Repurchase Agreements

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Introduction

- Repo: a contract to sell and subsequently repurchase securities at a specified date and price, often the next day (BIS)
  - US tri-party repo market is $1.5 trillion (daily)
  - Monetary policy conducted through repos
- What is the role of the (short-term) repo market ?
  - Counterparty risk (Mills and Reed) ?
  - Private information on the asset return (Chiu-Koeppl) ?
- Repo can meaningfully co-exist with sales... even with full information and no risk exposure

Introduction

- Motive for trade is the random valuation for the asset
- Repos have no role in a Walrasian market
- With matching frictions
  - Traders exploit *intertemporal* gains from trade using asset sales
  - Traders exploit *intratemporal* gains from trade using repos
- Persistence of the valuation shock explains markets size:
  - Repos volume is strictly decreasing in persistence
  - Sales volume is hump-shape in persistence

The Model

- Version of KMT (2011) and LR (2010)
- $t = 0, 1, \ldots,\ \beta \in (0,1)$
- Two subperiods: trade and settlement
- Asset in fixed supply A and numeraire good
- Continuum of traders: Measure 1/2 of traders h and ℓ
- Trader $i = h, \ell$ utility
$$u_i(a) + d$$
where $u_h'(a) \geq u_\ell'(a)$ for all a
- Traders switch types with probability $1 - \pi \in [0, \frac{1}{2}]$

Walrasian Benchmark

- Walrasian markets for purchases and repos at price p and p^r
  - Repo: Enjoy the service from holding the asset this period only
  - Asset sales: Enjoy the service from holding the asset any time
- Commitment to terms of trade
- Clearinghouse implements the transfer of the asset
- Numeraire is transferred in the settlement stage

Walrasian Benchmark

- Trader $i = h, \ell$ solves

$$\begin{aligned} W_i(a) &= \max_{a_i, q_i^r} u_i(a_i + q_i^r) - d + \beta E_{k|i} W_k(a_i) \\ &\text{s.t. } d + pa = pa_i + p^r q_i^r \end{aligned}$$

with FOCs and envelope

$$\begin{aligned} u_i'(a_i + q_i^r) + \beta E_{k|i} W_k'(a_i) &= p \\ u_i'(a_i + q_i^r) &= p^r \\ W_i'(a) &= p \end{aligned}$$

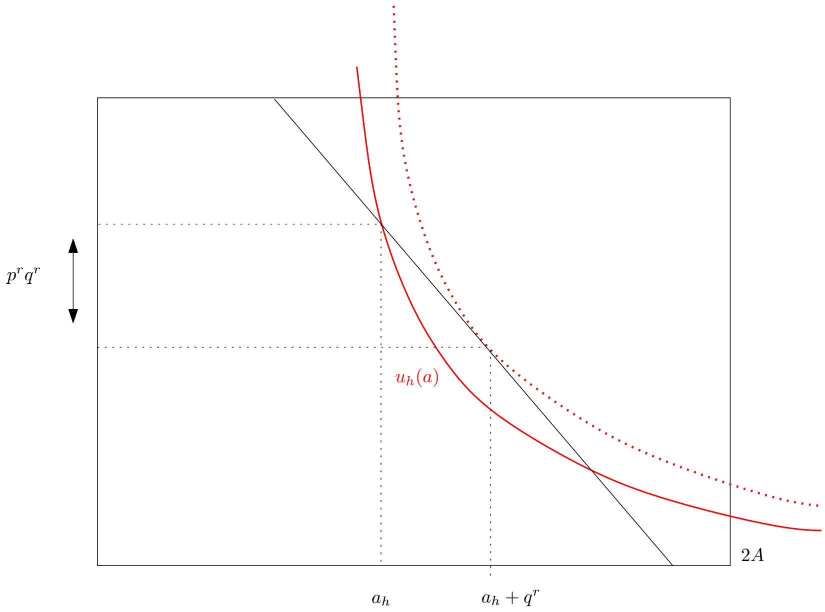
Walrasian Benchmark

- Equilibrium prices and quantities satisfy

$$\begin{aligned}(1-\beta)p &= p^r \\ u_h'(a_h+q_h^r) &= p^r \\ u_\ell'(a_\ell+q_\ell^r) &= p^r \\ (a_h+q_h^r)+(a_\ell+q_\ell^r) &= 2A\end{aligned}$$

- Anything goes for repos (in particular $q_h^r = q_\ell^r = 0$).

