Enriching information to prevent bank runs*

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Abstract

Sequential service in the banking sector, as modeled by Diamond and Dybvig (1983), is a barrier to full insurance and potential source of financial fragility against which deposit insurance is infeasible (Wallace, 1988). In this paper, we pursue a different perspective, viewing the sequence of contacts as opportunities to extract information through a larger message space with commitment to richer promises. As we show, if preferences satisfy a separating property then the desired elimination of dominated strategies (Green and Lin, 2003) occurs even when shocks are correlated. In this manner the sequential service promotes stability.

1 Introduction

In the influential work of Bryant (1980) and Diamond and Dybvig (1983) the question of whether bank runs result from opportunistic behavior is critical. By emphasizing welfare optimality in face of private information, the literature that evolved appeals to the revelation principle and focuses on direct mechanisms. In the context analyzed by Wallace (1988), the question has a trivial answer unless the planner is forced to make irreversible transfers in sequence as it gathers information about liquidity needs. But then, with

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this kind of *sequential service*, bank runs, defined as suboptimal equilibria attained by misrepresentation strategies, have been shown to exist.[1] In this paper, we argue that the dual societal goal of achieving efficiency *and* uniqueness is poorly addressed by stringent applications of the revelation principle.[2] We show that a larger message space can explore the sequential service in order to promote bank stability.

In a way, our results generalize the findings in Green and Lin (2003). Assuming finite population and independent, private shocks to preferences, they have changed the view that the Diamond-Dybvig environment is intrinsically exposed to runs. Under sequential service, they notice, the last trader is offered a consumption plan defined by what others have done, which is history, and by what that trader's type-announcement is. And at this last node in the event tree there is no future opportunistic behavior to consider. Hence, the fact that the planner has computed constraints under the assumption of truthful revelation (by all players) is not an issue for the last traders. Their constraints still represent individual rationality even if someone has lied in the past.

Green and Lin (2003) go further to deduce a chain of dominant strategies. That is, conditional on incoming traders revealing their types, truth-telling is the best reply to all past actions. As Andolfato et al. (2007) show, the argument is invalid when shocks are correlated because truth-telling constraints are no longer invariant to past lies since the planner is led to the wrong conditional distribution of incoming types.

In this paper, we argue that runs can actually be ruled out in a large set of

[1] The main result in this paper does not apply directly to the Peck-Shell (2003) model (the first run examples with sequential service). We show, however, runs being eliminated in one related specification. As mentioned in our final section, the propositions do apply to another cornerstone case of financial fragility identified by Ennis and Keister (2009). In addition to references therein, a related literature is found in survey papers by Cavalcanti (2010), and Ennis and Keister (2010).

[2] We think that progress in this matter, at the heart of *implementation theory*, is not likely to benefit from a generic result in the field. As Jackson (2001) asserts in his survey, existing Bayesian-implementation predictions extend Nash-implementation results, and the mechanisms used to prove the classic Maskin (1999) theorem rely heavily on comparing announcements across individuals, something infeasible with sequential service. The reader may also appreciate that we pursue implementation with a modest-sized message space, without highly inefficient outcomes standing off the equilibrium path.

Aside from more abstract theory, Basseto and Phelan (2008) provide an equally simple implementation mechanism in a taxation environment (without sequential service), which requires the population size to be sufficiently high.

economies, even if the stochastic process generating types is not restricted. In order to prevent runs, preferences are required to satisfy certain conditions and welfare may have to be reduced by an arbitrarily small number. But with such qualifications we can show that direct-revelation mechanisms are not sufficiently broad in the case of correlated shocks. In order to make this point, we ask each trader to send two messages to the planner.

The idea is easily understood with shocks producing impatient traders with zero marginal utility of "late" consumption, and patient traders that can substitute "early" and "late" consumption perfectly. Let us examine the response to typical transfers based on a type announcement. Notice that lies correspond to patient traders consuming early and receiving no future transfers. But, after consuming, anyone in such a corner situation would not mind to reveal their true type in a second message, an information that the planner can then use to adjust the contract to incoming traders. Allocations then become functions of a larger history of announcements. But because the planner is always applying the correct conditional distribution of shocks, a recursive elimination of run strategies follows.

With more general preferences, requiring all types to consume in all dates, we find that the planner's ability to commit can be explored further, in light of a *separation* property detailed below.

The rest of the paper is divided as follows. Section 2 presents the environment with weak assumptions on preferences and their stochastic process. Section 3 defines the usual direct mechanisms. Section 4 introduces the concept of relative optimality and the assumption of separation. Section 5 introduce mechanisms that provide the desired information update. Section 6 discusses alternative assumptions supporting the main results. Section 7 concludes with final remarks. The appendix contains all proofs.

2 The environment

The economy has two dates and is inhabited by a finite population of size $\bar{n}$. The first date is divided into $\bar{n}$ (sub) periods. There is a single resource constraint for providing consumption across periods and dates. In particular, the economy is endowed with $Y > 0$ date-1 goods that can be stored throughout the first $\bar{n}$ periods without costs. There is also a linear technology for transforming date-1 leftovers into date-2 consumption goods at rate-of-return $R > 1$.

Each person is randomly paired with a date-1 period and, in addition, is hit by a privately observed preference shock that can be either 0 (impatient) or 1 (patient). 'Person n' is someone matched with period n at date 1. If, for that person, consumption is c_{n1} at date 1, and c_{n2} at date 2, then the derived utility is $u_{\omega_n}(c_{n1}, c_{n2})$ if the shock realization is ω_n. The functions u_0 and u_1 are assumed increasing, continuous, concave and differentiable.

