Adverse Selection and Liquidity Distortion in Decentralized Markets

Briana Chang

Northwestern University

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Overview

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- Question: Why markets remain illiquid even w/ positive gain from trade?
- This paper:
    - An equilibrium model of illiquidity
    - Endogenous market segmentation

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- Liquidity : How fast a seller can find a buyer to cash his asset?
- Key feature: decentralized trading market with
  - Search frictions: eg, Over-the-Counter market (OTC)
  - Adverse Selection: sellers have private info about their asset quality
  - Example: Asset-backed securities, housing market, Corporate assets

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- Two possible dimensions of market distortion:
  - price discount?
  - illiquidity?

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  - The higher the dispersion (range), the more illiquid the market
- Result 2 (+Unknown motives for sale):
  - A submarket with price discount coexists with illiquid submarkets
- Predictions on price, liquidity(trade volume), market segmentation
- Relation to Guerrieri, Shimer and Wright (2010)
  - a dynamic setting in asset trading market
  - a mechanism design approach
  - a semi-pooling Eq may arise

Related Literature

- Asset market with search friction:
  - Over-the-Counter Market: Duffie (2005,2007), Weill (2008)....
  - Monetary Search: Williamson and Wright (1994), Trejos and Wright (1995)...
- Asset market with adverse selection: Akerlof (1970), Eisfeldt (2004)
- Competitive Search Equilibrium:
  - Complete information: Moen (1997), Mortensen and Wright (2002)
  - Decentralized price competition: Kircher (2010)
  - w/ Adverse Selection: Guerrieri, Shimer and Wright (2010)

Roadmap

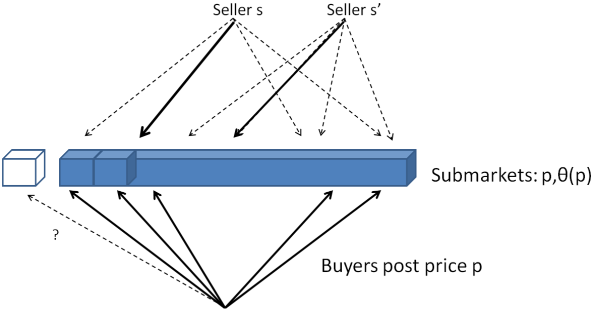
- Setup
- Basic Model
- Generalization
- Obscure motives for sale
- Conclusion

Setup

- Players:
  - A continuum of sellers with asset quality s, $s \in S = [s_l, s_h]$ with $G(s)$
  - A continuum of homogenous buyers (more than sellers)
- *Flow* payoff of owning an asset s
  - Sellers: $s - c$
  - Buyers: s
- Setup: continuous time, risk-neutral, indivisible asset
- Competitive Search:
  - *Buyers* post trading prices p, at a flow cost $k > 0$
  - Sellers *direct* their search toward their preferred market
  - Traders meet randomly at each market
  - The meeting rate depends on buyer-seller ratio $\theta(p)$:
    - $m(\theta) = \theta^\rho$ for sellers ($\rho < 1$)
    - $\frac{m(\theta)}{\theta}$ for buyers

Competitive Search Equilibrium

- Each submarket is characterized by $(p, \theta(p))$



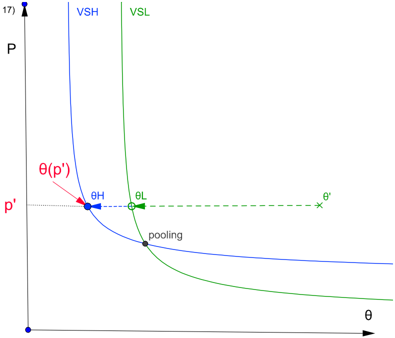
- Equilibrium Conditions:
  - A Seller *directs* their search optimally, given $(p, \theta(p))$
  - A Buyer is indifferent among all submarkets $(p, \theta(p))$, expecting assets quality:

$$\int \frac{\tilde{s}}{r} \mu(\tilde{s}|p) d\tilde{s}$$

  - No profitable deviation for buyers by posting a new price

Off-Path Belief of Buyers

- Gale (1992), Guerrieri, et (2010)
- Opening new submarkets by posting p' :
  - Take $V^*(s)$ as given: *Market utility property*
  - Form a belief about $\theta(p')$ and the types he will attract $T(p')$ offpath



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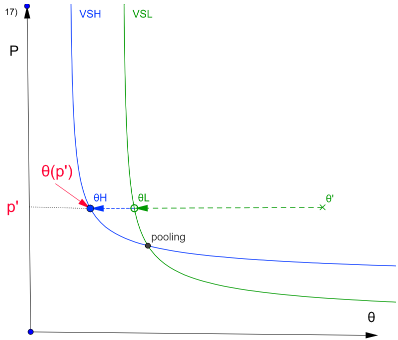