Walrasian Allocations



Bilateral Trade and Settlement

- Agents meet pairwise in the trading stage
- Each agent h meets an agent ℓ
- Core-allocation or bargaining

Allocations

- An allocation is a triple $\{q^s(a_h, a_\ell), q^r(a_h, a_\ell), d(a_h, a_\ell)\}$
- We only focus on stationary and symmetric allocations
- An allocation is feasible if

$$\begin{aligned} q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \\ q^r(a_h, a_\ell) + q^s(a_h, a_\ell) &\in [-a_h, a_\ell] \end{aligned}$$

Distributions

- $(\mathbf{q}^\mathbf{s}, \mathbf{q}^\mathbf{r}, \mathbf{d})$ defines distribution of asset $\mu_i(a)$
- Concentrate on invariant distributions

Value functions

$$\begin{aligned} V_h(a) &= \pi \int [u_h(a+q^s(a,a_\ell)+q^r)-d+\beta V_h(a+q^s)]d\mu_\ell(a_\ell) \\ &+ (1-\pi)\int [u_\ell(a-q^s(a_h,a)-q^r)+d+\beta V_\ell(a-q^s)]d\mu_h(a_h) \end{aligned}$$

$$\begin{aligned} V_\ell(a) &= \pi \int [u_\ell(a-q^s(a_h,a)-q^r)+d+\beta V_\ell(a-q^s)]d\mu_h(a_h) \\ &+ (1-\pi)\int [u_h(a+q^s(a,a_\ell)+q^r)-d+\beta V_h(a+q^s)]d\mu_\ell(a_\ell) \end{aligned}$$

Core Allocations

$$\begin{aligned} (q^s, q^r, d) = \quad & argmax[u_h(a_h + q^s + q^r) - d + \beta V_h(a_h + q^s)] \\ s.t. \quad & q^s \in [-a_h, a_\ell], \; q^s + q^r \in [-a_h, a_\ell] \\ & u_\ell(a_\ell - q^s - q^r) + d + \beta V_\ell(a_\ell - q^s) \geq \lambda U_\ell(a_\ell) \\ & u_h(a_h + q^s + q^r) - d + \beta V_h(a_h + q^s) \geq U_h(a_h) \end{aligned}$$

where $U_i(a_i) = u_i(a_i) + \beta V_i(a_i)$ and with $\lambda \geq 1$.

FOC:

$$V_h'(a_h + q^s) \geq V_\ell'(a_\ell - q^s) \; (= \text{if } q^s < a_\ell)$$

and

$$\begin{aligned} u_h'(a_h + q^s + q^r) = u_\ell'(a_\ell - q^s - q^r) \quad & \text{if} \quad \xi_i = 0, i = h, \ell \\ q^s + q^r = a_\ell \quad & \text{if} \quad \xi_\ell > 0 \\ q^s + q^r = -a_h \quad & \text{if} \quad \xi_h > 0 \end{aligned}$$

Core Allocations

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Random Matching - Special Cases

Proposition

With random matching and $\pi = 1/2$, the pairwise core allocations defines a unique invariant equilibrium characterized by a distribution of asset holdings for each type that are degenerate at $a^* = A$ with $q^s(a^*, a^*) = 0$, and $q^r(a^*, a^*) > 0$.

