Liquidity Hoarding

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Chicago Fed
August 9, 2011

Motivation

- Current crisis associated with illiquidity and freeze in markets.
- Lack of liquidity in the interbank market.
- Banks hoard liquidity rather than lend.
- Rationing and rates reaching historic highs.
- Unprecedented government interventions.
- Introduction of many liquidity facilities.

Empirical evidence

- Acharya and Merrouche (2009): 30% increase in UK banks' liquidity buffers in August 2007.
- Heider, Hoerova and Holthausen (2008) provide evidence of liquidity hoarding in the unsecured euro interbank market after September 28, 2007.
- Ashcraft, McAndrews and Skeie (2008): hoarding of reserves, reluctance to lend and extreme fed funds rate volatility between September 2007 to August 2008.
- Afonso, Kovner, Schoar (2010): rates spiked and terms were sensitive to borrower risk, but volume of lending remained stable after Lehman's collapse, possibly supply did not catch up with demand.

Motivation

- Liquidity hoarding: Lending vs. piling cash
- Idle cash
- Banks that demand cash cannot get it
- Inefficient early liquidations
- Inefficiently low level of lending (compared to a "benchmark")

Motivation

- Liquidity hoarding
- No credit risk
- Uncertainty about future liquidity need and access to markets.
- Motives for hoarding:
  - Precautionary motive
  - Speculative motive

Motivation

- Policy:
- Goodfriend & King (1988): With efficient interbank markets only lend to the market (OMO).
- Interbank market will distribute the liquidity.
- Hoarding incentives create inefficiency in the interbank market.
- Lending to individual institutions.

Questions

- Efficient allocation of liquidity: Hoarding
- Efficient level of liquidity in the financial system: Portfolio choice
- Policies:
- OMOs, Lender of Last Resort
- Liquidity Requirements

Related literature

- *Endogenous choice of liquidity*: Allen and Gale (2004a,b), Gorton and Huang (2004), Diamond and Rajan (2005), Acharya, Shin and Yorulmazer (2009), Diamond and Rajan (2009).
- Our paper differs in several respects: precautionary motive for liquidity hoarding; initial portfolio choice and later decision to lend; policy options.
- *Interbank markets*: Rochet and Tirole (1996), Allen and Gale (2000); Goodfriend and King (1988); Flannery (1996), Freixas and Jorge (2007), Bhattacharya and Gale (1987), Repullo (2005), Acharya, Gromb and Yorulmazer (2007).

Outline

- The planner's problem
  - constrained efficient outcome
- A laisser-faire equilibrium
- Constrained inefficiency of equilibrium
  - provision of liquidity: hoarding
  - level of liquidity: portfolio choice
- Policy analysis
  - LoLR
  - Other policies

The Model

Primitives I

- *Time*: Time is divided into four dates, indexed $t = 0, 1, 2, 3$
- *Assets*: Two assets:
  - liquid asset ('cash')
  - illiquid asset ('the asset')
- *Returns*:
  - cash pays a return of 1 at each date
  - asset pays a return of $R > 1$ at date 3

Primitives II

- *Bankers*: Ex ante identical, risk-neutral agents $i \in [0, 1]$
  - Has 1 unit of cash and 1 unit of asset at $t = 0$
  - Decide whether to hold cash or consume at $t = 0$
  - $U(c_0, c_3) = \rho c_0 + c_3$, with $\rho > 1$
- *Creditors*: Ex ante identical, risk-neutral agents $j \in [0, 1]$
  - Creditor j has 1 unit of debt with face value 1 in bank $i = j$
  - Uncertain about when to consume $t = 1, 2, 3$
  - At each date $t = 1, 2$ a fraction θ_t of the creditors receive a liquidity shock (at most once)
  - $V(c_1, c_2, c_3) = \theta_1 c_1 + (1 - \theta_1)\theta_2 c_2 + (1 - \theta_1)(1 - \theta_2) c_3$

Primitives III

- *Liquidity shocks*: Creditors that receive a liquidity shock demand repayment from the bank
- *Default*: On receiving a shock, a bank must either pay one unit of cash to discharge debt or default and suffer a loss of 100% of the value of his portfolio
- *Distributions*: $\theta_1 \sim f_1(\theta_1)$ and $\theta_2 \sim f_2(\theta_2)$ and *iid* with full support, i.e., $[0, 1]$

The Planner's Problem

The planner's problem

- We assume the planner cannot transfer assets between agents.
- The planner can only accumulate and distribute liquidity at the first three dates and reallocate payoffs at the last date.
- The planner has complete information (for now).
- The planner's policy consists of an cash balances $m_0, m_1(\theta_1), m_2(\theta_1, \theta_2)$ at date 0, at date 1 in state θ_1 and at date 2 in state (θ_1, θ_2), respectively.
- This defines the amounts $x_1(\theta_1) = m_0 - m_1(\theta_1)$ and $x_2(\theta_1, \theta_2) = m_1(\theta_1) - m_2(\theta_1, \theta_2)$ distributed at date 1 in state θ_1 and at date 2 in state (θ_1, θ_2), respectively.

The planner's problem

- $t = 0$: m_0 units of cash
- $t = 1$:
  - x_1 units distributed
  - $m_1 = m_0 - x_1$ carried to $t = 2$
- $t = 2$:
  - x_2 units distributed
  - $m_2 = m_1 - x_2$ carried to $t = 3$

Use of cash

- One unit of cash is always consumed by creditors.
- One unit of cash can save one unit of the asset generating an output of R.
- Hence, one unit of cash, if used to save an asset, generates $R + 1$.
- Planner maximizes total expected output.
- Efficiency requires using cash to save as many assets as possible.

Feasible policies

- A policy m_0, $m_1(\theta_1)$, $m_2(\theta_1, \theta_2)$ is feasible if

$$m_0 \geq 0,\ x_1(\theta_1) \geq 0,\ x_2(\theta_1, \theta_2) \geq 0 \tag{1}$$

and

$$x_1(\theta_1) + x_2(\theta_1, \theta_2) \leq m_0, \tag{2}$$

for any (θ_1, θ_2).

- The planner chooses a feasible policy to maximize the total surplus

$$E_0\left[R\left\{x_1(\theta_1) + x_2(\theta_1, \theta_2) + (1-\theta_1)(1-\theta_2)\right\} + m_0(1-\rho)\right]$$

Efficiency

- Efficiency requires saving as many assets as possible.
- *Date 2:* Amount of cash at date 2 is m_1. The optimal policy is

$$x_2(\theta_1, \theta_2) = \min\{(1-\theta_1)\theta_2, m_1\}$$

- *Date 1:* Amount of cash at date 1 is m_0. The optimal policy is

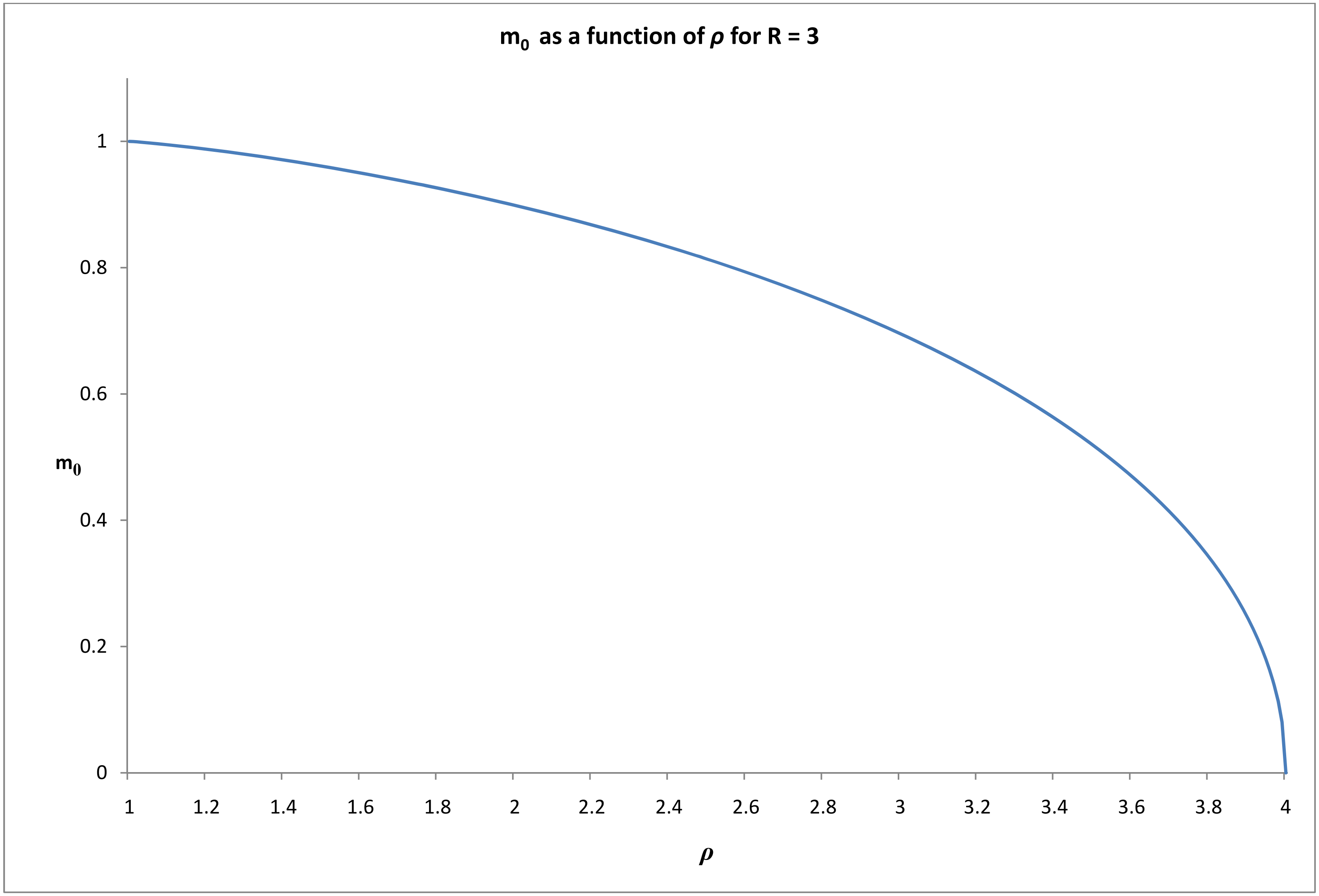
$$x_1(\theta_1) = \min\{\theta_1, m_0\}$$

