Financial Risk Capacity

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Abstract

I study a growth model with a financial sector that deals with asymmetric information in capital markets. Entrepreneurs sell assets under asymmetric information to obtain funding for new projects. The financial sector buys these assets and resells them as a single pool. Financial intermediation is risky because the distribution of asset quality is uncertain.

When the quality of assets is surprisingly bad, financial firms incur in losses which they cover by liquidating their own assets. When a sequence of negative shocks hits the financial system, it looses the capacity to bear further losses. With weaker balance sheets, the volume of intermediation shrinks and adverse-selection effects kick-in. These episodes reduce the profitability of the financial system up to a point where it prevents external re-capitalization. These phenomena lead to persistent drops in the growth rate of the economy which are overcome as the financial sector slowly strengthens its net-worth via retained earnings.

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1 Introduction

The financial sector plays a key role for the reallocation of resources in the economy. For this reason, business cycle fluctuations are partly attributed to the financial system's capacity to provide intermediation services. Economic theory also stresses that one of the main functions of financial intermediation is to mitigate problems of asymmetric information that plague capital markets. This paper studies the connection between risk and intermediation in financial markets with asymmetric information. The purpose of this paper is to understand the implications of this type of intermediation on economic growth and the optimal regulation of the financial institutions.

Although there is a large body of theoretical and empirical research on how asymmetric information and financial market intermediation may affect the business cycle, until now, these aspects have not been studied jointly. There are two reasons that explain why this connection may be key to understand business cycles. Beginning with the seminal contribution of Stiglitz and Weiss [1981], we know that asymmetric information can cause credit rationing phenomena that affect economic performance.[1] Yet, the literature abstracts from the fact that most of everyday transactions occur through financial institutions (FIs). Gorton (2009), for example, argues that "The essential function of banking is to create a special kind of debt, debt that is immune to adverse selection by privately informed agents.". Due to their size, FIs posses the technology to dilute part of the risks that stem from asymmetric information. Nevertheless, financial intermediation remains a risky business. The capacity to tolerate financial intermediation risk (the *financial risk capacity*) is tied to the conditions of FIs. As a consequence, intermediation in markets with asymmetric information will respond not only to underlying information structure of an economy, but also to the financial system's financial risk capacity. This paper explains how shocks that adversely affect the financial sector propagate and spill-over to the rest of the economy by exacerbating adverse-selection phenomena, although the underlying information structure of the economy remains unchanged.

The second reason is of a dynamic character. The connection between asymmetric information and financial intermediation can explain why recoveries are particularly slow after the occurrence of banking crises. A theory that links financial intermediation to the balance sheet of FIs must also explain why the financial sector is not quickly recapitalized after such episodes.[2] After all, competition arguments suggest that financial intermediation should be more profitable when less intermediation services are provided, which ultimately attract funds to this sector. This paper shows that in presence of informational asymmetries, the profitability of financial intermediation may decrease as the aggregate volume of intermediation shrinks. This means that an individual FI may be less profitable when its competitors find themselves in weaker conditions. This characteristic provides an explanation as to why the private recapitalization of FIs was so slow during the last financial crisis and why did lending took time to take-off during its aftermath. During his only television interview, FED Chairman, Ben Bernanke, was asked when he would consider the crisis to be over. He answered "When banks start

[1] Other classical examples in corporate finance include Myers and Majluf [1984], who explain how asymmetric information in the quality of assets has implications for their use as collateral and the availability of funds to finance investment projects.

[2] This aspect has concerns both theorists and policy makers. See for example Duffie [2010], for a recent account.

raising capital on their own.". This paper answers why this didn't happen.

I take these insights and build a growth model where the purpose of financial intermediaries is to mitigate asymmetric information in financial markets but the capacity to do so is bounded by the net-worth of the financial system. The theory explains: (1) The interaction between risky financial intermediation and asymmetric information. (2) The implications of these features for the amplification and propagation of real shocks. (3) The type of externalities that are found in this environment. Finally, (4), the effects of financial regulation instruments aimed at resolving the related market inefficiencies.

I introduce a financial sector with the features described into an endogenous growth model where financial intermediation is the engine of economic growth. The financial system is modeled as a large competitive sector that provides intermediation services for the reallocation of capital between small agents in two sectors of the economy. The first group is composed by entrepreneurs in need of funding to generate new investment projects. The second is a group of agents that lack access to investment opportunities but, in contrast to their counterparts, does have the resources to carry out these projects. In the process of intermediation, FIs raise capital from the later group to provide funding to investors. In turn, investors are required to use their existing capital whose quality is privately known as collateral for these transactions. The setup of the model serves the interpretation of FIs as brokers or as commercial banks.

In frictionless environments, the economy runs efficiently without the need of intermediation. Under asymmetric information, FIs serve the purpose of diluting the idiosyncratic risk faced by small agents who trade directly under asymmetric information. To embody this role, I allow the FIs to exploit the law of large numbers in a way that allows them to pool idiosyncratic risk together. Nevertheless, aggregate shocks prevents all risk from being entirely wiped out.

The model has two key ingredients. First, the entire distribution of collateral quality (which is private information) is, *a priori*, unknown. This source of uncertainty introduces risk into financial intermediation. If, in contrast, the financial sector knew the actual distribution of capital quality, financial intermediation would not be risky at all: by trading with a large number of sellers, the financial sector could perfectly infer the quality of assets traded (rational expectations).[3] The assumption here is that the financial system purchases a portfolio of assets with only an inexact prior on what the actual distribution of what their underlying quality is. Mechanically, the financial sector buys a particular percentile of the distribution of the private sectors assets but is uncertain about the quality of that tail. Changes in the distribution of firm revenues have been found to precede recessions (see Bloom [2009] and Bloom et al. [2009]). This is a realistic way of modeling financial risk when these shocks stem from private information on the side firms.[4]

The second ingredient is, as in the real world, that FIs face a limited liability constraint. Thus, when the quality of assets bought is surprisingly worse than expected, a FI must resort to its own

[3] One can say more than this. In a competitive environment, the law of large numbers would take care of driving profits to 0: the conditional expected quality of assets bought at a given price would converge to the actual quality sold in equilibrium.

[4] Very recently, a number of studies have provide several theories on the propagation of sector or firm specific shocks to the rest of the economy. See for example, Arellano et al. [2010] or Gilchrist et al. [2010].

funds to finance operational losses. This restriction makes net-worth the key state variable because today's net-asset position affects financial intermediation tomorrow. This paper underscores how this sensitive feedback-loop between asymmetric information and the evolution of net-worth works.

To illustrate the mechanics of the model, let me guide you through a hypothetical simulation. Think about a particular sequence of distributions of capital quality. Assume that these are surprisingly below the financial system's expectations (leading to a sequence of financial losses). Financial losses are financed by liquidating part of the FI's own assets. For a sufficiently adverse sequence, a FI's net-worth is depleted up to the point in which it runs out of capital to finance future losses under the same market conditions. In such states, FIs cut-back on lending, requiring a larger discount between the expected value of assets and the actual purchase price. These phenomenon exacerbates adverse-selection effects leading to larger contractions in volumes of intermediation and further increases in lending premia. Depending on the how the quality of collateral responds to changes in prices, the profitability of financial intermediation may decrease in spite of higher premia. Low profitability in the financial industry precludes the external recapitalization of FIs. Instead, the financial system is recapitalized only through retained earnings but this process is slow since volumes of intermediation and expected profits are low to begin with. In the paper, I show that this ratchet effect is a distinctive feature of adverse-selection in capital markets.

Because in the model, capital reallocation is the engine for economic growth, it is capable of explaining 3 salient features about financial crises. First, their *recurrence*, as documented by Allen and Gale [1998] or Reinhart and Rogoff [2009], to name a few recent studies.[5] Second, financial crises are associated with severe drops in the volume of intermediation and increases in discount rates. According to Brunnermeier [2009] or Gorton and Metrick [2010], these characteristics are the symptoms of *adverse selection* effects. Third, financial crises have *persistent* effects on output, a feature that is in line with recent evidence found by Cerra and Saxena [2008].

The environment leads me to identify a novel pecuniary externality: financial firms may fail to internalize that their individual decisions have effects on price that ultimately may exacerbate adverse selection in the future. I use the model to evaluate the effects of dividend taxes and capital requirements in the financial system and explain that a benevolent planner will balance increasing inefficiencies in normal times with a reduction in the chances of falling to adverse selection driven crisis.

I label the financial system's capacity to bear risk and, therefore, to provide intermediation services as *financial risk capacity*, in resemblance to the theories of physical capacity constraints.[6]

The paper is organized as follows. The following section discusses how the paper relates to the literature. Section 2 introduces the model. Section 3 provides banking theory interpretation of the model. Section 4 characterizes equilibria. Section 5 presents two examples in closed form solution with the purpose of underscoring the role of asymmetric information in this economy. Section 6 presents a quantitative exercise. Section 7 introduces several extensions to the model and Section 8 provides a discussion on government policies aimed at resolving market imperfections. I conclude in section 9.

[5]For a particular definition of financial crisis, Reinhart and Rogoff [2009] document that the U.S. experienced a crisis during 13% out of the 200 years they study. Similar conclusions are found in Allen and Gale [1998].

[6]A similar concept labeled financial capacity is used in Gertler [1992] to refer to the capacity of firms in the productive sector to obtaining credit.

Every proof is relegated to the appendix.

1.1 Notes on the literature

One of the observations from the recent financial crises is that the balance-sheet of the financial sector may be key to understand the reallocation of resources in the economy. This observation has motivated a substantial amount of research. The literature explains the relevance of the financial system's asset position for the efficient allocation of resources from two different perspectives.

Intermediaries net-worth and bank runs. Following Diamond and Dybvig [1983], a first stream of research focuses on the role of the financial sector's capital structure for the occurrence of bank runs caused by maturity mismatches. In these models, financial crises arise when an exogenous shocks to the demand and supply of short run funds affects financial institutions. This literature underscores that the availability of funds (for the whole financial system) determines the effectiveness of internal insurance mechanisms against idiosyncratic shocks. From this perspective, the financial system's asset position because it can discourage bank-run phenomena caused by fear of reduction on bank solvency. Other seminal contributions in this area include Holmstrom and Tirole [1998] or Allen and Gale [1998].

Intermediaries net-worth and agency costs. A second branch of the literature stresses that the financial system's net-worth reduces agency costs involved in financial intermediation. Among others, in Holmstrom and Tirole [1997], agency costs arise because financial intermediaries may put insufficient effort in monitoring loans, or may simply divert funds. The financial sector's net-worth reduces agency costs since larger participation in bank operations, have more incentives to monitor loans. Following this insight, in the aftermath of the recent financial crisis, several studies have introduced a financial sector with this features into business cycle models. Recent examples include Gertler and Karadi [2009], Gertler and Kiyotaki [2010], and Curdia and Woodford [2009] who use these models to understand the effects of government policies that target the balance-sheet of financial intermediaries.

On the other hand, Brunnermeier and Sannikov [2009] and He and Krishnamurthy [2008] study aggregate shocks that affect the financial system's wealth disproportionately in comparison with the rest of the economy. They study how such shocks exacerbate agency costs in the model, are amplified by fire-sale spirals and carry effects on output when the allocation of resources is affected.

This paper is closer to this stream of models for two reason. First, the financial system's net-worth is also key to provide incentives which is key for the efficient allocation of resources. Here, the financial sector's net-worth matters because of limited liability, which can be motivated as stemming from moral hazard. A novelty of this papers is that the systematic risk faced by the financial system is exacerbated by asymmetric information. This features has the opposite implications for the balance-sheet structure of the financial system. Under moral hazard only, as net-worth is depleted, agency costs increase. One should expect the marginal benefit of increasing the firms capital to be greater as financial institutions suffer a hit. Moreover, with free-entry in the financial system, less competition attracts capital from other sectors of the economy into the financial system. Thus, one should expect a counterbalancing market forces injecting capital to the financial system in times of crisis. All of the aforementioned papers close-up this channel by restricting free-entry: they endow intermediaries with a special monitoring ability. Here, in contrast, adverse selection reduces the profitability of financial equity which is why ultimately, the system is not recapitalized.

