Housing and Liquidity

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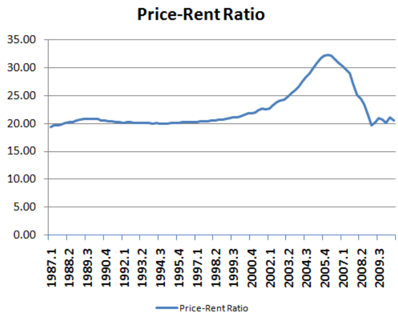
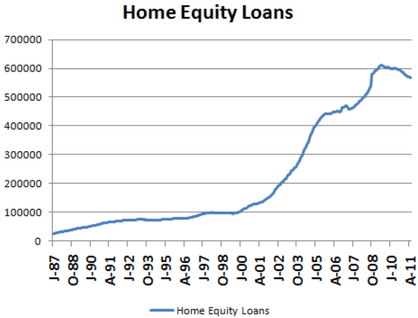
Motivation



(a) Price-Rent Ratio



(b) Home Equity Loans

Figure

Motivation

- Reinhart and Rogoff (2009) argue that securitization allowed consumers "to turn their previously illiquid housing assets into ATM machines." Ferguson(2008) also says this "allowed borrowers to treat their homes as cash machines."
- Greenspan (2007) says that home equity withdrawal financed about 3% of personal consumption from 2001 to 2005.
- According to Federal Reserve Survey of Terms of Business Lending (September 2006)
    - 49.7% US households had home-secured debt in 2004.
    - 46.9% commercial and industrial loans secured by collateral. Typical collateral is real estate.

What we do

- Idea: Liquidity or pledgability plays a role in the price of housing.
- Model the liquidity or pledgability component explicitly (Kiyotaki-Moore; Holmström-Tirole)
- Whether liquidity premium exists when housing stock is high or low depends on the preference.
- Study the dynamics of housing prices.
- Investigate the interaction between other liquid assets, including money, and housing.
- Baseline Model: Rocheteau-Wright (2005, 2010) with housing
  - See the paper for other references, including housing literature.

Guideline

- A basic model with fixed housing supply and no money
- Augment the basic model with endogenous housing supply
- A monetary model with housing

Model Setup

- Time
  - Discrete time, infinite horizon
  - Two subperiods: First, DM opens, then CM opens.
- A continuum $[0, 1]$ of households.
- A continuum $[0, \overline{n}]$ of firms owned by the households.
- A fixed stock of housing H for now.

Model Setup(Continued)

- Decentralized Market
  - Captures the idea that sometimes credit is not perfect.
  - Loans require collateral, here this is home equity.
- Centralized Market
  - Active firms invest 1 unit of CM good to produce.
  - Firms issue bonds s to finance the investment.
  - Households work, consume CM good, settle debt, trade bonds and housing.
- Technology
  - In CM, labor is transformed to CM good one for one.
  - Active DM firms get assess to technology $x = f(1 - y)$.

Households Problem

- Households utility

$$\mathscr{U}(x,y,h,\ell) = U(x,h) + u(y) - \ell$$

- CM problem

$$W_t(s_t,d_t,h_t) = \max_{h_{t+1},\ell_t,x_t} \{U(x_t,h_t) - \ell_t + \beta V_{t+1}(s_{t+1},0,h_{t+1})\}$$

$$\text{s.t. } x_t + s_{t+1} + d_t + \psi_t h_{t+1} = \ell_t + R_t s_t + T_t + e_t.$$

- $e_t = \psi_t h_t$ is the home equity.

Households Problem with home production

- We can also model house as an input to produce home good.
- CM problem

$$W_t(s_t, d_t, h_t) = \max_{h_{t+1}, \ell_t^M, \ell_t^H, c_t^M, c_t^H} \left\{ U\left(c_t^M, c_t^H\right) - \ell_t^M - \ell_t^H + \beta V_{t+1}(s_{t+1}, 0, h_{t+1}) \right\}$$

$$\text{s.t. } c_t^H + s_{t+1} + d_t + \psi_t h_{t+1} = \ell_t^M + R_t s_t + T_t + e_t$$

$$c_t^H = F\left(\ell_t^H, h_t\right)$$

Decentralized Market

- Active firms match with households with matching function $M(1,n)$ with n being the number of active firms.
- Implies $\alpha_h = M(1,n)$ and $\alpha_f = M(1,n)/n$
- Trade happens bilaterally.
- A seller extends loans d_t to a buyer.
- House used as collateral for loans up to limit $D(e_t)$.

