A Uniqueness Proof for Monetary Steady State*

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Abstract

The framework in Lagos and Wright (2005) combining decentralized and centralized markets is used extensively in monetary economics, for several reasons: it has relatively solid microfoundations; it is tractable; it integrates naturally with other theories; and it is easily quantifiable. Much is known about this model, but there is one loose end: only under very special assumptions about bargaining or preferences has it been shown there is a unique monetary steady state. For general decentralized market utility and bargaining, I prove uniqueness, for generic parameters using fiat money and all parameters using commodity money. I also derive monotone comparative static results.

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1 Introduction

The model in Lagos and Wright (2005), hereafter LW, combining some decentralized and some centralized trade, has been used extensively in the recent monetary economics literature. One reason is that the LW model has relatively solid microfoundations for the role of a medium of exchange (due to decentralized markets) and yet is extremely tractable (due to centralized markets). Another reason is that is allows a relatively easy integration of modern monetary theory and other branches of economics, including standard theories of growth, unemployment, taxation, banking, and even new Keynesian macro (again due to the combination of decentralized and centralized trade). Yet another reason is that it is not hard to quantify the model using either calibration or estimation techniques.[1]

Much is known about the properties of the LW model and its set of equilibria, but there is a loose end: only under severe conditions on bargaining (take-it-or-leave-it offers) or nonstandard assumptions on preferences (log-concave marginal utility) in the decentralized market has it been previously shown there exists a unique monetary steady state, and hence, for the may papers using the model only a very incomplete characterization of equilibria is available. In this note, I prove uniqueness of steady state with valued fiat money for generic parameter values, using only standard assumptions on decentralized market preferences, and general Nash bargaining power. I also show uniqueness for all parameter values in a commodity-money version, where real assets are used as media of exchange. As a corollary, I derive monotone comparative statics – e.g. the value of money is unambiguously decreasing the rate of monetary expansion.

[1] The point is that the results presented here should be of fairly wide interest because the LW framework is one that people are actually using – as opposed to a random selection from the plethora of model in the journals. See the bibliography at http://www.ssc.upenn.edu/~rwright/courses/LW-bib.pdf, which as of today has 67 entries, showing how the model can be applied to substantive issues and substantiating the claims in the text that it can be eaily calibrated or estimated, and extended to incorporate growth, opitmal taxation, sticky prices, and so on.

2 The Standard Model

Time is discrete and continues forever. Alternating over time are two types of markets: a frictionless centralized market CM, and a decentralized market DM with anonymous trade and a double coincidence problem detailed below. These two frictions in the DM make barter and credit impossible, and this makes a medium of exchange essential. Different goods X and x are traded in the CM and DM, and are produced one-for-one using labor H and h in the two markets. In the baseline model, utility over a period encompassing one CM and one DM is additively separable and linear in H,

$$\mathcal{U} = \mathcal{U}(X, H, x, h) = U(X) - H + u(x) - c(h),$$

where U, u and c satisfy the usual monotonicity and curvature properties, and there is a discount factor $\beta \in (0, 1)$ between one CM and the next DM, but not between the DM and CM.[2]

The good DM x is not storable: it must be produced and consumed simultaneously. The CM good may be storable but is not portable: if one invests k units of X in some storage technology in the CM it yields $f(k)$ in the next CM, but in between it is *illiquid* in the sense that it cannot be brought into the DM. The usual LW model has $f(k) = 0$, but nothing changes if the CM good is storable, as long as it is not portable – in fact, the only role $f(k)$ plays here is that it provides a way to measure the return on illiquid assets. Note that a similar notion of liquidity underlies much work in economics, including e.g. research using Diamond-Dybvig (1983), where resources invested in projects cannot be traded or consumed until the projects come to fruition. Also note that agents in the DM cannot trade claims on $f(k)$ to be paid off in a future CM, any more than they can trade

[2] It is easy to relax many of these assumptions to allow e.g. discounting between the CM and DM, many goods in the CM or the DM, and nonlinear technologies. One can make utility linear in X instead of H without changing anything of substance. One can also relax separability and write e.g. $\mathcal{U} = \mathcal{V}(X, x, h) - H$. Sometimes, Inada conditions like $u'(0) = \infty$ are assumed, which are useful for existence but play no role for uniquess.

other IOU's, because anonymity implies they could renege on such claims without fear of punishment.[3]