Fire-sales and strategic behavior. Pending paragraph. Rajan-Diamond(2011), Schleifer-Vishny(1992, JF) and Bolton, Santos, Sheinkman (2010). Vayanos and gromb. Strategic behavior.

Lorenzoni [2008] combines some features of this literature with limited commitment in common to this paper. That paper identifies a fire-sale externalities that stem from excessive risky intermediation (with relation to a social optimal). The pecuniary externality here from intermediaries not internalizing the adverse selection effects that are activated when they suffer losses.

Asymmetric information. This paper also shares some features with other general equilibrium models with adverse selection in capital markets. Stiglitz and Weiss [1981] is the first paper to study credit rationing when the quality of projects is private information. Carlstrom and Fuerst [1997] introduce a version of this model into general equilibrium.[7]

Eisfeldt [2004] studies a general equilibrium model where the quality of existing assets is private information. In this environment, assets are sold under asymmetric information risk-sharing motives and shows how asymmetric information limits risk-sharing. Bigio [2009] and Kurlat [2009] study models in which assets are sold under asymmetric information to relax financial constraints and explain how shocks that affect exacerbate adverse-selection can cause recessions. In the present paper, the action is placed on the financial sector whose net worth determines the market outcomes. In other models that study lemons markets, such as Hendel and Lizzeri [1999], Kurlat [2009] or Daley and Green persistence is a product of learning mechanisms. Here, persistence is caused by the slow recovery of the financial intermediaries when asymmetric.

The interaction between limited liability constraints and asymmetric information studied here is novel within the context of a macroeconomics model but it is not entirely new. There has been several decades of research on how limited liability constraints affect insurance markets. In particular, liability insurance markets share a common striking feature with financial markets: crises in this sector are recurrent, are characterized by large swings in insurance premia and by falls in the volume of issued policies.

In parallel to the sub-prime crisis, these events typically followed episodes of large negative profits for insurance firms, suggesting the hypothesis that limited liability constraints where relevant.[8] Gorton and Metrick [2010] documents a similar pattern in the collapse of asset backed security markets.

The relationship between financial and insurance market crises is not surprising if one thinks of the financial sector as a large clearing house that pools risky transactions and funds itself by issuing liabilities backed by pools of assets, as in this paper. Indeed, the financial sector in this paper resembles the insurance sector with limited liability constraints studied by Winter [1991a]. The present paper introduces a similar sector into a growth model, and adds two layers to set-up the model in general equilibrium: (1) capital reallocation is key for the economy because, as in Kiyotaki and Moore [2008], not all agents have access to savings instruments, (2) the value of capital is explicitly affected by adverse selection and finally, (3) losses in the financial sector are caused by changes in the quality of

[7] Martin [2009] compares pooling with separating equilibria in a similar context in which the quantity of collateral interest rates are used as screening devices.

[8] See Winter [1991b] for a survey on this literature. Gron [1994a] and Gron [1994b] provide empirical investigations that tested the tested the implications of the insurance sector's net worth on insurance premiums and volumes.

assets, as in Bigio [2009].

Empirical Literature. In fact, during the recent crisis, authors such as Brunnermeier [2009] or Gorton and Metrick [2010] suggest that both asymmetric information and the financial condition of the banking system played a major role.

2 Model

2.1 Environment

The model is formulated in discrete time with an infinite horizon. There are two goods: a perishable consumption good (the *numeraire*) and capital. Every period is divided into two stages, $s \in \{1, 2\}$. There are two aggregate shocks, a TFP shock $A_t \in \mathbb{A}$ and a shock $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, ..., \phi_N\}$ that determines the distribution of capital quality. (A_t, ϕ_t) form a joint Markov process that evolves according to a transition probability $\chi : (\mathbb{A}\times\Phi) \times (\mathbb{A}\times\Phi) \to [0, 1]$ with the standard assumptions.

Notation. I use the notation $x_{t,s}$ to refer to the value of a variable x in period t stage s when the variable changes values between stages. Otherwise, if the variable remains constant throughout the period, I use the time subscript only.

Demography. There are two populations of agents: entrepreneurs and financial intermediaries. Each population has a unit mass but financial intermediaries are assumed to be bigger in a sense to be clear below. Intermediaries face an exogenous constant probability of dying. When an intermediary dies, he is immediately replaced by a newborn intermediary. The purpose of introducing stochastic survival is to obtain analytic examples but intermediaries are treated as long-lived in the numerical exercises.

Entrepreneurs. Entrepreneurs are identified by a number $z \in [0, 1]$ and carry their capital stock $k_t(z)$ as their individual state variable. At the beginning of the first stage, entrepreneurs are randomly segmented into two groups: investors and producers. I also refer to these types as i-entrepreneurs and p-entrepreneurs. Entrepreneurs become investors with a probability π independent of time and z. As a consequence, every period, there are masses π of i-entrepreneurs and $1 - \pi$ of p-entrepreneurs.

Investors have access to an investment technology that allows them to create new capital units using consumption goods but cannot use their capital stock for the production of consumption goods. In contrast, producers can use capital to produce consumption goods, but lack the possibility of generating capital directly. This segmentation induces a need for trade: i-entrepreneurs have access to the investment technology but lack the input to operate it. p-entrepreneurs produce consumption goods but lack access to an investment technology that allow them to accumulate capital. Financial intermediaries provide intermediation services between these two groups. Randomizing across activities is introduced for tractability since, otherwise, the relative wealth of each group of entrepreneur would become a state variable.

Entrepreneurs have log-preferences over consumption streams and evaluate these according to an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}\beta^t \log(c_t)\right]$$

where c_t is consumption.

Intermediaries. Intermediaries are identified by some $j \in [0, 1]$. At the beginning of every period, they receive a large exogenous endowment of consumption goods $\bar{e}_t(j)$. In addition, they carry a stock of consumption goods $n_{t,1}(j)$ held within financial institutions (henceforth, banks) of their property. $n_{t,1}(j)$ is the intermediaries individual state variable which is interpreted as the bank's net-worth. During the first stage, intermediaries can alter the composition of their financial wealth by injecting equity from their personal endowment to their banks or do the opposite by paying dividends. After equity injections and dividends their banks' net-worth evolves from $n_{t,1}(j)$ to $n_{t,2}(j)$.

Financial intermediaries participate in capital markets purchasing capital units from i-entrepreneurs in the first stage and reselling them to p-entrepreneurs during the second stage. They purchase capital by issuing tradeable riskless IOUs that entitle the holder to a unit of consumption in the second stage.[9] For now, I implicitly assume that managerial incentives cannot be met, so intermediaries face no strategic decisions between holding on to capital purchases or selling immediately.[10]

Since ϕ_t arrives between stages, the value of the purchased capital is random. This randomness makes financial intermediation a risky business. In particular, the intermediary may suffer losses if his purchase cost (issued IOUs) exceeds the value of his capital purchases. When the intermediary experiences financial losses, he is forced to draw funds from his bank's equity in order to settle these claims.

In principle, financial losses could be financed via the intermediaries personal endowment. Instead, I assume, as in real world banks, that financial intermediation is subject to a *limited liability constraint* (LLC) such that the intermediaries personal endowment is not liable to his banks losses.[11] This condition implies that losses from financial intermediation cannot exceed their bank's net-worth, $n_{t,2}(j)$. As a consequence of the LLC, the bank's net-worth will affect the intermediaries capacity to engage in more or less transactions. In that sense, a bank's net-worth acts as a cushion to absorb potential losses. For this reason, there is a distinction between the intermediary's personal endowment and his bank's equity: net-worth relaxes the LLC constraint whereas the personal endowment does not. The LLC can be obtained endogenously as a result of a commitment problem on the side of financial intermediaries. If intermediaries cannot be forced to inject equity into their financial firms to cover losses, this constraint would show up as an ex-post incentive compatibility condition.[12] Otherwise, one can take the LLC as an institutional constraint.

When an intermediary dies, his bank can be bought by a newly born intermediary. The presence of dividend taxes implies that intermediaries will rather buy a bank of dying entrepreneur than starting a new bank. The death probability is a constant, $\rho \in [0, 1]$. When, $\rho = 0$, intermediaries are characterized by an infinite horizon problem. When $\rho = 1$, one can solve the model analytically.

[9] An alternative interpretation that resembles common bank practices is explained below.

[10] The model can be extended in this direction. The outcome involves a non-trivial fire-sale behavior for financial firms which in turn may have implications for the behavior of the financial system. Fire-sales may induce a different type of externality than the one caused by adverse-selection which is the focus of this paper. Diamond-Rajan, for example. **Note: Douglas Gale suggested exploring this direction.**

[11] Scotland..banking era.

[12] **Otherwise, this constrained can be obtained as a high penalty for defaulting in IOU's. Like loss in reputation for example.**

Intermediaries have linear preferences over consumption streams and evaluate these according to an expected utility criterion:

$$\mathbb{E}\left[\sum_{t\geq 0}(\beta^f)^t c_t\right]$$

where c_t is consumption.

Technology. The investment technology transforms one unit of consumption into an efficiency unit of capital available for production the following period. A p-entrepreneur that holds a capital stock of $k_t(z)$, will produce consumption goods according to a linear technology $A_t k_t(z)$. In addition, financial intermediaries have access to a storage technology that transforms 1 unit of consumption into R^b units of consumption. In principle, one can think of this technology as a risk-less government bond financed through lump-sum taxes but I leave the interpretation open. I assume, R^b is smaller than the expected return to capital. This assumption implies that it is never efficient to have capital in hands of the financial sector and in a frictionless economy, the financial sector would disappear. I also assume that $\beta^f R^b < 1$ so that intermediaries would liquidate their banks if they did not face the LLC constraint.

Capital. At the beginning of every period, capital held by each entrepreneur is divisible into a continuum of units. Each unit is identified by a quality $\omega \in [0,1]$. There is a deterministic increasing and differentiable function $\lambda(\omega) : [0,1] \to \mathbb{R}_+$ that determines the corresponding efficiency units that evolve from an ω-quality unit. Efficiency units can also be interpreted as random depreciation shock but they are not restricted to be less than 1.

In addition, there is a set of measures over qualities that depends on the realization of ϕ_t. In particular, at t, these measures are defined by an absolutely continuous function $f_{\phi_t} : [0,1] \to \mathbb{R}_+$. which, in turn, is a function of ϕ_t. Between periods, each piece is transformed into future efficiency units by scaling pieces by $\lambda(\omega) f_\phi(\omega)$. Thus, $\lambda(\omega) f_\phi(\omega) k(\omega)$ efficiency units remain from an $\omega - quality$ next period. To simplify the analysis, I assume that these measures are the same across entrepreneurs but change through time depending on the sequence of realizations of ϕ_t. Once capital units are scaled by their corresponding efficiency units, pieces are merged back into a single homogeneous capital unit. This unit will be divided again in the same way in the following period and this process is repeated indefinitely. Thus, by the end of the second stage, the effective capital stock that remains from an original stock k is $k \int \lambda(\omega) f_\phi(\omega) d\omega$. Distinguishing between $(\lambda(\omega)$ and $f_\phi(\omega))$ only provides an interpretation to these shocks. For allocations, only their product matters.