We shall use letters in bold, without subscripts or superscripts to denote vectors of size $\bar{n}$. Hence $\boldsymbol{\omega}$ stands for a full history of shocks $(\omega_1, ..., \omega_{\bar{n}})$ identified with a queue, that is, it describes the type of the person matched with each period in date 1. When we want to refer to a history after n we use $\boldsymbol{\omega}_n^+ = (\omega_{n+1}, ..., \omega_{\bar{n}})$. For a history before n we use $\boldsymbol{\omega}_n^- = (\omega_1, ..., \omega_{n-1})$.

We let $\Omega = \{0, 1\}^{\bar{n}}$ and assume that a state $\boldsymbol{\omega} \in \Omega$ occurs with probability $P(\boldsymbol{\omega}) > 0$. We assume the following sequence of events. After $\boldsymbol{\omega}$ is drawn according to P, people are matched to the date-1 periods and learn their values of (n, ω_n). Then, they are called to sequentially (announce a message and) consume in date 1, according to the queue order, without knowing what previous traders (those in periods $1, ..., n-1$) have done. Since types are private information, any mechanism for transferring resources to person n at date 1 cannot depend on information provided by person n' if $n' > n$. Since individuals are identical *ex ante*, the planner objective is maximization of expected utility.

3 Direct mechanisms

We consider first the standard application of the revelation principle in the form of direct mechanisms. We start defining transfer functions. They map types (or announcements about types) into consumption allocations, according to constraints imposed by sequentiality. A transfer function is $\boldsymbol{c} = (c_1, .., c_{\bar{n}})$ where, for each n, $c_n = (c_{n1}, c_{n2})$ maps the set of histories Ω into $\mathbb{R}_+^2$. A transfer function $\boldsymbol{c}$ satisfies the sequentiality requirement if c_{n1} is constant on coordinates $n+1, ..., \bar{n}$. The set of such functions, which also satisfies the feasibility constraint

$$\sum_n \left(c_{n1} + R^{-1} c_{n2}\right) \leq Y \tag{1}$$

at all points of its domain, is the set of direct mechanisms denoted $\boldsymbol{D}$.

The ex ante utility attained by $\boldsymbol{c} \in \boldsymbol{D}$ is

$$\sum_{\boldsymbol{\omega} \in \Omega} \sum_{n} P(\boldsymbol{\omega}) u_{\omega_n}(c_n(\boldsymbol{\omega})). \tag{2}$$

There is also a definition of truth-telling constraints, associated to $\boldsymbol{c} \in \boldsymbol{D}$, for position n:

$$\sum_{\boldsymbol{\omega} \in \Omega} p_n(\boldsymbol{\omega}; \omega) \left(u_\omega(c_n(\boldsymbol{\omega})) - u_\omega(c_n(\boldsymbol{\omega}_{-n}, \omega')) \right) \geq 0 \ \forall (\omega, \omega') \in \{0,1\}^2, \tag{3}$$

where $p_n(\boldsymbol{\omega}; \omega)$ is the probability of event $\boldsymbol{\omega}$ conditional on person n drawing ω. The notation $\boldsymbol{\omega}_{-n}$ has the standard meaning

$$\boldsymbol{\omega}_{-n} = (\omega_1, .., \omega_{n-1}, \omega_{n+1}, ., \omega_{\bar{n}})$$

while $(\boldsymbol{\omega}_{-n}, \omega')$ denotes the profile where the n-th entry of $\boldsymbol{\omega}$ is replaced with ω'.

The benchmark optimality problem is that of choosing $\boldsymbol{c} \in \boldsymbol{D}$ in order to maximize (2) subject to truth-telling constraints (3) for all positions. A solution $\boldsymbol{c}^*$ is said to be optimal.

Every element $\boldsymbol{c}$ of $\boldsymbol{D}$ defines a game of announcements. A (direct) strategy for person n is a function $s_n : \{0,1\} \to \{0,1\}$, and that for the whole population is $\boldsymbol{s}$. The vector $\boldsymbol{s}(\boldsymbol{\omega})$ defines an announcement profile at point $\boldsymbol{\omega}$. A Bayesian-Nash equilibrium for $\boldsymbol{c}$ is $\boldsymbol{s}$ such that, for all n and all ω,

$$\sum_{\boldsymbol{\omega} \in \Omega} p_n(\boldsymbol{\omega}; \omega)[u_\omega(c_n(\boldsymbol{s}(\boldsymbol{\omega}))) - u_\omega(c_n(\boldsymbol{s}'(\boldsymbol{\omega})))] \geq 0 \tag{4}$$

for strategies $\boldsymbol{s}'$ such that $\boldsymbol{s}'_{-n} = \boldsymbol{s}_{-n}$.

4 Relative optimality

With direct mechanisms, the announcements that determine current transfers are also used to inform the planner about future distributions of tastes. As we shall see, it is useful to study behavior when the planner uses one announcement to organize transfers and another to gather information about the future. Before such a mechanism is defined explicitly in the next section, we present a refinement of the concept of optimality for abstract games that requires person-n to be offered an "efficient" contract, in the perspective of a subgroup called *truncation* n.

