Federal Reserve Bank of Chicago

# Interest Spreads and Margins in Collateral Equilibrium with Heterogeneous Beliefs

*Robert Barsky, Avery Bogus, and Matthew Easton*

August 18, 2022

WP 2022-36

https://doi.org/10.21033/wp-2022-36

# Interest Spreads and Margins in Collateral Equilibrium with Heterogeneous Beliefs

Robert Barsky, Federal Reserve Bank of Chicago

Avery Bogus, Uptake Technologies, Inc.

Matthew Easton, Columbia University

The authors thank Gadi Barlevy, Cyril Monnet, Stefan Nagel, and Alp Simsek for helpful comments on earlier drafts and related presentations

Abstract

There continues to be substantial interest in models combining heterogeneous beliefs about asset values with leverage generated by loans from pessimists to the optimistic natural buyers of the asset. This paper determines the size of the interest spread and margin on the loan as a function of the downside risk perceived by the lender, and the amount of risk capital put forward by the borrower. We show that in a continuous state version of a model of collateral equilibrium with high initial leverage, most of the burden of adjustment to increases in such risk are borne by an increase in the interest spread and not the margin or “haircut”. This is contrary both to the predictions of the much-discussed binomial asset pricing model and the stylized facts in empirical data from the bilateral repo market.

## I. **Introduction**

There continues to be substantial interest in models of asset pricing combining heterogeneous beliefs about asset values with leverage generated by loans from pessimists to optimists. (See, e.g. the recent survey in Simsek, 2021). In a series of papers, Geanakoplos and various coauthors stressed that markets for collateralized risky loans clear on two dimensions - an interest rate (or a spread above the riskless rate) and a specification of the amount of collateral per dollar of lending. The latter is summarized by the margin or "haircut" associated with the loan. (see Geanakoplos, 2012, for a particularly clear discussion). At the empirical level, Geanakoplos (2012) stresses the strong association of major booms and busts in financial markets with substantial movements in haircuts, a phenomenon he calls the "leverage cycle." Interest spreads, on the other hand, show far more modest time-variation. Put differently, financial crises seem to be first and foremost periods in which the quantity of leverage falls and only secondarily periods in which the cost of obtaining leverage is high. In an evocative reference to the loan at the center of The Merchant of Venice, Geanakoplos (2012) comments: ".... Shakespeare explained that to take out a loan one had to negotiate both the interest rate and the collateral level. It is clear which of the two Shakespeare thought was the more important. Who can remember the interest rate Shylock charged Antonio? (It was 0%.) But everybody remembers the pound of flesh that Shylock and Antonio agreed on as collateral."

There are a number of reasons to regard the issue of "spreads vs. haircuts" (or more generally, the question of "prices vs. quantities" in financial markets) as one of first order importance. If financial market scares manifest themselves in increased haircuts without large increases in risky interest rates, the monitoring of spreads alone would provide insufficient warning of financial stress. Likewise, the policy implications for issues such as the lender of last resort function of central banks might well depend on the relative importance of spreads vs. haircuts as equilibrating mechanisms, as increased margins manifest themselves as a variant of “credit rationing" (Geanakoplos, 2010; Fostel and Geanakoplos, 2013). Further, margins and spreads are key statistics on which to evaluate the empirical relevance of models of collateralized risky lending. Finally, the question of how markets clear, including cases in which non-price in addition to price mechanisms are at work, is at the very core of economists' underlying intellectual agenda.

Geanakoplos (2003) and Geanokoplos and Zame (2014, although unpublished drafts appeared years earlier) pioneered the formal theory of collateral in general equilibrium, the basis of which is that the only penalty for default is the loss of collateral, with one of several important concomitant implications being that assets that serve a strong collateral function sell at a premium over otherwise equivalent assets}. Fostel and Geanakoplos (2015; FG) provide an elegant theory of a collateralized loan market with heterogeneous beliefs, in which haircuts are always sufficient to preclude equilibrium default and all lending normally occurs at the riskless rate. Their version of the theory of collateral equilibrium is constructed in the context of “binomial economies,” in which there are only two continuation states. Of course FG and many other papers by Geanakoplos and various coauthors make clear that the binomial model is a special case, but it is the basis of the bulk of their presentation, which is important because of its very strong implication that the margin is always large enough to prevent equilibrium default[1]. Simsek (2013) studies a closely related model in which there is a continuum of states (although only two agents) and finds that the equilibrium features default in some states of the world and that collateralized loans consequently trade at spreads above the riskless interest rate. Simsek's model has in common with FG's baseline model that essentially only one contract is traded in equilibrum. In the section of Simsek's paper on pure debt contracts, his primary concern is re-examination and refinement of Miller's (1977) conclusion that in the absence of opportunities for short selling (which is an essential assumption of the benchmark pure debt contracts model in the literature on heterogenous beliefs), the increased belief heterogeneity unambiguosly increases asset prices. His main conclusion thatt the specific nature of the belief heterogeneity is more important than the overall degree of heterogeneity and that asset prices are affected less by disagreement about default than about disagreement about states in the nondefault region. Our focus on the determination of haircuts and interest rates and the relative burden that they bear in adjustment to a run is clearly a fundamentally different application of Simsek's basic model. The question is ultimately quantitative, which is not the province of the theoretical papers of FG and Simsek on which we build.

The experiment on which we focus is a worsening in the lender's perceived probability of downside risk (impacting default probability and loss given default). We call this a "scary shock," evoking Geanakoplos' notion of "scary bad news" without suggesting that we have a dynamic model (such as the three-period model in

[1]The defense of the binomial specification that it is appropriate for short intervals because it approximates a diffusion process ignores the fact that in the financial crisis of 2008, in particular, large discrete drops in asset values and indeed financial business failures occurred over night.