Entrepreneurs won't necessarily hold on to all of their capital units. On the contrary, they may choose to sell particular units during the first stage to obtain consumption goods. These decisions are represented by an indicator function $\mathbb{I}(\omega) : [0,1] \to \{0,1\}$ that takes a value of 1 when units of quality ω are sold. Because each quality has 0 measure, the restriction to all-or-nothing sales is without loss of generality. When choosing $\mathbb{I}(\omega)$, the entrepreneur transfers $k \int \mathbb{I}(\omega) d\omega$ units of capital to the financial intermediaries. These units evolve into $k \int \lambda(\omega) \mathbb{I}(\omega) f_{\phi_t}(\omega) d\omega$ efficiency units of t+1 capital. Simple accounting shows that the efficiency units that remain with the entrepreneur are $k \int \lambda(\omega) [1 - \mathbb{I}(\omega)] f_\phi(\omega) d\omega$. Taking into account possible investments and purchases of capital, the entrepreneur's capital stock evolves according to:

$$k' = i + k^{b} + k \int \lambda(\omega) [1 - \mathbb{I}(\omega)] f_{\phi}(\omega) d\omega, \tag{1}$$

where i are the capital units created by exploiting the investment technology (when available) and k^b are purchases of efficiency units of t+1 capital. I impose some structure on the quality distributions $\{f_\phi\}$:

Assumption 1 *The set $\{f_\phi\}_{\phi \in \Phi}$ satisfies that $\mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^*] \equiv \int I_{[\omega<\omega^*]} \lambda(\omega) f_\phi(\omega) d\omega$ is weakly decreasing in ϕ for any ω^*.*

This condition states that the average efficiency unit that remains from qualities under some ω^* is decreasing in ϕ. In equilibrium, intermediaries will always purchase lower-tail qualities, so this assumption implies that the amount of effective capital held by intermediaries decreases with ϕ regardless of the amounts of capital purchased. This implies that conditional expectation satisfy a first order dominance condition and provides an ordering to the shocks with the interpretation that they adversely affect intermediaries.[13]

Let $\bar{\lambda}(X) \equiv \mathbb{E}_\phi [\lambda(\omega)]$ be the unconditional effective depreciation in state X. The setup is general to accommodate to two polar cases of particular interest. The first case is when all qualities are the same conditional on a shock, $\lambda(\omega) = \lambda^*$, but efficiency units are decreasing in ϕ (f_ϕ integrateS to smaller number as ϕ is larger). In this case, ϕ has the effects of permanent capital depreciation shock and also induces risk into financial intermediation. This polar case is used to study risky financial intermediation without asymmetric information.

The second case of interest is when $\int \lambda(\omega) f_\phi(\omega) d\omega = \bar{\lambda}(X)$ for any $\phi \in \Phi$. This condition states that ϕ is mean preserving (MPS). This implies that the production possibility of the economy is independent of ϕ. Under the information structure described below, the second case corresponds to an environment with ex-ante adverse selection. This feature has the connotation that if ϕ has any effect on allocations, it is because these shocks affect *equilibrium* but not the *feasible* set of allocations.

Information. There are two endogenous aggregate states, $K_t = \int k_t(z) dz$ and $N_{t,s} = \int n_{t,s}(j) dj$, the capital stock held by entrepreneurs and the net-worth of the financial sector respectively. It will be shown that in order to characterize equilibria, it is only necessary to keep track of their ratio $\kappa_{t,s} \equiv N_{t,s}/K_t$ as a unique state variable. Thus, the state of this economy is $X_{t,1} = \{A_t, \phi_{t-1}, \kappa_{t,1}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$ and $X_{t,2} = \{A_t, \phi_t, \kappa_{t,2}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$. $X_{t,s}$ is common knowledge.

On the other hand, the $\omega-$quality behind a capital unit is only known to its owner. This means that financial firms can observe only the volume of capital units purchased $k \int \mathbb{I}(\omega) d\omega$ but ignore, $k \int \lambda(\omega) \mathbb{I}(\omega) f_{\phi_t}(\omega) d\omega$, the efficiency units that remain from this purchase. In contrast, i-entrepreneurs also face uncertainty about the realization of ϕ_t but know exactly which ω units are

[13]Note that Assumption 1 is neither a general nor a particular case of first or second order stochastic dominance. The standard definitions of stochastic dominance are related to properties of the distribution of qualities, f_ϕ. Instead, here, the condition refers to the properties of functions that are the conditional expectation under a threshold quality (where the threshold is the argument of the function). The condition in Assumption 1 implies that *conditional expecations* (and not distributions) satisfy first order dominance.

being sold. Thus, conditional on ϕ_t, the owner knows $\int \lambda(\omega)\mathbb{I}(\omega) f_{\phi_t}(\omega)d\omega$. For convenience, I assume that the entrepreneur's type is also observable. This assumption, is enough to ensure that, in equilibrium, p-entrepreneurs are excluded from selling capital which is convenient for tractability. Intermediaries are informed about their survival, at the beginning of the second stage.

Markets. Markets are incomplete: there exists no insurance against type-risk and i-entrepreneurs cannot sell claims against new investment projects. Instead, the only possible transaction are purchases and sales of capital from or to financial intermediaries. This follows from the assumption that financial firms are larger than entrepreneurs. In this context, being means that intermediaries can engage in many more transactions and therefore have the advantage of exploiting the law of large numbers to dilute the idiosyncratic risk faced when participating in a small number of transactions (under asymmetric information). Thus, implicitly, I assume that banks fully diversify financial contracts (or mutually insure against idiosyncratic risk). Consequently, profits/losses from financial intermediation are perfectly correlated across intermediaries.

There are two markets for capital. The first is the market for capital units sold by i-entrepreneurs and bought by banks and where units are sold under asymmetric information. This market opens during the first stage and satisfies the following assumption:

Assumption 2 *Financial firms are* ***competitive*** *and capital markets are* ***anonymous*** *and* ***non-exclusive****.*

Anonymity is key to guarantee that the market in the first stage is a pooling one. Without anonymity, financial firms pay a different price depending on the capital traded by the entrepreneur. With exclusivity, intermediaries could use dynamic incentives as screening devices. Assumption 2, therefore, implies that because quantity cannot be used as a screening device. Hence, there will be a unique pooling price p_t in the first stage market for capital.

The second market is the one in which financial intermediaries sell back the purchased units while revealing the actual efficiency units behind capital. In essence, this is a market for t+1 capital units traded in period t. This market opens during the second stage and clears at a price q_t.

Timing. (1) At the beginning of the period, $X_{t,1}$ is realized and i-entrepreneurs choose $\mathbb{I}(\omega)$. Intermediaries choose an amount of capital purchases, equity injections and dividend pay-outs. (2) During the second stage ϕ_t is realized and $X_{t,2}$ is updated. Intermediaries learn the average quality behind the pool purchased capital and resell as a homogeneous units of $t+1$ capital. Entrepreneurs choose over consumption and purchases of capital and, simultaneously, i-entrepreneurs also decide their investment. By the end of the period, intermediaries redeem all issued IOUs and realize profits from intermediation.

The timing of the model is summarized by Figure 1. The following sections describes the problem faced by agents in this economy and the corresponding market clearing conditions that define equilibria. This economy has a recursive representation so from now on, I drop the time subscripts. I use x' to denote the value of a variable x in the subsequent stage.

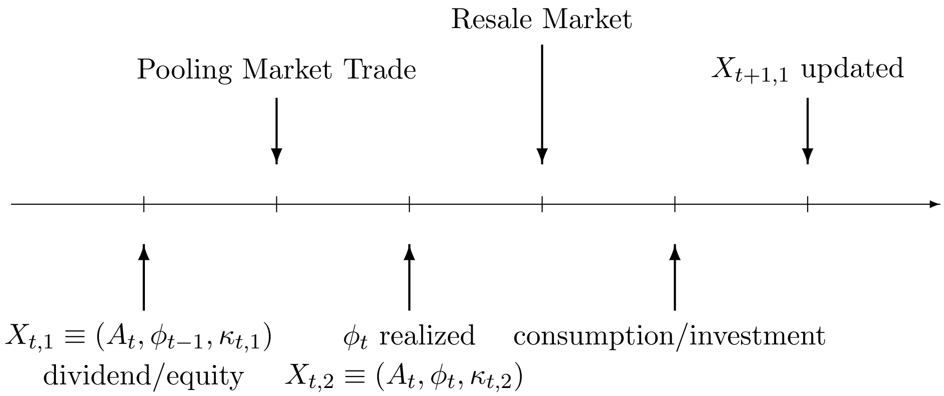

Figure 1: Timing

2.2 First Stage Problems

Investor's first stage. During the first stage, entrepreneur's enter the period with a capital stock k. The i-entrepreneur's choose which capital units to sell in exchange for consumption goods:

Problem 1 (i-entrepreneur's s=1 problem) *The i-entrepreneur's problem at the first stage is:*

$$V_1^i(k, X) = \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[V_2^i\left(k'\left(\phi'\right), x, X'\right)|X\right]$$

$$s.t.\ x = pk \int_0^1 \mathbb{I}(\omega)\, d\omega \ \ and\ k'\left(\phi'\right) = k\int \lambda(\omega)\left[1 - \mathbb{I}(\omega)\right] f_{\phi'}(\omega)\, d\omega$$

The first equation is the entrepreneurs budget constraint were x represents the quantity of consumption goods available during the second stage. x is obtained by selling capital in the pooling market. The second equation accounts for the capital kept by the entrepreneur which depends on the qualities sold and ϕ. This term corresponds is the last term in equation (1). The solution to this problem determines a supply schedule for capital units in the pooling market.

Since producers are excluded from capital markets during the first stage, their value function is the expected value of their second stage value function:

Problem 2 (p-entrepreneur's s=1 problem) *The p-entrepreneur's problem at the first stage is:*

$$V_1^p(k, X) = \mathbb{E}\left[V_2^p\left(k'\left(\phi'\right), x, X'\right)|X\right]$$

$$s.t.\ x = Ak \ \ and\ k'\left(\phi'\right) = k\int \lambda(\omega)\, f_{\phi'}(\omega)\, d\omega$$

Intermediaries first stage. A financial intermediary enters the period with n consumption goods stored in his bank and $\bar{e}$ as personal endowment. Intermediaries will choose an amount of his endowment e as equity injections to his bank. He can do the opposite transferring d consumption units as dividend payoffs to consume in the current period after paying a linear dividend tax τ.[14] Equity injections and dividends are limited by their sources: $e \in ([0, \bar{e}])$ and $d \in [0, n]$. The intermediaries consumption during the period is $c = (\bar{e} - e) + (1 - \tau) d$. and bank's net-worth in the next stage is $n' = n + e - d$. The presence of a dividend tax is important because it affect the dynamics of the net-worth of financial institutions.

Let Q be the quantity of capital purchased by the intermediary. He purchases this amount by issuing pQ in marketable IOUs (inside liquidity). During the second stage, the value his capital purchases is $q \cdot \mathbb{E}_{\phi'} [\lambda(\omega) | X]$. $\mathbb{E}_{\phi'} [\lambda(\omega) | X]$ is the expected depreciation rate under the distribution $f_{\phi'}$ (consistent with the sales decisions $\mathbb{I}(\omega)$). The LLC states that the amount of issued IOUs cannot exceed the bank's net-worth plus the value of its capital:

$$pQ \leq q\mathbb{E}_{\phi'} [\lambda(\omega) | X] Q + n' \text{ for any } \left(X', X\right) \in \mathbb{X} \times \mathbb{X}$$

Let $\Pi(X, X') \equiv q\mathbb{E}_{\phi'} [\lambda(\omega) | X] - p$ be the intermediaries profits per unit of capital purchased. $\Pi(X, X')$ is a function of X and X' since, as we shall see, q is a function of both X and X'. The intermediaries problem is summarized by,

Problem 3 *The financial intermediaries problem at the first stage is:*

$$V_1^f f(n, X) = \max_{Q, e \in [0, \bar{e}], d \in [0, n]} c + \mathbb{E}\left[V_2^f (n' + \Pi\left(X, X'\right) Q, X') | X\right]$$

$$\begin{aligned} s.t.\ -\Pi\left(X, X'\right) Q &\leq n',\ \forall X' \\ c &= (\bar{e} - e) + (1 - \tau) d \\ n' &= n + e - d \end{aligned} \tag{2}$$

The first constraint of this problem is the LLC, the second is the intermediaries budget constraint and the last is the evolution of his bank's balance net-worth.

