Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality

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Introduction

- adverse selection as a source of illiquidity
  - sellers can always sell an asset for a low price
  - owners of good assets demand a high price in an illiquid market
- one possible explanation for fire sales in asset markets in 2007–2008
- asset purchase program can raise prices and alleviate illiquidity
- contrast this with a more standard “pooling” equilibrium

Some Literature

- adverse selection with pooling:

  Eisfeldt (2004), Kurlat (2009), Daley and Green (2010), Chari, Shourideh, and Zetlin-Jones (2010), Tirole (2011)

- adverse selection with separation:

  De Marzo and Duffie (1999), Guerrieri, Shimer and Wright (2010), Chang (2010)

- illiquidity and search frictions:

  Duffie, Garleanu and Pederson (2005), Weill (2008), Lagos and Rocheteau (2009)

Model

Model

- unit measure of risk-neutral, infinitely-lived consumers
  - stochastic discount factor, i.i.d.
  - β_s with probability π_s, $s \in \{l, h\}$
  - later we allow for a Markov process
- fixed supply of heterogeneous trees
  - type $j \in \{1, \ldots, J\}$ tree produces δ_j units of fruit per period
  - $\delta_{j+1} > \delta_j > 0$, measure K_j of type j trees
- fruit is perishable
- low β consumers sell trees to high β consumers
- the owner of a tree knows its type j, but no one else does

Timeline

- each agent owns a portfolio of trees $\{k_j\}$
- trees produce fruit
- discount factors are realized
- buyers and sellers choose prices $p \in \mathbb{R}$
- trade occurs
- agents consume their remaining fruit

Key Equilibrium Objects

- $\Theta(p) \in [0, \infty]$: buyer-seller ratio at price p
  - sell a tree at p with probability $\min\{\Theta(p), 1\}$
  - buy a tree at p with probability $\min\{\Theta(p)^{-1}, 1\}$
- $\Gamma(p) \in \Delta^J$: probability distribution over types at price p
  - $\gamma_j(p)$ is the fraction of type j trees offered at price p
- $\mathbb{P}$: set of prices with trade
- F: cumulative distribution of prices

Equilibrium

Equilibrium

- can solve everything on a per-tree basis (Proposition 1)
  - $v_{s,j}$: value of a type j tree to a consumer in preference state s
  - $\bar{v}_j = \pi_h v_{h,j} + \pi_l v_{l,j}$: continuation value
- equilibrium is a vector $(v_h, v_l, \Theta, \Gamma, \mathbb{P}, F)$

Equilibrium

- buyers' optimality:

$$v_{h,j} = \max_p \left(\min\{\Theta(p)^{-1}, 1\} \frac{\delta_j}{p} \beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'} + (1 - \min\{\Theta(p)^{-1}, 1\}) \delta_j \right) + \beta_h \bar{v}_j$$

Equilibrium

- buyers' optimality:

$$v_{h,j} = \delta_j \max_p \left(\min\{\Theta(p)^{-1}, 1\} \frac{\beta_h \sum_{j'} \gamma_{j'}(p)\bar{v}_{j'}}{p} + (1 - \min\{\Theta(p)^{-1}, 1\}) \right) + \beta_h \bar{v}_j$$

Equilibrium

- buyers' optimality:

$$v_{h,j} = \delta_j \lambda + \beta_h \bar{v}_j,$$

where

$$\lambda \equiv \max_p \left(\min\{\Theta(p)^{-1}, 1\} \frac{\beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'}}{p} + (1 - \min\{\Theta(p)^{-1}, 1\}) \right)$$

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- active markets: $p \in \mathbb{P} \Rightarrow p$ solves the above problem

Equilibrium

- sellers' optimality:

$$v_{l,j} = \delta_j + \max_p \Big(\min\{\Theta(p), 1\} p + \big(1 - \min\{\Theta(p), 1\}\big) \beta_l \bar{v}_j \Big)$$

Equilibrium

- sellers' optimality:

$$v_{l,j} = \delta_j + \max_p \Big(\min\{\Theta(p), 1\}p + \big(1 - \min\{\Theta(p), 1\}\big)\beta_l \bar{v}_j \Big)$$

- rational beliefs: if $\Theta(p) < \infty$ and $\gamma_j(p) > 0$,

$$v_{l,j} = \delta_j + \min\{\Theta(p), 1\}p + (1 - \min\{\Theta(p), 1\})\beta_l \bar{v}_j$$

Equilibrium

- all sellers' trees are offered for sale at some price $p \in \mathbb{P}$:

$$\frac{K_j}{\sum_{j'} K_{j'}} = \int_{\mathbb{P}} \gamma_j(p) dF(p)$$

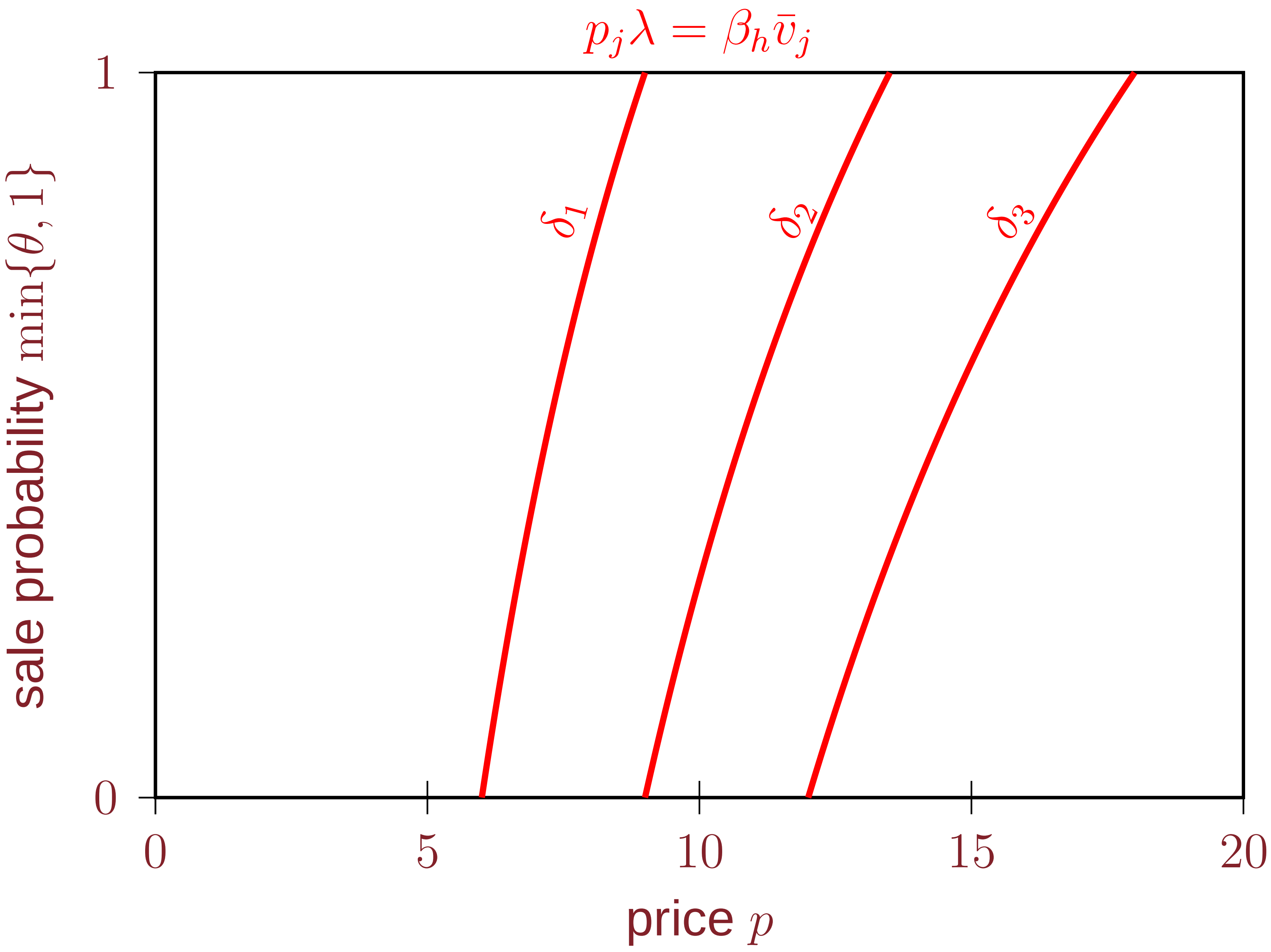
- fruit market clears:

$$\pi_h \sum_j \delta_j K_j = \pi_l \sum_j K_j \int_{\mathbb{P}} \Theta(p) p dF(p)$$

