}_t). \quad (13)$$

Note that equations (12) and (13) pin down a dynamic for the Lagrange multiplier μ_t

on the implementability constraint similar in spirit to the one of [Aiyagari, Marcet, Sargent, and Sappala \(2002\)](#).

The first-order condition with respect to wage w_t is

$$A\nu \cdot \mu_t \cdot u_c(c_t) + \lambda_t^\pi = 0. \quad (14)$$

This condition captures the trade-off between the marginal effect that wage has on the implementability constraint (11) through government's surplus and the New-Keynesian Phillips Curve (8). The remaining first-order conditions with respect to consumption c_t and inflation π_t are reported in Appendix A.1, and they are given by equations (61) and (62), when maturity N is set to 1.

Special Case We consider a special case with risk-neutral households $u(c_t) = c_t$ and no lending limits $\underline{\lambda}_t = \underline{\Lambda}_t = 0$. In this case equation (13) becomes

$$\mu_t = \mathbb{E}_t[\mu_{t+1}] + \beta^{-1}\bar{\lambda}_t.$$

Since the Lagrange multiplier on the borrowing limit is non-negative $\bar{\lambda}_t \geq 0$, then $\mu_t \geq \mathbb{E}_t[\mu_{t+1}]$. We can use the submartingale convergence theorem: μ_t converges almost surely. This last condition and result is equivalent to [Aiyagari, Marcet, Sargent, and Sappala \(2002\)](#): in the long-run the government eventually accumulates enough *real assets* that it never needs to tax again. Differently from [Aiyagari, Marcet, Sargent, and Sappala \(2002\)](#), the simultaneous presence of both nominal and real bonds requires an extra condition to be satisfied. This is given by the optimal policy of nominal government debt (12). If, for explanation purposes, we further assume there are no lending and borrowing limits - i.e., $\underline{\lambda}_t = \underline{\Lambda}_t = \bar{\lambda}_t = \bar{\Lambda}_t = 0$ - we can combine (12) and (13) to get

$$\text{Cov}_t(\pi_{t+1}^{-1}, \mu_{t+1}) = 0. \quad (15)$$

Intuitively, this condition states that under risk-neutrality and in absence of lending and borrowing limits it is ex-ante optimal for the government to create policies such that, averaging on all future states, inflation is not used to relax the implementability constraint.

4.7 Inspecting the Mechanism: Two-Period Model

The Ramsey problem we lay out can be thought of as a dynamic portfolio choice problem with incomplete markets in which the planner looks for the optimal government debt allocations of two securities, namely non state-contingent nominal and real bonds. To provide intuition about the determinants of these allocations, we now examine stylized examples in which the objective of the planner is most transparent, namely specifications in which the economy can be in two states only. In such an environment, the planner's objective is to choose a portfolio of non state-contingent bonds that replicates Arrow-Debreu securities. That is, the planner aims at implementing the complete markets allocation.

We thus ask how the government can use inflation fluctuations to replicate a portfolio of Arrow-Debreu securities?

Consider a two-period $t = 0, 1$ version of the model where $u(c) = c$ and disutility for labor $v(h) = h^2/4$. Moreover, assume that at time 1 there are two realizations of the exogenous shocks, i.e. a low state (π_1^L, g_1^L) and a high state (π_1^H, g_1^H) .⁴ Assume each realization happens given a joint conditional probability $f(\pi_1, g_1 | \pi_0, g_0)$. Under these conditions, the household optimality conditions imply $Q_0 = \beta \mathbb{E}_0 [\pi_1^{-1}]$, $q_0 = \beta$ and $h_t = 2(1 - \tau_t)w$. Firms take the exogenous sequence of prices as given and chooses labor such that $w = A$, which we further assume normalized to a unitary value.⁵ The resource constraint of the economy $c_t = h_t - g_t - \frac{\varphi}{2}(\pi_t - \pi)^2$ yields expressions for consumption. We follow the primal approach to get the following implementability constraints

$$\begin{aligned} \frac{B_0}{\pi_0} + b_0 + g_0 &= h_0 \left(1 - \frac{h_0}{2}\right) + \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 + \beta b_1, \\ \frac{B_1}{\pi_1} + b_1 + g_1 &= h_1 \left(1 - \frac{h_1}{2}\right). \end{aligned}$$

The optimal policy under full commitment requires to find $\{h_0, h_1, B_1, b_1, \mu_0, \mu_1\}$ such that (i) the implementability constraints hold, (ii) nominal and real debt are chosen optimally

$$\mu_0 \mathbb{E}_0 [1/\pi_1] = \mathbb{E}_0 [\mu_1/\pi_1], \quad (16)$$

$$\mu_0 = \mathbb{E}_0 [\mu_1], \quad (17)$$

⁴Note that $\{\pi_t\}_{t=0}^1$ can be chosen such that the Taylor rule $1 = \pi \left(\frac{\pi_0}{\pi}\right)^{\phi\pi} \mathbb{E}_0 \left[\frac{1}{\pi_1}\right]$ is satisfied.

⁵In the spirit of the New-Keynesian Phillips Curve, this would be equivalent to $w = A(1 - \nu)/\nu$ with $\nu = 1/2$.

and (iii) labor is chosen optimally

$$1 - h_t/2 - \mu_t(h_t - 1) = 0. \quad (18)$$

Proposition 1 (*Debt Management, Labor and Tax Smoothing*). *Given initial conditions B_0, b_0, g_0, π_0 , optimal nominal and real debt management and tax management are such that smoothing of taxes and leisure is achieved across states*

$$l_1^H = l_1^L \iff \tau_1^H = \tau_1^L, \quad (19)$$

where l_1^L and l_1^H denote leisure at time 1 in the low and high state, respectively. Moreover, smoothing of taxes and leisure is achieved across time

$$l_1^x = l_0 \iff \tau_1^x = \tau_1^0, \quad (20)$$

where $x \in \{L, H\}$.

Proof. Equation (19) follows from equation (17), combined with a formula for $\mu_t(h_t)$, which can be derived directly from equation (18). Apply the definition of expectation to get $l_1^H = l_1^L \kappa$, with $\kappa = \frac{f^H}{f^L} \cdot \frac{\mathbb{E}_0 \frac{1}{\pi_1} - \frac{1}{\pi_1^H}}{\frac{1}{\pi_1^L} - \mathbb{E}_0 \frac{1}{\pi_1}} = 1$. Similarly, equation (20) follows from equation (16), combined with the formula for μ . Apply the definition of expectation to get $l_1^L = l_0 \eta$, with $\eta = f^L + f^H \frac{1}{\kappa} = 1$. ■

Equation (19) reveals that nominal debt is used for smoothing taxes across states. In order to see this, use the implementability constraints to express leisure in function of the portfolio choices⁶

$$l_0 = \sqrt{1 - 2 \left(\frac{B_0}{\pi_0} + b_0 + g_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 - \beta b_1 \right)}, \quad (21)$$

$$l_1 = \sqrt{1 - 2 \left(\frac{B_1}{\pi_1} + b_1 + g_1 \right)}. \quad (22)$$

Substitute equation (22) in equation (19), to get the following cross-states smoothing con-

⁶Note that the each implementability constraint yields two solutions for labor. For simplicity, we pick the one such that $h_t \leq 1$. Moreover, we also checked analytically that picking the other solution would lead to the same exact formulas for nominal and real bonds as they appear in proposition 2.

dition

$$\frac{B_1}{\pi_1^L} + g_1^L = \frac{B_1}{\pi_1^H} + g_1^H, \quad (23)$$

which does not contain real debt. Similarly, substitute equations (21), (22) in equation (20), to get the following inter-temporal smoothing condition

$$\frac{B_0}{\pi_0} + b_0 + g_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 - \beta b_1 = \frac{B_1}{\pi_1^x} + b_1 + g_1^x, \quad (24)$$

where $x = \{L, H\}$. Note that since equation (24) needs to hold for both $x = \{L, H\}$, you can arbitrary choose, and without loss of generality, the x that matches the realization of the shock at time 0, to further simplify

$$\frac{B_0 - B_1}{\pi_0} + b_0 - \beta \mathbb{E}_0 [\pi_1^{-1}] B_1 = (1 + \beta)b_1. \quad (25)$$

These considerations lead us to formulate the following proposition.

Proposition 2 (Optimal Nominal and Real Debt Management). *Given initial conditions B_0, b_0, g_0, π_0 , optimal nominal debt management is such that*

$$B_1 = B_1^* \equiv \frac{g_1^H - g_1^L}{\pi_1^H - \pi_1^L} \cdot \pi_1^L \pi_1^H, \quad (26)$$

satisfy the intra-temporal (cross-states) smoothing condition (19). Given equation (26), optimal real debt management is such that

$$b_1 = b_1^* \equiv \frac{1}{1 + \beta} \left[\frac{B_0}{\pi_0} + b_0 - \left(\frac{1}{\pi_0} + \beta \mathbb{E}_0 \left[\frac{1}{\pi_1} \right] \right) B_1^* \right], \quad (27)$$

satisfy the inter-temporal smoothing condition (20).

Proof. Equation (26) follows directly from equation (23). Equation (27) follows directly from equation (25). ■

Note that if shocks are *inflationary*, i.e. $\pi_1^H > \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is positive. Viceversa, if shocks are *deflationary*, i.e. $\pi_1^H < \pi_1^L$ and $g_1^H > g_1^L$, then B_1^* is negative. The sign for real debt depends on the initial amount of outstanding liabilities $B_0 \pi_0^{-1} + b_0$. In order to gain intuition, we assume the government has a zero net holdings of initial real liabilities, i.e. $B_0 \pi_0^{-1} + b_0 = 0$. Under this condition, equation (27) reveals that b_1^* is

negative with *inflationary* shocks and positive with *deflationary* shocks.

The intuition is simple. Holding a leveraged portfolio position enables the planner to achieve insurance without re-adjusting the debt structure.

On the one hand, with *inflationary* shocks, it is optimal to have positive nominal debt since it maintains the option to get inflated away when a positive government expenditure shock hits, inducing the government liability to fall. This mechanism insures the government with an economic force that tends to relax the implementability constraint when it is most needed, by counterbalancing the high government expenditure shock with a falling liability. We call this a *risk management* motive: borrow with the most volatile asset (nominal debt), that falls in inflationary times. Moreover, with *inflationary* shocks, the position of real debt b_1^* should be negative, i.e. the government should accumulate real assets to smooth labor and taxes through time in high government expenditure inflationary times. We call this a precautionary motive: buy assets that pay in inflationary times (real assets).

Viceversa, the opposite applies with *deflationary* shocks. For example, in this case, the government chooses to optimally hold nominal assets which appreciate in period of high government expenditures helping relaxing the implementability constraint.

4.8 Quantitative Model with Long-Term Bonds

For the sake of clarity, we introduced the model with short-term bonds. Before turning to the quantitative analysis, we augment the model by introducing long-term nominal and real bonds when both instruments exhibit a generic, but same, maturity N .⁷ We then proceed to formulate the Ramsey problem and characterize the optimal policy.