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- $T(p')$: The types which are most likely to come
- **No pooling**

Equilibrium Characterization

- Equilibrium: Def
  - Sellers *direct* their search toward their preferred market
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  - Buyers is indifferent among all submarkets $U_b = 0$
  - No profitable deviation p' for buyers, expecting $\theta(p')$ and $T(p')$
- **A Mechanism Design Approach:**a market designer $\{\theta(\cdot), p(\cdot)\}$
- On the sellers' side:
  - Promise a seller who reports his type $\hat{s} \in S$, with the pair $(p(\hat{s}), \theta(\hat{s}))$
- On the buyers' side: **(In a matching environment)**
  - feasibility constraint (free-entry)
  - the recommended posting price p must be optimal for buyers

Two Steps

- **Step 1:** Characterize the set of *feasible* mechanism $\alpha = (p^{\alpha}, \theta^{\alpha}, V^{\alpha}) \in A$
  - IC for sellers $V^{*}(s) = \max_{\hat{s}} V(\theta(\hat{s}), p(\hat{s}), s)$, IR, free entry
- **Proposition 1:** The pair of function $\{\theta(\cdot), p(\cdot)\}$ satisfies sellers' IC condition *if and only if*

$$\frac{1}{r + m(\theta^{*}(s))} \text{ is non-decreasing} \qquad \text{(M)}$$

$$V^{*}(s) = \frac{s - c + p^{*}(s) \cdot m(\theta^{*}(s))}{r + m(\theta^{*}(s))} \qquad \text{(ICFOC)}$$

$$= V^{*}(s_L) + \int_{s_l}^{s} V_s(\theta^{*}(\tilde{s}), \tilde{s}) d\tilde{s}$$

(Milgrom and Segal (2002))

Buyers' Optimality Condition

- **Step 2:** no profitable deviation by posting a new price, given $\alpha \in A$
- Lemma 1: pin down the type which is mostly likely to come lemma1
- The necessary condition for which $\alpha \in A$ can be decentralized
  - No pooling $\Longrightarrow p(s) = \frac{s}{r} - \frac{k\theta(s)}{m\theta(s)}$
  - $V^*(s_L) = V^{FB}(s_L)$

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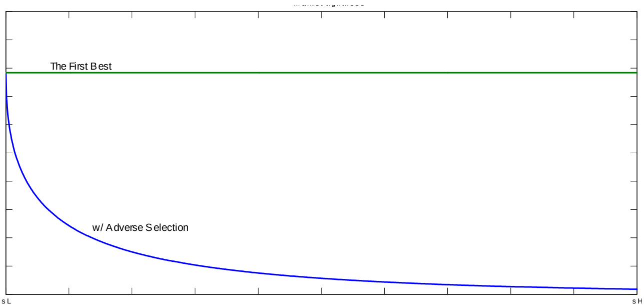
  - only when buyers' willingness to pay matches with sellers' waiting preference

Solution

- $\theta^*(s)$ is the solution to the differential equation DE



Equilibrium $\theta^*(s)$

- Initial condition: $\theta^*(s_L) = \theta^{FB}(s_L)$ & price schedule: $p^*(s) = \frac{s}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))}$
- Downward Distorted market tightness (for better assets)

Short Summary

- Endogenous market illiquidity
  - A phenomenon of buyers' strike
  - Liquidity works as a screening device (Guerrieri and Shimer (2011))
  - Independent of assumed distribution
  - $\theta^*(s)$ crucially depends on the range of underlying asset quality

Short Summary

- Endogenous market illiquidity
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  - $\theta^*(s)$ crucially depends on the range of underlying asset quality
- Implications:
  - different severities of the adverse selection: $(y + \sigma_i s) \Rightarrow \theta^*(s; \sigma_i)$
  - assets paying similar cash flow can differ significantly in their liquidity
  - capital reallocation is low when underlying dispersion is high
- Can easily incorporate:
  - A general payoff function
  - Resale