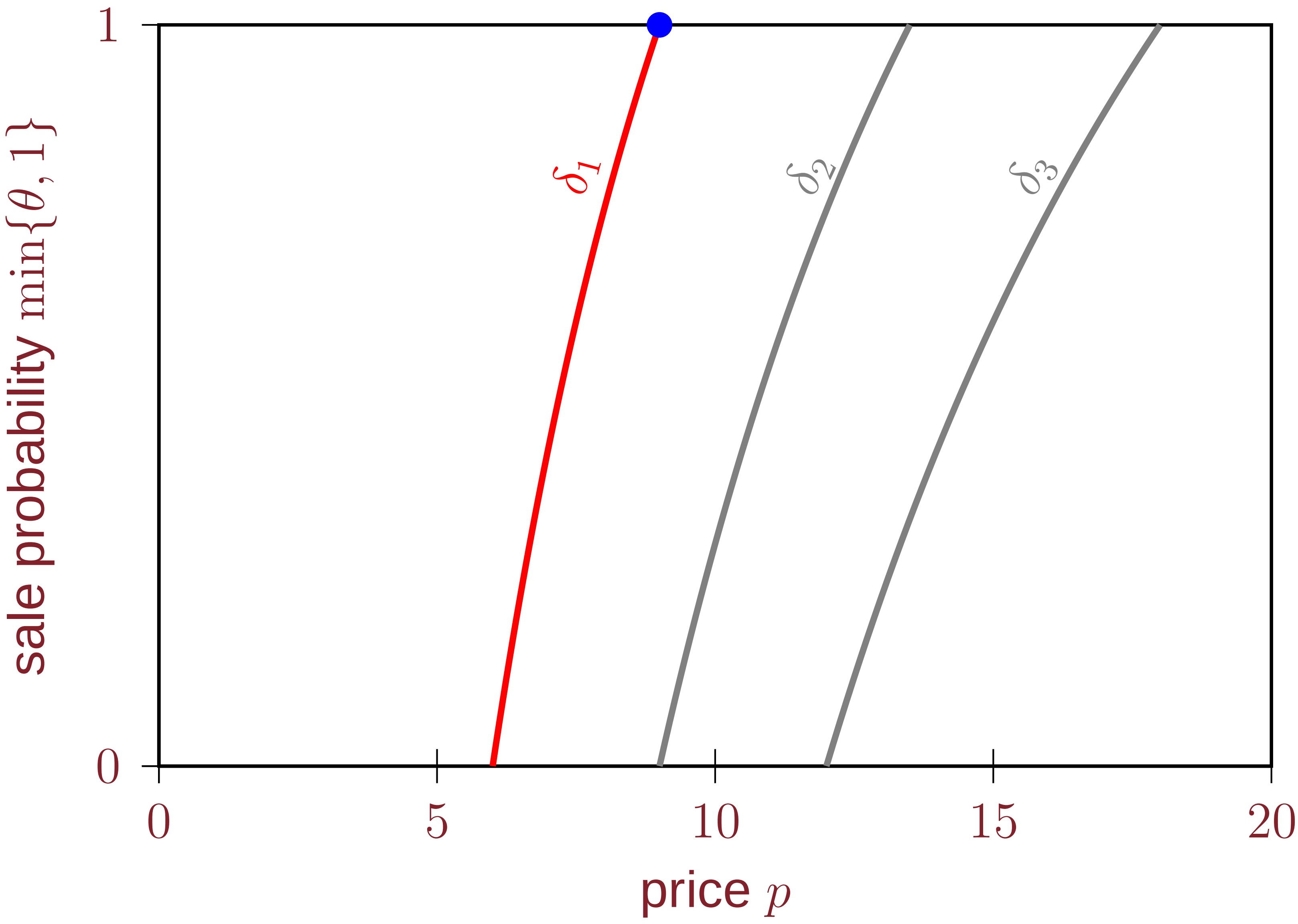
Characterization

- equilibrium exists and is unique
- equilibrium is separating
- algorithm for finding an equilibrium
  - fix $\lambda \in [1, \beta_h/\beta_l]$
  - find a “partial equilibrium”
  - check if fruit-market clears
- next: algorithm to find a partial equilibrium

Buyers' Indifference Curves



Sellers' Indifference Curves



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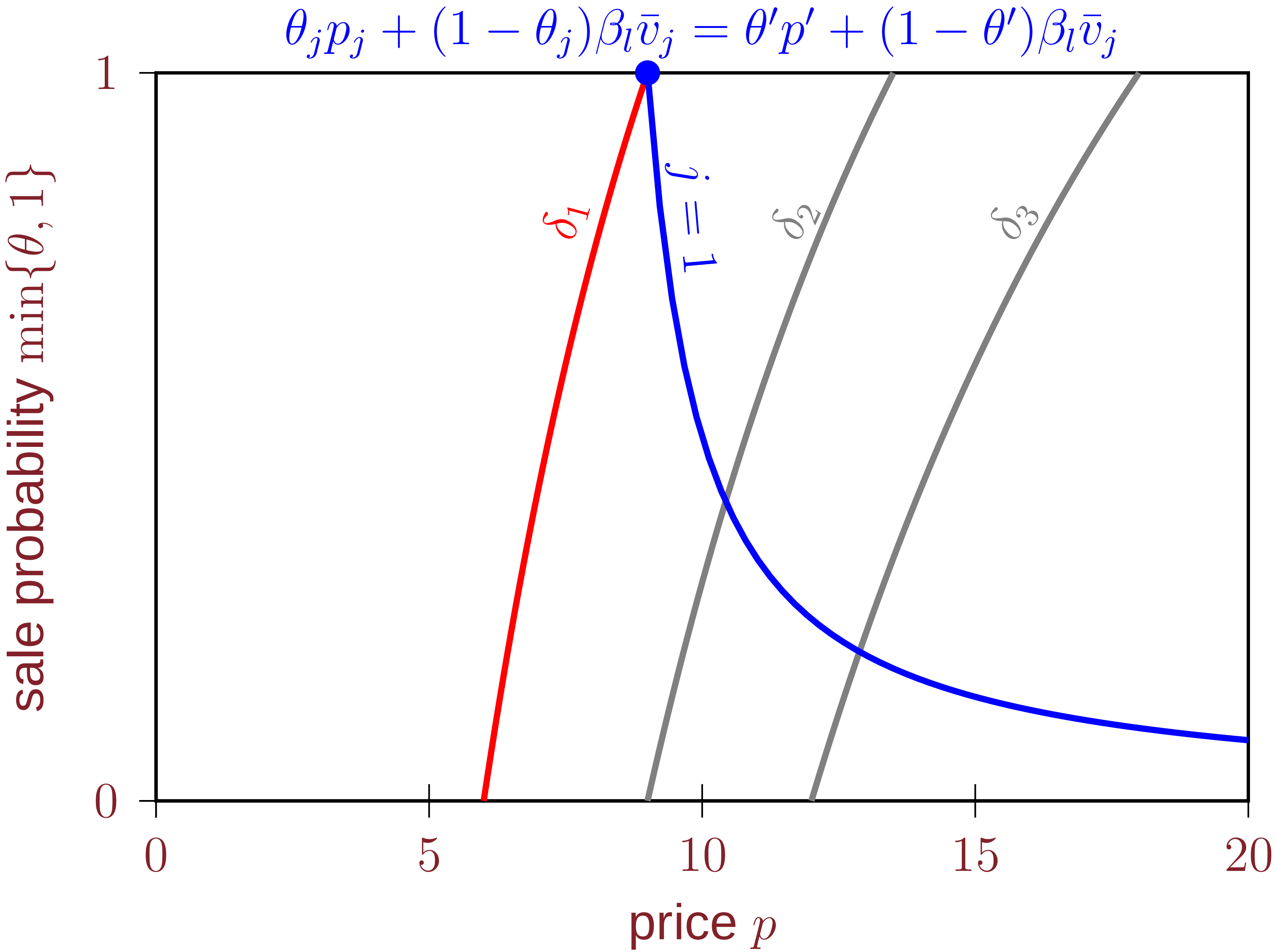

