A Comment on Monetary Policy and Rational Asset Price Bubbles

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A Comment on Monetary Policy and Rational Asset Price Bubbles*

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Abstract

Galí (2014) showed that a monetary policy rule that raises interest rates in response to bubbles can paradoxically lead to larger bubbles. This comment shows that a central bank that wants to dampen bubbles can always do so by raising interest rates aggressively enough. This result is different from the Miao, Shen and Wang (2019) comment on Galí's paper. They argue Galí's model contains additional equilibria in which more aggressive rules dampen bubbles. We show that for these equilibria, more aggressive rules involve threats to raise interest rates more than actual rate increases.

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Introduction

Policymakers have long argued about how to respond to a surge in asset prices that cannot be explained by changes in fundamental factors. This is sometimes framed as the *lean* versus *clean* debate regarding asset bubbles, i.e., whether policymakers should lean against asset prices by raising interest rates (or by restricting credit) or wait to see if asset prices fall and, if necessary, clean up afterwards.

In a provocative paper, Galí (2014) questioned the very premise underlying this debate. He observed that while raising the real interest rate will act to lower the fundamental value of an asset due to the higher discounting of dividends, there is no analogous reason why a higher interest rate should reduce the gap between an asset's price and its fundamental. Equilibrium only requires the expected gap to grow at the rate of interest. A higher interest rate should be associated with faster growing bubbles rather than smaller ones. Galí then presented a model in which a rule that raises the interest rate more aggressively in response to shocks that generate unexpectedly large bubbles results in even larger bubbles when such shocks occur.

Miao, Shen, and Wang (2019) argued Galí's results depend on the particular equilibrium he focused on. They consider a different equilibrium in his model and show that a rule which raises the interest rate more aggressively in response to shocks that lead to unexpectedly large bubbles results in smaller bubbles when such shocks occur. They note that their equilibrium is expectations stable (e-stable) and Pareto dominates the equilibrium Galí studied, so is arguably more natural for agents to coordinate on.

The apparent takeaway from these papers is that whether a central bank can reduce the size of bubbles by setting a high interest rate depends on which equilibrium the economy is at. This comment argues that if a central bank wants to dampen a bubble, it can always do so by setting a sufficiently higher interest rate than markets expect. A sufficiently aggressive rule can lead to smaller bubbles in response to shocks that generate unexpectedly large bubbles even starting with the equilibria that Galí (2014) studied.

Intuitively, the model Galí studied admits multiple equilibrium asset price paths for a given path of earnings. When the central bank adopts a more aggressive rule, it can leave earnings unchanged and select equilibria with larger bubbles. However, it cannot do this indefinitely. Eventually, a sufficiently aggressive rule will select the equilibrium with the largest possible bubble for a given earnings path. At that point, a more aggressive rule will cause earnings to fall when interest rates rise, as is standard in models where goods prices are rigid. Lower earnings leave agents with less to spend on assets, resulting in smaller bubbles.

While we argue that sufficiently tighter monetary policy can dampen bubbles, we do not analyze whether dampening bubbles is desirable. That requires theoretical and quantitative analysis beyond what we do.[1]

[1] On the theoretical front, Barlevy (2022) surveys recent work which shows that leaning against a bubble can be welfare improving by mitigating a recession that would be more severe if the bubble were allowed to grow and then collapse. Whether this benefit exceeds the cost of tighter policy requires a quantitative analysis.

1 Economic Environment

While we can illustrate all of our results using the original Galí (2014) framework, as we show in an appendix, we prefer to use a distinct but related model that can be solved in closed form and which can admit a unique equilibrium under certain conditions. These features help convey our results more transparently.

We begin with a single asset vintage before incorporating multiple asset vintages as in Galí's setup. Consider an overlapping generations economy in which agents live for two periods. The economy features a single consumption good. The cohort born at date t receives an endowment of e_t goods when young but values consumption when old. Specifically, their utility equals their consumption c_{t+1} when old. This endowment economy should be interpreted as a reduced form for a production economy that we explicitly write down in an appendix in which the values of e_t represent endogenously-determined earnings.

There is also a cohort of old born at date 0 whose utility is linear in consumption. These old agents are endowed not with goods but with a unit of an asset they can exchange for goods. For now, this is the only asset in the economy. As in Galí, the asset pays no dividends, although at various points below we will consider a variation in which the asset pays a small dividend. Let p_t denote the price of this asset at date t.

Each cohort needs to convert their endowment when young into consumption when old. We assume that they have two ways to do so. One is to buy assets at date t and trade them for goods at date $t+1$. Alternatively, we assume they can store their endowment and consume it one-for-one when old. Let s_t denote the amount of their endowment that agents store. The cohort born at date t thus solves

$$\max_{s_t \in [0, e_t]} E_t \left[c_{t+1} \right] \tag{1}$$

s.t.

$$c_{t+1} = s_t + \frac{p_{t+1}}{p_t} \left(e_t - s_t \right)$$

Depending on how $\frac{p_{t+1}}{p_t}$ compares to 1, agents will either store all of their endowment ($s_t = e_t$), use it all to buy assets ($s_t = 0$), or will be indifferent between the two options.[2]

When the endowment e_t grows over time, storage will be dynamically inefficient. That is, suppose

$$e_t = (1+g)^t e_0 \tag{2}$$

where $g > 0$. In this case, if young agents gave their endowment to the old agents they overlap with rather than store it, each cohort could consume the endowment of the next generation which is larger than their own. As Tirole (1985) demonstrated, dynamic inefficiency allows for the possibility of equilibrium bubbles, or situations in which the price of an asset p_t exceeds the present discounted value of its dividends.

[2] In Galí's model, agents value consumption in both periods of life and cannot store goods. They must therefore choose between buying the asset and consuming while young. This mirrors the choice between buying the asset and storing goods in our model. The linearity in our setup makes it possible to solve for an equilibrium in closed form.

An equilibrium in this economy is a path of prices $\{p_t\}_{t=0}^{\infty}$ such that when agents choose s_t optimally, the amount of assets the young want to buy at each date t is equal to the amount of assets the previous cohort wants to sell. One equilibrium is for each cohort to spend all of its endowment on the asset and store nothing. Since the supply of the asset is 1, this would imply

$$p_t = e_t \tag{3}$$

At these prices, the return to buying the asset is $\frac{p_{t+1}}{p_t} = 1 + g$ and so buying the asset dominates storage. The path $p_t = e_t$ thus constitutes an equilibrium given the young buy all assets sold by the old.

There are many other equilibrium paths for p_t besides (3). For example, $p_t = 0$ for all t is an equilibrium: If agents expect $p_{t+1} = 0$, they would not be willing to pay anything for the asset at date t. Figure 1 plots some deterministic equilibrium price paths. We fully characterize the set of such equilibria in Appendix A. These equilibria have the property that once the price p_t falls below e_t, the price p_t stops growing. Intuitively, if p_t fell below e_t at some date t, agents that period would both buy the asset and store goods. This requires $\frac{p_{t+1}}{p_t} = 1$, which means p_{t+1} will remain below e_{t+1} given the endowment grows over time.

Since our model admits equilibria in which $p_t > 0$ even though the asset pays no dividends, it gives rise to bubbles and allows us to study the effects of policy on bubbles. However, policy analysis with multiple equilibria can be tricky. We will therefore consider a variation of the model that admits a unique equilibrium that is also a bubble. It turns out that if the asset pays a dividend of $d > 0$ goods per period instead of 0, the equilibrium price path will be unique and still correspond to a bubble.[3] Intuitively, even a small dividend ensures the return on the asset $\frac{d+p_{t+1}}{p_t}$ must be positive. In that case, buying the asset dominates storage, and agents will spend all their endowment on the asset. Among the equilibria in Figure 1, only the red line corresponding to $p_t = e_t$ remains an equilibrium when $d > 0$. No other path survives as an equilibrium. We prove this formally in Appendix A.

Proposition 1: If $d > 0$, then there is a unique equilibrium price path, namely $p_t = e_t$.

To confirm that this equilibrium represents a bubble, observe that $p_t = e_t$ implies $\lim_{t\to\infty} p_t = \infty$. At the same time, the fundamental value of the asset is given by the value of the dividends d per period, discounted by the return on savings $1 + r_t = \frac{d+p_{t+1}}{p_t}$ that agents earn in equilibrium. That is,

$$f_t = \sum_{j=1}^{\infty} \left(\prod_{i=0}^{j-1} \frac{1}{1 + r_{t+i}} \right) d \tag{4}$$

[3] Tirole (1985) already showed there can be a unique equilibrium that is also a bubble; see part (c) of his Proposition 1. While we highlight the role of $d > 0$, Tirole emphasized the importance of assuming a zero (or negative) return on storage. Both conditions are in fact necessary. If storage converted a unit of goods at date t into $1 + z$ units at date $t + 1$ for $z > 0$, the equilibrium in our model would no longer be unique even if $d > 0$.

Since $r_t = g + \frac{d}{e_t} > g$ for all t, we have

$$f_t < \sum_{j=1}^{\infty} \left(\frac{1}{1+g} \right)^j d = \frac{d}{g}$$

The fundamental value of the asset is bounded at all dates, while the price of the asset $p_t = (1+g)^t e_0$ grows without bound.[4] Appendix A shows that p_t exceeds f_t for all t and not just asymptotically.

Proposition 2: When $d > 0$, the bubble term $b_t = p_t - f_t > 0$ for all dates t.

2 Reduced-Form Monetary Policy

We now turn to policy. Monetary policy is irrelevant in an endowment economy like the one above. But it will matter in a production economy where e_t is endogenous and firms set prices before knowing what the monetary authority does. We present such a model in Appendix B based on Allen, Barlevy, and Gale (2022), which in turn combines Adam (2003) and Galí (2014). Rather than present that full setup here, we use the endowment economy above to sketch a reduced-form version of how monetary policy operates.

Suppose a monetary authority sets a nominal interest rate i_t each period at which it will freely borrow or lend money at date t in exchange for monetary payments at date $t+1$. We want to know which variables will be affected by a change in i_t. Let p_t denote the real price of the asset and $1 + r_t = \frac{d+p_{t+1}}{p_t}$ denote the real return on the asset, where d can be a positive real payment or 0. Let P_t denote the price of goods at date t and $\Pi_t \equiv \frac{P_{t+1}}{P_t}$ the growth in the price of goods between t and $t+1$. If $1 + r_t > \frac{1+i_t}{\Pi_t}$, there would be infinite demand to borrow from the monetary authority to buy assets, yet asset supply is finite. If $1 + r_t < \frac{1+i_t}{\Pi_t}$, agents would prefer lending to the central bank over the asset, so no one would buy the assets the old want to sell. Even without the full structure of the model, then, it follows that in equilibrium,

$$1 + r_t = \frac{d + p_{t+1}}{p_t} = \frac{1 + i_t}{\Pi_t} \tag{5}$$

If i_t changes, either the real rate r_t or inflation Π_t must adjust for (5) to hold. This is where the model in Appendix B comes in. If goods prices are fully flexible, the model implies changing i_t has no effect on real variables: p_t and $1 + r_t$ are unchanged while Π_t adjusts to equate $\frac{1+i_t}{\Pi_t}$ with $1 + r_t$. If instead goods prices are rigid and Π_t is fixed, the real return $1 + r_t$ must change with i_t. The question is how: Must earnings e_t change for p_t to change, or can p_t change when earnings do not? We will show that a small increase in $1+r_t$ can raise p_t and leave earnings e_t unchanged, but a large increase in $1+r_t$ must lower both p_t and e_t.

[4] The finiteness of f_t requires $\lim_{t\to\infty} d/e_t = 0$. Rhee (1991) showed that $\lim_{t\to\infty} d/e_t > 0$ implies bubbles cannot arise.