- *Date 0:* There is an interior solution if $1 < \rho < R + 1$ and m_0 is characterized by the first-order condition

$$R\left(1 - \underbrace{\int_0^{m_0} F_2\left(\frac{m_0-\theta_1}{1-\theta_1}\right) f_1(\theta_1)\, d\theta_1}_{\text{IDLE CASH}}\right) + 1 = \rho$$

$$\Pr(\text{idle cash}) = \Pr(\theta_1 \leqslant m_0 \text{ and } (1-\theta_1)\theta_2 \leqslant m_0 - \theta_1).$$

Figure 6a: Planner's choice m_0 as a function of ρ for R=3

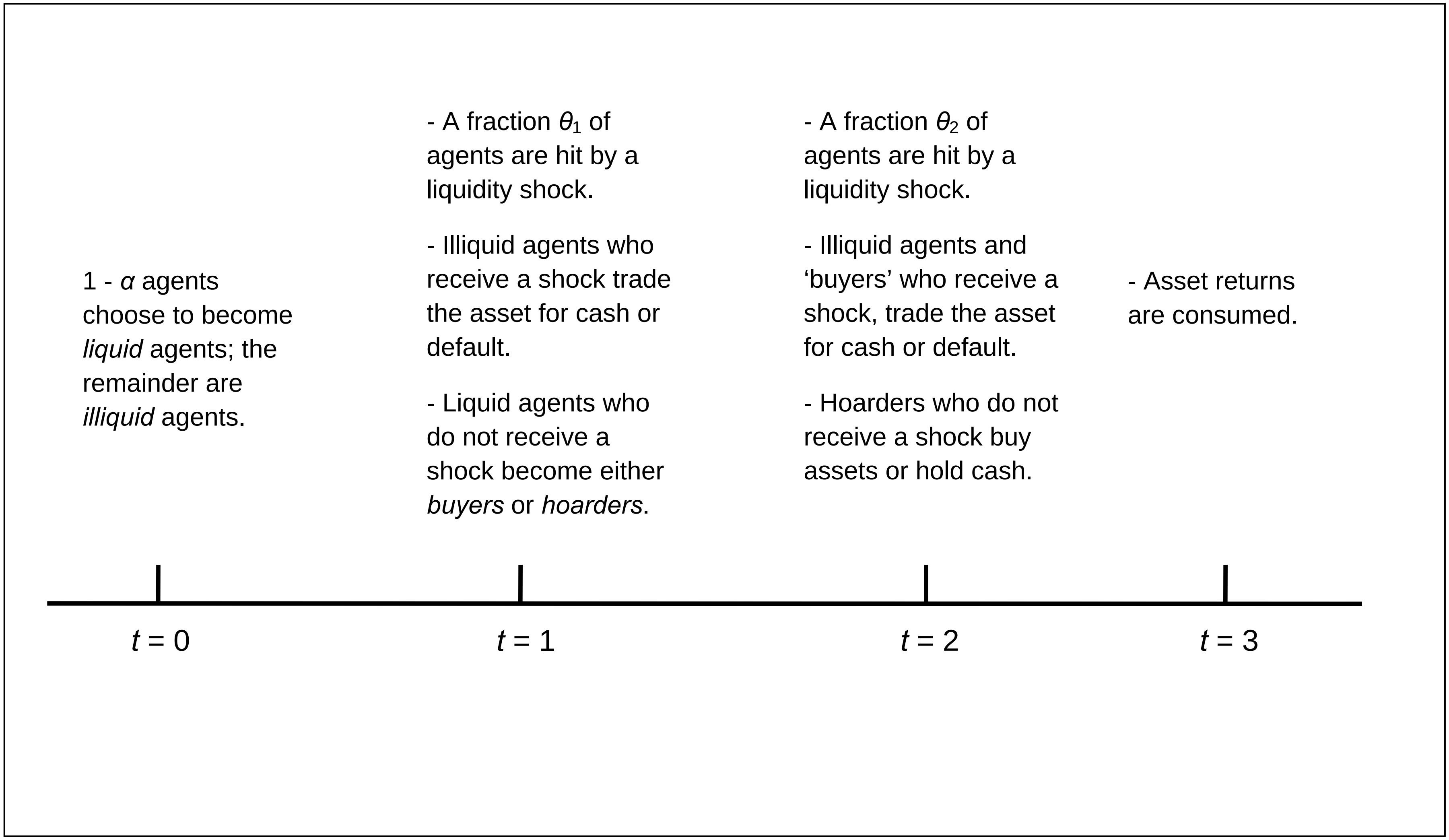


Laisser-Faire Equilibrium

The laisser-faire economy

- At date 0, bankers decide whether to hold liquidity, that is, whether to become "liquid" bankers $(1 - \alpha)$ or remain "illiquid" (α)
- At date 1, there is a spot market on which the asset can be traded for cash
- Some bankers receive a liquidity shock (θ_1) that requires them to pay one unit of cash to creditors; failure to do so leads to default and liquidation
- At date 2, some of the bankers who have not already received a shock may receive a liquidity shock $((1 - \theta_1)\theta_2)$
- At date 3, solvent bankers receive the returns from the assets they hold and remaining debts are paid