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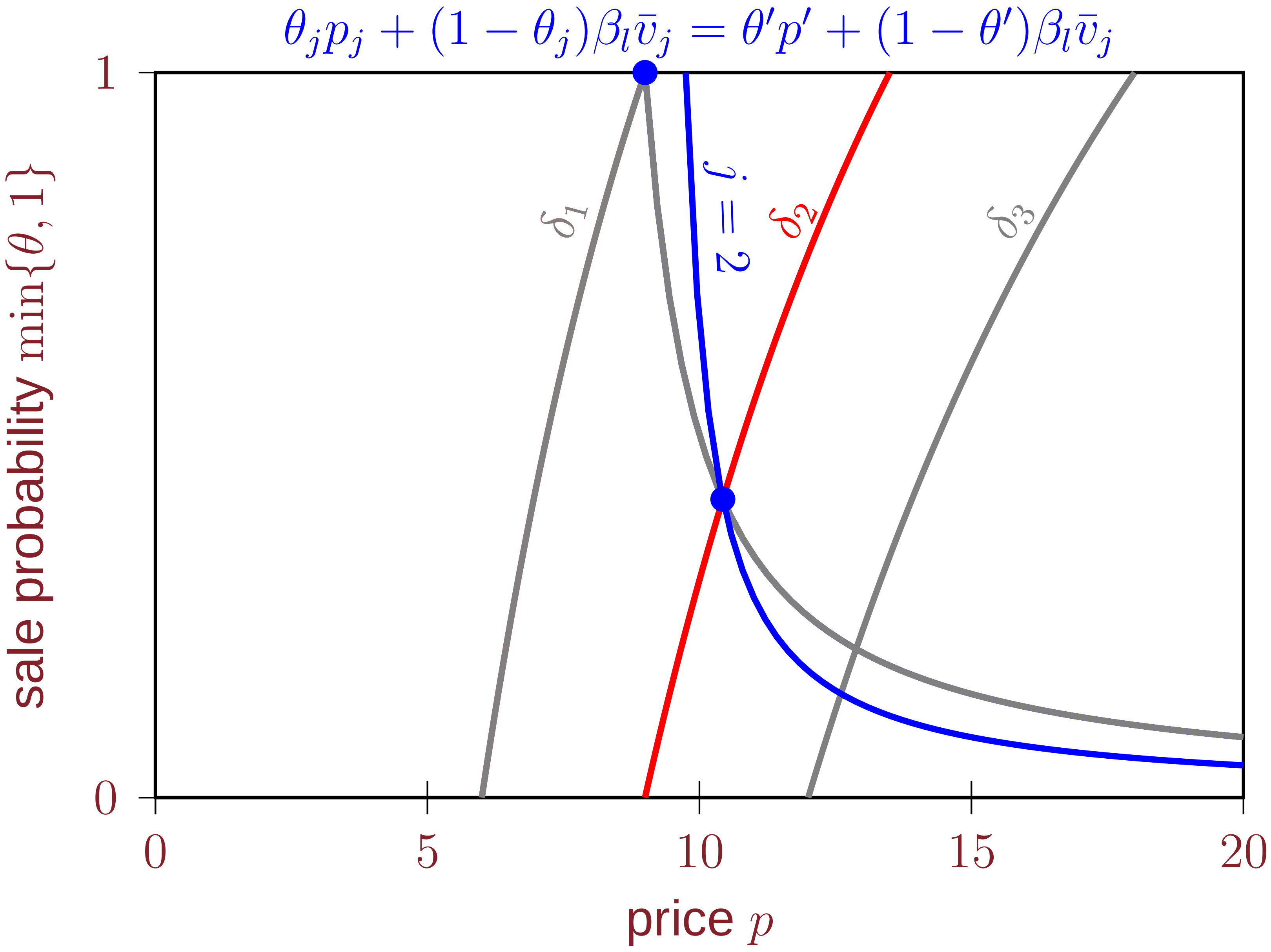

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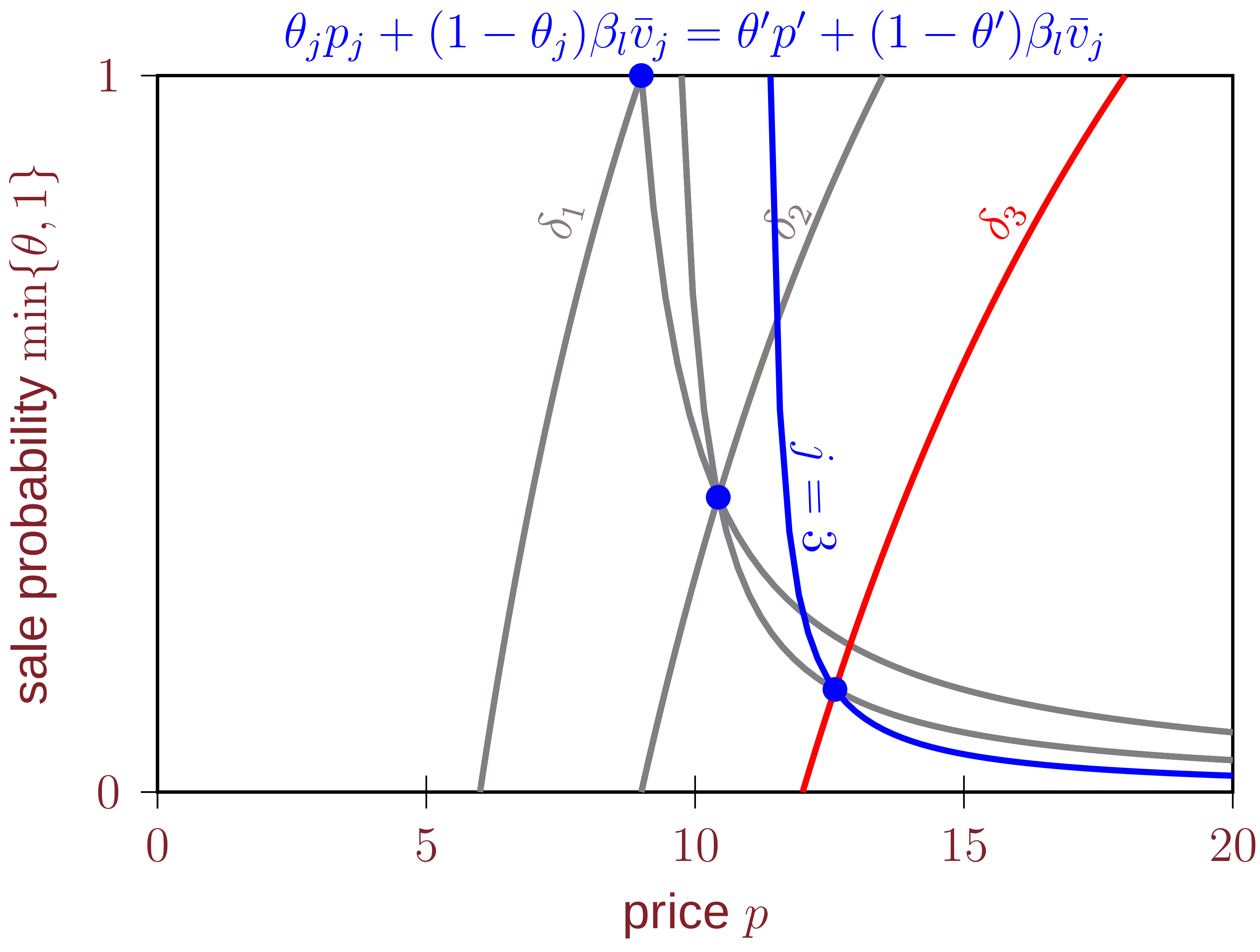
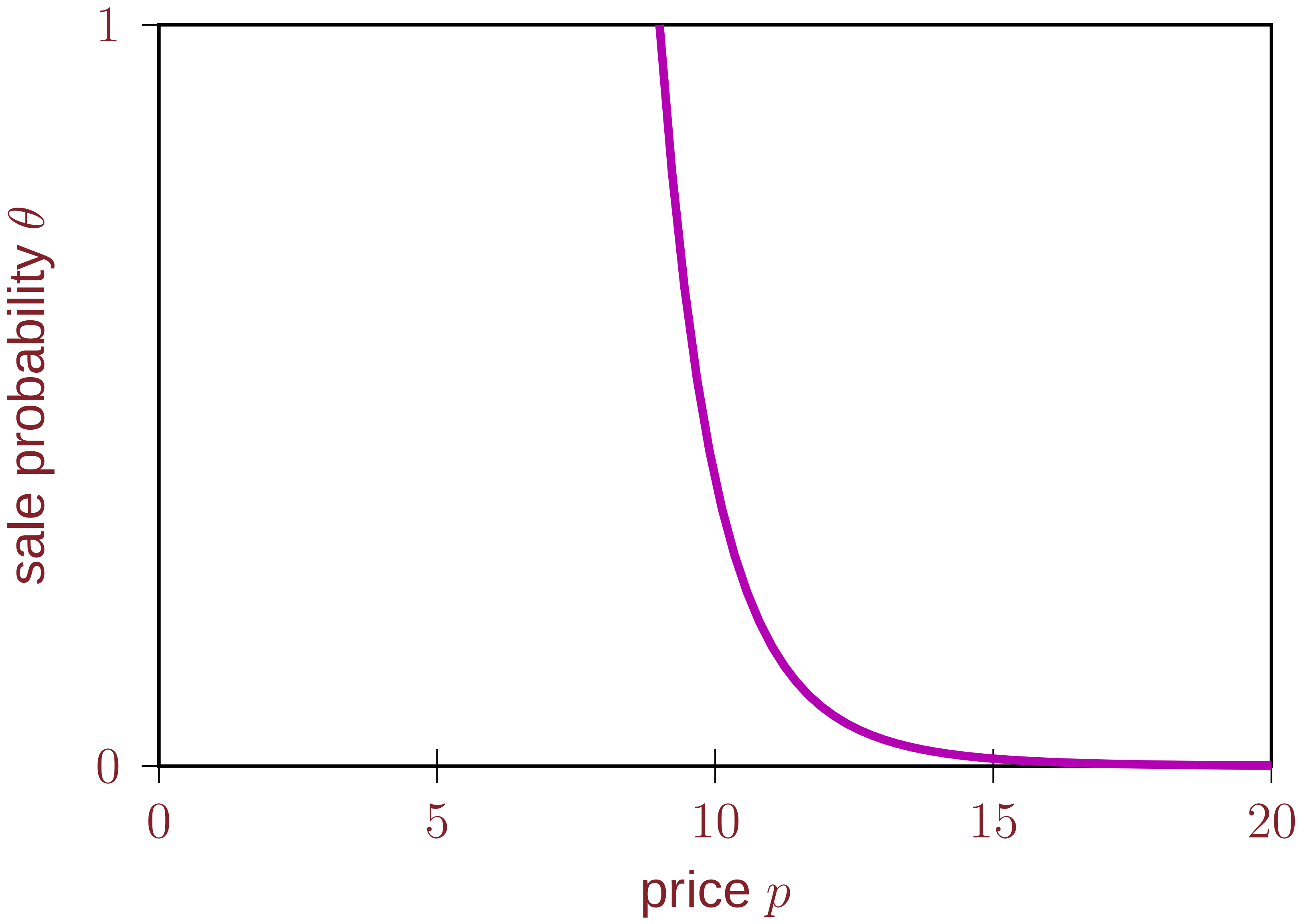


