Trade Dynamics in the Market for Federal Funds

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The market for federal funds

A market for loans of reserve balances at the Fed.

The market for federal funds

- What's traded?

  Unsecured loans (mostly overnight)

- How are they traded?

  Over the counter

- Who trades?

  Commercial banks, securities dealers, agencies and branches of foreign banks in the U.S., thrift institutions, federal agencies

Why is the fed funds market interesting?

- It is an interesting example of an OTC market
  (Unusually good data is available)

- Reallocates reserves among banks
  (Banks use it to offset liquidity shocks and manage reserves)

- Determines the interest rate on the shortest maturity instrument in the term structure

- Is the "epicenter" of monetary policy implementation

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- Warren

In this paper we ...

(1) Develop a model of trade in the fed funds market that explicitly accounts for the two key OTC frictions:

- Search for counterparties
- Bilateral negotiations

In this paper we ...

(2) Use the theory to address some elementary questions:

- Positive:
  - What are the determinants of the fed funds rate?
  - How does the market reallocate funds?
- Normative:

  Is the OTC market structure able to achieve an efficient reallocation of funds?

In this paper we ...

(3) Calibrate the model and use it to:

- Assess the ability of the theory to account for empirical regularities of the fed funds market:
  - Intraday evolution of reserve balances
  - Dispersion in fed funds rates and loan sizes
  - Skewed distribution of number of transactions
  - Skewed distribution of proportion of intermediated funds

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  - Intraday evolution of reserve balances
  - Dispersion in fed funds rates and loan sizes
  - Skewed distribution of number of transactions
  - Skewed distribution of proportion of intermediated funds
- Conduct policy experiments:

  What is the effect on the fed funds rate of a 25 bps increase in the interest rate that the Fed pays on reserves?

The model

- A trading session in continuous time, $t \in [0, T]$, $\tau \equiv T - t$
- Unit measure of *banks* hold reserve balances $k(\tau) \in \mathbb{K} = \{0, 1, \ldots, K\}$
- $\{n_k(\tau)\}_{k \in \mathbb{K}}$: distribution of balances at time $T - \tau$
- Linear payoffs from balances, discount at rate r
- Fed policy:
  - U_k : payoff from holding k balances at the end of the session
  - u_k : flow payoff from holding k balances during the session
- Trade opportunities are bilateral and random (Poisson rate α)
- Loan and repayment amounts determined by Nash bargaining
- Assume all loans repaid at time $T + \Delta$, where $\Delta \in \mathbb{R}_+$

Institutional features of the fed funds market

Model	**Fed funds market**
• Search and bargaining	

Institutional features of the fed funds market

Model	Fed funds market
• Search and bargaining	• Over-the-counter market

Institutional features of the fed funds market

Model

- Search and bargaining
- $[0, T]$

Fed funds market

- Over-the-counter market

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- 4:00pm-6:30pm

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- 4:00pm-6:30pm
- Distribution of reserve balances at 4:00pm
- Transactions sizes

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- Transactions sizes
- Reserve requirements, interest on reserves...

Bank with balance k contacts bank with balance k' at time $T - \tau$

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- The set of feasible post-trade balances is:

$$\Pi\left(k,k'\right)=\left\{\left(k+k'-y,y\right)\in\mathbb{K}\times\mathbb{K}:y\in\left\{0,1,\ldots,k+k'\right\}\right\}$$

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- The set of feasible loan sizes is:

$$\Gamma\left(k,k'\right)=\left\{b\in\{-K,...,0,...,K\}:\left(k-b,k'+b\right)\in\Pi\left(k,k'\right)\right\}$$

Bank with balance k contacts bank with balance k' at time $T - \tau$

- The set of feasible post-trade balances is:

$$\Pi\left(k, k'\right) = \left\{\left(k + k' - y, y\right) \in \mathbb{K} \times \mathbb{K} : y \in \left\{0, 1, \ldots, k + k'\right\}\right\}$$

- The set of feasible loan sizes is:

$$\Gamma\left(k, k'\right) = \left\{b \in \{-K, ..., 0, ..., K\} : \left(k - b, k' + b\right) \in \Pi\left(k, k'\right)\right\}$$

- $V_k\left(\tau\right)$: value of a bank with balance k at time $T - \tau$

Bargaining

Bank with balance k contacts bank with balance k' at time $T-\tau$.

The *loan size* b, and the *repayment* R maximize:

$$\left[V_{k-b}(\tau)+e^{-r(\tau+\Delta)}R-V_k(\tau)\right]^{\frac{1}{2}}\left[V_{k'+b}(\tau)-e^{-r(\tau+\Delta)}R-V_{k'}(\tau)\right]^{\frac{1}{2}}$$

$$\text{s.t.} \qquad b\in\Gamma\left(k,k'\right), \quad R\in\mathbb{R}$$

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$$\text{s.t.} \qquad b\in\Gamma\left(k,k'\right), \quad R\in\mathbb{R}$$

$$b^* \in \arg\max_{b\in\Gamma(k,k')}\left[V_{k'+b}(\tau)+V_{k-b}(\tau)-V_{k'}(\tau)-V_k(\tau)\right]$$

$$e^{-r(\tau+\Delta)}R^*=\frac{1}{2}\left[V_{k'+b^*}(\tau)-V_{k'}(\tau)\right]+\frac{1}{2}\left[V_k(\tau)-V_{k-b^*}(\tau)\right]$$

Value function

$$rV_i(\tau) + \dot{V}_i(\tau) =$$

$$= u_i + \frac{\alpha}{2} \sum_{j,k,s \in \mathbb{K}} n_j(\tau)\, \phi_{ij}^{ks}(\tau) \left[V_k(\tau) + V_s(\tau) - V_i(\tau) - V_j(\tau) \right]$$

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with $V_i(0) = U_i$, and

$$\phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k,s) \in \Omega_{ij}[\mathbf{V}(\tau)] \\ 0 & \text{if } (k,s) \notin \Omega_{ij}[\mathbf{V}(\tau)] \end{cases}$$

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with

$$\Omega_{ij}[\mathbf{V}(\tau)] \equiv \arg \max_{(k',s') \in \Pi(i,j)} \left[V_{k'}(\tau) + V_{s'}(\tau) - V_i(\tau) - V_j(\tau)\right]$$

where $\tilde{\phi}_{ij}^{ks}(\tau) \geq 0$ and $\sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} \tilde{\phi}_{ij}^{ks}(\tau) = 1$

Time-path for the distribution of balances

For all $k \in \mathbb{K}$,

$$\begin{aligned} \dot{n}_k(\tau) &= \alpha n_k(\tau) \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i(\tau) \phi_{ki}^{sj}(\tau) \\ &\quad - \alpha \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{s \in \mathbb{K}} n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) \end{aligned}$$

Definition

An equilibrium is a value function, $\mathbf{V}$, a path for the distribution of reserve balances, $\mathbf{n}(\tau)$, and a path for the distribution of trading probabilities, $\boldsymbol{\phi}(\tau)$, such that:

(a) given the value function and the distribution of trading probabilities, the distribution of balances evolves according to the law of motion; and

(b) given the path for the distribution of balances, the value function and the distribution of trading probabilities satisfy individual optimization given the bargaining protocol.

Assumption A. For any $i, j \in \mathbb{K}$, and all $(k, s) \in \Pi(i, j)$, the payoff functions satisfy:

$$u_{\lceil \frac{i+j}{2} \rceil} + u_{\lfloor \frac{i+j}{2} \rfloor} \geq u_k + u_s$$

$$U_{\lceil \frac{i+j}{2} \rceil} + U_{\lfloor \frac{i+j}{2} \rfloor} \geq U_k + U_s, \text{ “} > \text{” unless } k \in \left\{ \left\lfloor \frac{i+j}{2} \right\rfloor, \left\lceil \frac{i+j}{2} \right\rceil \right\}$$

where for any $x \in \mathbb{R}$,

$$\lfloor x \rfloor \equiv \max \{k \in \mathbb{Z} : k \leq x\}$$

$$\lceil x \rceil \equiv \min \{k \in \mathbb{Z} : x \leq k\}$$

Proposition

Let the payoff functions satisfy Assumption A. Then:

(i) An equilibrium exists. The paths $\mathbf{V}(\tau)$ *and* $\mathbf{n}(\tau)$ *are unique.*

(ii) The equilibrium path for $\boldsymbol{\phi}(\tau) = \{\phi_{ij}^{ks}(\tau)\}_{i,j,k,s\in\mathbb{K}}$ *is*

$$\phi_{ij}^{ks}(\tau) = \begin{cases} \tilde{\phi}_{ij}^{ks}(\tau) & \text{if } (k,s) \in \Omega_{ij}^* \\ 0 & \text{if } (k,s) \notin \Omega_{ij}^* \end{cases}$$

where $\tilde{\phi}_{ij}^{ks}(\tau) \geq 0$ *and* $\sum_{(k,s)\in\Omega_{ij}^*} \tilde{\phi}_{ij}^{ks}(\tau) = 1$*, with*

$$\Omega_{ij}^* = \begin{cases} \left\{\left(\frac{i+j}{2}, \frac{i+j}{2}\right)\right\} & \text{if } i+j \text{ even} \\ \left\{\left(\left\lfloor\frac{i+j}{2}\right\rfloor, \left\lceil\frac{i+j}{2}\right\rceil\right), \left(\left\lceil\frac{i+j}{2}\right\rceil, \left\lfloor\frac{i+j}{2}\right\rfloor\right)\right\} & \text{if } i+j \text{ odd.} \end{cases}$$

Proposition

Let the payoff functions satisfy Assumption A. Then, the equilibrium supports an efficient allocation of reserve balances.