We define optimality of $\boldsymbol{c} \in \boldsymbol{D}$, relative to arbitrary strategies $\boldsymbol{s}$ and $\boldsymbol{t}$, that need not be equilibria, by checking a series of optimization problems. Truncation n is the subgroup formed by people in positions $n, ..., \bar{n}$. We need the definition of interim expected utility for truncation n, conditional on $\boldsymbol{t}_n^-(\boldsymbol{\omega}) = (t_1(\omega_1), ..., t_{n-1}(\omega_{n-1}))$, which is

$$\sum_{\boldsymbol{\omega}\in\Omega} P(\boldsymbol{\omega}|\boldsymbol{t}_n^-(\boldsymbol{\omega})) \sum_{i=n}^{\bar{n}} u_{\omega_i}(c_i(\boldsymbol{\omega})). \tag{5}$$

The series of problems correspond to interim-utility maximization subject to stochastic endowments. The case $n = 1$ corresponds to the standard problem that gives rise to optima of the previous section. For $n \geq 2$, a planner maximizes interim utility of truncation n, subject to truth-telling and resource constraints analogous to those of the original economy, but with total endowment reduced by commitments made at truncation $n - 1$ and parametrized by $(\boldsymbol{s}, \boldsymbol{t})$. More formally, let $\boldsymbol{c}^1 \in \boldsymbol{D}$ solve the benchmark optimality problem henceforth called (P1). In recursive terms, problem (P$j+1$) is defined as that of choosing $\boldsymbol{c}^{j+1} \in \boldsymbol{D}$ so as to maximize (5) for $n = j+1$, subject to truth-telling constraints (3) for $n = j+1, ..., \bar{n}$, and subject to the promise-keeping constraint

$$c_n^{j+1} = c_n^j \circ \boldsymbol{s}, \;\; \forall n \leq j, \tag{6}$$

where $\boldsymbol{c}^j \in \boldsymbol{D}$ solves (Pj).

We say that $\boldsymbol{c}$ is *optimal relative to* $(\boldsymbol{s}, \boldsymbol{t})$ if there exists $\{\boldsymbol{c}^1, ..., \boldsymbol{c}^{\bar{n}}\}$ solving (P1-P$\bar{n}$) and such that $c_n = c_n^n$ for $n = 1, ..., \bar{n}$. We let $\mathcal{R}$ denote the set of $\boldsymbol{c}$ that is optimal relative to some pair of strategies. It follows that

Lemma 1 *$\mathcal{R}$ is a subset of $\boldsymbol{D}$.*

There are two assumptions about preferences that we need. They are satisfied by the preferences assumed in Green and Lin (2003) and Peck and Shell (2003), as discussed in Section 6. We shall state them as indirect requirements that can be satisfied by more general preferences, and provide later a direct restriction on utility functions. Let $\mathcal{D}$ denote the set of direct mechanisms that satisfy all truth-telling constraints with strict inequality.

Assumption (interiority) $\mathcal{R}$ is a subset of the closure of $\mathcal{D}$.

The relevance of interiority can be appreciated in the following proposition.

Proposition 2 *Truth telling is a best reply to truth-telling strategies in any* $\boldsymbol{c} \in \mathcal{R}$*, and is the unique best reply in some* $\tilde{\boldsymbol{c}} \in \mathcal{D}$ *arbitrarily close to* $\boldsymbol{c}$*.*

Proposition 2 provides some elements for the construction of a mechanism inducing truth-telling in a strong sense. If the planner could learn the true history $\boldsymbol{\omega}_n^-$ and plan truncation-n consumption conditional on future $\boldsymbol{\omega}_n^+$, so as to satisfy (3), then revelation is a truncation equilibrium regardless of how much resources are tied up by previous actions. In order to design a way of learning $\boldsymbol{\omega}_n^-$ for sure, we need more structure.

The next assumption refers to the impact of small changes in contracts achieved by reallocations across dates. Suppose that $\boldsymbol{c}$ is optimal relative to $(\boldsymbol{s}, \boldsymbol{t})$, and that the pair (n, ω_n) is given. If $a \in \mathbb{R}$ and, for a given rate of return $\rho > 0$, $c_n(\boldsymbol{s}(\boldsymbol{\omega})) \geq (a, -\rho a)$, then we can define

$$U_n(a, \omega_n) \equiv \sum_{\boldsymbol{\omega} \in \Omega} p_n(\boldsymbol{\omega}; \omega_n) u_{\omega_n}(c_n(\boldsymbol{s}(\boldsymbol{\omega})) + (-a, \rho a)).$$

In the assumption below, the notation N_0 stands for an open neighborhood of 0.

Assumption (separation) There exists $\rho > 0$ such that for any given $(\boldsymbol{c}, n, N_0)$, with $\boldsymbol{c} \in \mathcal{R}$ and $n < \bar{n}$, one can find $a, b \in N_0$ satisfying

$$\begin{aligned} s_n(1) &= 0 \text{ implies } U_n(a,0) < U_n(0,0) \text{ and } U_n(a,1) > U_n(0,1); \\ s_n(0) &= 1 \text{ implies } U_n(b,0) > U_n(0,0) \text{ and } U_n(b,1) < U_n(0,1). \end{aligned}$$

5 Twofold mechanisms

One motivation for categorizing transfers according to truncations is the possibility that the planner is able to infer the true history with richer message spaces. In this section, we seek that in what we call *twofold* mechanisms.

The augmented set of histories is $\Gamma = \{(h_1, ..., h_{\bar{n}}) : h_n \in \{0,1\}^2\}$ and has typical element $\boldsymbol{h}$. A twofold transfer function is $\boldsymbol{x} = (x_1, .., x_{\bar{n}})$ where, for each $n = 1, ..., \bar{n}$, $x_n = (x_{n1}, x_{n2})$ maps Γ into $\mathbb{R}^2_+$. At each period n, a person is called to make an announcement $h_n \in \{0,1\}^2$. These transfer functions are also restricted by sequentiality: x_{n1} must be constant on $\boldsymbol{h}_n^+$. The set $\boldsymbol{X}$ of such functions which also satisfies the feasibility constraint

$$\sum_n \left(x_{n1} + R^{-1}x_{n2}\right) \leq Y \tag{7}$$

at all points of its domain is the set of twofold mechanisms. In a similar fashion as with direct mechanisms, twofold mechanisms define a two-part revelation game where a strategy for person n is a pair of functions $(s_n, t_n) : \{0,1\} \to \{0,1\}^2$. Thus a strategy for the economy is a pair of direct strategies $(\boldsymbol{s}, \boldsymbol{t})$. A Bayesian-Nash equilibrium is defined in the obvious way.