Continuous Types and Continuous Time

Continuous Types



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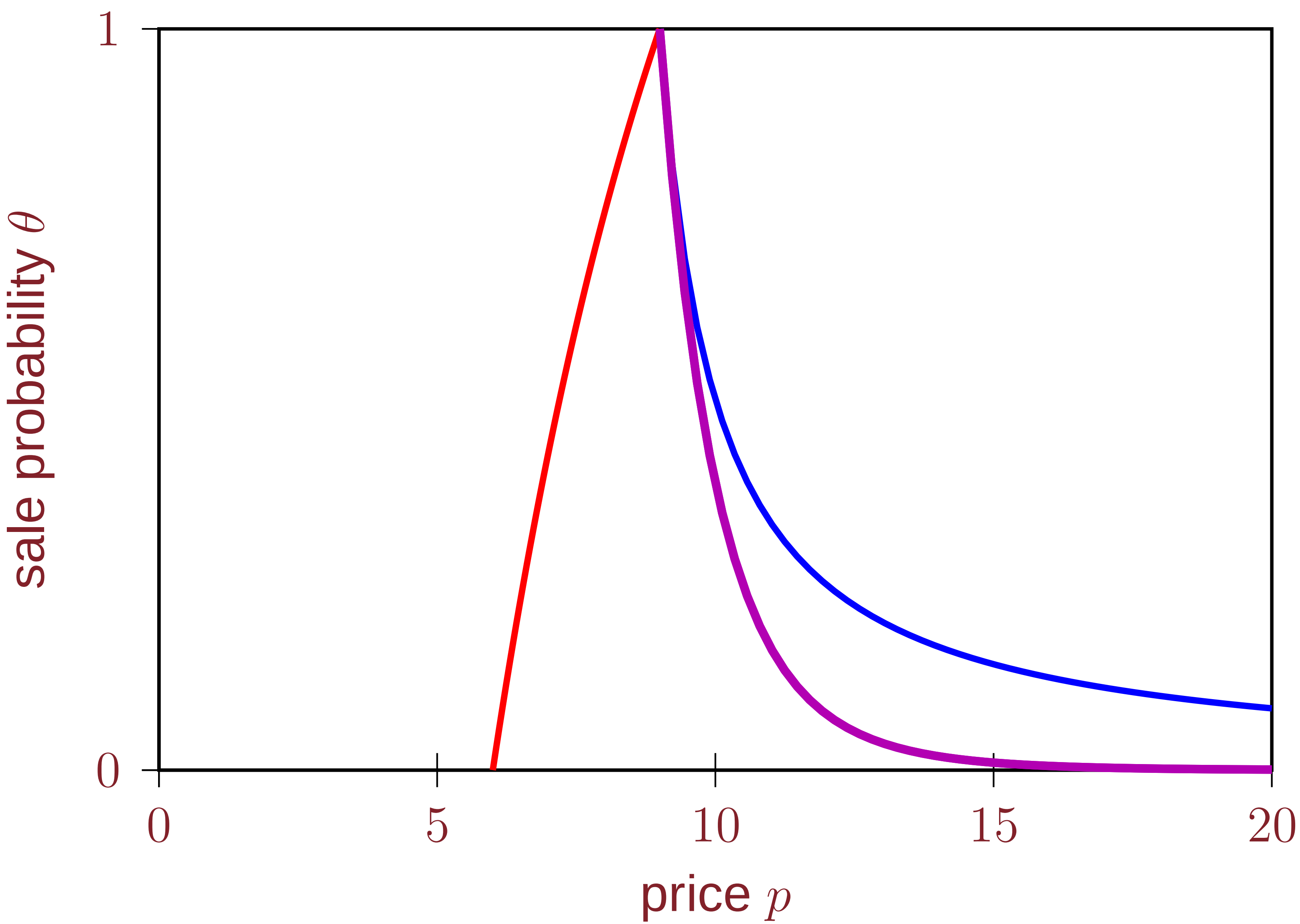

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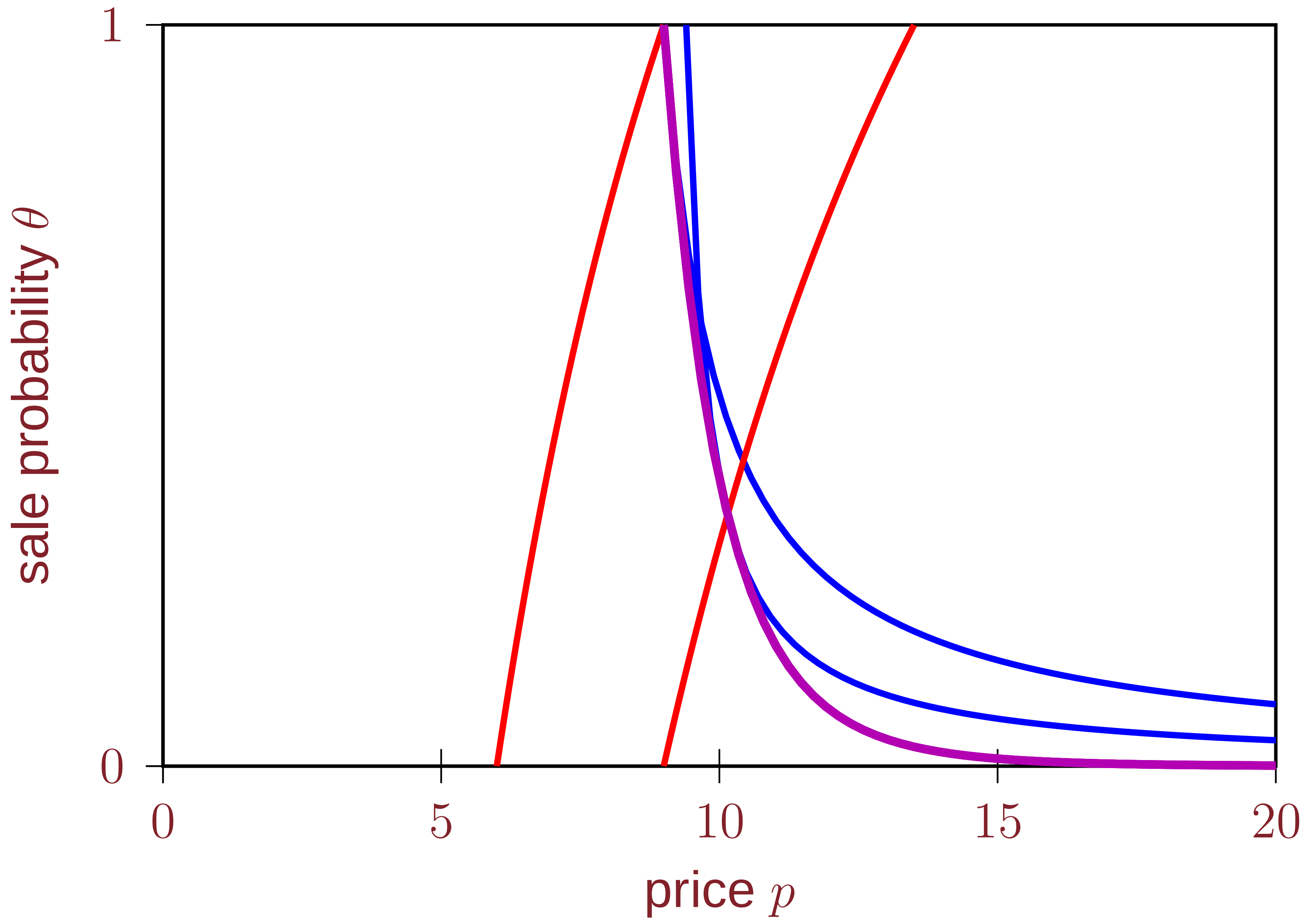