Geanakoplos, 2010) in which an event in period two is the source of the revised view that the asset has particularly serious downside risk. A bearish revision in pessimists' beliefs and the resultant run are often cited as an important causal channel in financial crises, not least the panic of 2007-2008. There are of course other important features and determinants of this and other financial crises. However, as Geanakoplos (2010) stresses, other shocks that have been identified as key causal channels have the wrong implications for the response of margins. In particular, a loss of risk capital on the part of the natural buyers of a risky asset (which shows up in the collateral equilibrium model as a reduction in the optimists' endowment) increases the hunger on the part of those buyers for further leverage, causing equilibrium margins to *fall*, even as asset prices also fall. (Interest rate spreads of course rise).

The focus of the present paper is on the level of interest rates and margins that we can expect in a stylized example of collateral equilibrium, and the extent to which each of the two bear the burden of adjustment. A key finding of this paper is that the extent to which the burden of adjustment to a scary shock falls on the interest rate rather than the margin depends on how leveraged the investment is, which in turn depends negatively on the endowments of natural buyers, i.e. optimists. The greater the initial leverage, the more the incremental effect of a scary shock shows up in a rise in the interest rate spread. With sufficiently low endowments on the part of the optimistic natural buyers, our results suggest that the theory points to an interest rate cycle rather than a leverage cycle. This is unfortunate, because the stylized facts of repo and similar markets accord much more closely with the theoretical implications of FG. Empirically, margins do much more of the heavy lifting than the default premia embodied in interest rates. In the process of developing the argument, we provide a transparent exposition of the reason for the sharp FG result in the two-state case and why the continuous case is so different. Throughout the paper we stick with a single functional form that admits a simple and relatively transparent analytical solution. Operating on the unit interval, we postulate that optimists are certain that the payoff from the asset will be unity, while pessimists have a uniform distribution over $[\theta,1]$, where $0\leq \theta<1$ is seen as the worst possible realization. We will focus on the equilibrium effects of what we will call a "scary shock", a reduction in $\theta$, which generates a flight from lending against the risky asset on the part of the pessimists.

The plan of this short paper is as follows. Section II provides empirical motivation. We use the Gorton-Metrick data on haircuts and interest rate spreads in the bilateral repo market during 2007-2008 to illustrate the stylized fact that *in the data,* interest spreads vary far less than haircuts. Section III constructs a streamlined derivation of the Simsek model, with special attention to the equilibrium haircuts and interest rates on which Simsek did not explicitly focus. Referring the reader to Simsek (2013) for proofs of the isomorphism between the general equilibrium model and a simple principal-agent problem (with a very important additonal condition that we discuss below) as well as existence and uniqueness, we transform the axes in order to produce an alternative diagram in $(r,m)$ space, so that the equilibrium risky interest rate and margin on the loan can be read directly off the graph. We then discuss by contrast the two-state case and explain why there can be no risky borrowing unless the asset price were to fall to the pessimist's valuation - this setup's version of the FG Binomial Nondefault Theorem.

Section IV presents analytical results for the particular parameterization of the Simsek model mentioned above, in which the pessimist's beliefs about the asset's payoffs are characterized by a uniform distribution, while the optimist is confident enough to have a Dirac delta at unity. Representing beliefs by this simple configuration on the unit interval, we derive simple closed-form solutions for our model's equilibrium variables in terms of exogenously determined variables. We compute equilibrium loan size, interest rates, margins, and the price of the risky asset, and present diagramatic representations of the underlying workings of the loan market equilibria that determine them. Then, using particularly transparent numerical examples, we compare the uniform case to the binomial case that is analagous in the sense that the mean payouts perceived by the pessimist are the same in both cases. These numerical examples illustrate the substantial difference in the results of the model with discrete versus continuous belief distributions. While in the binomial case the interest rate is always equal to the exogenously set riskless rate of zero and all adjustments to changes in the pessimists' perceived risk show up in variations in the haircut or margin, the analogous uniform case yields positive and potentially quite high interest rates (associated with potentially high perceived default probabilities for the pessimist), which can also variably substantially with the key parameters. Figure II reveals in a transparent and intuitive way the essential reason for the striking difference in results.

In Figure III, perhaps the central exhibit of the paper, we illustrate the dependence of the relative variation of margins and interest rates on the size of the optimist's endowment. In our example, the value of this endowment for which the interest rate and haircut rise at approximately the same rate as $\theta$ falls, is about .5, half the value $s^{\max} = 1$ that the optimist is certain will occur. We show that the smaller the cash endowment of the optimist, the more quickly the risky interest rate rises with the extent of downside risk perceived by the pessimist, in itself and relative to the rise in the margin. For the very small endowments that are associated with the high initial leverage ratios in the Gorton-Metrick data, and apparently many other assets that were at the center of the 2007-2008 financial crisis, we see that default premia and hence interest rates vary dramatically with $a$, while haircuts show far smaller movements.

Section V concludes with a brief discussion of whether there may yet be a way to more closely match the stylized fact in the data that haircuts are larger, and far more variable, than interest spreads. We consider several mechanisms - two different forms of bankruptcy costs, increased pessimism on the part of *both* agents, and finally collateral calls presumably financed by the sale of relatively riskless assets that are outside of the model. The general finding is that the less realistic Fostel-Geanakoplos model fits the data well, while the theoretically more natural Simsek version does not, in the absence of embellishments that are out of the spirit of collateral equilibrium models.