3 Monetary Policy Shocks

Galí modelled i_t as being set according to a rule that can be applied when there are multiple asset vintages. We do the same in the next section. But we can convey the key intuition using monetary policy shocks and a single asset vintage. Suppose the central bank announces a path $\{i_t^*\}_{t=0}^{\infty}$ but then unexpectedly sets $i_0 > i_0^*$ at date 0. It is convenient to treat the change in i_0 as an unexpected surprise, although Appendix B considers the case where i_0 is random and looks at the effect of a high *realization* of i_0, which yields qualitatively similar results. An unexpectedly high i_0 corresponds to a monetary policy shock akin to those identified in Christiano, Eichenbaum, and Evans (1999), Kuttner (2001), Romer and Romer (2004), Gürkaynak, Sack, and Swanson (2005), Gertler and Karadi (2015), and Nakamura and Steinsson (2018).

We assume goods prices are fully rigid within a period so that Π_0 is fixed as i_0 varies. A shock to i_0 will change the real rate r_0. Since $1 + r_0 = \frac{d+p_1}{p_0}$, either p_0 or p_1 must change. To see how this can occur in equilibrium, we first consider the case where $d > 0$ and the equilibrium $\{p_t\}_{t=0}^{\infty}$ is unique. We then consider the case where $d = 0$ and there are multiple equilibria $\{p_t\}_{t=0}^{\infty}$ for a fixed earnings path $\{e_t\}_{t=0}^{\infty}$.

Start with $d > 0$. We use a star to denote equilibrium values when i_t is equal to its anticipated value i_t^*. The model in Appendix B features exogenous productivity growth at rate g, ensuring that when $i_t = i_t^*$ for all t, earnings $e_t^* = (1+g)^t e_0^*$ as in the endowment economy. Proposition 1 then implies that the unique equilibrium asset price path is $p_t^* = e_t^*$ for all t. The real interest rate at date 0 is

$$1 + r_0^* \equiv \frac{e_1^* + d}{e_0^*} = 1 + g + \frac{d}{e_0^*} \tag{6}$$

If Π_0 cannot vary with i_0, a positive shock to i_0 will raise r_0 above $r_0^* = g + \frac{d}{e_0^*}$. Since $r_0 > r_0^* > 0$, young agents will spend all of their earnings to buy assets at date 0 and p_0 will equal equilibrium earnings e_0. The expected return on the asset will be $1 + r_0 = \frac{d+p_1}{e_0}$. Since agents in the model reoptimize at the start of each period, a shock at date 0 has no effect on real variables beyond date 0, including $p_1 = e_1^*$. The only way $1 + r_0 = \frac{d+e_1^*}{e_0}$ can rise above $\frac{d+e_1^*}{e_0^*}$, then, is if e_0 falls below e_0^*. Appendix B fills in the details of how earnings e_0 fall at date 0. This temporary contractionary effect of an unexpectedly higher interest rate on earnings and output is standard whenever goods prices are rigid.

An unexpectedly high interest rate i_0 thus depresses earnings e_0 and the asset price p_0. To determine the effect on the bubble term $b_0 = p_0 - f_0$, we can use the definition of r_0 to solve for the asset price at date 0:

$$p_0 = \frac{d + p_1}{1 + r_0} \tag{7}$$

At the same time, the fundamental value of the asset satisfies the recursive equation

$$f_0 = \frac{d + f_1}{1 + r_0} \tag{8}$$

Rearranging and subtracting (8) from (7), together with the fact that $b_t = p_t - f_t$, yields

$$b_1 = (1 + r_0)\, b_0 \tag{9}$$

Since a shock to i_0 has no effect on real variables after date 0, neither the real asset price p_1 nor the real fundamental value f_1 depend on i_0. The shock thus has no effect on b_1. If r_0 increases and b_1 is unchanged, then $b_0 = \frac{b_1}{1+r_0}$ must fall. Hence, when $d > 0$, an unexpectedly high value of i_0 dampens the bubble at date 0. Intuitively, if monetary policy is neutral in the long run, it will not affect the long-run value of the bubble. If the bubble grows more rapidly but ends up at the same level, its initial value must fall.

What about the case where $d = 0$ and, as in Galí, there can be multiple equilibrium paths $\{p_t\}_{t=0}^{\infty}$ for a given earnings path $\{e_t\}_{t=0}^{\infty}$? Once again, when Π_0 is fixed, an unexpectedly high $i_0 > i_0^*$ requires the real interest rate $1 + r_0 = \frac{p_1}{p_0}$ to rise above $1 + r_0^*$. Since $r_0^* \geq 0$, this means $r_0 > 0$. Young agents at date 0 will then spend all of their earnings on the asset, and so p_0 will equal e_0. As before, a shock to i_0 has no effect on real variables after date 0. Earnings at date 1 will therefore remain equal to e_1^*. But this no longer pins down the price p_1 as when $d > 0$. One way to achieve a higher r_0 is for p_1 to remain equal to p_1^* and for $p_0 = e_0$ to fall, just as when $d > 0$. Another possibility is for earnings e_0 to remain fixed at e_0^* and for p_1/p_0 to rise as we switch to a different equilibrium path $\{p_t\}_{t=0}^{\infty}$ for the same earnings profile $\{e_t^*\}_{t=0}^{\infty}$.

The latter scenario is illustrated in Figure 2. The black lines in Figure 2 correspond to different equilibrium price paths for the fixed earnings path $\{e_t^*\}_{t=0}^{\infty}$. These are the same paths as in Figure 1. Suppose that the real interest rate absent a monetary policy shock is $r_0^* < g$. The light blue line in Figure 2 represents one such equilibrium (in which r_0^* happens to be 0). A positive nominal interest rate shock leads to a higher real rate $r_0 > r_0^*$. As long as this real rate $r_0 \leq g$, we can find a different equilibrium in which earnings e_0 are fixed at e_0^* and $\frac{p_1}{p_0} = 1 + r_0$. The dark blue line in Figure 2 is the unique deterministic equilibrium with $\frac{p_1}{p_0} = 1 + r_0$ for $r_0 < g$. An interest rate shock in this case steers the economy from the light blue line to the dark blue line without affecting earnings. While the growth rate $\frac{p_1}{p_0}$ is necessarily higher in the new equilibrium, the path also features higher asset prices, i.e., $p_t > p_t^*$ for all t. Since the fundamental value of the asset is 0 when $d = 0$, this means an unexpectedly higher interest rate $i_0 > i_0^*$ leads to larger bubbles.

More generally, since $r_0 > r_0^* \geq 0$, the young would spend all of their endowment on the asset and p_0 must equal e_0. If increasing the rate to $i_0 > i_0^*$ had no effect on earnings, we would have $e_0 = e_0^*$. It follows that the asset price with an unexpectedly high interest rate satisfies $p_0 = e_0^*$. Since agents cannot spend more than their earnings, the price p_0^* under the original interest rate i_0^* could not have exceeded e_0^*. It follows that $p_0 = e_0^* \geq p_0^*$. This inequality becomes strict beyond date 0, since

$$
\begin{aligned}
p_1 &= (1 + r_0)\, p_0 \\
&> (1 + r_0^*)\, p_0 \\
&\geq (1 + r_0^*)\, p_0^* = p_1^*
\end{aligned}
$$

This is the key result in Galí (2014): A higher nominal interest rate can magnify bubbles. Monetary policy in this case works not by depressing earnings but by selecting a different equilibrium path $\{p_t\}_{t=0}^{\infty}$ even as earnings are unchanged. The path in which asset prices grow more rapidly also features larger values of p_t.

However, the result that a higher interest rate can lead to larger bubbles breaks down for large increases in i_0. Suppose i_0 rises enough so that $\frac{1+i_0}{\Pi_0} > 1+g$. Since the implied $r_0 > g > 0$, young agents will spend all of their earnings on the asset at date 0 and the equilibrium price p_0 will equal e_0. But it is no longer possible for e_0 to stay unchanged at e_0^*. For suppose $e_0 = e_0^*$. Since $p_0 = e_0^*$ and $p_1 = (1+r_0)\, p_0$, we would have $p_1 > (1+g)\, e_0^* = e_1^*$. The unexpected shock to i_0 has no effect beyond date 0, so $e_1 = e_1^*$. If e_0 stayed equal to e_0^*, the asset would be worth more at date 1 than buyers can pay. So e_0 cannot remain at e_0^*. Instead, since $p_0 = e_0$ and $p_1 \leq e_1^*$, we have that

$$1 + r_0 = \frac{p_1}{p_0} \leq \frac{e_1^*}{e_0}$$

Rearranging, we have $e_0 \leq \frac{e_1^*}{1+r_0}$. If $r_0 > g$, then e_0 must fall below e_0^*. A large interest rate shock must depress earnings e_0, just as in the case where $d > 0$. Since $p_0 \leq e_0$, depressing earnings enough will depress the asset price p_0. A sufficiently large interest rate shock must reduce the bubble at date 0:

Proposition 3: If $d = 0$, for any p_0^*, a sufficiently large real interest rate r_0 will push $p_0 = e_0$ below p_0^*.

Figure 3 shows this result graphically. An unexpected shock to i_0 has no effect beyond date 0, so the set of equilibrium price paths from date 1 on is the same as in Figures 1 and 2. However, the set of equilibria in period 0 changes. The dashed lines between dates 0 and 1 show paths that are equilibria for the original earnings path $\{e_t^*\}_{t=0}^{\infty}$. The set of equilibria when $r_0 > g$ corresponds to the solid lines between dates 0 and 1. In all of these equilibria, $\frac{p_1}{p_0}$ is equal to $1 + r_0$. The different paths correspond to different values of p_1, which recall is not uniquely determined. Starting from the equilibrium corresponding to the same light blue line as in Figure 2, a sufficiently large shock at date 0 would lead to a set of equilibrium prices p_0 that are all below p_0^*, since all are below $\frac{e_1^*}{1+r_0}$. For r_0 large enough, $\frac{e_1^*}{1+r_0}$ will fall below any original p_0^*.

4 Monetary Policy Rules and Multiple Asset Vintages

In the previous section, we showed that a central bank can always dampen a bubble by setting the nominal interest rate sufficiently above what agents expect. But if agents understand this policy, they should expect higher rates when bubbles arise. To deal with this, Galí considered stochastic bubbles and let the central bank commit to raising i_t when the realized value of bubble assets is larger than expected. To consider this case, we follow Galí and Miao et al by allowing multiple asset vintages in our model. We use this version to illustrate Galí's finding that a more aggressive rule can lead to larger bubbles in response to shocks that lead to unexpectedly large bubbles. We then show that a sufficiently aggressive response will lead to smaller bubbles. The logic is similar to the case of interest rate shocks with a single vintage.

Suppose that instead of a single asset endowed to old agents at date 0, each new cohort of old is endowed with its own unit supply of assets. At date t, there will be $t + 1$ vintages that were endowed to the old at dates $s = 0, 1, ..., t$, respectively. Dividends on any vintage are 0. Asset vintages are distinguishable, and vintages can potentially trade at different prices even though their payouts are identical. Agents cannot sell these assets short; they can only sell assets they already own. Let $p_{s|t}$ denote the price at date t of the assets that originated at date s, and let $b_t = \sum_{s=0}^{t} p_{s|t}$ denote the total value of all asset vintages available at date t. An equilibrium now constitutes a path of prices $\{p_{s|t}\}_{t\geq s}$ for each vintage $s = 0, 1, 2, ...$ that is consistent with market clearing and optimal choice.[5]

At each date t, we can partition assets into vintages whose price $p_{s|t}$ is 0 and vintages whose price $p_{s|t}$ is positive. Vintages where $p_{s|t} = 0$ at date t must continue to trade at 0 beyond date t. If not, there would be infinite demand at some point for free assets that yield a positive expected payoff, but only a finite supply of each vintage. Vintages where $p_{s|t} > 0$ at date t must all be expected to grow at the same rate for agents to buy them. One way to satisfy these conditions is to assume the initial price $p_{t|t}$ can assume two values, either 0 or positive, and then let any vintage that starts at 0 stay at 0 and any vintage that started at a positive price grow at a common rate. That is, we look for paths $\{p_{s|t}\}_{t\geq s}$ where

$$p_{t|t} \begin{cases} = 0 & \text{with probability } q \\ > 0 & \text{with probability } 1-q \end{cases}$$

and

$$p_{s|t+1} = (1 + r_t)\, p_{s|t} \text{ for all } s \leq t \tag{10}$$

where r_t is the common (possibly random) return on all assets with positive prices at date t. Since $p_{t|t}$ is random and r_t can be random, the total value of all bubbles $b_t = \sum_{s=0}^{t} p_{s|t}$ at date t can be random.