Positive implications

The theory delivers:

(1) Time-varying distribution of trade sizes, trade volume

(2) Time-varying distribution of fed fund rates

(3) Endogenous intermediation

Trade volume

- Flow volume of trade at time $T - \tau$:

$$\bar{v}(\tau) = \sum_{i \in \mathbb{K}} \sum_{j \in \mathbb{K}} \sum_{k \in \mathbb{K}} \sum_{s \in \mathbb{K}} v_{ij}^{ks}(\tau)$$

where

$$v_{ij}^{ks}(\tau) \equiv \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) |k - i|$$

Trade volume

- Flow volume of trade at time $T-\tau$:

$$\bar{\upsilon}(\tau) = \sum_{i\in\mathbb{K}} \sum_{j\in\mathbb{K}} \sum_{k\in\mathbb{K}} \sum_{s\in\mathbb{K}} \upsilon_{ij}^{ks}(\tau)$$

where

$$\upsilon_{ij}^{ks}(\tau) \equiv \alpha n_i(\tau) n_j(\tau) \phi_{ij}^{ks}(\tau) |k-i|$$

- Total volume traded during the trading session:

$$\bar{\upsilon} = \int_0^T \bar{\upsilon}(\tau)\, d\tau$$

Fed funds rate

- If a bank with i borrows $k-i=j-s$ from bank with j at time $T-\tau$, the interest rate on the loan is:

$$\rho_{ij}^{ks}\left(\tau\right)=\frac{\ln\left[\frac{R_{ij}^{ks}(\tau)}{k-i}\right]}{\tau+\Delta}=r+\frac{\ln\left[\frac{V_j(\tau)-V_s(\tau)}{j-s}+\frac{\frac{1}{2}S_{ij}^{ks}(\tau)}{j-s}\right]}{\tau+\Delta}$$

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- The daily average (value-weighted) fed funds rate is:

$$\bar{\rho}=\frac{1}{T}\int_0^T\bar{\rho}(\tau)\,d\tau$$

where

$$\bar{\rho}(\tau)\equiv\sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}}\omega_{ij}^{ks}(\tau)\,\rho_{ij}^{ks}(\tau)$$

$$\omega_{ij}^{ks}(\tau)\equiv v_{ij}^{ks}(\tau)/\bar{v}(\tau)$$

Endogenous intermediation

- Cumulative purchases: $O^p = \sum_{n=1}^{N} \max\{k_n - k_{n-1}, 0\}$
- Cumulative sales: $O^s = -\sum_{n=1}^{N} \min\{k_n - k_{n-1}, 0\}$

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Bank-level measures of intermediation

- *Excess funds reallocation:*

$$X = O^p + O^s - |O^p - O^s|$$

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Bank-level measures of intermediation

- *Excess funds reallocation:*

$$X = O^p + O^s - |O^p - O^s|$$

- *Proportion of intermediated funds:*

$$\iota = \frac{X}{O^p + O^s}$$

- Analytics for special case with $\mathbb{K} = \{0, 1, 2\}$
- Intuition for efficiency result
- Frictionless limit
- Figures

Payoff functions

$$e^{r\Delta_f} U_k = \begin{cases} k + i_f^r \bar{k} + i_f^e (k - \bar{k}) & \text{if } \bar{k} \leq k \\ (1 + i_f^r) k - \min(i_f^w - i_f^r, i_f^c) (\bar{k} - k) & \text{if } k < \bar{k} \end{cases}$$

$$u_k = k^{1-\epsilon} i_+^d \quad \text{with} \quad \epsilon \approx 0$$

Payoff functions

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Baseline parameters

T	Δ_f	Δ	i_+^d	i_f^r	i_f^e	i_f^w	i_f^c	θ	α	r
$\frac{2.5}{24}$	$\frac{2.5}{24}$	$\frac{22}{24}$	$\frac{10^{-7}}{360}$	$\frac{.0025}{360}$	$\frac{.0025}{360}$	$\frac{.0075}{360}$	$\frac{.0175}{360}$	$\frac{1}{2}$	50	$\frac{0.0001}{365}$

Small-scale simulations: $\mathbb{K} = \{0, 1, 2\}$

$$\bar{k} = 1$$

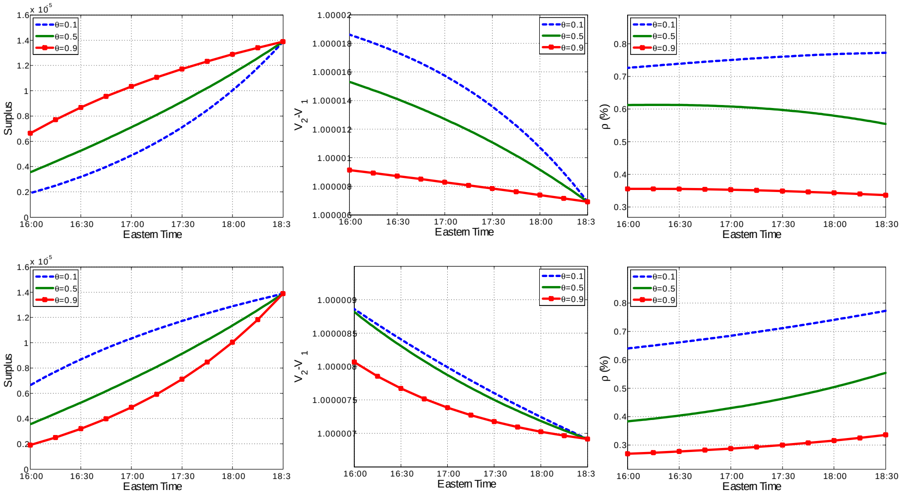
Two scenarios

$\{n_0^H(T), n_2^L(T)\}$	$\{n_0^L(T), n_2^H(T)\}$
$\{0.6, 0.3\}$	$\{0.3, 0.6\}$

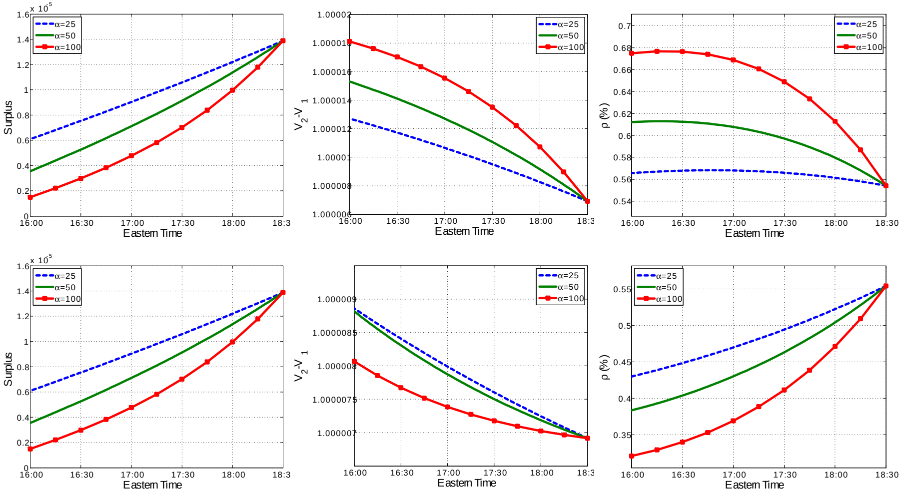
Experiments

Bargaining Power (θ)			Discount Rate (i_f^w)			Contact Rate (α)		
0.1	0.5	0.9	$\frac{.0050}{360}$	$\frac{.0075}{360}$	$\frac{.0100}{360}$	25	50	100