There is an asset m that is liquid, in that it can be brought into the DM and used in trade. In general one should think of m as the standard "tree" in Lucas's (1978) asset-pricing model, bearing "fruit" $\delta \geq$ as dividends in each CM, in units of X. The special case usually studied is the one where m is fiat money, $\delta = 0$. By contrast, I refer to the case $\delta > 0$ as commodity money. When $\delta > 0$ the total supply of the asset is taken to be fixed at M, so that real resources are constant and it makes sense to consider steady state. When $\delta = 0$ and m is fiat money there can be a steady state even if M is growing at rate π, as long as its price ϕ falls at the same rate. In this case, π is the inflation rate, and the nominal interest rate i is given by the Fisher equation $1 + i = (1 + \pi)/\beta$. Here π and i are exogenous policy instruments, and I assume $i > 0$ but do consider the limit as $i \to 0$.

Let $W(m, k)$ and and $V(m, k)$ be the value functions for agents in the CM and DM with portfolio (m, k). The CM problem is

$$\begin{aligned} W(m,k) &= \max_{X,H,\hat{m},\hat{k}} \left\{ U(X) - H + \beta \hat{V}(\hat{m}, \hat{k}) \right\} \\ \text{s.t. } X &= H + (\phi + \delta)m - \phi \hat{m} + f(k) - \hat{k} - T \end{aligned}$$

where $(\hat{m}, \hat{k})$ is the portfolio taken out of the CM, since a "hat" indicates a variable next period, and T is a lump-sum tax that can be used to vary M. Assume an interior solution for H (sufficient conditions are discussed in LW), and substitute from the budget equation to get

$$\begin{aligned} W(m,k) &= (\phi + \delta)m + f(k) - T + \max_X \{U(X) - X\} \\ &\quad + \max_{\hat{m},\hat{k}} \left\{ -\phi \hat{m} - \hat{k} + \beta \hat{V}(\hat{m}, \hat{k}) \right\}. \end{aligned} \tag{1}$$

This immediately yields several results: $X = X^*$ where $U'(X^*) = 1$; the

[3] See Kocherlakota (1988), Wallace (2001), Arauojo (2004), or Aliprantis et al. (2007) for more on the role of anonymity in monetary theory.

portfolio choice $(\hat{m}, \hat{k})$ is independent of (m, k); $W_m = \phi + \delta$; and $W_k = f'(k)$.[4]

In the DM with probability σ an agent want to consume x but cannot produce it, in which case he is called a buyer; with probability σ he can produce but does not want to consume, in which case he is called a seller; and with probability $1 - 2\sigma$ he does neither. Buyers and sellers are then paired off (anonymously). This generates a double coincidence problem equivalent to the usual one coming from random matching and specialized goods.[5] Each trade involves a seller giving x to the buyer in exchange for $d \leq m$, where m is the liquid asset the buyer brought to the DM, which is assumed to be observed by the seller. The agents bargain over the terms of trade (x, d) according to the generalized Nash solution, with bargaining power of the buyer given by θ and threat points given by continuation values from not trading. As in LW, it is easy to show the outcome depends on the buyer's m iff the constraint $d \leq m$ binds, and does not depend on his k or the portfolio of the seller at all.

To facilitate the presentation, begin with the case of fiat money, since $\delta > 0$ involves some technical complications that seem best to avoid for the main point (see below). Now, as is standard, in equilibrium a buyer with m in the DM wants to spend it all. Inserting $d = m$ into the Nash product and maximizing wrt x, the FOC can be rearranged to give the usual result in such models: $\phi m = g(x)$, where

$$g(x) \equiv \frac{\theta c(x) u'(x) + (1 - \theta) u(x) c'(x)}{\theta u'(x) + (1 - \theta) c'(x)}.$$

[4] These results follow directly from the quasi-linearity of $\mathcal{U}$, and keep the analysis tractable, as compared to e.g. related models in Green and Zhou (1998,2002), Camera and Corbae (1999), Zhu (2003,2005) or Molico (2006). Things are actually the same if one dispenses with quasi-linearity and instead uses nonconvexities and lotteries, as in Rocheteau et al. (2007). A different path to tractability is described in Shi (1997).

[5] A minor difference is that LW has some double coincidence meetings where both agents consume and produce. One can add something similar here, but it is completely irrelevant for the results. Other specifications in the literature (e.g. agents know whether they will be buyers or sellers before they enter the DM) also work.