Following Galí, we let the central bank set the nominal interest rate i_t as an increasing function of $b_t - E_{t-1}[b_t]$. This rule can be understood as leaning against unexpectedly large bubbles. The driving source of randomness in our setup is the price of the latest vintage $p_{t|t}$, specifically whether $p_{t|t} = 0$ or $p_{t|t} > 0$. This implies a two-point distribution, so the interest rate rule features at most two rates, i^H and i^L, where the higher rate i^H is applied if b_t exceeds $E_{t-1}[b_t]$. A more aggressive rule corresponds to a higher ratio $\frac{1+i^H}{1+i^L}$. Galí (2014) and Miao et al (2019) study the effect of more aggressive rules for different equilibria of the same underlying model. We now do the same in our setup.

4.1 Revisiting Galí (2014)

We begin with the equilibria that Galí (2014) studied. To describe these equilibria, let us start with the case where goods prices are fully flexible. Our model in Appendix B implies that interest rate rules have no effect on real variables in this case, meaning e_t equals e_t^* regardless of the rule. Just as this earnings path allowed multiple equilibrium paths $\{p_t\}_{t=0}^{\infty}$ with a single asset, it allows multiple equilibrium paths for $\{b_t\}_{t=0}^{\infty}$ when there are multiple assets. The equilibria Galí considered have two distinguishing features. First, they are interior equilibria in which the value of all assets b_t at any date t is below the maximal

[5] Galí also assumed that a fraction δ of the assets that traded at a positive price last period collapse to 0. Assets from the same vintage can thus trade at different prices. For simplicity, we set $\delta = 0$ so there is a single price $p_{s|t}$ for each s.

possible value for b_t. In our model, this would imply $b_t < e_t$, i.e., agents do not spend all of their earnings on assets. However, if b_t were below e_t for all t, the real interest rate r_t would always equal 0. This is at odds with the second feature of the equilibria Galí studied, namely that the total value of all assets b_t and the real interest rate r_t at date t are both higher if $p_{t|t} > 0$ than if $p_{t|t} = 0$. A higher r_t if $p_{t|t} > 0$ requires that r_t be strictly positive in this case. Young agents would then spend all of their earnings on assets, so $b_t = e_t$. In our setup, then, we can only consider equilibria that are interior if $p_{t|t} = 0$ rather than in all states of the world. The equilibria in our model that are most similar to those in Galí (2014) are those where (i) the initial price $p_{t|t}$ for any date t is distributed according to

$$p_{t|t} = \begin{cases} e_t - \sum_{s=0}^{t-1} p_{s|t} & \text{with probability } q \\ 0 & \text{with probability } 1-q \end{cases} \tag{11}$$

and (ii) the prices of all existing vintages between dates t and $t+1$ grow at a common rate r_t that is equal to 0 if $p_{t|t} = 0$ (to ensure that b_t is interior) and some $r \in (0, g)$ if $p_{t|t} > 0$, i.e.,

$$p_{s|t+1} = \begin{cases} (1+r)\, p_{s|t} & \text{if } p_{t|t} > 0 \\ p_{s|t} & \text{if } p_{t|t} = 0 \end{cases} \qquad \text{for all } s \leq t \tag{12}$$

Equations (11) and (12) describe a family of equilibria indexed by a parameter $r \in (0, g)$. We can confirm these paths are equilibria. When $p_{t|t} > 0$, young agents will spend all of their earnings on assets given $r > 0$. When $p_{t|t} = 0$, young agents are indifferent between storage and assets. Since all asset prices grow at the same rate, agents are indifferent among vintages. The black line in Figure 4 illustrates a sample path b_t of such an equilibrium assuming $p_{t|t} > 0$ for $t = 2$, 6, and 7. In these periods, agents are willing to pay a positive price for the latest vintage. When that happens, existing vintages trade at the same as if the new vintages had a price of zero, but the total value of all bubble assets b_t is higher, as is the real return r_t.

We next consider the effect of changing the policy rule $\frac{1+i^H}{1+i^L}$ when goods prices are rigid for equilibria that satisfy (11) and (12). The formal analysis is in Appendix B, but we provide a sketch here. We focus on equilibria in which earnings e_t equal e_t^* regardless of $\frac{1+i^H}{1+i^L}$. Such equilibria exist as long as $\frac{1+i^H}{1+i^L} < 1+g$.

Under (11) and (12), the total value of assets b_t equals e_t^* if $p_{t|t} > 0$. If $p_{t|t} = 0$, then b_t is equal to either b_{t-1} or $(1+r)\, b_{t-1}$, depending on the realization of $p_{t-1|t-1}$. Either way, since $r \in (0, g)$, we would have $b_t \leq (1+r)\, b_{t-1} < (1+g)\, e_{t-1}^* = e_t^*$. This means $b_t = e_t^* > E_{t-1}[b_t]$ if $p_{t|t} > 0$ and $b_t < E_{t-1}[b_t]$ if $p_{t|t} = 0$. A monetary policy rule that raises i_t when $b_t > E_{t-1}[b_t]$ would therefore set $i_t = i^H$ when $p_{t|t} > 0$ and $i_t = i^L$ when $p_{t|t} = 0$. When goods prices are fully rigid and Π_t is the same if $p_{t|t} = 0$ or $p_{t|t} > 0$, we have

$$\frac{1+i^H}{1+r_t^H} = \frac{1+i^L}{1+r_t^L} = \Pi_t \tag{13}$$

We can rearrange this to get

$$\frac{1+r_t^H}{1+r_t^L} = \frac{1+i^H}{1+i^L} \tag{14}$$

Since $b_t < e_t^*$ when $p_{t|t} = 0$, the real interest rate r_t^L must equal 0 when $p_{t|t} = 0$ to ensure young agents are willing to store goods. Substituting into (14) implies $1+r_t^H = \frac{1+i^H}{1+i^L}$. Restricting attention to equilibria from the family above, a more aggressive rule corresponding to a higher $\frac{1+i^H}{1+i^L}$ would leave the real interest rate r_t^L unchanged at 0 whenever $p_{t|t} = 0$ but would raise the real interest rate r_t^H when $p_{t|t} > 0$. Essentially, a more aggressive rule corresponds to an equilibrium with a higher value of $r \in (0, g)$ from the family of equilibria that satisfy (11) and (12). Figure 4 illustrates this graphically: Holding fixed when $p_{t|t}$ is positive or zero, raising $\frac{1+i^H}{1+i^L}$ shifts the path of b_t from the black line to the blue line. A more aggressive rule implies the total value of bubble assets b_t will be weakly larger under a more aggressive rule than a less aggressive rule. This is similar to the way an unexpectedly high interest in Figure 2 resulted in a weakly higher price path for the single intrinsically worthless asset in that case. It is one of the key results in Galí (2014).

The analogy between monetary policy rules and monetary policy shocks suggests that a sufficiently aggressive rule will not be able to select from the family of equilibria for the fixed earnings path $\{e_t^*\}_{t=0}^{\infty}$, and would instead force the path of earnings $\{e_t\}_{t=0}^{\infty}$ to change. Consider what happens when $\frac{1+i^H}{1+i^L}$ exceeds $1+g$. Suppose there was an equilibrium with $e_t = e_t^*$ for all t. We want to produce a contradiction. Generalizing (5), agents will neither borrow infinite amounts from the central bank nor prefer lending to the central bank over buying assets when the *expected* return $E[1+r_t]$ on the asset equals $\frac{1+i_t}{\Pi_t}$, i.e.,

$$E[1+r_t] = \frac{E_t\left[p_{s|t+1}\right]}{p_{s|t}} = \frac{1+i_t}{\Pi_t} \tag{15}$$

This generalization allows for equilibria in which agents are uncertain at date t about the price $p_{s|t+1}$ at which they can sell an asset in $t+1$. Since Π_t does not depend on the realization of i_t, it follows that

$$\frac{E\left[1+r_t^H\right]}{E\left[1+r_t^L\right]} = \frac{1+i^H}{1+i^L}$$

Since agents must always earn at least the return on storage, $E\left[1+r_t^L\right] \geq 1$. This implies

$$E\left[1+r_t^H\right] \geq \frac{1+i^H}{1+i^L} > 1+g$$

Young agents will therefore prefer buying assets to storage when $i_t = i^H$. Since they spend all of their earnings e_t on the asset, we must have $b_t = e_t = e_t^*$ when $i_t = i^H$. If $E\left[1+r_t^H\right] > 1+g$, there exists a realization of $1+r_t^H$ that exceeds $1+g$. But then, $\left(1+r_t^H\right) b_t > (1+g)\, e_t^* = e_{t+1}^*$, which means in some state of the world, spending on the asset at date $t+1$ exceeds the wealth agents have to buy assets. Earnings e_t thus cannot remain unchanged at e_t^* under a sufficiently aggressive rule. In Appendix B, we show that setting $\frac{1+i^H}{1+i^L}$ sufficiently high will push e_t arbitrarily close to 0 when $i_t = i^H$:

Proposition 4: For any equilibrium in which b_t at date t has a two-point distribution, earnings e_t^H when $i_t = i^H > i^L$ will be arbitrarily close to 0 if $\frac{1+i^H}{1+i^L}$ is sufficiently high.

In terms of Figure 4, the above result implies that at $t = 2, 6$, and 7, earnings e_t can be made arbitrarily close to 0 by pursuing a sufficiently aggressive rule. Since $b_t \leq e_t$, the value of bubble assets will similarly be arbitrarily small. In Appendix C, we derive an identical result for Galí's original setting.

