Optimal Unconventional Monetary Policy

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Introduction

- Liquidity problems occur because there is a mismatch between assets and liabilities
- Rarely, such mismatches lead to financial crises

Introduction

Observation:

- Federal Reserve began purchasing private securities in 2008 (for example, Maiden Lane I, II, and III)
- QE I: $1128 billion in purchase of mortage-backed securities (February 2009 to July 2010) and $169 billion in purchase of agency securities (September 2008 to April 2010)

Introduction

Question:

- Is this "new" practice optimal?
- Our aim is to build a framework that can answer this question.

Related literature

- Unconventional monetary policy: Williamson (2010), Gertler and Karadi (2011)
- Framework draws on settlement friction a la' Freeman (1996)
- Diamond-Dybvig (1983) type of liquidity shock

Environment

Time, locations and agents

- Infinite sequence of discrete time periods
- Three islands: creditor, debtor, settlement
- Both creditor and debtor live for 3 periods (OG)
- Continuum of measure one of each type born each period

Environment

Endowments

- Each young creditor endowed with κ units of generation-specific capital
- Each young debtor endowed with 1 unit of labor
- Labor is a general input

Environment

Technologies

- Young debtor can instantaneously and linearly transform labor into units of any consumption good
- Young debtor employs capital in either a short term or long term production process
  - short term technology $f(k)$ with properties: one period maturity $f' > 0, f'' < 0$
  - long term technology $Af(k)$ with properties: two period maturity $A > 1$

Environment

Preferences

- Debtor: $-g(l_t) + u(x_{2,t+1} + x_{3,t+2})$
  - labor is costly and generation-specific consumption good enjoyed either when middle aged or old
- Creditor: $v(q_{2,t+1} + q_{3,t+2})$
  - enjoy generation-specific consumption good either when middle aged or old
  - perfect substitutes

Environment

Timing

- Young debtor travels to creditor island (acquires capital from young creditors)
- Young debtor returns and capital is employed in either short term or long term technology
- Young debtor's labor is used to produce consumption good
- Young creditors stay home

Environment

Timing

- Middle-aged creditors arrive at settlement island
- Short-term production is completed
- All middle-aged debtors arrive at settlement island
- All middle-aged leave for debtor island

Environment

Timing

- Long-term production is completed
- All old debtors arrive at settlement island
- $1 - \alpha$ measure of old creditors arrive at settlement island
- All old leave for debtor island

Planner's stationary allocation

Consider stationary allocations

$$\max_{\substack{l_1, l_1^*, x_2, x_2^*, x_3, x_3^* \\ q_2, q_2^*, q_3, q_3^*, k, k^*, \lambda}} \theta \left\{ \begin{array}{c} \lambda \left[-g\left(l_1\right) + u\left(x_2 + x_3\right)\right] + \\ \left(1-\lambda\right)\left[-g\left(l_1^*\right) + u\left(x_2^* + x_3^*\right)\right] \end{array} \right\}$$

$$+ \left(1-\theta\right) \left\{ \begin{array}{c} \left(1-\alpha\right) v\left(q_2 + q_3\right) + \\ \alpha v\left(q_2^* + q_3^*\right) \end{array} \right\}$$

s.t.

$$\begin{array}{l} \lambda \left[l_1 + f\left(k\right)\right] + \left(1-\lambda\right)\left[l_1^* + Af\left(k^*\right)\right] = \\ \quad \left(1-\alpha\right)\left(q_2 + q_3\right) \quad + \alpha\left(q_2^* + q_3^*\right) + \lambda\left(x_2 + x_3\right) + \left(1-\lambda\right)\left(x_2^* + x_3^*\right) \\ \kappa = \lambda k + \left(1-\lambda\right) k^* \\ 0 \le \lambda \le 1 \end{array}$$

Planner's stationary allocation

The allocation is characterized by

$$\begin{aligned}
\hat{\lambda} &= 0 \\
\hat{k}^* &= \kappa \\
\hat{q}_2 + \hat{q}_3 &= \hat{q}_2^* + \hat{q}_3^* \\
g'\left(\hat{l}_1^*\right) &= u'\left(\hat{x}_2^* + \hat{x}_3^*\right) \\
g'\left(\hat{l}_1^*\right) &= \frac{1-\theta}{\theta} v'\left(\hat{q}_2^* + \hat{q}_3^*\right)
\end{aligned}$$