The primacy of haircuts rather than spreads as the equilibrating mechanism shows up clearly in the Gorton-Metrick (2012) data from bilateral repo markets during the dramatic 2007-2008 episode. Figure I displays eight panels, each of which corresponds to a class of relatively risky collateral assets. For each asset class, the figure shows data on average haircuts and repo spreads from three periods: i) the pre-crisis first half of 2007; ii) the second half of 2007, which might be thought of as the period of the relatively contained “subprime crisis"; and iii) 2008 as a whole, the period of the general financial crisis centered on the shadow banking system

Across all eight asset classes, bilateral repo in the first half of 2007 appears nearly riskless, with both haircuts and spreads close to zero. In the second period, there are modest increases in the repo spread, and larger (in some cases an order of magnitude so) but still not dramatic increases in haircuts. Finally, the third period shows truly striking spikes in haircuts, as high as 60% for some private label assets (see Gorton and Metrick for details). While the interest spread also rises sharply in period 3, this increase is easily an order of magnitude less than the rise in haircuts, with the highest repo spreads on the order of 200 basis points. Computing descriptive statistics, pooling the time series and cross section data in any number of ways, indicates that whether one focuses on means, variances, or range, haircuts trump variation in spreads in every respect. While Krishnamurthy et al, (2014) cast doubt on the notion that the same dramatic increase in haircuts characterized the larger triparty repo market, one might just as readily argue that the complete disappearance of repo for many kinds of collateral in the triparty market is equivalent to a haircut of 100 percent

It is worth mentioning that although we find the most transparent example to be the bilateral repo market, at a broad brush level, the tendency for financial market scares to manifest themselves in a sharp drop in the equilibrium *quantity* of lending against risky assets without a commensurately sharp increase in the *price* of (new) loans seems to go far beyond repo. For instance a key feature of the 2007-2008 crisis was the collapse of the large market for asset-backed commercial paper (Krishnamurthy et al, 2014.). Here as well, one might well ask why the quantity of securitized lending fell so dramatically instead of contracting more modestly, with a greater share of the adjustment to increased default risk occurring through spikes in interest rates on the risky loans.

## III. Interest Rates and Haircuts with Continuous Belief Distributions

We follow closely the model of "pure debt contracts" in Simsek (2013). The model has two dates {0,1} and two types of risk neutral agents {0, 1}, denoting pessimists and optimists, respectively. There is a continuum of each type of agent. There are two assets - a risky asset that we will call a tree, and a consumption good that we will call fruit. Fruit can be stored at a constant real return of 0, so that it functions as a riskless asset, which can be thought of as "cash." Agents receive endowments of cash at date 0 but consume at date 1 only. The endowments of optimists and pessimists are denoted $n_1$ and $n_0$, respectively. Optimists use their endowments as risk capital, which they combine with borrowings from pessimists to buy trees in period 0. The trees are endowed to unmodeled agents who sell them in exchange for fruit, which is subsequently consumed in period 2. The aggregate quantity of trees is normalized to unity.

Two assumed inequalities concerning the size of the cash endowments of the two groups guarantee that the optimists will hold all the trees in equilibrium *and* that this can be accomplished only with at least some risky borrowing:

$$n_1 + s^{\min} < E_1[s] < n_0 + n_1 \tag{1}$$

These together ensure that the set of possible equilibrium prices for the risky trees will lie strictly between the optimists' full price (the maximum they would pay for the asset, which is the integral of the possible payoffs weighted by their perceived probabilities and discounted at the riskless rate) and the price at which the pessimists' short sale constraint ceases to bind (the "pessimists' price"). In some of our discussion, however, we will be interested in the case where the optimists' endowment is indeed insufficient to keep the price from falling to the pessimists' price - a phenomenon that as suggested in Geanakoplos (2010), might well be regarded as a financial collapse.

Next we turn to the characterization of the beliefs of the two kinds of agents. The optimists first and foremost believe that the expected payoff from the trees is higher than do the pessimists: $E_1[s] > E_0[s]$. Sufficient conditions for the existence and uniqueness of the solution to the principal-agent problem described below also require the relative optimism of the optimist to be increasing in the state $s$, expressed as the weak hazard inequality: $\frac{f_1(s)}{1-F_1(s)} \leq \frac{f_0(s)}{1-F_0(s)}$, for all $s \in [s^{min}, s^{max}]$,which is trivially satisfied for the uniform distributions with which we work.

The "simple debt contracts," which are the sole contracts in the current paper, are characterized by an amount borrowed per unit of collateral and an associated interest rate. Barring the limiting case where the price of trees collapses to the pessimists' level (something ruled out in Simsek's model, but periodically entertained in our discussion), pessimists confine their period zero activities to storage and lending; they will not buy trees they regard as overpriced, and short contracts are not available. If we define the contractual interest rate on a loan of size $b$ collateralized by one tree as $r(b)$,the payment received by lenders in state $s$ is $min[s, \phi]$, where $\phi = b(1 + r(b))$. Simsek reformulates the determination of collateral equilbrium as a principal-agent problem. The principal agent problem will in general lead to a continuum of solutions depending on the division of the surplus. The crucial additional observation that selects the particular solution to the principal agent problem that maps into the unique general equilibrium is that there exists a large enough supply of the riskless asset that someone must hold it in edquilibrium (see also the footnore immediately below). Thus under risk neutrality, arbitrage between storage and lending establishes the size of the loan collateralizable with one tree as $E_0[\min(s, \phi)]$.