4.2 Revisiting Miao, Shen, and Wang (2019)

We now turn to Miao, Shen, and Wang (2019). To describe the equilibria they focus on, we again start with the case of fully flexible goods prices. Recall that $e_t = e_t^*$ for all t in this case. The distinguishing feature of the equilibria in Miao et al (2019) is that b_t attains its maximal value at each date. In our framework, this means agents spend all their earnings on assets, i.e., $b_t = e_t$ for all t. At date 0, that requires

$$p_{0|0} = e_0 \tag{16}$$

For $t > 0$, we assume that the new asset vintage at date t can either trade at $p_{t|t} = 0$ or at some positive price $p_{t|t} > 0$. In particular,

$$p_{t|t} = \begin{cases} \varepsilon e_t & \text{with probability } q \\ 0 & \text{with probability } 1-q \end{cases} \tag{17}$$

where $0 < \varepsilon < \frac{g}{1+g}$. This restriction on ε implies $(1-\varepsilon)(1+g) > 1$. As we show below, this will ensure the return on older vintages is always positive when $e_t = e_t^*$ for all t. For asset prices to grow at the same rate and total spending on all assets b_t to add up to e_t, we need

$$p_{s|t+1} = \begin{cases} (1-\varepsilon)\frac{e_{t+1}}{e_t} p_{s|t} & \text{if } p_{t+1|t+1} > 0 \\ \frac{e_{t+1}}{e_t} p_{s|t} & \text{if } p_{t+1|t+1} = 0 \end{cases} \tag{18}$$

When $e_t = e_t^*$ for all t, the ratio $\frac{e_{t+1}}{e_t}$ equals $1+g$. We can verify that the prices in (16), (17), and (18) constitute an equilibrium when $e_t = e_t^*$ for all t. Unlike the equilibria in the previous subsection, the return on assets bought at date t now depends on the realization of $p_{t+1|t+1}$: It will be $1+g$ if $p_{t+1|t+1} = 0$ and $(1-\varepsilon)(1+g)$ if $p_{t+1|t+1} > 0$. Since this return always exceeds 1 given our assumption on ε, young agents will spend their entire earnings on assets each period, i.e., $b_t = e_t$ for all t. Spending on new assets crowds out spending on existing vintages but leaves total spending on all assets b_t constant at e_t^*. This is a contrast to equilibria defined by (11) and (12) where spending on existing vintages $\sum_{s=0}^{t-1} p_{s|t}$ was the same whether $p_{t|t} = 0$ or $p_{t|t} > 0$ while total spending on all assets b_t was higher when $p_{t|t} > 0$ than when $p_{t|t} = 0$. Figure 5 illustrates a sample path for b_t as well as the prices of individual vintages for this equilibrium assuming $p_{t|t} > 0$ for $t = 2$, 6, and 7. Once again, agents are willing to pay a positive price for new vintages at these dates. When that happens, the realized return $1+r_t = \frac{p_{s|t+1}}{p_{s|t}}$ on existing assets is lower than if new vintages had a price of zero, but the total value of all bubble assets b_t remains equal to earnings e_t.

What happens for this class of equilibria as we vary $\frac{1+i^H}{1+i^L}$ and goods prices are rigid? Miao et al show that if $p_{t|t}$ are independent across time, a more aggressive rule will not lead to larger bubbles as with Galí's equilibria. Instead, it does nothing. This is true in our setting as well. If $i^H = i^L$, monetary policy is perfectly predictable and equilibrium earnings e_t are equal to e_t^*. If we set $\frac{1+i^H}{1+i^L}$ above 1, the nominal and real interest rate would have to rise if $b_t > E_{t-1}[b_t]$. For equilibria where $b_t = e_t$ for all t, this means the nominal and real interest rate would rise if $e_t > E_{t-1}[e_t]$. Since the only source of uncertainty is whether $p_{t|t} = 0$ or $p_{t|t} > 0$, the real interest rate will vary with e_t only if the value of e_t depends on whether $p_{t|t} = 0$ or $p_{t|t} > 0$. Working through the model in Appendix B, such variation is only possible if the ratio $\frac{e_t}{e_t^*}$ can assume two values at each date, one below 1 and one greater than or equal to 1. The real interest rate would be lower if $e_t \geq e_t^*$ at date t, since this is when agents spend more to buy assets whose price in period $t+1$ is independent of e_t. But this is inconsistent with the requirement that the real and nominal rate be higher when $e_t > E_{t-1}[e_t]$. The only equilibrium when $\frac{1+i^H}{1+i^L} > 1$ and $b_t = e_t$, then, is for $b_t = e_t^*$ for all t. In that case, $b_t = E_{t-1}[b_t]$ for all t, b_t is never unexpectedly large, and i_t never varies. A promise to be more aggressive if $b_t > E_{t-1}[b_t]$ has no effect on the realized value of b_t. Intuitively, for the equilibria Miao et al study, a rule that raises the interest rate if b_t exceeds $E_{t-1}[b_t]$ serves to stabilize both e_t and b_t at e_t^*. When $i^H = i^L$, e_t and b_t are already equal to e_t^* for all t. A more aggressive rule thus has nothing left to do: Committing to stabilize the bubble even more aggressively will have no additional effect.

This does not mean that raising interest rates cannot serve to dampen bubbles. To see this, consider a rule that sets i_t to a high value when $p_{t|t} > 0$ rather than when $b_t > E_{t-1}[b_t]$. In that case, the equilibrium consistent with (16), (17), and (18) would imply that $e_t < e_t^*$ when $p_{t|t} > 0$ and $e_t \geq e_t^*$ when $p_{t|t} = 0$. Since $b_t = e_t$, the value of bubble assets would be lower when the rule calls for a higher interest rate. In line with Proposition 4, the bubble can be made arbitrarily small when $p_{t|t} > 0$.

Rather than study an alternative monetary policy rule, Miao et al allow for correlated shocks to $p_{t|t}$. In particular, they consider equilibria where the price of new assets $p_{t|t}$ depends on whether new bubbles were created in period $t-1$. Suppose the value of ε in (17) depends on $p_{t-1|t-1}$, so that if $p_{t|t} > 0$, then

$$p_{t|t} = \begin{cases} \overline{\varepsilon} e_t & \text{if } p_{t-1|t-1} > 0 \\ \underline{\varepsilon} e_t & \text{if } p_{t-1|t-1} = 0 \end{cases}$$

where $\overline{\varepsilon} \geq \underline{\varepsilon}$. If $i^H = i^L$, the interest rate will be perfectly forecastable and earnings e_t will equal e_t^* for all t. If $\overline{\varepsilon} = \underline{\varepsilon} = \varepsilon^*$, the expected real return on assets purchased at date t will be $(1 - q\varepsilon^*)(1+g)$. By contrast, when $\overline{\varepsilon} > \underline{\varepsilon}$, the expected real return on assets will be $(1 - q\underline{\varepsilon})(1+g)$ if $p_{t|t} = 0$, which exceeds the expected real return $(1 - q\overline{\varepsilon})(1+g)$ if $p_{t|t} > 0$. We can use this difference to construct an equilibrium in which e_t is slightly lower when $p_{t|t} > 0$ than when $p_{t|t} = 0$ but the expected return on assets remains higher when $p_{t|t} = 0$ and $e_t > e_t^*$. If the central bank adopts a rule that responds more aggressively to $b_t - E_{t-1}[b_t]$, it will raise i_t more when $e_t > E_{t-1}[e_t]$ and lower it more when $e_t > E_{t-1}[e_t]$. This will serve to reduce $b_t = e_t$ whenever $b_t > E_{t-1}[b_t]$. However, the rule does this not by driving b_t and e_t to arbitrarily low values as in Proposition 4, but by reducing the extent to which realized earnings e_t deviate from e_t^*. In the limit as $\frac{1+i^H}{1+i^L} \to \infty$, earnings e_t equal e_t^* regardless of the realization of $p_{t|t}$. In that case, $b_t = E_{t-1}[b_t] = e_t^*$ for all t. As a result, i_t will not fluctuate over time. Miao et al's analysis can thus be viewed as illustrating the effects of a threat to raise i_t rather than of necessarily raising i_t.

Conclusion

This comment argues that the results in Galí (2014) and Miao, Shen, and Wang (2019) should not be interpreted to mean that whether policymakers can dampen bubbles by setting a higher interest rate when bubbles are large depends on the initial equilibrium the economy is at. We show that while a rule that leans more aggressively against unexpectedly large bubbles can select equilibria in which bubbles are the same or larger on impact, setting rates sufficiently above what markets expect will inevitably suppress bubbles regardless of what equilibrium we start with.

The fact that policymakers *can* rely on higher interest rates to dampen bubbles does not mean they *should*. In the model of bubbles Galí proposed that our setup borrows from, bubbles arise when the economy is dynamically inefficient and bubbles help mitigate that inefficiency. As such, there is no benefit of deflating bubbles. However, there are other settings in which deflating bubbles may be useful. For example, Biswas, Hanson, and Phan (2020) consider a related model of bubbles to the one in Galí but based on credit market frictions in which bubbles help improve resource allocation. They show that when wages are rigid, a stochastic bubble that bursts may leave agents worse off because the collapse of the bubble can lead to a prolonged recession that offsets the benefits from overcoming credit market frictions. Allen, Barlevy, and Gale (2022) show that bubbles based on information frictions may reduce welfare by encouraging socially costly speculation. Once we acknowledge that leaning against bubbles can dampen them, we need to decide on which model of bubbles is best suited for studying whether we should in fact dampen them. We leave this analysis and the broader question of the desirability of leaning against bubbles to future work.

Appendix A: Proofs

We first characterize the set of deterministic equilibria for our model when $d = 0$.

Proposition A1: The set of deterministic equilibrium asset prices $\{p_t\}_{t=0}^{\infty}$ is defined by a cutoff date $0 \leq t^* \leq \infty$ such that

$$p_t = \begin{cases} e_t & \text{if } t < t^* \\ p_{t^*} & \text{if } t > t^* \end{cases} \tag{19}$$

and p_{t^*} can assume any value in $[e_{t^*-1}, e_{t^*}]$, where $e_{-1} \equiv 0$.

Proof: Let r_t denote the rate of return that those who buy the asset at date t anticipate to earn from it in equilibrium. Since the asset yields no dividends, the return to buying the asset at date t is just the rate at which the price of the asset grows between dates t and $t + 1$, i.e.,

$$1 + r_t = \frac{p_{t+1}}{p_t} \tag{20}$$

Suppose the real interest rate r_t at date t was positive, implying $p_{t+1} > p_t$. Then the cohort born at date t would strictly prefer buying the asset to storage. Since the total amount spent on the asset is e_t, and since the supply of the asset is 1, we have

$$p_t = e_t$$

In any period t in which $r_t > 0$, then, the equilibrium price of the asset is uniquely determined.

Next, suppose $r_t = 0$. This implies $p_{t+1} = p_t$. The young are indifferent between storage and buying the asset, meaning any $s_t \in [0, e_t]$ is optimal. Since the amount agents spend on the asset is indeterminate, we cannot pin down the asset price p_t at date t. However, when $r_t = 0$, we have

$$p_{t+1} = p_t \leq e_t < e_{t+1}$$

Hence, if $r_t = 0$, the young at date $t + 1$ will not spend all of their earnings on the asset and must store some of it. This requires that $r_{t+1} = 0$ to leave the young at date $t + 1$ indifferent between storage and buying the asset. A zero real interest rate is thus an absorbing state: Once the real interest rate falls to 0, it will remain there forever and the price of the asset will remain constant from that point on.

Finally, we rule out the case where $r_t < 0$. If $r_t < 0$, the young at date t would choose storage rather than buying assets. We now argue this cannot be an equilibrium for any p_t. First, if $p_t > 0$, the old would want to sell their assets while the young would only store goods, so this cannot be an equilibrium. If $p_t = 0$, then the return r_t would be undefined unless p_{t+1} is also equal to 0, in which case $r_t = 0$, which is a contradiction. Finally, a negative price $p_t < 0$ cannot be an equilibrium, since then the young would want to buy assets given they don't have to sell them and earn $r_t < 0$, but the old would refuse to sell.

It follows equilibrium is associated with a cutoff date $0 \leq t^* \leq \infty$, before which the interest rate is positive and after which it is zero. The asset price must equal e_t before date t^* and must be constant (and below e_t) after date t^*. At the cutoff date t^*, the price can assume any value between e_{t^*-1} and e_{t^*}.

Figure 1 provides a graphical illustration of the set of deterministic equilibria. If $p_0 < e_0$, the asset price will be constant at its initial level forever. If the price starts at e_0, it will grow with the endowment for some time, but if it ever falls below the endowment, the price will remain flat from that point on. The continuum of equilibrium paths can therefore be indexed by the long-run limiting price they settle to, $\lim_{t\to\infty} p_t$. Note that one possible equilibrium is where $p_t = 0$ for all t, in which case there is no bubble. ■

Next, we prove results stated in the text that concern the case where the asset pays out dividends.

