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Credit Risk and Disaster Risk

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Abstract

Credit spreads are large, volatile and countercyclical, and recent empirical work suggests that risk premia, not expected credit losses, are responsible for these features. Building on the idea that corporate debt, while safe in ordinary recessions, is exposed to economic depressions, this paper embeds a trade-off theory of capital structure into a real business cycle model with a small, exogenously time-varying risk of economic disaster. The model replicates the level, volatility and cyclicality of credit spreads, and variation in the corporate bond risk premium amplifies macroeconomic fluctuations in investment, employment and GDP.

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The widening of credit spreads during the recent financial crisis has drawn attention to their important allocative role: for many large corporations, the bond market, much more than the equity market, is the "marginal source of finance". Consistent with this view, credit spreads are significantly negatively correlated with investment (-0.60), as depicted in Figure 1.[1] This relation underscores the need for a framework linking macroeconomic aggregates and credit spreads. The corporate bond market is of interest both because of its absolute size (around 5 trillion dollars, or one-third of GDP, in the United States as of 2012) and because, while many firms do not access the corporate bond market directly and instead rely on bank loans, many of these loans are securitized and trade on a market similar to that of corporate bonds.

To be compelling, the framework must also be consistent with two facts recently emphasized in empirical finance literature. First, the "credit spread puzzle": credit spreads are larger than expected credit losses (the product of the probability of default and the expected loss conditional on default).

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[1] See Gilchrist, Yankov and Zakrajek (2009) or Mueller (2009) for more detailed examinations of the empirical power of credit spreads to forecast GDP or investment.

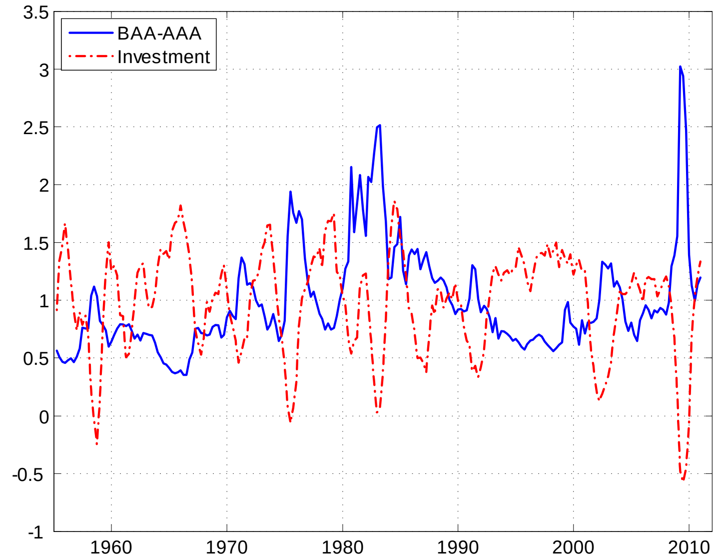


Figure 1: Credit spread (BAA-AAA) and growth rate of fixed nonresidential investment (rescaled).

For instance, an investment grade (BAA) bond defaults with probability around 0.4% per year, and the recovery upon default is around 50%, hence expected losses are about 20 basis points, but the BAA-AAA spread averages close to 100bps, or five times more.[2] Hence, corporate bonds sell at a discount relative to expected cash flows, i.e. they present the investor with an average excess return, or corporate bond risk premium, similar conceptually to the well-known equity risk premium. Second, this corporate bond premium appears to account for the bulk of the movements in credit spreads, and for the movements correlated with investment.[3]

The contribution of this paper is to propose a tractable, quantitative, macroeconomic framework that reproduces these key features of credit spreads, and to study its implications for business cycles. By their very nature, corporate bonds are safe in normal times, and suffer only from limited default during ordinary recessions, but are exposed to the risk of a very large downturn such as the Great Depression. This suggests that the corporate bond risk premium reflects compensation for bearing "tail risk", i.e. low probability events with disastrous consequences. Building on this idea, I embed a simple model of capital structure, where the choice of defaultable debt is driven by taxes and bankruptcy costs, into a real business cycle (RBC) model, and incorporate a small risk of an economic "disaster", following the work of Rietz (1988), Barro (2006), Gabaix (2012), and Gourio (2012). The risk of disaster captures the possibility of a very large recession such as the Great Depression, and is assumed to vary exogenously over time.[4]

Introducing a capital structure choice modifies the standard RBC equilibrium in two ways. First,

[2]See for instance Huang and Huang (2003). In the data, there is also a substantial spread between AAA and Treasuries, but this spread is probably driven in large part by the special liquidity of Treasuries.

[3]See Gilchrist and Zakrajsek (2012).

[4]This probability may vary over time because of time-varying rational beliefs, but an alternative "behavioral" interpretation is that it reflects time-varying pessimism ("animal spirits"). This simple modeling device captures the idea that aggregate uncertainty is sometimes high, and that some asset price changes are not obviously related to current or future productivity.

the standard Euler equation is adjusted to reflect that investment is financed using both debt and equity, and the user cost of capital hence takes into account expected discounted bankruptcy costs as well as the tax advantage of debt. Second, an additional equation determines the endogenous leverage choice, by equating the marginal expected discounted (tax) benefits and (bankruptcy) costs of debt. The model remains highly tractable and intuitive, which allows a simple quantitative evaluation of the role of defaultable debt on quantities and prices. In particular, the model nests the standard real business cycle model in the limiting case of an all-equity financed firm.

The paper generates four central results. First, as in Gourio (2012), an exogenous increase in the probability of disaster leads to a recession driven by a reduction of investment and employment: uncertainty leads agents to save less in risky capital. Second, credit spreads are large, volatile and countercyclical, consistent with the data outlined above. Third, and also consistent with the data, the level, volatility and countercyclicality in credit spreads are driven by the risk premium rather than by expected credit losses. To understand how the model replicates these features of the data, note that when the probability of economic disaster exogenously increases, there are two offsetting effects on the endogenous probability of default. First, holding constant leverage, a higher probability of disaster of course leads to a higher probability of default since the likelihood of bad outcome increases. However, higher disaster risk leads investors to cut back on leverage, which reduces the probability of default. Overall, the probability of default is roughly uncorrelated with investment. However, defaults are now expected to be more systematic, i.e. more likely to be triggered by a bad aggregate shock rather than a bad idiosyncratic shock; as a result expected *discounted* bankruptcy costs rise. This makes corporate bonds less attractive as an investment for households and increases the corporate bond risk premium and hence credit spreads, without a significant change in actual default probability.

The fourth result is that debt financing amplifies substantially – by a factor of about three – the response of the economy to an increase in the disaster probability. The higher corporate bond risk premium leads firms to use less debt and to substitute for equity – but debt is cheaper because of its tax advantage. As a result, the user cost of capital goes up by more with debt financing. Consistent with the extant literature (e.g. Cordoba and Ripoll (2004), Khan and Thomas (2011), Kocherlakota (2000)), this amplification effect does not arise if the economy is subjected to TFP shocks.

The model has several implications. First, eliminating the deductibility of interest expenses from taxable corporate income leads to a reduction in leverage and in macroeconomic volatility. Second, making debt payments contingent on disaster realizations (as has been recently suggested by several commentators) reduces volatility substantially, by eliminating the amplification effect of financial frictions.

The importance of potential "default waves" for corporate bonds has been noted before in the literature. Giesecke et al. (2011) document, using long-term U.S. data, a series of large corporate default waves, including the Great Depression. Using more recent data, Das et al. (2007) document "excess clustering" of defaults, and Duffie et al. (2009) estimate a significant probability of large default losses on portfolios of corporate bonds.

An alternative interpretation of the data is that the variation in credit spreads, in particular during the 2008 financial crisis, is driven by the balance sheets of financial institutions, such as insurance companies, which may be the "marginal investors" in these markets (Brunnermeier and Sannikov (2012) or Krishnamurthy and He (2012)). Under this interpretation, the credit spread reflects a time-varying intermediation (or liquidity) wedge rather than an aggregate risk premium. While more research is needed to disentangle the importance of each factor, the risk premium explanation is attractive *a priori* because corporate bonds are not exotic assets that have to be intermediated: any household can buy directly a low-cost, diversified portfolio of corporate bonds through a mutual fund or an ETF. Moreover, the simultaneous appearance of large spreads (low prices) in many different markets is suggestive of an aggregate risk premium.

In contrast to most of the macroeconomic literature, which focuses on small entrepreneurial firms which cannot raise equity easily and rely on bank or debt finance, the model is designed to capture the richer margins that large US corporations use to fund their assets. In my model, firms always pay dividends (unless they default), and no borrowing constraint binds. The relative attractiveness of debt and equity finance varies over time, leading to variation in the user cost of capital. This theory is attractive because it escapes the standard critique that most firms do pay dividends and are "thus" unconstrained. Nor does the model rely on a significant heterogeneity between small, productive, constrained firms on the one hand, and large, unproductive, unconstrained firms on the other hand.

Organization of the paper

The rest of the introduction discusses the related literature. Section 1 sets up the model, and Section 2 discusses the solution method and parameter choices. Section 3 presents the results, and Section 4 considers some implications and extensions of the baseline model. Section 5 concludes. An online appendix provides additional results and details the numerical method.

Related literature

The paper is related to four different branches of literature. First, the paper builds on the large macroeconomic literature studying general equilibrium business cycle models with financing constraints, as exemplified by Bernanke, Gertler and Gilchrist (1999). In that model however, there is no corporate bond risk premium: credit spreads are essentially equal to expected credit losses. Hence, model estimation that replicates credit spread variation (such as Christiano, Motto and Rostagno (2009) and Gilchrist, Ortiz and Zakrajek (2009)) imply a counterfactually high level and volatility of default probability, which makes it difficult to assess the quantitative relevance of the mechanism. Some recent studies attempt to replicate more closely the behavior of asset prices, in particular Gomes and Schmid (2010), Mendoza (2010), Miao and Wang (2010), and Liu, Wang and Zha (2010). Also related is the work of Amdur (2010), Covas and Den Haan (2009), and Hennessy and Levy (2007) who study the cyclicality of capital structure. The most closely related paper is Philippon (2009), who demonstrates how to link bond prices and real investment. While his results does not require him to make assumptions on the stochastic discount factor, my model provides a plausible general equilibrium framework where variation in risk premia feed through corporate bond spreads and investment, as Philippon emphasized.[5]

Second, the paper relates to the vast finance literature on credit risk models and the "credit spread

[5] A significant difference is that Philippon works under the Modigliani and Miller theorem, whereas bankruptcy costs play a key role in my analysis.

puzzle" (e.g. Leland (1994), Collin-Dufresne, Goldstein and Martin (2001), Collin-Dufresne and Goldstein (2001), Huang and Huang (2003), Hackbardt, Miao and Morellec (2006), Chen, Collin Dufresne and Goldstein (2009), Bhamra, Kuehn and Strebulaev (2009a, 2009b), Chen (2010)). As discussed in the introduction, this literature documents that it is difficult to reconcile credit spreads with observed credit losses. Perhaps surprisingly, there is, to my knowledge, no study measuring the contribution of disaster risk to the credit spread puzzle.[6] Moreover, this literature has exogenous cash flows, no investment, and is not set in general equilibrium, making it difficult to evaluate the macroeconomic impact of the financial frictions. On the other hand, these papers consider richer cash flow dynamics and long-term debt.

Third, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Gabaix (2012), Wachter (2012)). In particular, the model is a direct, but significant, extension of Gourio (2012), who studied a frictionless real business cycle model with time-varying disaster risk.

Fourth, the paper considers the real effects of a particular shock to uncertainty – a change in the probability of disaster. The negative effect of uncertainty on output has been studied most recently by Bloom (2009), who emphasizes the "wait-and-see" effect driven by lumpy hiring and investment behavior. My model focuses on changes in *aggregate* uncertainty (as in Fernandez-Villaverde et al. (2011)), and the mechanism is different: higher uncertainty lowers desired investment by increasing the risk premium on capital and by exacerbating financial frictions. A related mechanism has recently been explored in the studies of Arellano, Bai and Kehoe (2010), Chugh (2010), and Gilchrist, Sim and Zakrajek (2010); I discuss these in more detail in section 4.

1 Model

This section presents the firm and household problems and defines the equilibrium.

