Information aggregation and ex post optimality in a large, two-stage, Cournot market

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Challenge

Devise trading mechanisms under dispersed and incomplete information that

- resemble markets
- achieve good outcomes

Information aggregation with strategic trading

Reny and Perry (2006) obtains ex post efficiency

- unit supplies and demands
- quasi-linear preferences
- 'values' depend on both idiosyncratic signals and aggregate states
- large but finite population

Work that generalizes Reny-Perry environment to divisible goods

- mechanisms have one stage
- Cournot-quantity-game type fails to obtains ex post optimality (Vives 1988)
- double-auction type uses demand functions as strategies (Cripps and Swinkels, 2006; Vives, 2011)

Two-stage Cournot quantity game

- inspired by parimutuel betting
- agents submit quantities at both stages
  - first stage: provisional demands which are then announced
  - second stage: new offers made by all but a small random subset of people
  - payoffs: from new offers using a trading post; for those without new offers, by first-stage demand
- our two-stage mechanism resembles those used in experiments (Axelrod, Kulich, Plott, and Roust, 2009)

Our results

Generic existence of fully revealing equilibrium whose outcome is (almost) ex post efficient

- first stage (almost) reveals the aggregate state
- second-stage outcome is (almost) competitive with known aggregate state
- works for both continuum population or finite but large populations

Generic uniqueness?

Environment

- unit measure of agents
- two-good economy: $u(q; x, z) + y$, u' increases in both x and z
  - q quantity of asset
  - y quantity of numeraire
  - z aggregate states drawn from $Z = \{z_1, ..., z_n\}$
  - x informative signal about z drawn from $X = \{x_1, .., x_n\}$
- information structure
  - $\mu(x|z)$ is the conditional probabilities
  - $z \neq z'$ implies that there exists $x \in X$ so that $\mu(x|z) \neq \mu(x|z')$
- each agent is endowed with a units of the asset

Trading mechanism: first stage

- nature draws z, which no one observes
- each agent receives a private signal
- each agent submits a demand q for the asset
- each agent then receives a trading status
  - with probability η, inactive: no second-stage action
  - with probability $1-\eta$, active: allowed to make second-stage offers
- the distribution of demands from active traders, ν, is announced

Trading mechanism: second stage

Actions

- α: offer of the asset
- λ: offer of the numeraire
- $\alpha \cdot \lambda = 0$

Aggregates

- A and Y denotes the aggregate offers of asset and numeraire
- $p = \frac{Y+\varepsilon_y}{A+\varepsilon_a}$

Payoffs

- to agents with active trading status: offer (α, λ) implies $(q, y) = (a - \alpha + \frac{\lambda}{p}, \alpha p - \lambda)$
- to agents with inactive trading status: demand q implies $(q, y) = (q, p(a - q))$

Strategies and beliefs

Strategies

- first-stage strategies: $q : X \to \mathbb{R}_+$
- second-stage strategies:
  - asset offer $\alpha : X \times \Delta(\mathbb{R}_+) \to [0, a]$;
  - numeraire offer $\lambda : X \times \Delta(\mathbb{R}_+) \to \mathbb{R}_+$

Beliefs: $\varphi : X \times \Delta(\mathbb{R}_+) \to \Delta(Z)$

- belief depends on the private signal and the announced distribution
- in the continuum case, only belief about the state is relevant

Sequential Equilibrium

Strategy profile (q, α, λ) and belief φ is a sequential equilibrium if

- $(\alpha(x, \nu), \lambda(x, \nu))$ solves

$$\max_{\alpha,\lambda} \sum_{z \in Z} \varphi(x, \nu)(z)\{u(a - \alpha + \frac{\lambda}{p(z)}; x, z) + \alpha p(z) - \lambda\}$$

- $p(z)$ is determined by the ratio of aggregate offers (including $(\varepsilon_a, \varepsilon_y)$) at each state z
- $q(x)$ solves $\max_q \sum_{z \in Z} \mathbb{P}(z|x)[u(q; x, z) + p(z)(a - q)]$
- φ is determined by Bayes rule whenever possible

Competitive equilibrium with z known

Allocation $q_z^{ce} : X \to \mathbb{R}_+$ and price p_z^{ce} is a CE with $(\varepsilon_a, \varepsilon_y)$ and η and for the state z if

- $q_z^{ce}(x)$ maximizes $u(q; x, z) + p_z^{ce}(a - q)$
- $\sum_{x \in X} \mathbb{P}(x|z) q_z^{ce}(x) + \frac{\varepsilon_y}{(1-\eta)p_z^{ce}} = a + \frac{\varepsilon_a}{1-\eta}$

Properties

- for each z and $(\varepsilon_a, \varepsilon_y)$ and η, there exists a **unique** competitive equilibrium for the state z
- equilibrium allocation and price is continuous with respect to $(\varepsilon_a, \varepsilon_y)$ and η and hence is (almost) **efficient**

Existence of fully revealing equilibrium

An equilibrium (q, α, λ) is fully revealing if $q(x) \neq q(x')$ whenever $x \neq x'$

Theorem

If ε_y is sufficiently small, then there is generic existence of a fully revealing equilibrium, which is almost ex post efficient

Outline of the proof

- if $q(x) \neq q(x')$ for $x \neq x'$, then for each z, the second-stage outcome is the CE allocation and the price is the CE price
- it remains to show that if the second-stage prices are those CE prices, then the solution to the first-stage problem satisfies $q(x) \neq q(x')$ for $x \neq x'$

The genericity argument

Why genericity?

- first-stage problem: $\max_q \sum_{z \in Z} \mathbb{P}(z|x)[u(q;x,z) + p(z)(a-q)]$
- higher x implies higher marginal utility, but also higher expected price
- hence, $q(x)$ may be the same for different x's

Fully revealing equilibrium exists generically by perturbing marginal utilities

Perturbation of u

$v(q; x, z, b) = u(q; x, z) + w(q; x, z, b)$, where

- $b = (b_x)_{x \in X}$
- $w'(q; x, z, b) = b_x$ for $q \in [0, q_+]$
- $v(\cdot; x, z, b)$ is strictly increasing, concave, and satisfies Inada conditions
- $w(q; x, z, b) = 0$ when $b = 0$

Lemma

Suppose that $q_x^*(b)$ solves the first-stage problem, assuming second-stage outcome is a CE. For sufficiently small ε_y, $\frac{\partial}{\partial b_x} q_x^*(b) > 0$ and $\frac{\partial}{\partial b_x} q_{x'}^*(b) < 0$ for $x' \neq x$.

- increase in b_x decreases $q_{x'}$ through increase in $p(z)$'s
- two opposite effects of increasing b_x on q_x:
  - increase in marginal utility increases q_x
  - increase in $p(z)$'s decreases q_x
  - the first effect is stronger

The lemma implies that the set of b's for which $q_x(b) = q_{x'}(b)$ has measure 0 in a neighborhood of $b = 0$

Generic uniqueness

The conjecture: generically, every equilibrium is fully revealing

- suppose that the first-stage problem is not fully revealing
- the second-stage prices are determined by F.O.C.'s
- because there are more equations than unknowns in an equilibrium that is not fully revealing, such an equilibrium is not generic

Arguments yet to be completed

Conclusion

Two-stage Cournot game as a trading mechanism for financial assets

- parallel market for each asset (and money)
- have to choose periodic trading windows for each stage
- financing the ε's and trades of inactive agents
- prevent information disclosure between two stages

Limitations of our model

- no cash-in-advance constraints
- no dynamic consideration beyond the two stages, which would be relevant with periodic markets