Proof of Proposition 1: We prove a more general result: The equilibrium is unique for any dividend sequence $\{d_t\}_{t=0}^{\infty}$ such that $\sum_{t=0}^{\infty} d_t = \infty$. For suppose $d_t \geq 0$ for all t and $\sum_{t=0}^{\infty} d_t = \infty$. This includes the scenario in the text, where $d_t = d > 0$, as a special case.

Let r_t^e denote the expected return on the asset at date t, which allows p_{t+1} to be random. We first rule out the possibility that $r_t^e < 0$. If $p_t = 0$ and $r_t^e < 0$, then there would have to be negative realizations of p_{t+1}. But this cannot be an equilibrium, since the asset owners would refuse to sell while young agents would want to buy. If $p_t > 0$ and $r_t^e < 0$ then the old at date t would want to sell the asset but the young would refuse to buy. So this cannot be an equilibrium either.

Next, we rule out the possibility that $r_t^e = 0$. If $r_t^e = 0$, then there must be a realization of the price at date $t+1$ in which $p_{t+1} \leq p_t - d$. In that realization, we would have $p_{t+1} < e_{t+1}$. This implies $r_{t+1}^e = 0$. By induction, there must be a path of price realizations in which the asset price declines by at least d_t. Since $\sum_{t=0}^{\infty} d_t = \infty$, there must be some date along this path in which the realized price of the asset is negative. But this is incompatible with equilibrium.

It follows that $r_t^e > 0$ at all dates. Storage is dominated, and the unique equilibrium price is $p_t = e_t$ for all t. ■

Proof of Proposition 2: Again, we prove a more general result: The equilibrium corresponds to a bubble for any path of nonnegative dividends $\{d_t\}_{t=0}^{\infty}$ where $\sum_{t=0}^{\infty} \frac{d_t}{(1+g)^t} < \infty$. To see this, recall that the equilibrium interest rate r_t in Proposition 2 implies that

$$p_t = \frac{d + p_{t+1}}{1 + r_t}$$

At the same time, the fundamental value f_t satisfies

$$f_t = \frac{d + f_{t+1}}{1 + r_t}$$

Subtracting the latter expression from the former reveals that the difference $b_t \equiv p_t - f_t$ must satisfy

$$b_t = \frac{b_{t+1}}{1 + r_t}$$

By repeated substitution, it follows that $b_0 = \left(\prod_{t=0}^{T-1} \frac{1}{1+r_t}\right) b_T$ for any $T > 0$. Hence, if $\lim_{T\to\infty} b_T > 0$, then $b_0 > 0$. ■

Appendix B: Modelling Monetary Policy

This Appendix describes a production economy with nominal price rigidities for which the endowment economy in the paper can serve as a reduced form. We first consider the case where agents can trade a single asset, available from date 0, that yields a constant dividend $d > 0$ per period. We then consider the case in which a new vintage of assets is introduced each period, all of which pay a dividend of 0.

B.1 Production, and Earnings

Our model is similar to the one in Allen, Barlevy, and Gale (2022), which in turn builds on Galí (2014) to model production and price rigidity but follows Adam (2003) in allowing for elastic labor supply. Galí assumes labor supply is inelastic, and monetary policy has real effects by redistributing consumption between young and old rather than changing total output.

As in the endowment economy in the text, agents live for two periods and care only about consumption when old, i.e.,

$$u(c_t, c_{t+1}) = c_{t+1}$$

Young agents are endowed with 1 unit of labor rather than goods. They can use their labor endowment to produce goods at home or work for another agent. They cannot work as their own employee. If a household allocates ℓ units of time to home production, they will produce $(1+g)^t h(\ell)$ consumption goods, where $h(\ell)$ is concave in ℓ, i.e., $h''(\ell) < 0$. We further assume $h'(0) = 1$ and $h'(1) = 0$.

We index agents by $i \in [0, 1]$. Each can produce a distinct intermediate good by employing the services of other workers. All intermediate goods involve the same linear production technology: If producer i hires n_{it} units of labor at time t, she can produce $x_{it} = (1+g)^t n_{it}$ units of intermediate good i.

Any agent can combine intermediate goods to form final consumption goods according to a constant elasticity of substitution (CES) production function. That is, given x_{it} of each $i \in [0, 1]$, the amount of final goods X_t that can be produced is

$$X_t = \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{21}$$

with $\sigma > 1$. Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. An agent who purchases intermediate goods to produce and sell final goods at price P_t would solve

$$\max_{x_{it}} P_t \left(\int_0^1 x_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to x_{it} yields the producer's demand for each intermediate good

$$x_{it} = X_t \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{\sigma}} \tag{22}$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Setting $X_t = 1$, this means P_t must equal $\int_0^1 P_{it} \left(\frac{P_{it}}{P_t}\right)^{-\frac{1}{\sigma}} di$, which yields the familiar CES price aggregator:

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{23}$$

Let W_t denote the wage per unit labor and $w_t = (1+g)^{-t} W_t$ denote the cost of producing one unit of an intermediate good at time t. Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (22) and production costs w_t. To allow for the possibility that producers have to set their price before knowing what the monetary authority does, we condition producer i's choice on her information set Ω_{it}. Each producer will thus set P_{it} to solve

$$\max_{P_{it}} E\left[(P_{it} - w_t) X_t \left(\frac{P_{it}}{P_t}\right)^{-1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is given by

$$P_{it} = \frac{E\left[w_t X_t | \Omega_t\right]}{(1-\sigma) E\left[X_t | \Omega_t\right]} \tag{24}$$

This price implicitly determines the amount agents will hire in their capacity as intermediate goods producers. By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$X_t = (1+g)^t \left(\int_0^1 n_t^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} = (1+g)^t n_t$$

Each agent will optimally choose to work n_t to satisfy

$$(1+g)^t h'(1-n_t) = \frac{W_t}{P_t}$$

or, alternatively, to satisfy

$$h'(1-n_t) = \frac{w_t}{P_t}$$

Each agent earns $(1+g)^t \frac{w_t}{P_t} n_t$ worth of goods in wages, $(1+g)^t \left(1 - \frac{w_t}{P_t}\right) n_t$ worth of goods in profits, and produces $(1+g)^t h(1-n)$ goods at home. Their income measured in goods is thus given by

$$e_t = (1+g)^t \left[n_t + h(1-n_t)\right]$$

Observe that real earnings e_t are increasing in n_t for $n_t \in [0,1]$, and are maximized when $h'(1-n_t) = 1$. Given our restrictions on $h(\cdot)$, this maximums occurs at $n_t = 1$.

B.2 Savings

Agents who earn e_t while they are young will want to convert it into consumption when old. As in the endowment economy, they can store goods to consume when old. But they can also use their earnings to buy assets. We therefore need to take a stand on the set of assets agents can trade. For now, we assume the only asset available for agents is the one endowed to the old at date 0 and so is available in unit supply. We further assume the asset pays a fixed dividend $d > 0$, in line with the case discussed in the text where the equilibrium is unique. Let p_t denote the real price of the asset relative to goods. Then the return to buying the asset is

$$1 + r_t = \frac{d + p_{t+1}}{p_t}$$

We further allow agents to borrow or lend money from the central bank at a nominal interest rate of $1 + i_t$ that the central bank announces at date t. To ensure that agents do not desire to borrow or lend infinite amounts to the central bank, the expected real rate paid by the central bank must be equal to the expected real rate from buying the asset, i.e.,

$$(1 + i_t) E_t \left[\frac{P_t}{P_{t+1}} \right] = E_t [1 + r_t] \tag{25}$$

When the monetary authority changes the nominal interest rate i_t, either the inflation rate $\Pi_t \equiv P_{t+1}/P_t$ or the expected return $1 + r_t^e$ or both will have to adjust. Given this indifference condition, we can assume wlog that households do not trade with the central bank, and their decisions can be reduced to choosing the amount to store s_t that maximizes

$$\max_{s_t} E_t [c_{t+1}]$$

s.t.

$$c_{t+1} = s_t + \frac{d + p_{t+1}}{p_t} (e_t - s_t)$$

Households will choose to spend all of their income to buy assets if the return on the asset exceeds the return on storage, i.e., if $\frac{d+p_{t+1}}{p_t} > 1$.

B.3 Defining Equilibrium

Given a path of nominal interest rates $\{1 + i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, w_t, p_t\}_{t=0}^{\infty}$ for goods, wages, and the one asset that agents can trade, together with a path for employment $\{n_t\}_{t=0}^{\infty}$ and a path for savings $\{s_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five conditions for these five variables:

$$\begin{array}{ll}
\text{(i) Optimal pricing:} & P_t = \dfrac{E\left[w_t X_t | \Omega_t\right]}{(1-\sigma)\, E\left[X_t | \Omega_t\right]} \\
\text{(ii) Optimal labor supply:} & h'\left(1-n_t\right) = w_t / P_t \\
\text{(iii) Optimal savings:} & s_t = \begin{cases} e_t & \text{if } \frac{d+E_t[p_{t+1}]}{p_t} > 1 \\ \in [0, e_t] & \text{if } \frac{d+E_t[p_{t+1}]}{p_t} = 1 \\ 0 & \text{if } \frac{d+E_t[p_{t+1}]}{p_t} < 1 \end{cases} \\
\text{(iv) Asset market clearing:} & p_t = e_t - s_t \\
\text{(v) Money market clearing:} & (1+i_t)\, E_t\left[\frac{P_t}{P_{t+1}}\right] = E_t\left[\frac{d+p_{t+1}}{p_t}\right]
\end{array}$$

B.4 Equilibrium with Flexible Prices

Consider the benchmark where intermediate goods producers can set their prices P_{it} after observing the wage W_t rather than before, so prices are flexible. Producers can deduce what other producers will do and the labor workers will supply, and so can perfectly anticipate total output X_t. Their information set is thus $\Omega_t = \{w_t, X_t\}$. It follows that $E\left[w_t X_t | \Omega_t\right] = w_t X_t$ and $E\left[X_t | \Omega_t\right] = X_t$. The optimal pricing rule (i) then implies

$$P_t = \frac{w_t}{1-\sigma}$$

The real wage w_t / P_t divided by productivity is thus constant and equal to $1-\sigma$. From the optimal labor supply, we can solve for employment:

$$h'\left(1-n_t\right) = 1-\sigma \tag{26}$$

Hence, $n_t = n^*$ for all t, as are real earnings $e_t = n_t + h\left(1-n_t\right)$, just as in the endowment economy in the text. Using Proposition 2, we know that in this case, optimal savings are $s_t = 0$ for all t and $p_t = e_t$ for all t. Finally, we can use (v) to pin down the inflation rate at date t, i.e.

$$\Pi_t \equiv \frac{P_{t+1}}{P_t} = \frac{(1+i_t)\, e_t}{d+e_{t+1}}$$

The initial price level P_0 is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules.

B.5 Equilibrium with Rigid Prices

We now consider the case where intermediate goods producers set prices before the monetary authority sets $1+i_t$. If monetary policy is deterministic, producers can perfectly anticipate it and the equilibrium w_t. This implies $\Omega_t = \{w_t, X_t\}$ and $w_t / P_t = 1-\sigma$ as before.

To allow for unexpected monetary policy, we assume monetary policy at date 0 depends on the realization of a sunspot variable ω_0 that is unrelated to the fundamentals of the economy and follows a binomial distribution, i.e.,

$$\omega_0 = \begin{cases} H & \text{w/prob } q \\ L & \text{w/prob } 1-q \end{cases}$$

Since there is only uncertainty at date 0, the equilibrium from date 1 on will be the same as in the flexible price economy. All we need is to solve for the equilibrium at date 0.