Notice g is differentiable, and $\partial\hat{x}/\partial\hat{m} = \hat{\phi}/g'(\hat{x}) > 0$. Also,

$$\begin{aligned} V(m,k) &= \sigma\left[u(x) + W(0,k)\right] + \sigma[W(m+\tilde{d},k) - c(\tilde{x})] + (1-2\sigma)W(m,k) \\ &= \sigma\left[u(x) - \phi m\right] + \sigma\left[\phi\tilde{d} - c(\tilde{x})\right] + W(0,k) + \phi m, \end{aligned}$$

using $W_m = \phi$, where $\left(\tilde{x},\tilde{d}\right)$ is the trade when an agent is a seller, which as I emphasized above does not depend on (m,k).

Inserting $\hat{V}$, the CM portfolio problem in (1) can be summarized by

$$\max_{\hat{m},\hat{k}}\left\{-\phi\hat{m} - \hat{k} + \beta(1-\sigma)\hat{\phi}\hat{m} + \beta\sigma u(\hat{x}) + \beta\hat{W}(0,\hat{k})\right\}, \tag{2}$$

ignoring terms that do not depend on $(\hat{m},\hat{k})$, where it is understood that $\hat{x}$ and $\hat{\phi}\hat{m}$ are constrained via the bargaining solution $\phi m = g(x)$. Since $\hat{W}_k(\hat{m},\hat{k}) = f'(\hat{k})$, the FOC for $\hat{k}$ implies $1 \leq \beta f'(\hat{k})$, $=$ if $\hat{k} > 0$. Let k^* be the solution to this condition; given $k = k^*$ it can be dropped as a state variable from now on. The FOC for $\hat{m}$ is $\phi = \beta\hat{\phi}\left[1 - \sigma + \sigma u'(\hat{x})/g'(\hat{x})\right]$, assuming $\hat{m} > 0$, since the focus here is on monetary equilibria. Using the Fisher equation this FOC reduces to $i = \ell(\hat{x})$, where

$$\ell(x) \equiv \sigma\frac{u'(x) - g'(x)}{g'(x)}. \tag{3}$$

As is standard, $\ell(x)$ is the marginal value of liquidity. Solving $\ell(x) = i$ for x, then using market clearing $m = M$, one can recover real balances $z \equiv \phi M = g(x)$, the nominal price level $p \equiv \phi^{-1} = M/g(x)$, and the rest of the equilibrium. Discussing the existence of a solution to $\ell(x) = i$ is routine. In terms of uniquness, however, there is unfortunately no way to guarantee $\ell(x)$ is monotone without special assumptions because it depends on third derivatives of utility.[6] Indeed there are versions of the model where is $\ell(x)$ is

[6] As mentioned, there are results for special cases. Thus, $\theta = 1$ implies $g(x) = c(x)$, so $\ell'(x) < 0$ and there cannot be multiple solutions to $\ell(x) = i$; but $\theta = 1$ is very special. Also, normalizing $c(x)$ to be linear and assuming $\log u'(x)$ is concave, one can show $\ell'(x) < 0$ even though it depends on u'''; but log-concave marginal utility is a non-standard assumption, and is violated for many common functions.

definitely not monotone.[7] Hence, the literature contains only an incomplete characterization of equilibria. At this point, however, note that $\ell(x) = i$ is a necessary but not a sufficient condition for equilibrium. This observation is pursued below.

3 The Argument

Setting $\hat{k} = k^*$ and ignoring terms that do not depend on $\hat{m}$, (2) becomes

$$\max_{\hat{m}} \left\{ -\frac{\phi\hat{\phi}\hat{m}}{\hat{\phi}\beta} + (1-\sigma)\hat{\phi}\hat{m} + \sigma u(\hat{x}) \right\}.$$

Let real balances be $\hat{z} = \hat{\phi}\hat{m}$ and use the Fisher equation to reduce this to

$$\max_{\hat{m}} \left\{ -\hat{z}(i+\sigma) + \sigma u(\hat{x}) \right\},$$

where again one should interpret $\hat{x}$ and $\hat{z}$ as constrained via the bargaining solution $\hat{z} = g(\hat{x})$. Inserting this constraint, one gets something that looks like a simple (basically static) decision problem

$$\max_{\hat{x}} \left\{ -(i+\sigma)g(\hat{x}) + \sigma u(\hat{x}) \right\}. \tag{4}$$