1.1 Firms

I first give an overview of the structure of the firm problem, then I describe the mathematical formulation.

1.1.1 Summary

There is a continuum of perfectly competitive firms, which are all identical ex-ante and differ ex-post only in their realization of an idiosyncratic shock. For expositional simplicity, firms are assumed to live only for two periods. (I discuss at the end of this section how to relax this.) At the end of period t, new firms are born and purchase capital K_{t+1}^{w} in a competitive market, for use in period $t+1$. This investment is financed by issuing equity S_t and debt B_{t+1} claims. In period $t+1$, the aggregate shocks and the idiosyncratic shock are revealed, firms decide on employment and production, and then sell back their capital to a new generation of firms. Two cases arise at this point: (1) the firm value V_{t+1} is greater than outstanding debt B_{t+1}: the debt is then repaid in full and the residual value goes to equityholders as dividends; or (2) the firm value is less than outstanding debt: in this case the absolute

[6] See however the work in progress of Bhamra and Strebulaev (2011).

priority rule applies: equityholders are "wiped out", and bondholders capture the firm's value, net of some bankruptcy costs. In all cases, these firms disappear after production in period $t+1$ and new firms are created, which will raise funds and invest in period $t+1$, and operate in period $t+2$.

Since firms are ex-ante identical, they all make the same choices. Because both production and financing technologies exhibit constant return to scales, the size distribution of firms is indeterminate, and has no effect on aggregate outcomes.

1.1.2 Production

All firms have the same productivity and operate the same constant returns to scale Cobb-Douglas production function using capital and labor:

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha},$$

where z_t is aggregate total factor productivity (TFP), and K_{it},N_{it} and Y_{it} are the individual firm capital stock, employment and output. Both input and output markets are competitive and frictionless.

1.1.3 Productivity shocks

To model the possibility of large recessions, I assume that the aggregate TFP process in this economy is driven not only by the usual "small" normally distributed shocks standard in RBC theory, but also by rare large shocks, which I call "disasters". Formally,

$$\log z_{t+1} = \log z_t + \mu + \sigma_e e_{t+1} + x_{t+1} b_{z,t+1},$$

where $\{e_{t+1}\}$ is *i.i.d.* $N(0,1)$; x_{t+1} is an indicator equal to 1 if a disaster happens, and 0 otherwise; and $b_{z,t+1}$ is a random variable that defines the size of the disaster, with $b_{z,t+1}$ *i.i.d.* $N\left(\overline{b_z} - \sigma_z^2/2, \sigma_z^2\right)$.

Hence, a disaster realization affects total factor productivity permanently by a level factor. The realization of disaster also simultaneously affects the capital stock, as explained in the next paragraph. The probability that a disaster occurs at time $t+1$ is denoted $p_t = \Pr(x_{t+1} = 1)$, and this probability itself follows a Markov chain with transition matrix Q.

1.1.4 Depreciation shocks

Firms purchase capital at time t, but the actual quantity of capital that they will have to operate at time $t+1$ is random, and is affected both by realizations of aggregate disasters x_{t+1} as well as an idiosyncratic shock ε_{it+1}. Specifically, if a firm i purchases K_{it+1}^{w} units of capital at time t (where w stands for wish), it actually has $K_{it+1} = K_{it+1}^{w} e^{x_{t+1} b_{k,t+1}} \varepsilon_{it+1}$ to operate in period $t+1$, and $(1-\delta)K_{it+1}$ units of capital to resell. The shock $b_{k,t+1}$ is *i.i.d.* $N\left(\overline{b_k} - \sigma_k^2/2, \sigma_k^2\right)$. The idiosyncratic shock ε_{it+1} is *i.i.d.* across firms and across time, and drawn from a cumulative distribution function H, with mean unity. The idiosyncratic shock's sole purpose is to generate a smooth distribution of firm value, so that some firms default and some don't. Hence, technically, there are five aggregate shocks $\{e_{t+1}, x_{t+1}, p_{t+1}, b_{z,t+1}, b_{k,t+1}\}$, which are are assumed to be independent, conditional on p_t.[7]

[7] By definition, the distribution of both x_{t+1} and p_{t+1} depends on p_t; but the realization of x_{t+1} and p_{t+1}, given p_t are independent.

1.1.5 Discussion of the assumptions regarding disasters

The quantitative relevance of rare events is demonstrated by Barro (2006) and Barro and Ursua (2008). These authors construct a long country panel dataset and identify numerous large negative macroeconomic shocks, which are usually caused by wars or economic depressions. In a standard neoclassical model there are two simple ways to model these macroeconomic disasters – as destruction of the capital stock, or as a reduction in total factor productivity.

TFP appears to play an important role during economic depressions (Kehoe and Prescott (2007)). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies, such as expropriation, confiscatory taxes, or trade policies. They may also be caused by disruptions in financial intermediation, if these lead to inefficient capital allocation.

Capital destruction is clearly realistic for wars or natural disasters, but not for economic depressions. A broader interpretation is that it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions. This "capital quality" shock interpretation is also used in recent work on financial frictions (e.g. Gertler and Karadi (2011)).

My formulation introduces both features simultaneously, because the two are needed to generate realistic implications, as explained in Section 4. At heart, the model mechanism requires two ingredients: (1) that disasters are clearly bad events, with high marginal utility of consumption; (2) that the realized return on capital is low during disasters. These assumptions are certainly realistic. Introducing a large TFP shock is the simplest way to obtain (1) in a neoclassical model, and introducing a depreciation shock is the simplest way to obtain (2). An alternative to depreciation shocks is to introduce steep adjustment costs: since investment falls significantly during disasters, the price of capital would also fall, generating endogenously a low return on capital during disasters. I do not pursue this strategy in the paper as it is difficult to calibrate adjustment costs to generate this effect while maintaining realistic business cycle dynamics.

For parsimony and tractability, these rare disasters are modeled here as instantaneous, permanent jumps; Gourio (2012) shows that the key results are largely unaffected if disasters are modeled as smaller shocks that are persistent, and are followed by recoveries, provided that risk aversion is increased somewhat.

1.1.6 Capital structure choice

Capital structure is driven by expected default costs and by the tax advantage of debt. Bondholders recover a fraction θ of the firm value upon default, where $0 < \theta < 1$. On the other hand, a firm which issues debt at a price q receives χq, where $\chi > 1$; i.e. for each dollar that the firm raises in the bond market, the government gives a subsidy $\chi - 1$ dollar. For simplicity, I assume that the subsidy takes place at issuance.[8] An alternative interpretation of χ is that it is a reduced form for the various advantages that debt has over equity; for instance the corporate finance literature emphasizes that debt

[8] In reality, interest payments are deductible from taxable corporate income, hence the implicit subsidy takes place when firms' earnings are taxed.

disciplines managers, or that it is more efficient when information is asymmetric between firm insiders and outsiders (see Tirole (2005), chapters 5 and 6).

The bond price q is determined at time of issuance, taking into account default risk, and hence depends on the firm's choice of debt and capital as well as the economy's state variables. Equity issuance is assumed to be costless. This assumption is natural given that the largest source of equity is retained earnings.[9]

When $\chi = \theta = 1$, the capital structure is indeterminate and the Modigliani-Miller theorem holds. When $\chi = 1$, the firm finances only through equity, since debt has no advantage. As a result, there is no default, and the model degenerates to the the RBC model with disaster risk studied in Gourio (2012). When $\theta = 1$, or more generally $\theta\chi \geq 1$, the firm finances only through debt, since default is not costly. I will hence assume $\chi\theta < 1$, a necessary assumption to generate an interior choice for the capital structure.

1.1.7 Employment, Output, Profits, and Firm Value

To solve the optimal financing choice, we first need to determine the profits and the firm value. (The probability distribution of firm value determines the likelihood of default and hence the lending terms the firm can obtain ex-ante.) The labor choice is determined through static profit maximization, given the realized values of productivity, the capital stock, and the aggregate wage W_t:

$$\pi\left(K_{it}, z_t; W_t\right) = \max_{N_{it} \geq 0} \left\{ K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} - W_t N_{it} \right\},$$

which leads to the labor demand

$$N_{it} = K_{it} \left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t} \right)^{\frac{1}{\alpha}}, \tag{1}$$

and the output supply

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha} = K_{it} \left(\frac{z_t(1-\alpha)}{W_t} \right)^{\frac{1-\alpha}{\alpha}}.$$

These equations can then be aggregated. Define aggregate capital, output and employment as $K_t = \int_0^1 K_{it} di$, $Y_t = \int_0^1 Y_{it} di$, $N_t = \int_0^1 N_{it} di$, we obtain that $Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}$, i.e. an aggregate production function exists, and it has exactly the same shape as the microeconomic production function. The law of motion for capital is obtained by summing over i the equation $K_{it+1} = K_{it+1}^{w} e^{x_{t+1} b_{k,t+1}} \varepsilon_{it+1}$. Since all firms are identical ex-ante, and they will make the same investment choice $K_{it+1}^{w} = K_{t+1}^{w}$, and since ε_{it+1} has mean unity, idiosyncratic shocks average out, leading to

$$K_{t+1} = K_{t+1}^{w} e^{x_{t+1} b_{k,t+1}}.$$

Profits equal

$$\pi_{it+1} = Y_{it+1} - W_{t+1} N_{it+1} = \alpha Y_{it+1} = \alpha K_{it+1} \left(\frac{z_{t+1}(1-\alpha)}{W_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} = K_{it+1} \alpha \frac{Y_{t+1}}{K_{t+1}},$$

i.e. each firm receives factor payments for its capital, proportionally to the (idiosyncratic) quantity of capital it has, and to the aggregate marginal product of capital $\alpha \frac{Y_{t+1}}{K_{t+1}}$. The total firm value at the end

[9] It is easy to incorporate additional equity costs through a reduced form cost function, as in Gomes (2001).

of the period is the sum of profits and the proceeds from the sale of undepreciated capital:

$$V_{it+1} = \pi_{it+1} + (1-\delta)K_{it+1} = K_{it+1}\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right). \tag{2}$$

An alternative expression for firm value can be obtained by defining the return on capital. Let $R_{t+1}^K = e^{x_{t+1}b_{k,t+1}}\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)$; this is the familiar expression for the unlevered physical return on capital, adjusted to reflect the possibility of disasters and ensuing capital destruction. The individual return on capital is also affected by the idiosyncratic shock ε_{it+1}; hence define $R_{it+1}^K = \varepsilon_{it+1}R_{t+1}^K$. The firm value is thus the quantity of capital invested multiplied by the idiosyncratic return on capital:

$$V_{it+1} = R_{it+1}^K K_{t+1}^w = \varepsilon_{it+1} R_{t+1}^K K_{t+1}^w.$$

1.1.8 Investment and Financing Decisions

To find the optimal choice of investment and financing, we first need to find the likelihood of default, and the loss-upon-default, for any possible choice of investment and financing. This determines the price of corporate debt. Taking as given this bond price schedule, the firm can then decide on optimal investment and financing.

The firm defaults if its realized value V_{it+1}, is too low to repay the debt it is due to repay B_{t+1}. This occurs if the firm's idiosyncratic shock ε is less than a cutoff value, which itself depends on the state of the economy (i.e., on the aggregate state variables). Mathematically, default occurs at time $t+1$ if and only if

$$\varepsilon_{it+1} < \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w} \stackrel{def}{=} \varepsilon_{t+1}^*.$$

If a disaster is realized ($x_{t+1} = 1$), the realized aggregate return on capital R_{t+1}^K is lower, the default threshold ε_{t+1}^* is higher, and more firms default. (Note that ε_{t+1}^* is closely related to leverage, defined as $L_{t+1} = B_{t+1}/K_{t+1}^w$, and which is decided at time t.)