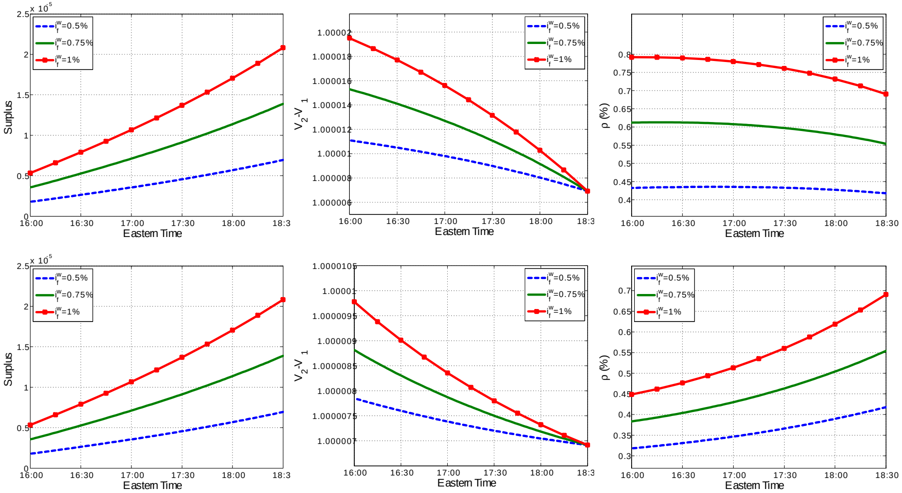
Bargaining power



Contact rate



Discount-Window lending rate



Large-scale simulations: $\mathbb{K} = \{0, 1, ..., 49\}$

$$\bar{k} = 1$$

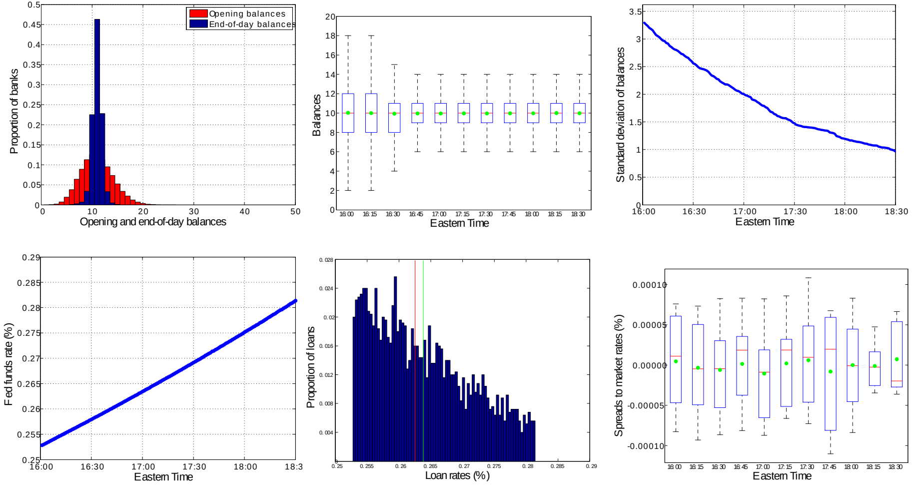
Initial distribution of balances:

$$n_k(T) = \frac{\lambda^k e^{-\lambda}}{k! \sum_{j=0}^{49} n_j(T)} \quad \text{with} \quad \lambda = 10$$

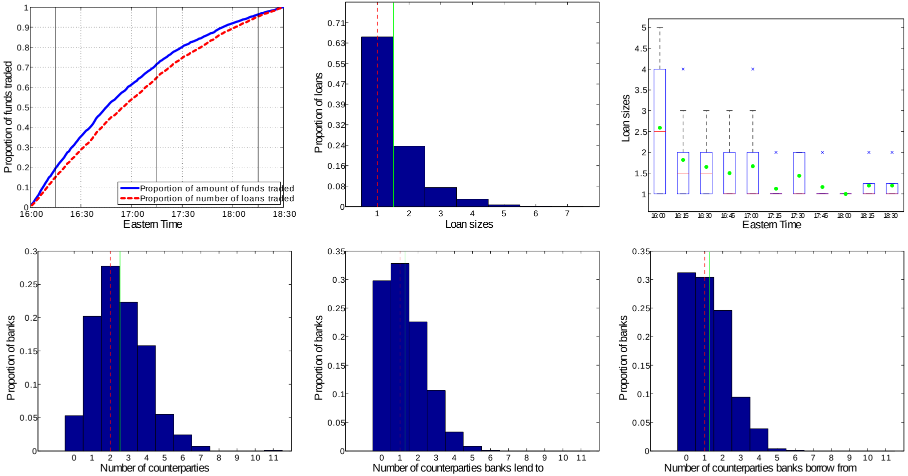
$$\Rightarrow$$

$$Q = \sum_{j=0}^{49} k n_k(T) \approx 10$$

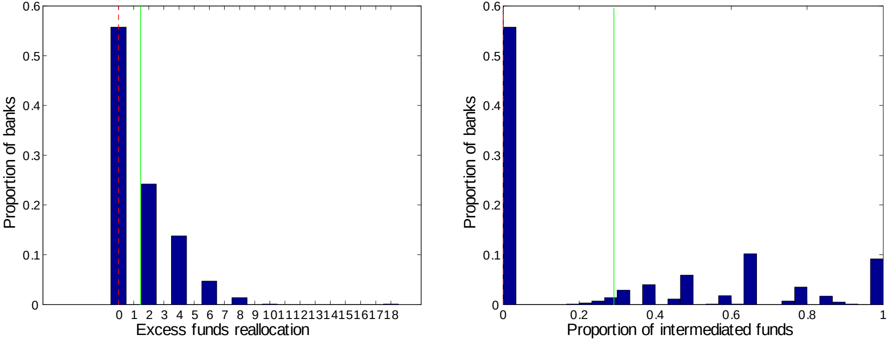
Reserve balances and fed funds rates



Size distribution of loans and distributions of trading activity



Intermediation



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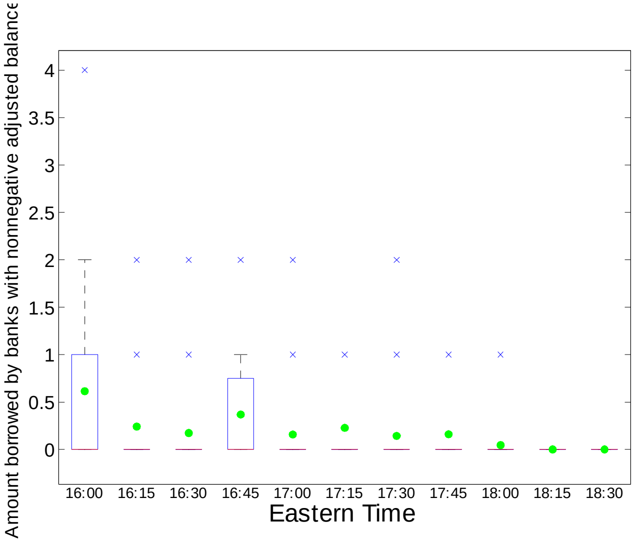


i_f	$Q/\bar{k} = 0.50$	$Q/\bar{k} = 1.00$	$Q/\bar{k} = 1.67$
0	76	38	1
25	76	51	26
50	76	63	51
75	76	76	76

i_f^w	$Q/\bar{k} = 0.50$	$Q/\bar{k} = 1.00$	$Q/\bar{k} = 1.67$
25	26	26	26
50	51	38	26
75	76	51	26
100	101	63	26

Corridor system

(i_f, i_f^w)	$Q/\bar{k} = 0.50$	$Q/\bar{k} = 1$	$Q/\bar{k} = 1.67$
$0 - 50$	51	26	1
$25 - 75$	76	51	26
$50 - 100$	101	76	51
$75 - 125$	126	101	76
$100 - 150$	151	126	101

IOR Policy intuition from the analytical example

Proposition

If $r \approx 0$,

$$\rho_f(\tau) \approx \beta(\tau) i_f^e + [1 - \beta(\tau)] i_f^w \qquad \text{where}$$

1. *If* $n_2(T) = n_0(T)$, $\beta(\tau) = \theta$
2. *If* $n_2(T) < n_0(T)$, $\beta(\tau) \in [0, \theta]$, $\beta(0) = \theta$ *and* $\beta'(\tau) < 0$
3. *If* $n_0(T) < n_2(T)$, $\beta(\tau) \in [\theta, 1]$, $\beta(0) = \theta$ *and* $\beta'(\tau) > 0$.

Figures

Ex-ante heterogeneity

We also extend the model to allow for:

1. Heterogeneity in contact rates
2. Heterogeneity in bargaining powers
3. Heterogeneity in target balances
   (or non-bank participants, e.g., GSEs)

More to be done...

- Fed funds brokers
- Banks' portfolio decisions
- Random "payment shocks"
- Sequence of trading sessions
- Quantiative work with ex-ante heterogeneity

The views expressed here are not necessarily reflective of views at the Federal Reserve Bank of New York or the Federal Reserve System.