At first blush, this observation does not help much, since the FOC from (4) is $(i+\sigma)g'(x) = \sigma u'(x)$, which reduces to $i = \ell(x)$ as before, naturally. Thus, one cannot say much about the set of solutions, since one cannot say much about the monotonicity of $\ell(x)$. Still, re-interpreting $i = \ell(x)$ as coming from a decision problem, rather than an equilibrium condition, allows one to impose some structure that can be exploited, as Cavalcanti and Puzzello (2007) point out in a different context. In particular, consider the example in Figure 1, where $\ell(x)$ is non-monotone. Consider first the limiting case $i = 0$, and notice there are three solutions to $\ell(x) = 0$, all of which are

[7] Silveira and Wright (2007) e.g. introduce a small twist that guarantees $\ell(0) = 0$, and since $\ell(x) \to 0$ as $x \to \infty$, if there is a solution to $\ell(x) = i$ there are generically multiple solutions (the twist is that there is a positive probability of being able to augment one's liquid balances after becoming a buyer; also, Inada conditions are not assumed).

candidate equilibria (the discussion obviously generalizes to any number of candidates). We now show that generically only one is an equilibrium.

A routine calculation shows that the SOC is $(i+\sigma)g''(x)+\sigma u''(x)\leq 0$, which holds at $\ell(x)=0$ iff $\ell'(x)\leq 0$. This eliminates candidate x_0: it is in fact a local minimizer for (4), while x_1 and x_2 both constitute local maximizers. For generic values of the parameters (e.g. for almost all θ) only one of the two constitutes a global maximizer. It is easy to determine which one: moving from x_1 to x_2 entails a loss and a gain, indicated respectively by the area between the curve and the axis from x_1 to x_0 and the area between the axis and the curve from x_0 to x_2.[8] Hence, in this example, when $i\to 0$ the global max occurs at x_2. Now as i increases slightly the following should be obvious: the value of the objective function falls continuously; there are still two local maximizers and a local minimizer, say $x_1(i)$, $x_2(i)$ and $x_0(i)$, with the former decreasing and the latter increasing in i; and the global maximizer is still $x_2(i)$.

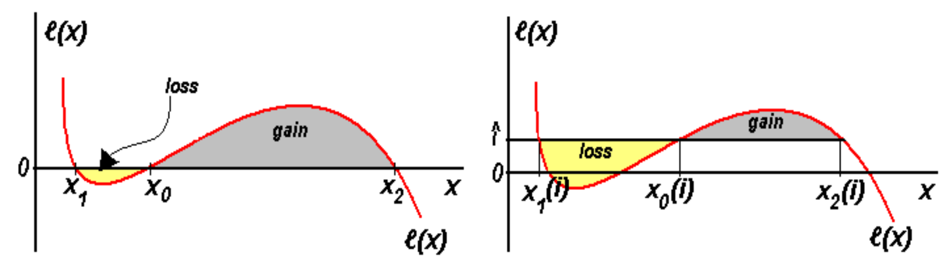

Figure 1: At $i=0$ global max occurs at x_2; at $\hat{\imath}$ it jumps from $x_2(\hat{\imath})$ to $x_1(\hat{\imath})$.

At some point as i increases, however, we may reach a point like $i=\hat{\imath}$ in the right panel, where the value of the objective function is the same at $x_2(\hat{\imath})$ and $x_2(\hat{\imath})$ because the loss between $x_1(\hat{\imath})$ and $x_0(\hat{\imath})$ just equals the

[8] This is easy to understand, since the integral of the FOC is the objective function, but note the reason the marginal value of liquidity $\ell(x)$ can go negative is a holdup problem in the bargaining. This is of no consequence for the general point however – one can start with any $i_0>0$, rather than $i=0$, and never even mention that $\ell(x)$ can be negative.

gain between $x_0(\hat{\imath})$ and $x_2(\hat{\imath})$. As i increases beyond $\hat{\imath}$, the global maximizer jumps from $x_2(i)$ to $x_1(i)$. It is clear from the diagram (or from the envelope theorem) that any jump must be to the left, not the right. So the global maximizer, say $x(i)$, decreases with i, with possiblly but not necessarily some jumps to the left. At a high enough value of i it is also possible that $x(i) = 0$ (at least, if we do not impose Inada conditions on u). These observations imply, given only that $\ell(x)$ is continuous, as i varies the global maximizer $x(i)$ is decreasing and continuous except possibly when it jumps to the left, which occurs for at most a countable number of values for i.