Given this default rule, the bond issue is priced ex-ante using the representative agent's stochastic discount factor M_{t+1} :

$$q_t = E_t\left(M_{t+1}\left(\mathbb{I}_{B_{t+1}\le V_{it+1}} + \mathbb{I}_{V_{it+1}<B_{t+1}}\theta\frac{V_{it+1}}{B_{t+1}}\right)\right),$$

where $\mathbb{I}$ is an indicator function, the first term captures the non-default states, where the bond payoff is one, and the second term the default states, where the bond payoff is the recovery parameter θ times the firm value, divided among all the bondholders. This can be explicitly written as:

$$q_t = E_t\left(M_{t+1}\left(\int_{\varepsilon_{t+1}^*}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_0^{\varepsilon_{t+1}^*} \varepsilon R_{t+1}^K K_{t+1}^w dH(\varepsilon)\right)\right). \tag{3}$$

Note that the threshold between default and non-default, ε_{t+1}^*, depends on aggregate realizations, notably disasters, and so does the return on capital R_{t+1}^K and the discount factor M_{t+1}.

We can finally set up the firm decision problem. It chooses at time t how much to invest and how much debt and equity to issue, so as to maximize the expected discounted net equity value:

$$\max_{B_{t+1},K_{t+1}^w,S_t} E_t\left(M_{t+1}\max\left(V_{it+1}-B_{t+1},0\right)\right) - S_t, \tag{4}$$

subject to the funding constraint:

$$\chi q_t B_{t+1} + S_t = K^w_{t+1}, \tag{5}$$

and the definition of the equity value $V_{it+1} = \varepsilon_{it+1} R^K_{t+1} K^w_{t+1}$, as well as the bond price (3). The objective function (4) takes into account the option of default for equityholders. Note that, given constant return to scale, this net equity value will equal zero in equilibrium, reflecting free entry.

Substituting equation (5) into the objective (4) shows that this problem is equivalent to maximizing the following total firm value expression:

$$\begin{aligned}
& E_t \left(M_{t+1} \max \left(V_{it+1} - B_{t+1}, 0 \right) \right) + \chi q_t B_{t+1} - K^w_{t+1} \\
= \; & E_t \left(M_{t+1} \mathbb{I}_{B_{t+1} \leq V_{it+1}} \left(V_{it+1} - B_{t+1} \right) \right) \\
& + \chi E_t \left(M_{t+1} \left(\mathbb{I}_{B_{t+1} \leq V_{it+1}} + \mathbb{I}_{V_{it+1} < B_{t+1}} \theta \frac{V_{it+1}}{B_{t+1}} \right) B_{t+1} \right) - K^w_{t+1}, \\
= \; & E_t \left(M_{t+1} R^K_{t+1} K^w_{t+1} \right) + \left(\chi - 1 \right) E_t \left(M_{t+1} B_{t+1} \left(1 - H \left(\varepsilon^*_{t+1} \right) \right) \varepsilon^*_{t+1} \right) \\
& - \left(1 - \theta \chi \right) E_t \left(M_{t+1} R^K_{t+1} K^w_{t+1} \Omega \left(\varepsilon^*_{t+1} \right) \right) - K^w_{t+1},
\end{aligned}$$

where $\Omega(x) = \int_0^x s dH(s)$. This firm value is the sum of (i) expected discounted value of capital, $E_t \left(M_{t+1} R^K_{t+1} K^w_{t+1} \right)$, and (ii) the tax savings of debt, net of (iii) expected discounted bankruptcy costs (since $\theta\chi < 1$ by assumption) and (iv) the investment cost K^w_{t+1}. By contrast, in a frictionless model ($\chi = \theta = 1$), the firm would simply maximize $E_t \left(M_{t+1} R^K_{t+1} K^w_{t+1} \right) - K^w_{t+1}$. While bankruptcy costs are borne by debt holders ex-post, expected bankruptcy costs are passed on into debt prices ex-ante, implying that equity holders actually bear the costs of default.

To solve this problem, we can simply take first order conditions with respect to K^w_{t+1} and B_{t+1}, taking into account that $\varepsilon^*_{t+1} = B_{t+1} / \left(R^K_{t+1} K^w_{t+1} \right)$. The first-order condition with respect to K^w_{t+1} yields a modified investment Euler equation,

$$E_t \left(M_{t+1} R^K_{t+1} \lambda_{t+1} \right) = 1, \tag{6}$$

where

$$\lambda_{t+1} = 1 + \left(\chi - 1 \right) \varepsilon^*_{t+1} \left(1 - H \left(\varepsilon^*_{t+1} \right) \right) - \left(1 - \theta \chi \right) \Omega(\varepsilon^*_{t+1}). \tag{7}$$

In a model without financial frictions, the standard Euler equation is $E_t \left(M_{t+1} R^K_{t+1} \right) = 1$; here, equation (6) is modified to take into account the tax shield (the second term), which reduces the user cost of capital if there is no default, and expected discounted bankruptcy costs (the third term) which increase it if there is default. In the case $\chi = \theta = 1$, we obtain the standard equation, which corresponds to an unlevered firm (i.e. $\lambda_{t+1} = 1$).

Note that firms always have the possibility to rely solely on equity, i.e. set $B_{t+1} = 0$ and consequently never default ($\varepsilon^*_{t+1} = 0$). This implies that firms always have access to cheaper financing than in the frictionless (all-equity financed) model. As a result, the steady-state capital stock is always higher when $\chi > 1$ than in the frictionless version. The model hence features "overaccumulation" of capital, in contrast to many financial friction models (e.g. Bernanke, Gertler and Gilchrist (1999)) where capital is lower than the first best.

The first order condition with B_{t+1} is

$$(1-\theta)\, E_t\left(M_{t+1}\varepsilon_{t+1}^{*} h\left(\varepsilon_{t+1}^{*}\right)\right) = \frac{\chi-1}{\chi} E_t\left(M_{t+1}\left(1-H\left(\varepsilon_{t+1}^{*}\right)\right)\right). \tag{8}$$

This equation determines the optimal financing choice between debt and equity.[10] The left-hand side is the marginal cost of debt: an additional dollar of debt will increase the likelihood of default, and the associated bankruptcy costs. The right-hand side is the marginal benefit of debt, i.e. the higher tax shield in non-default states. Importantly, both the marginal cost and the marginal benefit are discounted using the stochastic discount factor M_{t+1}. The importance of this risk-adjustment is consistent with the empirical work by Almeida and Philippon (2007), who note that corporate defaults are more frequent in "bad times" and as a result the ex-ante marginal cost of debt is higher than a risk-neutral calculation would suggest. This risk-adjustment will play a substantial role in the analysis below: for a given debt level, an increase in the probability of disaster increases expected *discounted* default costs, not because defaults become more likely, but also they are more likely to occur during disasters, which are times of high marginal utility.

Reinterpretation of firms as infinitely lived

I have presented the firms as living two periods, but we could allow them to continue operating beyond the second period. In this case, the problem reads

$$\begin{aligned} V(K_{it}, s_t) &= \max\left\{0, \varepsilon_{it} R_t^K K_t^w - B_t + W_t\right\}, \\ W_t &= \max_{K_{t+1}^w, B_{t+1}} \left\{\chi q_t B_{t+1} - K_{t+1}^w + E_t\left(M_{t+1} V(K_{it+1}, s_{t+1})\right\}, \end{aligned}$$

where s_t denotes the aggregate states. But it is clear that this problem has the exact same solution as the problem of the two-period firms, since $W_t = 0$. The net present value of operating in the future is zero, given constant return to scale, i.i.d. idiosyncratic shocks, and no physical adjustment costs: there are no net future profits to staying in the industry, so firms are indifferent between staying and exiting. As a result, the default and investment decisions are unaffected by the two-period assumption. Obviously, the assumptions that make the two-period and infinite horizon problem equivalent are unrealistic, and future work should relax them.

1.2 Household

The representative household has preferences over consumption and leisure, following Epstein and Zin (1989):

$$U_t = \left((1-\beta)(C_t^{\upsilon}(1-N_t)^{1-\upsilon})^{1-\psi} + \beta E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}. \tag{9}$$

Here ψ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and γ measures risk aversion towards static gambles over the bundle. When $\psi = \gamma$, the model collapses to expected utility. While the additional flexibility of Epstein-Zin utility is useful in calibrating the model, the key qualitative results can be obtained with standard CRRA preferences (See section 4).

[10] As in Bernanke, Gertler and Gilchrist (1999), a second order condition is required to ensure that this FONC is sufficient. Some regularity condition must be imposed on the distribution H, e.g. the function $z \to \frac{zh(z)}{1-H(z)}$ is increasing. Most distributions (such as the log-normal distribution used below) satisfy this assumption.

The household supplies labor in a competitive market, and trades stocks and bonds issued by the corporate sector.[11] The budget constraint reads

$$C_t + n_t^s P_t + q_t B_t \leq W_t N_t + \varrho_t B_{t-1} + n_{t-1}^s D_t - T_t, \tag{10}$$

where B_{t-1} is the aggregate quantity of debt issued by the corporate sector in period $t-1$ at price q_{t-1}, each unit of which is redeemed in period t for ϱ_t, n_t^s is the quantity of equity shares, P_t is the price of equity, D_t is the payoff to equityholders (there is no capital gains term $n_{t-1}^s (D_t + P_t)$ since firms live only two periods), and T_t is a lump-sum tax. The number of equity shares n_t^s is normalized to one. In the absence of default, $\varrho_t = 1$, but $\varrho_t < 1$ if some bonds are not repaid in full. The household takes the process of ϱ_t as given, but it is determined in equilibrium by firms' default decisions as shown in the previous section, so that $\varrho_t = 1 - H(\varepsilon_t^*) + \Omega(\varepsilon_t^*)\theta R_t^K K_t^w / B_t$.

Utility maximization yields the familiar labor supply condition:

$$W_t = \frac{(1-\upsilon)C_t}{\upsilon(1-N_t)}. \tag{11}$$

Intertemporal choices are determined by the stochastic discount factor (a.k.a. marginal rate of substitution), which prices all assets:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\psi)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\psi)} \frac{U_{t+1}^{\psi-\gamma}}{E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{\psi-\gamma}{1-\gamma}}}, \tag{12}$$

and the Euler equations

$$\begin{aligned} E_t\left(M_{t+1} R_{t+1}^c\right) &= 1, \\ E_t\left(M_{t+1} R_{t+1}^e\right) &= 1, \end{aligned}$$

where $R_{t+1}^c = \frac{\rho_{t+1}}{q_t}$ is the return on corporate bonds, and $R_{t+1}^e = \frac{D_{t+1}}{P_t}$ is the return on equity.[12]

1.3 Equilibrium

The equilibrium definition is standard: first, firm and household optimization imply (6) and (8) with the stochastic discount factor given by (9) and (12); next the labor market clears:

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon(1-N_t)}. \tag{13}$$

Last, the goods market clears, i.e. $C_t + I_t = Y_t$. This last equation reflects an assumption that defaults cost $(1-\theta)\Omega(\varepsilon_t^*)R_{t+1}^K K_{t+1}^w$ are transfers rather than real resources costs.[13] If they are real resource costs, default realizations induce a negative wealth effect. The assumption is fairly innocuous because

[11] It is possible to introduce government bonds as well. If the government finances this debt using lump-sum taxes and transfers, Ricardian equivalence holds, and government debt policy does not affect the equilibrium allocation and prices.

[12] Technically, the Euler equations hold for any individual firm's equity return or bond return. Given that firms are ex-ante identical, they are written here for the aggregate equity and bond returns (i.e. the return on a diversified portfolio of stocks or bonds).

[13] For instance, the default costs may be transferred to the government, which then rebates it to household using lump-sum transfers (T_t in equation 10).

these wealth effects are fairly small, and it helps clarify that it is not default realizations, but ex-ante default risk, that matter for the results.

Overall, equations (6) and (8) are the only departures of our model from the standard real business cycle model: first, the Euler equation needs to be adjusted to reflect the tax shield and bankruptcy costs; second, the optimal leverage is determined by the trade-off between costs and benefits of debt finance.

2 Numerical solution and calibration

This section discusses the solution method and the parameter values.

2.1 Recursive representation and solution method

It is useful, both for conceptual clarity and to implement a numerical algorithm, to consider a recursive formulation of the equilibrium. First, note that the equilibrium can be entirely characterized from time t onwards given the values of the realized aggregate capital stock K_t, the probability of disaster p_t, and the level of total factor productivity z_t, i.e. these are the three state variables.[14] Second, examination of the first-order conditions shows that they can be rewritten solely as a function of the detrended capital $k_t = K_t/z_t$ and p_t. This is a standard simplification in the stochastic growth model when technology follows a unit root. As a result the equilibrium policy functions (consumption, investment, employment, leverage and the household value, which matters for the stochastic discount factor) can be expressed as functions of two state variables only, k and p.