Environment The model is identical to section (4) except that the representative household saves through: (i) a N -period non-contingent nominal debt B_t^N traded at a price Q_t^N and (ii) a N -period non-contingent inflation-protected debt b_t^N traded at a price q_t^N . The government issues both types of debt, collects revenues in the current period and repays

⁷Note that the model with long-term bonds collapses to the short-term formulation when $N = 1$. Alternatively, we could have introduced maturities through long-term perpetuities with decreasing coupon rates. With our approach with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions. With perpetuities it would have required 8 state variables and 14 policy functions. With perpetuities, the additional 4 policy functions for bonds prices and associated Lagrange multipliers are required, since nominal and real bonds prices are expressed recursively and would not be substitutable directly in the implementability constraint. We chose our methodology since the stochastic simulation approach we adopted is scalable in function of the state variables but less effective and stable the more policies need to be solved jointly at each time step. Note also that with 8 state variables a stochastic simulation approach would still be needed.

debt at maturity. In particular, the government repays nominal maturing debt at a unitary price and real maturing debt at a price $\prod_{j=1}^N \pi_{t-j+1}$. As before, the government levies a distortionary labor tax τ_t on labor income. The representative household, conjointly with government financial needs, make savings decisions in long-term nominal and real debts.

In every period t , the representative household receives labor and investment income according to the following budget constraint

$$c_t + Q_t^N B_t^N + q_t^N b_t^N + p_t S_{t+1} = (1 - \tau_t) w_t h_t + \frac{B_{t-N}^N}{\prod_{j=1}^N \pi_{t-j+1}} + b_{t-N}^N + (p_t + d_t) S_t. \quad (28)$$

Household Optimality Households maximize utility (1) subject to their budget constraint (28). The intratemporal labor-consumption margin and the firm's stock pricing equation are identical to those of section (4). The Euler equations that price long-terms bonds are

$$u_c(c_t) \cdot Q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}) \cdot \left(\prod_{j=1}^N \pi_{t+j} \right)^{-1}, \quad (29)$$

$$u_c(c_t) \cdot q_t^N = \beta \mathbb{E}_t u_c(c_{t+N}), \quad (30)$$

Government The government needs to finance spending g_t using labor income taxes and bonds, subject to the following budget constraint:⁸

$$Q_t^{N-1} \frac{B_{t-1}^N}{\pi_t} + q_t^{N-1} b_{t-1}^N = \tau_t A h_t w_t - g_t + Q_t^N B_t^N + q_t^N b_t^N. \quad (31)$$

Implementability Substitute τ , Q_t^N , and q_t^N in equation (31) using equations (4), (29), and (30) to get sequences of implementability constraints

$$\mathbb{E}_t \left[\frac{u_c(c_{t+N-1})}{\prod_{j=1}^{N-1} \pi_{t+j}} \right] \frac{B_{t-1}^N}{\pi_t} + b_{t-1}^N \mathbb{E}_t [u_c(c_{t+N-1})] = u_c(c_t) s_t + B_t^N \mathbb{E}_t \left[\frac{u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right] + b_t^N \mathbb{E}_t [u_c(c_{t+N})], \quad (32)$$

where s_t is surplus as defined in subsection 4.5.

Optimal Policy with Full Commitment We now consider optimal debt management and fiscal policy under the assumption that the government has Full Commitment and issue

⁸We implicitly assume that the government can buy back both nominal and real bonds from the private sector. As documented in the OECD report by [Blommestein and Hubig \(2012\)](#), more than 80% of countries engage in some forms of debt buyback and some of them they do so on a regular basis.

long-term nominal and real bonds. The government chooses stochastic sequences of allocations $\{c(g^t), h(g^t)\}_{t=0}^{\infty}$ and prices $\{w(g^t), \pi(g^t)\}_{t=0}^{\infty}$, and stochastic sequences of nominal and real non state-contingent bonds $\{B^N(g^{t-1}), b^N(g^{t-1})\}_{t=0}^{\infty}$ to maximize the household's utility (1) subject to the resource constraint (2), with associated Lagrange multiplier λ_t , the implementability constraint (32), with multiplier μ_t , the New-Keynesian Phillips Curve (8), with multiplier λ^π , the Taylor Rule (10), with multiplier λ^T , the bounds $(\bar{B}, \underline{B}, \bar{b}, \underline{b})$, with multipliers $(\bar{\Lambda}, \underline{\Lambda}, \bar{\lambda}, \underline{\lambda})$.

The first order conditions with respect to nominal bond B_t and real bond b_t are

$$\mu_t \cdot \mathbb{E}_t [\Pi_{j=1}^N \pi_{t+j}^{-1} \cdot u_c(c_{t+N})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+N}) \cdot \Pi_{j=1}^N \pi_{t+j}^{-1}] + \beta^{-1} (\bar{\Lambda}_t - \underline{\Lambda}_t), \quad (33)$$

$$\mu_t \cdot \mathbb{E}_t [u_c(c_{t+N})] = \mathbb{E}_t [\mu_{t+1} \cdot u_c(c_{t+N})] + \beta^{-1} (\bar{\lambda}_t - \underline{\lambda}_t). \quad (34)$$

Note that equations (33) and (34) collapse to (12) and (13) when $N = 1$. The first order condition with respect to wage is identical to equation (14). The remaining first-order conditions with respect to consumption c_t and inflation π_t , together with further details about the computational methodology, can be found in Appendix A.

5 Quantitative Analysis

In this section, we calibrate our model and discuss our quantitative results when considering an optimal policy under Full-Commitment.

5.1 Calibration and Solution Method

We parameterize the utility function as follows: $u(c) \equiv \frac{c^{1-\gamma}}{1-\gamma}$ and $v(l) \equiv \chi \frac{l^{1-\eta}}{1-\eta}$, with $\eta = 1.8$ to match a unitary Frisch elasticity of labor supply and $\chi = 4.3276$ to normalize average labor to 1/3 of the time endowment in the stochastic simulation of the Full-Commitment model. The production function is linear, with: $F(k, l) \equiv Ah$, and A is normalized to a unit value.

We calibrate fiscal parameters using data from [Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez \(2015\)](#). In particular, we use measures of government expenditures and labor tax rate for the period 1971-2013.⁹ We also use this data to compute the

⁹We convert the data from a quarterly to an annual frequency, obtained as average values in each year. The data can be found at the following link:

<https://www.openicpsr.org/openicpsr/project/112890/version/V1/view>.

average ratio of government spending to GDP, which is around 20%.

We calibrate the Markov process for g_t as an AR(1) in logs, formally: $\log g_{t+1} = (1 - \rho_g) \log \mu_g + \rho_g \log g_t + \epsilon_t$, with ϵ_t normally distributed with mean zero and standard deviation σ_g . We then match the average ratio of government spending to GDP, as well as the standard deviation and autocorrelation of linearly detrended (log) government spending, using the data described above. We do not discretize this process, since we adopt a stochastic simulation approach.

We set the maturity of government debt N equal to 5 years for both nominal and real bonds. This is close to the average maturity of US federal debt (~ 5.5 years).

We set parameter the price elasticity of demand $1/\nu$ to 10, which is a standard value used in the literature. The Taylor rule responds only to deviations from the steady state inflation rate. We set the steady state inflation rate to 2%, which is the Fed target level. Moreover, ν and φ are such that the quarterly slope of the New Keynesian Phillips Curve, given by $h/(4\nu\varphi)$ is on average ~ 0.041 , calculated as average of a long stochastic simulation under Full Commitment. This in the range of estimates provided in [Gali and Gertler \(1999\)](#).¹⁰

Table 1: PARAMETER VALUES

		Parameter	Value
Preferences	Discount factor	β	0.96
	Risk aversion	η_c	2
	Labor disutility	χ	4.3276
	Labor elasticity	η_l	1.8
Firm	Price elasticity	$1/\nu$	10
	Adjustment cost	φ	20
Government	Average g	μ_g	0.068
	Volatility of $\log(g)$	σ_g	0.016
	Autocorr. of $\log(g)$	ρ_g	0.977
	Maturity (Years)	N	5

Notes: The table reports the parameter values.

We solve the optimal policy under Full Commitment using an algorithm similar in spirit to the Parameterized Expectations Algorithm ([den Haan and Marcet, 1990](#)).¹¹ We

¹⁰[Gali and Gertler \(1999\)](#) directly estimate a Phillips curve with a slope in the range 0.018-0.047, as reported in Table 1.

¹¹Note that we use this method only for the optimal policy under Full Commitment. In order to solve for

provide extensive details on the solution method in Appendix A, which has been proposed by Valaitis and Villa (2019). This method relies on a neural network to approximate the forward looking terms in the optimality conditions, as functions of the state vector. Note that in appendix A.3, we conduct robustness checks of our results by changing the seed through which the stochastic simulation is generated and by changing the variance of the g_t process.

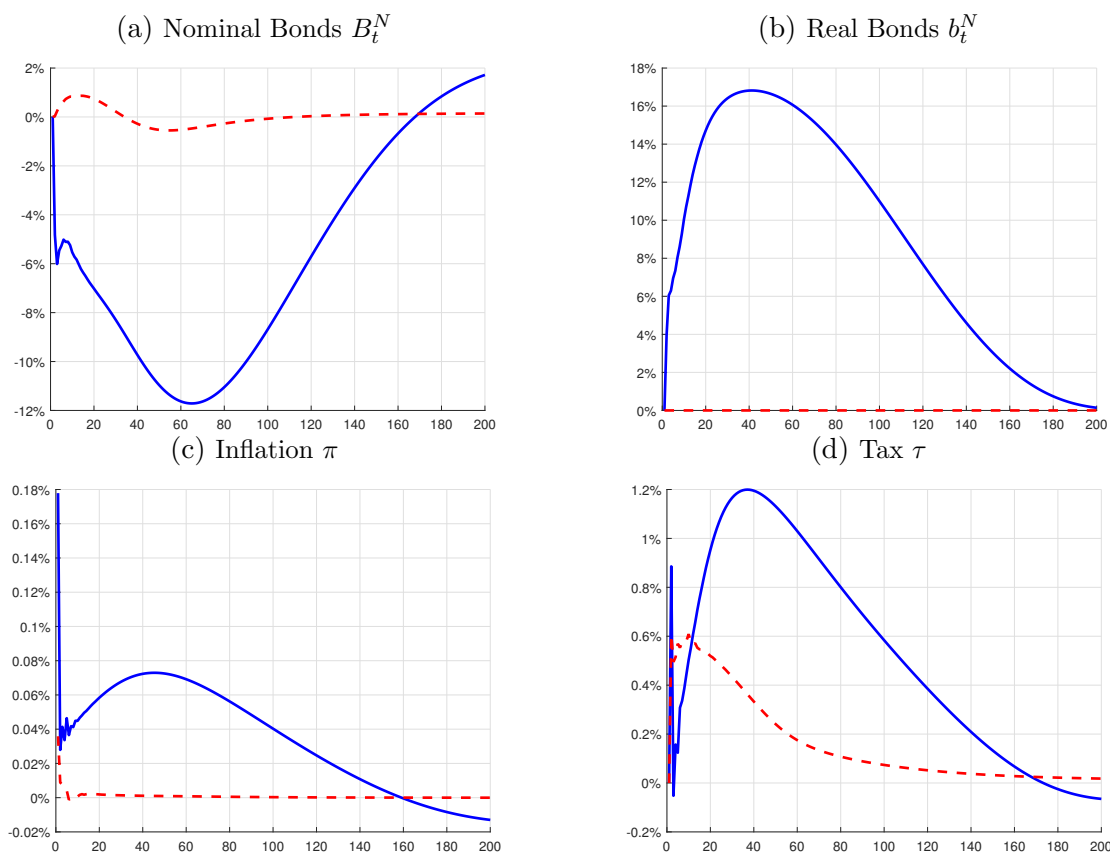
5.2 Baseline Results

We begin by comparing our calibrated model to a counterfactual scenario where the government can only issue nominal bonds. When the government cannot issue TIPS, the Ramsey planner faces a trade-off between responding to shocks using distortionary taxes versus inflation. On the one hand, by inflating away nominal debt, the government can finance the additional expenditure without increasing labor taxes. On the other hand, by raising expected inflation, the planner reduces the value of household savings and decreases the price of government nominal bonds. Therefore, both the current and the future price of nominal bonds fall. In addition to that, inflation distorts firms' production decisions as price adjustment is costly. The presence of TIPS in the government debt portfolio affects this trade-off in two ways.