Planner's stationary allocation

- Planner allocates all capital to long-term technology
- Creditor's consumption is independent of travel schedule

Additional issues

- Middle-aged and old agents are endowed with fiat money M
- Money is universally verifiable
- IOUs are verifiable but only on settlement island
- IOUs are state contingent with short term interest rate normalized to one and long-term rate equal to $\gamma \geq 1$
- IOUs redeemed by either money or goods
- There a secondary market for IOUs, the price of unredeemed IOUs sold here is denoted $\rho \leq 1$

Travel pattern

- Young debtor exchanges IOU for young creditor's capital
- Young debtors accept money for labor-produced goods sold to middle-aged and old agents
- Short-term producers settle all debts with middle-aged creditors
- Non-returning (middle-aged) creditors sell unredeemed IOUs
  - potential buyers are short-term producers and returning creditors
- Long-term producers settle all debts with IOU holders

Creditor's problem

$$\max \quad (1-\alpha)\, v\,(q_2+q_3)+\alpha\, v\,(q_2^*+q_3^*)$$

s.t.

$$\begin{aligned} \left[\rho\,(1+\gamma)\,(1-a)+a\right] p_k \kappa &= p_x\,(q_2+q_3) \\ \frac{\rho\,(1+\gamma)\,(1-a)+a}{\rho}\, p_k \kappa &= p_x\, q_3^* \end{aligned}$$

Lemma 1

In equilibrium, $\rho\left(1+\gamma\right)=1$

–the non-arbitrage condition

Short-term producer's problem

$$\max -g\left(l_1\right) + u\left(x_2 + x_3\right)$$

s.t

$$f\left(k\right) + l_1 - \frac{p_k}{p_x}k = \rho\left(x_2 + x_3\right)$$

FOCs:

$$\begin{aligned} f'\left(k\right) - \frac{p_k}{p_x} &= 0 \\ g'\left(l_1\right) - \frac{1}{\rho}u'\left(x_2 + x_3\right) &= 0 \end{aligned}$$

Long-term producer's problem

$$\max -g\left(l_1^*\right) + u\left(x_2^* + x_3^*\right)$$

s.t.

$$Af\left(k^*\right) + (1+\gamma)\left(l_1^* - \frac{p_k}{p_x}k^*\right) = x_3^*$$

FOCs:

$$\begin{aligned} Af'\left(k^*\right) - \frac{1}{\rho}\frac{p_k}{p_x} &= 0 \\ g'\left(l_1^*\right) - \frac{1}{\rho}u'\left(x_3^*\right) &= 0 \end{aligned}$$

Equilibrium

Definition:
(i) debtors and creditors maximize expected lifetime utility, taking prices as given;
(ii) all markets clear; and
(iii) the subjective distribution of production types is equal to the objective distribution of production types

Market clearing conditions

- Usual goods market, capital market, money market and loan market clearing conditions.
- IOU resale market clearing condition:
  $\lambda f(k) + l_1^* \geq (1-\alpha)\rho A f'(k^*)\kappa$
- The inequality holds if and only if $\rho = 1$.

Proposition

Proposition 1: The equilibrium measure of long-term producers is strictly positive, or $\lambda_t < 1$. With $\lambda_t > 0$, both short-term producers and long-term producers choose $k = k^* = \kappa$.

Intuition: If all debts are paid off in short term, there will be no profit in the IOU resale market. A debtor can be better off by choosing the long term production.