We use superscripts H and L to denote the value of a variable for a given realization of the sunspot. We use the convention that $i_0^H > i_0^L$, so H denotes the state in which the nominal interest rate is higher. The optimal price setting condition (i) is given by

$$P_0 = \frac{qn_0^H w_0^H + (1-q)\, n_0^L w_0^L}{(1-\sigma)\left(qn_0^H + (1-q)\, n_0^L\right)}$$

Rearranging, this implies that the weighted average of the real wage $\frac{w_0^\omega}{P_0^\omega}$ at date 0 across the two realizations must equal $1-\sigma$, i.e.

$$\alpha \frac{w_0^H}{P_0} + (1-\alpha)\frac{w_0^L}{P_0} = 1-\sigma \tag{27}$$

where $\alpha \equiv \frac{qn_0^H}{qn_0^H + (1-q)n_0^L}$. The implication is that the real wage is weakly higher than $1-\sigma$ in one state of the world and weakly lower than $1-\sigma$ in the other. The optimal labor supply condition (ii) then implies

$$h'(1-n_0^\omega) = \min\left\{\tfrac{w_0^\omega}{P_0}, 1\right\} \tag{28}$$

where we take into account that when the real wage is very high, households may be at a corner and supply all of their labor services. Employment will be weakly higher than n^* in the state where the real wage is higher than $1-\sigma$, and weakly lower than n^* in the state where the real wage is lower than $1-\sigma$. Real earnings are given by

$$e_0^\omega = n_0^\omega + h(1-n_0^\omega)$$

which is increasing in n_0^ω. Hence, in the state of the world in which the real wage is higher, real earnings will be higher. Using conditions (iii) and (iv), we can solve for the price of the asset in each state ω. In particular, if $\frac{d+e_1}{e_0^\omega} > 1$, then $p_0^\omega = e_0^\omega$. Otherwise, equilibrium requires that $\frac{d+e_1}{p_0^\omega} = 1$. Hence, the asset price p_0^ω in each state is given by

$$p_0^\omega = \min\{e_0^\omega, e_1 + d\}$$

Finally, we can use condition (v) to recover the inflation for each state ω, i.e.,

$$\Pi_0^\omega \equiv \frac{(1+i_0^\omega)\, p_0^\omega}{e_1 + d} \tag{29}$$

While all real variables at date 1 are the same regardless of ω, the same need not be true for nominal variables. The fact that the initial price level is indeterminate in the flexible price equilibrium implies P_1^ω can vary with ω.

Without any additional restrictions on how the price level P_1^ω varies with ω_0, the equilibrium would be indeterminate. For example, even though prices are rigid, if the price level varies with ω in such a way that $\frac{P_1^H}{P_1^L} = \frac{1+i_0^H}{1+i_0^L}$, monetary policy will have no effect on the real economy, meaning $n_0^H = n_0^L = n^*$. More generally, inflation can adjust to allow a higher nominal interest rate to be either contractionary, meaning $n_0^H < n_0^L$, or expansionary, meaning $n_0^H > n_0^L$.

One restriction we can impose is to assume price-setters do not set their prices as a function of things that happened in the past, either because they are irrelevant or because they cannot observe them. This implies that inflation between dates 0 and 1 does not depend on ω_0, i.e., $\Pi_0^H = \Pi_0^L$, although it does not restrict the level of inflation. From (29), we have

$$\frac{\left(1+i_0^H\right)p_0^H}{e_1+d} = \frac{\left(1+i_0^L\right)p_0^L}{e_1+d}$$

Using the equilibrium condition for p_0^ω, we can rewrite this as

$$\frac{p_0^L}{p_0^H} = \frac{\min\left\{n_0^L + h\left(1-n_0^L\right), e_1+d\right\}}{\min\left\{n_0^H + h\left(1-n_0^H\right), e_1+d\right\}} = \frac{1+i_0^H}{1+i_0^L} \tag{30}$$

Given $i_0^H > i_0^L$, it follows that $n_0^L > n_0^H$. Combining (30) with (27) and (28) yields four equations for four unknowns and allows us to solve for the unique equilibrium. Increasing the ratio $\frac{1+i_0^H}{1+i_0^L}$ leads to more variable employment and output.

Galí does not explicitly impose the restriction that $\Pi_0^H = \Pi_0^L$ in his setup. However, he does assume that the central bank uses an interest rule that is a function of past inflation and future expected inflation and that places enough weight on past inflation. As the sensitivity to past inflation goes to ∞, inflation must be the same regardless of the realization of ω. So our assumption is in the same spirit as what he does.

B.6 Equilibrium with Rigid Prices and an Expanding Set of Assets

Finally, we consider the case in which a new vintage of assets arrives at each date. As in the second part of the text, we assume all vintages are intrinsically worthless and yield no dividends. Each period is associated with its own sunspot variable that follows a binomial distribution, i.e.,

$$\omega_t = \begin{cases} 0 & \text{w/prob } q \\ p & \text{w/prob } 1-q \end{cases}$$

This variable governs whether the initial price of the date-t vintage is zero or positive: $p_{t|t} > 0$ if $\omega_t = p$ and $p_{t|t} = 0$ if $\omega_t = 0$.

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^\infty$ where $i_t = i^{\omega_t}$, an equilibrium consists of a path of prices $\{P_t, w_t\}_{t=0}^\infty$ for goods and wages, a path for asset prices $\left\{p_{s|t} : s \leq t\right\}_{t=0}^\infty$ for all assets that trade at date t, a path for employment $\{n_t\}_{t=0}^\infty$ and a path for savings $\{s_t\}_{t=0}^\infty$ such that agents behave optimally and markets clear.

In terms of an equilibrium, the optimal pricing condition (i) and optimal labor supply condition (ii) are unchanged. However, the optimal savings, asset market clearing, and money market clearing conditions must be revised to take into account that there are multiple asset vintages. In particular, the optimal savings condition (iii) and the money market clearing condition (v) involve the expected return on any particular vintage, $\frac{p_{s|t+1}}{p_{s|t}}$, while the asset market clearing condition involves spending on all assets. The revised conditions are as follows.

$$\begin{array}{ll} \text{(iii) Optimal savings:} & s_t = \begin{cases} e_t & \text{if } E_t\left[\frac{p_{s|t+1}}{p_{s|t}}\right] > 1 \\ \in [0, e_t] & \text{if } E_t\left[\frac{p_{s|t+1}}{p_{s|t}}\right] = 1 \\ 0 & \text{if } E_t\left[\frac{p_{s|t+1}}{p_{s|t}}\right] < 1 \end{cases} \\ \text{(iv) Asset market clearing:} & \sum_{s=0}^{t} p_{s|t} = e_t - s_t \\ \text{(v) Money market clearing:} & (1+i_t)\, E_t\left[\frac{P_t}{P_{t+1}}\right] = E_t\left[\frac{p_{s|t+1}}{p_{s|t}}\right] \end{array}$$

Finally, we impose that producers do not set prices based on anything that happened in the past, which implies inflation does not vary with the state, i.e.,

$$\Pi_t^0 = \Pi_t^p$$

The optimal price setting condition (i) still implies that the weighted average of the real wage $\frac{w_t}{P_t}$ at date t across the two realizations for ω_t must equal $1 - \sigma$, i.e.

$$\alpha_t \left(\frac{w_t^p}{P_t}\right) + (1 - \alpha_t)\left(\frac{w_t^0}{P_t}\right) = 1 - \sigma$$

where $\alpha_t \equiv \frac{q n_t^p}{q n_t^p + (1-q) n_t^0}$. Note that P_t does not depend on ω_t, in line with the assumption that producers set prices before ω_t is revealed. The optimal labor supply condition (ii) then implies

$$h'\left(1 - n_t^\omega\right) = \min\left\{\frac{w_t^\omega}{P_t}, 1\right\} \tag{31}$$

Real earnings are given by

$$e_t^\omega = (1+g)^t \left[n_t^\omega + h\left(1 - n_t^\omega\right)\right]$$

which is increasing in n_t^ω. Finally, define $1 + r_t^{e,\omega} \equiv E_t\left[\frac{p_{s|t+1}}{p_{s|t}}\right]$. Condition (v) pins down the implied inflation for each state ω as

$$\Pi_t^\omega \equiv \frac{1 + i^\omega}{1 + r^{e,\omega}} \tag{32}$$

Since inflation Π_t^ω does not depend on ω, we have

$$\frac{1 + r_t^{e,p}}{1 + r_t^{e,0}} = \frac{1 + i^p}{1 + i^0} \tag{33}$$

In other words, the ratio of nominal interest rates set by the monetary authority for different realizations of ω will equal the ratio of expected real interest rates for these realizations.

Define $i^H = \max\left\{i^0, i^p\right\}$ as the higher interest rate, without taking a stand on when the monetary authority sets a higher nominal interest rate. Since $i^H > i^L$, then (33) implies $r_t^{e,H} > r_t^{e,L}$. Since the real interest rate cannot be negative given the possibility of storage, it follows that $r_t^{e,H} > 0$. This implies that $b_t = \sum_{s=0}^{t} p_{s|t}$ must equal e_t when $i_t = i^H$. We now have the tools to prove Proposition 4.

Proof of Proposition 4: The equilibrium condition (33) implies that

$$\frac{1 + r_t^{e,H}}{1 + r_t^{e,L}} = \frac{1 + i^H}{1 + i^L}$$

Since storage restricts $r_t^{e,L} \geq 0$, we can set $1 + r_t^{e,H}$ to be arbitrarily large by setting $\frac{1+i^H}{1+i^L}$ to be sufficiently high. Since $r_t^{e,H} > 0$, then $b_t^H = e_t^H$ given that young agents will prefer buying assets to storage. Let $\overline{r}_t^H$ denote the maximal return agents could earn at date t when $i_t = i^H$, and let $\overline{\omega}_{t+1}$ denote the realization of ω_{t+1} at date $t+1$ that would yield this return. By construction, $\overline{r}_t^H \geq r_t^{e,H}$. Since agents at date $t+1$ must be willing to buy the assets held by the young, in the state of the world ω_{t+1} in which the realized return on assets is $\overline{r}_t^H$, we must have

$$\left(1 + \overline{r}_t^H\right) e_t^H \leq e_{t+1}^{\overline{\omega}_{t+1}}$$

which implies

$$\left(1 + r_t^{e,H}\right) e_t^H \leq e_{t+1}^{\overline{\omega}_{t+1}}$$

which we can rearrange to get

$$e_t^H \leq \frac{e_{t+1}^{\overline{\omega}_{t+1}}}{1 + r_t^{e,H}}$$

Total earnings $e_{t+1}^{\overline{\omega}_{t+1}}$ are bounded given finite labor supply. We can thus make the bound on e_t^H arbitrarily close to 0 by setting $\frac{1+i^H}{1+i^L}$ to be sufficiently high. ■

Appendix C: Results for the Galí (2014) Miao et al (2019) Setup

This Appendix confirms our results for the Galí (2014) setup. Briefly, Galí considered a related overlapping generations economy where agents live for two periods. Each cohort in his model is a mass 1 of identical agents. The utility of the cohort born at age t over consumption $C_{1,t}$ when young and $C_{2,t+1}$ when old is

$$\ln C_{1,t} + \beta \ln \left(C_{2,t+1}\right)$$

Agents are endowed with a unit of labor when young they supply inelastically. When agents turn old, they acquire the knowledge to use labor to produce differentiated intermediate goods that can be combined into a final good. Labor productivity ensures final goods output $Y_t = 1$, which is distributed as wages W_t to the young and profits $1 - W_t$ to the old. Young agents are also endowed with new intrinsically worthless assets worth U_t in goods, where U_t is random. Note it is young agents who are endowed with new bubbles, unlike our setup where it is the old. Young agents allocate their income $W_t + U_t$ to consumption and assets. Let B_t denote the value of assets inherited from before date t, so the young spend $B_t + U_t$ on assets.