Example

$D(e) = D_0 + D_1 e.$

Decentralized Market

- DM problem

$$V_t(s_t, 0, h_t) = \alpha_h \{u(y_t) + W_t(s_t, d_t, h_t)\} + (1 - \alpha_h) W_t(s_t, 0, h_t)$$

where d_t and y_t are determined through an abstract trading mechanism in general.

Firm's Problem

- Profit

$$\begin{aligned}\pi_t &= \alpha_f \left[d_t + f(1-y_t)\right] + (1-\alpha_f) f(1) - R_t \\ &= \alpha_f d_t - \alpha_f c(y_t) + f(1) - (1+r)\end{aligned}$$

  Here $r = 1/\beta - 1$.
- $c(y) = f(1) - f(1-y)$ is the opportunity cost of selling y.
- Assume $f(1) > 1 + r$. All $\overline{n}$ firms are active.

Trading Mechanism

- Let g be some non-negative non-decreasing function with $g(0) = 0$. Let y^* solve $u'(y^*) = c'(y^*)$ and $d^* = g(y^*)$ .The terms of trade is determined by a general mechanism

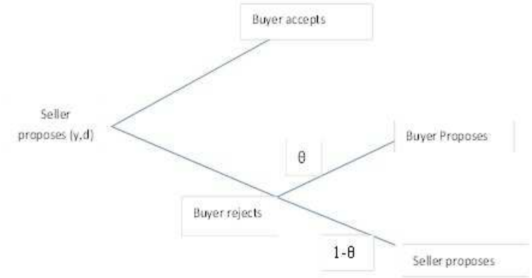
$$y(D) = \begin{cases} g^{-1}(D) & \text{if } D < d^* \\ y^* & \text{if } D \geq d^* \end{cases}, d(D) = \begin{cases} D & \text{if } D < d^* \\ d^* & \text{if } D \geq d^* \end{cases}$$

- Generalized Nash bargaining, proportional Kalai bargaining, our strategic bargaining game and competitive Walrasian pricing are all special cases with different g functions.
- e.g. With Proportional Bargaining

$$g(y) = (1-\theta)u(y) + \theta c(y)$$

Trading Mechanism

- Consider three DM trading mechanisms:
  - Walrasian Pricing,
  - Proportional Bargaining,
  - A pricing game.
- The pricing game



Euler Equation

$$\psi_t = \underbrace{\beta \psi_{t+1}}_{\text{Resale value}} + \underbrace{\beta U_2(x_{t+1}, h_{t+1})}_{\text{Marginal utility}} + \underbrace{\alpha_h \beta \psi_{t+1} \mathscr{L}(e_t)}_{\text{Liquidity Premium}}$$

with $\mathscr{L}(e) = \begin{cases} \left[\frac{u'(y)}{g'(y)} - 1\right] D'(e) & \text{if } D(e) < d^* \\ 0 & \text{if } D(e) \geq d^* \end{cases}$ and $y = y[D(e)]$.

Assume $\mathscr{L}$ is decreasing. This is true for price taking and proportional bargaining.

Steady State Equilibrium

Proposition

There is a unique steady state equilibrium.