$\pi = 1/2$ implies $V_h(a) = V_\ell(a) = V(a)$ for all a.

$$V'(a_h + q^s) = V'(a_\ell - q^s)$$
$$q^s = \frac{a_\ell - a_h}{2}$$

and

$$u_h'(a_h + q^s + q^r) = u_\ell'(a_\ell - q^s - q^r)$$
$$u_h'(\frac{a_\ell + a_h}{2} + q^r) = u_\ell'(\frac{a_\ell + a_h}{2} - q^r)$$

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$\pi = 1/2$ implies $V_h(a) = V_\ell(a) = V(a)$ for all a.

$$\begin{aligned} V'(a_h + q^s) &= V'(a_\ell - q^s) \\ q^s &= \frac{a_\ell - a_h}{2} \end{aligned}$$

and

$$\begin{aligned} u_h'(a_h + q^s + q^r) &= u_\ell'(a_\ell - q^s - q^r) \\ u_h'(\frac{a_\ell + a_h}{2} + q^r) &= u_\ell'(\frac{a_\ell + a_h}{2} - q^r) \end{aligned}$$

Random Matching - Special Cases

Proposition

With random matching and $\pi = 1$, the pairwise core allocations define an equilibrium characterized by a distribution of asset holdings for each type that are degenerate at some level a_h^* and a_ℓ^* with $a_h^* > a_\ell^*$ where $q^s(a_h^*, a_\ell^*) = 0$ and $q^r(a_h^*, a_\ell^*) = 0$.

(guess and verify) $\pi = 1$ and $q^s = 0$ imply for $i = h, \ell$,

$$V_i(a_i) = \frac{u_i(a_i + q_i^r)}{1 - \beta}$$

and

$$\begin{aligned} u_h'(a_h + q^s + q^r) &= u_\ell'(a_\ell - q^s - q^r) \\ V_h'(a_h + q^s) &= V_\ell'(a_\ell - q^s) \end{aligned}$$

imply $q^r(a_h, a_\ell) = 0$ is an equilibrium and that a_h^* and a_ℓ^* are uniquely given by $a_h^* + a_\ell^* = 2A$ and

$$u_h'(a_h^*) = u_\ell'(a_\ell^*)$$

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(guess and verify) $\pi = 1$ and $q^s = 0$ imply for $i = h, \ell$,

$$V_i(a_i) = \frac{u_i(a_i + q_i^r)}{1 - \beta}$$

and

$$\begin{aligned} u_h'(a_h + q^s + q^r) &= u_\ell'(a_\ell - q^s - q^r) \\ V_h'(a_h + q^s) &= V_\ell'(a_\ell - q^s) \end{aligned}$$

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$$u_h'(a_h^*) = u_\ell'(a_\ell^*)$$

Directed Search

- Corbae, Temzelides and Wright (2003): the matching rule is an equilibrium matching if no coalition consisting of 1 or 2 agents can do better by deviating
- Matching rule: min a_h matched with max a_ℓ, etc.
- An equilibrium with an invariant distribution is characterized by degenerate supports a_h^* and a_ℓ^* where $q^s(a_\ell^*, a_h^*) = a_h^* - a_\ell^*$ and $q^s(a_h^*, a_\ell^*) = 0$, and

$$\begin{aligned}
a_h^* + a_\ell^* &= 2A \\
u_h'(a_h^* + q^r) &= u_\ell'(a_\ell^* - q^r) \\
u_h'(a_h^* + q^r) &= \alpha\lambda U'(a_\ell^*) + (1-\alpha-\beta)\lambda U'(a_h^*)
\end{aligned}$$

where $\alpha = \frac{(1-\beta\pi)\pi}{(2\pi-1)}$

Directed Search and Bargaining

$$\max_{q^s,q^r,d} [u_h(a_h + q^s + q^r) - d + \beta V_h(a_h + q^s) - U_h(a_h)]^\theta$$
$$\times [u_\ell(a_\ell - q^s - q^r) + d + \beta V_\ell(a_\ell - q^s) - U_\ell(a_\ell)]^{1-\theta}$$