Mathematically, define detrended output, consumption, investment, etc. as $y = Y/z$, $c = C/z$, $i = I/z$, etc. and detrended utility as $u = U^{1-\psi}/z^{\upsilon(1-\psi)}$. A recursive equilibrium is a list of functions, $y(k,p)$, $N(k,p)$, $c(k,p)$, $i(k,p)$, $L(k,p)$, $u(k,p)$, such that $y(k,p) = k^{\alpha} N(k,p)^{1-\alpha}$, $c(k,p) + i(k,p) = y(k,p)$, and

$$(1-\alpha)\frac{y(k,p)}{N(k,p)} = \frac{(1-\upsilon)\,c(k,p)}{\upsilon\,(1-N(k,p))},$$

$$k' = k'(k,p,\omega') = \frac{e^{x' b'_k}\left((1-\delta)\,k + i(k,p)\right)}{e^{x' b'_z + \mu + \sigma_e e'}},$$

where we denote $\omega' = (x', e', b'_k, b'_z, p')$ the aggregate shock realizations; the return on capital is $R^K(k,p,\omega') = e^{x' b'_k}\left(1-\delta+\alpha\frac{y(k',p')}{k'}\right)$, and

$$E_{\omega'|p}\left(M\left(k,p,\omega'\right) R^K(k,p,\omega') \left(\begin{array}{c} 1+(\chi\theta-1)\,\Omega\left(\varepsilon^*(k,p,\omega')\right) \\ +(\chi-1)\,\varepsilon^*(k,p,\omega')\left(1-H\left(\varepsilon^*(k,p,\omega')\right)\right)\end{array}\right)\right) = 1,$$

$$E_{\omega'|p}\left(M\left(k,p,\omega'\right)\left(\begin{array}{c} \chi\,(\theta-1)\,\varepsilon^*(k,p,\omega')h\left(\varepsilon^*(k,p,\omega')\right) \\ +(\chi-1)\left(1-H\left(\varepsilon^*(k,p,\omega')\right)\right)\end{array}\right)\right) = 0,$$

[14] Perhaps surprisingly, the level of outstanding debt B_t at the beginning of period is not a state variable. This is because of two assumptions: (1) defaults take place after production, (2) and bankruptcy costs are a tax (i.e. not in the resource constraint). Hence, the outstanding debt, and the number of firms in default today, does not affect the equilibrium today or in the future.

where $\varepsilon^*(k,p,\omega') = \frac{L(k,p)}{R^K(k,p,\omega')}$ is the default threshold, and the stochastic discount factor is given by

$$\begin{aligned} M(k,p,\omega') &= \beta e^{(\upsilon(1-\gamma)-1)\left(\mu+\sigma_e e'+x'b_z'\right)} \\ &\times \left(\frac{c(k',p')}{c(k,p)}\right)^{\upsilon(1-\psi)-1}\left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\psi)} \\ &\times \frac{u(k',p')^{\frac{\psi-\gamma}{1-\psi}}}{E_{\omega'|p}\left(e^{\upsilon(1-\gamma)(\mu+\sigma_e e'+x'b_z')}u(k',p')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{\psi-\gamma}{1-\gamma}}}, \end{aligned}$$

and utility is defined through

$$\begin{aligned} u(k,p) &= (1-\beta)c(k,p)^{\upsilon(1-\psi)}(1-N(k,p))^{(1-\upsilon)(1-\psi)} \\ &+\beta e^{\mu\upsilon(1-\psi)}\left(E_{\omega'|p}e^{\upsilon(1-\gamma)\left(\sigma_e e'+x'b_z'\right)}u(k',p')^{\frac{1-\gamma}{1-\psi}}\right)^{\frac{1-\psi}{1-\gamma}}. \end{aligned}$$

A computational advantage of this formulation is that the variables are stationary since we take out the stochastic trend, which of course facilitates the numerical analysis. Compared to the standard RBC model, we have an additional equilibrium policy function to solve for: leverage $L(k,p)$, and correspondingly, we have an additional first-order condition.

Given the nonlinear form of the model, and the focus on risk premia, it is important to use a nonlinear solution method. The policy functions are approximated using Chebychev polynomials and solved for using projection methods.[15]

2.2 Parametrization

The model is solved and simulated at the annual frequency, using the parameters listed in Table 1. (Section 4 and the numerical appendix provide detailed sensitivity analysis.) A first set of fairly uncontroversial parameters $(\alpha, \delta, \upsilon, \beta, \mu, \sigma_e)$ follows the business cycle literature (see for instance Cooley and Prescott (1995)) . Next, the intertemporal elasticity of substitution of consumption (IES) is set at 2. We discuss below the role of this parameter, and why it is difficult to reconcile the model with the evidence if the IES is low.[16]

A critical part of the calibration regards the size and average probability of disasters. Barro and Ursua (2008) and Barro and Jin (2012), using international data, estimate an average probability of disaster of 3.8% per year, with a mean disaster size of 21% for consumption and a Pareto distribution of disaster size b. However, the United States are probably less likely to be affected by disasters of this size than most other countries. To reflect this, I set conservatively the probability of disaster to 2% per year, and the mean disaster size to 15%. (The Great Depression – the largest disaster experienced by the US –

[15] The appendix details the computational method, and the Matlab code that implements it is available on the author's webpage.

[16] Of course, there is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Gruber (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES implies, counterintuitively, that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. Moreover, in my model, a simple regression of consumption growth on interest rates leads to a large bias in the estimated IES: it is around 0.3, because states with high probability of disaster have low interest rates and high expected consumption growth.

Table 1: Parameter values for the benchmark model

Parameter	Symbol	Value	Parameter	Symbol	Value
Capital share	α	0.30	Mean disaster size	$\overline{b_z} = \overline{b_k}$	0.15
Depreciation rate	δ	0.08	Std dev. of disaster size	$\sigma_z = \sigma_k$	0.30
Utility weight on C	υ	0.30	Mean log prob. of disaster	$\log(\overline{p})$	-4.15
Discount factor	β	0.987	Persistence of $\log(p)$	ρ_p	0.75
Trend growth of TFP	μ	0.01	Std. dev. of $\log(p)$	σ_p	0.46
Std. dev. TFP shock	σ_e	0.015	Mean prob. of disaster		0.02
IES utility	$1/\psi$	2	Debt advantage (BAA)	$\chi - 1$	0.0420
Risk aversion	γ	10	Debt advantage (AAA)	$\chi_{aaa} - 1$	0.0163
Std. dev. idiosyncratic	σ_ε	0.19	Bankruptcy losses	$1 - \theta$	0.30

Note: The time period is one year.

led to a GDP decline of 33%, but it was not permanent.) I also set the standard deviation of log disaster size to 30%, reflecting the large uncertainty about the size of disaster. Another important assumption is that the capital destruction and the TFP reduction are equal, i.e. $b_k = b_z$. There is unfortunately little data on TFP and capital stocks (rather than output). Section 4 relaxes and discusses this assumption.

A second critical element is the process for disaster probability, which is a Markov chain, which is picked to approximate the following AR(1): $\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \sqrt{1 - \rho_p^2} \varepsilon_{p,t+1}$,where $\varepsilon_{p,t+1}$ is $i.i.d.$ $N(0,1)$. Given the lack of direct evidence on σ_p, this parameter is chosen so that the model reproduces the volatility of credit spreads. The persistence ρ_p of the probability of disaster is set to .75; this parameter is relatively unimportant.

The risk aversion parameter γ is picked to reproduce approximately the level of equity premia. Note that γ is the risk aversion over the consumption-hours bundle. Since $\gamma = 10$ and the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 3.33 (Swanson (2012)), a low value by the standards of the asset pricing literature. As I explain below, this risk aversion parameter plays an important role in determining business cycle dynamics.

The last important element of the calibration pertains to the capital structure. I use a log-normal distribution for H, the distribution of idiosyncratic shocks. This leaves me with three parameters: the tax shield χ, the bankruptcy cost $1-\theta$, and the standard deviation of idiosyncratic shocks σ_ε. There are two reasonable targets for θ : on the one hand, we know that the recovery rate that lenders get on their defaulted debt is around 45% (the loss given default); on the other hand, estimates of default costs are usually of 10-20% or even less (e.g. see the discussion in Van Binsbergen et al. (2010)). It is impossible to match both targets simultaneously; I compromise and set $1 - \theta = .3$, which implies an (endogenous) recovery rate on debt $\frac{\theta}{\varepsilon_t^* H(\varepsilon_t^*)} \int_0^{\varepsilon_t^*} \varepsilon dH(\varepsilon)$ equal to 65%; i.e. the "loss given default" is 35%.[17]

The parameters σ_ε and χ are then picked jointly to match the average default rate for BAA firms in normal times (i.e. outside disasters), 0.5% per year, and average leverage, 0.55[18] . This implies a

[17] An interesting extension, studied in the online appendix, is to allow the recovery rate θ to be lower in disaster states.

[18] In the data leverage is somewhat smaller, perhaps 0.45. It is difficult to replicate all the features of the data with a leverage equal to 0.45. I interpret this as reflecting the low volatility of firm profits and value in this model. In reality

volatility σ_ε of about 19%, and a tax subsidy χ of 4.2 cents per dollar of debt issued. This measure can be compared to the actual tax advantage of debt. With a marginal corporate income tax rate of 35%, a nominal interest rate of 7% (the average of the BAA rate in the US) implies a tax subsidy of 2.45 cents on the dollar, which is lower than what I find. (This assumes that households are taxed at the same rate on debt and equity income.) One interpretation is that that there are other advantages to debt than the tax shield.

Because of its stylized nature, my model does not generate a cross-section of firms: all firms are ex-ante identical and borrow at the same interest rate. I calibrate the model so that this "representative firm" has a BAA rating. However, to compare the model implication to the data, I need to define the AAA asset. To do, I introduce a fringe of AAA firms, which do not affect aggregates, but can be used to calculate the AAA bond price.[19] The question is in what dimension do these firms differ for them to be less risky. Paradoxically, assuming that they have lower idiosyncratic shocks (a lower σ_ε) does not necessarily guarantee that they have lower yields, because they may lever more. Hence, I simply assume that these firms face lower benefits of debt, i.e. have a lower χ, which leads them to lever less and hence be less risky. I set χ_{aaa} to replicate the (outside disasters) default rate of AAA which is around 4bps per year (20bps over 5 years); this leads to $\chi_{aaa} = 1.63\%$ and a leverage ratio of 45%.

3 Results

This section first explains the response of the model economy to the different shocks, then discusses the amplification effect of debt financing, and the time-varying systematic risk generated by the model, and finally compares the model implications to the data.

3.1 Impulse responses

I first discuss the response of the model economy to a standard productivity shock and to a disaster realization.[20] I then turn to the novel shock of this model: variation in disaster probability.

3.1.1 TFP shock

The standard TFP shock has nearly identical effects as in the RBC model: higher TFP leads to higher investment, output, consumption and employment, and to higher equity excess returns (on impact) and interest rates (persistently).Furthermore, the default rate falls on impact, since higher profits increase the likelihood that firms can repay their debt, but this effect is very small. There is essentially no effect on leverage or credit spreads, because a change in TFP has little effect on the (tax) benefits or (default) costs of debt.

firms face fixed costs, and some production factors are hard to adjust, which imply a higher effective leverage than the financial leverage. This motivates my higher target for leverage.

[19] A previous version of this paper (Gourio (2010)) defined the AAA asset as a risk-free asset, with similar results.

[20] The figures corresponding to these impulse response functions are in the online appendix.

3.1.2 Disaster realization

A disaster realization generates an immediate, once-and-for-all decline in productivity, and given the assumption $b_k = b_z$, capital falls by the same amount. As a result, output, investment, consumption all decline once-and-for-all by the same amount endogenously, while employment does not change. The reason is that the decline in productivity leads to a lower optimal stock of capital. But with $b_k = b_z$, the capital stock destruction is exactly in line with the reduced capital demand. Mathematically, the detrended capital stock $k = K/z$ is unaffected by the disaster realization since the numerator and the denominator are affected in the same way. Economically, the economy shifts from one steady-state of the neoclassical model to another steady-state. Leverage does not change following the disaster since firms instantaneously rebalance and issue a lower debt, in line with their now lower capital stock.