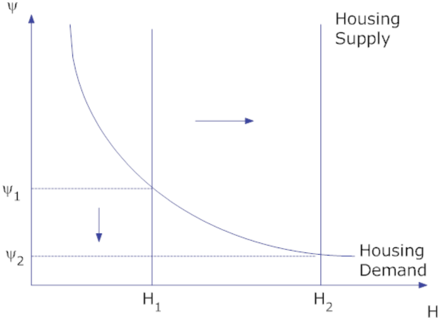


Figure: Steady State with Fixed Housing Supply

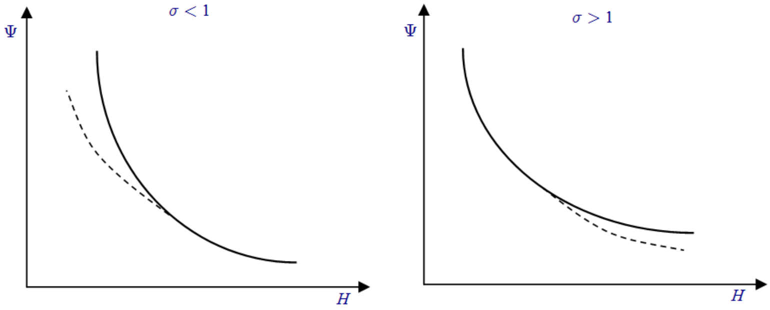
Comparative Statics

- How does debt limit influence the house price?
- With $D(e) = D_0 + D_1 e$,

$$\frac{d\psi}{dD_1} = \frac{\alpha\psi\left[\frac{\mathscr{L}}{D_1} + \psi H \mathscr{L}'\right]}{r - \alpha\mathscr{L} - \alpha\psi\mathscr{L}' D_1 H} \gtreqless 0.$$

Comparative Statics

- Suppose $U(x,h) = \tilde{U}(x) + \frac{h^{1-\sigma}}{1-\sigma}$.



(a) (b)

Figure

Comparative Statics

- The simple idea is that credit limit depends on the VALUE of the housing stock, and this value can go up or down with the quantity depending on whether the elasticity of demand is above or below 1.
- As a consequence, total welfare could decrease if housing stock increases.

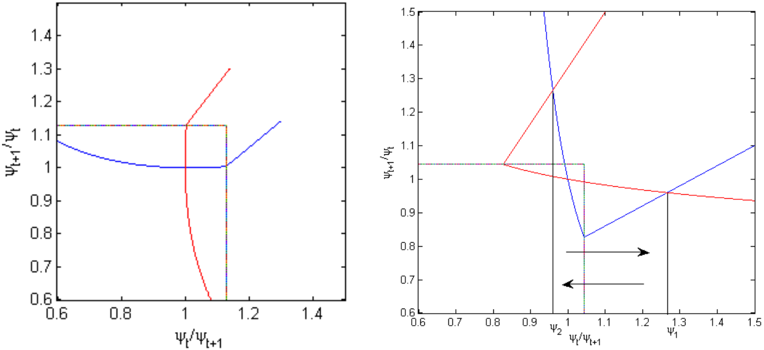
House Price Dynamics

- Housing prices can exhibit exotic dynamics even when everything is stationary.
- Set $c(y) = y$, $H = 1$, $U(x,h) = \tilde{U}(x) + \kappa \frac{h^{1-\sigma}}{1-\sigma}$, $u(y) = \eta \frac{(y+\varepsilon)^{1-\gamma} - \varepsilon^{1-\gamma}}{1-\gamma}$ and $D(e) = e$. Then $y^* = \eta^{\frac{1}{\gamma}} - \varepsilon$.
- In nonstationary equilibrium, house price is a sequence satisfying

$$\begin{aligned}\psi_t &= F(\psi_{t+1}) \\ &= \beta(1-\alpha)\psi_{t+1} + \beta\kappa + \alpha\beta\psi_{t+1}\frac{\eta(y_{t+1}+\varepsilon)^{-\gamma}}{(1-\theta)\eta(y_{t+1}+\varepsilon)^{-\gamma} + \theta}\end{aligned}$$

with $y_{t+1} = \min\left\{g^{-1}(\psi_{t+1}),\ y^*\right\}$.