2.3 Second Stage Problems

Investor's second stage. During the first stage, i-entrepreneurs sell part of their capital stock in exchange for an amount x of consumption goods brought into the following stage. Their individual state vector are consumption goods available this period x and k units of the following periods capital. They solve,

Problem 4 (i-entrepreneur's s=2 problem) *The i-entrepreneur's problem at the second stage is:*

$$V_2^i (k, x, X) = \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta \mathbb{E}\left[V_1^j \left(k', X'\right) | X\right], j \in \{i, p\}$$

[14] A distinction between cost of equity injections and dividend taxes is common in the dynamic corporate finance literature. See for example Hennessy and Whited [2005] or Palazzo [2010] among others. In this environment, only their ratio of the cost of equity and dividend taxes matters, so I normalize the tax rate to account for this differences.

$$c + i + qk^b = x \text{ and } k' = k^b + i + k$$

The budget constraint says that the i-entrepreneur uses his available consumption goods x, to consume c, invest i, or purchase k^b capital at price q. The capital accumulation equation is consistent with (1) since k already incorporates sales and the effective depreciation from the previous stage.

Producers second stage. The p-entrepreneur's problem is identical to the i-entrepreneur's except that he is restricted to set $i \leq 0$:

Problem 5 (p-entrepreneurs s=2 problem) *The p-entrepreneur's problem at the second stage is:*

$$V_2^p(k, x, X) = \max_{c \geq 0, i \leq 0, k^b \geq 0} \log(c) + \beta \mathbb{E}\left[V_1^j(k', X')\,|X\right], j \in \{i, p\}$$

$$c + i + qk^b = x \text{ and } k' = k^b + i + k$$

Intermediaries second stage. The intermediaries only action during the second stage consists on reselling all the capital units purchased during the first stage while revealing their actual depreciation. Thus, their value function in this stage is $V_2^f(n, X) = \beta^F \mathbb{E}\left[V_1^f(R^b n, X')|X\right]$ if they remain alive or $V_2^f = (1-\tau)\beta^F R^b n$ if they will die during the next period.

2.4 Market Clearing Conditions and Equilibrium

Notation. I append terms like ${}^j(k, X)$ to variables that indicate the policy function of an entrepreneur of type j in state (k, X). I will also use $\mathbb{I}(\omega, k, X)$ to refer to an i-entrepreneur's decision to sell an $\omega-$quality when his state is (k, X) .

In every period and stage, there is a measure over capital holdings across the population of i and p-entrepreneurs. I denote these measures by $\Gamma(k, i)$ and $\Gamma(k, p)$ respectively. By independence, the measures satisfy:

$$\int \Gamma(dk, i) = \pi K \text{ and } \int \Gamma(dk, p) = (1-\pi) K. \quad (3)$$

Their evolution is consistent with individual decisions and type process. In addition, there is also a distribution $\Lambda(n)$ of net-worth across financial intermediaries. These measures are functions of the state but are not state variables.

First stage. In the first stage, market clearing requires that the demand for capital by intermediaries to equal the supply of capital by i-entrepreneurs. This condition is given by:

$$\int_0^\infty Q(n, X)\Lambda(dn) = \int_0^\infty k \int_0^1 \mathbb{I}(\omega, k, X)\, d\omega \Gamma(dk, i)$$

Second stage. In the second stage, market clearing requires that the supply for capital by intermediaries be equal to the demand of purchases of efficiency units by all entrepreneurs. The demand for efficiency units by p and i-entrepreneurs are respectively,

$$D^p(X', X) \equiv \int_0^\infty k^b \left(x(k, X), k\int_0^1 \lambda(\omega) f_{\phi'}(\omega)\, d\omega, X'\right)\Gamma(dk, p)$$

and

$$D^i\left(X',X\right)\equiv\int_0^\infty k^b\left(x\left(k,X\right),k\int_0^1\lambda\left(\omega\right)\left[1-\mathbb{I}\left(\omega,k,X\right)\right]f_{\phi'}\left(\omega\right)d\omega,X'\right)\Gamma\left(dk,i\right)$$

and supply is given by:

$$S\left(X',X\right)\equiv\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|X\right]\int_0^\infty Q\left(n,X\right)\Lambda\left(dn\right)$$

Market clearing requires $S\left(X',X\right)=D^p\left(X',X\right)+D^i\left(X',X\right)$. Equilibria is defined in the following way:

Definition 1 (Recursive Competitive Equilibrium) *A recursive competitive equilibrium (RCE) is (1) a set of price functions, $\{q\left(X',X\right),p\left(X\right)\}$, (2) a set of policy functions for p-entrepreneurs $c^p\left(x,k,X\right)$, $k^{b,p}\left(x,k,X\right)$, $i^p\left(x,k,X\right)$, a set of policy functions for i-entrepreneurs $c^i\left(x,k,X\right)$, $k^{b,i}\left(x,k,X\right)$, $\mathbb{I}^i\left(\omega,k,X\right)$ and a set of policy functions for financial intermediaries $Q\left(n,X\right)$, $e\left(n,X\right)$, $d\left(n,X\right)$, (3) sets of value functions, $\left\{V_1^l\left(k,X\right)\right\}_{l=p,i}$, $\left\{V_2^l\left(x,k,X\right)\right\}_{l=p,i}$ and $\left\{V_s^f\left(n,X\right)\right\}_{s=1,2}$,(4) a law of motion for the aggregate state X, such that for any distributions $\Gamma\left(k,p\right)$, $\Gamma\left(k,i\right)$ and $\Lambda\left(n'\right)$ satisfying the consistency condition (3), the following hold: (I) The entrepreneurs' policy functions are solutions to their problems taking $q\left(X',X\right)$, $p\left(X\right)$ and the law of motion for X as given. (II) $Q\left(n,X\right)$, $e\left(n.X\right)$, $d\left(n.X\right)$ are the solutions to the financial intermediaries problem taking as given $q\left(X',X\right)$, $p\left(X\right)$ and the law of motion for X as given. (III) Capital markets clear at the first stage. (IV) Capital markets clear at the second stage. (V) The law of motion X is consistent with policy functions and the transition function χ. All expectations are consistent with the law of motion and agent's policies.*

The definition of equilibria does not depend on the measure over asset holdings because this economy admits aggregation. There is an important detail. Nothing in the definition of equilibrium precludes multiplicity of prices. In particular, we may find two price function $p\left(X\right)$ consistent with the definition of equilibria. This is a common feature in models with asymmetric information like Stiglitz and Weiss [1981], for example. As prices increase, both the average quality of capital sold and the quantity increase so expected profits for financial firms may potentially be non-monotonic which may yield multiple equilibria. Although this multiplicity is interesting in itself, it is not the focus of this paper. Thus, for the rest of the paper, I introduce an equilibrium refinement:

Definition 2 (Pareto un-improvable Equilibrium) *A RCE is Pareto un-improvable if given the law of motion for X, $\forall X$, there does not exist any $p^o>p\left(X\right)$, such that p^o satisfies market clearing in the first stage, and induces a second stage market clearing price $\tilde{q}\left(p,X'\right)$ that is consistent with the agents policy functions and the LLC.*

This refinement selects the RCE where the volume of intermediation is the largest possible. A later section shows that beyond its name, such equilibria, indeed, cannot be Pareto improved upon. We need to show some intermediate results first.

Before proceeding to the characterization, I provide a description of alternative interpretations of the LLC constraint and financial intermediation.

3 Interpretation

3.1 Accounting and Financial Constraints

Bank accounting. During the first stage, intermediaries enter period with net-worth n (in consumption goods) kept in their banks. At this state, they may alter banks balance sheet by injecting equity or paying out dividends. In addition, the bank accumulates assets by pQ holding claims to risky capital units and buys these issuing pQ in IOU's. Their banks' balance sheet evolves like this in the first stage:

Assets	**Liability**
n	
	Net-worth
	n

Beginning of period Balance Sheet

$\Rightarrow$

Assets	**Liability**
$n+e-d$	pQ
pQ	**Net-worth**
	$n+e-d$

End of stage 1 Balance Sheet

$\Leftrightarrow$

Assets	**Liability**
n'	pQ
pQ	**Net-worth**
	n'

End of stage 1 Balance Sheet

After ϕ is realized, the value of the assets in the balance sheet of financial firms adjusts as the both the quality and the price of this units is adjusted. Thus, the capital on the asset side may differ from the original owed amount of IOUs. Accounting for the change in value leads to the following balance sheet:

Assets	**Liability**
n'	pQ
$q\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\|X\right]Q$	**Net-worth**
	$n'+\Pi Q$

End of stage 2 Balance Sheet

Thus, net-worth is affected by losses or gains from financial intermediation $\Pi(X,X')$.

LLC as Value-at-Risk (VaR). The LLC resembles a Value-at-Risk constraint. The LLC constraint states that $n+e-d+\Pi(X',X)Q \geq 0$ for any possible realization of ϕ'. Given X, the worst outcome is realized with some probability η, that depends on the evolution of the exogenous states. Thus, the LLC states that the value of net-worth at its $\eta-$percentile must be greater than 0, or, that the η Value-at-Risk of n' must be greater than 0. Thus, $VaR\left(n',\eta\right) \geq 0$ where $VaR(b,\eta)$ is the value of a random variable b at its $\eta-$percentile. **Note 1 on Regulation: Basel-III imposes some $VaR(b,\eta)$ constraints**.

LLC as a leverage constraint. The LLC can also be expressed as a constraint on leverage. Arranging the LLC constraint leads to,

$$\frac{\overbrace{Q}^{\text{Traded Capital}}}{\underbrace{n'}_{\text{Net-worth}}} \leq \underbrace{\frac{1}{-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right]}}_{\text{Marginal Leverage Constraint}},\ \forall X'.$$

The term on the right is the marginal leverage because if when this constraint binds, a additional unit of net-worth allows the entrepreneur to increase Q in that amount.[15] Multiplying both sides by $p(X)$ and adding 1 leads to an equivalent constraint expressed in terms of the bank's actual leverage:

$$\underbrace{\underbrace{\frac{\overbrace{p(X)Q+n'}^{\text{Assets}}}{\underbrace{n'}_{\text{Net-worth}}}}}_{\textit{Leverage}} \leq \underbrace{\frac{p(X)}{-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right]}+1}_{\text{Total Leverage Constraint}},\ \forall X'.$$

The left-had side of this constraint is the bank's leverage. **Note 2 on Regulation: Basel-III also imposes leverage constraints**.

3.2 Banking Interpretation

So far the model can be interpreted as investment banking because banks here perform activities as brokers or underwriters, taking positions in their trades. The model can be perturbed to interpret is a model of commercial banking.

Two minor alterations are made for this purpose. The first is that the way in which ϕ enters in this economy is different. f_ϕ is no longer the same for all entrepreneurs. Instead, entrepreneurs draw random draws over a particular f_ϕ. The aggregate shock simply affects the probability of these draws. Thus, before everyone had either a good f_ϕ or a bad f_ϕ. In this world, entrepreneurs will draw either one or the other, but the probabilities change. The second difference is on the type of contracts that are available. Financial firms do the following. They give the entrepreneur and IOU in exchange for part of his capital stock. There's an implicit interst rate in this contract. If the entrepreneur pays back for the loan, his capital is returned. If the entrepreneur defaults, then his pledged assets are seized, and the entrepreneur looses them.