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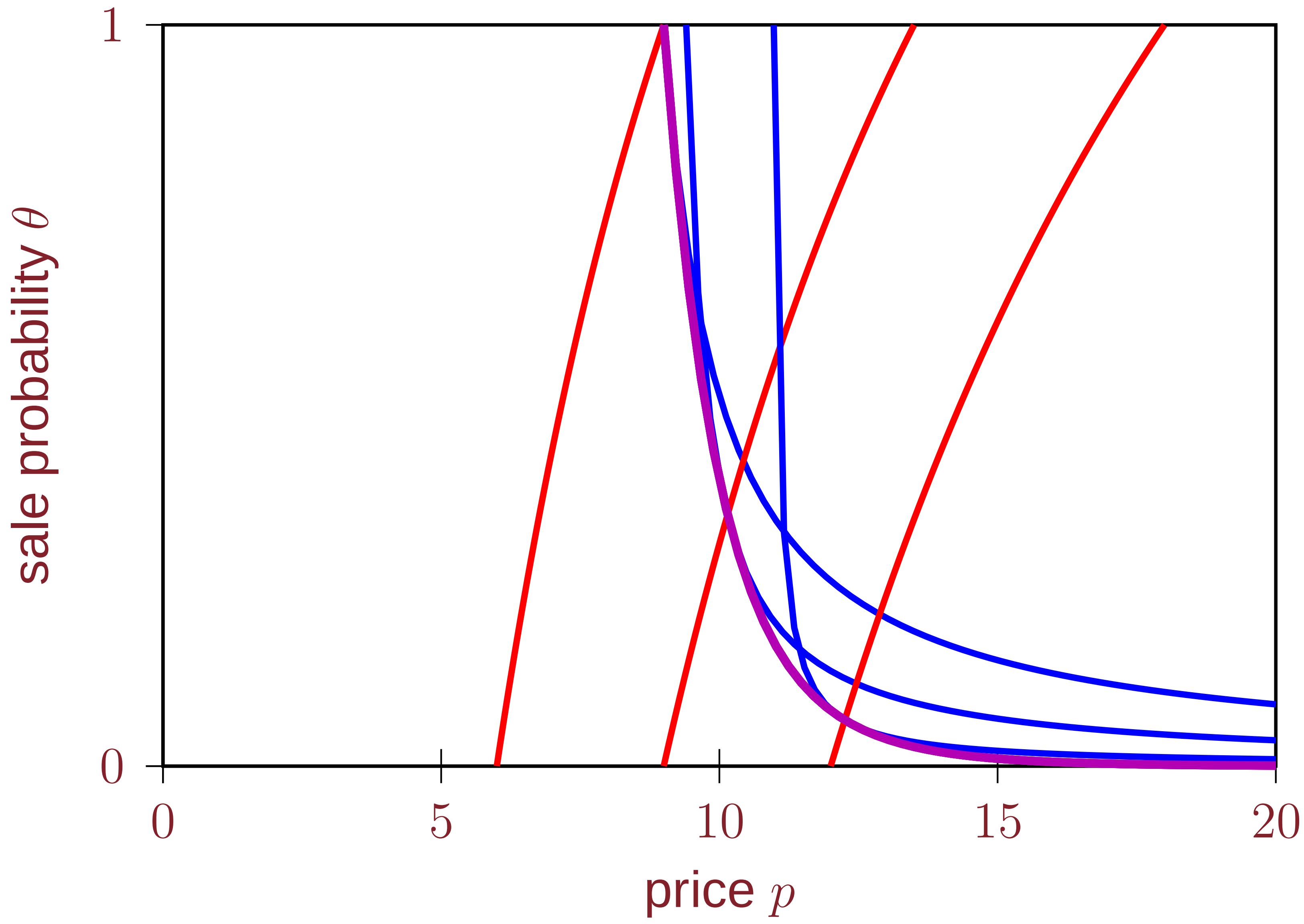


Closed-Form Solution

- lowest price: $P(\underline{\delta}) = \dfrac{\underline{\delta}\beta_h(\pi_l + \pi_h\lambda)}{\lambda - \beta_h(\pi_l + \pi_h\lambda)}$

- sale probability: $\Theta(p) = \left(\dfrac{P(\underline{\delta})}{p}\right)^{\frac{\beta_h}{\beta_h - \beta_l\lambda}}$

- rate of return decreasing in Θ, hence increasing in δ:

$$\frac{\delta + P(\delta)}{P(\delta)} = \frac{\lambda + (\beta_h - \lambda\beta_l)(1 - \Theta(P(\delta)))(1 - \pi_h)}{\beta_h(\pi_l + \lambda\pi_h)}$$

Persistent Types and Continuous Time

- allow preferences to follow a first order Markov process: $\pi_{ss'}$
- useful for taking a continuous time limit of the model
  - q_{hl} and q_{lh} are transition rates for preferences
  - $\rho_h < \rho_l$ are discount rates
- in continuous time, buyers contact sellers at a Poisson rate $\alpha(p)$
- for example, if tree types are dense on $[\underline{\delta}, \bar{\delta}]$ and $\lambda = 1$:

$$\alpha(p) = \frac{q_{hl} + q_{lh} + \rho_l}{\left(\frac{p}{P(\underline{\delta})}\right)^{\frac{q_{hl}+q_{lh}+\rho_l}{\rho_l-\rho_h}} - 1}$$

- real trading delays even if trading opportunities are abundant
  - contrast with search theoretic models of illiquidity