Stationary equilibrium

In stationary equilibrium,

$$g'\left(l_1^*\right) = \frac{1}{\rho} u'\left(x_3^*\right), \; x_2^* = 0$$

$$l_1 = l_1^*, \; x_2 = x_2^*, \; x_3 = x_3^*$$

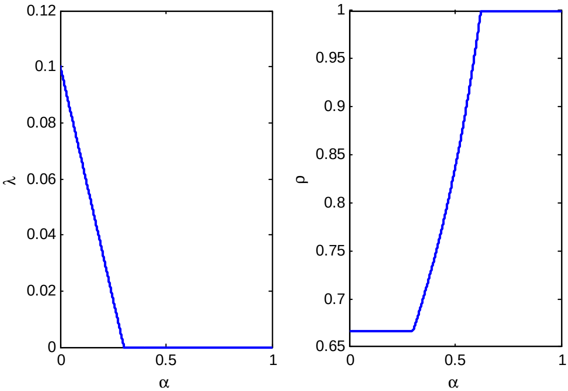
$$k = k^* = \kappa$$

Three mutually exclusive and complementary cases:

- Case 1: $\lambda = 0$, $\rho = 1$, and $l_1^* \geq (1-\alpha) Af'(\kappa)\kappa$. (ample liquidity)
- Case 2: $\lambda = 0$, $\rho \in (1/A, 1)$, and $(1-\alpha) f'(\kappa)\kappa \leq l_1^* < (1-\alpha) Af'(\kappa)\kappa$. (scarce liquidity)
- Case 3: $\lambda = \frac{(1-\alpha)f'(\kappa)\kappa - l_1^*}{f(\kappa)}$, $\rho = \frac{1}{A}$, and $l_1^* < (1-\alpha) f'(\kappa)\kappa$. (very scarce liquidity)

Stationary equilibrium – an example

Utility fcns: $-g(l_1) + u(x_2 + x_3) = \phi\sqrt{1 - l^2} + \frac{(x_2+x_3)^{1-\sigma}}{1-\sigma}$,
$v(q_2 + q_3) = \frac{(q_2+q_3)^{1-\sigma}}{1-\sigma}$, $\sigma = 1.5$, $\phi = 4$.
Production fcn: $f(k) = k^{1/3}$, $A = 1.5$, $\kappa = 1$.



Proposition

Proposition 2: The stationary equilibrium in the decentralized economy achieves the planner's allocation for some welfare weight θ if and only if the equilibrium is in case 1.

A three-step central bank operation

1. The central bank issues money to buy IOUs at $\rho = 1$.
2. IOUs are redeemed by long-term producers using consumption goods next period.
3. The central bank sells consumption goods on the debtor island to redeem money.
   Also
4. A lump-sum tax-transfer scheme to achieve the welfare weights.

A three-step central bank operation

With the central bank policy

- No profit in the IOU resale market. No short term production.
- No distortion in debtor's intertemporal marginal substitution.
- The total money stock is constant.
- As long as $\hat{l} \leq (1-\alpha) A f'(\kappa) \kappa$, there is no active central bank trading in the IOUs resale market.

Fisherian creditors

- The preference of the non-returning creditors is modified to $u\left(q_{2t+1}\right)$. All others remain the same.
- Old-age consumption cannot substitute for middle age consumption.
- The stationary equilibrium in the decentralized economy without policy intervention does not change.

Planner's allocation

- Additional constraint: $\lambda \left[l_1 + f\left(k\right) \right] + \left(1 - \lambda\right) l_1^* \geq \left(1 - \alpha\right) q_2$

Three mutually exclusive and complementary cases:

1. perfect risk sharing
2. rationing with all long-term production
3. mixed short-term and long-term production

Optimal policy

To implement the planner's allocation, the central bank use the same 3-step operations,

- except that $\rho = \frac{u'\left(\hat{x}_3^*\right)}{g'\left(\hat{l}_1^*\right)}$
- the lump-sum tax-transfer scheme is bounded from implementing transfers to the early-settling creditors
- there is no active central bank trading if $\hat{q}_2 \leq Af'\left(\kappa\right)\kappa$ in PA I and $\hat{q}_2 = \frac{u'\left(\hat{x}_3^*\right)}{g'\left(\hat{l}_1^*\right)} Af'\left(\kappa\right)\kappa$ in PA II and III.

Conclusion

- We construct a model to think about unconventional monetary policy.
- Unconventional monetary policy alleviates liquidity problem in the private debt market.
- Monetary policy is not only about the total quantity of money (aggregate money growth rate), it is also about the amount of liquidity in a specific market.