First, higher inflation has less impact on the cost of current and future borrowing, since it does not affect the price of inflation protected bonds. Second, the use of inflation becomes more costly because the planner needs to compensate households holding real bonds.

the optimal policy under No Commitment, described in section 6, we use a different methodology similar in spirit to the one introduced by Clymo and Lanteri (2020). A detailed description of the solution algorithm under No Commitment can be found in Appendix A.2.

Figure 4: IMPULSE RESPONSE FUNCTIONS



Notes: The figure shows the impulse response functions to a government expenditure shock of +3% of GDP. X-axes report time t . Solid blue line: baseline model. Dashed red line: model without TIPS bonds. The panels for inflation and taxes report percentage points difference. The panels for bonds show percentage points difference expressed as a ratio of GDP.

We investigate the workings of the model using impulse response functions in figure 4 where we shock the economy with a one-time government expenditure shock equal to 3% of GDP. We find that real bonds play a substantial role in shaping the optimal policy. The optimal policy prescribes: (i.) the accumulation of nominal liabilities and real assets in good times, and, (ii.) inflating away nominal liabilities and financing government expenditures using real assets in bad times. This stands in contrast to a counterfactual model without TIPS bonds, where government accumulates nominal liabilities in bad times and decumulates it otherwise. Because the government chooses to borrow in nominal bonds in response to shocks, it tried to keep the current nominal bond price high and, therefore, inflation plays a minor role in this counterfactual economy.

Reallocation to TIPS bonds in bad times is supported by moments from model sim-

ulation reported in table 2. It shows that TIPS bonds are countercyclical and nominal bonds are procyclical, while the total debt portfolio is countercyclical in both models.¹² On average, the optimal policy features lower levels of tax, inflation and short rates, but a higher responsiveness of these policy tools to government expenditure shocks.

Table 2: SUMMARY OF MOMENTS

Description	Moments	No Tips	Baseline
Avg. Inflation	$\mathbb{E}(\pi)$	2.00	1.702
Avg. Tax	$\mathbb{E}(\tau)$	23.75	21.97
Avg. Short Nom. Rate	$\mathbb{E}(i)$	6.25	5.88
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	-0.28
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.40	0.24
Std. Inflation	$\sigma(\pi)$	~ 0	0.002
Std. Tax	$\sigma(\tau)$	0.066	0.124
Corr. Inflation and Gvt. Exp.	$\rho(\pi, g)$	0.66	0.84
Corr. Tax and Gvt. Exp.	$\rho(\tau, g)$	0.88	0.79
Corr. Nominal and Gvt. Exp.	$\rho(B^N, g)$	0.70	-0.64
Corr. Real and Gvt. Exp.	$\rho(b^N, g)$	-	0.75
Corr. Debt and Gvt. Exp.	$\rho(B^N + b^N, g)$	0.70	0.42
Corr. Inflation Vol. and Gvt. Exp.	$\rho(\sigma(\pi), g)$	0.55	0.24

Notes: The table reports sample moments from simulating model equilibrium dynamics for 5000 periods. The simulation is initialized at $b^N = B^N = 0$ and we drop the first 100 periods before calculating moments.

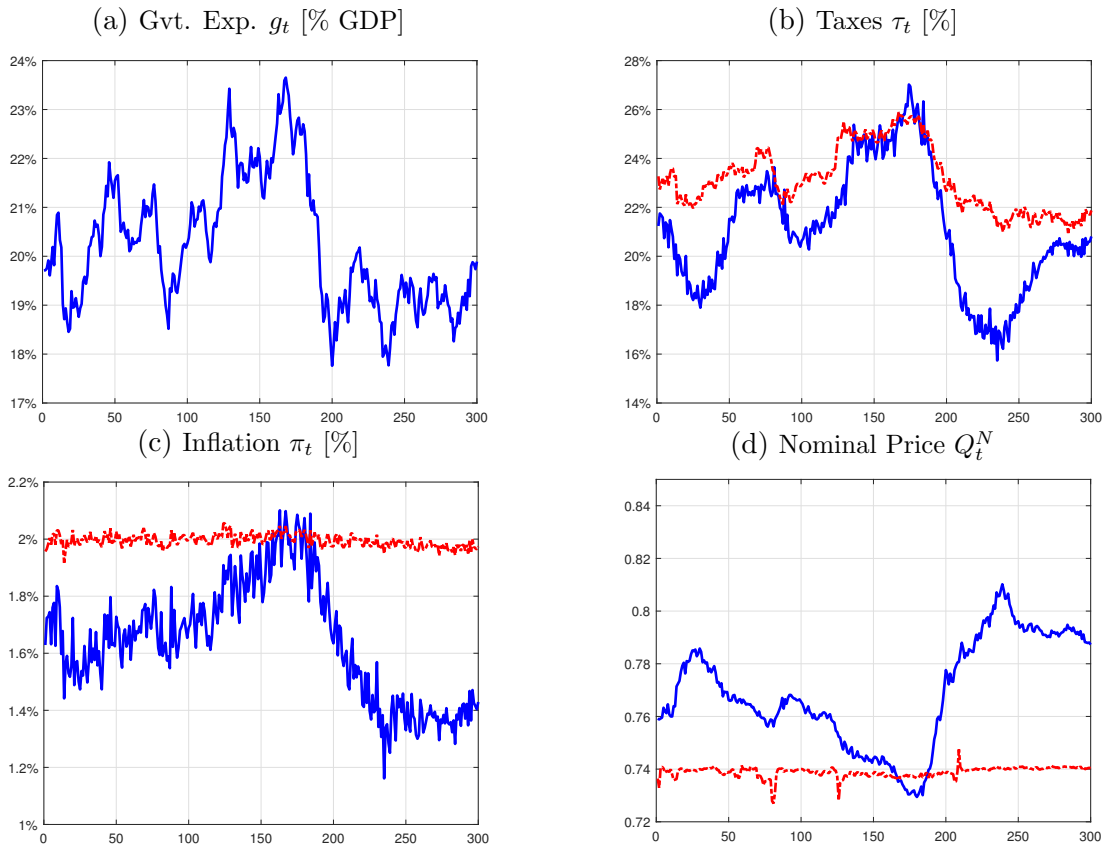
5.3 Example: Simulation With Prolonged Period of High Government Expenditures

In figure 5, we present an extract from the model simulation with a prolonged period of high government expenditure. The top left panel shows the exogenous process for government expenditure, which starts to increase around period 100 and remains high for around 100 periods. The other three panels show policy variables in the baseline model (solid blue line) and the model without TIPS bonds (dashed red line). Inflation and taxes are on average lower in the baseline model but more responsive to increases in government expenditure. Because inflation is on average lower, nominal bond prices tend to be higher in the baseline

¹²An increase in government expenditure indicates economic downturn. Therefore negative correlation with g_t means that a variable is procyclical.

model. By keeping the average inflation below the steady state target of 2%, the Ramsey planner incurs real costs, which are compensated by the marginal benefits of having higher bond prices. Likewise, the Ramsey planner internalizes that higher volatility of inflation translates into more volatile nominal bond price - it drops by 2 percentage points during the period of high expenditure but then recovers from around 0.74 to around 0.80. The nominal bond price volatility has a relatively small cost for the planner in the baseline model, since it is always possible to refactor the portfolio toward real bonds, if nominal bonds have to sell at a high discounts. This substitution is impossible in the one bond model. Therefore, inflation features very little volatility in the counterfactual economy.

Figure 5: SIMULATION: POLICY VARIABLES

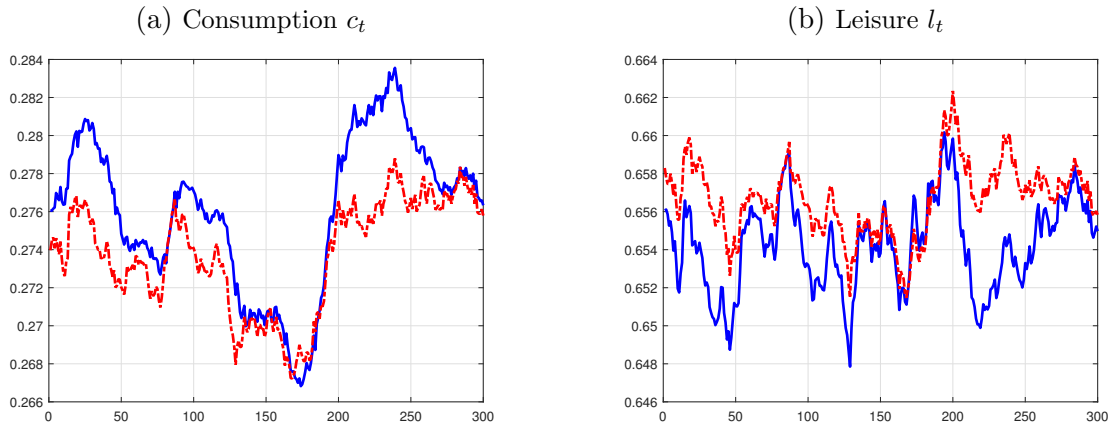


Notes: The figure shows an excerpt from the simulation of model equilibrium dynamics. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 2.

As shown by figure 6, higher welfare is achieved through higher consumption and less volatile leisure. Compared to the benchmark model, consumption increases by an average

$\sim 0.8\%$ and leisure volatility falls by 6.64% . In fact, in the baseline model taxes are on average lower and household tends to work more. At the same time, labor supply is less elastic and it does not fluctuate as much even when in presence of a more volatile labor tax rate in the baseline model. Overall, compared to the benchmark model, the higher consumption and lower leisure volatility leads to a consumption equivalent welfare gain of 0.223% . The next session analyzes the role of outstanding debt in shaping the optimal policy.