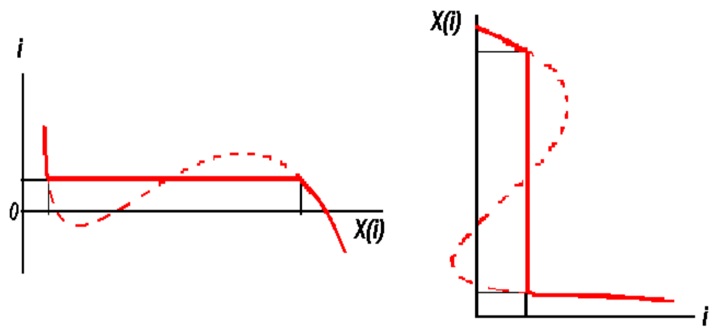

Figure 2: Inverse demand and demand curves, $X = X(i)$

The function $x(i)$ can be interpreted as an individual demand for x. Aggregate demand $X(i)$ looks similar, except one can fill in the gaps between any jumps, since if there are multiple global maximizers one can assign any fraction of the population to each. From Figure 1 one can easily see the shape of the inverse demand, or the demand curve, as depicted in the left and right panels of Figure 2. We have proved that $X(i)$ is continuous and downward sloping, with possibly some vertical (in the right panel) segments, and possibly a horizontal segment at coinciding with the i axis for high i. Steady state is fully characterized by the intersection of $X(i)$ with supply, which is vertical (in the right panel) at the exogenous i. It is now obvious that for generic i there is a unique steady state equilibrium. Given this, comparative static results follow easily and are left as exercises. Summarizing, we have proved the following:[9]

Theorem 1 *When $\delta = 0$, for generic values for i, aggregate demand $X(i)$ is single valued, and hence there cannot be mutiple steady state monetary equilibrium. Given $x > 0$, x and $z = g(x)$ are decreasing in $i = (1+r)(1+\pi)$ and increasing in θ and σ.*

4 Generalization: Commodity Money

We now study $\delta > 0$, with a fixed M so it makes sense to consider steady state. When $\delta > 0$, it is known that agents may want to hold more $\hat{m}$ than they bring to the DM, because the terms of trade can turn against you if you have a lot of liquidity at hand, as in Lagos and Rocheteau (2007) or Geromichalos et al. (2007). To model this, let agents in the CM choose $(\hat{m}_1, \hat{m}_2)$, as well as $k = k^*$, where $\hat{m}_1$ is brought to the DM and $\hat{m}_2$ is left in the CM, and let $z_j \equiv (\phi + \delta)m_j$. Then the generalized version of (**??**) is

$$\max_{\hat{z}_1, \hat{z}_2} \left\{ -\frac{\phi(\hat{z}_1 + \hat{z}_2)}{\beta(\hat{\phi} + \delta)} + \hat{z}_2 + (1-\sigma)\hat{z}_1 + \sigma u(\hat{x}) \right\},$$

s.t. the bargaining constraint $z_1 = g(x)$.

The FOC for z_2 is $-\phi/\beta(\hat{\phi} + \delta) + 1 \leq 0$, $= 0$ if $z_2 > 0$. Thus, if agents leave anything in the CM then $\phi = \beta(\hat{\phi} + \delta)$, which in steady state means assets are priced by fundamentals, $\phi = \beta\delta/(1-\beta)$, and agents are indifferent about how much they hold. The FOC for $z_1 > 0$ can be written

$$\frac{\phi}{\beta(\hat{\phi} + \delta)} - 1 = \ell(x). \tag{5}$$

To interprety this, let i_1 and i_0 be the interest rates on liquid and illiquid assets – i.e. the returns on z_1 and on either z_2 or k. Then define the *spread*

[9] The focus here is exclusively on steady state equilibria. Lagos and Wright (2003) study dynamics, from which it is clear there is no hope of generating uniqueness once one leaves the realm of steady state (as in virtually any monetary model).

by $s \equiv (i_0 - i_1)/(1+i_1)$. In equilibrium $1+i_0 = 1/\beta$ and $1+i_1 = (\hat{\phi}+\delta)/\phi$, implying LHS of (5) is exactly s. Hence, (5) equates the marginal benefit of liquidity to the spread, which is the marginal cost.