with first order conditions

$$\begin{aligned} V_h'(a_h + q^s) &= V_\ell'(a_\ell - q^s) \\ u_h'(a_h + q^s + q^r) &= u_\ell'(a_\ell - q^s - q^r) \end{aligned}$$

$$\begin{aligned} d(a_h, a_\ell) &= (1-\theta)[u_h(a_h + q^s + q^r) + \beta V_h(a_h + q^s) - U_h(a_h)] \\ &\quad + \theta[U_\ell(a_\ell) - u_\ell(a_\ell - q^s - q^r) - \beta V_\ell(a_\ell - q^s)] \end{aligned}$$

Directed Search and Bargaining

$$\max_{q^s,q^r,d}[u_h(a_h+q^s+q^r)-d+\beta V_h(a_h+q^s)-U_h(a_h)]^{\theta}$$
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$$\begin{aligned} V_h'(a_h+q^s) &= V_\ell'(a_\ell-q^s) \\ u_h'(a_h+q^s+q^r) &= u_\ell'(a_\ell-q^s-q^r) \end{aligned}$$

$$\begin{aligned} d(a_h,a_\ell) &= (1-\theta)[u_h(a_h+q^s+q^r)+\beta V_h(a_h+q^s)-U_h(a_h)] \\ &\quad +\theta[U_\ell(a_\ell)-u_\ell(a_\ell-q^s-q^r)-\beta V_\ell(a_\ell-q^s)] \end{aligned}$$

Directed Search and Bargaining

Proposition

With directed search and bargaining, an equilibrium with an invariant distribution is characterized by a degenerate distribution of asset holdings for each type, at some level a_i^* with $i = h, \ell$ with

$$q^s(a_h^*, a_\ell^*) = 0, q^s(a_\ell^*, a_h^*) = a_h^* - a_\ell^*$$

and

$$q^r(a_h^*, a_\ell^*) = q^r(a_\ell^*, a_h^*) = q^r$$

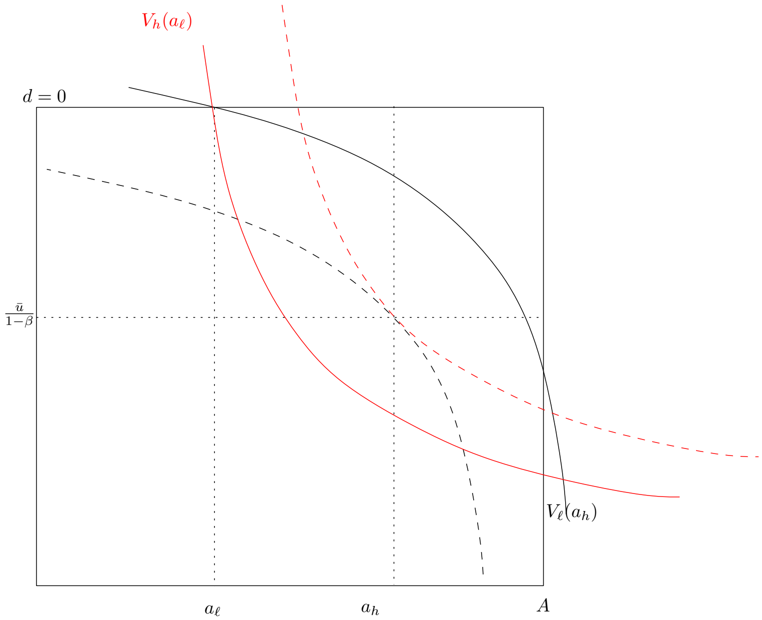
where q^r solves $u_h'(a_h^* + q^r) \geq u_\ell'(a_\ell^* - q^r)$ (with equality if $q^r < a_\ell^*$), and