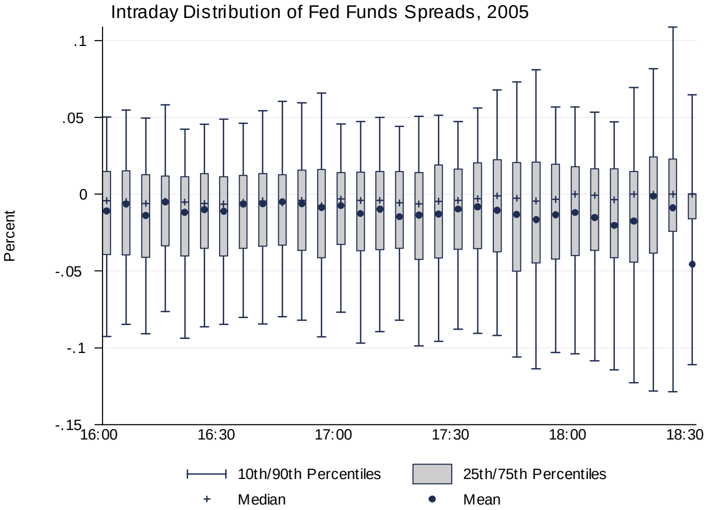
Theoretical and empirical rates

Data	**Model**
$1 + i_f^r$	$e^{i^r \Delta_f}$
$1 + i_f^e$	$e^{i^e \Delta_f}$
$1 + \rho_f(\tau)$	$e^{\rho(\tau)(\tau+\Delta)}$

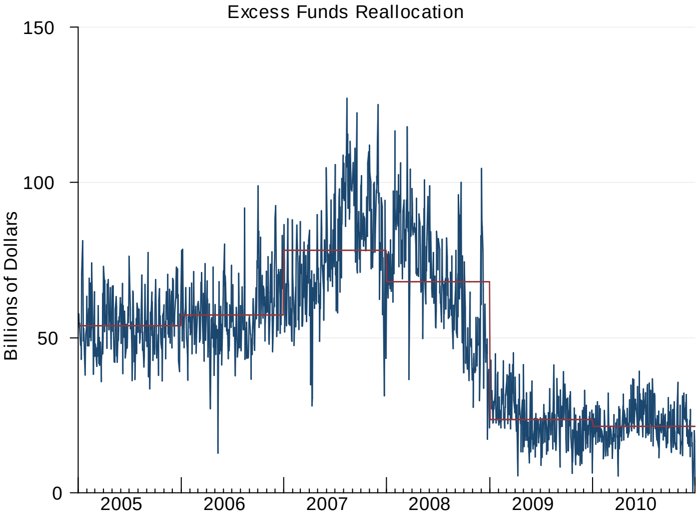
Evidence of OTC frictions in the fed funds market

- Price dispersion
- Intermediation
- Intraday evolution of the distribution of reserve balances
- There are banks that are "very long" and buy
  There are banks that are "very short" and sell

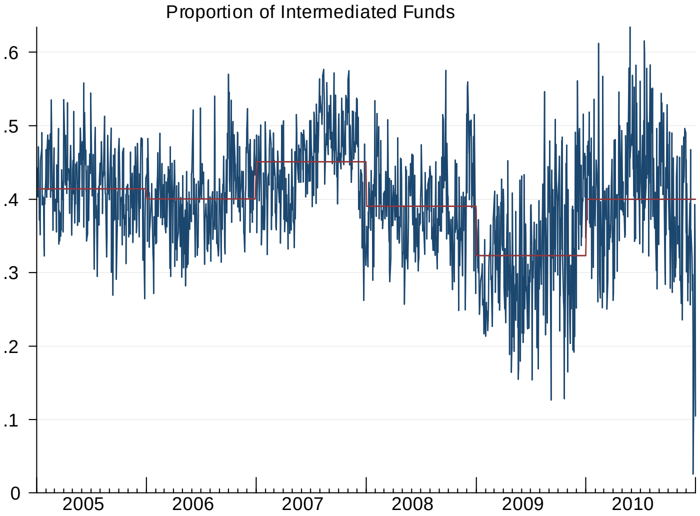
Price dispersion



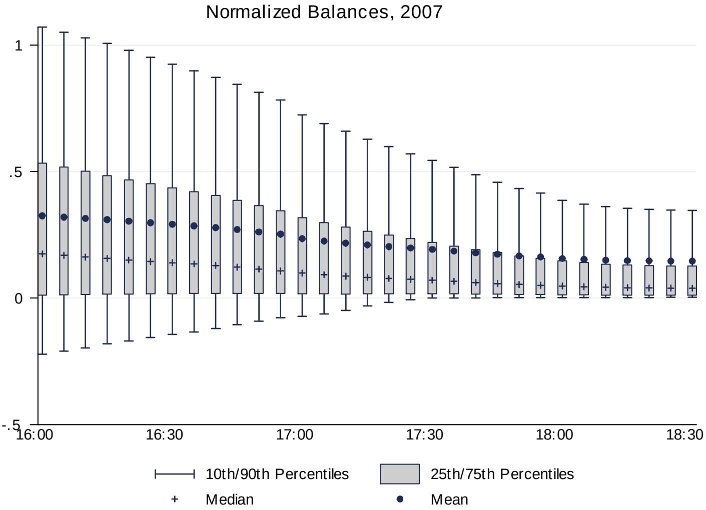
Intermediation: excess funds reallocation



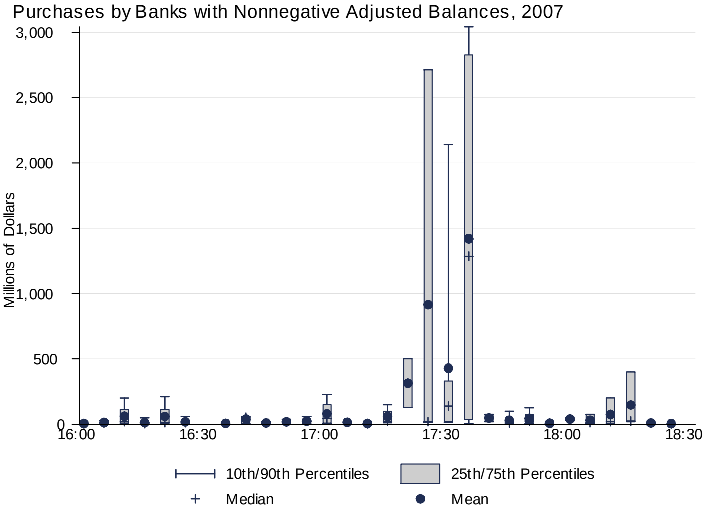
Intermediation: proportion of intermediated funds



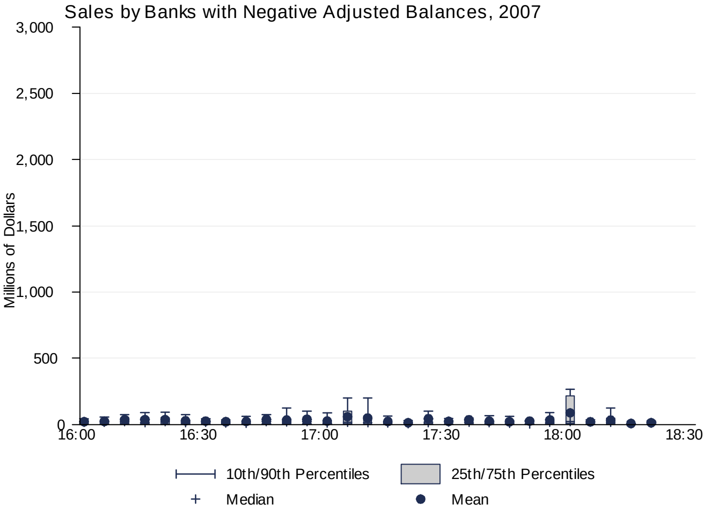
Intraday evolution of the distribution of reserve balances



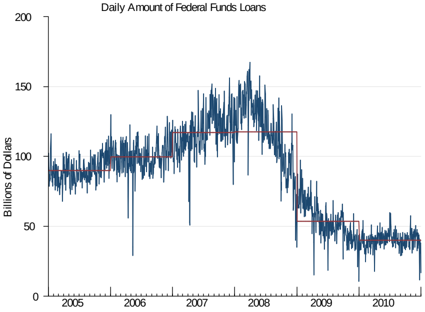
Banks that are “long” ...and buy...

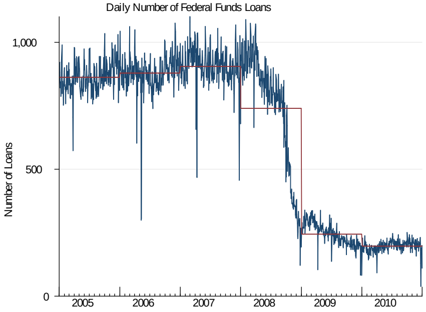


Banks that are "short"...and sell...