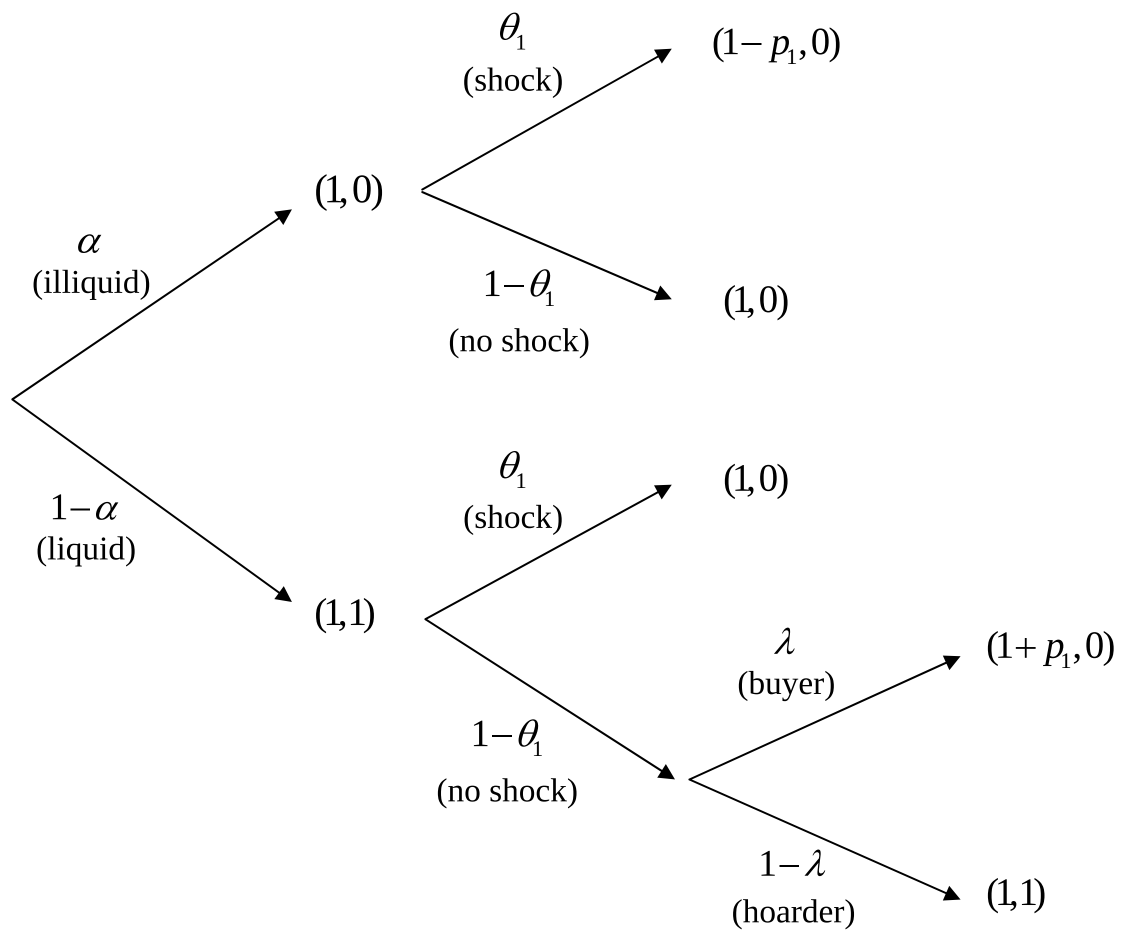
Figure 1: Timeline



Allocations I

- At date 0 a fraction $1-\alpha$ of bankers decide to hold liquidity (one unit)
- At date 1, a fraction θ_1 of the bankers receive a liquidity shock
- A measure $(1-\alpha)\,\theta_1$ of liquid bankers use their own cash to discharge the debt; a measure $\alpha\theta_1$ of illiquid bankers must either sell p_1 assets for liquidity or default
- **Buyers:** $(1-\alpha)\,(1-\theta_1)\,\lambda$ of liquid bankers choose to buy assets
- **Hoarders:** $(1-\alpha)\,(1-\theta_1)\,(1-\lambda)$ choose to hoard cash

Figure 2: Allocations at dates 0 and 1



Allocations II

- At date 2, several types remain inactive:
  - those already received a shock at date 1;
  - hoarders who receive a shock at date 2,
  - buyers who do not receive a shock at date 2
  - illiquid bankers who do not receive a shock at date 2
- Demand for liquidity:
  - buyers who receive a shock at date 2
  - illiquid bankers who receive a shock at date 2
- Supply: Hoarders who do not receive a shock at date 2