$$d(a_\ell^*, a_h^*) = d(a_h^*, a_\ell^*) + \frac{\bar{u}}{1-\beta}$$

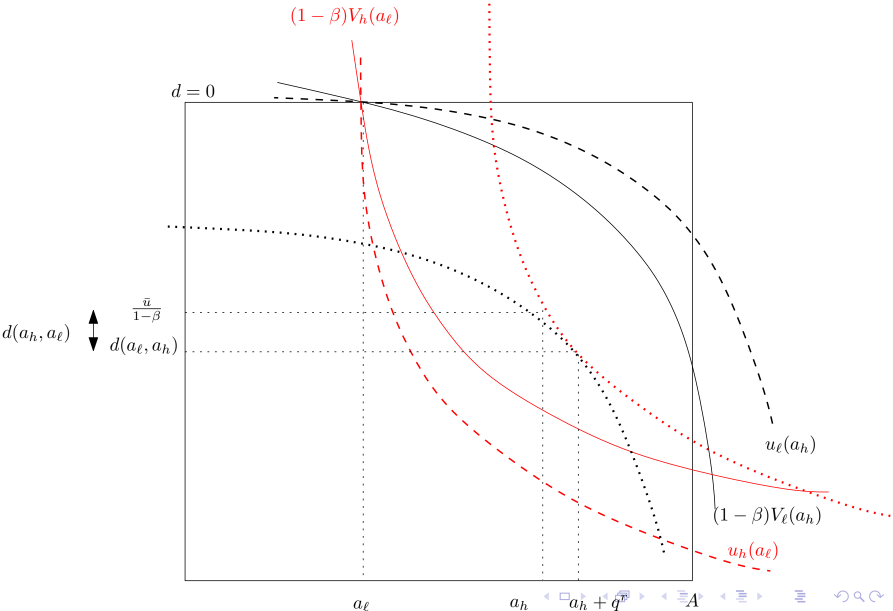
where

$$\bar{u} = (1-\theta)[u_h(a_h^*) - u_h(a_\ell^*)] + \theta[u_\ell(a_h^*) - u_\ell(a_\ell^*)]$$