Simsek reformulates the determination of collateral equilbrium as a principal-agent problem. The optimization problem faced by optimists can be written as:

$$\max_{(a_1,\phi)\in\mathbb{R}^2_+} a_1\mathbb{E}_1[s] - a_1\mathbb{E}_1[\min(s, \phi)] \tag{2}$$

$$\text{s.t. } a_1 p = n_1 + a_1\mathbb{E}_0[\min(s, \phi)] \tag{3}$$

Thus the optimists choose an amount $a_1$ of risky trees to purchase with collaterized loans (in addition to their cash endowment) by maximizing their expected payoff from the trees net of expected debt repayment, subject to a budget constraint that takes the price of trees as given and the interest rate as increasing in borrowing per tree in order to satisfy the lender's "participation constraint" $b = \mathbb{E}_0[\min(s, \phi)]$.

[1]This is of course an "as if" approach to computing the collateral equilibrium. The appearance that optimists can choose any amount of borrowing as long as they are willing to pay a sufficiently

Writing the Lagrangian and the associated first-order conditions, we have:

$$\mathcal{L} = a_1(\mathbb{E}_1[s] - \mathbb{E}_1[\min(s,\phi)]) - \lambda\{a_1(p - \mathbb{E}_0[\min(s,\phi)]) \ - n_1\} \tag{4}$$

$$\frac{\mathbb{E}_1[s] - \mathbb{E}_1[\min(s,\phi)]}{p - \mathbb{E}_0[\min(s,\phi)]} = \lambda \tag{5}$$

$$\lambda = \frac{1 - F_1(\bar{s})}{1 - F_0(\bar{s})} = \frac{1}{1 - F_0(\bar{s})} \tag{6}$$

$$\mathcal{L}_n = \lambda \tag{7}$$

Some re-arranging (see Appendix to Simsek, 2013) gives what Simsek calls the *optimality curve:*

$$p^{opt}(\bar{s}) \equiv \int_{s^{min}}^{\bar{s}} s dF_0 + (1 - F_0(\bar{s})) \int_{\bar{s}}^{s^{max}} \frac{s dF_1}{1 - F_1(\bar{s})} \tag{8}$$

Substituting $a_1 = 1$ into the budget constraint gives Simsek's *market clearing curve:*

$$p^{mc} = n_1 + E_0[\min(s,\phi)]$$

high interest rate is not a characteristic of the equilibrium. The general equilibrium is unique, so that only a single contract charaterized by one particular combination of interest rate and haircut is traded. It is crucial to point out that the principal agent problem has a continuum of solutions, while the parallel general equilbirum has a unique solution. Some agents have to store the riskless asset in equilibrium, and those agents will not be the optimists, who value a unit of output more than pessimists because it allows the optimists to purchase the risky asset that they value so highly. Once we know that the pessimists store some of the riskless asset, we can pin down the general equilibrum via the observation that pessimists must be indifferent between lending and storing.

The *risky interest rate* $r$ and the margin $m$ are given by:

$$1 + r = \frac{\overline{s}}{E_0[\min(s, \overline{s})]} \tag{9}$$

$$m = \frac{n_1}{p} = \frac{n_1}{n_1 + E_0[\min(s, \overline{s})]} \tag{10}$$

When the time comes for graphical analysis, we will construct our version of the optimality curve as a variant of Simsek's diagram. Instead of drawing the curves in $(\overline{s}, p)$ space, we make a transformation to $(r, \frac{n_1}{p})$ space so that the equilibrium margin and interest rate - the variables of prime interest - can ultimately be read directly off of the diagram. The transformation for the horizontal axis makes use of equation (12). Since it is easily verified that $\frac{d}{ds}(\frac{\overline{s}}{E_0[\min(s,\overline{s})]}) = E_0[\min(s, \overline{s})] - \overline{s}(1 - F(\overline{s}) > 0$, and that $r$ is thus monotonically increasing in $\overline{s}$, we have a unique mapping

$$r(\overline{s}) = \frac{\overline{s}}{E_0 \min(s, \overline{s})} - 1. \tag{11}$$

Note that after the transformation the optimality curve is upward-sloping, while the transformed market-clearing curve is downward-sloping. As we move up our transformed optimality curve, we should think of a parametrically lower asset price with fixed endowment $n_1$. At the point of the curves' intersection, $\frac{n_1}{p}$ is the equilibrium margin $m$.

A simple example that can be solved analytically characterizes beliefs using the uniform distribution. Assuming the optimistic beliefs are a point mass at 1 and the pessimistic beliefs are uniform across $[\theta, 1]$, where $\theta$ is the lower bound of the pessimists' belief distribution, we can solve analytically for the equilibrium price, interest rate, and margin in terms of only the exogenous variables $\theta$ and $n_1$

$$p = \min(\frac{1+\theta+n_1}{2}, 1) \leqq 1 \tag{12}$$

$$r = \frac{2(1-\sqrt{n_1(1-\theta)})}{1+\theta-n} - 1 \quad \text{for} \quad \theta, n_1 \geqq 0 \quad \text{and} \quad \theta + n_1 \leqq 1 \tag{13}$$

$$m = \min[\max(\frac{2n}{1+\theta+n_1}, n_1), 1] \tag{14}$$

In contrast with the above case of the uniform distribution, suppose, as do FG, that there are just two discrete states, H and L. Optimists believe state H will occur with probability $\pi_{H,1}$ and pessimists believe that it will occur with probability $\pi_{H,0}$. If an optimist were to purchase an additional unit of the asset with a collaterized loan, the expected gain would be $\pi_{H,1}[H - (1+r)P]$. The maximum interest rate that an optimist is willing to pay is thus $\frac{H}{P}$. However, the interest rate that makes the pessimist indifferent between lending and storing is given by:

$$1 + r = \frac{1}{\pi_{_{H,0}}} \frac{p - (1-\pi_{_{H,0}})L}{p}. \tag{15}$$

As long as $p$ exceeds the pessimists' equity evaluation, the minimum interest rate that would be acceptable to a pessimist is too high to leave any surplus for the optimists. Thus the endogenous collateral constraint in FG can be interpreted as the result of optimists' refusal of contracts with risky borrowing due to what they regard as an excessive interest rate, a point made previously by Geanakoplos and coauthors (see, e.g. Fostel and Geanakoplos, 2014). *In summary, as long as the the* *asset price exceeds the pessimist's valuation, there can be no risky borrowing.*[2] This is the special case of the Fostel-Geanakoplos nondefault theorem that occurs in our setup.