We now construct a particular $\boldsymbol{x}$ recursively, starting with an optimal direct $\boldsymbol{c}^*$. We then pursue the following separation scheme for some $\rho > 0$ and small $\varepsilon > 0$. For $n = 1$, we fix $c_1^1(\boldsymbol{\omega}) = c_1^*(\boldsymbol{\omega})$. If $h_{n1} = 0$, the individual is offered a perturbation $(-a_n, \rho a_n)$ with $|a_n| < \varepsilon$ that only a type-1 would accept. Alternatively, if $h_{n1} = 1$, the individual is offered a perturbation $(-b_n, \rho b_n)$ with $|b_n| < \varepsilon$ that only a type-0 would accept. Otherwise, no adjustment is offered. The responses are captured as a second type announcement h_{n2}. This gives, for $n = 1$,

$$x_n(\boldsymbol{h}) = \begin{cases} c_n^n(h_{11}, ..., h_{\bar{n}1}) + (-a_n, \rho a_n), & \text{if } h_{n1} = 0 \text{ and } h_{n2} = 1; \\ c_n^n(h_{11}, ..., h_{\bar{n}1}) + (-b_n, \rho b_n), & \text{if } h_{n1} = 1 \text{ and } h_{n2} = 0; \\ c_n^n(h_{11}, ..., h_{\bar{n}1}), & \text{otherwise.} \end{cases} \tag{8}$$

The induction proceeds with the definition of x_{n+1} given x_n. Let $\boldsymbol{e} : \Omega \to \Omega$ denote the identity function. The mechanism plans consumption for person $n+1$ as in the maximization problem (P$n+1$) of the previous section with $\boldsymbol{s} = \boldsymbol{e}$ but using $\boldsymbol{t} = (\boldsymbol{h}_{n+1,2}^-, \boldsymbol{e}_n^+)$. Notice that the way that $\boldsymbol{t}_n^+$ is chosen has no consequence for the solution c_{n+1}^{n+1} of such maximization problem. Next, we set $x_{n+1}(\boldsymbol{h}) = c_{n+1}^{n+1}(h_{11}, ..., h_{\bar{n}1})$ and apply (8) for suitably chosen (a_n, b_n) with $\max\{|a_n|, |b_n|\} < \varepsilon$.

Finally, for $n = \bar{n}$, having computed $c_{\bar{n}}^{\bar{n}}$ as above, we set

$$k_0(\boldsymbol{h}) = (1 - \frac{\rho}{R}) \sum_{n=1}^{\bar{n}-1} [h_{n2}(1 - h_{n1})a_n + h_{n1}(1 - h_{n2})b_n],\ k_1(\boldsymbol{h}) = Rk_0(\boldsymbol{h})$$

and

$$x_{\bar{n}}(\boldsymbol{h}) = \begin{cases} c_{\bar{n}}^{\bar{n}}(h_{11}, ..., h_{\bar{n}1}) + k_0(\boldsymbol{h}), & \text{if } h_{\bar{n}1} = h_{\bar{n}2} = 0; \\ c_{\bar{n}}^{\bar{n}}(h_{11}, ..., h_{\bar{n}1}) + k_1(\boldsymbol{h}), & \text{if } h_{\bar{n}1} = h_{\bar{n}2} = 1; \\ (0,0), & \text{otherwise.} \end{cases} \tag{9}$$

That is, make the person $\bar{n}$ choose $s_{\bar{n}} = t_{\bar{n}}$ and pay for the social cost of extracting information.

Lemma 3 *There exists a twofold mechanism featuring satisfaction of truth-telling constraints with strict inequality and welfare close to optimal.*

Since ε and hence the costs k_i are arbitrarily small then consumption of person $\bar{n}$ is nonnegative in all histories. In addition, due to the interiority assumption, there is a mechanism close to $\boldsymbol{c}^*$ satisfying truth-telling constraints with strict inequality. Consequently, one can construct a twofold mechanism arbitrarily close to $\boldsymbol{x}$ and such that truth-telling constraints in the problems defining c_n^n never bind. These are the elements of the proof of the following proposition.

Proposition 4 *Let a twofold mechanism satisfying Lemma 3 be fixed. Then all of its Bayesian-Nash equilibria feature truth-telling in the second announcements.*

We conclude this section with our main result.

Proposition 5 *Truth-telling in all announcements is the only Bayesian-Nash equilibrium for a Proposition-4 twofold mechanism.*

6 Alternative assumptions and extensions

In this section, we provide direct restrictions on utility functions that imply interiority and separation. We also present a numerical example, in the spirit of Peck and Shell (2003), for an alternative environment where strategies cannot depend on positions.

6.1 Restricting utility functions

Consider the following preference structure.