Figure 6: SIMULATION: ALLOCATIONS



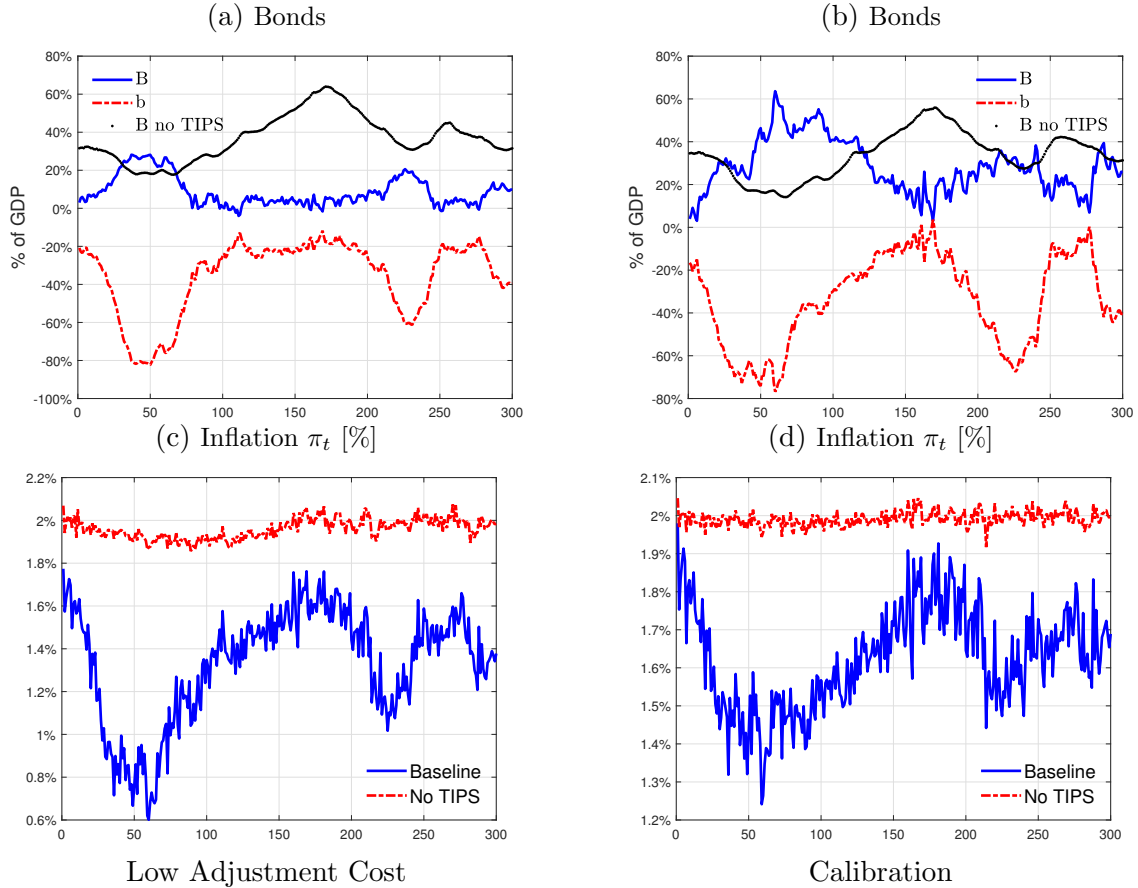
Notes: The figure shows an excerpt from the simulation of model equilibrium dynamics. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 2.

5.4 Role of Nominal Rigidities

We next turn to study the role of nominal rigidities for bonds positions and inflation volatility. [Chari and Kehoe \(1999\)](#) show that in the model with flexible prices the planner relies heavily on inflation to absorb the expenditure shocks. But, as shown in [Siu \(2004\)](#), if the model is calibrated to match the empirically realistic degree of price rigidity, the real cost of inflation on firms pricing decisions begins to outweigh the benefits of relaxing the budget constraint and there is little incentive to use inflation in a model, where only nominal bonds are available. Our results are consistent with [Siu \(2004\)](#). In figure ?? we compare the bond positions and inflation in our baseline model and the model with only nominal bonds. In addition to that, we analyze a counterfactual where we resolve both models with a much lower degree of nominal rigidity, controlled by the parameter Φ_π . We find that, indeed, the size of nominal rigidity affects inflation volatility in both models but

its' role is more pronounced in the one-bond model. Compared to the baseline calibration, in the counterfactual with low inflation adjustment costs, inflation increases by 78% in the baseline model and by 267% in the one-bond model.

Figure 7: ROLE OF NOMINAL RIGIDITIES



Notes: The figure shows an excerpt from the simulation of model equilibrium dynamics. X-axes report time t . Solid blue line: baseline model. Dashed red line: benchmark model without TIPS bonds. Both models are simulated with the same realization of government expenditure shocks. The same simulation was used to calculate moments in table 2.

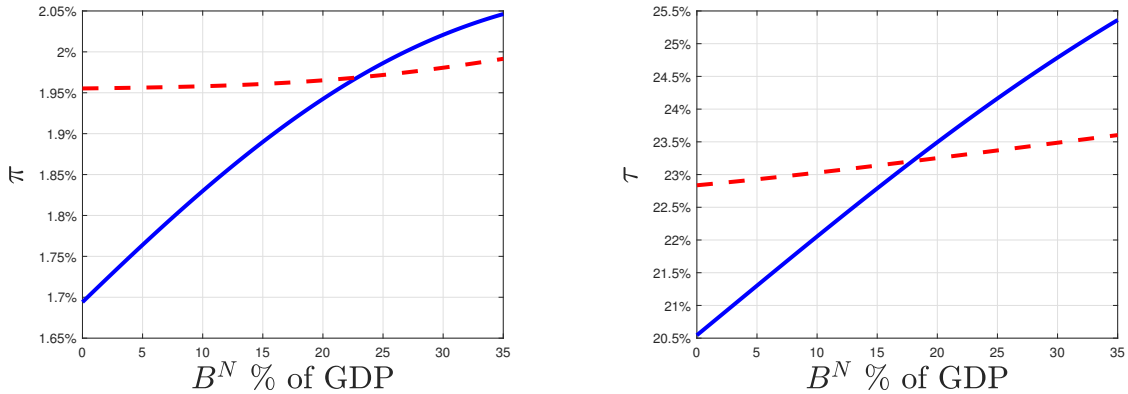
5.5 Role of Initial Debt

In this section we analyze the relation between outstanding debt and the use of inflation when TIPS bonds are available. Specifically, we ask whether more debt causes more inflation. By using inflation, the Ramsey planner weights the benefits of inflating away nominal liabilities against two types of costs. First, by rational expectations, higher inflation eventually gets reflected in nominal bond prices (equation 6) and new nominal bonds need to

sell at a higher discount. Second, inflation has real costs as it distorts firms’ pricing decisions (equation 8). The reason that we observe more volatile inflation in the baseline model is because inflations’ effect on nominal prices is not relevant for the Ramsey planner when the TIPS bonds are available. In this section we ask if high outstanding nominal debt can lead to high inflation.

The level of outstanding nominal debt changes the trade-off between inflation of nominal liabilities and real distortions. When the outstanding nominal debt is high, the same inflation rate allows to achieve a greater reduction in nominal liability while incurring the same distortion. At the same time, the trade-off between nominal liability effect and inflations’ effect on nominal bond prices does not change. The same inflation rate allows to inflate more liabilities but more bonds need to be reissued in the next period. This together suggests that more nominal debt should lead to higher inflation.

Figure 8: ROLE OF NOMINAL DEBT



Notes: The figure plots policy functions of inflation and taxes in function of nominal debt. Other state variables are fixed at their mean values. Left panel: inflation. Right panel: tax rate. Solid blue line: baseline model. Dashed red line: model without TIPS bonds.

We investigate the role of nominal debt in models with and without TIPS bonds by looking at the policy functions of inflation and taxes in figure 8, which plots optimal inflation and taxes in function of nominal debt by keeping other state variables at their average levels.¹³ The left panel shows that inflation responds positively to nominal debt in both models but the response in the baseline model is much larger. As the outstanding nominal debt increases from 0 to 75% of the GDP, inflation rate increases from 1.4% to 2.9% holding everything else fixed. In contrast, inflation in the one bond model moves from

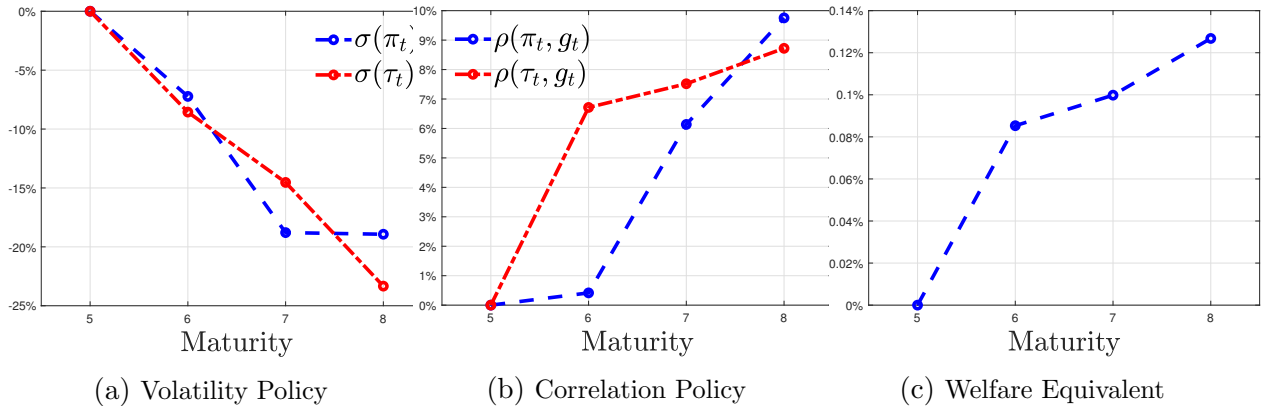
¹³Since we solve the model using the parameterized expectations algorithm, we are not solving for the policy functions explicitly. Instead, we use the model simulated data and the neural network to fit the relation between the policy and the state variables.

1.9% to 2.05%. If real misallocation was the main cost of the use of inflation, one would expect that optimal inflation would respond to outstanding nominal debt similarly in both models. However, we observe that inflation responds little to shocks or outstanding debt in a one bond model, consistent with [Siu \(2004\)](#) and [Marcet, Oikonomou, and Scott \(2013\)](#). Yet the reason for this lack of response is that the Ramsey planner mostly cares about the effect that inflation has on nominal bond prices. Since this concern is close to irrelevant in the model with TIPS bond, here the Ramsey planner uses inflation more aggressively.

5.6 Role of Maturity

In this section we analyze the role of maturity on optimal inflation and taxes. In general, longer maturity brings greater benefits of using inflation. As maturity increases, both inflation and taxes become less volatile, as shown in the left panel of figure 9. Intuitively, longer maturity allows the planner to spread the inflation policy intervention across multiple periods. On the one hand, optimal policy prescribes lower volatility of taxes and inflation as maturity increases. but, on the other hand, higher responsiveness of these policy tools to government expenditures. As shown in the right panel of figure 9, increasing the maturity from five to eight years is associated with a consumption equivalent welfare gain of +0.13%.

Figure 9: ROLE OF MATURITY



Notes: The figure shows comparative statics when the bond maturity is exogenously increased from five to eight years in our baseline model. Each panel describes the relative values of respective moments relative to the counterpart in the model where maturity is five years. The left panel shows the volatility of inflation (dashed blue line) and the volatility of taxes (dotted-dashed red line). The middle panel shows the correlation of inflation with government expenditures (dashed blue line) and the correlation between taxes and government expenditures (dotted-dashed red line). The right panel shows the welfare increase relative to the model where the bond maturity is five years.