## IV. Parametric Solution for the Uniform Distribution and its Binomial Analogue

a. *Comparison of the Binomial Case with the Analogous Uniform*

There are two states, $s = \{.5, 1\}$. Optimists believe the high state will occur with probability one, while pessimists place equal probability on the two outcomes. Fostel-Geanakoplos reasoning tells us that the asset price must be $P = \max(.5 + n_1, .75)$. In particular, let $n_1 = .26$, so the asset will be held entirely by optimists at $p = .76$, yielding $m = .34$. If instead the optimists continue to believe with certainty that the realization of $s$ will be 1, while pessimists believe that $s$ is distributed unifomly over $[.5, 1]$, equations (11), (12), and (13) tell us that the equilibrium is characterized by: $p \cong .88$, $m \cong .30$, and $r \cong .03$. Risky borrowing increases the price by .12, the margin falls by about 13%, and the interest rate rises to a nontrivial 3%.

Now let us repeat the above exercise, but with gloomier pessimists who believe that the maximum possible loss is 100%. In the binomial version, the states are s = {0,1}; the optimist is again confident that the high state will prevail, while the pessimist believes that there is a 50% chance that realized $s$ will be zero. F-G

[2] If we were to reduce the optimist's endowment enough that he asset price falls to the pessimists' valuation, there may be multiple equilbria in the sense that risky borrowing at certain (often high) interest rates is admissable along with riskless borrowing. These equilbria are, as noted by FG, "essentially equivalent" in the sense that they produce the same price for the risky asset and provide the same consumption allocations for both optimists and pessimists as the allocations at the zero interest rate,

reasoning tell us that borrowing is zero, so the margin $m = 1$. Let $n_1 = .51$, the lowest endowment at which case the asset will be held entirely by optimists at $p = .51$. The continuous version has pessimists believing that s is uniformly distributed over [0,1]. Now the equilibrium is characterized by: $p \cong 0.76$, $m \cong 0.68$, and $r \cong 0.17$. Instead of responding to the prospect of complete default by abstaining from any lending, pessimists lend about a third of the asset value at an interest rate that provides a hefty default premium, supporting a price that is some 50% higher than in the analogous binomial case.

Figure II illustrates graphically the two examples above. Let's focus on the top panel, beginning with the market-clearing curve. Since the maximum "cash in the market" that can be generated with only riskless borrowing (which corresponds approximately to a margin of .34) is .76, that would have to be the market clearing price corresponding to r=0. The optimality curve, however, tells us that the price at which optimists would choose to limit themselves to riskless borrowing would be 1, corresponding to a margin of .26. If the price is less than 1, optimists want to push beyond riskless borrowing to purchase more of the highly valued risky asset, even while paying a default premium that they believe to be excessive. Thus $r = 0$ and $m = .26$ can't be an equilbrium. Optimists would increase their borrowing until the gap between the two curves is eliminated. This occurs at $r \cong .03$ and $m \cong .30$, which corresponds to a price just low enough to induce the optimists to take on the necessary extra leverage, while paying the default premium.

Now contrast this with the analogous binomial case, in which the only possible outcomes are $L = s^{\min} = .5$ and $H = s^{\max} = 1$, Thus the pessimist believes that each occurs with probability .5. Suppose for a moment that $p = .76$ (corresponding to $s^{\min} = .5$, $n_1 = .26$). Here, too, the optimist would be willing to pay well above the riskless rate - from III.b, we know that the maximum interest rate that the optimist is willing to pay is given by $\frac{s^{\max}}{P} - 1 \approxeq .32$. The pessimist, however, in order to do any risky lending at all, requires an interest rate of $r = \frac{1}{\pi_{H,0}} \frac{p-(1-\pi_{H,0})L}{p} - 1 = \frac{1}{2} \frac{.76-(.5)(.5)}{.76} - 1$ $\approxeq .335$. Thus the only equilibrium in the loan market features borrowing of .5 at an interest rate of zero. The total available cash in the market is thus $.5 + .26$, just financing the sale of the asset at $p = .76$.