Figure 3a: Allocations at date 2

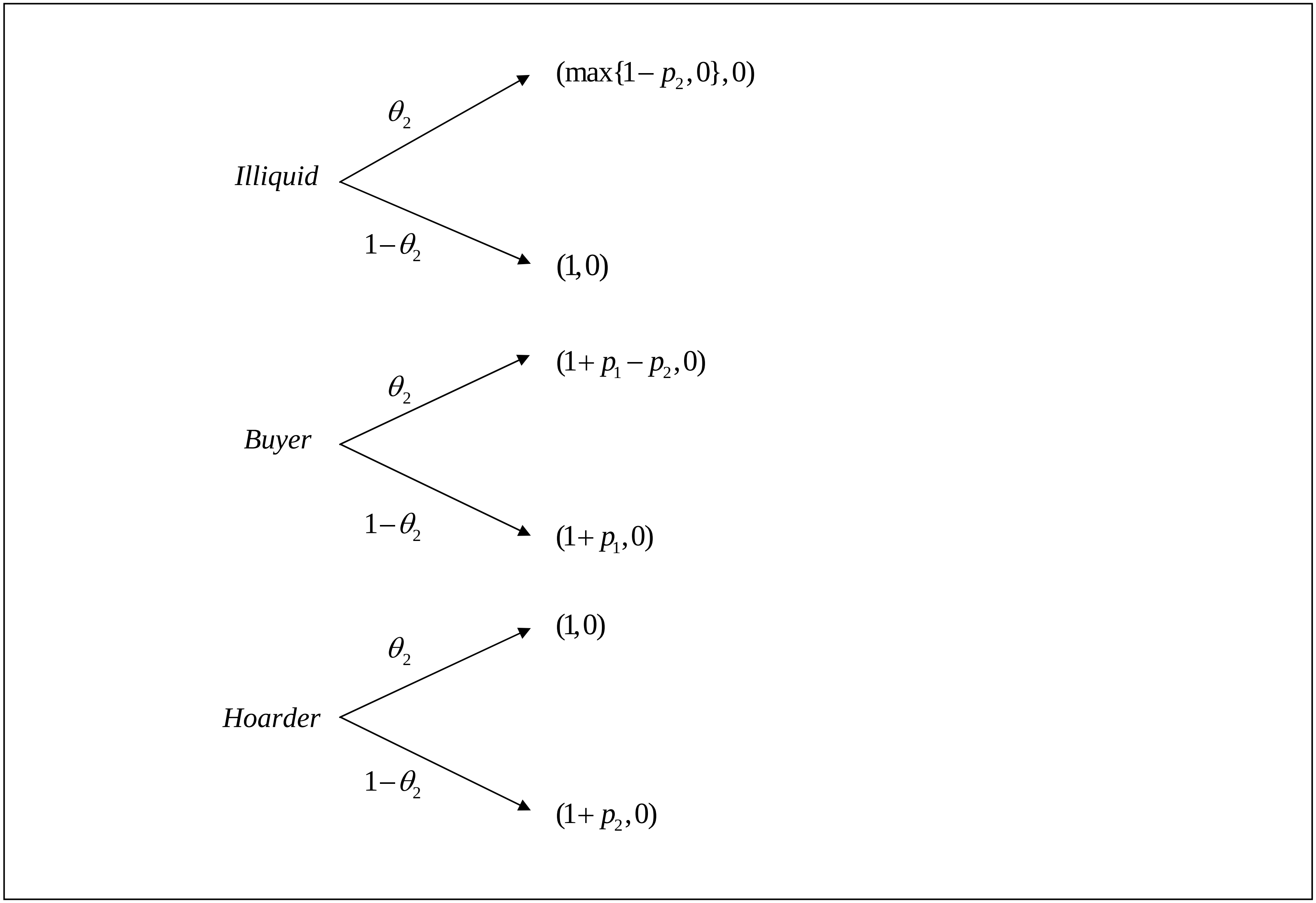


Figure 3b: Allocations at date 2

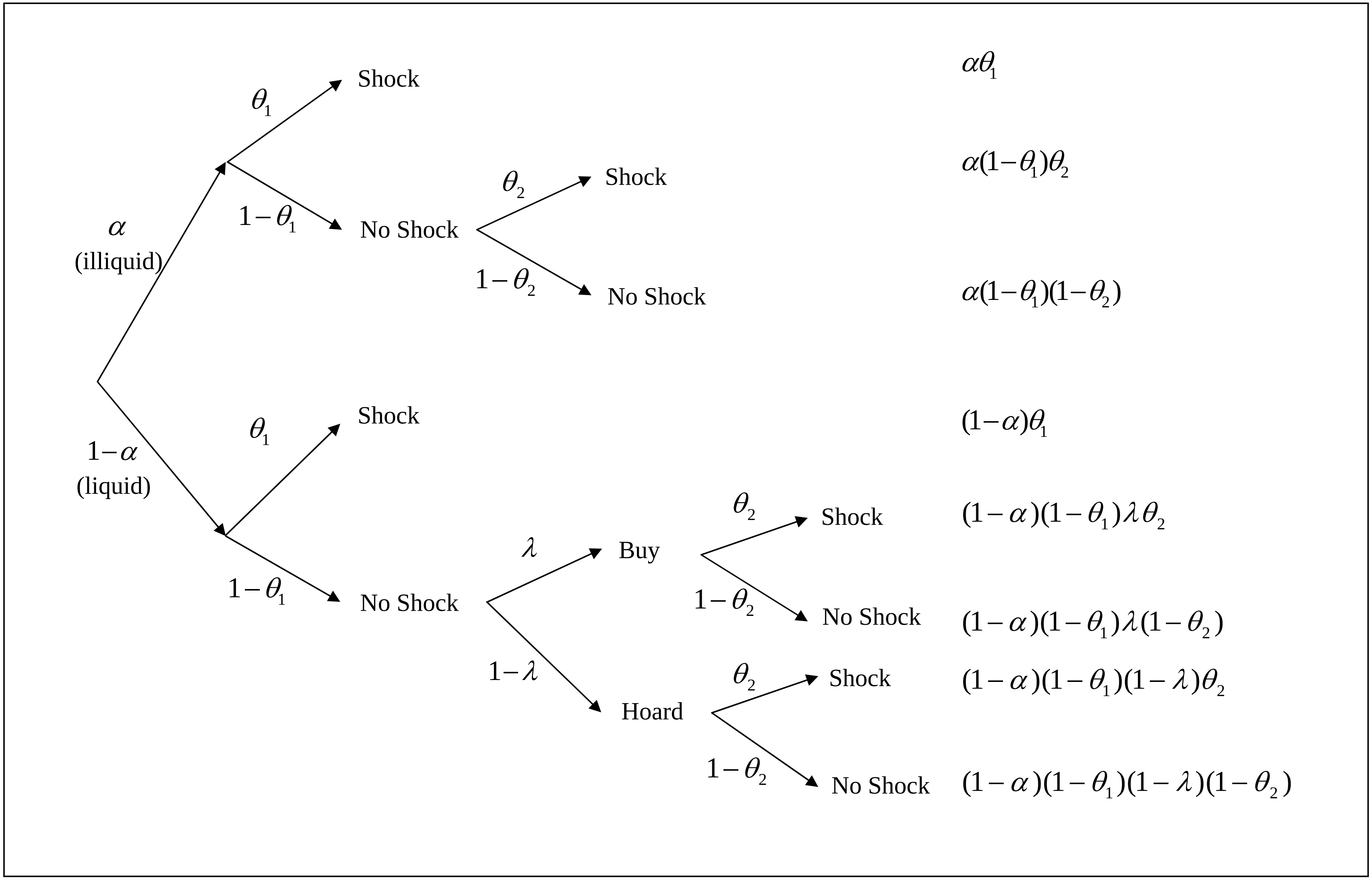
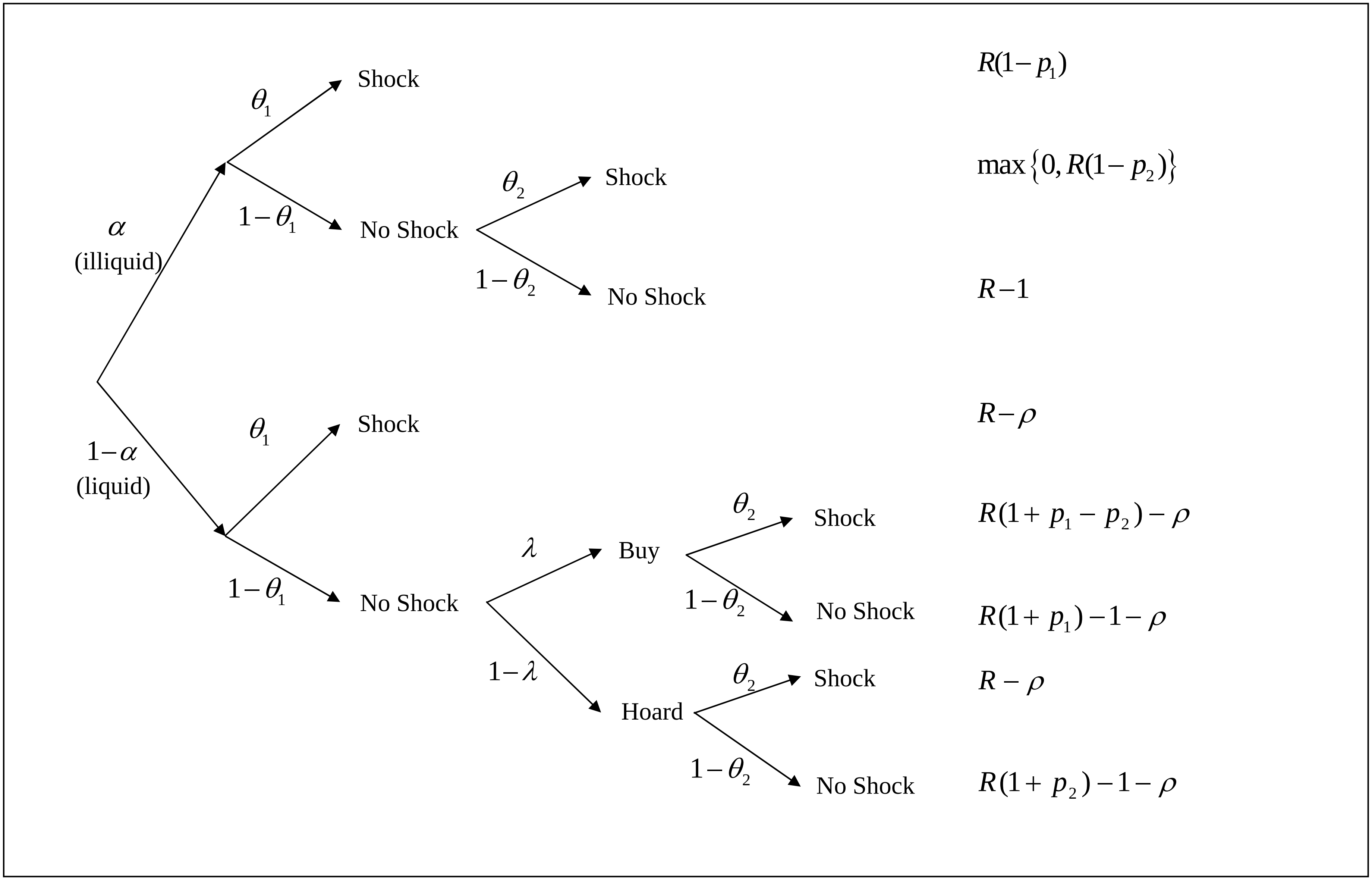


Figure 4: Terminal Payoffs



Market clearing I

- *Date 2:* Let θ_2^* and θ_2^{**} be defined by

$$\theta_2^* = (1-\alpha)(1-\lambda) \text{ and } \theta_2^{**} = 1-\lambda.$$

- There are three demand-and-supply regimes:

$$\theta_2 > \theta_2^{**} \text{ and } p_2 = 1 + p_1 \text{ (only buyers)}$$

$$\theta_2^* < \theta_2 < \theta_2^{**} \text{ and } p_2 = 1 \text{ (buyers + some illiquid)}$$