Intertemporal gains from trade

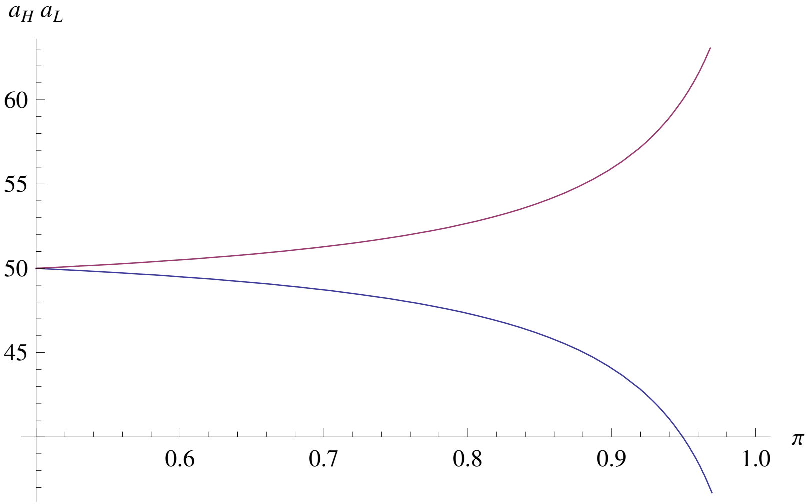


Intratemporal gains from trade

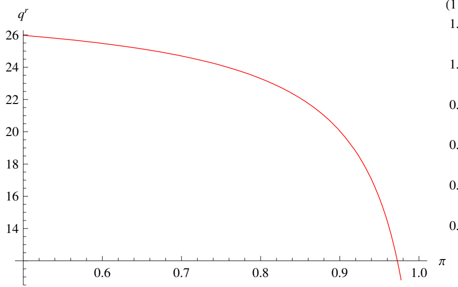


Quantities

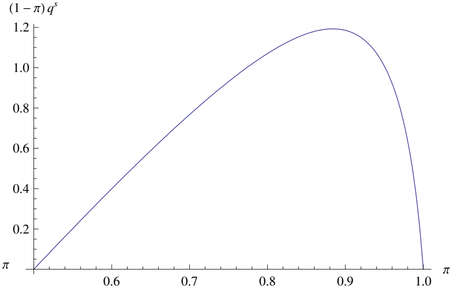
$u_h(a) = \frac{a^{1-\sigma}}{1-\sigma}$, $u_\ell(a) = \lambda u_h(a)$ where $\lambda \in (0,1)$,
$\theta = 0.5$, $\lambda = 0.1$, $\sigma = 2$, $\beta = 0.9$, and $A = 50$.



Quantities



q^r
26
24
22
20
18
16
14
0.6
0.7
0.8
0.9
1.0
π



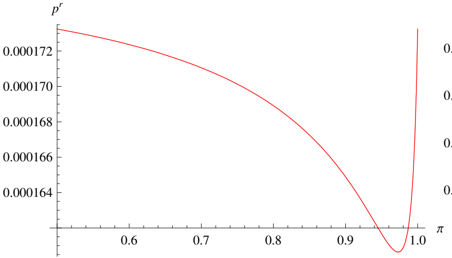
$(1-\pi)\,q^s$
1.2
1.0
0.8
0.6
0.4
0.2
0.6
0.7
0.8
0.9
1.0
π

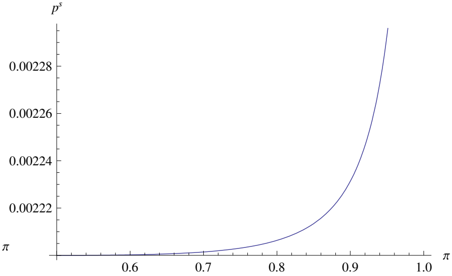
Prices

Let p^r be the price of a repo and p^s the price of a sale. Then

$$p^r = \frac{(1-\theta)[u_h(a_h^* + q^r) - u_h(a_h^*)] + \theta[u_\ell(a_\ell^*) - u_\ell(a_\ell^* - q^r)]}{q^r}$$

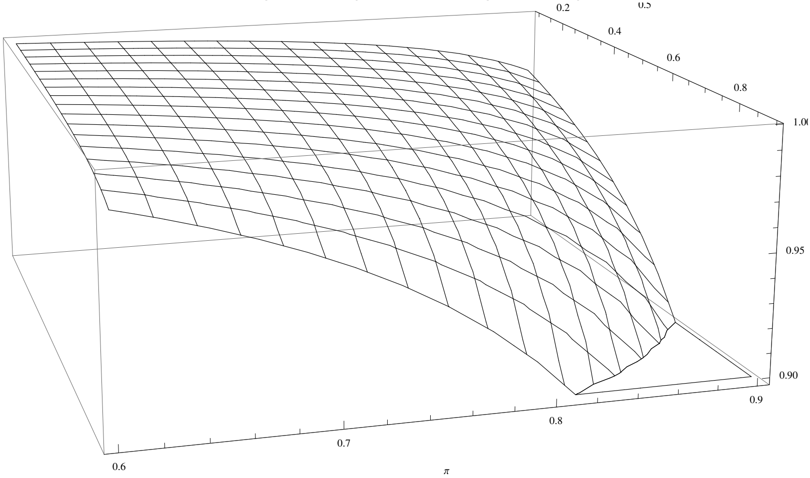
$$p^s = \frac{(1-\theta)[u_h(a_h^*) - u_h(a_\ell^*)] + \theta[u_\ell(a_h^*) - u_\ell(a_\ell^*)]}{(1-\beta)(a_h^* - a_\ell^*)}$$





Share of Repos from Asset Realocation

As a function of $\pi \in (0.6, 0.9)$ and $\theta \in (0.1, 0.9)$:



Conclusion

- Present an environment where repos help achieve efficiency
- Key element: Future and present valuations differ across and within types
- Ideas could be applied to non-financial assets as well (e.g. housing rental market, car rental market, labor market)

Why Repo?