The wage W_t that young agents earn must be compatible with the prices that intermediate goods producers set on their respective goods. These prices are set optimally before U_t is realized. The prices of intermediate goods determine the price of final goods given the production function for goods. Let $\Pi_t = P_t/P_{t-1}$ denote the gross inflation rate in the final goods price between dates $t-1$ and t. The central bank observes U_t, and then sets the real rate $R_t = \frac{1+i_t}{\Pi_{t+1}}$ by setting the nominal interest rate i_t. Here, we use the fact that since producers set prices date t information is revealed, then $E_t[\Pi_{t+1}] = \Pi_{t+1}$. The central bank's rule for R_t depends on how inflation Π_t compares with the central bank's inflation target Π and how the value of all bubble assets $B_t + U_t$ compares to the unconditional expectation $B + U \equiv E[B_t + U_t]$.

Given a stochastic process for U_t, an equilibrium is a path for $\{C_{1t}, C_{2t}, W_t, \Pi_t, R_t, B_t\}_{t=0}^{\infty}$ that satisfies the following six conditions at each date t:

$$C_{1,t} + C_{2,t} = 1 \tag{34}$$

$$C_{1,t} = W_t - B_t \tag{35}$$

$$\beta E_t\left[\frac{C_{1,t}}{C_{2,t+1}} R_t\right] = 1 \tag{36}$$

$$B_t + U_t = \beta E_t\left[\left(\frac{C_{1,t}}{C_{2,t+1}}\right) B_{t+1}\right] \tag{37}$$

$$E_{t-1}\left[\frac{1 - \mathcal{M}W_t}{C_{2,t}}\right] = 0 \tag{38}$$

$$R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{B_t + U_t}{B + U}\right)^{\phi_b} \tag{39}$$

where $\mathcal{M}$ is the markup firms set and depends on the elasticity of demand for each intermediate good, and R is the steady state real interest rate. To solve this system, Galí log-linearized the equilibrium conditions above in a neighborhood of a deterministic steady state. More precisely, Galí showed that for any fixed U, there exist two steady state values for B, one stable and one unstable. His approximation is near the stable steady state. For any value of ϕ_b, we can always choose small enough perturbations to ensure that the equilibrium remains within a neighborhood of the stable steady state. Galí showed that there exist equilibria near the stable steady state in which B_t will be unaffected by U_t regardless of ϕ_b while $E_t[B_{t+1}]$ is a function of U_t with a slope that increases with ϕ_b. A more aggressive rule will thus have no contemporaneous effect on the value B_t of existing assets but will lead to a larger bubble on average at date $t+1$ when $R_t > R$. We constructed equilibria with similar properties in our setup in Section 4.1.

For any value of ϕ_b, we can look for sufficiently small perturbations of U_t that ensure that $B_t + U_t$ remains within a neighborhood of $B + U$. This is what Galí did. However, to assess the effects of pursuing a more aggressive rule, we would need to know what happens for a given process U_t as we increase ϕ_b. This is a well-posed question given that U_t cancels out in the household's budget constraint (35) and so ϕ_b is irrelevant for what values that U_t can assume.

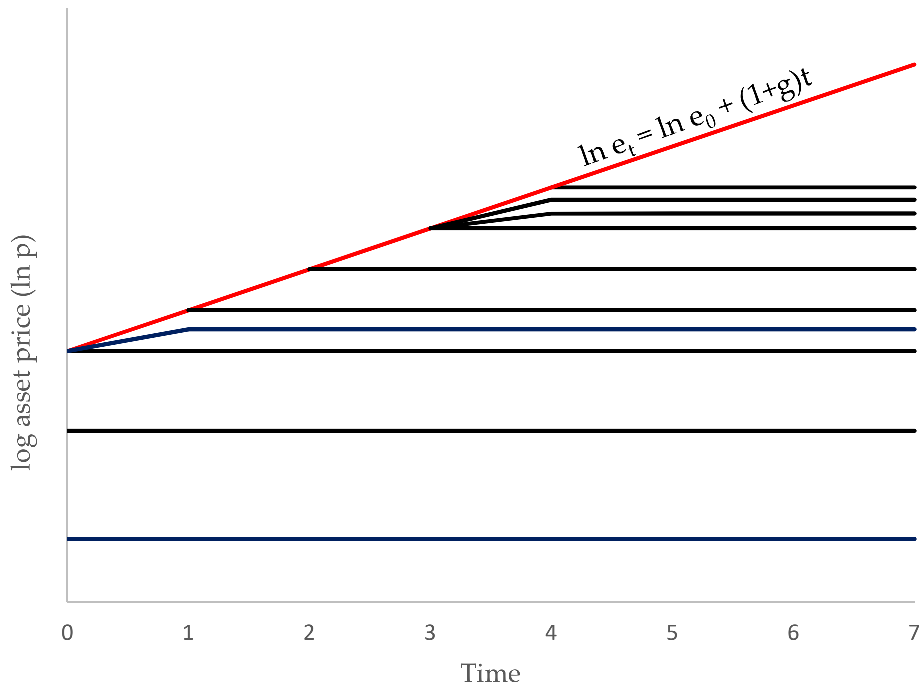
For a fixed process U_t, will there always exist an equilibrium in which B_t is unaffected by U_t regardless of ϕ_b while the sensitivity of $E_t[B_{t+1}]$ to U_t increases with ϕ_b? Suppose there was such an equilibrium. In that case, ϕ_b would have no effect on the distribution of $B_t + U_t$ at date t. Since B_t is independent of U_t, the distribution of $B_t + U_t$ must be nondegenerate: The same value of B_t will be associated with different values of U_t. This means there exist realizations of U_t for which $B_t + U_t > E[B_t + U_t]$. Increasing ϕ_b would lead to arbitrarily large values of R_t for these realizations of U_t. Since Π_t, B_t, and U_t are all known at date t, (39) implies that R_t is non-stochastic. For (36) to hold, $C_{1,t}/C_{2,t+1}$ must tend to zero for realizations of U_t that imply large values of R_t. This will be true for any realization of $C_{2,t+1}$. It follows that B_{t+1} would have to become arbitrarily large to satisfy (37) with B_t unchanged. However, $B_{t+1} \leq 1$, since young agents cannot spend on existing assets more than the total resources of the economy. For very aggressive rules, then, we cannot sustain an equilibrium in which B_t is unchanged regardless of U_t as $\phi_b \to \infty$.

What kind of equilibria are possible for a fixed process U_t as $\phi_b \to \infty$? The characteristics of an equilibrium depend on $\lim_{\phi_b \to \infty} \frac{\sup\{B_t+U_t\}}{B+U} \geq 1$. Suppose that $\lim_{\phi_b \to \infty} \frac{\sup\{B_t+U_t\}}{B+U} > 1$. In this case, as $\phi_b \to \infty$, the real interest rate R_t under (39) will become arbitrarily large for realizations of U_t for which $B_t + U_t > B + U$. The only way to satisfy (36) for these realizations is if $C_{1,t}/C_{2,t+1}$ tends to 0 in that case. Since $B_{t+1} \leq 1$ and thus bounded above, $E_t\left[\left(\frac{C_{1,t}}{C_{2,t+1}}\right) B_{t+1}\right]$ must tend to zero. From (37), this means that $B_t + U_t$ for this realization of U_t must tend to zero. Finally, since the maximal value of $B_t + U_t$ exceeds $E[B_t + U_t]$, it follows that $E[B_t + U_t] = E[B_t]$ must tend to zero. In other words, any equilibrium in which the distribution $B_t + U_t$ is nondegenerate, B_t will become arbitrarily small as ϕ_b becomes large.

However, there may be equilibria in which $\lim_{\phi_b \to \infty} \frac{\sup\{B_t+U_t\}}{B+U} = 1$. In this case, R_t will converge to a constant that is independent of ϕ_b. A more aggressive rule will then have no effect on equilibrium. This is the case Miao, Shen, and Wang (2019) consider when they log-linearize near the unstable steady state. In particular, Miao et al show that when U_t is iid, the only equilibrium that remains within a neighborhood of the unstable steady state is the one in which $B_t + U_t$ is constant for any realization of U_t. In this case, changing ϕ_b has no effect. They then consider the case where U_t is serially correlated. In that case, the ratio $\frac{B_t+U_t}{B+U}$ will not be identically equal to 1 for finite ϕ_b, but the ratio will tend to 1 as $\phi_b \to \infty$. In short, letting $\phi_b \to \infty$ will either reduce the value of existing bubbles or will stabilize the total value of all assets at some constant value regardless of the realization of U_t.

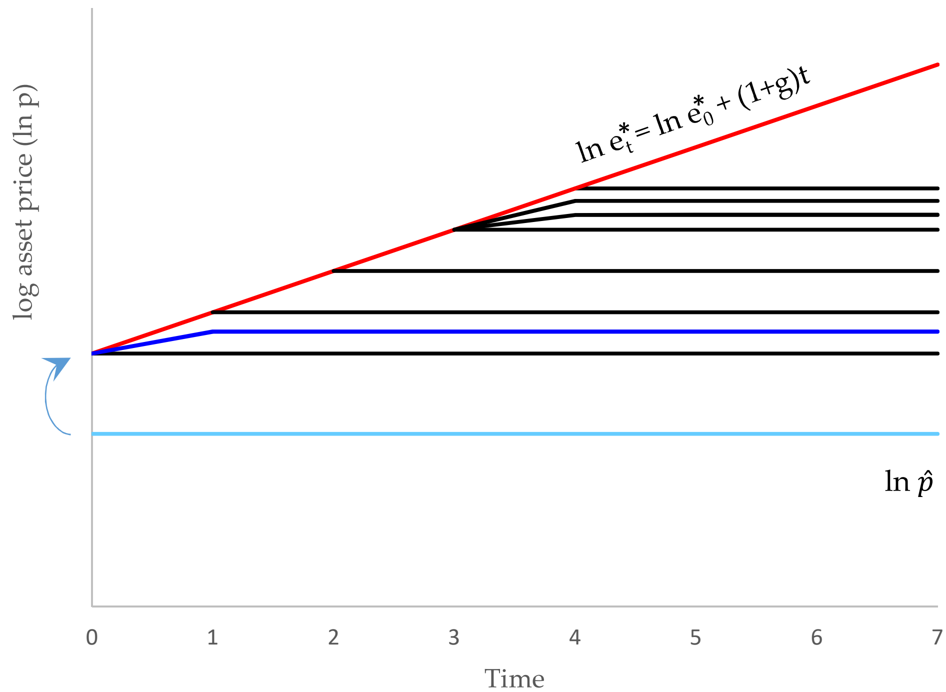
The irrelevance of monetary policy near the unstable steady state is a consequence of the particular rule that Galí and Miao et al consider. If we replaced the rule with $R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi}\left(\frac{U_t}{U}\right)^{\phi_b}$, then R_t will become arbitrarily large whenever $U_t > U$. This would imply that B_t must become arbitrarily small when $U_t > U$, as will $E[B_t] \leq B_t$, in line with our Proposition 4. Actually setting an interest rate above what markets expect, as opposed to only threatening to set such an interest rate, will depress bubbles.