$$\theta_2 < \theta_2^* \text{ and } p_2 = \frac{1}{R} \text{ (everyone)}$$

Figure 5A: Supply of cash at date 2

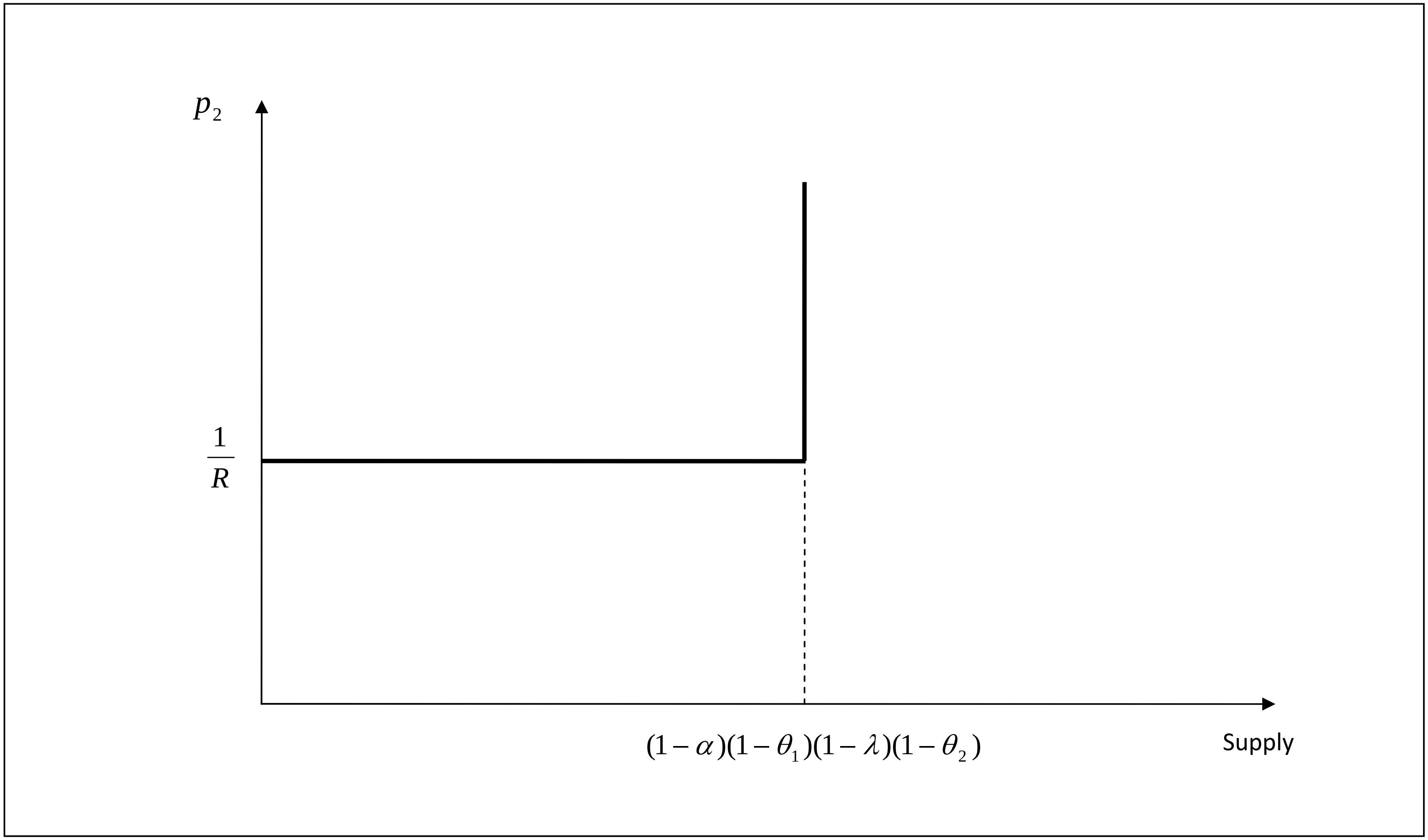


Figure 5B: Demand for cash at date 2

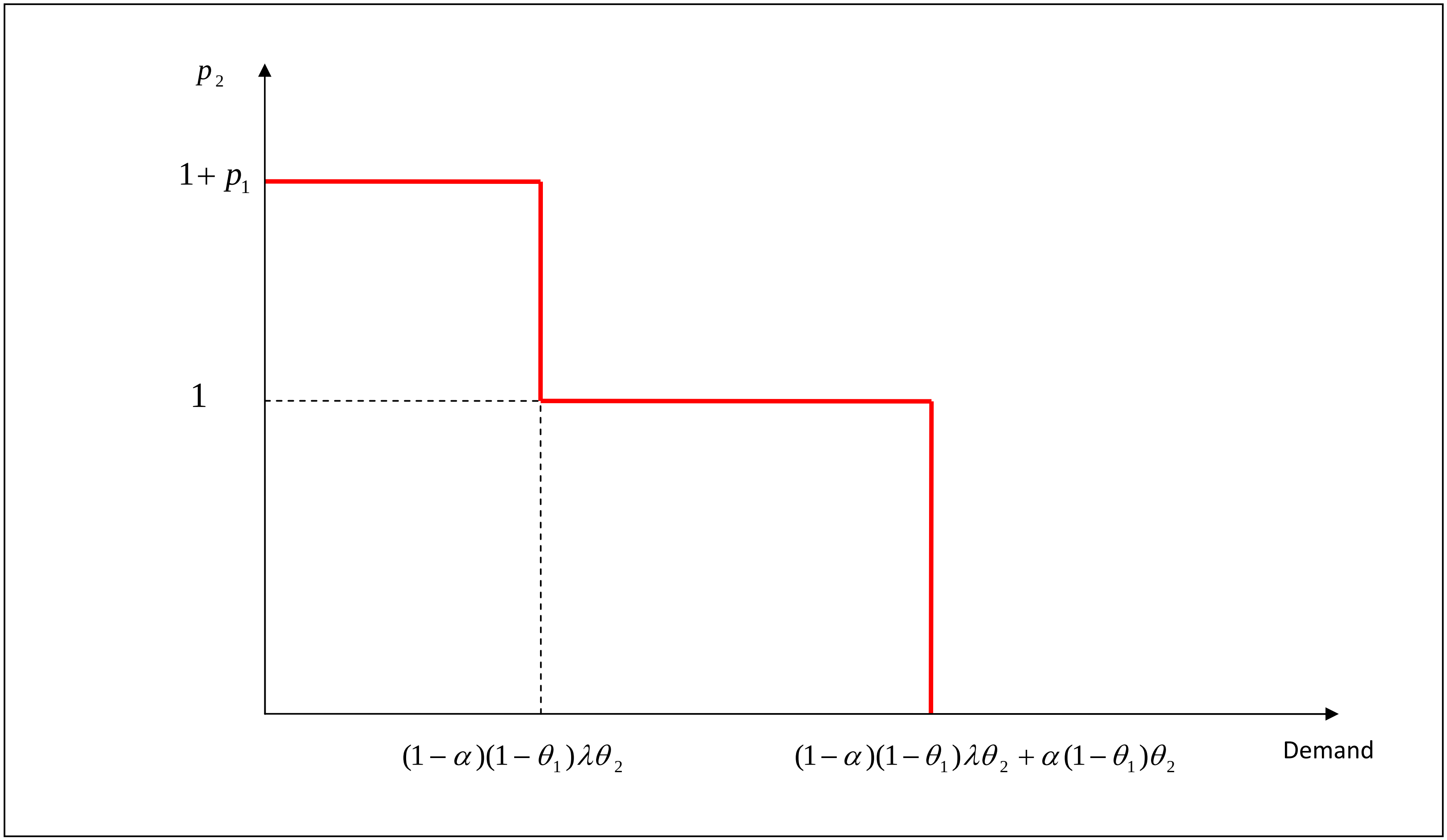
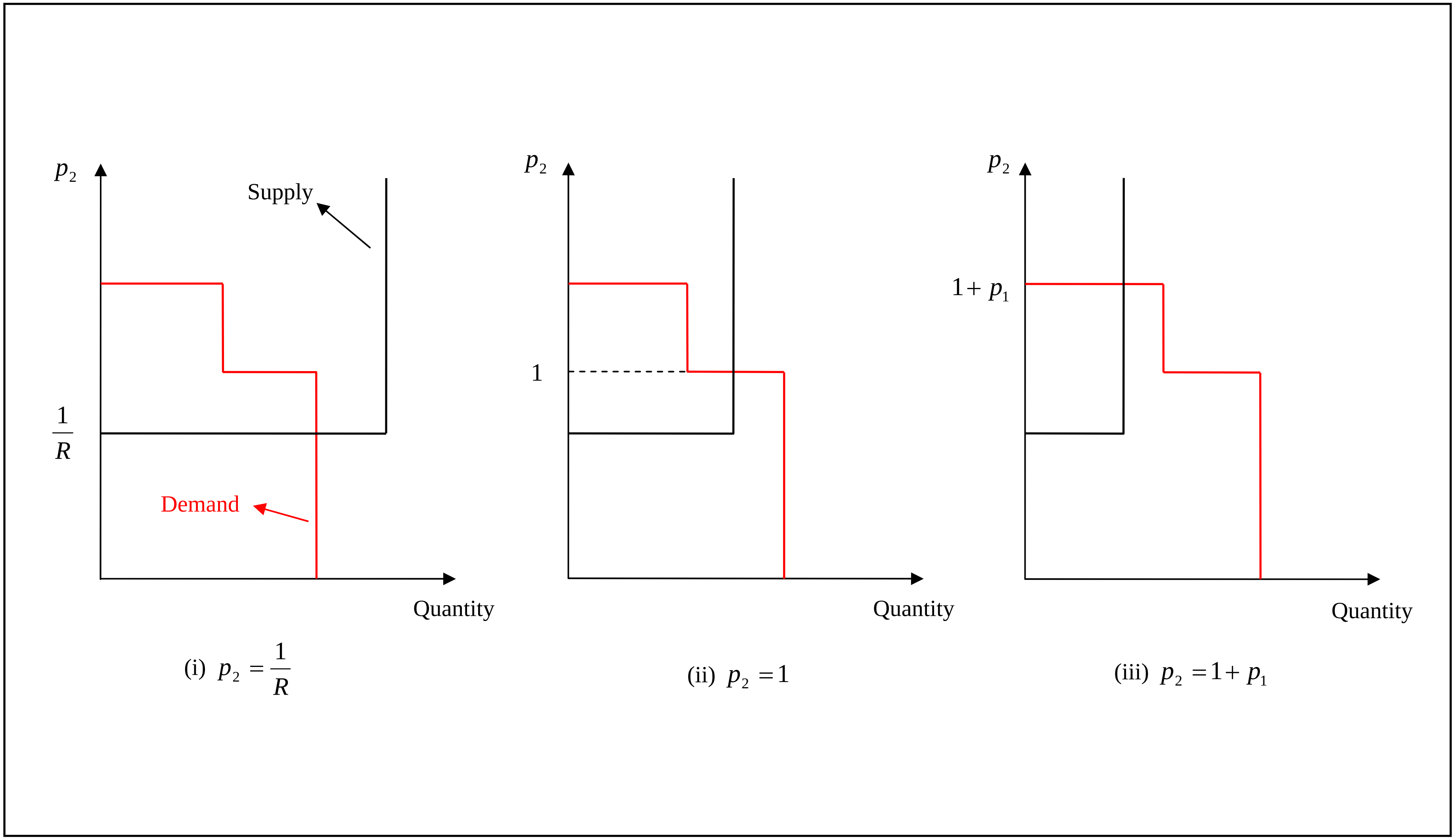


Figure 5C: Different demand and supply regimes



Market clearing II

- *Date 1:* For any θ_1, $\lambda(\theta_1)$ is the fraction of *buyers* (and the complement *hoarders)*
- Buying is optimal iff $p_1(\theta_1) \geqslant E[p_2(\theta_1, \theta_2)|\theta_1]$
- Hoarding is optimal iff $p_1(\theta_1) \leqslant E[p_2(\theta_1, \theta_2)|\theta_1]$

Market clearing II

- Suppose $p_1 > E[p_2]$ and everyone is a buyer $(\lambda = 1)$
- No cash at $t = 2$, $p_2 = 1 + p_1$. CONTRADICTION!

- Suppose $p_1 < E[p_2]$ and everyone is a hoarder $(\lambda = 0)$
- $p_1 = 1$ and no buyer so $p_2 \leqslant 1$. CONTRADICTION!