5.7 Alternative Shocks

In this subsection, we analyze the role of TFP shocks. As discussed, the government expenditure shocks analyzed in the previous subsections are *inflationary*. That is: period with high government expenditures are associated with high inflation. We consider an alternative model where the only source of stochasticity is given by total factor productivity shocks A_t . We assume that A_t follows an AR(1) process in logs. Table 3 shows the relevant moments of the alternative model together with the baseline model with government expenditure shocks. Columns 4 and 5 in table 3 reveal a similar pattern. In the model with TFP shocks, average real bond to GDP ratio is still negative and nominal bond to GDP ratio is still positive. Moreover, the two bonds remain negatively correlated negatively and inflation follows a similar pattern as in the baseline model. The difference is that now inflation increases in recession and nominal debt gets inflated away. At the same time, during recession, the government optimally finances it using real assets and borrows in real bonds, if necessary. This is a manifestation of the negative correlation between inflation and nominal bonds and the positive correlation between inflation and real bonds. It is worth noticing, that the presence of the Taylor rule imposes a constraint that the nominal interest rate needs to follow the dynamics of inflation. And the inverse of this rate also discounts the net present value of future government surpluses. Therefore, the net present value is likely to move in the same direction as inflation for most types of shocks, unless these shocks have drastically different implications for the expected government primary surpluses.

Table 3: COMPARISON WITH A MODEL WITH TFP SHOCKS

Description	Moments	No TIPS	Baseline	Baseline	Data
		<i>g shocks</i>	<i>g shocks</i>	<i>TFP shocks</i>	
Avg. Real to GDP	$\mathbb{E}(b^N/Y)$	-	-0.28	-0.37	0.06
Avg. Nominal to GDP	$\mathbb{E}(B^N/Y)$	0.40	0.24	0.40	0.86
Corr. Tax and GDP	$\rho(\tau, Y)$	0.54	0.3	-0.84	0.17
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.39	0.39	-0.66	0.18
Corr. Tax and Inflation	$\rho(\tau, \pi)$	0.84	0.96	0.81	0.08
Corr. Inflation and Real	$\rho(\pi, b^N)$	-	0.93	0.45	0.25
Corr. Inflation and Nominal	$\rho(\pi, B^N)$	0.68	-0.69	-0.22	-0.54
Corr. Real and Nominal	$\rho(b^N, B^N)$	-	-0.84	-0.70	0.14

Notes: The table shows the relevant moments from the model with TFP shocks only compared with two models with government expenditure shocks only. The third column (*No TIPS*) corresponds to the model with government expenditures shocks when nominal bonds are not available. The fourth column corresponds to the *baseline* model with both types of bonds and government expenditures shocks. The fifth column corresponds to the *baseline* model with both types of bonds and TFP shocks. To compute mean and correlations of government spending, taxes, and other data, we log and linearly detrend each variable, in both the model (where necessary) and data. Fiscal parameters such as labor taxes are taken from from Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) as described in section 5. The source of other data is the St. Louis FRED database. Given the recently adoption of TIPS we considered a time span from 1997 to 2013. Data about TIPS and debt can be found on the treasury website at the following link: <https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding>.

6 Optimal Policy with No Commitment

In this section, we turn our attention to a different assumption on the commitment technology. In particular, we view the public sector as a succession of decision makers—one government at each time t —with *no commitment* to future realized policies. The government in power at t seeks current level of labor tax rate and government expenditure, together with issuance of nominal and real non-contingent bonds that will be inherited by the future government. Consistent with our assumptions in the previous subsection, these bonds are non-contingent with respect to future shocks. We follow Debortoli and Nunes (2013), who discuss in depth the role of endogenous government expenditure. Similarly to Debortoli and Nunes (2013), we have chosen to combine the optimal time-consistent policy with endogenous government expenditure since we believe is a realistic feature, not

only because governments can opt to change expenditures levels, but also to capture disagreements among consecutive government about public expenditure. As a consequence, we consider a private sector with utility identical to (1), except for an additional public expenditure component

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t) + \theta_t \cdot v^g(g_t)], \quad (35)$$

where θ_t are preference shocks, and $v_g(\cdot)$ is a differentiable function such that $v_g^g(\cdot) > 0$ and $v_{gg}^g(\cdot) < 0$. Note that θ_t is low (high) when public expenditures are less (more) valuable to the private sector.

We focus on a symmetric Markov-perfect equilibrium and we denote the state of the economy at time t by $x_t \equiv (B_t, b_t, \theta_t)$. In this environment, let all future governments set their policy according to functions $\tilde{c}(x)$, $\tilde{h}(x)$, $\tilde{w}(x)$, $\tilde{B}(x)$, $\tilde{b}(x)$, $\tilde{g}(x)$, and $\tilde{\pi}(x)$.

Furthermore, let $\tilde{W}(x)$ be the present discounted value of government utility (35) as associated with the policy functions introduced above, given the state of the economy x . Using this notation, the government in power at time t chooses allocations and wage (c, h, w) , as well as policies (B', b', g, π) to maximize

$$u(c) + v(l) + \theta \cdot v^g(g) + \beta \mathbb{E} \tilde{W}(x'), \quad (36)$$

subject to the resource constraint

$$Ah - c - g - \Phi(\pi) = 0, \quad (37)$$

with associated multiplier λ , and the implementability constraint

$$u_c(c) \cdot s + \mathbb{E}_t \left[\beta u_c(\tilde{c}(x')) \cdot \left(\frac{B'}{\tilde{\pi}(x')} + b(x') \right) \right] - u_c(c) \left(\frac{B}{\pi} + b \right) = 0, \quad (38)$$

with multiplier μ , the New-Keynesian Phillips Curve (8)

$$\mathcal{N}(x, x') \equiv u_c(c) \left(Y \cdot \left(\frac{\nu - 1}{\nu} + \frac{w}{A} \right) - \Phi_\pi(\pi) \pi \right) + \mathbb{E}_t [u_c(\tilde{c}(x')) \cdot \Phi_\pi(\tilde{\pi}(x')) \tilde{\pi}(x')] = 0, \quad (39)$$

with multiplier λ^π and the Taylor Rule (10)

$$\mathcal{T}(x, x') \equiv \bar{\pi}^{1-\phi_\pi} \pi(x)^{\phi_\pi} \mathbb{E}_t [u_c(\tilde{c}(x')) \cdot \tilde{\pi}(x')^{-1}] - u_c(c) = 0, \quad (40)$$

with multiplier λ^T .¹⁴

It is important to note that our solution method is a global method that tackles directly problem (36), following a computational methodology similar in spirit to the one introduced by [Clymo and Lanteri \(2020\)](#). We do not work directly with the Generalized Euler Equations in our computational work, since the policy functions for debts could contain jumps. A detailed description of the solution algorithm can be found in [Appendix A.2](#). Even if not used in the solution method, we derive and interpret Generalized Euler Equations that characterize the optimal time-consistent policy as they reveal key distinctive feature of this problem with respect to Full Commitment.

Differentiable Markov-perfect We follow the literature on Markov-perfect fiscal policy (e.g., [Klein, Krusell, and Ríos-Rull, 2008](#); [Debortoli and Nunes, 2013](#); [Debortoli, Nunes, and Yared, 2017](#)) and we focus our attention on policies that are differentiable functions of the “natural” state space x . Under the assumption of differentiability, it is possible to derive and interpret Generalized Euler Equations that characterize the optimal time-consistent policy.

The first-order conditions with respect to consumption, labor, and wage are

$$\lambda = u_c(c) - \mu u_{cc}(c) \left(\frac{B}{\pi} + b \right) + \mu u_{cc}(c) s + \mu u_c(c) \frac{\partial s}{\partial c} + \lambda^\pi \mathcal{N}_c + \lambda^T \mathcal{T}_c, \quad (41)$$

$$v_l(l) = \lambda A + \mu u_c(c) \cdot \frac{\partial s}{\partial h} + \lambda^\pi u_c(c) A \cdot \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A} \right), \quad (42)$$

$$0 = \mu u_c(c) \frac{\partial s}{\partial w} + \lambda^\pi u_c(c) h. \quad (43)$$

In equation (41), the planner equalizes the marginal effect on the resource constraint today (λ) with the marginal utility gain of an additional unit of consumption today plus the impact of that additional unit of consumption through s in the implementability constraint today, plus the marginal impacts on the Phillips Curve and Taylor Rule, plus the second order effects of an additional unit of consumption on the future government’s implementability constraint. In equation (42), the planner offsets the marginal disutility of

¹⁴Note that since we dropped the subscript t from inflation and in order to avoid confusion we denote the inflation target as $\bar{\pi}$, instead of π .

labor with the marginal increase in production, the marginal effects on the implementability constraint through s , plus the marginal impact of h on the Phillips Curve. Finally, in equation (43) the planner sets the wage by equating the marginal effect of the wage on the implementability constraint (through government surplus s) with the marginal effect on the New-Keynesian Phillips Curve.

The first-order conditions with respect to nominal bond, real bond, inflation, and government expenditures are

$$0 = \beta \mathbb{E} \tilde{W}_B(x') + \mu \beta \mathbb{E}_t S_{B'} + \lambda^\pi \mathcal{N}_{B'} + \lambda^T \mathcal{T}_{B'}, \quad (44)$$

$$0 = \beta \mathbb{E} \tilde{W}_b(x') + \mu \beta \mathbb{E}_t S_{b'} + \lambda^\pi \mathcal{N}_{b'} + \lambda^T \mathcal{T}_{b'}, \quad (45)$$

$$0 = -\lambda \Phi_\pi + \mu u_c(c) \frac{B}{\pi^2} + \lambda^\pi \mathcal{N}_\pi + \lambda^T \mathcal{T}_\pi, \quad (46)$$

$$0 = \theta v_g^g(g) - \lambda - \mu u_c(c), \quad (47)$$

where $S(x') \equiv u_c(\tilde{c}(x')) \cdot (B' \tilde{\pi}(x')^{-1} + b(x'))$. To set the nominal and real bonds, the social planner balances the expected present discounted value of an additional unit of B or b on the future government's continuation value with the marginal impacts on the Taylor Rule and New-Keynesian Phillips Curve plus the expected marginal effect on the consumer's Euler equation (S). Inflation is optimally chosen by equating the marginal effects on the Taylor Rule and New-Keynesian Phillips Curve with the marginal effect on the implementability constraint (through the marginal utility of consumption and the amount of nominal bonds).

An important difference between these optimality conditions and their counterparts in the Full Commitment problem of the previous subsection is that past multipliers on the implementability constraint (11) are absent here, because the government disregards the effects of current policy on past decisions of the private sector, and in particular past bonds. Moreover, the derivatives of the future policy functions appear inside the terms $\mathbb{E}_t S_{B'}(x')$, $\mathbb{E}_t S_{b'}(x')$, $\mathcal{N}_{b'}(x, x')$, $\mathcal{T}_{b'}(x, x')$, rendering these optimality conditions Generalized Euler Equations.