As for asset prices, the return on physical capital is very low when the disaster hits due to capital destruction. The losses are further increased by default since $\theta < 1$. The equity return, which is a leveraged version of the return on capital, is hence very low, while the corporate bond return is also low as some firms go into default and their bonds are not paid back in full. Hence, both equity and corporate debt are risky assets, since their returns are low precisely in the states when consumption is low, i.e. marginal utility is high.

3.1.3 An increase in disaster probability

The important shock in this paper is the shock to the probability of disaster – i.e. an increase in the perceived risk of a very bad outcome. Figure 2 presents the responses of macroeconomic quantities to a one standard-deviation increase in the probability of disaster at time $t = 2$. The higher risk leads to a sharp reduction in investment. Employment simultaneously falls, generating a recession. Intuitively, there is less demand for investment and this reduces the need for production. Technically, the employment decline comes from intertemporal substitution of labor supply – the shadow risk-free interest rate is low, leading people to work less.

This low risk-free interest rate reflects a "flight to quality" – the unattractiveness of investment in real capital – and leads consumption to increase on impact.[21] Households would ideally like to save in safe assets. But there is no net supply of safe assets in this economy – only risky capital.[22] Finally, because risk increases, risk premia rise as the economy enters this recession: the expected equity excess return rises with the disaster probability.

Figure 3 presents the response of leverage and credit spreads: as risk increases, firms substitute out of debt and into equity. This deleveraging is consistent with the observed pattern for newly issued credit (Baker and Wurgler (2000)). In spite of this deleveraging, the BAA-AAA credit spread increases substantially. Hence, the model will generate the correct negative comovement of investment and credit

[21] This unattractive on-impact consumption response can be eliminated by introducing sticky prices or countercyclical markups (see Gourio (2012)).

[22] As I explain in Section 4, this response requires that (i) there is capital destruction, so that capital is a risky investment; (ii) the IES of consumption large enough. If these conditions are not both satisfied, households react to higher disaster risk by increasing investment and reducing consumption.

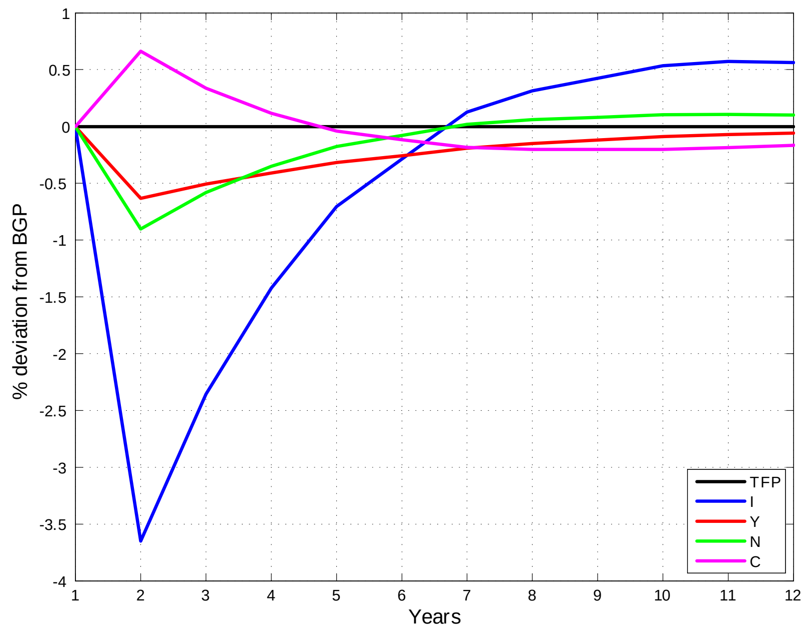


Figure 2: Impulse response of investment, output, consumption, employment and tfp to a one-standard deviation increase in the disaster probability

spreads.[23]

3.2 Amplification of Business cycles through debt finance

Importantly, the presence of debt financing amplifies macroeconomic fluctuations in the model. To establish this and measure the amplification, figure 4 superimposes the responses of investment to a shock to the probability of disaster for the all-equity financed (RBC) model ($\chi = \theta = 1$) and for the benchmark model. Investment is about three times more volatile with debt finance, which is significant – amplification is usually fairly limited in flexible price models. Similar amplification in response to the disaster probability shock holds for other variables such as employment or GDP. On the other hand, there is no amplification for TFP shocks.

To understand this amplification, note that disaster risk has both a direct effect on the risk-adjusted return on capital and an indirect effect. The direct effect is the only one at work in the RBC model ($\chi = \theta = 1$). As analyzed in Gourio (2012), this effect is to make the investment technology less attractive if the IES is larger than unity, and is equivalent to a change in the discount factor β of the household.

The indirect effect, which is novel to this paper, is that higher disaster risk increases expected *discounted* bankruptcy costs: holding debt fixed, default is (i) more likely and (ii) more likely to be systematic, i.e. default is more likely to occur in “bad times”. Higher expected bankruptcy costs in turn increase the user cost of capital, leading firms to cut back more on investment than in the frictionless model. Or to put it another way, these higher bankruptcy costs lead firms to substitute equity for debt,

[23] Interestingly, however, when disaster probability rises by a very large amount, the deleveraging can be so large that the total credit spread falls.

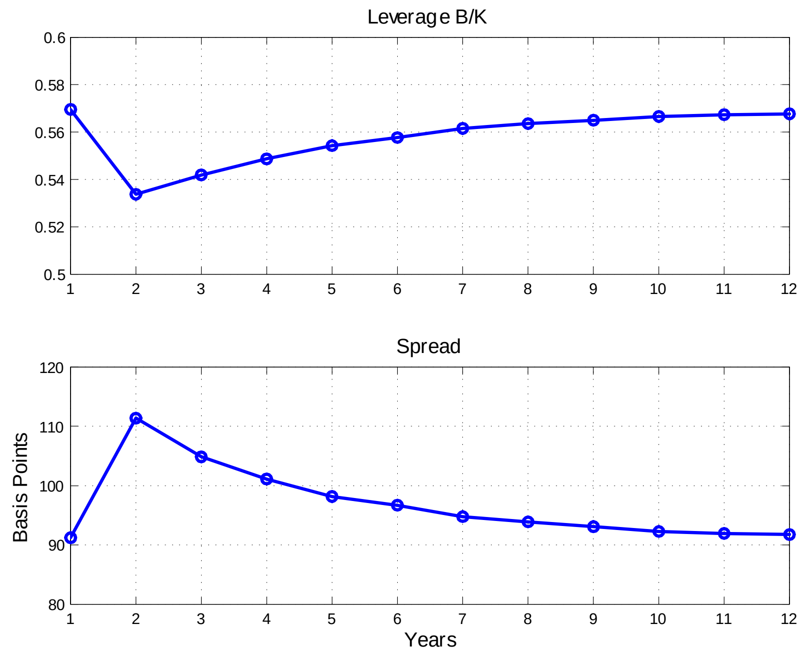


Figure 3: Impulse response of leverage and credit spreads to a one-standard deviation increase in the disaster probability.

and to lose the tax shield that makes debt financing cheaper. Overall, the presence of debt financing implies a more volatile user cost of capital, which generates the substantial amplification depicted in figure 4.

3.3 Time-varying systematic risk

To illustrate the increase in *systematic* risk that occurs when the disaster probability rises, figure 5 presents the correlation of defaults that is expected given the probability of disaster today, i.e. $Corr_t\left(def_{i,t+1}, def_{j,t+1}\right)$ for any two firms i and j in the benchmark model. In normal times, the probability of disaster is low, defaults are largely idiosyncratic, since aggregate TFP shocks generate only a limited comovement in defaults. The correlation becomes higher, however, when the probability of disaster rises. This is because defaults are now much more likely to be triggered by the realization of a disaster, which affects all the firms. This higher implied correlation would show up in some asset prices such as CDOs (collateralized debt obligations).[24]

This correlation is driven both by the exogenous increase in aggregate uncertainty, which mechanically increases the correlation (since idiosyncratic uncertainty is constant), but also by firms' endogenous leverage choices. Because firms cut back on leverage when disaster risk increases, their choices mitigate the increase in the correlation of defaults. This is illustrated in figure 5, where the line with crosses is the benchmark model, where firms choose their leverage each period, and the line with circles is a version of the model with constant leverage.[25] In the latter case, the correlation unambiguously increases, while

[24] See Coval, Jurek and Stafford (2009) and Collin-Dufresne, Goldstein and Yang (2012) for recent work on the importance of aggregate risk for the pricing of CDOs.

[25] That is, we assume that firms have to use a constant, exogenous value of leverage. The equations as the same as the

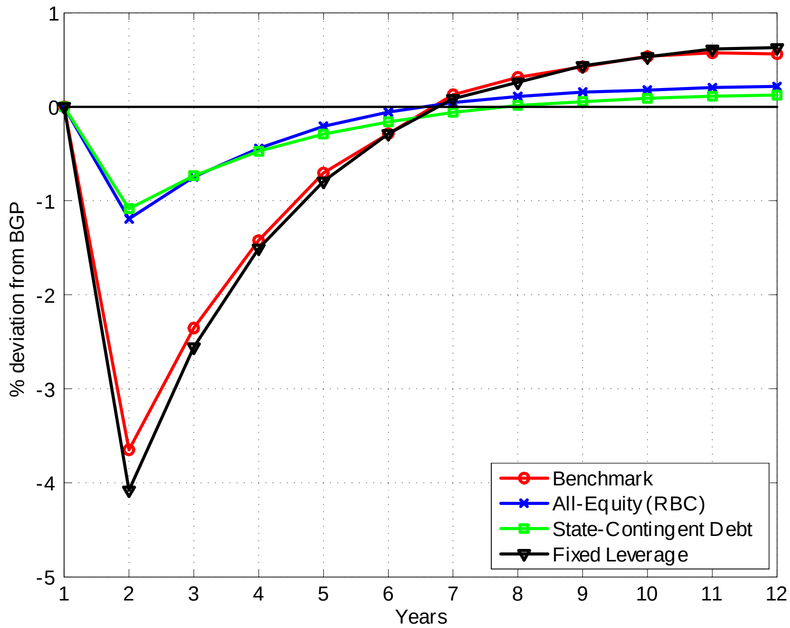


Figure 4: Impulse response of Investment to a one standard deviation shock to the disaster probability in four models: Fixed leverage, Benchmark model (with endogenous leverage), State-contingent debt, all-equity finance model (RBC)

in the former, firms' deleveraging leads the correlation to fall eventually.

We now turn to a quantitative examination of the model's implications.

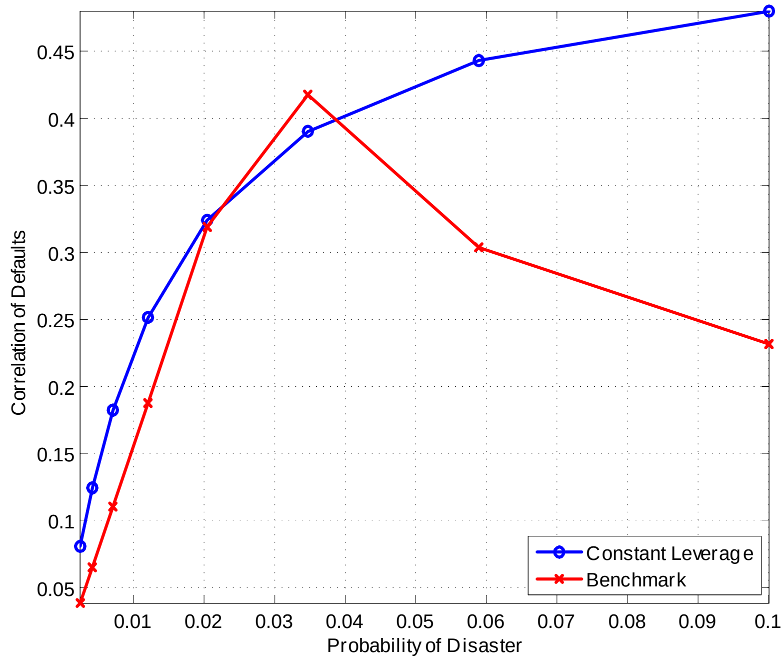
3.4 Business Cycles and Financial Statistics

Table 2 reports business cycle and asset returns statistics, while table 3 reports leverage, default rates, loss-given-default and credit spreads. The model statistics are calculated in a sample that does not include disaster realizations, to be comparable to the data sample (1947-2011) which is devoid of disasters.[26] To illustrate the role of constant disaster risk and of time-varying disaster risk, the tables present the results for three different assumptions about the structure of shocks hitting the economy: (i) no disaster risk, i.e. only TFP shocks, (ii) TFP shocks and a positive, but constant probability of disaster; (iii) TFP shocks and time-varying disaster risk. I also report results for both the all-equity RBC model ($\chi = \theta = 1$) and the benchmark model, with debt financing ($\theta < 1$ and $\chi > 1$).