Firesales, Flight to Quality, and Asset Purchase Programs

Firesales

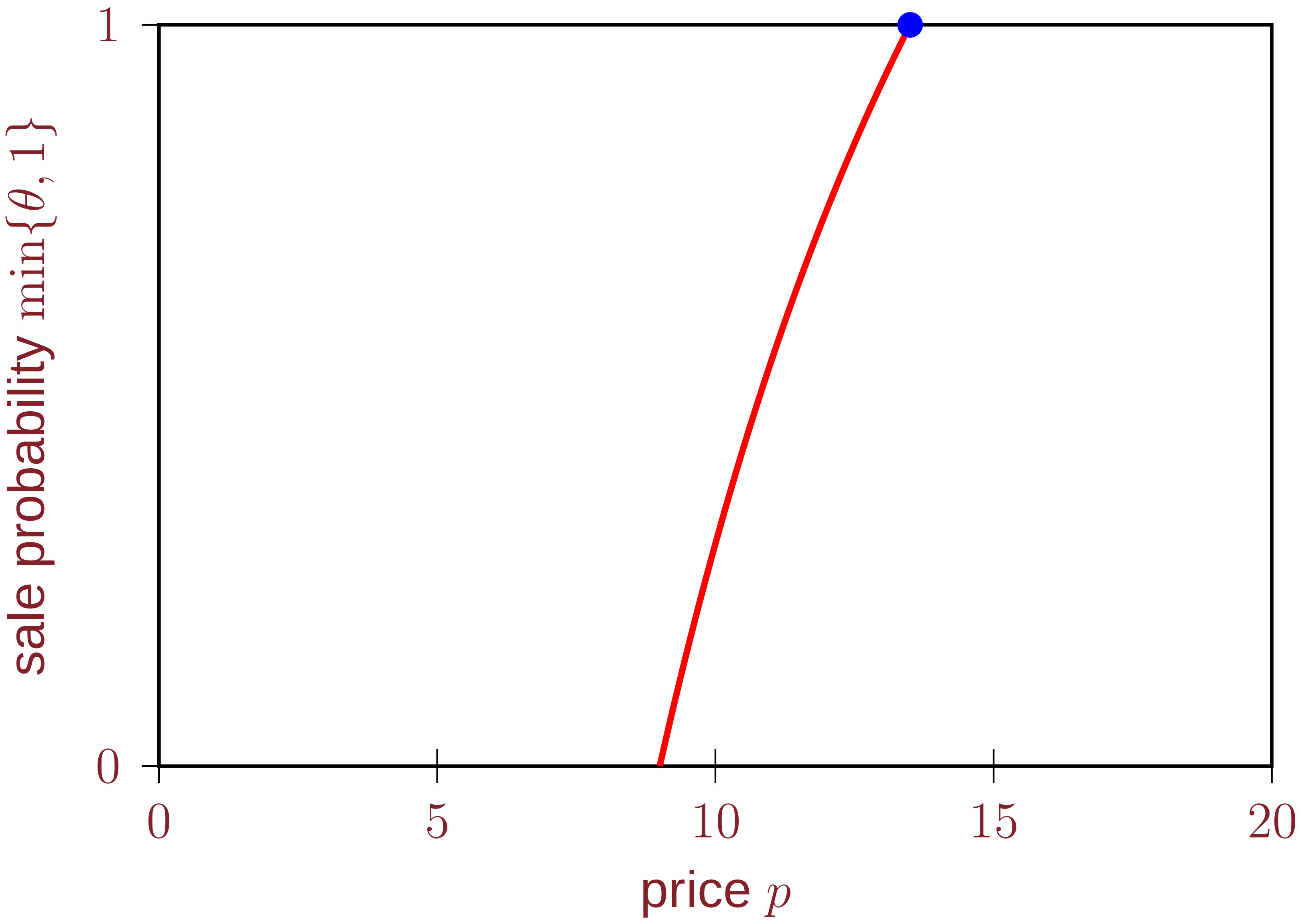
- possible explanation for fire sales in asset markets in 2007–2008

> “The crisis that can occur with debt is due to the fact that the debt is not riskless. A bad enough shock can cause information insensitive debt to become information sensitive, make the production of private information profitable, and trigger adverse selection. Instead of trading at the new and lower expected value of the debt given the shock, agents trade much less than they could or even not at all. There is a collapse in trade. The onset of adverse selection is the crisis.”
> – Dang, Gorton, and Holmström (2009)

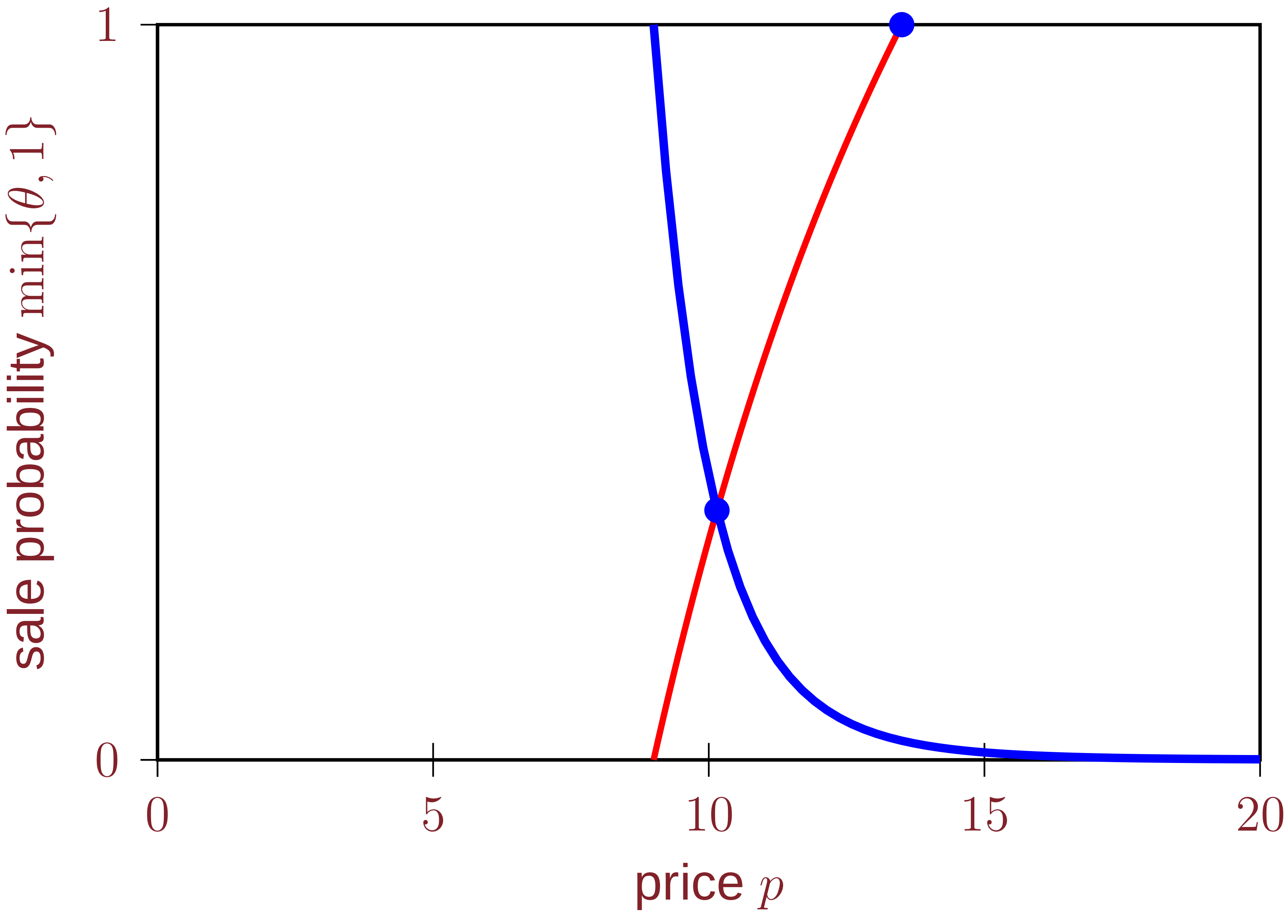
Firesales

- suppose initially everyone believes all trees are worth δ_0
- suddenly we learn there is dispersion in tree quality
  - expected value is δ_0, but $\underline{\delta} < \delta_0$
  - value function $v_{s,j}$ is convex, so everyone wants to learn δ
  - trees become illiquid, possibly reducing all tree prices