Capital. Capital has the same properties in this environment, where it is again divided into pieces $\omega \in [0,1]$. As before, there is a set of measures over qualities that depend on the realization of ϕ. And in particular, these measures are defined by an absolutely continuous function $f_\phi : [0,1] \to \mathbb{R}_+$. which, in turn, is a function of ϕ_t. These measures are fixed and have the same property. Between periods, each capital piece is transformed into future efficiency units by scaling pieces by $\lambda\left(\omega\right) f_\phi\left(\omega\right)$. Thus, $\lambda\left(\omega\right) f_\phi\left(\omega\right) k\left(\omega\right)$ efficiency units remain from an $\omega - quality$ next period.

The key here is that the the entrepreneur will have a random draw that determines some $\phi \in \{\phi_1, \phi_2, ..., \phi_N\}$ for his firm. The novelty here is that the entrepreneur draws his asset quality distribution depending on some $\delta_t \in \Delta^N$.

4 Characterization

4.1 Policy Functions

Entrepreneur's second stage policies. I begin the characterization of equilibria describing the value function of the p-entrepreneur. As a result of log-preferences specification, policy functions are linear

[15] Note that when $-\left[q\left(X',X\right)\lambda\left(\phi\right)-p(X)\right] < 0$, the constraint is trivially satisfied.

functions of the entrepreneur's virtual wealth, $W^p(k,x,X) \equiv \left(A + q\bar{\lambda}(X)\right)k$. Formally, we have,

Proposition 1 *In any RCE, the p-entrepreneur's policy functions are* $k^{p,\prime}(k,x,X) = \beta\frac{W^p(k,x,X)}{q}$ *and* $c^p(k,x,X) = (1-\beta)W^p(k,x,X)$ *and the value function is of the form* $V_2^p(k,x,X) = \psi^p(X) + \log(W^p(k,x,X))$ *where* $\psi^p(X)$ *is a function of the aggregate state.*

Policy functions for i-entrepreneur's are also linear functions of their corresponding virtual wealth, $W^i(k,x,X) \equiv (x + q^c\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*(X)])k$. When $q > 1$, i-entrepreneurs find it cheaper to augment their capital stock producing investment units. Otherwise, they may find it convenient to purchase capital sold by banks rather than investing directly. For this reason, q^c is the effective cost of generating capital units for an i-entrepreneur, and is given by $q^c = \min\{1,q\}$. The following Proposition describes the policy functions for i-entrepreneur's.

Proposition 2 *In any RCE, the i-entrepreneur's policy functions are* $k^{i,\prime}(k,x,X) = \beta\frac{W^i(k,x,X)}{q^c}$ *and* $c^i(k,x,X) = (1-\beta)W^i(k,x,X)$ *and the value function is of the form* $V_2^i(k,x,X) = \psi^i(X) + \log\left(W^i(k,x,X)\right)$ *where* $\psi^i(X)$ *is a function of the aggregate state.*

This proposition implies that we can describe the supply of capital by i-entrepreneurs by solving a simple portfolio problem.

Entrepreneurs first stage policies. From Proposition 2, we have that the i-entrepreneur's problem is given by,

$$\begin{aligned} V_1^i(k,X) &= \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\log\left[p(X)\int_0^1 \mathbb{I}(\omega)\,d\omega \ + q^c\left(X,X'\right)\int \lambda(\omega)\left[1-\mathbb{I}(\omega)\right]f_{\phi'}(\omega)\,d\omega\right]|X\right] \\ &\quad +\psi^i(X) + \log(k). \end{aligned}$$

In this expression I replaced the definition of x and $\mathbb{E}_\phi[\lambda(\omega)|\omega < \omega^*(X)])$. The solution to this problem is characterized by the following proposition,

Proposition 3 *In any RCE, the i-entrepreneur's policy function in the first stage is given by,* $\mathbb{I}^*(\omega,k,X) = 1$ *if* $\omega < \omega^*$ *and 0 otherwise. The cut-off rule solves,*

$$\omega^* = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left[p\tilde{\omega} \ + q^c\left(X,X'\right)\int_{\tilde{\omega}}^1 \lambda(\omega)\,f_{\phi'}(\omega)\,d\omega\right]|X\right] \tag{4}$$

ω^* *is increasing in* p.

This proposition states that the solution to the entrepreneur's problem during the first stage is characterized by a unique cut-off rule for which all units of inferior quality are sold. ω^* is chosen by solving a portfolio problem. This portfolio problem has an intuitive interpretation: ω^* is the fraction of the entrepreneurs capital stock that is transferred to financial firms by the end of this stage in exchange for IOUs. This fraction of entrepreneur's capital is valued at private information price $p(X)$. ω^* can also be interpreted as the fraction of risk-less wealth. The fraction of capital that remains with the entrepreneur is $(1-\omega^*)$. This fraction has an average quality of $\mathbb{E}_{\phi'}[\lambda(\omega)|\omega > \omega^*]$ corresponding to the right tail of the distribution of qualities from the $\omega^* - percentile$. The efficiency units kept by entrepreneurs are valued at the opportunity cost of generating a unit a unit of capital, $q^c(X, X')$. Because, entrepreneurs agree on selling capital before they know the actual distribution of capital quality, this portion of their wealth is risky. This portfolio problem is standard except that the portfolio weights affect the expected return of the risky portion of wealth. Nonetheless, ω^* is increasing in $p(X)$ so supply is well behaved.

Intermediaries first stage policies. At the beginning of every period, intermediaries choose e, d and Q to maximize expected profits. The following Lemma shows that the value function and policies of the intermediary are homogeneous of degree 1 in n:

Lemma 1 *The value function is a linear function of n: $V_1^f(n, X) = v_1^f(X)\, n$ where $v_1^f(X)$ is the marginal value of financial equity and depends on the aggregate state. Also, $V_2^f(n, X) = v_2^f(X)\, n$ where $v_2^f(X) = \beta^F \mathbb{E}\left[v_1^f(X) R^b\right]$ if the intermediary remains alive and $v_2^f(X) = \beta^F R^b n$ if he dies. $v_1^f(X)$ satisfies the following Bellman equation:*

$$v_1^f(X) = \max_{Q \geq 0, e \in [0, \bar{e}], d \in [0,1]} (1-\tau)\, d - e + \mathbb{E}\left[v_2^f(X')\left(\Pi\left(X, X'\right) Q + n'\right) |X\right] \tag{5}$$

subject to,

$$\begin{aligned} -\Pi\left(X, X'\right) Q &\leq n', \forall X' \\ n' &= 1 + e - d \end{aligned}$$

The Bellman equation (5) has linear optimal policiesm $e^*(X), d^*(X)$ and $Q^*(X)$ given by:

$$e(n, X) = e^*(X)\, n,\ \ d(n, X) = d^*(X)\, n,\ \ Q(n, X) = Q^*(X)\left(1 + e^*(X) - d^*(X)\right) n$$

The value function of the intermediary is linear in his net-worth because he is risk-neutral. In equilibrium, there may be multiple solutions to $d(n, X)$ and $e(n, X)$ for some states but without loss of gerenality, I restrict the attention to linear policies. The following proposition describes the intermediaries policies:

Proposition 4 *Taking n' as given, $Q^*(X)$ solves,*

$$Q = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f(X')\, \Pi(X, X')\, |X\right] \tilde{Q} \text{ subject to } \Pi\left(X, X'\right) \tilde{Q} \leq n', \forall X'.$$

In equilibrium, $\min_{\tilde{X}'} \Pi\left(X, \tilde{X}'\right) < 0$, and $\left(e^*(X), d^*(X)\right)$ satisfy:

$$\begin{aligned} e &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0\right\}\right] \geq 1 \\ d &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0\right\}\right] \leq (1-\tau). \end{aligned}$$

When inequalities are strict, $e = \bar{e}$ and $d = 1$. e and d are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied.

This proposition states that the constraint on the volume of intermediation $Q(X)$ is binding whenever $\mathbb{E}\left[v_2^f(X')\,\Pi(X,X')|X\right] > 0$. $v_2^f(X')\,\Pi(X,X')$ corresponds to the value of earnings for the intermediary which are equal to profits times the value of his bank's equity. The reason why $v_2^f(X')$ fluctuates over time is because the value of equity carries an additional value by relaxing the capacity constraints on the bank. When, $\mathbb{E}\left[v_2^f(X')\,\Pi(X,X')|X\right] = 0$, Q is indeterminate and 0 in the case of negative expected profits.

The proposition also pins down the conditions for capital injections and dividend payoffs. These policies depend on the marginal value of keeping equity in the bank:

$$\tilde{v}(X) \equiv \beta^F \left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)}, 0\right\}\right]$$

$\tilde{v}(X)$ is the expected marginal value of equity inside the bank and functional form has a very intuitive interpretation. β^F shows up in this expression because it is the discount of future utility. The first term inside the bracket, $\mathbb{E}[v_2^f(X')]$, corresponds to the future expected value of Bank equity. The second term represents the shadow value that equity has because it relaxes the LLC constraint of the bank. As explained before, the inverse of the worst case scenario losses is the marginal leverage $\frac{1}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)}$ on the bank. By increasing a unit of equity, the bank can issue this amount in IOUs expanding its scale of operations. This term multiplies $\mathbb{E}[v_2^f(X')\Pi(X,X')]$ which represents the expected profits which are scaled by the marginal value of bank equity. The max simply takes care of the fact that the intermediary does not issue IOUs when $\mathbb{E}[v_2^f(X')\Pi(X,X')]$ is negative.

When $\tilde{v}(X) < (1-\tau)$, the intermediary prefers to pay-out dividends, equity is valued at $\tilde{v}(X)$ but after tax dividends yield higher instant utility. In contrast, the intermediary injects equity to his bank when the value of holding equity in the bank exceeds 1, his cost of equity injections. This result implies that banks have (s,S)-bands for their dividend policies where $[(1-\tau), 1]$ is the inaction region.

One of the key insights of this paper is that $\tilde{v}(X)$ is non-monotonic in κ, the size of the financial sector. Later on, I will show how in absence of asymmetric information, $\tilde{v}(X)$ is in monotone decreasing in κ. but if asymmetric information is sufficiently severe, $\tilde{v}(X)$ this monotonicity is lost. This feature is important because it explains why the financial sector is not recapitalized externally after a negative sequence of shocks. This result, relies also in that $\Pi(X,X')$ is non-monotone and, for this, we need to find out the functional form of $p(X)$ and $q(X,X')$. The details are explained in the following section.

4.2 Prices and Returns

Prices. The linearity of policy functions allows aggregation. Integrating across the entrepreneur's capital stock yields:

$$D^p\left(X,X'\right) \equiv \int k^{p,\prime}(k,x,X)\,\Gamma(dk,p) - \int k\Gamma(dk,p) = \left[\frac{\beta\left(A + q(X,X')\,\bar{\lambda}\right)}{q(X,X')} - \bar{\lambda}\left(X'\right)\right](1-\pi)\,K.$$

The second equality uses that $\int k\Gamma(dk,p)$ is equal to $(1-\pi)\bar{\lambda}(X')K$ (since k for the p-entrepreneur's is the undepreciated portion of his capital stock). The demand for capital units by i-entrepreneurs may be characterized in a similar way. In this case, $x^i(X,k) = p(X)\omega^*(X)k$. Their aggregate demand function is broken up into three regions that depend on the value of q:

$$D^i(X,X') = \left\{ \begin{array}{cl} \beta\frac{p(X)\omega^*(X)}{q(X,X')} & \text{if} \quad q(X,X')<1 \\ {[0,\beta p(X)\omega^*(X)\pi K]} & \text{if} \quad q(X,X')=1 \\ 0 & \text{if} \quad q(X,X')>1 \end{array} \right.$$

The first section is downwards sloping when $q>1$. This region is ex-post inefficient as there is under supply of capital for the saving entrepreneurs. The second region is a flat demand at $q=1$. Equilibrium is ex-post efficient at the minimum quantity where $q=1$ and corresponds to the demand in a frictionless benchmark. Quantities above this level, are repurchased by investing entrepreneurs. The last region corresponds to equilibria where the demand curve is again downwards sloping and where wealth is redistributed from the financial intermediaries to i-entrepreneurs.