Numerical Examples with Walrasian Pricing



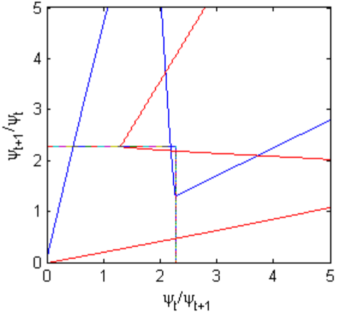
(a) Example 1 (b) Example2

Figure

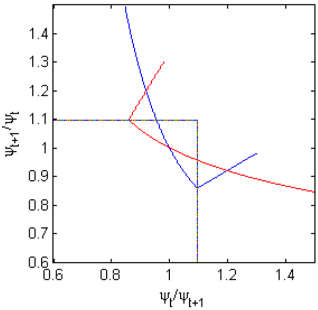
Numerical Examples with Walrasian Pricing

- In exmaple 2, there exist 3-cycles. One is $\psi^1 = 0.8680$, $\psi^2 = 1.5223$ and $\psi^3 = 1.1134$.
- By Sarkovskii theorem and Li-Yorke theorem, there exist cycles of all orders and also chaotic dynamics.

Proportional Bargaining and Pricing Game



(a) Proportional Bargaining



(b) Pricing Game

Figure

Endogenize Housing Supply

- In CM, a construction firm can produce house using CM good with cost function $v(h)$.
- House stock depreciates with rate δ.
- Depreciation of house stock happens after households derive utility from house, or, from consumption of home goods produces using h as an input in CM.

Steady State Equilibrium

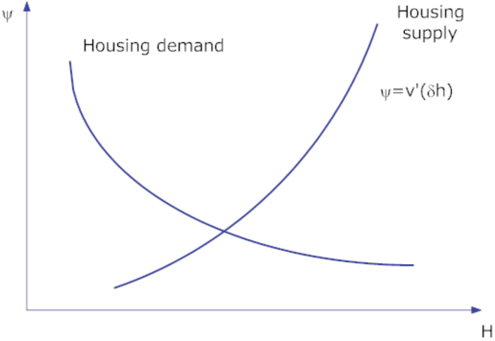
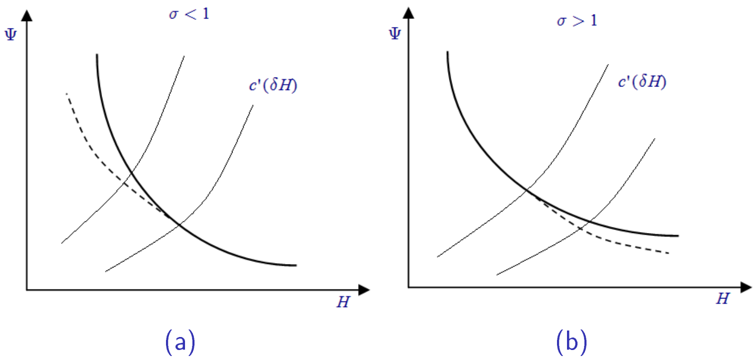


Figure: Endogenous Supply of Housing

Proposition

There is a unique steady state equilibrium.

Steady State Equilibrium



Figure

Dynamics

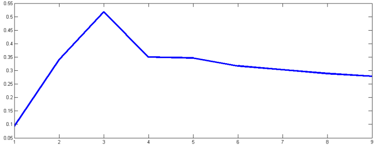
- Dynamics is characterized by

$$\psi_t = \beta (1-\delta) \psi_{t+1} + \beta U_2 [x(h_{t+1}), h_{t+1}] + \alpha_h \beta (1-\delta) \psi_{t+1} \mathscr{L}(e_{t+1})$$

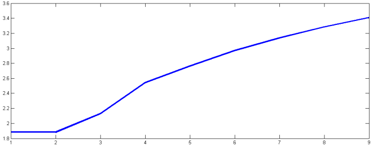
$$h_{t+1} = (1-\delta) h_t + v'^{-1}(\psi_t)$$

- There might exists many dynamic equilibria.

Dynamics with Walrasian Pricing



(a) Housing Price



(b) Housing Stock

Figure: Particular Selection of Equil Transition Path

Model Setup

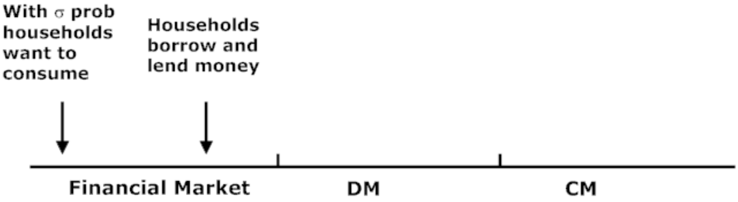


Figure: Time Line

- Add a third subperiod: FM
- In FM, DM buyers and non-buyers borrow and lend money with interest rate ρ_t, like BCW
- Debt each buyer can take is less than $D(\psi_t H)/(1+\rho_t)\phi_t$ due to lack of commitment, i.e. households need collateral, home equity, to borrow cash from a BCW bank.