Firesale



Firesale



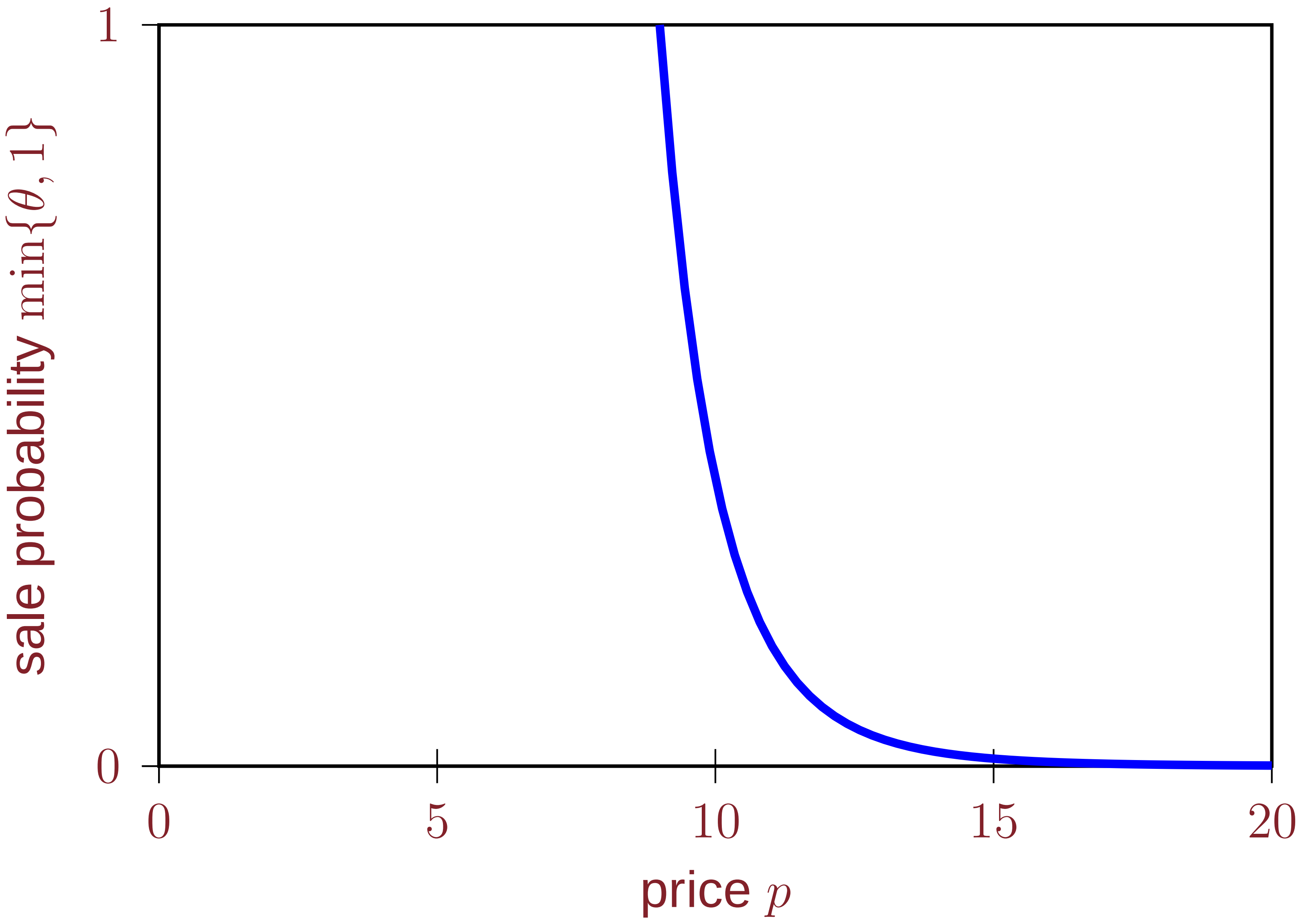
Flight to Quality

- imagine there are two types of trees
  - potential adverse selection problem for type a trees
  - no adverse selection problem for type b trees
  - all fruit are perfect substitutes
- emergence of adverse selection reduces λ if originally $\lambda > 1$
  - the price of type b trees increases
  - interpret this as a flight to quality

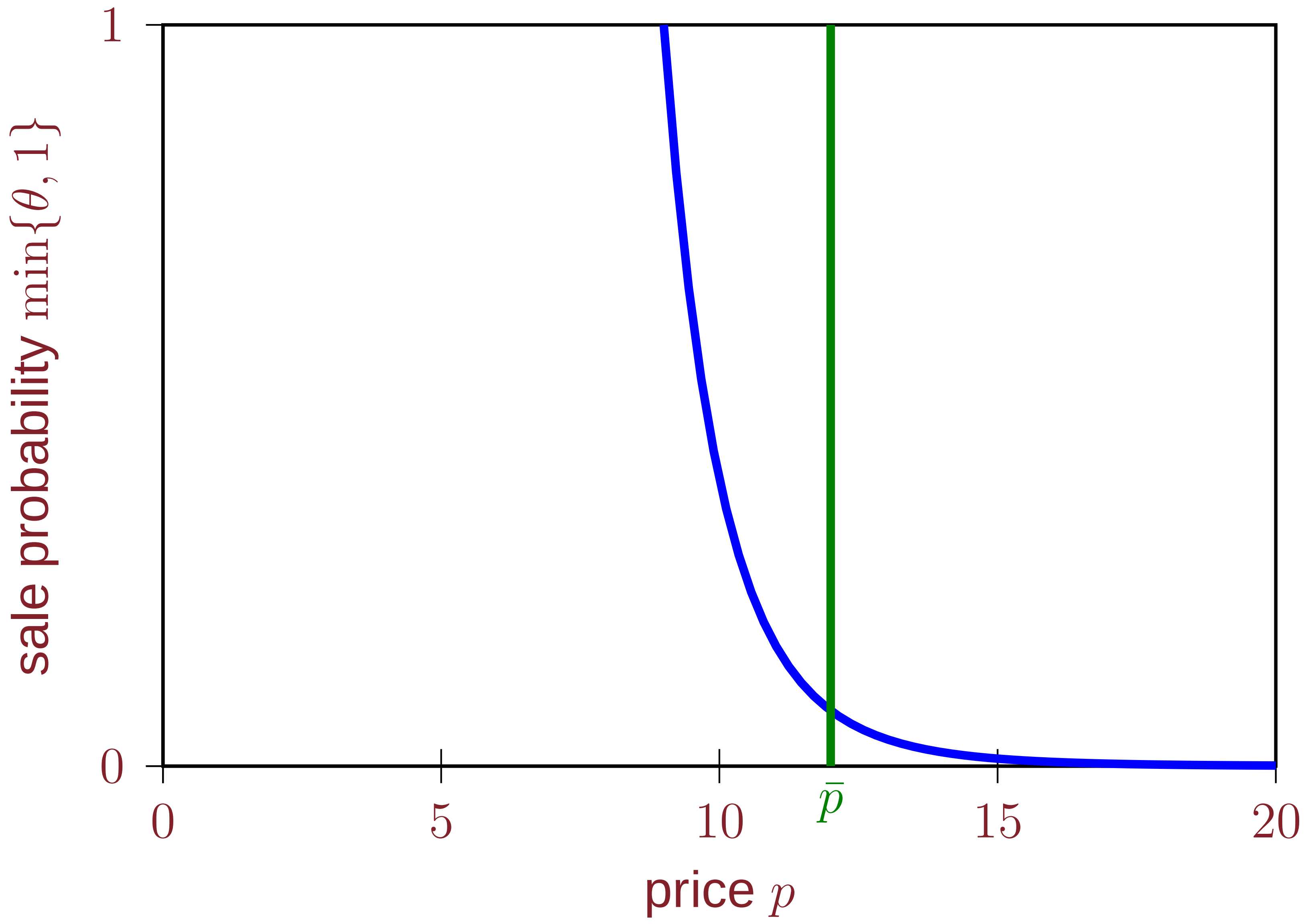
Asset Purchase Program

- suppose "government" can offer to pay $\bar{p} > \underline{p}$ for any tree
- new equilibrium:
  - minimum price in private market is $\bar{p}$
  - government buys trees which, if completely liquid, are worth less
  - other trees stay in the private market, prices & liquidity increase
  - price of another type of tree (without adverse selection) falls
- if government previously owned some trees, can even be profitable

Asset Purchase Program



Asset Purchase Program



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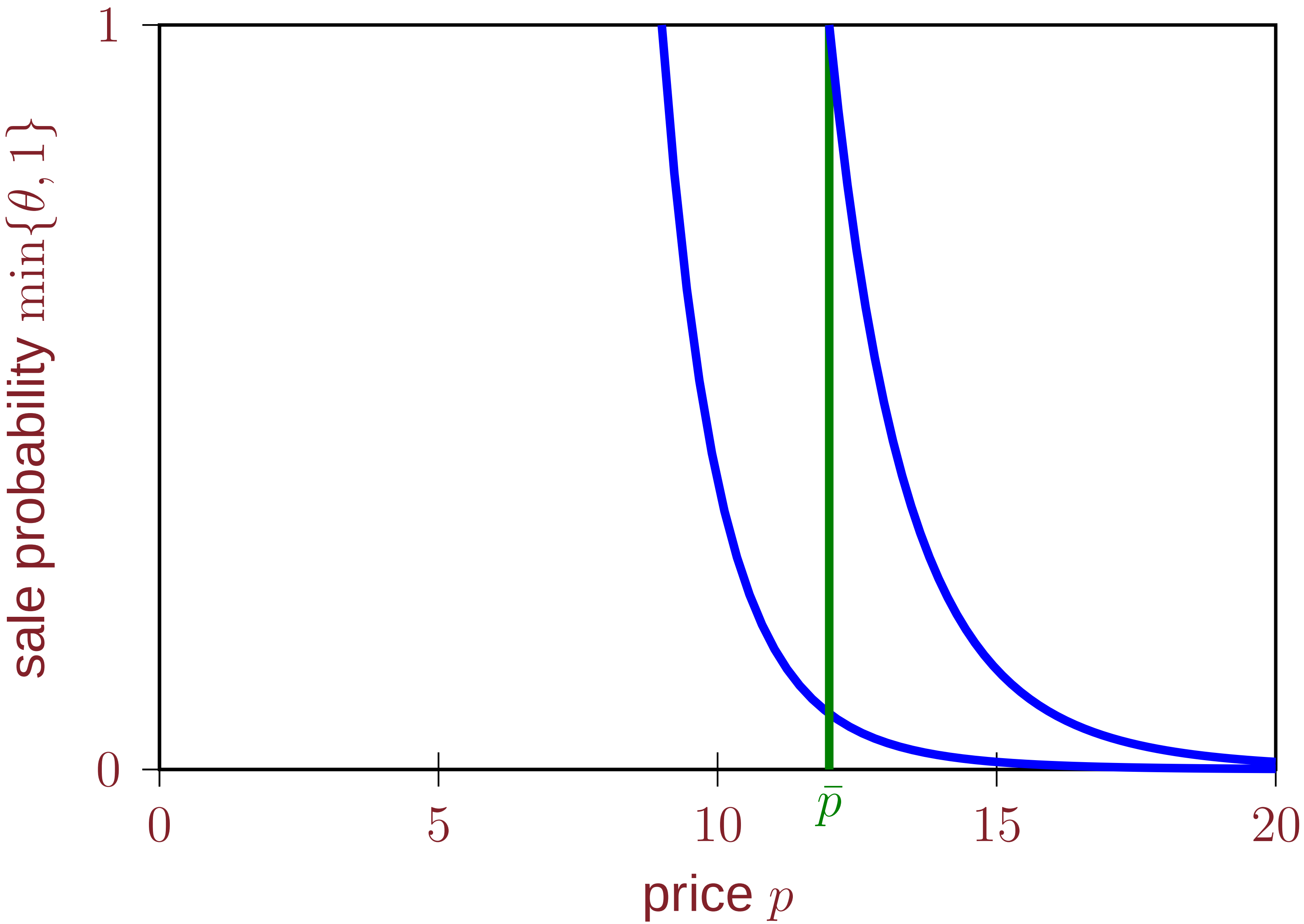