Aggregate investment is the difference between the entrepreneur's total demand for capital and the demand for used units:

$$I^i(X,X') = \beta\frac{p(X)\omega^*(X)}{q(X,X')}\pi K - D^i(X,X'),$$

so there's no investment when $q<1$. When $q=1$, $D^i(X,X')$ is obtained as the residual of the difference between the supply of used units minus the purchases by p-types.

The capital supplied by intermediaries during the second stage, $S(X,X')$, is given by the amount of capital sold by i-entrepreneurs, $\omega^*(X)$, (at price $p(X)$) and the average quality given ϕ':

$$S(X,X') = \mathbb{E}_{\phi'}[\lambda(\omega)|\omega<\omega^*(X)]\,\omega^*(X)\pi K.$$

During the first stage, supply for capital units is random because it depends ϕ, which determines the average quality of the tail the financial system bought. The equilibrium price of capital, $q(X,X')$, that clears out capital markets in the second stage: $D^i(X,X')+D^p(X,X')=S(X,X')$. In equilibrium, $S(X,X')$ depends on $p(X)$ indirectly because $p(X)$ affects $\omega^*(X)$. Thus, market clearing conditions in the second stage, depend on X through p. One needs to solve for this price to characterize equilibria.

In equilibrium, market clearing in the first stage implies that:

$$Q(X)\int n'(X)\,d\Lambda(n) = \omega^*(X)\int k\Gamma(k,i) \Longleftrightarrow Q(X)=\omega^*(X)\kappa.$$

Since, $Q(X)$ and $\omega^*(X)$ are function of p and, in equilibrium, the first stage capital market clears at this price. The following proposition shows that p is well defined and monotone in the size of the financial sector:

Proposition 5 *Given a state X and a law of motion for the state, there exists some $p(X)$ satisfying market clearing. In addition, in any Pareto un-improvable equilibrium, $p(X)$ is weakly increasing in* *the financial sector's capacity κ.*

We use this result to provide an expression for $q(X, X')$ as an function of X' and $p(X)$.

Proposition 6 *In equilibrium q is given by,*

$$q(X, X') = \max\left\{\left[\frac{\beta A}{\pi\omega^*(X)\,\mathbb{E}_\phi[\lambda(\omega)\,|\omega < \omega^*(X)] + (1-\pi)(1-\beta)\bar{\lambda}(X')}\right], g(p(X), X')\right\}$$

where

$$g(p(X), X') \equiv \min\left\{1, \frac{\beta(\pi p(X)\omega^*(X) + A(1-\pi))}{(\beta\omega^*(X)\,\mathbb{E}_\phi[\lambda(\omega)\,|\omega < \omega^*(X)]\,\pi + (1-\beta)\bar{\lambda}(X))}\right\}$$

$q(X, X')$ is obtained by equating the supply with the demand for efficiency units. This expression depends on the the previous and current states. It is immediate to show that $q(X, X')$ is increasing in $\omega^*(X)$ although this is not true (in general) about profits.

Returns to financial assets. Given that we have a functional form for q, we also have an expression for $\Pi(X, X')$:

$$\Pi(X, X') = \begin{cases} \frac{\beta(\pi p(X)\omega^*(X)+A(1-\pi))}{\left(\beta\omega^*(X)\pi+(1-\beta)\frac{\bar{\lambda}(X)}{\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*(X)]}\right)} & \text{if} \quad q(X, X') < 1 \\ \mathbb{E}_\phi[\lambda(\omega)\,|\omega < \omega^*(X)] & \text{if} \quad q(X, X') = 1 \\ \left[\frac{\beta A}{\pi\omega^*(X)+(1-\pi)(1-\beta)\frac{\bar{\lambda}(X)}{\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*(X)]}}\right] & \text{if} \quad q(X, X') > 1 \end{cases}$$

In addition, profits are decreasing in ϕ'. In general, is not $\Pi(X, X')$ won't be monotone in κ. The reason is that $Q(X)$ is weakly monotone in κ. In equilibrium, it must be the case that as $Q(X)$ increase, $\omega^*(X)$ is weakly increasing. We know that $p(X)$ is an increasing function of $Q(X)$, but the value of the financial sector's assets is not increasing in the volume of intermediation. The reason for this is that as the volume of intermediation increases, there are two effects. On the one hand, there is a standard quantity effect. As in standard consumer theory, the greater the volume of capital supplied, the the lower its price. On the other hand, if there is a quality effects, as the volume also affects the quality of assets sold. This can be seen by analyzing the expression for profits. Observe that these $\omega^*(X)$ in the denominator, but the average quality $\mathbb{E}_\phi[\lambda(\omega)\,|\omega > \omega^*(X)]$ also increases with $\omega^*(X)$. Clearly, this non-monotonic behavior holds if $\lambda(\omega)$ is sufficiently sensitive to $\omega^*(X)$. The fact that profits may be non-monotone in the financial sector's capacity is the determinant factor that prevents the recapitalization of the financial system.

Equilibrium profits $\Pi(X, X')$ are distributed according to some stationary distribution consistent with the law of motion of the aggregate state. $\Pi(X, X')$ is bounded by some interval $[\Pi_L(X), \Pi_H(X)]$ because $q, \mathbb{E}_{\phi'}[\lambda(\omega)\,|\omega > \bar{\omega}]$, and p are also bounded. We are now ready to study the evolution of the financial sector's financial risk capacity.

4.3 Evolution of Financial Risk Capacity

Equity injections and dividends. Suppose the economy is at state X. This state can be attained if, it satisfies $\tilde{v}(X) \in [(1-\tau), 1]$. When, $\tilde{v}(X) > 1$, equity is injected into the financial system up to the point where κ is adjusted so that the value of equity inside the banks is 1. If this were not the case, intermediaries would have incentives to alter the financial composition of their banks which, in turn, implies that these value for κ cannot be reached in equilibrium. Such states are reflected immediately to a new state where $\tilde{v}(X) = 1$.

Similarly, if $\tilde{v}(X) < (1-\tau)$, dividends are payed-out by banks, up to the point where κ is reduced to a new point where $\tilde{v}(X) = (1-\tau)$. This means that there is an inaction region characterized by those states in which $\tilde{v}(X) \in [1-\tau, 1]$ and states where $\tilde{v}(X) \in \{(1-\tau), 1\}$, become reflecting barriers where states where $\tilde{v}(X) \notin \{(1-\tau), 1\}$, are immediately reflected onto. This barriers impose bounds on the value of κ.

Thus, at the beginning of the first stage, κ evolves according to:

$$\kappa'|\kappa = (1 + e^*(X) - d^*(X))\,\kappa$$

Between the first stage and the second, stage to the second, κ evolves according to:

$$\kappa'|\kappa = R^b \left[1 + \max\left\{\frac{[\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0\right\}\right] \frac{\kappa'}{\gamma(X, X')}$$

and where $\gamma(X, X')$ is the growth rate of the capital stock:

$$\gamma\left(X, X'\right) = \pi \frac{[p(X)\omega^*(X) + q^c(X, X')\,\mathbb{E}_\phi[\lambda(\omega)\,|\omega > \omega^*]]}{q^c(X, X')} + (1-\pi)\,\beta \frac{[A + q(X, X')\,\lambda(X)]}{q(X, X')} - \lambda(X)$$

Marginal value of equity. We are ready to obtain an expressions for $v(X)$ and $\tilde{v}(X)$. We can obtain a recursive expression for $\tilde{v}(X)$, that depends on the transition function from states X to X'' and is independent of equity/dividend policies. The marginal value of bank equity at any state X is:

$$v_1^f(X) = \min\left\{\max\left\{\beta^F R^b \mathbb{E}[\tilde{v}(X)\,|X], (1-\tau)\right\}, 1\right\}. \tag{6}$$

Because we are using the transition function before equity injections or dividends, we know that it satisfies the Feller property. This condition is enough to guarantee the uniqueness of $\tilde{v}(X)$, given a Markov process for profits. The value of equity before dividend or equity injections is $v(X) = \min\{\max\{\tilde{v}(X), 1-\tau\}, 1\}$. In any RCE, the state space maybe devided into several regions of interest, one of which is interpreted as a financial crises regime. The following section describes them.

4.4 States of the Financial Industry

Holding (A, ϕ) fixed, before equity injections or dividends, κ will fall into 1 of 4 possible regimes depending defined by the value of $\tilde{v}(X)$ and $\tilde{v}'_{\kappa}(X)$.

Dividend-Payoff - Reflecting Barrier: For values of κ, such that $\tilde{v}(X) < (1-\tau)$, dividends are payed out so that κ adjusts up to the closest reflecting barrier where $v(X) = (1-\tau)$. For example, for sufficiently large values of κ, is Q large enough so that $\mathbb{E}[\Pi(X', X)]$ is decreasing in κ (because the quantity effect dominates the quality effect). This implies that the value of equity in the financial sector is relatively low and there is a lot of intermediation. With low expected marginal profits, intermediaries prefer to pay-out dividends. Because $\Pi(X', X)$ is a submartingale, κ is expected to enter the dividend pay-off region infinitely often. The dividend-payoff region can also appear for intermediate values of κ, if the quality effect brings down marginal profit from intermediation.

Equity Injection and Reflecting Barrier: When κ falls in states where $v(X) > 1$, expected profits are large enough the value of equity such that equity injections are expected. This region is the equity injection region. When κ falls in this region, equity injections send the state to its reflecting barrier, where $v(X) = (1-\tau)$. This region is the counterpart of the *dividend pay-off* region. As with the dividend-payoff region, there may be multiple intervals for which κ, falls in this region.

Innaction-Competitive Regime: The the third region is characterized states in which two conditions are met. The first condition is that the marginal value of equity is above the marginal benefit of dividends and but under the cost of equity injections: $v(X) \in ((1-\tau), 1)$. In such states, κ is at an innaction region.

The second condition is that the marginal value of equity is decreasing in κ ($v'_{\kappa}(X) \leq 0$) and the sign of this derivative does not change for any $\tilde{\kappa} > \kappa$. This condition implies that the financial industry is behaving competitively since the expected value of equity is decreasing in κ. This happens when expected profits from financial intermediation are decreasing in the volume of intermediation. We shall see in the next section, that economies where assymetric information is mild, every innaction region $v(X) \in ((1-\tau), 1)$ has this property but this property breaks down with asymmetric information.

In this region, expected profits can be 0 if the financial system is large enough to bear losses under worst case scenario outcomes. Although expected profits are 0, $v(X)$ may be above $(1-\tau)$ since small negative shocks may lead to positive expected profits in the near future. Below a certain level, κ is small enough so that 0 expected profits are no longer compatible with the LLC as capacity constraints bind.

Financial Crisis Region: The interesting region are financial crises. Financial crises are characterized by states that fall in neither of the three previous regions. In this region, κ is so low that it triggers adverse-selection effects: net-worth is low enough that the LLC binds, and since financial firms don't have the capacity to provide much intermediation, market clearing implies that the price of capital units is low. Low prices and volumes, imply that the average quality of capital is also bad. This leads to a market where expected mark-ups are very high but where the volume of intermediation is small enough that expected profits are low and, for this reason, the value of equity is low enough that it prevents recapitalization.

In this region, it is the case that for some $\kappa' > \kappa$, the value of a financial firm's net-worth is increasing in its competititors size ($v'_\kappa(X) > 0$). Larger κ, bank's have more capacity to support fianancial risk, and therefore can sustain more intermediation, which reverts adverse-selection effects. Intermediaries thus, face a coordination failure. They would be willing to recapitalize their banks to increase expected profits but they fail to do so.