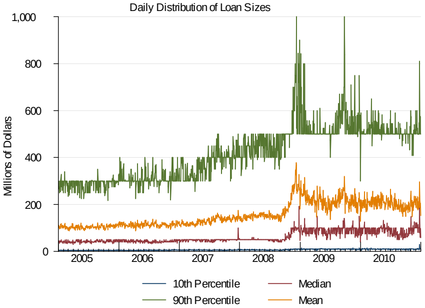


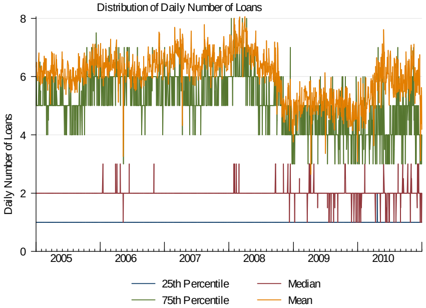
Daily volume



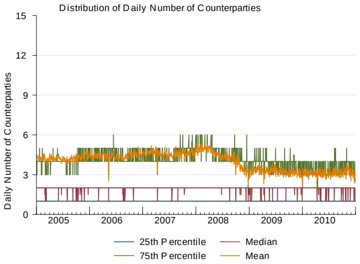


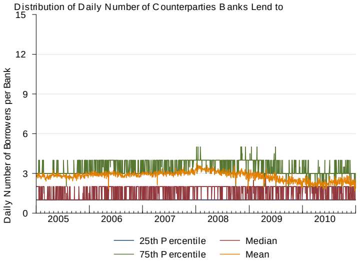
Daily volume (size distribution)

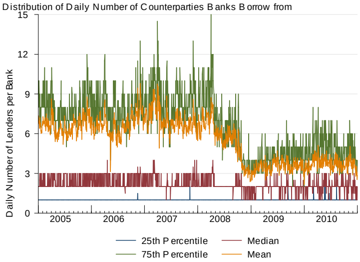




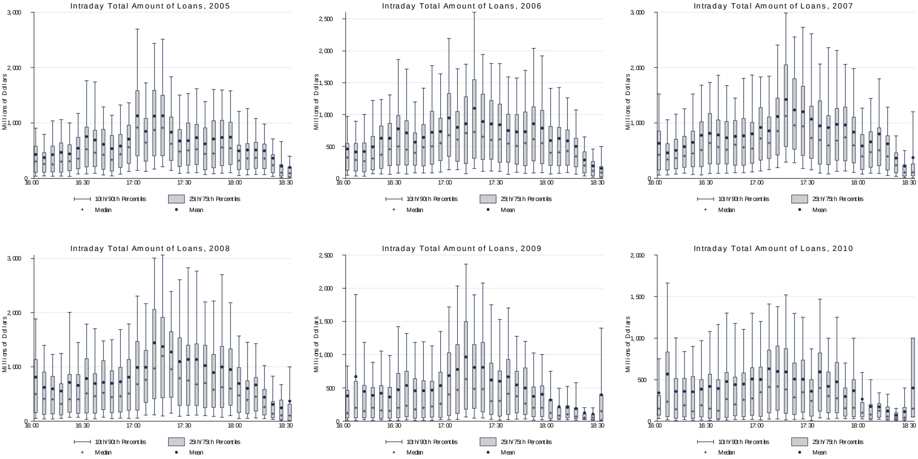
Daily distribution of the number of counterparties



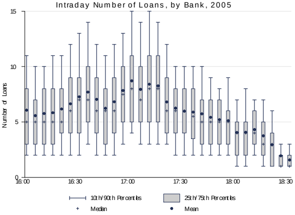


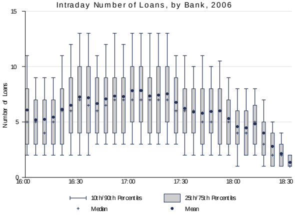


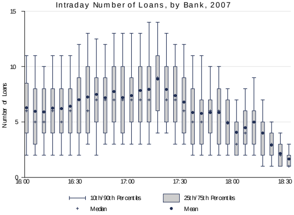
Intraday volume (dollar amount)

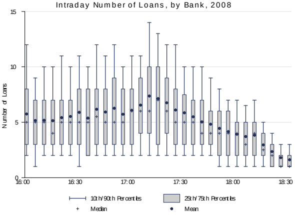


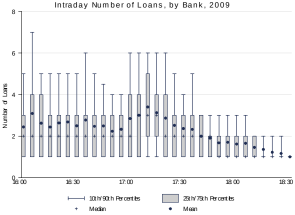
Intraday volume (number of loans)

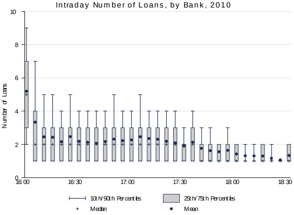




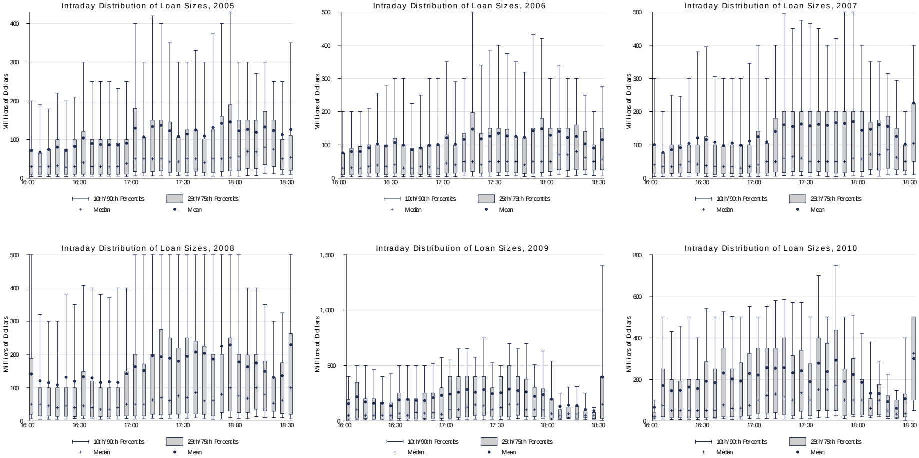




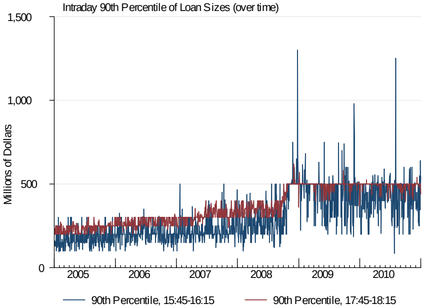


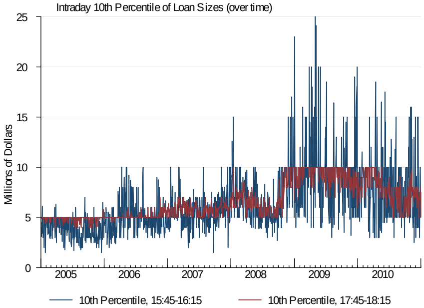


Intraday size distribution of loans

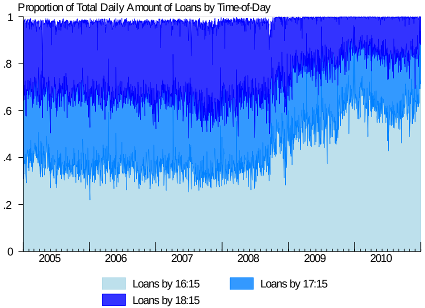


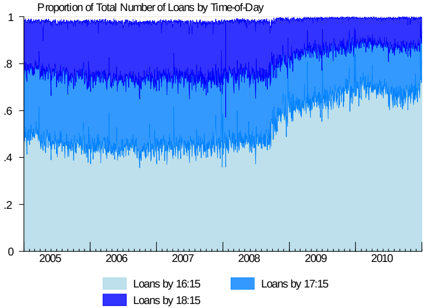
Intraday size distribution of loans



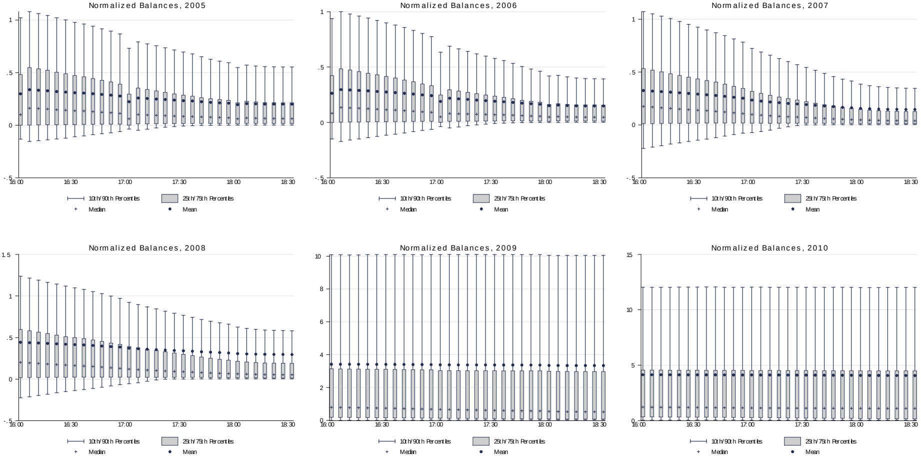


Trading activity by time-of-day

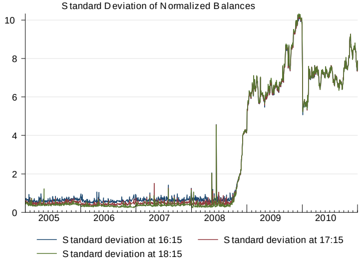


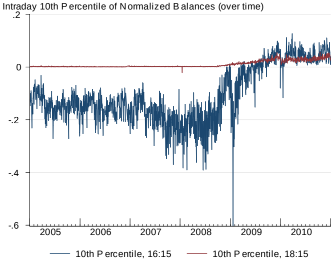


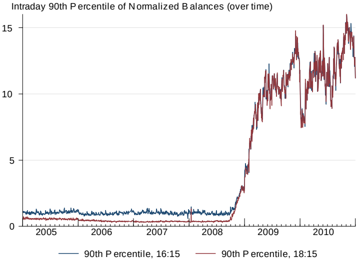
Intraday evolution of the distribution of reserve balances