$$rJ(s) = s + \delta(V(s) - J(s))$$

  - Heterogenous buyers

Obscure Motives for Sale

- Sellers have different liquidity position c and it is unobserved by the market
- Two dimensions sellers' type (s_i, c_i)
- The type who are willing to wait longer $=$? the more valuable assets
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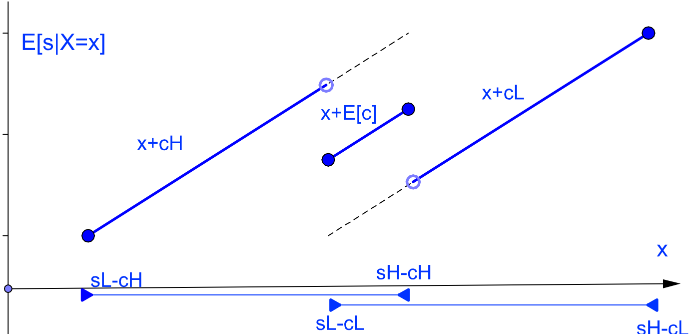
- The original screening mechanism must adjust
- A Seller's liquidity preference is determined by his flow payoff

$$x = s_i - c_i$$

- The key condition: is $E[s|x]$ monotonically increasing?
  - Yes $\Longrightarrow$ can be nested in our general model $h(x) \equiv E[s|X = x]$
    - eg: $c_i \tilde{} U[c_L, c_H]$
  - No $\Rightarrow$ Buyers' willingness to pay doesn't align with sellers' liquidity preference
    - A semi-pooling equilibrium

An Example of Non-Monotonicity

- $s_i \in S$ and $c_i \in \{c_H, c_L\}$ and $P(c_H|s) = \lambda$

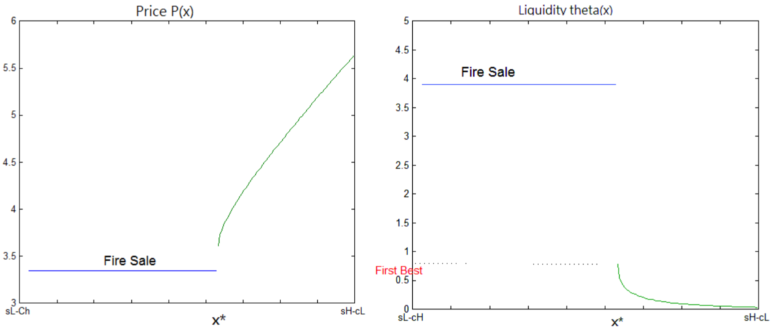


Buyers' value $h : X \to R$

- X-axis : sellers' value of holding the asset $x = s_i - c_i$
- Y-axis : how much is x actually worth to buyers

An Equilibrium with Fire Sale

- Constructing a semi-pooling EQ x_1 example



- $x < x_1$: a pooling market in which buyers get $E[s|x < x_1]$
  - Liquid market with price discount
- $x \geq x_1$: separated submarkets in which buyers get $x + c_L$
  - Liquidity distortion (as before)
- Apply Proposition 1 and Lemma 1 lemma1
  - Key conditions: $V^*(x_1) = V^{FB}(x_1)$ and $V^*(x_L) \geq V^{FB}(c_H, s_L)$

Conclusion

- Two important dimensions in the trading market: *Price* and *Liquidity*
  - Standard Lemon Model: high types subsidize low types (pooling)
  - Basic Model: Liquidity Distortion (full separation)
  - Unobserved selling motives: Price + Liquidity distortion (semi-pooling)
- Different market distortion arise endogenously: Price discount? illiquid risk?
  - Sellers' liquidity preference: $\begin{array}{c}\text{asset quality}\\ \text{(commone value)}\end{array} + \begin{array}{c}\text{liquidity position}\\ \text{(private value)}\end{array}$
  - Buyers' willingness to pay
- Jointly determination of price, liquidity and market segmentation

Off-Path

- Off-path: A buyer open up new submarkets by posting $p' \notin P^*$ back
- Market utility property (take $V^*(s)$ as given)
  - Belief about market tightness $\theta(p')$

$$\begin{aligned} \theta(p', s) &\equiv \inf\{\tilde{\theta} > 0 : U(p', \tilde{\theta}, s) \geq V(s)\} \\ \theta(p') &\equiv \inf_s \theta(p', s) \end{aligned}$$

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  - Expecting to attract the type s

$$T(p') = \arg \inf_{s \in S} \{\theta(p', s)\}$$

$$\mu(s|p') = 0 \text{ if } s \notin T(p') \tag{1}$$

- There does not exist any p' such that $U_b(p, \theta(p'), \mu_{p'}) > 0$, where $\theta(p')$ and $\mu(s|p')$ satisfy restriction above.