- For every value of θ_1,

$$0 < \lambda(\theta_1) < 1$$

in equilibrium at date 1, and hence,

$$p_1(\theta_1) = E[p_2(\theta_1, \theta_2) | \theta_1].$$

Market clearing III

- We know p_2:

$$\theta_2 > \theta_2^{**} \text{ and } p_2 = 1 + p_1$$

$$\theta_2^{*} < \theta_2 < \theta_2^{**} \text{ and } p_2 = 1$$

$$\theta_2 < \theta_2^{*} \text{ and } p_2 = \frac{1}{R}$$

- In equilibrium, we have $p_1 = E\left[p_2\right]$, so that we can derive p_1 as a function of λ:

$$\tilde{p}\left(\lambda\right) = \frac{1 + F_2\left(\left(1-\alpha\right)\left(1-\lambda\right)\right)\left(1-R^{-1}\right)}{F_2\left(1-\lambda\right)}$$

Market clearing III

- In equilibrium, we have $p_1 = E[p_2]$.
- For low shocks θ_1, $(1-\alpha)(1-\theta_1)\lambda = \alpha\theta_1$, and $p_1 = E[p_2]$.
- As θ_1 increases, if everyone gets cash, little cash left for $t = 2$.
- p_2, therefore $E[p_2]$ and p_1 increase.
- At some point p_1 reaches the maximum value 1.
- If lending continues at $t = 1$, we cannot satisfy $p_1 = E[p_2]$ since $p_1 = 1$ but p_2 continues to increase.
- So lending at $t = 1$ has to stop.
- There is a unique value of λ, call it $\bar{\lambda} \in (0, 1)$, such that $\tilde{p}(\bar{\lambda}) = 1$.

Market clearing IV

- Hence, the equilibrium value of $\lambda(\theta_1)$ is given by

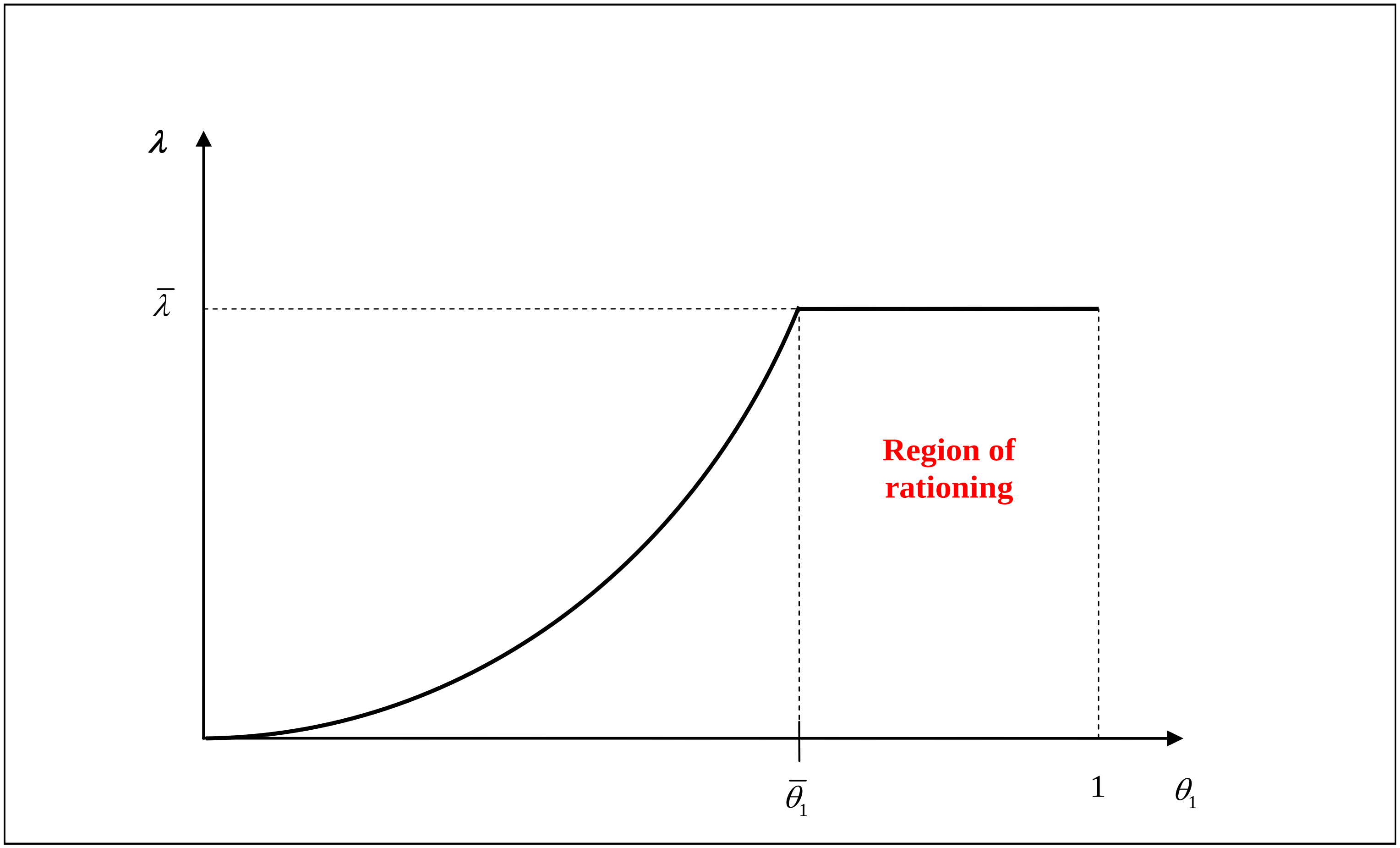
$$\lambda(\theta_1) = \min\left\{\frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}, \bar{\lambda}\right\},$$

for every value of θ_1, and the equilibrium value of $p(\theta_1)$ is given by

$$p_1(\theta_1) = \min\left\{\tilde{p}\left(\frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}\right), 1\right\},$$

for every value of θ_1.

Figure: Equilibrium λ as a function of θ_1



Market clearing V

- *Date 0:* In equilibrium at date 0, $0 < \alpha < 1$, which implies that bankers must be indifferent between acquiring liquidity and not acquiring it.

- Bankers are indifferent if and only if

$$\int_0^1 p_1 \left\{1 + (1-\theta_1)(1 - F_2(\theta_2^{**}))E\left[\theta_2 \,|\theta_2 > \theta_2^{**}\right]\right)\right\} f_1(\theta_1) d\theta_1 = \frac{\rho}{R}.$$

Equilibrium

An equilibrium is described by the endogenous variables α, $\lambda(\theta_1)$, $p_1(\theta_1)$, and $p_2(\theta_1, \theta_2)$ satisfying the following conditions:

- at date 2, for every value of (θ_1, θ_2), $p_2(\theta_1, \theta_2)$ is the market clearing price, given the values of α, $\lambda(\theta_1)$ and $p_1(\theta)$
- at date 1, for every value of θ_1, $\lambda(\theta_1)$ and $p_1(\theta)$ satisfy the market clearing conditions, given the value of α
- at date 0, agents are indifferent between acquiring liquidity and not acquiring it

Liquidity insurance I

- Let $\{\alpha, \lambda(\theta_1), p_1(\theta_1), p_2(\theta_1, \theta_2)\}$ be an equilibrium and consider the effect of opening a market for liquidity insurance at date 0
- At date 0, bankers enter into forward contracts to deliver or receive liquidity under specified conditions
- Suppliers acquire one unit of liquidity at date 0; demanders do not
- At dates $t = 1, 2$, each banker is required to report his type, that is, whether or not he has received a liquidity shock
- Suppliers who report "shock" and demanders who report "no shock" do not trade

Liquidity insurance II

- At date 1,
  - a supplier who reports "no shock" receives $(-1, \hat{p}_1(\theta_1))$
  - a demander who reports "shock" receives $(1, -\hat{p}(\theta_1))$
- At date 2,
  - a supplier who reports "no shock" for the second time and has not traded receives $(-1, \hat{p}_2(\theta_1, \theta_2))$
  - a demander who reports "shock" for the first time receives $(1, -\hat{p}_2(\theta_1, \theta_2))$

Incentive compatibility

- If $\hat{p}_1(\theta_1) > p_1(\theta_1)$, a demander who receives a shock will report "no shock" and buy on the spot market; if $\hat{p}_1(\theta_1) < p_1(\theta_1)$, a supplier who did receive a shock will report "shock" and sell on the spot market
- Thus, incentive compatibility at date 1 requires

$$\hat{p}_1(\theta_1) = p_1(\theta_1), \text{ for every } \theta_1$$

- Similarly, incentive compatibility at date 2 requires

$$\hat{p}_2(\theta_1, \theta_2) = p_2(\theta_1, \theta_2), \text{ for every } (\theta_1, \theta_2)$$

Policy Analysis

Sources of inefficiency

- At $t = 2$, hoarders who receive a shock use their liquidity to discharge their own debt rather the buyers'
- At $t = 1$, hoarders do not internalize the welfare losses resulting from early liquidations
- At $t = 0$, agents do not internalize the social value of paying off their debt

Central Bank sole provider of liquidity I

- Can the central bank achieve the allocation from the planner's problem?
- Suppose that Central Bank is the sole provider of liquidity $(\alpha = 1)$.
- Central Bank holds m_0 units of liquidity and pursues the socially optimal.
- At date 2, the market-clearing price is denoted by $p_2(\theta_1, \theta_2)$ and defined by

$$p_2(\theta_1, \theta_2) = \begin{cases} 1 & \text{if } (1-\theta_1)\theta_2 > \max\{m_0^* - \theta_1, 0\} \\ R^{-1} & \text{if } (1-\theta_1)\theta_2 < \max\{m_0^* - \theta_1, 0\} \end{cases}$$

- At date 1, the market clearing price is *assumed* to be

$$p_1(\theta_1) = \begin{cases} 1 & \text{if } \theta_1 > m_0^* \\ E[p_2(\theta_1, \theta_2) \mid \theta_1] & \text{if } \theta_1 < m_0^* \end{cases}$$

- We show that $\alpha = 1$ is privately optimal.