*b) Graph of Interest Rates and Margins as a Function of* $a$ *and* $n_1$

Figure III shows margins and interest rates for the analytical example of Section III-a as a function of the pessimist' perceived lower bound $a$ for $\mathrm{n}_1 = .5$ and for $\mathrm{n}_1 = 05$. For a given $\mathrm{n}_1$, the margin and interest rate both rise as $a$ falls and the pessimist perceives greater default risk. If $n_1$ =.5, the interest rate and haircut rise at approximately the same rate in response to falling $a$. For $\mathrm{n}_1 = .05$ on the other hand, the rise in the interest rate is for more dramatic than the increase in margins. For the very small endowments that are associated with the high leverage ratios in the Gorton-Metrick data, and many other assets that were at the center of the 2007-2008 financial crisis, we see that default premia and hence interest rates vary dramatically with $a$, while haircuts show far smaller movements.. The underlying economics is not hard to deduce. Optimists value the risky asset highly, and are willing to pay even an interest rate that they regard as actuarially quite unfair in order to get their hands on the asset. As the first-order conditions given above that underlay the optimality curve imply, the less risk capital optimists have, the greater the premium they place on borrowing. The slope of the secant line that represents the average derivative of our transformed optimality curve is given by

$$\frac{n_1(\frac{1}{E_0[s]} - \frac{1}{E_1[s]})}{\frac{s^{\max}}{E_0[s]} - 1} \tag{16}$$

and is strictly rising in $n_1$.Thus for low values of the optimist's endowment the transformed optimality curve is quite flat. Straightforward but slightly messy differentiation (available on request) shows that at the equilibrium, in which the transformed optimality curve intersects the transformed market clearing curve, the derivatives of the two curves are equal in absolute value. Consequently, for small $n_1$ both curves are very flat in the region of the equilibrium, and shifts in the curves occasioned by a change in $a$ cause very large variation in the interest rate alongside miniscule movements in margins.

## V. **Can it be Fixed?**

Is it possible to find a way to more closely match the stylized fact that haircuts on repos (and presumptively other loans collateralized by risky assets) are higher and when initial leverage is high, far more variable than interest spreads. We consider several mechanisms: First, two different specifications of bankruptcy costs. The first is a loss of some fraction of the salvage value in the case of default. Because the pessimist's loss given default is even greater in this case, it goes the wrong way and worsens the conundrum. The second variety of bankruptcy cost is a fixed cost. If large enough, this can certainly do the job - no optimal bargain would allow default in any state. This seems out of the spirit of collateral equilibrium, and is probably best interpreted as a form of institutional risk aversion associated with a particular business model that characterizes the typical conservative lenders in the repo market (e.g. money market funds, that strive to avoid "breaking the buck")..That said, it may ultimately be the answer. An alternative approach is to allow the optimist as well as the pessimist to become more pessimistic. By reducing desired borrowing, this goes in the right direction relative to the case where only the pessimist becomes more pessimistic. However, its extent is limited. If compression of beliefs substantially reduces heterogeneity, the optimists' price will get close to the pessimists' level without a very strong increase in the haircut. A final possibility is to go outside the strict general equilibrium interpretation of $n_1$ as the optimists' endowments, and instead think of a scary shock as triggering a collateral call funded by the sale of other, unmodelled assets held by the optimist. Because a rise in $n_{1,}$ all else equal, lowers leverage, (see Geanakoplos, 2010, and our discussion at the end of section IV), this helps both by lowering the interest rate and raising the haircut. The implication for the asset price, however, may go the wrong way. The scary shock puts downward pressure on the price, but the rise in $n_1$ tends to increase it. In summary, the overall prediction of the continuous state collateral equilibrium model is that contrary to the data, which features very high initial leverage, interest rates rather than haircuts are likely to bear the bulk of the adjustment to a scary shock.

**References**


1. Fostel, Ana and John Geanakoplos (2014). "Endogenous Collateral Constraints and The Leverage Cycle."*Annual Review of Economics* (May).

2. Fostel, Ana and John Geanakoplos (2015). "Leverage and Default in Binomial

Economies: A Complete Characterization." *Econometrica*, 83(6): 2191-2229.

3. Geanakoplos, John (2010). “The Leverage Cycle." *NBER Macroeconomics Annual,* 24(1): 1-66.

4. Geanakoplos, J. (2003). "Liquidity, default and crashes: Endogenous contracts in general equilibrium". In *Eighth World Econometric Society Conference, Vol. II*, pages 170–205. *Econometric Society Monographs*.

5. Geanakoplos, John and Zame, William R. (2014) "Collateral Equilibrium I: A Basic Framework", *Economic Theory*, 56:443–492.

6. Gorton, Gary and Andrew Metrick (2012). “Securitized banking and the run on repo." *Journal of Financial Economics*, 104(3): 425-451.