Characterization

- back Substituting payment schedule $p(s) = \frac{h(s)}{r} - \frac{k\theta(s)}{m(\theta(s))}$ into $(ICFOC)$:

$$V(s) = \frac{u(s) + (\frac{h(s)}{r} - \frac{k\theta}{m(\theta)})m(\theta^*(s))}{r + m(\theta^*(s))} = V(s_l) + \int_{s_l}^{s} U_s(\theta^*(\tilde{s}), \tilde{s})d\tilde{s}$$

$\Longrightarrow$differential equation of $\theta^*(s)$:

$$\frac{d\theta^*(s)}{ds} = \frac{-\frac{\theta h_s(s)}{\rho r}(r + m(\theta))}{[(h(s) - u(s)) + \frac{k}{\rho}((\rho - 1)\theta - \frac{r\theta}{m(\theta)})]} \equiv f(\theta, s)$$

Equilibrium

Definition

An equilibrium consists of P^*, a function of $V^* : S \rightarrow R_+$, a market tightness function $\theta(\cdot) : P \rightarrow [0, \infty]$, the conditional distribution of sellers in each submarket $\mu : S \times P^* \rightarrow [0, 1]$, such that the following conditions hold back:
$E1$ (optimality for sellers): let

$$V^*(s) = \max\{\frac{s-c}{r}, \max_{p' \in P^*} V(p', \theta(p'), s)\}$$

and for any $p \in P^*$ and $s \in S$, $\mu(s|p) > 0$ implies

$$p \in \arg\max_{p' \in P^* \cup \varnothing} V(p', \theta(p'), s)$$

$E2$ (optimality for buyers and free-entry): for any $p \in P^*$

$$0 = U_b(p, \theta(p), \mu_p)$$

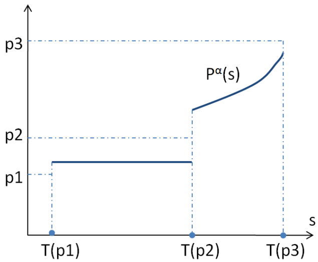
;and there does not exist any $p' \in P$ such that $U_b(p', \theta(p'), \mu_{p'})) > 0$

The types which are most likely to come

Lemma

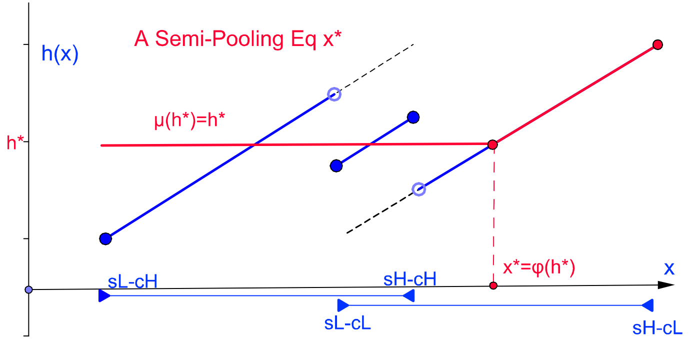
Given any mechanism $\alpha = (p^\alpha, \theta^\alpha, V^\alpha) \in A$, for any price $p' \notin$ range of p^α, the unique type $T(p')$ attracted by p' is given by back

$$
\begin{aligned}
T(p') &= s^+ \cup s^- \\
\textit{where } s^- &= \inf\{s \in S | p' < p^\alpha(s)\} \\
s^+ &= \sup\{s \in S | p' > p^a(s)\}
\end{aligned}
$$



Constructing a Semi-pooling EQ

- Reconstruct $h(\cdot)$ by bunching types: h^* solves $\int_{\tilde{s}_L}^{\phi_1(h)} h(x)dx = h$



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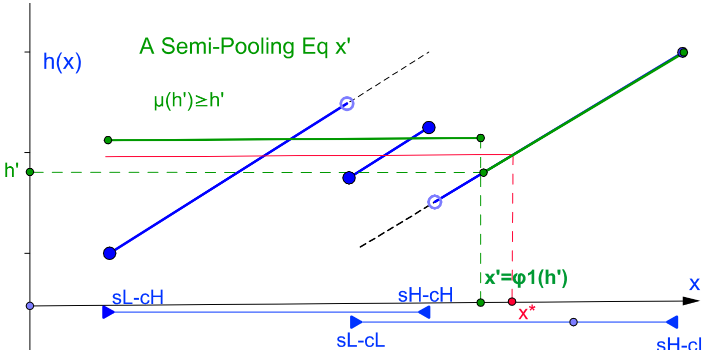
- Also: $\mu(h) = \int_{\tilde{s}_L}^{\phi_1(h)} h(x)dx \geq h$ back



- The set of Eq: the marginal type $x_1 \in (s_H - c_H, x^*]$

The Marginal Type

- The case when $\mu(h) > h$