Condition (primitive) The utility functions u_0 and u_1 are such that

$$\inf_{(a,b)\geq 0} \frac{\frac{\partial u_0}{\partial c_1}(a,b)}{\frac{\partial u_0}{\partial c_2}(a,b)} > \sup_{(a,b)\geq 0} \frac{\frac{\partial u_1}{\partial c_1}(a,b)}{\frac{\partial u_1}{\partial c_2}(a,b)}.$$

Proposition 6 *If preferences satisfy the primitive condition then separation holds.*

Example Consider the utility function u_ω, where $u_0(c_1, c_2) = Av(c_1)$ and $u_1(c_1, c_2) = v(c_1+c_2)$, for $A > 0$, and v increasing, continuous, concave, differentiable and satisfying Inada conditions.

It follows that u_ω as in the above example satisfies the interiority and separation assumptions.

6.2 Restricting the strategy space

Consider now that traders commit to announcement strategies before learning their assigned positions. We allow for mixed strategies of the form $\pi : \{0,1\} \to [0,1]$ with the understanding that π defines the probability of announcement $\boldsymbol{m} \in \Omega$ as

$$Q^\pi(\boldsymbol{m}|\boldsymbol{\omega}) = \prod_n [m_n \pi(\omega_n) + (1 - m_n)(1 - \pi(\omega_n))].$$

Likewise, that probability with deviation τ in position n is

$$Q^{\pi\tau}_{-n}(\boldsymbol{m}|\boldsymbol{\omega}) = (m_n\tau(\omega_n)+(1-m_n)(1-\tau(\omega_n))\prod_{i\neq n}[m_i\pi(\omega_i)+(1-m_i)(1-\pi(\omega_i))].$$

If $\boldsymbol{c} \in \boldsymbol{D}$ then a Bayesian-Nash equilibrium for $\boldsymbol{c}$ is π such that for all ω and all alternative strategy τ,

$$\sum_{\boldsymbol{\omega},n,\boldsymbol{m}} \frac{1}{\bar{n}} p(\boldsymbol{\omega};\omega)[Q^\pi(\boldsymbol{m}|\boldsymbol{\omega}) - Q^{\pi\tau}_{-n}(\boldsymbol{m}|\boldsymbol{\omega})]u_\omega(c_n(\boldsymbol{m})) \geq 0,$$

where $p(\boldsymbol{\omega};\omega)$ is the probability of event $\boldsymbol{\omega}$ conditional on $\omega_n = \omega$ for some n. The truth-telling strategy is π such that $\pi(0) = 0$ and $\pi(1) = 1$. The definition is easily extended to twofold mechanisms by changing the support of π to $\{0,1\}^2$.

We now construct a basic twofold mechanism $\boldsymbol{z}$, starting again with an optimal direct $\boldsymbol{c}^*$. We let $\boldsymbol{h}_{\cdot i}$ for $i \in \{1,2\}$ stand for $(h_{1i}, ..., h_{\bar{n}i})$. Likewise, $\boldsymbol{h}^+_{ni}$ and $\boldsymbol{h}^-_{ni}$ denote $(h_{n+1,i}, ..., h_{\bar{n}i})$ and $(h_{1i}, ..., h_{n-1,i})$, respectively. We then set

$$z_n(\boldsymbol{h}) = \begin{cases} c^*_n(\boldsymbol{h}_{\cdot 1}), & \text{if } \boldsymbol{h}_{\cdot 1} = \boldsymbol{h}_{\cdot 2}\,; \\ c^*_n(\boldsymbol{h}^-_{j+1,1}, \boldsymbol{h}^+_{j2}) + (0, f_{nj}(\boldsymbol{h})), & \text{if } n < j,\ \boldsymbol{h}^-_{j1} = \boldsymbol{h}^-_{j2},\ h_{j1} \neq h_{j2}; \\ c^*_n(\boldsymbol{h}^-_{n+1,1}, \boldsymbol{h}^+_{n2}), & \text{if } \boldsymbol{h}^-_{n1} = \boldsymbol{h}^-_{n2},\ h_{n1} \neq h_{n2}; \\ (0,0), & \text{otherwise;} \end{cases}$$

where

$$f_{nj}(\boldsymbol{h}) = \begin{cases} \sum_{i=j+1}^{\bar{n}} \frac{Rc^*_{i1}(\boldsymbol{h}^-_{j+1,1}, \boldsymbol{h}^+_{j2}) + c^*_{i2}(\boldsymbol{h}^-_{j+1,1}, \boldsymbol{h}^+_{j2})}{\sum_{n'=1}^{j-1} h_{n'1}}, & \text{if } h_{n1} = 1; \\ 0, & \text{otherwise.} \end{cases}$$

In words, this benchmark mechanism follows the optimal transfers until a lie is detected at position j. In this case, person n, for $n = j$, gets paid $c^*_n(\boldsymbol{h}^-_{n+1,1}, \boldsymbol{h}^+_{n2})$ and triggers a change in transfers to incoming traders. In particular, traders after j receive zero transfers at both dates. Then, according to the definition of the function f, the resources not spent with traders after j are rebated back, at date 2, to traders before j that announced the patient type.

The idea is that the planner is informed as soon as a lie occurs. In order to induce incoming traders to reveal their types it suffices to promise a small ε of consumption after a lie and to run the separation scheme of the previous section. For simplicity we have constructed $\boldsymbol{z}$ with a transfer of zero to incoming traders. The function f was designed to give an extra incentive for patient individuals to tell the truth, and other functions could be considered. For example, some transfers could be made to those that declare to be impatient in positions $1, ..., j-1$ if their marginal utility for consumption at date 2 is positive.