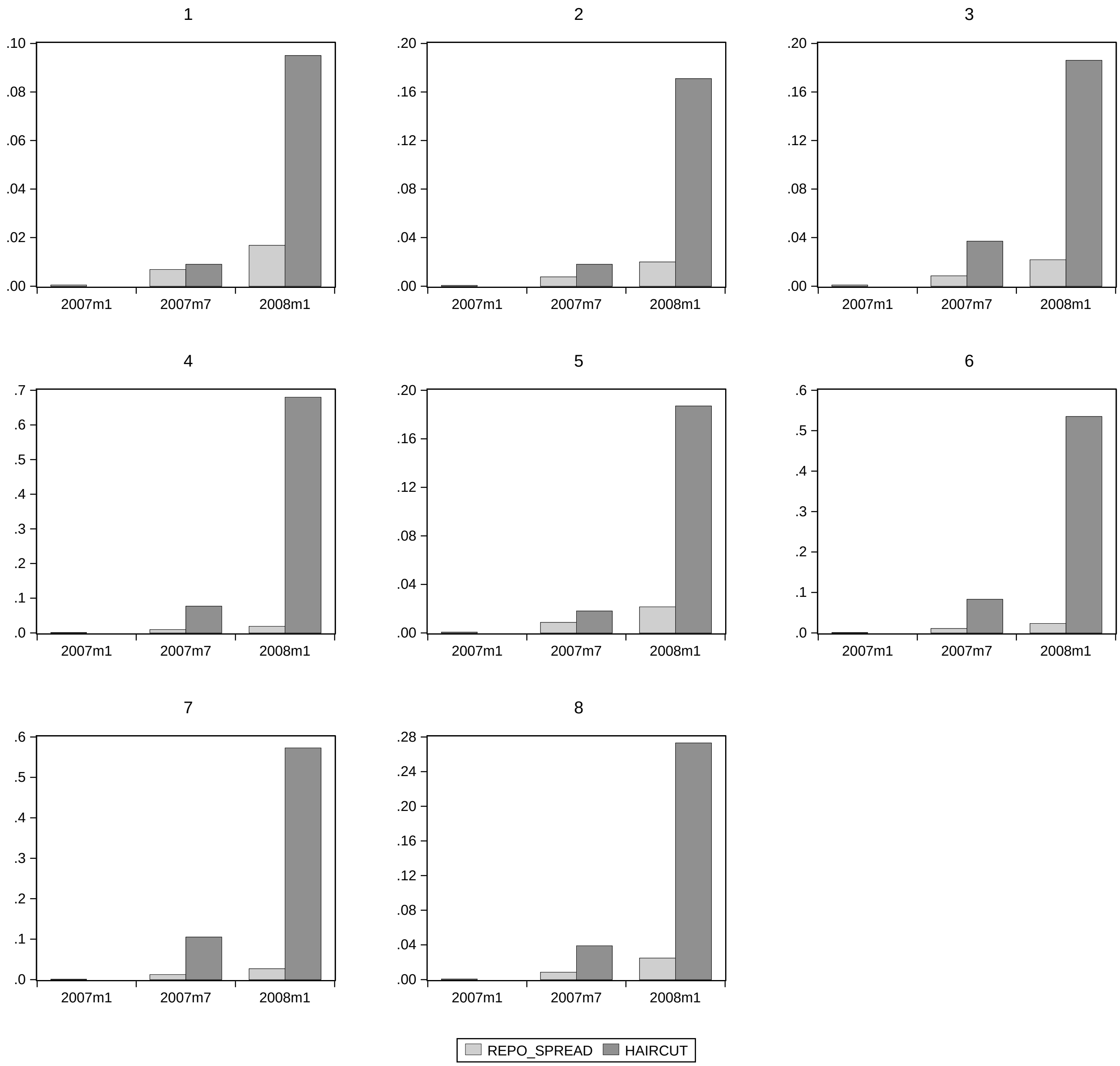
7. Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov (2014). “Sizing Up Repo." *The Journal of Finance*, 69: 2381-2417.

8. Miller, Edward M., 1977, “Risk Uncertainty, and Divergence of Opinion,” *The Journal of Finance*, 32(4), September, 1151-1168.

9. Simsek, Alp (2013). “Belief Disagreements and Collateral Constraints." *Econometrica*, 81(1):

10. Simsek, Alp (2021) "The Macroeconomics of Financial Speculation" *Annual Review of Economics,* Vol. 13:335-369 (August)

Figure I:  Mean Repo Spread and Haircut for Eight Asset Classes in Bilateral Repo Market: First Half of 2007, Second Half of 2007, All of 2008




Source: Gorton and Metrick (2012)