Central Bank II

- An illiquid banker's payoff is

$$E\left[\theta_1 R\left(1-p_1\left(\theta_1\right)\right)+\left(1-\theta_1\right)\theta_2 R\left(1-p_2\left(\theta_1,\theta_2\right)\right)\right.$$
$$\left.+\left(1-\theta_1\right)\left(1-\theta_2\right)R\right]$$
$$=E\left[R-\left(\theta_1+\left(1-\theta_1\right)\theta_2\right)p_2\left(\theta_1,\theta_2\right)R\right]$$

- A liquid banker's payoff is

$$E\left[R+\left(1-\theta_1\right)\left(1-\theta_2\right)p_2\left(\theta_1,\theta_2\right)R\right]-\rho$$

- Then it is optimal to be illiquid if and only if

$$E\left[p_2\left(\theta_1,\theta_2\right)R\right]\leq\rho$$

Central Bank III

- The first-order condition for the planner's problem is

$$R\left(1-\int_0^{m_0} F_2\left(\frac{m_0-\theta_1}{1-\theta_1}\right) f_1\left(\theta_1\right) d\theta_1\right)+1=\rho.$$

- From the definition of $p_2\left(\theta_1,\theta_2\right)$,

$$\begin{aligned} E\left[p_2\left(\theta_1,\theta_2\right)\right] &= R^{-1}F_2\left(\frac{m_0^*-\theta_1}{1-\theta_1}\right)+\left(1-F_2\left(\frac{m_0^*-\theta_1}{1-\theta_1}\right)\right) \\ &= 1-(1-R^{-1})F_2\left(\frac{m_0^*-\theta_1}{1-\theta_1}\right). \end{aligned}$$

$$\begin{aligned} E\left[p_2\left(\theta_1,\theta_2\right)R\right] &= R-(R-1)\int_0^{m_0^*} F_2\left(\frac{m_0^*-\theta_1}{1-\theta_1}\right) f_1\left(\theta_1\right) d\theta_1 \\ &\leq R\left(1-\int_0^{m_0^*} F_2\left(\frac{m_0-\theta_1}{1-\theta_1}\right) f_1\left(\theta_1\right) d\theta_1\right)+1 \\ &\leqslant \rho \end{aligned}$$

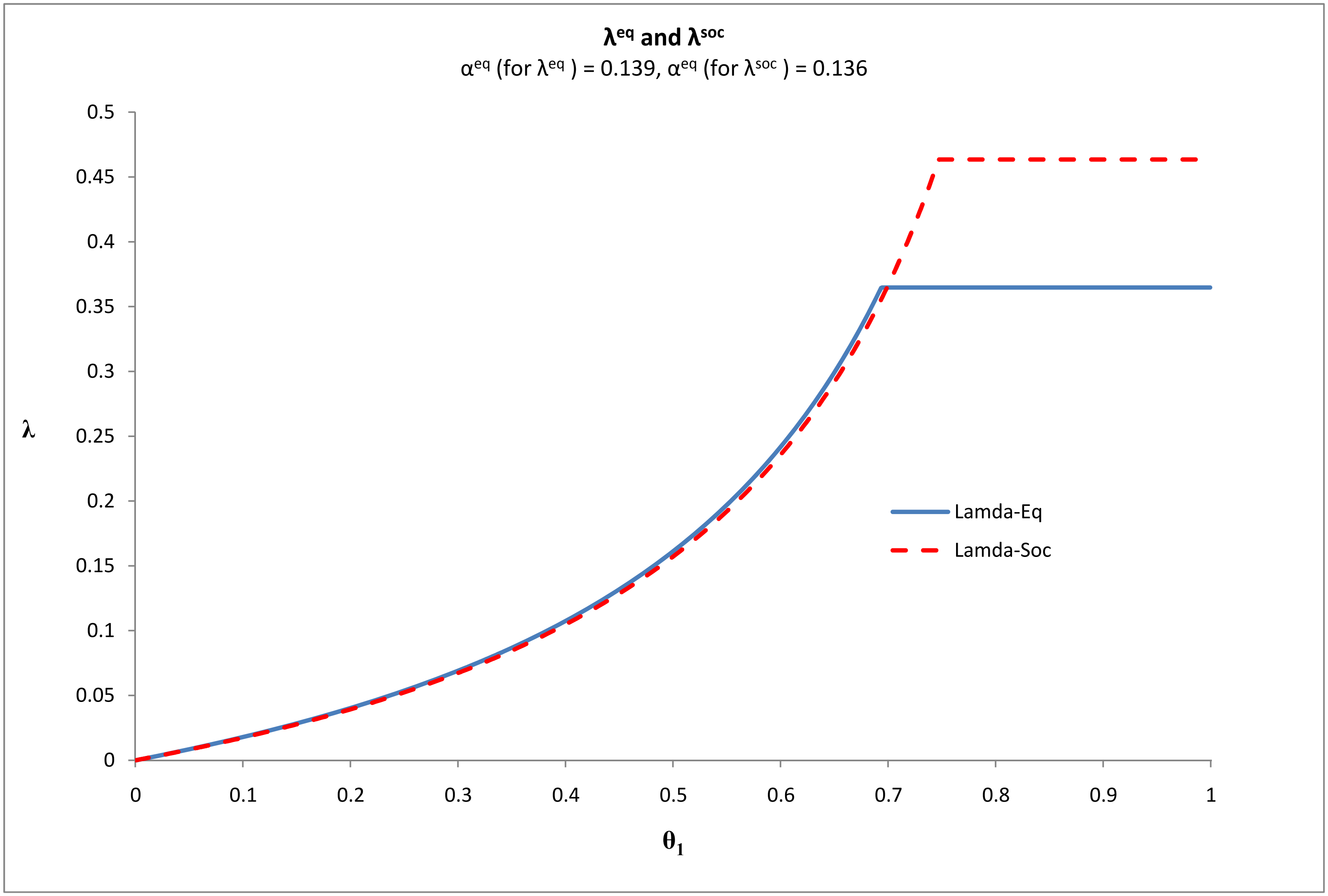
Policy with private liquidity (date 1)

- Choose socially optimal λ at $t = 1$ while allowing markets to clear at other dates
- Liquidity facilities
- The socially optimal level of λ^{soc} has the same structure as the equilibrium λ but is larger:

$$\lambda^{soc} = \min\left\{\frac{\alpha\theta_1}{(1-\alpha)(1-\theta_1)}, \widetilde{\lambda}\right\}, \text{ where } \widetilde{\lambda} > \bar{\lambda}$$

- Policy mitigates hoarding at $t = 1$.

Figure 6b: Equilibrium and constrained efficient levels of λ as a function of θ_1 for R=3 and ρ=2



Policy with private liquidity (date 0)

- Choose the socially optimal α at $t = 0$ while allowing markets to clear at other dates
- Liquidity requirements (Basel III)
- The optimal value of α^{soc} is smaller than the equilibrium level

Figure 6c: Equilibrium and constrained efficient levels of α as a function of ρ for R=3