The envelope conditions are

$$\tilde{W}_B(x) = \mu \cdot u_{cc}(c) \cdot \pi^{-1}, \quad (48)$$

$$\tilde{W}_b(x) = \mu \cdot u_{cc}(c). \quad (49)$$

The envelope conditions on the government's continuation function \tilde{W} for B and b synthesize the second order effects on consumption $u_{cc}(c)$ expressed in real terms by dividing by

inflation (in the case of nominal bonds). Imposing symmetric Markov-perfect equilibrium we can use the envelope conditions to back out the Generalized Euler Equations

$$0 = \beta \mathbb{E} \mu(x') u_{cc}(\tilde{c}(x')) \pi'^{-1} + \mu \beta \mathbb{E}_t S_{B'} + \lambda^\pi \mathcal{N}_{B'} + \lambda^T \mathcal{T}_{B'}, \quad (50)$$

$$0 = \beta \mathbb{E} \mu' u_{cc}(\tilde{c}(x')) + \mu \beta \mathbb{E}_t S_{b'} + \lambda^\pi \mathcal{N}_{b'} + \lambda^T \mathcal{T}_{b'}. \quad (51)$$

Formally, a differentiable symmetric Markov-Perfect equilibrium, is a set of policy functions for allocations and wage $c(x) = \tilde{c}(x)$, $h(x) = \tilde{h}(x)$, $w(x) = \tilde{w}(x)$, for bonds, inflation and government expenditure $B'(x) = \tilde{B}'(x)$, $b'(x) = \tilde{b}'(x)$, $\pi(x) = \tilde{\pi}(x)$, $g(x) = \tilde{g}(x)$, and for the Lagrange multipliers $\lambda(x)$, $\mu(x)$, $\lambda^\pi(x)$, $\lambda^T(x)$ that solve equations (37)-(43), (46), (47), (50), (51).

6.1 Inspecting the Mechanism: Two-Period Model

Consider the same two-period model of section 4.7, except assume that there are two different governments in power at period 0 and 1. In particular, the government in power at period 1 can choose inflation optimally. Under these conditions, the optimal choice of π_1 of the government at time 1 is given by the optimality condition:

$$-\Phi_\pi(\pi_1) + \mu_1 \frac{B_1}{\pi_1^2} = 0. \quad (52)$$

This has an intuitive interpretation. On the one hand, the government at time 1 faces the marginal cost of the nominal rigidities which, through the resource constraint, tends to lower consumption. On the other hand, the government internalizes the marginal benefit of inflating away nominal debt B_1 , which is inherited as a choice of the government in power at $t = 0$. Assume $u(c) = c$, a generic disutility for labor $v(h) = \chi h^2/2$, and a linear cost $\Phi = \varphi(\pi_1 - \pi)$. Under these conditions, expression (52) yields the following relationship between nominal debt and inflation at period 1:

$$\pi_1 = \sqrt{\frac{\mu_1}{\varphi} B_1}. \quad (53)$$

Now we turn our attention to the government in power at time 0. This government chooses b_1 and B_1 in order to best respond to the government at time 1. The Generalized Euler

Equations

$$\mu_0 \left(Q + \frac{\partial Q}{\partial B_1} \right) = \beta \mathbb{E}_0 \left[\frac{\mu_1}{\pi_1} \right], \quad (54)$$

$$\mu_0 \left(q + \frac{\partial q}{\partial b_1} \right) = \beta \mathbb{E}_0[\mu_1], \quad (55)$$

capture the inter-temporal strategic interactions among governments and lead us to formulate the following proposition that benchmarks the optimal policy under No Commitment against Full Commitment (see two-period model under Full commitment in subsection 4.7).

Proposition 3 (*Optimal Nominal and Real Debt Management*). *Given initial conditions B_0, b_0, g_0, π_0 , and under all the assumptions of this subsection, optimal nominal and real debt management is such that the intra-temporal (cross-states) smoothing condition is strategically biased*

$$\beta \text{Cov}_0 \left(\mu_1, \frac{1}{\pi_1} \right) = \mu_0 \frac{\partial Q}{\partial B_1}, \quad (56)$$

and the inter-temporal smoothing condition is satisfied without wedge

$$\mu_0 = \mathbb{E}_0[\mu_1]. \quad (57)$$

Proof. First, note that $q = \beta$. Hence, $\frac{\partial q}{\partial b_1} = 0$ and equation (55) collapses to its counterpart under Full Commitment (17), which corresponds to equation (57). Combine equation (57) with equation (54) to get equation (56) directly. ■

Under risk-neutrality, it is intuitive that the first-order condition with respect to real debt collapses to equation (17), consistent with the fact that under these circumstances the government loses at time 0 any power to manipulate time 0 real rates, since they are not forward-looking.

Equation (55) is clearly different from its counterpart under Full Commitment (15), since it contains the additional strategic terms $\frac{\partial Q}{\partial B_1}$ through which the government at time 0 internalizes the effects that its nominal debt choice has on time 0 nominal rates. Recall that $Q = \beta \mathbb{E}_0 [\pi_1^{-1}]$ to compute the strategic bias term contained in equation (56):

$$\frac{\partial Q}{\partial B_1} = -\beta \mathbb{E}_0 \left[\frac{1}{\pi_1^2} \frac{\partial \pi_1}{\partial B_1} \right] < 0,$$

since $\frac{\partial \pi_1}{\partial B_1} = \frac{1}{2\varphi\pi_1} \left(\mu_1 + B_1 \frac{\partial \mu_1}{\partial B_1} \right) > 0$, as clear from equation (53).¹⁵ Therefore, the sign of the strategic bias contained in equation (56) is negative which reveals that, under no commitment, the optimal policy is such that

$$\text{Cov}_0 \left(\mu_1, \frac{1}{\pi_1} \right) < 0. \quad (58)$$

Equation (58) suggests that, in general under No Commitment, the government does not reach a perfect smoothing across states even with just two realizations of shocks as in subsection 4.7. In fact, given the assumptions of subsection 4.7, equation (58) implies that¹⁶

$$\begin{cases} \mu_1^H > \mu_1^L \text{ and } \pi_1^H > \pi_1^L, \\ \mu_1^H < \mu_1^L \text{ and } \pi_1^H < \pi_1^L, \end{cases}$$

whereas, under Full Commitment, the planner was explicitly seeking to achieve $\mu_1^H = \mu_1^L$ as implied by equation (19) of proposition 2.

This is manifestation that the government at time 0 internalizes the effects that its nominal debt choice has on time 0 nominal rates. Intuitively, the time 0 government is facing a trade-off between: (i) diminishing its nominal borrowing costs and (ii) smoothing fiscal policy. This tension drives the optimal portfolio allocations under the optimal time-consistent policy: the hedging portfolio achievable with levered positions under Full Commitment is typically a sub-optimal choice under No Commitment. In fact, it would be an expensive financial choice ex-ante and accentuate the dilemma posed by the lack of commitment ex-post. That is, it would give incentive to the future government to use inflation excessively ex-post. Notably, our results are similar in spirit to those of [Debortoli, Nunes, and Yared \(2017\)](#), although we analyze a different problem.

We now turn our attention to the infinite-horizon model described in section 6, which we calibrate to capture these forces quantitatively.

¹⁵Since $\mu_1 > 0$ and $\frac{\partial \mu_1}{\partial B_1} > 0$, since more debt chosen at time 0 renders the government at time 1 more constrained. In general, μ_t is such that the first-order optimality condition with respect to labor $1 - \chi h_t - \mu_t(2\chi h_t - 1) = 0$ is satisfied.

¹⁶Consistently also with equation (53).

6.2 Quantitative Results

We now discuss our quantitative results for the case in which the government lacks commitment and, among other policies, issue one-period non state-contingent nominal and real bonds.

We calibrate the Markov process θ_t as an AR(1) in logs, formally: $\log \theta_{t+1} = (1 - \rho_\theta) \log \mu_\theta + \rho_\theta \log \theta_t + \epsilon_\theta$, with $\rho_\theta = \rho_g$, $\mu_\theta = -.005$, and ϵ_θ normally distributed with mean zero and standard deviation $\sigma_\theta = 0.03$. This calibration allows us to match an average ratio of government spending to GDP, as well as the standard deviation and autocorrelation of linearly detrended (log) government spending, in agreement with the data described in section 5, also used to solve for the optimal policy under Full Commitment.¹⁷ We discretize this process with an 11-valued Markov-Chain. More details can be found in the computational appendix A.2. For simplicity, we choose $v^g(g) = \log g$ and we calibrate the model imposing zero lower bounds on nominal and real debts, which are occasionally binding. All the other parameters we use to solve the model are identical to Full Commitment, as reported in table 1, except for φ . We increase φ from 20 to 50 to match a quarterly slope of the New Keynesian Phillips curve of ~ 0.018 , calculated as average of a long stochastic simulation under No Commitment with $\sigma_l = 1.8$.¹⁸ As shown in figure 7 of subsection 5.4, a higher level of nominal rigidities, which results in a flatter New Keynesian Phillips Curve, helps increasing both bonds positions, calculated as average of a long stochastic simulation. We solve the model and present results in the third column of table 4. Table 4 shows that, differently from Full Commitment, it is possible to calibrate the NC model to obtain average positive portfolio weights (with underlying positive debt to GDP ratios). The intuition that drives this result is explained in the two-period model presented in subsection 6.1. Under no commitment, a government needs to simultaneously tackle the need of smoothing fiscal policy and the need to lessen its borrowing costs. This latter force helps achieve the calibration with positive nominal and real bonds presented in table 4, whereas it is difficult to attain it under Full Commitment given the strong need for levered bond positions in order to smooth fiscal policy. As a concluding exercise, we lower σ_l from 1.8 to 1. As reported in table 4, this results in a $\sim 50\%$ increase in the Frisch elasticity. Intuitively, this parameter regulates the quantitative trade-off between reducing the average borrowing costs and reducing fiscal policy volatility. In particular, a higher Frisch elasticity is linked

¹⁷In particular, with the calibration reported in Table 1 we get an average ratio of government spending to GDP of $\sim 23\%$ and an implied $\sigma_g = 0.0167$, which is aligned with 0.016 we used in section 5.

¹⁸Note that this is still in the range 0.018-0.047 provided by Gali and Gertler (1999).

to higher welfare gains from smoothing fiscal policy. As shown in the fourth column of table 4, the optimal time-consistent policy is such that more leveraged positions are adopted resulting in portfolio weights close to those observe in reality.

Table 4: No Commitment and Portfolio Shares

Description	Moments	$\sigma_l = 1.8$	$\sigma_l = 1$	Target/Data
Real Portfolio Weight	$\mathbb{E}[b/(b + B)]$	0.39	0.08	0.07
Nominal Portfolio Weight	$\mathbb{E}[B/(b + B)]$	0.61	0.92	0.93
Frisch Elasticity	$\mathbb{E} \left[\frac{\partial h}{\partial w} \cdot \frac{w}{h} \right]$	1.04	1.53	2
Corr. Tax and GDP	$\rho(\tau, Y)$	0.84	0.89	0.17
Corr. Inflation and GDP	$\rho(\pi, Y)$	0.64	0.92	0.18
Corr. Inflation and Real	$\rho(\pi, b)$	0.83	0.77	0.25
Corr. Inflation and Nominal	$\rho(\pi, B)$	-0.69	-0.57	-0.54

Notes: The table reports the first portfolio weights of real and nominal bonds, together with the implied Frisch elasticity, and salient correlations among monetary and fiscal policy instruments. All moments are calculated in a simulation with $T = 10000$ periods. The first column refers to the NC model with $\sigma_l = 1.8$; the second row refers to the model with to the NC model with $\sigma_l = 1$. For the Frisch Elasticity we consider a reference value of 2, as it is standard in the literature. To compute mean and correlations of other variables, we log and linearly detrend each variable, in both the model (where necessary) and data. The source of other data is the St. Louis FRED database. Given the recently adoption of TIPS we considered a time span from 1997 to 2013. Data about TIPS and government debt in general can be found on the treasury website at the following link: <https://fiscaldata.treasury.gov/datasets/monthly-statement-public-debt/summary-of-treasury-securities-outstanding>.