Because $\mathbb{E}[\Pi(X', X)] > 0$ but close to zero, no capital injections occur. This property implies that κ is expected to exit the financial crises region, but the exits time may be very long. Thus, financial crises, as defined here, are characterized by above average financing premia, low intermediation, low expected profits (but positive) and a long waiting time for the recovery of the bank balance sheet and equity value.

Efficient Intermediation. The economy may also fall into states where intermediation is efficient. To be precise,

Definition 3 *The volume of intermediation is efficient in state X if $\mathbb{E}[\Pi(X', X)|X] = 0$.*

This definition of efficiency is consistent to the volume of intermediation of a financial sector without the LLC constraint, or equivalently, this is the volume of intermediation provided by a planner subject to an expected budget balance constraint where the LLC constraint does not bind. There are some conditions required for the occurrence of efficient states. This definition is important because it may be a benchmark for financial regulation as a central planner may want the economy to be at such states as often as possible.

Proposition 7 *If $(1-\tau) > \beta^F R^b$, then intermediation is never efficient. States with efficient intermediation may occur if $\beta^F R^b$ is sufficiently close to 1.*

We return to this point when we discuss government policies.

4.5 Computation of Equilibria

This section outlines the method used to compute Pareto un-improvable equilibria. The method involves 2 main steps. First, for each exogenous state vector (A, ϕ), one can find, for any possible volume of intermediation, a first stage a dn second stage prices, and expected and worst case financial profits/losses. The second step consists on computing the volumes of intermediation associated with each κ that occur in equilibrium consistent with $\tilde{v}(X)$ and a law of motion for κ.

Notation. Equilibrium objects in the model are functions of X but first we need to find the equilibrium volume of intermediation associated with each X. In order to do so, we can use the equilibrium conditions of the model to find prices for any arbitrary volume of intermediation as a partial equilibrium. I use bold letters to refer to prices and quantities that are indexed by some arbitrary volume, ω.

Step 1 - Intermediation and Profits. Any volume of intermediation $\omega \in [0, 1]$. By Proposition 3 one can find a first stage price $\mathbf{p}(\omega)$ associated to this volume of intermediation. By, Proposition 6, for any pair $(\omega, \mathbf{p})$ and a shock (A, ϕ) we can find the $\mathbf{q}(\omega, A, \phi)$ consistent with market clearing. We then compute financial proifts, $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi)$, associated with each level of intermediation and a shock ϕ. For the largest value of ϕ, we compute $\min_\phi \mathbf{\Pi}(\omega, \mathbf{p}, A, \phi)\,\omega$ to obtain the worst case scenario losses associated with each volume of intermediation. When we are done with these calculations, we have functions of expected profits and worst case losses associated with any possible volume of intermediation. This step is performed once.

Step 2.1 - Equilibrium κ and ω. Using the results obtained in the first step we can compute the equilibrium volume of intermediation expected profits, and worst case losses associated with each κ. To do so, we begin with a guess for a marginal value for bank equity $\tilde{v}(X)$ that to be updated in the following step.

For each κ and shock (A, ϕ), we find the largest volume of intermediation that yields 0 discounted profits and the largest ω such that the worst case losses equal κ :

$$\omega^{o} = \arg\max_{\omega} \mathbb{E}\left[\tilde{v}\left(X'\right)\mathbf{\Pi}\left(\omega, \mathbf{p}, A, \phi\right)|X\right] = 0 \text{ and } \omega^{\kappa} = \max\left(\omega : \kappa = \min_{\phi} \mathbf{\Pi}\left(\omega, \mathbf{p}, A, \phi\right)\omega\right).$$

The equilibrium ω is the largest feasible volume of intermediation,

$$\omega^{*}(X) = \begin{cases} \max(\omega^{\kappa}, \omega^{o}) & \text{if} \quad \min_{\phi} \mathbf{\Pi}\left(\omega, \mathbf{p}, \phi\right)\omega \leq \kappa \\ \omega^{\kappa} & \text{otherwise.} \end{cases}.$$

Given a size for the financial sector κ, $\omega^{*}(X)$ is the largest volume of intermediation consistent with equilibrium decisions by financial intermediaries.

Step 2.2 - Equilibrium $\tilde{\mathbf{v}}(X)$. Given $\omega^{*}(X)$ found in the previous step we find $\Pi(X, X')$. We then use the functional equation 6 to compute $\tilde{\mathbf{v}}(X)$. Steps 2.1 and 2.2 are iterated until convergence. Note that in the particular of $\rho = 1$, $\tilde{\mathbf{v}}(X) = 1$, so this step is carried out only once.

Appendix A, provides the algorithm used to compute Pareto un-improvable equilibria using the steps described above explains some numerical details. In the next section I implement this method and describe the main results obtained from the computation.

5 Examples

This section is provides two examples that illustrate how equity injections are a stabilizing forces behind financial markets and how, when asymmetric information is sufficiently severe, the incentives to recapitalize financial markets are weakened. For simplicity, assume that $\rho = 1$, so that we can obtain closed form solutions for the equity/dividend policies.

5.1 Example 1 - Risky intermediation without asymmetric information.

The first example presents an economy where there is financial risk on intermediation but asymmetric information is not present. Thus, assume that $\lambda(\omega) = \lambda^{*}$ so that all capital units have the same quality, but $\{f_{\phi}\}$ integrates to a smaller number as ϕ is larger. This example corresponds to an aggregate shock to effective depreciation $\bar{\lambda}(X)$. For this example ϕ belongs to either of two values, a good state, ϕ_G, and bad state, ϕ_B, that occur with i.i.d probability. In addition, I assume that R is constant.[16]

[16] The details of the rest of the calibration are detailed under the corresponding figures.

Given these assumptions, it is clear that for any ϕ, profits $\mathbf{\Pi}(\omega, \mathbf{p},\phi)$ are decreasing in ω, because there are no quality effects. This implies that $\tilde{\mathbf{v}}(X)$ is decreasing in ϕ, so there is a unique equilibrium interval for ω and, correspondingly, a unique equilibrium interval for κ.

Analysis. Figure 2 provides a graphical explanation of this result. The upper-left panel depicts 4 curves corresponding to the capital supply of capital, $\mathbf{p}(\omega)$, the value of bank assets in good and bad states, $\mathbf{q}(\omega,\phi_H)\lambda(\phi_H)$ and $\mathbf{q}(\omega,\phi_L)\lambda(\phi_L)$, and the expected value of the banks assets, $\mathbb{E}[\mathbf{q}(\omega,\phi)\lambda(\phi)]$. The difference between the green and black lines is the expected marginal profit from financial intermediation. Multiplying this amount by ω yields the expected bank profits: $[\mathbb{E}[\mathbf{q}(\omega,\phi)\lambda(\phi)] - \mathbf{p}(\omega)]\,\omega$. Expected bank profits are plotted at the bottom left panel. The bottom right panel plots worst case scenario profits. The top right panel plots the expected value of bank equity as a function of ω. The horizontal lines, correspond to the marginal costs of injecting equity and the marginal benefit of dividend pay-offs, the constants 1 and $(1-\tau)$. Following the steps described in the previous section, we know that an equilibrium ω must be such that financial firms no longer have the incentives to alter their balance-sheet composition. Thus, the equilibrium set of ω is characterized by volumes of intermediation for which the value of equity falls between the cost of equity injections and the benefit of dividend payouts, so that it is neither profitable to inject equity or to have dividend payouts. These volumes of intermediation correspond to the shaded area in the graphs.

Following step 2, we can back-out the equilibrium set for κ, by computing the maximal losses corresponding to the equilibrium set of ω. The figure shows that there is a unique equilibrium interval for κ. When shocks, lead κ to fall below this interval, equity injections send κ' back to the lower end of this equilibrium intervale. Dividends work in the opposite direction.

Figure 3 illustrates presents the behavior of financial variables as a functions of κ, at the begging of every period. The top left panel plots, e, d, and κ' as functions of κ. e, d always adjust so that κ' falls within the equilibrium set. The rest of the panels plot equilibrium variables. Figure 4 presents the computations of the real side variables of the model. The lessons learned from this example are summarized in the following proposition:

Proposition 8 *In any economy without asymmetric information, κ' fluctuates within a unique equlibrium interval $[\underline{\kappa},\overline{\kappa}]$. If $\kappa \leq \underline{\kappa}$, then $\kappa' = \underline{\kappa}$ and $\kappa \geq \overline{\kappa}$, then $\kappa' = \overline{\kappa}$. $v(X)$ and $\omega(X)$ are respectively decreasing and increasing in κ.*

Dynamics. This sample economy has very simple dynamic propoerties. The above proposition is useful to describe the dynamics of this economy: in equilibriumm worst case scenario losses are always negative. The converse would imply infinite profits for banks, but expected profits are positive. This implies that κ is a submartingale that shrinks when ϕ_B, is realized. Whenever the banking sector experiences losses such that $\kappa \leq \underline{\kappa}$, equity injections act as a stabilizing factor because the replenish the financial sector's capacity to provide intermediation services. Dividend pay-offs also stabilize the economy by preventing κ from growing two large. Thus, κ' fluctuates randomly in a unique interval and has an upward drift. This interval corresponds to the innaction region described earlier. As a consequence, the equilibrium volume intermediation, output growth, and the investment rate increase with κ. Since κ fluctuates within this interval, economic growth fluctuates around given intervals.

The next example shows how asymmetric information modifies the properties of this economy and allows for the existence of a financial crises regime.

5.2 Example 2 - Risky intermediation with asymmetric information.

By altering the assumptions on $\lambda(\omega)$ and $\{f_\phi\}$ to introduce asymmetri information. Thus, for this section, I assume $\lambda(\omega)$ is a striclty increasing function but $\{f_\phi\}$ integrates to 1 but in a way such that $\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*]$ is strictly decreasing in ϕ for any $\omega^* \notin \{0,1\}$. In particular, I calibrate $\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*]$ by picking lower and upper bounds for $\lambda(\omega)$, λ_L and $\lambda_H=\bar{\lambda}$, and then defining $\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*]=\lambda_L+(\lambda_H-\lambda_L)F_\phi(\omega^*)$, where F_ϕ is some CDF on $[0,1]$. When $\{F_\phi\}$, is some family of CDF's order by a first-order-stochastic dominance crieterion, we obtain $\mathbb{E}_\phi[\lambda(\omega)|\omega<\omega^*]$ with the desired properties. The rest of the calibration is similar to the one in the previous section.[17]

Graphical Explanation. Figure 5 describes the main features of the model in presence of asymmetric information. As with Figure 2, the upper-left panel describes 4 curves that correspond to $\mathbf{p}(\omega)$, $\mathbf{q}(\omega,\phi_H)\lambda(\phi_H)$, $\mathbf{q}(\omega,\phi_L)\lambda(\phi_L)$, and $\mathbb{E}[\mathbf{q}(\omega,\phi)\lambda(\phi)]$. This curves are no longer downward sloping in the volume of purchases because the average quality of capital also improves when ω is made larger. In fact, this curves are monotone for the chosen domain but are not necesarilly monotone everywhere. Total expected profits and worst case losses are again plotted in the bottom left panel and right panels.

The top right panel plots the value of bank equity as a function of financial intermediation ω. The value of equity is no longer decreasing in the volume of intermediation. The reason is that with asymmetric information, the value of equity can increase with more intermediation because the profitability of financial intermediation can also increase with scale of operations. For larger volumes of intermediation, adverse-selection effects revert.

The horizontal lines correspond to marginal cost of injecting capital and dividends benefits. In equilibrium, the set of ω is also found by finding the points where the the value of equity is whithin these thresholds. The key implication of the non-monotonic behavior of marginal profits is the possibility of multiple crossings. In this particular example, there are two intervals that characterize the equilibrium volumes of intermediation which determine the shaded area regions. The figure shows that there are two equilibrium sets: a first set that falls into the description of a financial crises regime. This set is characterized by low volumes of intermediation and a low bank equity (κ is low). Because, the profitability from intermediation is also low, intermediaries do not have the incentives to inject capital to banks.