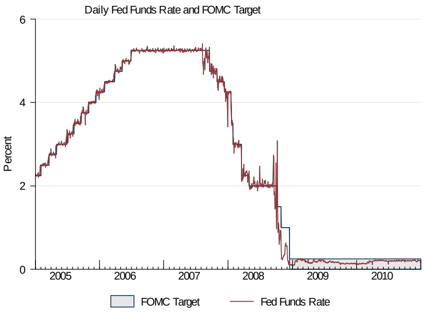
Intraday evolution of the distribution of reserve balances

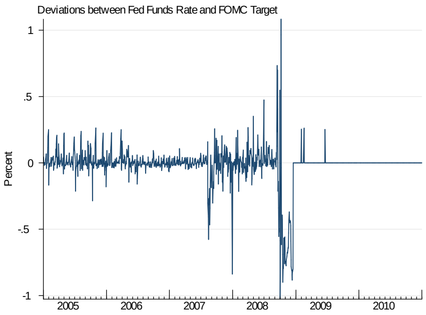




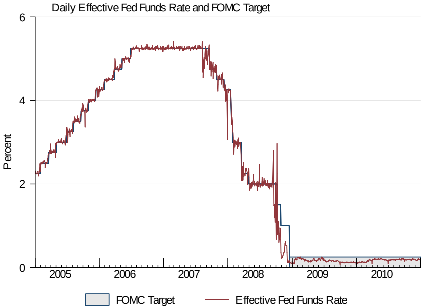


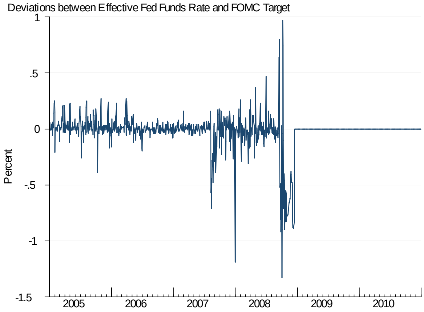
Daily fed funds rate vs. FOMC target



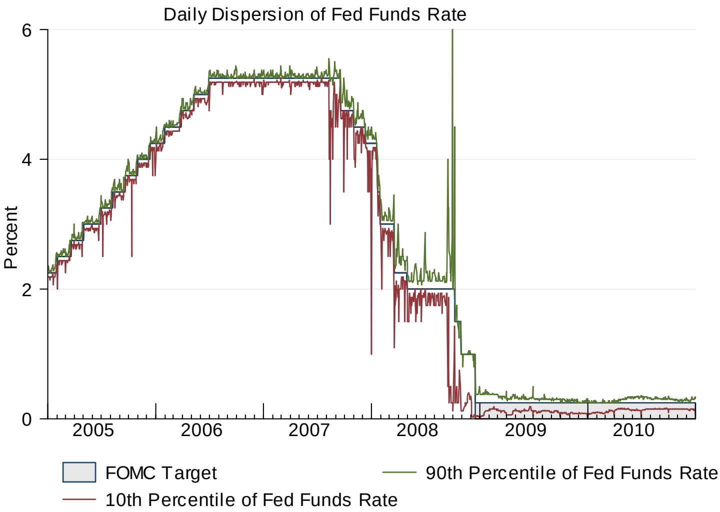


Daily effective fed funds rate vs. FOMC target

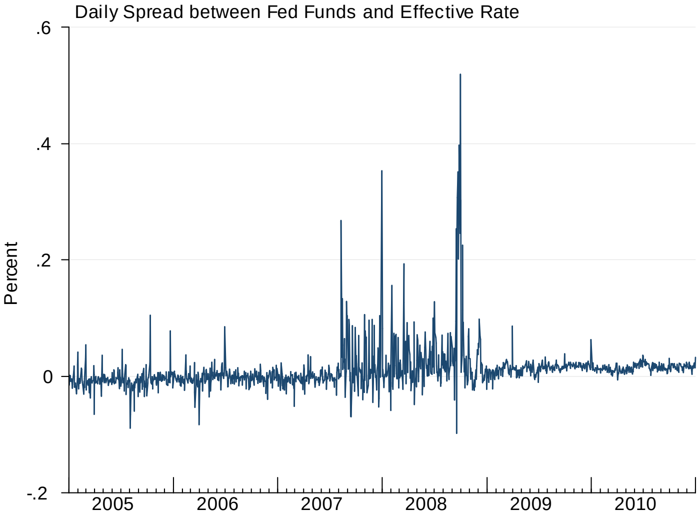




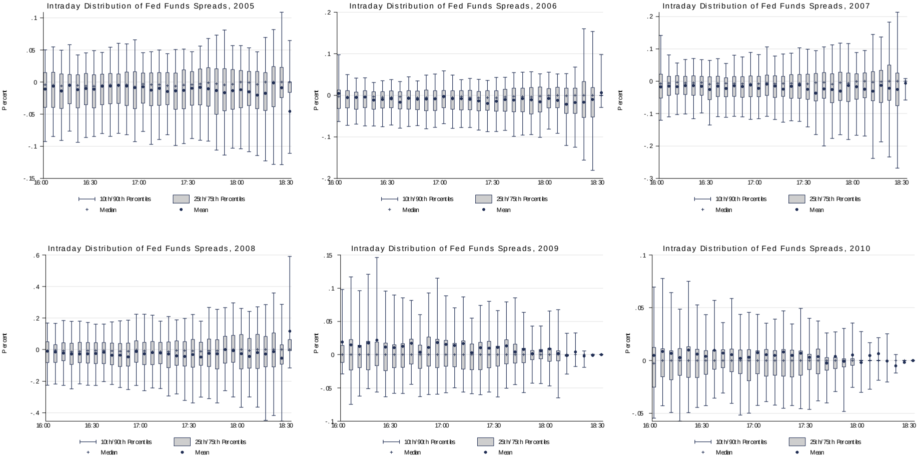
Daily fed funds rate dispersion



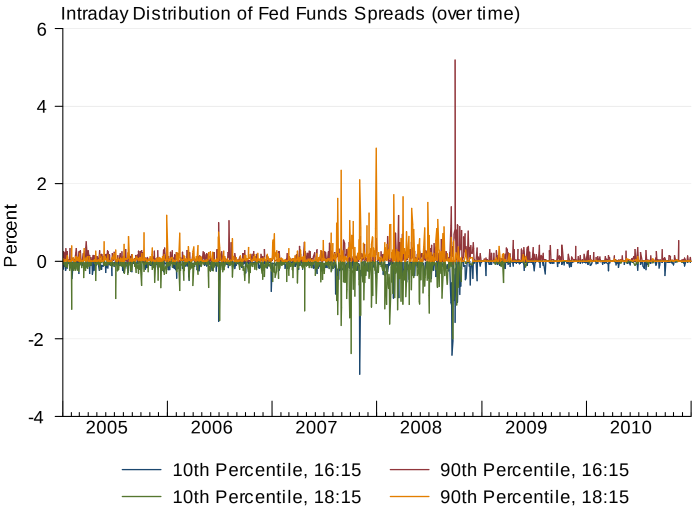
Fed funds rate vs. effective fed funds rate



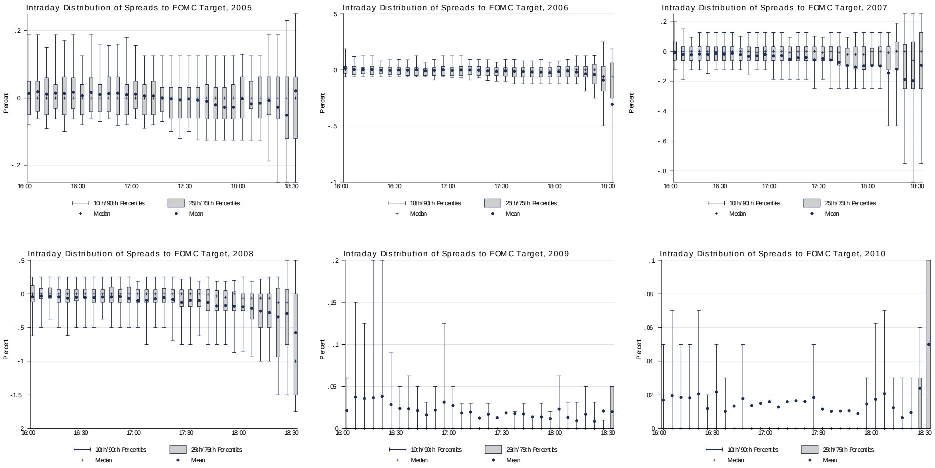
Intraday distribution of fed funds spreads