7 Conclusion

In the wake of rising inflation in the aftermath of unprecedented debt financed stimulus packages, controlling inflation has again moved to the forefront of governments attention. In this paper, we examine optimal government debt management in the presence of inflation concerns in a setting where i) the government can issue long-term nominal and real (TIPS) non state-contingent bonds, ii) the monetary authority sets short-term interest rates according to a Taylor rule, and iii) inflation has real costs as prices are sticky. Nominal debt can be inflated away, but bond prices reflect elevated inflation expectations. Real bond prices are higher, but such debt constitutes a real commitment ex post. In other words, nominal debt can be inflated away giving ex-ante flexibility, but real bonds constitute a real commitment ex-post. We analyze the optimal policy under full commitment and the optimal time-consistent policy without commitment.

The optimal policy under full commitment prescribes a leveraged portfolio of nominal liabilities and real assets, designed to exploit inflation fluctuations to smooth fiscal policy. In contrast, the optimal policy without commitment is strategically biased, designed not only to smooth fiscal policy but also to best respond to the future government in order to reduce borrowing costs. A hedging portfolio with levered positions would be an expensive financial choice ex-ante and exacerbate the dilemma posed by the lack of commitment ex-post. Notably, the tension is resolved by an optimal debt management policy that prescribes a realistic allocation to real liabilities. Intuitively, without commitment, the current government can reduce borrowing costs by strategically mitigating future governments' incentive to monetize nominal debt ex post, an incentive that households anticipate and price in.

Intriguingly, our model specification *without* commitment provides a remarkably accurate quantitative description of US data, quite in contrast to the specification with full commitment. This suggests that this framework realistically captures the relevant constraints actual governments face. We thus view modeling limited government commitment as a useful starting point for relevant policy design.

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A Computational Appendix

In this appendix we describe the computational procedure we used to solve the model under Full Commitment and No Commitment.

A.1 Algorithm under Full Commitment

We solve the model under Full Commitment using a generalization of the Parameterized Expectations Algorithm (den Haan and Marcet, 1990) proposed by Valaitis and Villa (2019). In this appendix, we describe how to adapt the methodology introduced by Valaitis and Villa (2019) in this context. At every instant t the information set is $\mathcal{I}_t = \{g_t, \{B_{t-k}^N\}_{k=0}^{N-1}, \{b_{t-k}^N\}_{k=0}^{N-1}, \{\mu_{t-k}\}_{k=1}^N, \{\lambda_{t-k}^T\}_{k=1}^N, \{\lambda_{t-k}^\pi\}_{k=1}^N\}$.¹⁹ Consider projections of the forward looking terms in the model onto \mathcal{I}_t . We model these relationships using a single hidden-layer artificial neural network $\mathcal{ANN}(\mathcal{I}_t)$ with 10 neurons in the hidden layer and as many neurons as many inputs and outputs in the input and output layers, respectively. Moreover, the activation functions we use are *hyperbolic tangent sigmoid* and the training algorithm is *Levenberg-Marquardt backpropagation*.

Before proceeding, we calculate the remaining first-order conditions with respect to consumption and inflation under Full Commitment, which were omitted in the main text.

The first-order condition with respect to consumption c_t is

$$u_{cc}(c_t) - v_l(l)A^{-1} + \mu_t \left(u_{cc}(c_t)s_t + \frac{\partial s_t}{\partial c_t} u_c(c_t) \right) + \tilde{b}_t u_{cc}(c_t)(\mu_{t-1} - \mu_t) \quad (59)$$

$$+ \lambda_t^\pi \left(\frac{\nu - 1}{\nu} + \frac{w_t}{A\nu} - \frac{u_{cc}(c_t)}{u_c(c_t)^2} \beta \mathbb{E}_t [u_c(c_{t+1}) \Phi_\pi(\pi_{t+1}) \pi_{t+1}] \right) + \quad (60)$$

$$\lambda_{t-1}^\pi \frac{u_{cc}(c_t)}{u_c(c_{t-1})} \Phi_\pi(\pi_t) \pi_t - \lambda_t^T \frac{1}{\pi} \left(\frac{\pi_t}{\pi} \right)^{-\phi_\pi} u_{cc}(c_t) + \lambda_{t-1}^T u_{cc}(c_t) \frac{1}{\beta \pi_t} = 0. \quad (61)$$

The first-order condition with respect to inflation π_t is²⁰

$$\frac{v_l(l_t)}{u_c(c_t)} \frac{\Phi_\pi(\pi_t)}{A} = \mu_t \frac{\partial s_t}{\partial \pi_t} + B_t \frac{\mu_t - \mu_{t-1}}{\pi_t^2} + \lambda_t^\pi \mathcal{H}_t + \frac{\lambda_{t-1}^\pi \mathcal{K}_t}{u_c(c_{t-1})} + \left(\frac{\pi_t}{\pi} \right)^{-\phi_\pi - 1} \frac{\lambda_t^T \phi_\pi}{\pi^2} - \frac{\lambda_{t-1}^T}{\beta \pi_t^2}. \quad (62)$$

We describe the procedure for a generic maturity N . In particular, when maturity

¹⁹For example, with $N = 5$ the problem requires to keep track of 26 state variables and solve for 10 policy functions.

²⁰Define $\mathcal{H}_t \equiv \left(\frac{\nu-1}{\nu} + \frac{w_t}{A\nu} \right) \Phi_\pi(\pi_t) - \mathcal{K}_t$ and $\mathcal{K}_t \equiv \varphi(2\pi_t - \pi)$.

$N \geq 2$, then we approximate all the following terms:

$$\begin{aligned}
\mathcal{ANN}_1 &= \mathbb{E}_t \left[\frac{u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right], \\
\mathcal{ANN}_2 &= \mathbb{E}_t \left[\frac{\mu_{t+1} u_c(c_{t+N})}{\prod_{j=1}^N \pi_{t+j}} \right], \\
\mathcal{ANN}_3 &= \mathbb{E}_t [u_c(c_{t+N})], \\
\mathcal{ANN}_4 &= \mathbb{E}_t [u_c(c_{t+N-1})], \\
\mathcal{ANN}_5 &= \mathbb{E}_t [u_c(c_{t+N}) b_{t+N}^N], \\
\mathcal{ANN}_6 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+N})], \\
\mathcal{ANN}_7 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+1}) b_{t+1}^N], \\
\mathcal{ANN}_8 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+N}) b_{t+1}^N], \\
\mathcal{ANN}_9 &= \mathbb{E}_t [\mu_{t+1} u_c(c_{t+N}) b_{t+N}^N], \\
\mathcal{ANN}_{10} &= \mathbb{E}_t [\mu_{t+N} u_c(c_{t+N}) b_{t+N}^N], \\
\mathcal{ANN}_{11} &= \mathbb{E}_t [\mu_{t+N-1} u_c(c_{t+N-1}) b_{t+N-2}^N], \\
\mathcal{ANN}_{12} &= \mathbb{E}_t [u_c(c_{t+1}) \{\varphi(\pi_{t+1} - \pi) \pi_{t+1}\}], \\
\mathcal{ANN}_{13} &= \mathbb{E}_t [\varphi(\pi_{t+1} - \pi) \pi_{t+1}], \\
\mathcal{ANN}_{14} &= \mathbb{E}_t [\lambda_{t+1}^\pi \{\varphi(\pi_{t+2} - \pi) \pi_{t+2}\}], \\
\mathcal{ANN}_{15} &= \mathbb{E}_t [u_c(c_{t+1}) \{\varphi(\pi_{t+1} - \pi) \pi_{t+1}\}], \\
\mathcal{ANN}_{16}^k &= \mathbb{E}_t \left[u_c(c_{t+N-k}) \left(\prod_{j=1}^{N-1} \pi_{t-k+j+1} \right)^{-1} \right], \quad \text{for } k \in \{1, 2, \dots, N-1\}, \\
\mathcal{ANN}_{17} &= \mathbb{E}_t \left[u_c(c_{t+1}) \frac{1}{\pi_{t+1}} \right], \\
\mathcal{ANN}_{18} &= \mathbb{E}_t [\{\varphi(\pi_{t+1} - \pi) \pi_{t+1}\} b_{t+1}^N].
\end{aligned}$$

The solution procedure is summarized by the following algorithm. Given starting values $\mathcal{I}_0 = \{g_0, \{B_{-k}^N\}_{k=0}^{N-1}, \{b_{-k}^N\}_{k=0}^{N-1}, \{\mu_{-k}\}_{k=1}^N\}$ and initial weights for the \mathcal{ANN} , perform a stochastic simulation $\{c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi, w_t\}_{t=1}^T$ as follows.²¹

1. Impose the Maliar moving bounds, see [Maliar and Maliar \(2003\)](#), on debt. These bounds are particularly important and need to be tight and open slowly since the ANN at the beginning can only make accurate predictions around zero debt - that is our initialization point. Proper penalty functions are used to approximate the

²¹The network can be initially trained imposing $\{b_t\} = 0$.

behavior of the Lagrange Multipliers (Λ, λ) which avoid out of bound solutions while the Maliar moving bounds are opening, see [Faraglia, Marcet, Oikonomou, and Scott \(2014\)](#) for more details.²²

- At every instant t , given the information set \mathcal{I}_t and the prediction $\mathcal{ANN}(\mathcal{I}_t)$, solve for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^r$, and w_t such that all the following equations are satisfied: the resource constraint (2), the implementability constraint (32), the New-Keynesian Phillips Curve (8), the Taylor Rule (10), the planner first-order condition with respect to nominal debt (33), the planner-first order condition with respect to real debt (34), the planner-first order condition with respect to wage (14), the planner-first order condition with respect to consumption (61), and the planner-first order condition with respect to inflation (62). Note that simply substituting predictions of the neural network in equations (33) and (34) such as

$$\begin{aligned}\mu_t &= \mathcal{ANN}_1(\mathcal{I}_t)^{-1} \left[\mathcal{ANN}_2(\mathcal{I}_t) + \frac{\bar{\Lambda}_t}{\beta} - \frac{\Lambda_t}{\beta} \right], \\ \mu_t &= \mathcal{ANN}_3(\mathcal{I}_t)^{-1} \left[\mathcal{ANN}_6(\mathcal{I}_t) + \frac{\bar{\lambda}_t}{\beta} - \frac{\lambda_t}{\beta} \right],\end{aligned}$$

render the system over-identified. We tackle this problem by using a Forward-States approach, as described in [Faraglia, Marcet, Oikonomou, and Scott \(2014\)](#). This involves approximating the expected value terms with the state variables that are relevant at period $t + 1$ and invoking the law of iterated expectations.²³ For example, equations (33) and (34) using the Forward-States approach are:

$$\begin{aligned}\mu_t &= [\mathbb{E}_t \mathcal{ANN}_1(\mathcal{I}_{t+1})]^{-1} \left[\mathbb{E}_t \mathcal{ANN}_2(\mathcal{I}_{t+1}) + \frac{\bar{\Lambda}_t}{\beta} - \frac{\Lambda_t}{\beta} \right], \\ \mu_t &= [\mathbb{E}_t \mathcal{ANN}_3(\mathcal{I}_{t+1})]^{-1} \left[\mathbb{E}_t \mathcal{ANN}_6(\mathcal{I}_{t+1}) + \frac{\bar{\lambda}_t}{\beta} - \frac{\lambda_t}{\beta} \right].\end{aligned}$$

- If the solution error is large, or a reliable solution could not be found, the algorithm automatically restores the previous period ANN and tries to proceed with a reduced Maliar bound.²⁴

²²We also find that including Λ and λ terms explicitly in the training set improves prediction accuracy.