The logic in the previous section allows us to characterize the demand for x as a function of s, although it is more convenient here to use the demand for z_1 (they are equivalent because $z_1 = g(x)$). As before, aggregate demand $Z^D(s)$ is downward sloping, with s on the vertical axis, has flat segments on at most a measure 0 set of values for s, and hits the horizontal axis at $\bar{z} = g(\bar{x})$ where $\bar{x}$ solves $\ell(\bar{x}) = 0$. At $\bar{z}$ agents are satiated in the real balances they bring to the DM. Since $s < 0$ violates no-arbitrage, if $s = 0$ then agents bring exactly $z_1 = \bar{z}$ to the DM and are willing to keep any amount z_2 in the CM, while if $s > 0$ then they bring $z_1 < \bar{z}$ to the DM and keep $z_2 = 0$ in the CM. This much is clear, given the analysis of the case $\delta = 0$. The difference from that case is that now the cost of liquidity s is endogenous, since it involves the price ϕ.[10]

To see what this entails, solve $s = \phi/\beta(\phi+\delta) - 1$ for ϕ and insert the result into $z_1 = (\phi+\delta)M$ to get

$$Z^S(s) = \frac{\delta M}{1-\beta(1+s)}.$$

This is aggregate supply: it gives the real value of liquidity as a function of s, since s maps into ϕ. Inverse supply (s as a function of Z^S) is strictly increasing, concave, goes to $-\infty$ as $Z^S \to 0$, goes to $r = (1-\beta)/\beta$ as $Z^S \to \infty$, and hits the horizontal axis at $\underline{z} = \delta M/(1-\beta)$.[11] Steady state solves $Z^S(s) = Z^D(s)$. In Figure 3, drawn for δM small enough that $\underline{z} < \bar{z}$, steady state occurs at $s > 0$, so agents take $z_1 \in (\underline{z}, \bar{z})$ to the DM and leave 0 in the CM. If δM is larger, however, then $\underline{z} > \bar{z}$ and $s = 0$ in equilibrium, whence agents take $z_1 = \bar{z}$ to the DM while $z_2 = \underline{z} - \bar{z}$ is kept in the CM. The

[10] In the case $\delta = 0$, with $\pi > 0$ the spread is the nominal interest rate $s = i$, and if $\pi = 0$ it is the real rate $s = r = (1-\beta)/\beta$.

[11] Also, when $\delta \to 0$, Z^S becomes a horizontal line at $s = r$, capturing the fiat money model as a limiting case.

important observation here is there is always a unique intersection between the decreasing function $Z^D(s)$ and the strictly increasing function $Z^S(s)$, for all parameter values.

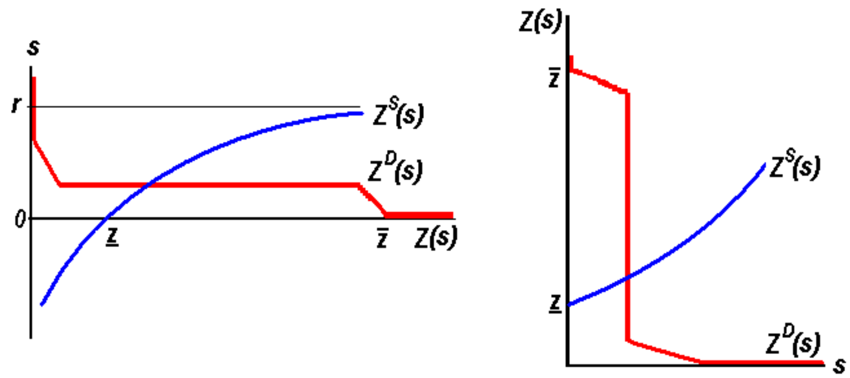

Figure 3: The commodity-money model

Theorem 2 *When $\delta > 0$ there is a unique steady state equilibrium.*

5 Conclusion

I proved uniqueness of monetary steady state for generic parameters in the standard LW model, and for all parameters in a commodity-money extension. There are two steps in the argument: first one reduces the equilibrium problem to something that looks like a simple demand problem; second, one show aggregate demand curves are continuous and at least weakly decreasing. Combined with a supply curve that is constant with fiat money and strictly monotone with commodity money, the results follow immediately. One might be surprised that demand curves are monotone – but recall the entire framework is based on quasi-linear utility in the CM, which keeps things tractable by harnessing the distribution of money balances. So perhaps the results are not so surprising. Nevertheless, since many people use the framework, the results may be useful to a broad audience.

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