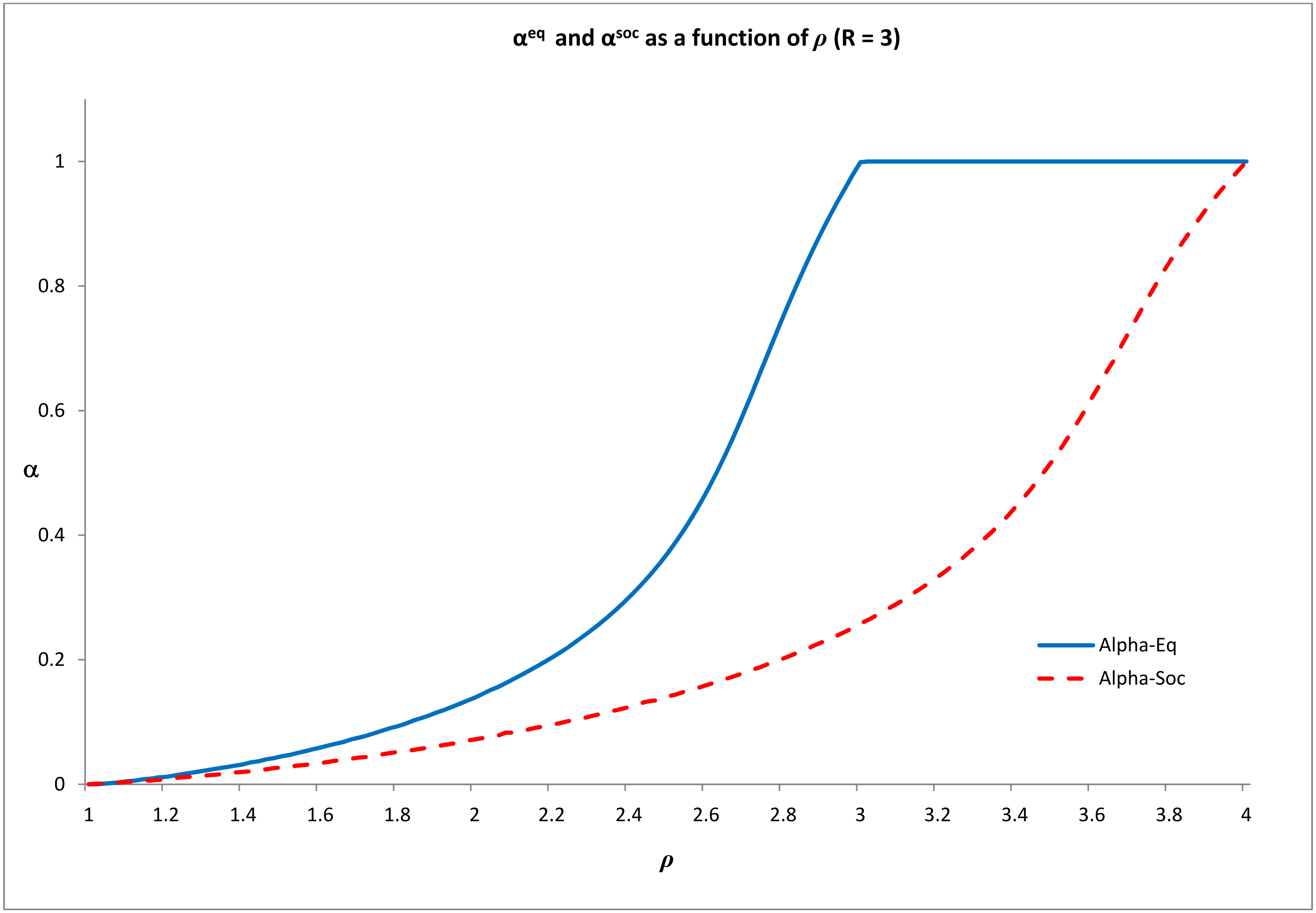
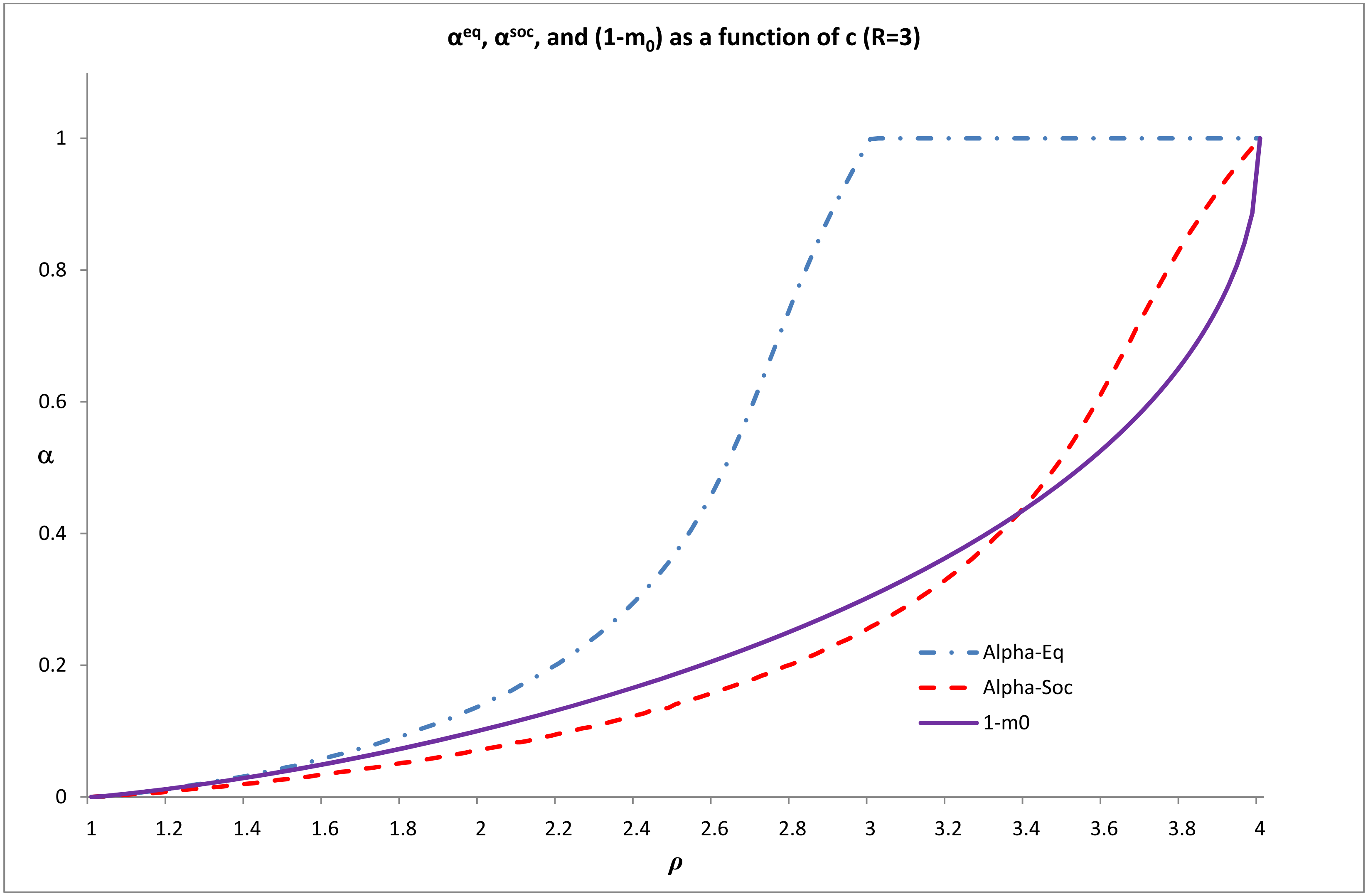


Figure 6d: Equilibrium and constrained efficient levels of α, and planner's choice ($1-m_0$) as a function of ρ for R=3



Comparative statics I

- How do the distribution and the volatility of shocks change equilibrium and socially optimal liquidity, and the wedge between the two?
- More likely liquidity shocks at $t = 2$: $g_2(\theta_2)$ FOSD $f_2(\theta_2)$, $G_2(\theta_2) \leqslant F_2(\theta_2)$
- Equilibrium requires $p_1 = E[p_2]$

$$\begin{aligned} F_2(1-\bar{\lambda}_f) + F_2((1-\alpha)(1-\bar{\lambda}_f))(1-R^{-1}) &= 1 \\ G_2(1-\bar{\lambda}_g) + G_2((1-\alpha)(1-\bar{\lambda}_g))(1-R^{-1}) &< 1 \end{aligned}$$

- This gives us $\bar{\lambda}_f > \bar{\lambda}_g$
- We can also show $\tilde{\lambda}_f > \tilde{\lambda}_g$

Comparative statics II

- Suppose θ_2 uniform over $[a, b]$.
- For $b' > b$, $f_2^{b'}(\theta_2)$ FOSD $f_2^{b}(\theta_2)$

$$\frac{1}{b-a}\left[(1-\bar{\lambda}) - a + ((1-\alpha)(1-\bar{\lambda}) - a)(1-R^{-1})\right] = 1$$

$$\bar{\lambda} = 1 - \frac{bR + a(R-1)}{R + (1-\alpha)(R-1)}$$

$$\frac{1}{b-a}\left[(1-\tilde{\lambda}) - a + (1-\alpha)(1-\tilde{\lambda}) - a\right] = 1$$

$$\tilde{\lambda} = 1 - \frac{b+a}{2-\alpha}$$

$$\frac{d(\tilde{\lambda}-\bar{\lambda})}{db} = \frac{1-\alpha}{(2-\alpha)(R+(1-\alpha)(R-1))} > 0$$

- The wedge increases as shocks become more likely.

Comparative statics II

- Effect of volatility of shocks
- Suppose θ_2 uniform over $[a, b]$ with $a + b = 1$ (symmetric around 1/2)
- For $b' > b$, $f_2^{b'}(\theta_2)$ is a mean-preserving spread of $f_2^{b}(\theta_2)$
- $$\bar{\lambda} = 1 - \frac{R - 1 + b}{R + (1-\alpha)(R-1)}, \text{ decreasing in } b.$$
$$\tilde{\lambda} = 1 - \frac{1}{2-\alpha}$$
$$\frac{d(\tilde{\lambda} - \bar{\lambda})}{db} = \frac{1-\alpha}{(2-\alpha)(R + (1-\alpha)(R-1))} > 0$$
- The wedge increases as volatility of shocks increases.
- Models using Knightian uncertainty.

Conclusion

- Goodfriend and King argued that it is sufficient to provide adequate liquidity to the system as a whole ...
- Yet, when agents are uncertain about future liquidity shocks, they hoard rather than lend.
- Inefficient (lack of) liquidity transfers.
- Freezes in markets.
- Reform of regulation of the financial sector.
- Role of Central Banks as LoLR.
- Liquidity requirements.