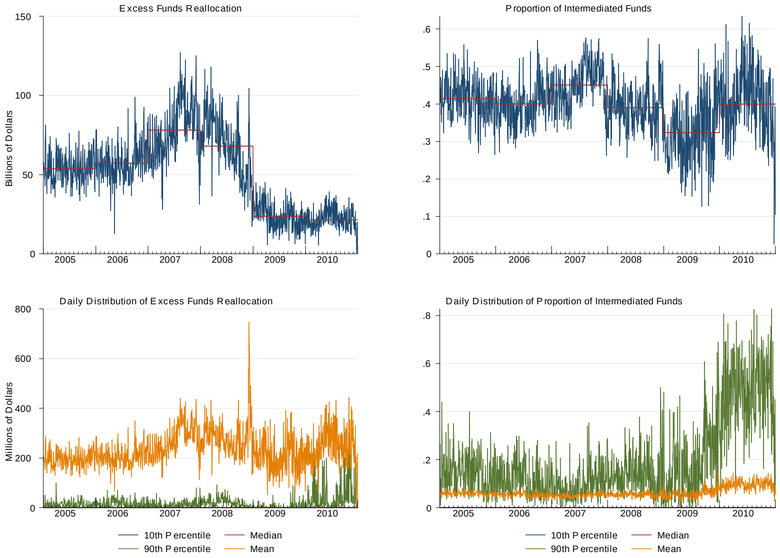
Intraday distribution of fed funds spreads (over time)



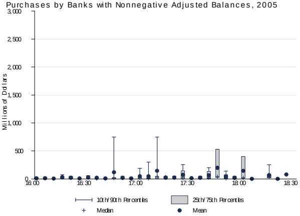
Intraday distribution of fed funds/FOMC target spreads

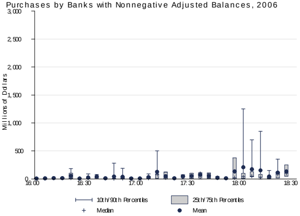


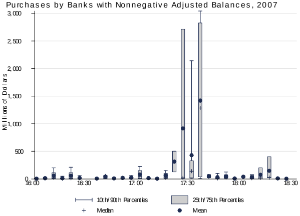
Daily intermediation

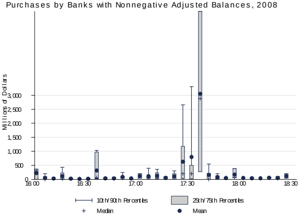


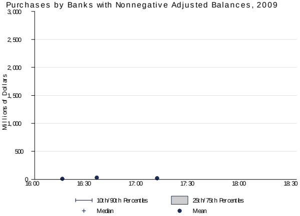
Banks that are "long"...and buy...

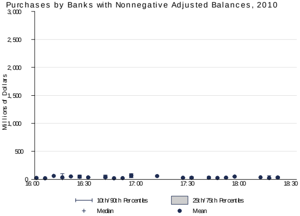




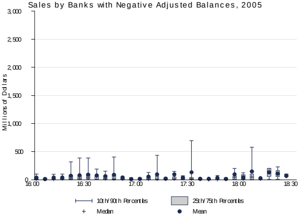


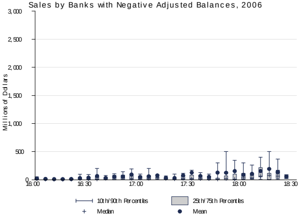


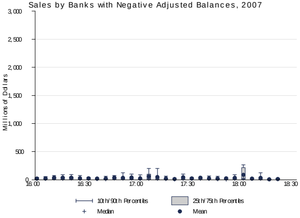


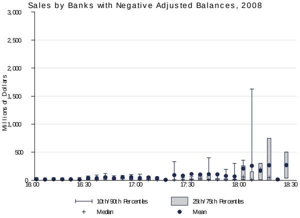


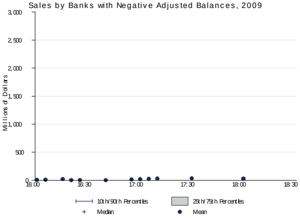
Banks that are "short"...and sell...

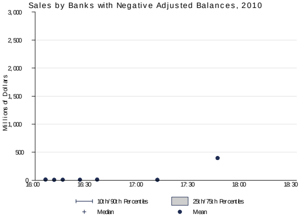




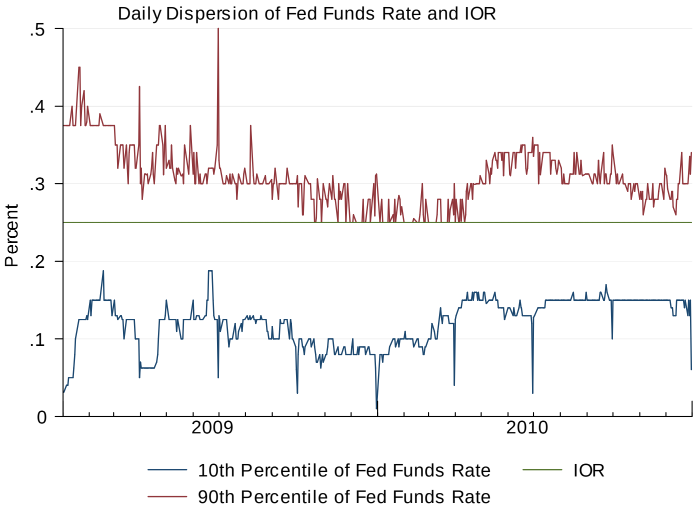




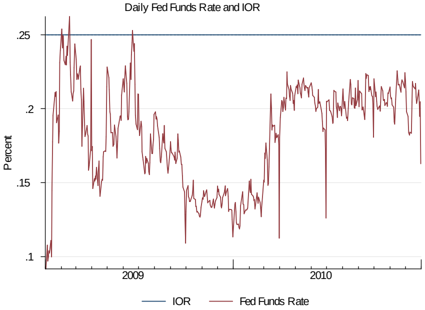


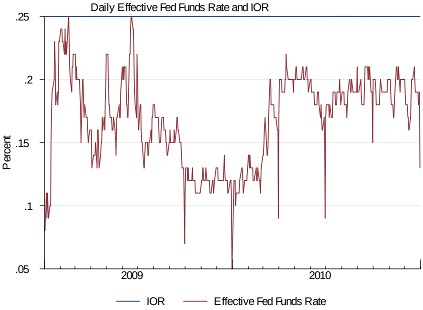


Daily fed funds rate vs. IOR



Daily FFR and daily effective FFR vs. IOR: a puzzle





Value function (derivation)

$$J_k(x,\tau) = \mathbb{E}\left\{ \int_0^{\min(\tau_\alpha,\tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau}\left(U_k + e^{-r\Delta}x\right) + \right.$$

$$\left. \mathbb{I}_{\{\tau_\alpha \leq \tau\}} e^{-r\tau_\alpha} \int J_{k-b_{\mathbf{ss}'}(\tau-\tau_\alpha)}\left(x + R_{\mathbf{s}'\mathbf{s}}\left(\tau-\tau_\alpha\right), \tau-\tau_\alpha\right) \mu\left(d\mathbf{s}', \tau-\tau_\alpha\right) \right\}$$

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- τ_α : time until next trading opportunity
- $b_{\mathbf{ss}'}(\tau)$: balance that bank $\mathbf{s} = (k, x)$ lends to bank $\mathbf{s}' = (k', x')$ at time $T - \tau$
- $R_{\mathbf{s}'\mathbf{s}}(\tau)$: repayment negotiated at time $T - \tau$ (due at $T + \Delta$)
- $\mu(\cdot, \tau)$: prob. measure over individual states, $\mathbf{s}' = (k', x')$

Bargaining

Bank with $\mathbf{s} = (k, x)$ meets bank $\mathbf{s}' = (k', x')$ at $T - \tau$.