We present a numerical example with a run equilibrium with $\boldsymbol{c}^*$ that does not have a run equilibrium with $\boldsymbol{z}$. The utility has a functional form as in the Example above, with a small modification: $u_0(c_1, c_2) = \max\{\lambda, Av(c_1)\}$ and $u_1(c_1, c_2) = \max\{\lambda, v(c_1 + c_2)\}$, where $v(c) = -\frac{1}{c}$ and λ is a lower bound so that the utility is finite at $(0, 0)$, but always greater than λ at the optimum. The conclusions would be the same if $\lambda = -\infty$, except that, with $\boldsymbol{z}$, the planner would have to transfer an arbitrarily small quantity of resources after a lie is detected.

Setting $A = 10$, $R = 1.05$, and $P(\cdot)$ constant on Ω, a consequence of assuming that shocks are *iid* and that the probability of being patient is the same as that of being impatient, we mimic one economy in Peck and Shell (2003, Appendix B), except for one key difference. We set $\bar{n} = 3$ and $Y = 9$ (instead of $\bar{n} = 2$ and total endowment of 6), so as to obtain the same endowment per capita. The importance of a larger $\bar{n}$ is that a deviation from running strategies, with $\boldsymbol{z}$, may generate a rebate with positive probability for the first trader (when the next trader announces a lie).[3]

[3] We did run simulations with $N = 2$ to test the code by reproducing findings in Peck

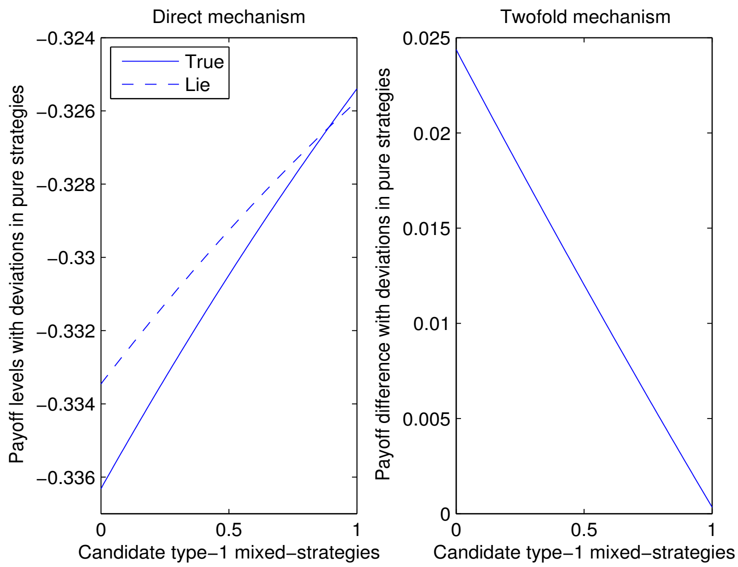

Figure 1: Direct and twofold mechanisms for a Peck-Shell example.

We use discrete approximations of the functions in $\boldsymbol{D}$ and compute the optimum $\boldsymbol{c}^*$. It features active truth-telling constraints. In the graph on the left in Figure 1 we plot the expected utility for a patient trader associated to choosing truth-telling ($\tau(1) = 1$) and misrepresentation ($\tau(1) = 0$), in response to a mixed strategy $\pi(1) \in [0, 1]$. The plot shows that misrepresentation is indeed the best reply to the run strategy $\pi(1) = 0$ because the expected utility of lying is above that of truth-telling at that point. In other words, there is a run equilibrium with $\boldsymbol{c}^*$. In the graph on the right, we plot the difference of the counterparts of the two curves with mechanism $\boldsymbol{z}$. We find that truth-telling is always the unique best response among all mixed strategies, although the plot refers to alternative pure strategies. Hence, for this particular economy, there are no run-equilibria for mechanisms close to $\boldsymbol{z}$.

We cannot show, however, that for general specifications the twofold mechanism of this subsection implements the optimum uniquely. The difficulty is illustrated with the following reasoning. Suppose that π is a candidate

and Shell (2003). As expected, the twofold mechanism does not eliminate bank runs in this case.

equilibrium such that $(\pi(0), \pi(1)) \neq (0, 1)$. Then, conditional on drawing a position after a lie has occurred, truth-telling is the best response for a small perturbation of the zero-transfer. Now, conditional on previous announcements being truthful, then truth-telling is the best response in two scenarios. First, if no lie becomes detected, implying that transfers follow the optimum and, consequently, that truth-telling constraints are satisfied. Second, if a lie is detected, but the planner saves resources so as to lift date-2 consumption of previous traders beyond optimal levels. But a third scenario cannot be ruled out, when there are no resources left to assure that previous traders receive the optimal allocation or better. This can happen, for instance, when a lie is detected at the last position.[4]

The preceding discussion is nevertheless useful for understanding the role of the "clock" assumption, that is, the assumption that strategies depend on positions which gives rise to the elimination of dominated strategies in the previous section. Our point is to illustrate that twofold mechanisms can reduce the set of equilibria relative to direct ones.

7 Final remarks

On a broader perspective, it is natural to assume that information is revealed sequentially in many economic contexts. But in models of banking, as we find out, there are particular implications of sequential services. On one hand, the sequential service is a barrier to full insurance and seems to be a source of fragility. On the other hand, it provides a series of opportunities for the planner to obtain information by exploring the ability to commit to future transfers. The latter helps to build a sound banking system and can be related to partial suspension schemes and historical episodes of bank holidays.[5]

Figuring out the dominant effect on bank stability is not straightforward, however, since the availability and usefulness of information devices depend on details of the model. For instance, in the modification of the Diamond-Dybvig model pursued by Peck and Shell (2003), with strategies that do not depend on positions, the results are mixed. Direct mechanisms have runs in

[4]The difference in expected utilities falls in Figure 1, with the twofold mechanism, because the average rebate generated by misrepresentation falls as truth-telling increases.