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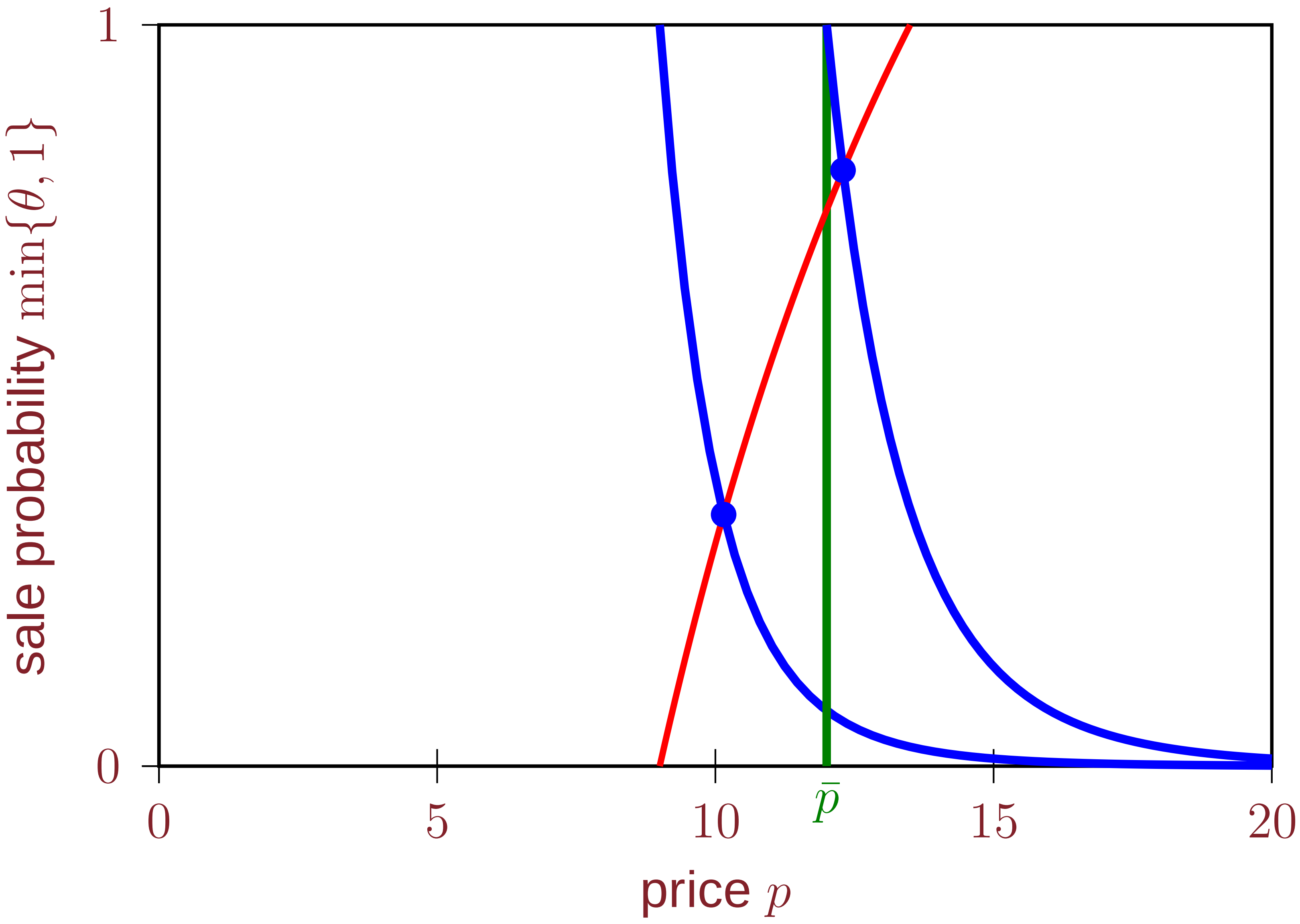

Pooling Environment

Environment

- focus with assets dense on $[\underline{\delta}, \bar{\delta}]$
- all trades occur at a common price p
- value functions: $v_h(\delta) = \delta\lambda + \beta_h \bar{v}(\delta)$ and $v_l(\delta) = \delta + \max\{p, \beta_l \bar{v}(\delta)\}$
- seller's optimality: $\zeta(\delta) = \begin{cases} 1 & \text{if } p > \beta_l \bar{v}(\delta) \\ 0 & \text{if } p < \beta_l \bar{v}(\delta) \end{cases}$
- buyers' optimality: $p\lambda = \beta_h \dfrac{\int_{\underline{\delta}}^{\bar{\delta}} \zeta(\delta)\bar{v}(\delta) d\Phi(\delta)}{\int_{\underline{\delta}}^{\bar{\delta}} \zeta(\delta) d\Phi(\delta)}$
- market clearing: $\pi_h \displaystyle\int_{\underline{\delta}}^{\bar{\delta}} \delta d\Phi(\delta) = \pi_l p \int_{\underline{\delta}}^{\bar{\delta}} \zeta(\delta) d\Phi(\delta)$

Key Outcome

- trees with $\delta < \delta^*$ are liquid, $\delta > \delta^*$ are illiquid

$$\delta^* = \frac{\beta_h(1-\pi_h\beta_h-\pi_l\beta_l)}{\beta_l(\lambda(1-\pi_h\beta_h)-\pi_l\beta_h)} \frac{\int_{\underline{\delta}}^{\delta^*} \delta d\Phi(\delta)}{\int_{\underline{\delta}}^{\delta^*} d\Phi(\delta)}$$

- possible nonuniqueness
  - but see Chari, Shourideh, and Zetlin-Jones
  - or assume $\int_{\underline{\delta}}^{\delta^*} \Phi(\delta) d\delta$ is log concave

Results

- notion of liquidity is dichotomous
- no link between price, dividend, and liquidity
- firesales: dispersion in tree quality weakly reduces the price
- asset purchase program
  - private market price must be $\bar{p}$
  - size of private market after intervention is indeterminate
  - odd behavior if the government caps the size of the program

Results

- notion of liquidity is dichotomous
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- firesales: dispersion in tree quality weakly reduces the price
- asset purchase program
  - private market price must be $\bar{p}$
  - size of private market after intervention is indeterminate
  - odd behavior if the government caps the size of the program
- using the correct notion of equilibrium matters

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Characterization Result

- solve the following sequence of problems (P-j):

$$v_{l,j} = \delta_j + \max_{p,\theta} \left(\min\{\theta, 1\} p + (1 - \min\{\theta, 1\}) \beta_l \bar{v}_j \right)$$

s.t. $p \le \beta_h \bar{v}_j$,

$v_{l,j'} \ge \delta_{j'} + \min\{\theta, 1\} p + (1 - \min\{\theta, 1\}) \beta_l \bar{v}_{j'}$ for all $j' < j$

$\bar{v}_j = \pi_h \left(\delta_j + \beta_h \bar{v}_j \right) + \pi_l v_{l,j}$

- solution is unique, except $\theta_1 \ge 1$ (Lemma 1)
- pin down θ_1 to ensure fruit market clears

$$\pi_h \sum_j \delta_j K_j = \pi_l \sum_j \theta_j p_j K_j$$

  - if this defines $\theta_1 < 1$, look for a different type of equilibrium