²³For a detailed description of the procedure using polynomial regressions see [Faraglia, Marcet, Oikonomou, and Scott \(2019\)](#) or [Faraglia, Marcet, Oikonomou, and Scott \(2014\)](#). Here we follow the same logic using the neural network.

²⁴If the unreliable solution has been detected in iteration i the algorithm restore the $i - 1$ environment and tries to proceed with $Bound_{i-1} = \alpha Bound_{i-1} + (1 - \alpha) Bound_{i-2}$.

4. If the solution calculated shrinking the bound at iteration $i - 1$ is not satisfactory, the algorithm does not go back another iteration but uses the same ANN and tries to lower the $Bound_{i-1}$ again towards $Bound_{i-2}$. Once a reliable solution is found, the algorithm proceeds to calculate the solution for iteration i again, but with $Bound_i = Bound_{i-1} + (Bound_{i-1} - Bound_{i-2})$. In this way, if an error is detected multiple times we guarantee that both $Bound_i$ and $Bound_{i-1}$ keep shrinking toward $Bound_{i-2}$ and there must exist a point close enough to $Bound_{i-2}$ such that the system can be reliably solved with both $Bound_{i-1}$ and $Bound_i$.
5. If the solution found at iteration i is satisfactory, the ANN enters the learning phase supervised by the implied model dynamics, the Maliar bounds are increased and a new iteration starts again.

Keep repeating until the ANN prediction errors converge below a certain small threshold and the simulated sequences for $c_t, \mu_t, B_t^N, b_t^N, \pi_t, \lambda_t^T, \lambda_t^\pi$, and w_t converge among iterations.

A.2 Algorithm under No Commitment

We now describe the key steps of the algorithm we use to compute the NC equilibrium of the model of Section 6. We solve the model using global methods and, specifically, an algorithm similar in spirit to [Clymo and Lanteri \(2020\)](#). Recall that the state space is $x \equiv (B, b, \theta)$.

1. We discretize the sets of B and b with 13 linear nodes each. Moreover, we discretize the AR(1) process for θ with Rouwenhorst with 11 nodes and a grid that spans 3 unconditional standard deviations.
2. We guess the future government policy functions $g(x)$, $B'(x)$, and $b'(x)$ as three-dimensional tensors with $13 \times 13 \times 11$ nodes and piece-wise linear interpolation. That is, $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$, with $1 \leq i \leq 13$, $1 \leq j \leq 13$, $1 \leq w \leq 11$. Evaluations of the policies outside of the specified indices are obtained through 3-D linear interpolation.
3. We define policy functions for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ on an augmented state space that includes both x and the additional space $x^g \equiv (B', b', g)$, that we use to evaluate all possible strategic interactions between current and future government.

Note that given $\pi(x, x^g)$ and $h(x, x^g)$, it is possible to back-out the associated policy for consumption

$$c(x, x^g) = Ah(x, x^g) - g(x, x^g) - \Phi(\pi(x, x^g)), \quad (63)$$

from the resource constraint equation (2), for wage

$$w(x, x^g) = \Phi_\pi(\pi(x, x^g)) \frac{\pi(x, x^g)}{h(x, x^g)} - \frac{1}{h(x, x^g)} \mathbb{E} \left[\beta \frac{u_c(c(x', x^{g'}))}{u_c(c(x, x^g))} \cdot \Phi_\pi(\pi(x', x^{g'})) \pi(x') \right] - A \cdot \left(\frac{\nu - 1}{\nu} \right), \quad (64)$$

through the NKPC equation (8), and for labor tax

$$\tau(x, x^g) = 1 - \frac{c(x, x^g)^{\eta_c}}{(1 - h(x, x^g))^{\eta_w} w(x, x^g)}, \quad (65)$$

from the intra-temporal consumption-labor substitution margin equation (4).

4. Given the guesses for the linearly-interpolated future government policy functions $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$ and expressions (63), (64), and (65); we solve with projection the implementability constraint, equation (38), and the Taylor Rule, equation (40) in order to find *augmented* policy functions for inflation $\pi(x, x^g)$ and labor $h(x, x^g)$ approximated using the following polynomial

$$\begin{aligned} P(x, x^g; \phi) \equiv & \phi(1) + \phi(2) \cdot B + \phi(3) \cdot b + \phi(4) \cdot B' + \phi(5) \cdot b' + \phi(6) \cdot \theta + \\ & + \phi(7) \cdot B^2 + \phi(8) \cdot b^2 + \phi(9) \cdot B'^2 + \phi(10) \cdot b'^2 + \phi(11) \cdot \theta^2 + \\ & + \phi(12) \cdot B \cdot \theta + \phi(13) \cdot b \cdot \theta + \phi(14) \cdot B' \cdot \theta + \phi(15) \cdot b' \cdot \theta + \phi(16) \cdot B \cdot b + \phi(17) \cdot B' \cdot b' \\ & + \phi(18) \cdot B \cdot B' + \phi(19) \cdot B \cdot b' + \phi(20) \cdot b \cdot B' + \phi(21) \cdot b \cdot b' + \\ & + \phi(22) \cdot g + \phi(23) \cdot g^2 + \phi(24) \cdot g \cdot B \cdot b + \phi(25) \cdot g \cdot B' \cdot b' + \phi(26) \cdot g \cdot \theta, \end{aligned}$$

with different sets of parameters ϕ^π and ϕ^h , respectively.

5. Given updated guess for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$, and an initial guess for the value function $\tilde{W}(x')$, and given all the other policies given by expressions (63), (64), and (65), solve the government problem described in equation (36) using one iteration of Value Function Iteration in order to find updated best responses for all government policies $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$.

Note that multiple iterations on the Value Function can be done, since we look for a symmetric MPE where all best responses and value functions converge to a fixed point, i.e. all governments are symmetric.

6. Use the updated best responses for all government policies $g(x) \simeq \tilde{g}(B_i, b_j, \theta_w)$, $B'(x) \simeq \tilde{B}(B_i, b_j, \theta_w)$, $b'(x) \simeq \tilde{b}(B_i, b_j, \theta_w)$, to restart from point 4. Iterate till convergence. At this step we use all policies to simulate an equilibrium sequence of T=10.000 periods. We declare the algorithm has converged when the maximum absolute errors of the simulated sequences for consumption, labor, bonds, and government expenditures between two consecutive iterations is in the order of 10^{-4} or lower.
7. At convergence, the *augmented* policy functions for $\pi(x, x^g) = P(x, x^g; \phi^\pi)$, $h(x, x^g) = P(x, x^g; \phi^h)$ can be reduced to standard policy functions just in function of the state space x by plugging the converged government optimal policies: $\pi(x) = P(x, (\tilde{B}(x), \tilde{b}(x), \tilde{g}(x)); \phi^\pi)$ and $h(x) = P(x, (\tilde{B}(x), \tilde{b}(x), \tilde{g}(x)); \phi^h)$.

A.3 Robustness

The following robustness checks refer to the Full Commitment solution with a φ of 4.8.

A.3.1 Changing the Seed

To see how our results depend on the specific realization of the g_t process we solve the model with 20 different seeds using the same starting point as in the main body of the paper. Overall, the main result is robust. Correlation between real and nominal bonds is on average -0.7904 and is negative for all realizations of g_t . Correlation between the difference of B^N and b^N is also negative on average and is only positive in two realizations. We also find that government issues nominal debt and holds real assets most of the time. The mean difference between B^N and b^N is 34.01% of GDP and has been on average negative for only one realizations. The results are summarized in table 5.

	$\rho(b_t^N, B_t^N)$	$\rho(B_t^N - b_t^N, g_t)$	$\mathbb{E}(b_t^N/Y_t)$	$\mathbb{E}(B_t^N/Y_t)$	$\mathbb{E}((B_t^N - b_t^N)/Y_t)$
Mean	-0.7904	-0.3733	-0.1465	0.1936	0.3401
Minimum	-0.9698	-0.8164	-0.3433	-0.2153	-0.0667
Maximum	-0.1315	0.5964	-0.0275	0.6289	0.697

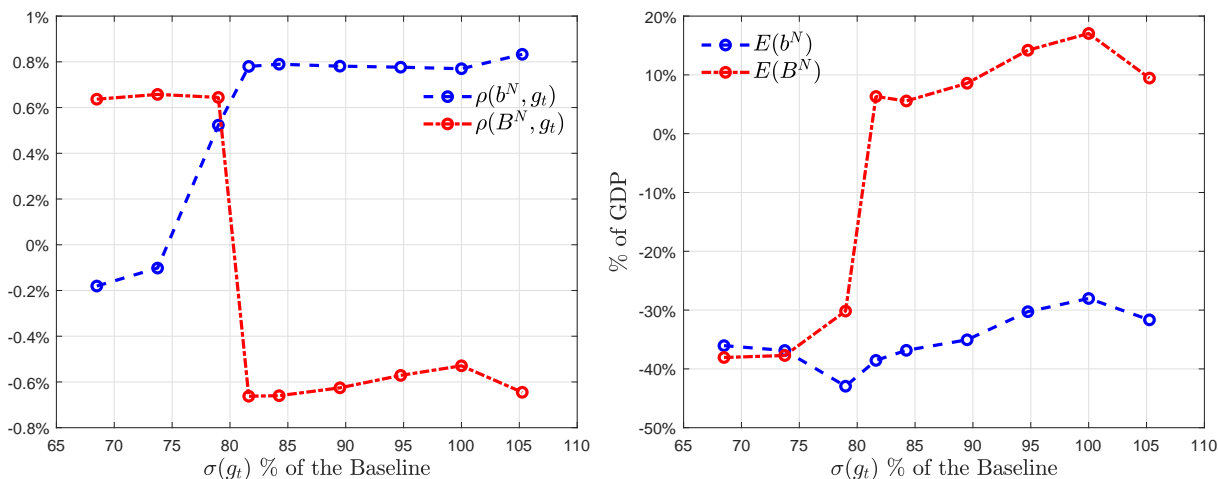
Table 5: Average moments across multiple realizations of g_t

Notes: Table shows the mean, minimum and maximum of selected moments when the model is solved with using different realizations of g_t .

A.3.2 Variance of g_t Process

In this subsection we analyze how the results depend on the variance of government expenditure. Specifically, we solve the model with the same seed but changing the variance of the shock process. We mainly find that the main result of accumulating nominal debt and real assets in good times is stronger when the government expenditure is more volatile. As shown in figure 10, the correlation between nominal bonds and g_t and the correlation between real bonds and g_t increases in absolute value as g_t becomes more volatile. Also, the government debt position becomes more leveraged as shown in the right panel.

Figure 10: ROLE OF VARIANCE OF g_t



Notes: The figure shows correlation of real and nominal bonds of g_t and average values of real and nominal bonds in function of the variance of g_t .

In addition to above, we find that 1. volatility of inflation is invariant and volatility of taxes increases in variance of g_t . 2. Correlation of total portfolio and g_t and the correlation between nominal and real bonds are stable. 3. Average inflation increases.