[5]Partial suspension schemes have been studied formally in model specifications without aggregate uncertainty.

one of their examples with two individuals, as well as in one of our examples with three individuals. Twofold mechanisms eliminate runs in the latter but not in the former case. Although our recursive elimination of dominated strategies does not apply in the Peck-Shell setting, future research can assess the performance of new mechanisms in a broader set of environments. Using elements of our approach, these mechanisms let the planner know when a run is taking place. Because the planner can shift transfers to a pattern less exposed to runs, this can eliminate the impulse to run fueled by panic that others are running.

Another sharp example of equilibrium runs, without the Peck and Shell (2003) modification, is due to Ennis and Keister (2009). Their specification has correlated shocks and preferences satisfying our separation and interiority assumptions. But because our larger mechanism eliminates runs in this case with relative ease, future research could address more systematically issues related to societal cost of identifying a run with more general preferences. It can also explore information devices not covered in our analysis. For instance, the planner could ask each trader to take a bet on the type of the last person in the queue. Since types are correlated and the last person always tells the truth, this device may offer yet another way to extract information at an arbitrarily low cost of setting up suitable lotteries.

The device that we did study formally has the planner opening borrowing and lending opportunities that depend on announcements.[6] The mechanism extracts information at zero cost in equilibrium because the opportunities are not taken under truth telling. But because they can distort incentives, the consequent effort to relax truth-telling constraints can spend resources, albeit in arbitrarily small quantities. If, however, the relevant off-equilibrium transfers in the Ennis-Keister specification satisfy truth-telling constraints strictly then our large mechanism would attain the exact optimal welfare. This possibility raises an awareness about limitations of the standard revelation-principle approach.

[6] The idea that depositors send two messages to their banks may seem "unpalatable" empirically. Of course, depending on the context being studied, the prices of speculative assets such as gold could work as proxies for additional messages.

References

[1] Andolfato, D., Nosal, E. and Wallace, N. 2007. The role of independency in the Green-Lin Diamond-Dybvig model. *Journal of Economic Theory* 137:709–715.

[2] Basseto, M. and Phelan, C. 2008. Tax riots. *The Review of Economic Studies* 75: 649-69.

[3] Bryant, J. 1980. A model of reserves, bank runs and deposit insurance. *Journal of Banking and Finance* 43:749–61.

[4] Cavalcanti, R. 2010. Inside-money theory after Diamond and Dybvig. *Federal Reserve Bank of Richmond Economic Quarterly* 96(1): 59-82.

[5] Diamond, D. and Dybvig, P. 1983. Bank runs, deposit insurance and liquidity. *Journal of Political Economy* 91:401–19.

[6] Ennis, H. and Keister, T. 2009. Run equilibria in the Green-Lin Model of financial intermediation. *Journal of Economic Theory* 144:1996–2020.

[7] Ennis, H. and Keister, T. 2009. On the fundamental reasons for bank fragility. *Federal Reserve Bank of Richmond Economic Quarterly* 96(1): 33-58.

[8] Green, E. and Lin, P. 2003. Implementing efficient mechanisms in a model of financial intermediation. *Journal of Economic Theory* 109:1–23.

[9] Jackson, M. 2001. A crash course in implementation theory. *Social Choice and Welfare* 18(4): 655-708.

[10] Maskin, E. 1999. Nash equilibrium and welfare optimality. *The Review of Economic Studies* 66: 23-38.

[11] Peck, J. and Shell, K. 2003. Equilibrium bank runs. *Journal of Political Economy* 111:103–123.

[12] Wallace, N. 1988. Another attempt to explain an illiquid banking system: The Diamond-Dybvig model with sequential service taken seriously. *Quarterly Review of the Federal Reserve Bank of Minneapolis* 12(4):3–16.

APPENDIX

Lemma 1 $\mathcal{R}$ is a subset of $\boldsymbol{D}$.

(The proof is straightforward and left to the reader).

Proposition 2 Truth telling is a best reply to truth-telling strategies in any $\boldsymbol{c} \in \mathcal{R}$, and is the unique best reply in some $\tilde{\boldsymbol{c}} \in \mathcal{D}$ arbitrarily close to $\boldsymbol{c}$.

Proof. Consider the games $\{\boldsymbol{c}^1, ..., \boldsymbol{c}^{\bar{n}}\}$ associated to $(\boldsymbol{c}, \boldsymbol{t}, \boldsymbol{s})$ when $\boldsymbol{c}$ is optimal relative to $(\boldsymbol{t}, \boldsymbol{s})$. By assumption, $\{\boldsymbol{c}^1, ..., \boldsymbol{c}^{\bar{n}}\}$ solves (P1-P$\bar{n}$) with $\boldsymbol{c}^1 = \boldsymbol{c}$. With n fixed, consider the best-reply correspondence for person n in the truncation-n mechanism $\boldsymbol{c}^n$. If the strategy of traders $n+1, ..., \bar{n}$ in the truncation-n game is truth-telling then the person-n reply problem is that of choosing $s' \in \{0, 1\}$ so as to maximize the expectation of $u_\omega(c_n^n(\boldsymbol{\omega}_{-n}, s'))$ with respect to $\boldsymbol{\omega}$, conditional on $\omega_n = \omega$. Since $\boldsymbol{c}^n$ solves (Pn), then c_n^n, the n-coordinate of $\boldsymbol{c}^n$, satisfies the truth-telling constraint (3) at (n, ω),