- Why all of the reallocation is not done via sale market?
- Directed Search equil. characterized by two conditions:

$$u_\ell'(c_\ell^*) = u_h'(c_h^*)$$

$$\left\{\beta + (1-\beta)\frac{\pi}{1-\pi}\right\} \times \left\{\theta\left[u_\ell'(c_\ell^*) - u_\ell'(c_\ell^* + q^{r*})\right] + (1-\theta)\left[u_h'(c_h^* - q^{r*}) - u_h'(c_h^*)\right]\right\} = \left\{\theta\left[u_\ell'(c_\ell^*) - u_\ell'(c_h^* - q^{r*})\right] + (1-\theta)\left[u_h'(c_\ell^* + q^{r*}) - u_h'(c_h^*)\right]\right\}$$

- The first expression guarantees the optimal allocation of consumption
- The second one determines what fraction of reallocation of assets, $c_h^* - c_\ell^*$, is done via repo, q^{r*}.

$$
\begin{aligned}
&\left\{\beta+(1-\beta)\frac{\pi}{1-\pi}\right\}\times \\
\left\{\theta\left[u_\ell'(c_\ell^*)-u_\ell'(c_\ell^*+q^{r*})\right]+(1-\theta)\left[u_h'(c_h^*-q^{r*})-u_h'(c_h^*)\right]\right\} &= \\
\left\{\theta\left[u_\ell'(c_\ell^*)-u_\ell'(c_h^*-q^{r*})\right]+(1-\theta)\left[u_h'(c_\ell^*+q^{r*})-u_h'(c_h^*)\right]\right\}
\end{aligned}
$$

- The LHS is increasing in q^r and RHS is decreasing in q^r, therefore
  - LHS is increasing in π, which makes q^{r*} decreasing in π: With more persistent types more reallocation is done via sale market.
  - LHS is decreasing in β, which makes q^{r*} increasing in β: With more patience more allocation is done via repo market.
- if $\left\{\beta+(1-\beta)\frac{\pi}{1-\pi}\right\}=1$ then $c_\ell^*+q^{r*}=c_h^*-q^{r*}$ and all of reallocation would be done via repo market.
- The effect of θ is not clear.
  - Note: $c_\ell^*<c_\ell^*+q^{r*}<c_h^*-q^{r*}<c_h^*$, hence

$$
\begin{aligned}
u_\ell'(c_\ell^*)-u_\ell'(c_\ell^*+q^{r*}) &< u_\ell'(c_\ell^*)-u_\ell'(c_h^*-q^{r*}) \\
u_h'(c_h^*-q^{r*})-u_h'(c_h^*) &< u_h'(c_\ell^*+q^{r*})-u_h'(c_h^*)
\end{aligned}
$$

Why Repo?

Rewrite the value functions $(1-\beta)V_h(\bar{a}_h)$ as

$$\underbrace{u_h(\bar{a}_h)+\theta S}_{h\text{-utility of holding } a_h} \quad -\frac{(1-\pi)}{1+\beta(2\pi-1)}\underbrace{[u_\ell(\bar{a}_h)+(1-\theta)\tilde{S}-u_h(\bar{a}_h)-\theta S]}_{\text{Loss from switching}}$$

S : gain from repo
$\tilde{S}$: gain from conducting asset purchase AND repo.
And $(1-\beta)V_\ell(\bar{a}_\ell)$ is

$$u_\ell(\bar{a}_\ell)+(1-\theta)S-\frac{(1-\pi)}{1-\beta(2\pi-1)}[u_\ell(\bar{a}_\ell)+(1-\theta)S-u_h(\bar{a}_\ell)-\theta\tilde{S}]$$

If only asset sales, then $\bar{a_h}$ would be farther apart from $\bar{a}_\ell$, thus increasing the loss from switching.

Weight on loss is minimized at $\pi=1$ and maximized at $\pi=1/2$.