Figure 1: Deterministic equilibrium price paths in endowment economy



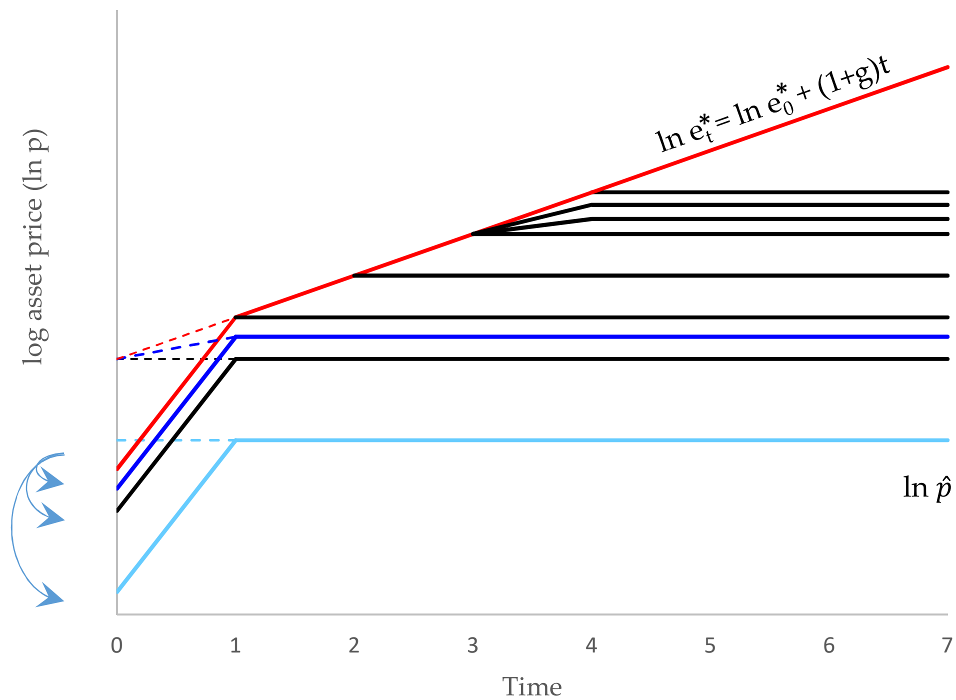
The red line corresponds to the log of the endowment $\ln e_t = \ln e_0 + (1+g)t$. Each black line represents a different equilibrium path for the price of the asset. Equilibria have the feature that if the price of the asset falls below e_t, the asset price will stop growing.

Figure 2: Effect of a surprise high nominal interest rate at date 0



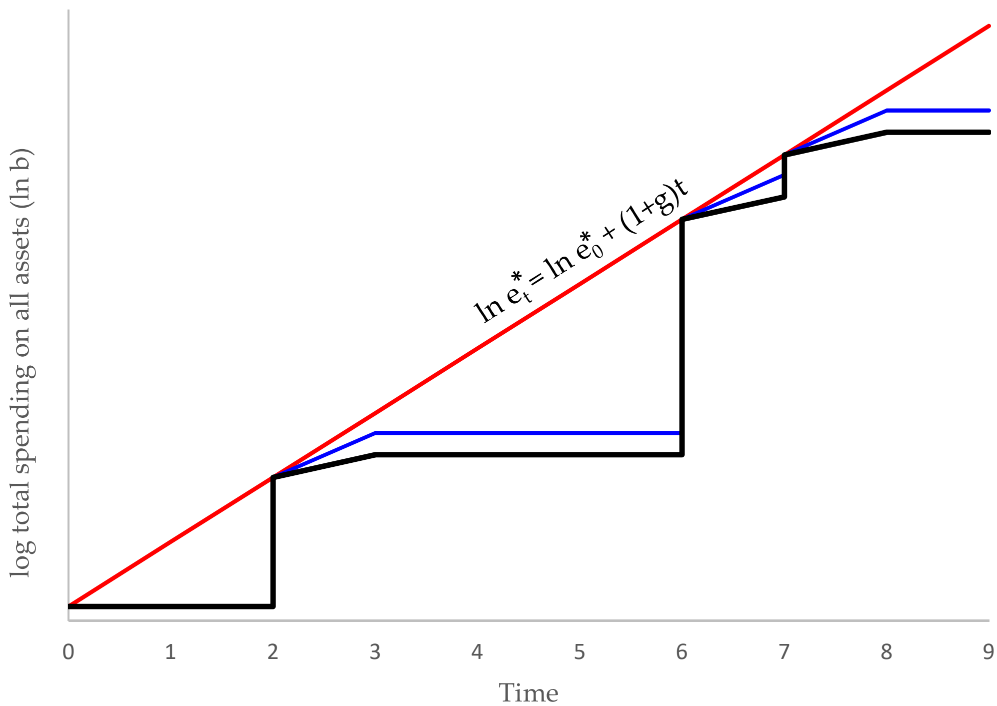
The red line corresponds to the log of earnings in the production economy if the central bank sets the nominal interest rate as expected. Each black line represents a different equilibrium path for the price of the asset from Figure 1. The light blue line, denoted $\ln \hat{p}_t$, represents the original equilibrium if the central bank set its nominal interest rate as expected. The dark blue line represents the equilibrium if the interest rate i_0 is unexpectedly high, earnings at date 0 stay unchanged at e_t^*, and the real interest rate rises but is below g. Under this equilibrium, a high i_0 results in larger bubbles in the long run.

Figure 3: Effect of a surprise high nominal interest rate at date 0 w/real rate $r_0 > g$



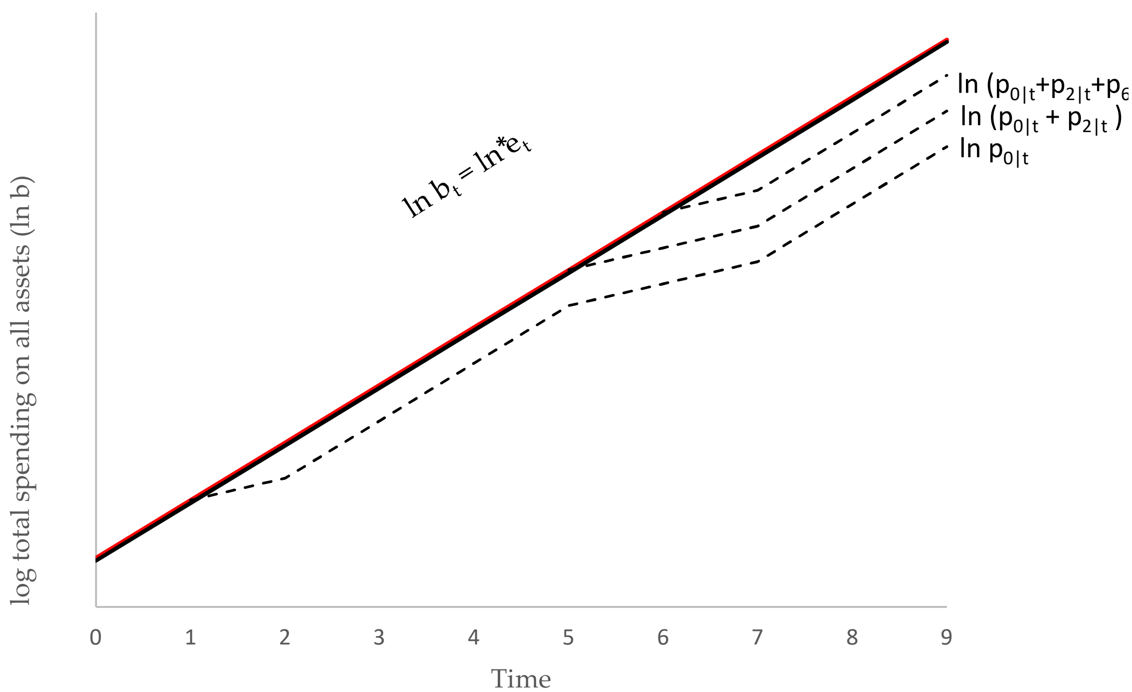
The red line corresponds to the log of earnings in the production economy if the central bank sets the nominal interest rate as expected. The dashed lines between dates 0 and 1 show paths that are equilibria for the original earnings path but not if the interest rate i_0 is unexpectedly high and $r_0 > g$. Potential equilibrium paths, including for earnings, are depicted by solid lines. The light blue line, both dashed and solid, denoted $\ln \hat{p}_t$, represents the original equilibrium if the central bank set its nominal interest rate as expected. The new equilibria can be any of the paths depicted as solid lines. All of these paths have a lower initial price p_0 than in the original equation $\hat{p}_0$.

Figure 4: Effect of more aggressive monetary policy rule in Gali (2014) model



The red line corresponds to the log of earnings in the production economy if when $i^H = i^L$ and the interest rate set by the central bank is perfectly predictable. The black line represents the realized path for the value of all assets b_t in one particular equilibrium in this case. The jumps at dates 2, 6, and 7 correspond to dates in which $p_{t|t} > 0$ and new assets are valued. These are the dates in which the real interest rate is positive and $b_t = e_t$. In all other dates, the real interest rate is positive and $b_t < e_t$. The blue line represents the path of b_t for the same shocks when $i^H > i^L$ so the central bank leans against bubbles. In this case, the real interest rate remains equal to 0 when $p_{t|t} = 0$ but is strictly higher when $p_{t|t} > 0$. The equilibrium requires that $\frac{1+i^H}{1+i^L} \leq 1 + g$. When this inequality is violated, the path of earnings can no longer remain equal to the red line.

Figure 5: Equilibria in Miao, Shan, and Wang (2019) model



The red line corresponds to the log of earnings in the production economy if when $i^H = i^L$ and the interest rate set by the central bank is perfectly predictable. The black line on top of the red line is, b_t which is equal to e_t in the equilibrium Miao et al consider. The jumps at dates 2, 6, and 7 correspond to dates in which $p_{t|t} > 0$ and new assets are valued. The dashed black lines show the values of individual asset vintages in equilibrium. When agents value new assets, they spend less on existing vintages, but their total spending is equal to e_t whether new assets are valued or not.

References

[1] Adam, Klaus, 2003. "Learning and Equilibrium Selection in a Monetary Overlapping Generations Model with Sticky Prices" Review of Economic Studies, 70(4), p887-907.

[2] Allen, Franklin, Gadi Barlevy, and Douglas Gale, 2022. "Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach" American Economic Journal: Macroeconomics, 14(2).

[3] Barlevy, Gadi, 2022. "Confessions of a Repentant Bubble Theorist" Economic Perspectives, 3, July.

[4] Biswas, Siddhartha, Andrew Hanson, and Toan Phan, 2020. "Bubbly Recessions" American Economic Journal: Macroeconomics, 12, p33-70.

[5] Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, 1999. "Monetary Policy Shocks: What Have We Learned and to What End?" Handbook of Macroeconomics, Vol 1, Part A, Elsevier, p65-148.

[6] Galí, Jordi, 2014. "Monetary Policy and Rational Asset Price Bubbles" American Economic Review, 104(3), p721–52.

[7] Gürkaynak, Refet, Brian Sack, and Eric Swanson, 2005. "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements," International Journal of Central Banking, 1, p55–93.

[8] Kuttner, Kenneth, 2001. ""Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market" Journal of Monetary Economics, 47, p523–544.

[9] Gertler, Mark and Peter Karadi, 2015. "Monetary Policy Surprises, Credit Costs, and Economic Activity," American Economic Journal: Macroeconomics, 7(1), p44–76.

[10] Miao, Jianjun, Zhouxiang Shen, and Pengfei Wang, 2019. "Monetary Policy and Rational Asset Price Bubbles: Comment" American Economic Review, 109(5), p1969-90.

[11] Nakamura, Emi and Jón Steinsson, 2018. "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect" Quarterly Journal of Economics, 133(3), p1283–1330.

[12] Rhee, Changyong, 1991. "Dynamic Inefficiency in an Economy with Land" Review of Economic Studies, 58(4), p791-7.

[13] Romer, Christina and David Romer, 2004. "A New Measure of Monetary Shocks: Derivation and Implications," American Economic Review, 94(4), p1055–1084.

[14] Sargent, Thomas and Neil Wallace, 1975. "Rational Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule" Journal of Political Economy, 83, p241–254.

[15] Tirole, Jean, 1985. "Asset Bubbles and Overlapping Generations" Econometrica, 53(6), p1499–528.