Figure II: Equilibrium with Uniform Distributions

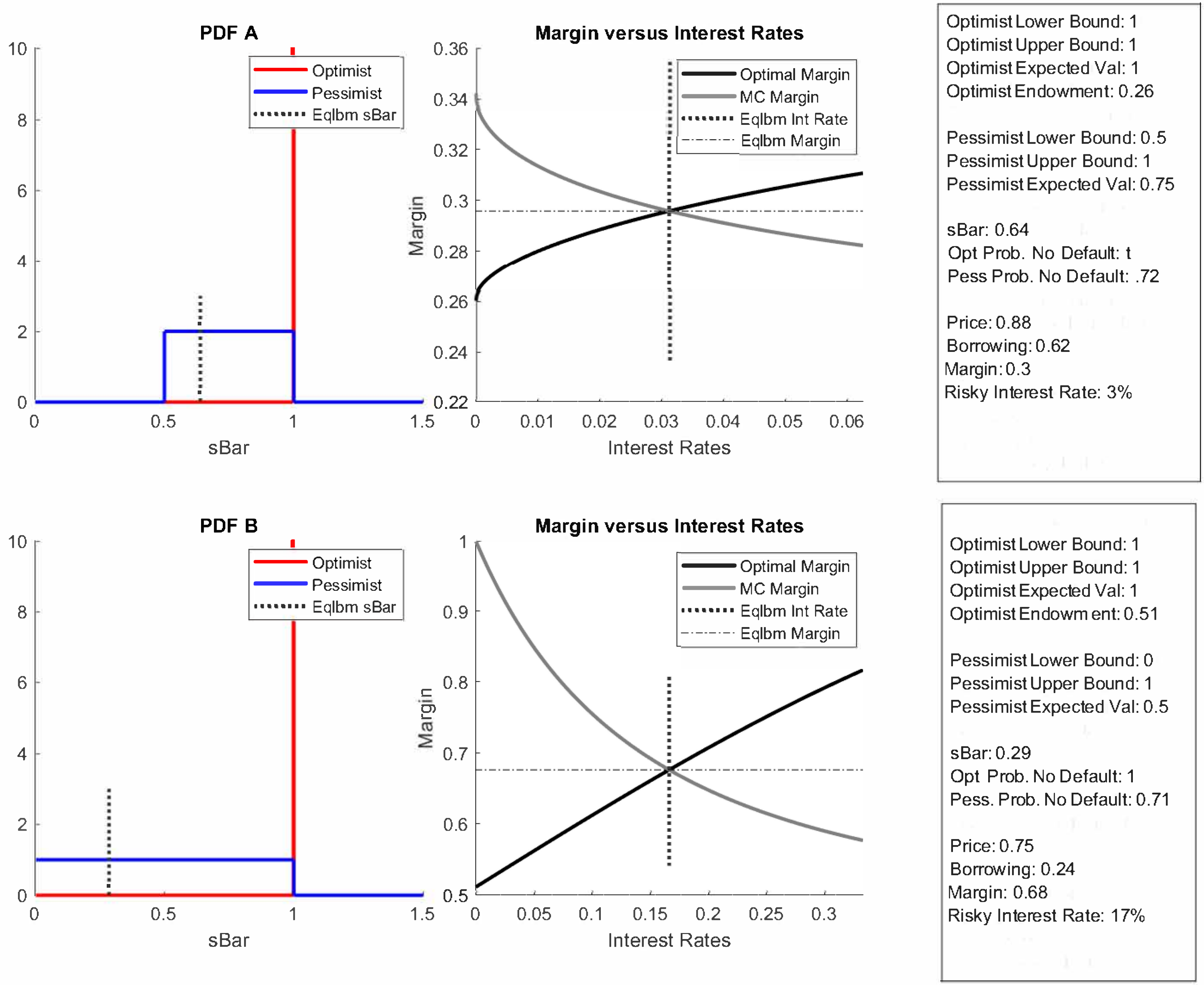
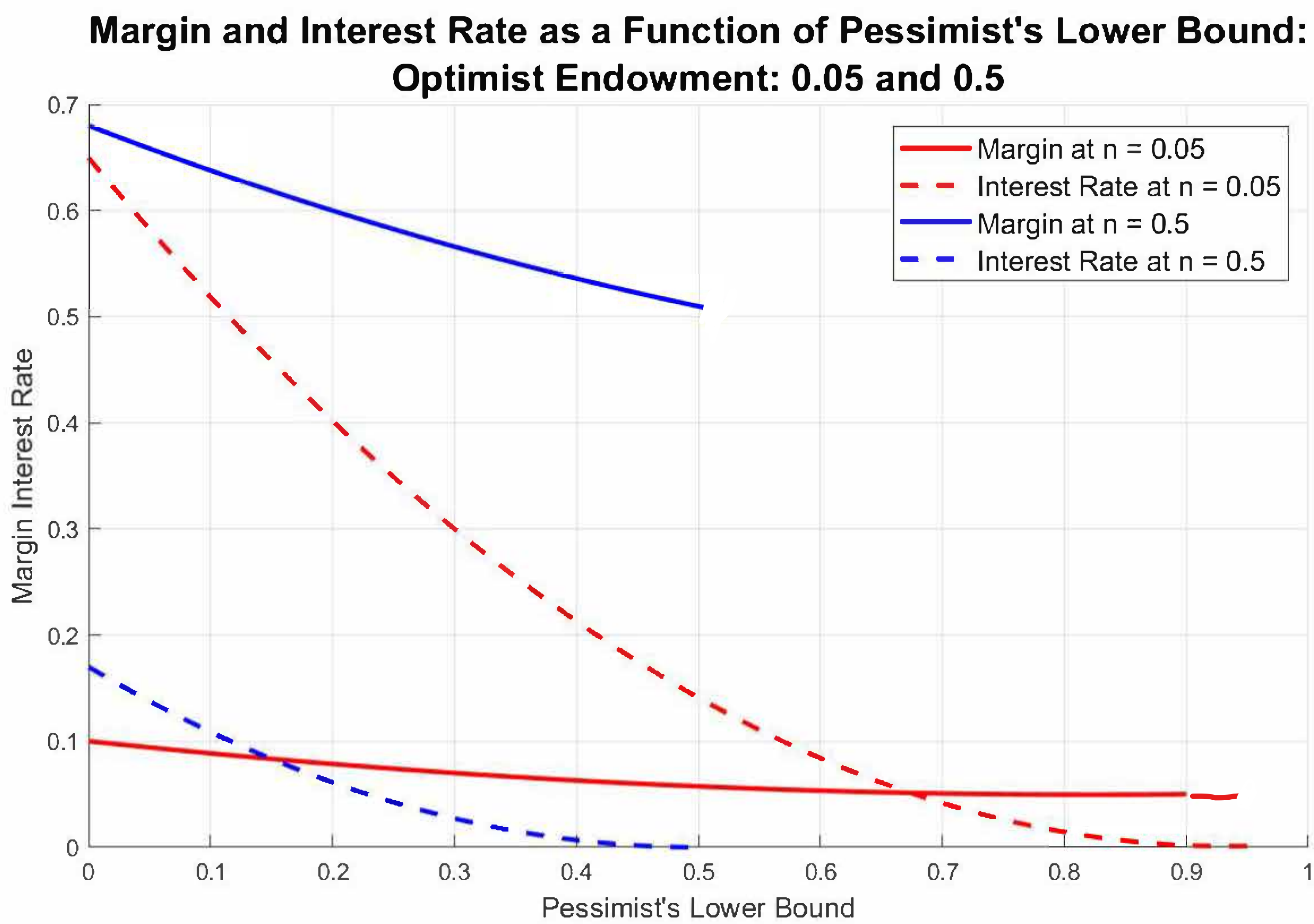


**Figure III**


Margin and Interest Rate as a Function of Pessimist's Lower Bound:
Optimist Endowment: 0.05 and 0.5
Margin at n = 0.05
Interest Rate at n = 0.05
Margin at n = 0.5
Interest Rate at n = 0.5
Margin Interest Rate
Pessimist's Lower Bound
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1