$$\sum_{\boldsymbol{\omega} \in \Omega} p_n(\boldsymbol{\omega}; \omega) \left(u_\omega(c_n^n(\boldsymbol{\omega}_{-n}, \omega)) - u_\omega(c_n^n(\boldsymbol{\omega}_{-n}, \omega'))\right) \geq 0$$

for all $\omega' \in \{0, 1\}$. This assures that $s' = \omega$ is a best reply for person n in mechanism $\boldsymbol{c}^n$. Now, since $\boldsymbol{c} \in \mathcal{R}$ then, under the assumption of interiority, there exists $\{\tilde{\boldsymbol{c}}^1, ..., \tilde{\boldsymbol{c}}^{\bar{n}}\} \subset \mathcal{D}$ arbitrarily close to $\{\boldsymbol{c}^1, ..., \boldsymbol{c}^{\bar{n}}\}$ but satisfying truth-telling constraints with strict inequality. Thus repeating the previous argument for the associated truncation-n transfer function $\tilde{\boldsymbol{c}}_n^n$ concludes the proof. ■

Lemma 3 There exists a twofold mechanism featuring satisfaction of truth-telling constraints with strict inequality and welfare close to optimal.

Proof. Consider the construction of the above twofold mechanism $\boldsymbol{x}$. Since the sequence of c_n^n is derived from small perturbations of relatively optimal mechanisms $\boldsymbol{c} \in \mathcal{R}$, then the interiority assumption implies that the construction can be redone by using instead $\tilde{\boldsymbol{c}} \in \mathcal{D}$ arbitrarily close to $\boldsymbol{c}$. ■

Proposition 4 Let a twofold mechanism satisfying Lemma 3 be fixed. Then all of its Bayesian-Nash equilibria feature truth-telling in the second announcements.

Proof. Let $\boldsymbol{x}$ be constructed as above and let us consider the best reply to arbitrary equilibrium strategy $(\boldsymbol{s}, \boldsymbol{t})$. Let (s_1', t_1') be a best response by person 1. Then t_1' must be the identity function, otherwise the separation assumption would be violated since, by construction, the underlying $(c_1^1, ..., c_{\bar{n}}^{\bar{n}})$ is optimal relative to $(\boldsymbol{s}, \boldsymbol{t})$. Since $(\boldsymbol{s}, \boldsymbol{t})$ is an equilibrium, t_1 is also the identity function. By induction, assuming that $\boldsymbol{t}_n^-$ is the identity function, the participation constraints used in the derivation of c_n^n for all $n < \bar{n}$ become written with the correct conditional distribution of future shocks. Thus the separation assumption implies the result also for all $n < \bar{n}$. That $t_{\bar{n}}$ is also the identity function follows from the fact that, for the last person, truth-telling is a dominant strategy with any relatively optimal mechanism and, by force of (9), there is no gain in choosing $t_{\bar{n}}' \neq s_{\bar{n}}'$. ■

Proposition 5 Truth-telling in all announcements is the only Bayesian-Nash equilibrium for a Proposition-4 twofold mechanism.

Proof. Let $\boldsymbol{x}$ be a Proposition-4 twofold mechanism. For such a mechanism it suffices to examine the best response to $(\boldsymbol{s}, \boldsymbol{e})$, that is, when the candidate equilibrium features the identity function $\boldsymbol{e}$ as the list of second announcements. Also, as argued in the proof of Proposition 1, in any equilibrium $s_{\bar{n}}$ must be the identity function. As a result, by construction of $\boldsymbol{x}$, the underlying $(c_1^1, ..., c_{\bar{n}}^{\bar{n}})$ is optimal relative to $(\boldsymbol{s}, \boldsymbol{e})$ and built with slacking truth-telling constraints. Because the conditional distribution used to construct $c_{\bar{n}-1}^{\bar{n}-1}$ is the correct one, then $s_{\bar{n}} = e_{\bar{n}}$ and the fact that truth-telling constrains slack imply that $s_{\bar{n}-1} = e_{\bar{n}-1}$ is the best response for person $\bar{n}-1$. Repeating the argument in a backward fashion demonstrates that $(\boldsymbol{e}, \boldsymbol{e})$ is the only Bayesian-Nash equilibrium of $\boldsymbol{x}$. ■

Proposition 6 If preferences satisfy the primitive condition then separation holds.

Proof. Take ρ such that

$$\inf_{(a,b)\geq 0} \frac{\frac{\partial u_0}{\partial c_1}(a,b)}{\frac{\partial u_0}{\partial c_2}(a,b)} > \rho > \sup_{(a,b)\geq 0} \frac{\frac{\partial u_1}{\partial c_1}(a,b)}{\frac{\partial u_1}{\partial c_2}(a,b)}.$$

Now note that for $\varepsilon > 0$ sufficiently small,

$$u_0\left((a,b) + (-\varepsilon, \rho\varepsilon)\right) \approx u_0(a,b) + \varepsilon\left(-\frac{\partial u_0}{\partial c_1}(a,b) + \rho\frac{\partial u_0}{\partial c_2}(a,b)\right) < u_0(a,b).$$

And

$$u_1\left((a,b)+(-\varepsilon,\rho\varepsilon)\right)\approx u_1\left(a,b\right)+\varepsilon\left(-\frac{\partial u_1}{\partial c_1}\left(a,b\right)+\rho\frac{\partial u_1}{\partial c_2}\left(a,b\right)\right)>u_1\left(a,b\right).$$

Therefore the patient person will never choose (a,b) if he can choose $(a,b)+(-\varepsilon,\rho\varepsilon)$. Reciprocally, the impatient prefer (a,b). Now if ε is instead negative and small we reverse the conclusion. ■