TFP shocks alone (rows 2 and 5) generate a decent match for quantity dynamics, as is well known from the business cycle literature. However quantities are not volatile enough (TFP volatility is too low) and this is especially true for employment. The model also generates small spreads for corporate bonds (22bps, see row 2 of Table 3), and these spreads simply account for the average default of corporate bonds: the average return on the BAA and AAA assets are almost equal, i.e. the excess return on

benchmark model, except that we discard the first-order condition for optimal leverage choice.

[26] The leverage and default probability data are taken from Chen, Collin-Dufresne, and Goldstein (2009). The other data (GDP, consumption, investment, and credit spreads) are from FRED. I use BAA-AAA as the credit spread measure, and obtain similar results as Chen, Collin-Dufresne, and Goldstein. All series are annualized.



Time-varying

Figure 5: systematic risk. Correlation of defaults across firms, as a function of the disaster probability, for the benchmark model and for the model with constant leverage.

Table 2: Business cycle statistics and Asset Returns (Annual)

Row			$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$E(R^{aaa})$	$E(R^{baa})$	$E(R^e)$
1	Data		2.78	1.81	7.01	2.67	1.70	2.50	7.30
2		No disaster risk	1.36	0.78	3.28	0.46	2.54	2.54	2.55
3	All-equity model (RBC)	Constant disaster risk	1.36	0.78	3.32	0.47	-0.22	-0.22	2.37
4		Time-varying disaster risk	1.37	0.81	3.67	0.56	-0.14	-0.14	2.37
5		No disaster risk	1.34	0.77	2.65	0.44	2.34	2.34	2.45
6	Benchmark model	Constant disaster risk	1.35	0.76	2.89	0.46	0.22	1.52	5.52
7		Time-varying disaster risk	1.53	1.12	5.28	1.17	0.36	1.14	5.13

Note: Annual volatility of the growth rates of investment, consumption, employment and output; and mean return on the AAA and BAA bonds and equity. Model statistics are computed in a sample without disasters.

Table 3: Leverage, Default probabilities, and credit spreads.

	Credit spread			Default rate	Loss given default	Leverage B/K	
	Mean	SD	Correl with Invt	Mean	Mean	Mean	SD
Data	0.94	0.41	-0.37	0.50	45	45	9
No disaster risk	0.22	0.00	0.99	0.94	34.28	62.75	0.09
Constant disaster risk	1.39	0.00	0.99	0.28	33.84	58.33	0.10
Time-varying disaster risk	0.90	0.40	-0.44	0.39	33.76	56.97	6.33

Note: Mean, volatility and correlation with investment of the credit spread; mean default rate; mean loss given default; mean and volatility of leverage. Annualized statistics. Model statistics are computed in a sample without disasters.

corporate bonds is close to zero. In general, the credit spread between the BAA asset and the AAA asset is the sum of the (physical) compensation for the different default risks, plus a risk premium:

$$\begin{aligned} E(y^{baa} - y^{aaa}) &= \Pr\left(\text{Default}^{baa}\right) \times E(LGD^{baa}) - \Pr\left(\text{Default}^{aaa}\right) \times E(LGD^{aaa}) \\ &\quad + E(R_c^{baa} - R_c^{aaa}), \end{aligned}$$

and here the last term is almost nil : spreads are completely accounted for by the (higher) probability of default of BAA ($0.22 = 0.94 \times 0.35 - 0.32 \times 0.34 + 0.00$). Moreover, these spreads are essentially constant. The risk premium for equity is also tiny.

Adding capital structure to the RBC model with only TFP shocks has only minimal effects on quantity dynamics, as can be seen by comparing rows 2 and 5 of Table 2. Hence, financial frictions do not amplify the response to TFP shocks.

When constant disaster risk is added to the model, the quantity dynamics are essentially unaffected (table 2, comparing rows 2 and 3 or 5 and 6). Table 3 reveals that credit spreads are significantly larger however, because defaults are much more likely during disasters, when marginal utility is high. The model generates a plausible credit spread of 139bps, much higher than the probability of default (28bps). The equity premium is also high, and it is higher in the model with capital structure, because of leverage. However, the volatility of spreads is still close to zero (and so is leverage). This motivates turning to the model with time-varying risk of disaster.

Rows 4 and 7 display the results for the models with time-varying disaster risk. The variation in disaster risk leads to volatile credit spreads in line with the data. The equity premium is comparable to the data, and similar to that of the model with constant probability of disaster. Introducing the time-varying risk of disaster also generates new quantity dynamics: investment and employment become more volatile, consistent with the impulse response function described in the previous section. Moreover, credit spreads are countercyclical: the model reproduces well the relation between investment or output and credit spreads emphasized in the introduction.

The amplification effect of disaster risk through financial frictions is visible in table 2: while the financial friction model exhibits less volatility than the RBC model when disaster risk is constant, it has more volatility than the RBC model when disaster risk is added. This is especially true for investment

Table 4: Model: Decomposition of Credit Spreads into expected losses and the risk premium

	$Spread_t$	$E_t(R_{c,t+1} - R_{f,t+1})$	$E_t\left(Loss_{t+1}\right)$
Mean	0.90	0.70	0.20
Std Deviation	0.40	0.42	0.06
Covariance with I/K	-0.08	-0.10	0.02

volatility, which goes from 3.28% in the RBC model without disaster risk, to 3.67% in the RBC model with disaster risk, to 5.28% in the capital structure model with disaster risk.

As explained in the introduction, there is substantial evidence that both the level and cyclicality of credit spreads are driven by the risk premium rather than by expected credit losses. Table 4 performs this decomposition of model credit spreads into expected credit losses and the risk premium. Not only is the largest share of the spread driven by the risk premium on average, but so is its variation over the business cycle. The covariance with investment are almost entirely driven by variation in the risk premium, consistent with Gilchrist and Zakrajsek (2012).

Moreover, in the data, investment are negatively correlated with spreads, but the *level* of the real interest rate is hardly correlated with investment: the correlation between investment and the BAA interest rate (net of inflation) is only -0.02. Hence, credit spreads seem to contain some specific information relevant for aggregate investment. The model implies a positive (rather than nil correlation): when disaster risk is high, interest rates and investment are low. This is because the interest rate falls more than the credit spread rises.

Finally, the model implies some volatility of leverage, but it falls somewhat short of the data. However, the one-period nature of firms in this model makes it difficult to interpret this statistic: the flow and stock of debt are equal in the model, while they behave differently in the data (Covas and Den Haan (2009)). The model prediction that leverage is procyclical is reasonable when applied to the flow of new debt.

I close this section by discussing some shortcomings of the model, due to its highly stylized nature. In particular, the correlation of consumption and output is too low, around 0.2. I abstract from many ingredients such as habits or sticky prices which may help with consumption comovement. Second, there is no default clustering outside disasters, and in particular states with high disaster probability do not generate high default rates. Finally, the equity return is fairly smooth in this model outside disasters. Equities are a one-period asset here, implying that the conditional volatility of equity returns equals the conditional volatility of dividends (i.e. there is only a cash flow effect and no discount rate effect).

4 Extensions and Comparative statics

This section considers some implications and extensions of the baseline model, and the sensitivity of the quantitative results to parameter changes.

4.1 State-contingent debt

In the aftermath of the 2008 financial crisis, several economists have proposed that private sector borrowers issue state-contingent debt with reduced payments conditional on large aggregate shocks (e.g., "contingent convertibles" or CoCos) rather than using standard debt contracts. This section evaluates this proposal by allowing firms in the model to issue debt contingent on the disaster realization and size (i.e. x and b). The model is easily modified; first, the funding constraint now reads,

$$K_{t+1}^{w} = S_t + \chi \left(q_t^{nd} B_{t+1}^{nd} + \sum_{b'} q_t^{d}(b') B_{t+1}^{d}(b') \right),$$

where B_{t+1}^{nd} (resp. $B_{t+1}^{d}(b')$) is the face value of the debt to be repaid in non-disaster states (resp. in disasters of size b') and q_t^{nd} (resp. $q_t^{d}(b')$) the associated price:

$$q_t^{nd} = E_t \left((1 - x_{t+1}) M_{t+1} \left(\int_{\varepsilon_{t+1}^{*}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}} \int_0^{\varepsilon_{t+1}^{*}} \varepsilon R_{t+1}^{K} K_{t+1}^{w} dH(\varepsilon) \right) \right),$$

and similarly for $q_t^{d}(b')$. Note that the default threshold ε_{t+1}^{*} depends on the state realized (disaster or not, and of which size), both because the firm value depends on the state (as in the benchmark model), but also because now the debt due depends on the state. Formally,

$$\varepsilon_{t+1}^{*} = \frac{\sum_{b'} \widetilde{x}_{t+1}(b') B_{t+1}(b') + B_{t+1}^{nd}(1 - x_{t+1})}{R_{t+1}^{K} K_{t+1}^{w}},$$

where $\widetilde{x}_{t+1}(b')$ is an indicator equal to 1 if a disaster of size b' is realized, and 0 otherwise. Maximizing equity value yields the first-order conditions for optimal debt choices:

$$\frac{\chi - 1}{\chi} E_t \left((1 - x_{t+1}) M_{t+1} \left(1 - H\left(\varepsilon_{t+1}^{*} \right) \right) \right) = (1 - \theta) E_t \left(M_{t+1} \varepsilon_{t+1}^{*} h\left(\varepsilon_{t+1}^{*} \right) (1 - x_{t+1}) \right), \quad (14)$$

$$\frac{\chi - 1}{\chi} E_t \left(\widetilde{x}_{t+1}(b') M_{t+1} \left(1 - H\left(\varepsilon_{t+1}^{*} \right) \right) \right) = (1 - \theta) E_t \left(M_{t+1} \varepsilon_{t+1}^{*} h\left(\varepsilon_{t+1}^{*} \right) \widetilde{x}_{t+1}(b') \right). \quad (15)$$

Rather than equating expected discounted marginal costs and benefits of debt over all the states together, the firm can now equate these expected marginal costs and benefits *conditional on a given disaster happening or not*. This added flexibility will lead the firm to issue little debt that is payable in disaster states, since bankruptcy is much more likely and costly in these states (through the state prices M_{t+1}). The following proposition demonstrates this for a reasonable special case.

Proposition 1 *Assume that p is constant, $b_k = b_z$, and that there are no TFP shocks ($\sigma_e = 0$). If corporations are allowed to issue disaster-contingent debt, they will structure their debt to make the probability of default equal in disaster states and non-disaster states, by issuing proportionately less debt in disasters.*

Proof. Given the assumptions, we can simplify the first-order conditions (14-15): the expectations are just expectations over the idiosyncratic shocks ε. Denoting default cutoffs in non-disaster states by ε_{t+1}^{nd*} and in disasters by $\varepsilon_{t+1}^{d*}(b')$, we have

$$\frac{\chi - 1}{\chi} \left(1 - H\left(\varepsilon_{t+1}^{nd*} \right) \right) = (1 - \theta) \Omega'(\varepsilon_{t+1}^{nd*}),$$

$$\frac{\chi - 1}{\chi} \left(1 - H\left(\varepsilon_{t+1}^{d*}(b') \right) \right) = (1 - \theta) \Omega'(\varepsilon_{t+1}^{d*}(b')),$$

which under the monotonicity assumption (footnote 12) implies that $\varepsilon^{d*}_{t+1} = \varepsilon^{nd*}_{t+1}(b')$ for all b', so that the probability of default $H\left(\varepsilon^{d*}_{t+1}\right) = H\left(\varepsilon^{nd*}_{t+1}(b')\right)$ is the same. The quantity of debt issued for each state follows from $\frac{B^{d}_{t+1}(b')}{K^{w}_{t+1}e^{b'}} = \frac{B^{nd}_{t+1}}{K^{w}_{t+1}}$, so that $B^{d}_{t+1}(b') = B^{nd}_{t+1}e^{b'}$. ■

Figure 4 compares the response to an increase in disaster risk in the model with state-contingent debt, with the response of the benchmark model, and the response of the model with an all equity-financed firm (RBC). The amplification effect largely disappears: the model with state-contingent debt implies now no more investment volatility than the frictionless RBC model.