The loan size b and the repayment R maximize:

$$\left[J_{k-b}(x+R,\tau) - J_k(x,\tau)\right]^{\frac{1}{2}} \left[J_{k'+b}(x'-R,\tau) - J_{k'}(x',\tau)\right]^{\frac{1}{2}}$$

$$\text{s.t.} \qquad b \in \Gamma(k, k')$$

$$R \in \mathbb{R}$$

Value function (derivation)

$$J_k(x,\tau) = V_k(\tau) + e^{-r(\tau+\Delta)}x \qquad \text{where}$$

$$V_k(\tau) = \mathbb{E}\left\{\int_0^{\min(\tau_\alpha,\tau)} e^{-rz}u_k dz + \mathbb{I}_{\{\tau_\alpha>\tau\}}e^{-r\tau}U_k + \mathbb{I}_{\{\tau_\alpha\leq\tau\}}e^{-r\tau_\alpha}\right.$$

$$\left.\sum_{k'\in\mathbb{K}} n_{k'}(\tau-\tau_\alpha)\left[V_{k-b_{kk'}(\tau-\tau_\alpha)}(\tau-\tau_\alpha) + e^{-r(\tau+\Delta-\tau_\alpha)}R_{k'k}(\tau-\tau_\alpha)\right]\right\}$$

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$$b_{kk'}(\tau) \in \arg\max_{b\in\Gamma(k,k')}\left[V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)\right]$$

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$$b_{kk'}(\tau) \in \arg\max_{b\in\Gamma(k,k')}\left[V_{k'+b}(\tau)+V_{k-b}(\tau)-V_{k'}(\tau)-V_k(\tau)\right]$$

$$\begin{aligned} e^{-r(\tau+\Delta)}R_{k'k}(\tau) &= \frac{1}{2}\left[V_{k'+b_{kk'}(\tau)}(\tau)-V_{k'}(\tau)\right]+ \\ &\quad \frac{1}{2}\left[V_k(\tau)-V_{k-b_{kk'}(\tau)}(\tau)\right] \end{aligned}$$

Special case with $\mathbb{K} = \{0, 1, 2\}$

- Bank with $i = 2$ is a *lender*, bank with $j = 0$, a *borrower*
- $\theta \in [0, 1]$: bargaining power of the borrower
- Only potentially profitable trade is between $i = 0$ and $j = 2$
- $S(\tau) \equiv 2V_1(\tau) - V_2(\tau) - V_0(\tau)$
- Conjecture $S(\tau) > 0$ for all $\tau \in [0, T]$ (to be verified later)

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Given $\{n_k(T)\}$, the distribution of balances follows:

$$\dot{n}_0(\tau) = \alpha n_2(\tau) n_0(\tau)$$
$$\dot{n}_2(\tau) = \alpha n_2(\tau) n_0(\tau)$$

Time-path for the distribution of balances

$$n_2(\tau) = n_2(T) - [n_0(T) - n_0(\tau)]$$

$$n_1(\tau) = 1 - n_0(\tau) - n_2(\tau)$$

$$n_0(\tau) = \frac{[n_2(T) - n_0(T)]\, n_0(T)}{n_2(T)\, e^{\alpha[n_2(T) - n_0(T)](T-\tau)} - n_0(T)}$$

Bargaining

The repayment R solves:

$$\max_{R} \left[V_1(\tau) - V_0(\tau) - e^{-r(\tau+\Delta)}R\right]^{\theta} \left[V_1(\tau) - V_2(\tau) + e^{-r(\tau+\Delta)}R\right]^{1-\theta}$$

$$\Rightarrow$$

$$e^{-r(\tau+\Delta)}R(\tau) = \theta\left[V_2(\tau) - V_1(\tau)\right] + (1-\theta)\left[V_1(\tau) - V_0(\tau)\right]$$

Value function

$$rV_0(\tau) + \dot{V}_0(\tau) = u_0 + \alpha n_2(\tau)\,\theta S(\tau)$$

$$rV_1(\tau) + \dot{V}_1(\tau) = u_1$$

$$rV_2(\tau) + \dot{V}_2(\tau) = u_2 + \alpha n_0(\tau)(1-\theta)\,S(\tau)$$

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$$\dot{S}(\tau) + \delta(\tau) S(\tau) = 2u_1 - u_2 - u_0$$

$$\delta(\tau) \equiv \{r + \alpha[\theta n_2(\tau) + (1-\theta) n_0(\tau)]\}$$

Surplus

$$S(\tau) = \left(\int_0^\tau e^{-\left[\bar{\delta}(\tau) - \bar{\delta}(z)\right]} dz \right) \bar{u} + e^{-\bar{\delta}(\tau)} S(0)$$

$$\bar{u} \equiv 2u_1 - u_2 - u_0$$

$$S(0) = 2U_1 - U_2 - U_0$$

$$\bar{\delta}(\tau) \equiv \int_0^\tau \delta(x)\, dx$$

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Fed funds rate

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Intuition for efficiency result

- Equilibrium:

  Gain from trade as perceived by borrower: $\theta S(\tau)$

  Gain from trade as perceived by lender: $(1-\theta) S(\tau)$

- Planner:

  Each of their marginal contributions equals $S^*(\tau)$

- $\delta^*(\tau) \geq \delta(\tau)$ for all $\tau \in [0, T]$, with "=" only for $\tau = 0$

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- Planner internalizes that searching borrowers and lenders make it easier for other lenders and borrowers to find partners
- These "liquidity provision services" to others receive no compensation in the equilibrium, so individual agents ignore them when calculating their equilibrium payoffs
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  E.g., too high if $(1-\theta)\,S(\tau) > S^*(\tau)$

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Frictionless limit

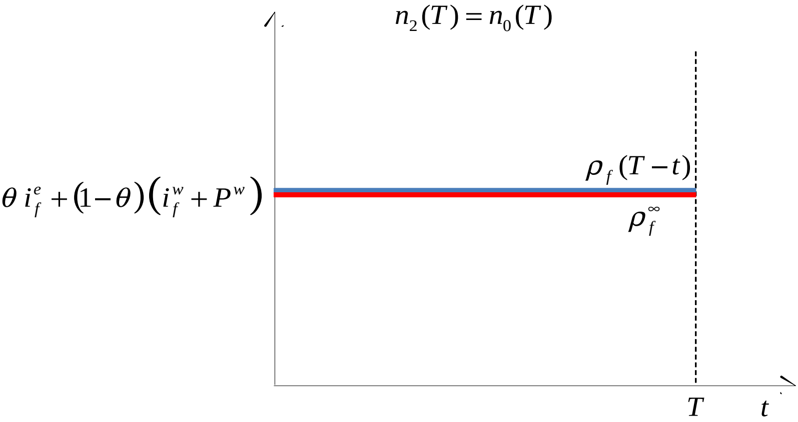
Proposition

Let $Q \equiv \sum_{k=1}^{K} kn_k(T) = 1 + n_2(T) - n_0(T)$.

For $\tau \in [0, T]$,

$$\rho^{\infty}(\tau) = \begin{cases} r + \frac{\ln\left[\left(1-e^{-r\tau}\right)\frac{u_1-u_0}{r}+e^{-r\tau}\left(U_1-U_0\right)\right]}{\tau+\Delta} & \text{if } Q < 1 \\ r + \frac{\ln\left[\left(1-e^{-r\tau}\right)\frac{u_1-u_0-\theta\bar{u}}{r}+e^{-r\tau}\left(U_1-U_0-\theta S(0)\right)\right]}{\tau+\Delta} & \text{if } Q = 1 \\ r + \frac{\ln\left[\left(1-e^{-r\tau}\right)\frac{u_2-u_1}{r}+e^{-r\tau}\left(U_2-U_1\right)\right]}{\tau+\Delta} & \text{if } 1 < Q. \end{cases}$$

IOR Policy: intuition from the analytical example



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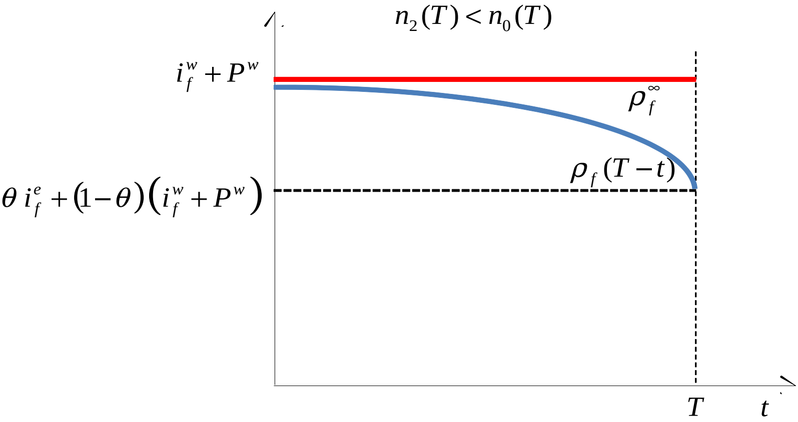

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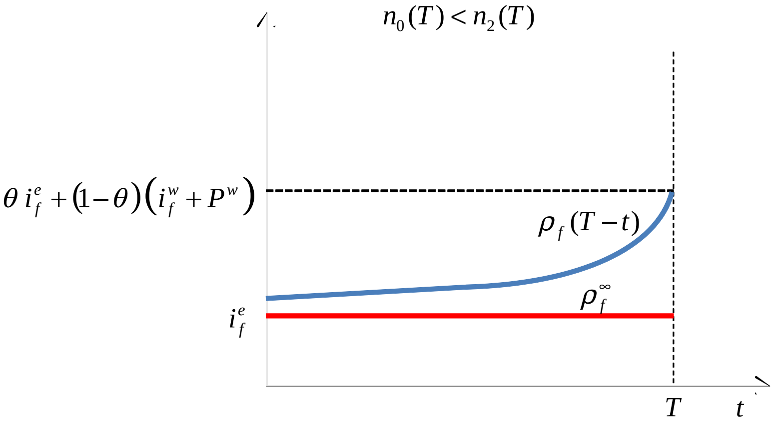

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