Case 1: Binding Borrowing Constraint

- Let $L(y) = \begin{cases} u'(y)/g'(y) - 1 & \text{if } y < y^* \\ 0 & \text{if } y > y^* \end{cases}$.
- Euler equations

$$\begin{aligned} \phi_t =& \beta\sigma\phi_{t+1} + \alpha_h \sigma\beta L(y_{t+1})\phi_{t+1} \\ &+ \beta(1-\sigma)(1+\rho_{t+1})\phi_{t+1} \end{aligned}$$

$$\begin{aligned} \psi_t =& \beta\psi_{t+1} + \beta U_2(x_{t+1}, h_{t+1}) \\ &+ \beta\sigma[\alpha_h L(y_{t+1}) - \rho_{t+1}] \frac{\psi_{t+1} D'(\psi_{t+1} h_{t+1})}{1+\rho_{t+1}} \end{aligned}$$

Case 2: Non-Binding Borrowing Constraint

- Euler equations

$$\begin{aligned} \phi_t = &\beta (1-\sigma)(1+\rho_{t+1})\phi_{t+1} + \beta\sigma\phi_{t+1} \\ &+ \alpha_h \sigma \beta L(y_{t+1})\phi_{t+1} \end{aligned}$$

$$\psi_t = \beta U_2(x_{t+1}, h_{t+1}) + \beta \psi_{t+1}$$

- Also we have,

$$\rho_t = \alpha_h L(y_t)$$

Type 1: Non-Binding BC

- For simplicity, assume $D(\psi h) = D_1 \psi h$.
- With non-binding borrowing constraint, house is priced at its fundamental.

$$\rho = i$$

$$\frac{i}{\alpha_h} = L(y)$$

$$r\psi = U_2[x(H), H]$$

Type 2: Binding BC and $\rho > 0$

- SS characterized by

$$
\begin{aligned}
i &= \alpha_h \sigma L(y) + (1-\sigma)\rho \\
r &= \alpha_h \sigma \frac{L(y) D_1}{1+\rho} - \frac{\sigma \rho D_1}{1+\rho} + U_2\left[x(H), H\right]/\psi \\
g(y) &= \frac{\phi_t M_t}{\alpha_h} = \frac{D_1 \psi H}{(1+\rho)(1-\sigma)}
\end{aligned}
$$

- If $[\alpha_h L(y) + 1]\, g(y)$ increasing in y, house price decreases as inflation goes up.
- Inflation reduces DM consumption.

Type 3: Binding BC and $\rho = 0$

- SS is characterized by

$$i = \alpha_h \sigma L(y),$$
$$r = \alpha_h \sigma L(y) D_1 + U_2[x(H), H]/\psi.$$

- Inflation reduces DM consumption and drives up house price.

- Only one of these three equilibrium exists at a time.
- Which equilibrium exists depends on the debt limit.
    - If the debt limit allows buyers to borrow up to the amount they want, the non-binding equ exists
    - If the debt limit allows buyers to borrow in all the money holdings of savers, $\rho > 0$
    - If the debt limit is very low, buyers cannot borrow all the money holdings of savers, $\rho = 0$

Summary

- We build a model where house can be used as collateral to get loans.
- Whether liquidity premium exists when housing stock is high or low depends on the utility.
- With the liquidity component, house prices can exhibit cycles and chaotic dynamics.
- With money in the economy, higher inflation leads to lower welfare but might drive up or drive down house price.
- The results are robust to trading mechanisms: strategic bargaining, axiomatic bargaining, competitive price taking, etc.
- We allow search frictions, with general matching technologies, free entry, etc.
- We strictly generalize existing models of imperfect credit markets, bringing them into the realm of search theoretic models.