This benefit of contingent debt in reducing volatility in response to shocks to disaster risk, comes on top of the obvious advantage that, should a disaster happen, there will be fewer defaults. (In the model, this does not matter since default realizations have no direct effect on GDP or other macroeconomic variables.) Overall, while the assumption that private contracts are not made contingent on aggregate realizations remains common in the literature, our results indicate that is far from innocuous.[27]

More generally, in this model an ex-post bailout of bondholders that was unexpected would not change the equilibrium, since default realizations do not matter; but a credible policy that firms in default receive transfers from the government to pay the bondholders in full would reduce credit spreads to zero, and take away the bankruptcy costs. This would lead aggregate investment to become less sensitive to disaster risk, and hence less volatile. Furthermore, investment would be higher since firms would now use 100% debt financing which is cheaper. Obviously, this policy is detrimental to welfare.

4.2 The tax advantage of debt and macroeconomic volatility

Following a large literature in corporate finance, the model features as a prime determinant of capital structure the tax subsidy to debt, or tax shield. The tax shield is inefficient in the model for two reasons: first, the tax shield lowers the user cost of capital and hence encourages capital accumulation. However, the competitive equilibrium of the model without taxes is already Pareto optimal, hence the subsidy leads to *overaccumulation* of capital. Second, the higher leverage created by the tax shield amplifies fluctuations in aggregate quantities, including consumption, and hence reduces welfare. To illustrate this, the left panel of Figure 6 depicts the mean leverage and the volatility of investment growth in the model, as a function of χ. For our benchmark calibration, removing the tax shield entirely would increase welfare significantly, equivalent to a permanent increase of consumption of 0.72%.[28]

4.3 Comparative statics

Tables 5 and 6 provide some comparative statics to illustrate the role of various assumptions.

4.3.1 Sample with disasters.

So far the results reported are calculated in samples which do not include disasters. Measured excess returns arise both through a standard risk premium and through sample selection (a "Peso problem")

[27] Krishnamurthy (2003) similarly found that allowing for conditionality in the Kiyotaki-Moore model reduces or eliminate the amplification effect of financial frictions.

[28] Glover et al. (2010) also study the effect of eliminating the tax shield. In their model, removing the tax shield has limited effects as firms may substitute operating leverage for financial leverage.

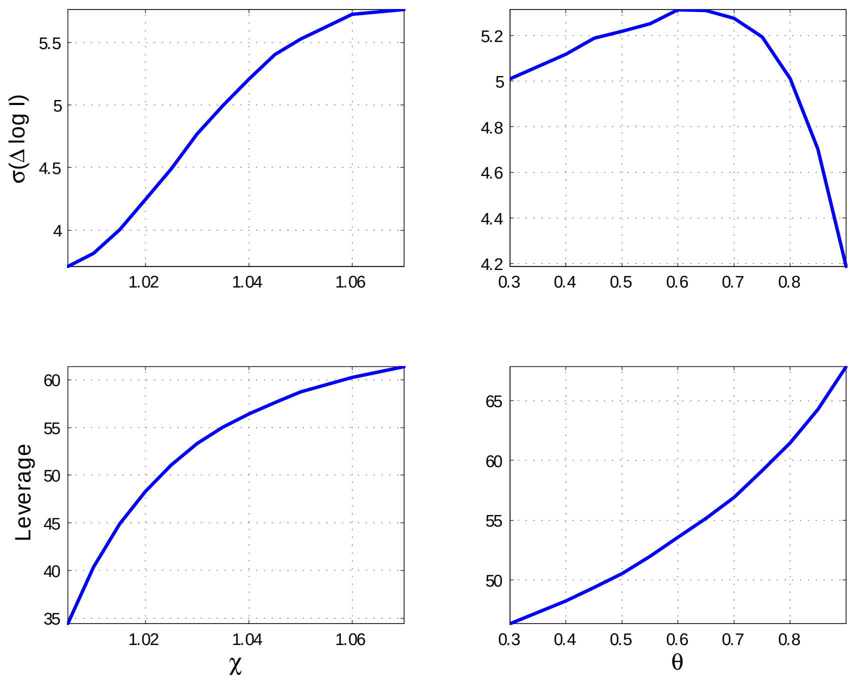


Figure 6: Impulse response of Investment to a one standard deviation shock to the disaster probability in four models: Fixed leverage, Benchmark model (with endogenous leverage), State-contingent debt, all-equity finance model (RBC)

since the sample does not include the lowest possible return realizations. Row 3 calculates the moments in population, i.e. in a sample that includes disaster realizations. The equity premium is reduced by about 60bps per year, and the corporate bond premium is reduced by about 10bps. The higher default rate still does not account for the observed credit spread. This shows that the peso problem is not the main source of excess returns. On the other hand, adding disasters to the sample obviously generates some significant volatility in macroeconomic quantities.

4.3.2 Fixed leverage

The model assumes that firms are able to adjust their leverage ratio each period costlessly. In reality, it may be difficult to issue new securities or to repurchase existing securities quickly. Row 4 shows the effect of imposing that firms keep a constant ratio of debt to capital stock, i.e. $L_t = B_{t+1}/K^w_{t+1} = \overline{L}$. This leads to more volatility of both macroeconomic quantities and credit spreads, as firms cannot delever in response to an increase in disaster risk. The amplification effect becomes larger than in the benchmark model (see figure 4). The other implications of the model are largely unaffected. Hence, the key model results are driven by the existence of leverage, and not by the particular adjustment of leverage that is obtained endogenously in the model.

4.3.3 Recursive Preferences: role of IES and risk aversion

Households are assumed to have recursive utility, but the model can also be solved in the special case of expected utility. When the elasticity of substitution is kept equal to 2, and the risk aversion is lowered to

.5 to reach expected utility, the *qualitative* implications are largely unaffected. Quantitatively however, because risk aversion is lower, all risk premia are lower, and the response of quantities to a probability of disaster shock is also smaller since agents care less about risk. For instance, the volatility of investment falls from 5.28% to 3.04%, the excess return on corporate bonds falls from 78bps to 11bps, and the credit spreads from 90bps to 31bps. This shows that risk aversion is a key determinant of both prices and business cycle quantities in this framework (unlike Tallarini (2000)).

In contrast, when the elasticity of substitution is reduced to .1 so as to reach the special case of expected utility while keeping $\gamma = 10$, a shock to the probability of disaster leads to starkly different *qualitative* effects. Investment, output and employment rise (rather than fall) as the probability of disaster rises. The intuition is that higher risk makes people save more, despite the fact that capital is more risky. In the all-equity model, the threshold value for the IES so that investment increases following a disaster probability shock is unity. In the model of this paper (with debt), higher uncertainty has a more negative effect on investment demand, and hence the threshold value for the IES is lower than unity. Hence, for a range of values of IES below unity, the model with debt finance implies that higher disaster risk lowers economic activity, while the all-equity model implies the opposite – an extreme example of the potential importance of financial frictions.

4.3.4 TFP vs. capital disasters

I model disasters as simultaneous declines in TFP and reduction in the capital stock. If a disaster only affects TFP, there are important qualitative and quantitative changes. First, in response to an increase in disaster probability, investment, output and employment rise, rather than fall - since capital is now a safe asset, it makes sense to engage in precautionary savings in anticipation of disasters. Hence, while investment is very volatile, it has the wrong comovement with credit spreads (and more generally with risk premia). Further, the low risk on capital implies a low equity risk premium and a low corporate bond premium. Hence, overall the model without capital shock is unable to replicate the evidence. (On the other hand, if there are no TFP shocks but only capital shocks, the model generates small risk premia, since agents anticipate a fast return to the steady-state.)

4.3.5 Capital structure parameters

Finally, in this model the level of idiosyncratic risk and bankruptcy costs affect business cycle dynamics by changing the incentives to use leverage. A lower idiosyncratic volatility leads to higher leverage, and hence to a larger sensitivity of macroeconomic quantities to disaster risk.

Higher bankruptcy deadweight cost (reducing θ) leads to lower leverage. The effect on spreads is ambiguous: lower recovery rates in themselves work to increase spreads, but lower leverage leads to lower spreads. Macroeconomic quantities are now a tiny bit less sensitive to disaster risk, as default during disasters are less costly. Figure 6 illustrates these comparative statics. Overall, these experiment underscore that leverage is not a sufficient statistic for the sensitivity of the economy to shocks: the underlying cause of high leverage (idiosyncratic volatility, risk aversion, deadweight costs) matters crucially for business cycle analysis.

Table 5: Business cycle statistics and Asset Returns (Annual)

Row		$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$E(R^{aaa})$	$E(R^{baa})$	$E(R^{e})$
1	Data	2.78	1.81	7.01	2.67	1.70	2.50	7.30
2	Benchmark	1.53	1.12	5.28	1.17	0.36	1.14	5.13
3	Sample with disasters	5.02	4.92	7.18	1.16	0.33	1.04	4.53
4	Fixed leverage	1.59	1.23	6.11	1.34	0.51	1.32	4.57
5	State-contingent debt	1.37	0.82	3.05	0.57	n.a.	n.a.	n.a.
6	$\theta = \gamma = .5$	1.37	0.82	3.07	0.58	3.04	3.15	3.83
7	$\theta = \gamma = 10$	1.14	1.23	3.00	0.75	0.53	1.32	5.46
8	$b_k = 0$	2.07	2.05	7.68	2.37	0.88	0.88	0.79
9	$\chi = 5.5$	1.61	1.28	5.68	1.39	0.29	1.36	5.35
10	$\theta = 0.5$	1.52	1.11	5.26	1.13	0.24	0.96	4.71
11	$\sigma_\varepsilon = 0.17$	1.56	1.19	5.57	1.25	0.35	1.22	5.30

Note: Volatility of the growth rates of investment, consumption, employment and output; and mean return on the AAA and BAA bonds and equity. Model statistics are computed in a sample without disasters, except where indicated.

Table 6: Leverage, Default probabilities, and credit spreads.

	Credit Spread			Default rate	Loss given default	Leverage B/K	
	Mean	SD	Correl with Invt	Mean	Mean	Mean	SD
Data	0.94	0.41	-0.37	0.50	45	45	9
Benchmark	0.90	0.40	-0.44	39.03	33.76	56.97	6.33
Sample with disasters	0.91	0.40	0.00	64.23	33.80	56.95	6.30
Fixed leverage	0.85	0.52	-0.79	14.48	33.65	56.00	0.00
$\theta = \gamma = .5$	0.31	0.09	-0.42	76.57	34.19	61.89	0.82
$\theta = \gamma = 10$	0.91	0.40	0.42	39.76	33.77	56.89	6.32
$b_k = 0$	0.22	0.00	0.43	94.62	34.28	62.12	0.19
$\chi = 5.5$	1.29	0.60	-0.57	64.10	34.01	59.58	4.98
$\theta = 0.5$	0.79	0.25	-0.09	11.71	52.33	50.47	7.74
$\sigma_\varepsilon = 0.17$	0.98	0.47	-0.39	33.47	33.31	58.98	7.44

Note: Mean, volatility and correlation with investment of the credit spread; mean default rate; mean loss given default; mean and volatility of leverage. Annualized statistics. Model statistics are computed in a sample without disasters, , except where indicated.

4.3.6 Comparison with uncertainty shocks

It is interesting to compare the results of the paper with several recent studies that also consider the effect of an increase in risk. Arellano, Bai and Kehoe (2009), Chugh (2009) and Gilchrist, Sim and Zakrajek (2010) all show that a temporary increase in σ_ε, the standard deviation of idiosyncratic shocks, leads to a recession by exacerbating financial frictions. The increase in disaster probability also leads to an increase in uncertainty and a recession, hence the two mechanisms are broadly similar. (My model has additional implications for the corporate bond premium, while these models have implications for other variables.)

A key difference is that an increase in disaster probability increases *aggregate* uncertainty rather than *idiosyncratic* uncertainty; as a result there are at least three different implications: (i) even in the frictionless case ($\chi = \theta = 1$), higher aggregate uncertainty creates a recession, whereas higher idiosyncratic risk has no effect; (ii) the response of the economy to a disaster probability shock depends on the risk aversion coefficient, whereas the response to an increase in σ_ε does not; (iii) increasing aggregate risk while keeping total risk constant (by reducing idiosyncratic risk) leads to a recession by increasing the correlation of firm defaults.

5 Conclusion

The key contribution of this paper is to construct a tractable general equilibrium model, consistent with the salient features of credit spreads and economic fluctuations. Corporate debt is exposed to tail risk, and the degree of this exposure changes substantially over the business cycle. This allows the paper to link the large macroeconomic literature concerned with financial frictions, with the large empirical finance literature that documents the importance of risk premia in credit markets.

One obvious limitation of the paper is the focus on short-term debt. Short-term debt rules out agency issues such as risk-shifting or debt overhang. Furthermore, in the model, defaults are driven only by insolvency and not by liquidity shortfalls. Integrating these economic features in a quantitative model with sizeable risk-premia seems an important target for future research.

References

[1] Almeida, Hetor, and Thomas Philippon, 2007. "The Risk-Adjusted Costs of Financial Distress". Journal of Finance, 62(6):2557:2586.

[2] Amdur, David, 2010. "A Business Cycle Model of Aggregate Debt and Equity Flows". Working Paper, Muhlenberg College.

[3] Arellano, Cristina, Yan Bai and Patrick Kehoe, 2010. "Financial Markets and Fluctuations in Uncertainty". Working Paper, U of Minnesota.

[4] Baker, Malcolm, and Jeffrey Wurgler, 2000. "The Equity Share in New Issues and Aggregate Stock Returns." Journal of Finance 55(5):2219–2257.

[5] Bansal, Ravi, and Amir Yaron, 2004. "Risks for the long run: A potential explanation of asset pricing puzzles". Journal of Finance, 59(4):1481–1509.

[6] Barro, Robert, 2006. "Rare disasters and asset markets in the twentieth century". Quarterly Journal of Economics, 121(3):823–866.

[7] Barro, Robert, Emi Nakamura, Jon Steinsson, and Jose Ursua, 2012. "Crises and Recoveries in an Empirical Model of Consumption Disasters". Forthcoming, AEJ:Macroeconomics.

[8] Barro, Robert, and Jose Ursua, 2008. "Macroeconomic Crisis since 1870". Brookings Papers on Economic Activity, 255-350.

[9] Barro, Robert, and Tao Jin, 2012. "On the Size Distribution of Macroeconomic Disasters". Econometrica, forthcoming.

[10] Bernanke, Benjamin, Mark Gertler, and Simon Gilchrist, 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework," in Handbook of Macroeconomics, ed. by J. B. Taylor, and M. Woodford, vol. 1, chap. 21, pp. 1341–1393. Elsevier.

[11] Bhamra, Harjoat, Lars-Alexander Kuehn and Ilyan Strebulaev, 2009a. "The Aggregate Dynamics of Capital Structure and Macroeconomic Risk". Review of Financial Studies, 23(12):4187–4241.

[12] Bhamra, Harjoat, Lars-Alexander Kuehn and Ilyan Strebulaev, 2009b. "The Levered Equity Risk Premium and Credit Spreads: A Unified Framework". Review of Financial Studies, 23(2):645–703.

[13] Bhamra, Harjoat and Ilya Strebulaev, 2011. "The Effects of Rare Economic Crises on Credit Spreads and Leverage". Working Paper, UBC.

[14] Van Binsbergen, Jules, John Graham, and Jie Yang, 2010. "The Cost of Debt". Journal of Finance, 65(6):2089-2136.

[15] Bloom, Nicholas, 2009. "The effect of uncertainty shocks". Econometrica, 77(3):623–685.

[16] Brunnermeier, Markus and Yuli Sanikov, 2011. "A Macroeconomic Model with a Financial Sector." Working Paper, Princeton University.

[17] Chen, Hui. 2010 “Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure”. Journal of Finance, 65(6):2171–2212.

[18] Chen Long, Pierre Collin-Dufresne, and Robert Goldstein, 2009. “On the Relation between Credit Spread Puzzles and the Equity Premium Puzzle”, Review of Financial Studies, 22(9):3367:3409.

[19] Chugh, Sanjay, 2010. “Firm risk and leverage-based business cycles.” Working Paper, Boston College.

[20] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno, 2009. “Financial Factors in Economic Fluctuations ” Working Paper, Northwestern University.

[21] Collin-Dufresne, Pierre, Robert S. Goldstein and Spencer J. Martin, 2001. “The Determinants of Credit Spread Changes”. Journal of Finance, 56(6):2177–2207.

[22] Collin-Dufresne, Pierre, and Robert S. Goldstein, 2001. “Do Credit Spreads Reflect Stationary Leverage Ratios?”. Journal of Finance, 56(5):1929–1957.

[23] Collin-Dufresne, Pierre, Robert S. Goldstein and Fan Yang. “On the Relative Pricing of long Maturity Options and Collateralized Debt Obligations”, forthcoming The Journal of Finance, 2012.

[24] Cooley, Thomas and Edward Prescott, 1995. “Frontiers of Business Cycle Research”. Princeton University Press.

[25] Cordoba, Juan Carlos, and Maria Ripoll, 2004. “Credit Cycles Redux”. International Economic Review, 45(4):1011–1046.

[26] Coval, Joshua D., Jakub W. Jurek, and Erik Stafford. 2009. "Economic Catastrophe Bonds." American Economic Review, 99(3): 628-666.

[27] Covas, Francesco and Wouter den Haan, 2009. “The Cyclical Behavior of Debt and Equity Finance”. American Economic Review, 101(2):877-899.

[28] Das, Sanjiv, Darrell Duffie, Nikunj Kapadia, and Leandro Saita, 2007. “Common Failings: How Corporate Defaults Are Correlated”. Journal of Finance, 62(1):93-118.

[29] Duffie, Darrell, Andreas Eckner, Guillaume Horel, and Leandro Saita, 2009. “Frailty Correlated Default”. Journal of Finance, 64(5):2089-2123.

[30] Epstein, Larry G. and Stanley E. Zin, 1989. “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework”. Econometrica, 57(4):937-969.

[31] Fernandez-Villaverde, Jesus, Pablo Guerron-Quintana, Juan Rubio-Ramirez, and Martin Uribe, 2011. “Risk matters: the real effect of volatility shock”. American Economic Review, 101(6): 2530-61.

[32] Hall, Robert, 1988. “Intertemporal Substitution in Consumption”, Journal of Political Economy, 96(2):339-357.

[33] Gabaix, Xavier, 2012. “Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance”. Quarterly Journal of Economics, 127(2):645-700.

[34] Gertler, Mark, and Peter Karadi, 2011. "A model of unconventional monetary policy.' Journal of Monetary Economics, 58(1):17-34.

[35] Giesecke, Kay, Francis A. Longstaff, Stephen Schaefer, and Ilya Strebulaev, 2011. "Corporate Bond Default Risk: A 150-Year Perspective". Journal of Financial Economics, 102(2):233-250.

[36] Gilchrist Simon, and Egon Zakrajsek, 2012. "Credit spreads and business cycle fluctuations." American Economic Review, 102(4): 1692-1720.

[37] Gilchrist Simon, Jae Sim and Egon Zakrajsek, 2010. "Uncertainty, Financial Frictions, and Investment Dynamics." Working Paper, Boston University.

[38] Gilchrist Simon, Alberto Ortiz and Egon Zakrajsek, 2009. "Credit Risk and the Macroeconomy: Evidence from an Estimated DSGE Model" Working Paper, Boston University.

[39] Gilchrist, Simon, Vlad Yankov and Egon Zakrajsek, 2009. "Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets", Journal of Monetary Economics, 56(4):471-493.

[40] Glover, Brent, Joao Gomes, Amir Yaron, 2010. "Corporate Taxes, Leverage, and Business Cycles.' Working Paper, Wharton.

[41] Gomes, Joao, and Lukas Schmid, 2009. "Equilibrium credit spreads and the macroeconomy." Working Paper, Wharton.

[42] Gomes, Joao, 2001. "Financing Investment," American Economic Review, 91, 1263–1285.

[43] Gourio, François, 2012. "Disaster risk and business cycles." American Economic Review, 102(6): 2734-2766.

[44] Gruber, Jonathan; 2006. "A Tax-Based Estimate of the Elasticity of Intertemporal Substitution." National Bureau of Economic Research Working Paper 11945.

[45] Hackbarth Dirk, Jianjun Miao, and Erwan Morellec, 2006. "Capital Structure, Credit Risk, and Macroeconomic Conditions" Journal of Financial Economics, 82(3):519–550.

[46] Hennessy, Christopher and A. Levy, 2007. "Why Does Capital Structure Choice Vary with Macroeconomic Conditions?". Journal of Monetary Economics, 54(6):1545:1564.

[47] Huang, J., and M. Huang, 2003. "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?". Working paper, Stanford University.

[48] Kehoe, Timothy and Edward C. Prescott, 2007. "Great Depressions of the Twentieth Century". Federal Reserve Bank of Minneapolis, Minnesota.

[49] Khan, Aubhik and Julia Thomas, 2011. "Credit shocks and aggregate fluctuations in an economy with production heterogeneity". Working Paper, Ohio State University.

[50] Kocherlakota, Narayana, 2000. "Creating Business Cycles Through Credit Constraints," Federal Reserve Bank of Minneapolis Quarterly Review, 24(3), 2–10.

[51] Krishnamurthy, Arvind. 2003. "Collateral constraints and the amplification mechanism". Journal of Economic Theory, 111(2):277-292.

[52] Krishnamurthy, Arvind, and Zhiguo He, 2012. "Intermediary Asset Pricing", American Economic Review forthcoming.

[53] Leland, Hayne, 1994. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure". Journal of Finance, 49(4):1213-1252.

[54] Liu Zheng, Pengfei Wang, and Tao Zha, 2012. "Do Credit Constraints Amplify Macroeconomic Fluctuations?". Econometrica, forthcoming.

[55] Philippon, Thomas, 2009. "The Bond Market's Q". Quarterly Journal of Economics, 124(3):1011-1056.

[56] Mendoza, Enrique, 2010. "Sudden Stops, Financial Crises, and Leverage". American Economic Review, 100(5):1941-1966.

[57] Merton, Robert, 1974. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates". Journal of Finance 29(2):449-447.

[58] Miao, Jianjun, and Pengfei Wang. 2010. "Credit risk and business cycles." Working Paper, Boston University.

[59] Mueller, Philippe, 2009. "Credit Spreads and Real Activity". Mimeo, LSE.

[60] Mulligan Casey, 2004: "Capital, Interest and Aggregate Intertemporal Substitution", Miimeo, University of Chicago.

[61] Rietz, Thomas, 1988. "The equity premium: A solution." Journal of Monetary Economics 22(1):117–131.

[62] Santoro, Marika and Chao Wei, 2011. "Taxation, investment and asset pricing." Review of Economic Dynamics, 14(3):443-454.

[63] Swanson, Eric, 2012. "Risk Aversion and the Labor Margin in Dynamic Equilibium Models". American Economic Review, 102(3):1663-1691.

[64] Tallarini, Thomas D. Jr., 2000. "Risk-Sensitive Real Business Cycles." Journal of Monetary Economics, 45(3):507-532.

[65] Tirole, Jean, 2005. "The Theory of Corporate Finance". Princeton U Press.

[66] Vissing-Jorgensen Annette, 2002. "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution", Journal of Political Economy 110(4):825-853.

[67] Wachter, Jessica, 2012. "Can time-varying risk of rare disasters explain aggregate stock market volatility?". Journal of Finance, forthcoming.

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