The second interval is an innaction competitive region. This region is characterized by a greater degree of financial intermediation and more capital than the financial crises region. Figure 6 illustrates of othre behavior of financial variables as a function of κ. Notice that in presence of asymmetric information, e and d adjust κ' only at intermediate values for κ. For very small values of κ, expected profits are so low that it is not convenient for intermediaries to inject capital into their banks. These states are attained when negative shocks hit banks. Figure 7 shows the computation of some of the real side variables and shows how the financial crises regime is characterized by much lower growth and investment rates. Formally,

[17] The details of the rest of the calibration are detailed under the corresponding figures.

Proposition 9 *For sufficiently severe asymmetric information, the return to financial assets is non-monotone in κ and there exists a financial crises regime.*

Dynamics. The dynamics of this economy have the property that negative shock ϕ_B reduce κ driving it to the financial crises regime. As adevere-selection effects kick-in, and the profitability no longer justifies the injection of capital on to the system so the economy may take a while to recover from this regime. The economy recovers as banks, slowly build equity through retained earinings. At some point, the banking system is strong enough to attract equity investments, and the economy rapidly returns to its inaction region. The rest of the paper studies the quantitative properties of a richer version of this model, introduces some extensions and discusses some policy implications.

6 Quantitative Exercise

6.1 Calibration

This section describes the calibration chosen for the quantitative exercise provided in the subsequent sections. β is based on a frictionless benchmark that yields a 2.5% annual risk free interest rate. The discount factor for the financial sector, β^f, is set to 0.96. The return to equity is set to $R^b = 1$.

$\bar{\lambda}$ is set to 0.9756 to obtain an annualized depreciation rate of 10% which is commonly found in the literature. The fraction of investors π is set to 0.1, which is consistent with the findings in Cooper et al. [1999] about plant investment. I assume A_t follows an $AR(1)$ process, with auto-correlation coefficient equal to 0.95. The mean of the process is set to deliver an average 3.5% annual growth rate of the economy. The standard deviation is set to 0.006.

I set τ such that it is consistent with a marginal cost of equity of 10% and tax-rate on dividends 30%. This numbers follows from the estimates of Hennessy and Whited [2005].

Qualities and dispersion shocks. At this stage, the calibration of dispersion shocks simply referential. I calibrate the distribution of asset qualities following $\mathbb{E}_{\phi}\left[\lambda\left(\omega\right)|\omega<\omega^{*}\right]=\lambda_{L}+\left(\bar{\lambda}-\lambda_{L}\right)F_{\phi}\left(\omega^{*}\right),$ where F_ϕ is the CDF of a Beta distributed random variable. $\lambda_L = 0$, so that the lowest qualities are worthless. The parameters of F_ϕ are evenly spaced between $[2, 4]$ and $[5, 10]$ corresponding to the parameters of a Beta. For example, when ϕ takes its lowest value, F_ϕ is a beta with parameters $(2, 5)$ and when ϕ takes its largest value the parameters are $(4, 10)$.

I assume that ϕ is independent of A, and has the same transition probability of a discretized $AR\left(1\right)$ process with autocorrelation of 0.2 and unit variance. The shocks are calibrated as in Bloom [2009] so that there is a twofold difference between the lowest and largest dispersion shocks.

Convex investment adjustment costs.

Convex Equity Adjustment Costs. The model is extended by enriching the features of financial institutions. Two extensions are particularly useful for calibrations. The first involves introducing convexity in the equity/dividend costs and benefits. Thus, in future versions of this paper I allow for the cost of equity to be given by,

$$C\left(e,n\right)=\left(\bar{e}-e\right)+\psi\left(\frac{e}{n}\right)e \text{ with } \psi'\geq 0,\psi''\geq 0 \text{ and } \psi\left(0\right)=0.$$

Dividends benefits are marginally decreasing,

$$B(d,n) = (1-\tau) - \varphi\left(\frac{d}{n}\right)d \text{ with } \varphi' \geq 0, \varphi'' \geq 0 \text{ and } \varphi(0) = 0.$$

This extension serves the purpose of smoothing equity injections and dividend payouts. The implications for the dynamics of the model is that the swings returning from financial crises regions to inaction regions will be slower.

In addition, for calibration purposes, it is convenient to introduce some sort of labor cost to financial intermediaries. This way, one can alter the volatility of the financial sector's profits.

Financial managers.

6.2 Results

6.2.1 Moments

Figure 8 presents a histogram constructed from the stationary distribution of the κ (integrating across the exogenous states). The heights of the bars represent the frequency at each level of κ. The histogram shows that κ is bounded by the largest and smallest reflecting barriers corresponding to the limits of the dividend pay-off and equity injection regions.

A salient feature of the histogram is that it shows that most of the time, the financial system's size fluctuates between 0.1 to 0.8. With some probability though, the financial system is reduced to less than 0.05. A sufficiently bad combination of shocks leads the economy to the financial crises regimes. In particular, the red colored (darker) bars account the frequency of states corresponding to the definition of a financial crisis. Note also that the frequency of financial crisis states is relatively high. This is a consequence of the long occupation times in the financial crisis regions. Financial crisis imply positive but very small expected profits. Without equity injections, the sector slowly builds κ through retained earnings.

The histogram shows intervals for κ that have no occurrence. This interval is an equity injection region that is between the financial crises region (concentrated near 0) and the inaction region. States in this region are transient because equity injections induce a reflection barrier for κ that takes it away from this region. When the financial system is in the inaction region and is hit by small shocks, κ falls into an equity injection region where adverse-selection effects are not strong enough to prevent external recapitalization via equity injections.

Table 1 describes some moments delivered by the model. These moments are computed with respect to the invariant distribution of X. The table also presents some referential historical statistics as a reference to contrast the model with some facts about financial crisis episodes.

The occupation time on financial crises regimes in the model is close to 12% of the whole sample. As reference, Reinhart and Rogoff [2009] calculate that, including the national banking era, banking crises occurred during 13% of the years in their sampel (according to particular definition). The average time the economy takes to leave a financial crisis region in the model is roughly 9 quarters.

The average growth rate of the economy is, by construction, close to the historical growth rate of the U.S. economy. During the financial crises regimes, the growth falls to -2.5%. Because, this is a model with endogenous growth, 100% of the growth rate is explained by capital accumulation (TFP fluctuates around a constant). During a financial crises, TFP is responsible for some portion of growth because TFP is mean reverting so since financial crisis are most likely to occur in periods of low TFP, TFP is expected to grow during these episodes.

The variance decomposition shows that in normal times, most of the volatility of output growth is due to movements in lending conditions. During a financial crisis, financial intermediation is responsible for a smaller share of this volatility. This property is a characteristic of the calibration (and not the model) and has to do with the elasticities of intermediation to ϕ at the financial crisis regime.

Financial crises episodes are also associated with a strong reduction in the volumes of financial intermediation. The volume of intermediation falls from an average ω^* of 0.73 in normal times to 0.58 during crisis episodes. In the model, intermediation is key to provide funds to investing entrepreneurs. Without these funds, they cannot carry out investment projects so capital is lost due to depreciation. Thus, the reduction in growth is close explained by the fall in the volume o f intermediation. Cerra and Saxena [2008] report a reduction in the growth rate of about 8% for a cross-country sample and while output in the U.S. fell about 4.1% during the great financial recession.

The model also delivers predictions about the GDP contribution of the financial sector, which in the model is close to 20% in normal times. This contribution falls to 0.7% during the crisis regime, essentially because the profits of the sector are low. κ falls dramatically during a crisis region to almost a negligible part of the capital stock.

Table 2 reports moments related to the performance of the financial variables. For example, the shows an average discount rate, (the difference between p and the expected value of capital purchases) to be close to 21%. Financing premia increase to 35% during a crisis regime. Corporate bond spreads are typically around 6% in the data reported in Gorton and Metrick [2010]. During financial crises though, spreads rose to almost 30% in REPO markets. There was a similar response on the average haircut during the great recession.

The leverage of financial institutions is 1.1 in normal times, which is excessively low when compared to the average historical leverage of about 9. During the financial crisis regimes, leverage increases to 1.5 because bank equity is depleted. Bank return to equity is therefore lower in normal times than during a crisis regime, 52% and 61% repectively. The return on assets falls from 9.7% in normal times to close to 0.5% during a crisis. Finally, the model predicts that the value of Bank equity is almost whiped out during a financial crisis.

6.2.2 Simulated Path

Figure 9 presents a typical sample path for the economy studied in this paper. The simulations cover 80 periods corresponding to a span of 20 years. The figure contains 9 plots. The episodes of financial crisis are identified by the gray areas behind of each plot. The top-left panel describes the evolution of, κ and κ', the size of the financial system before and after equity injections and dividends. In normal times, κ fluctuates between 60% and 90% as a fraction of the rest of the economy's capital. As one can see, whenever κ reaches some large value, dividends are payed-out (bottom left panel). In crisis episodes, the financial system's size is reduced severely shrinking almost to a negligible fraction. The middle left panel plots expected profits and actual profits. One can see from that figure that expected profits are on average above zero. Actual profits fluctuate around this quantity. During a financial crisis regime, expected profits shrink to almost zero. The large drops in κ are explained by the leverage ratio of FI's.

Financial crises are triggered when the purchase cost of capital exceeds the value of the collateral. This relation is plotted in the middle panel at the top of the figure. The plot to the right shows how the volume of intermediation falls during a crisis episode. During the periods after the surprise, the financial system looses the capacity to intermediate. Acting competitively, financial institutions demand large discount rates. The volume and the quality of collateral are reduced affecting both the margins. Because the margins of intermediation are large, the value of an additional unit of net-worth is above the average but not enough to justify the injection of capital to the financial system (see middle panel). We see that episodes of crises are characterized by inactivity in dividend pay-outs and equity injections.

The growth rate of the economy falls dramatically during crises episodes. This reduction shows seems like a break in the trend of output.

6.2.3 Response to Dispersion Shocks

Figure 10 shows the response of the economy to an increase in the dispersion of asset qualities. The initial effect is depicted in the top-right panel. The solid line corresponds to the change in the purchase price of capital (bank liabilities). The dashed line presents the value of assets. When the shock is realized, the value of assets falls below the value of liabiliteis. This discrepancy induces a reduction in profits. Whereas expected profits (middle panel) remained unchanged, actual profits fall substantially. The reduction in profits causes a reduction in the relative size of the financial sector (top right panel). In the subsequent periods, the price of capital falls below its average and consequently, there is a reduction in financial intermediation (middle left panel). The marginal value of a unit of equity in the financial system increases because the financial system is experiencing an increase in markups but since κ falls, the relative value of the financial industry also drops. On average, dividend payouts fall after the shock because equity becomes more valuable on the margin.

The real effects of the dispersion shock are depicted in the bottom row. The growth rate of the economy shrinks with the reduction in intermediation. The middle panel in the bottom shows the level of output with and without the shock. Because this is a linear growth model, this reduction in growth has a permanent effect. The bottom right panel shows the likelihood of falling into the financial crises regime after the shock hits. This probability increases immediately after the shock hits, and slowly returns to its expected value.

7 Extensions

This section discusses two extensions to the basic model with the purpose of enriching the discussion.

7.1 Spill-overs

So far, capital is ex-ante homogeneous so financial intermediation is a single activity. The model is unrealistic in that this dimension because in reality banks have a number of product lines and are engaged providing intermediation services in different sectors. Intermediaiton strategies in distinct sectors are differentiated according to information about profitability and risks involved. In reality, profitability and risks are assessed according to firm specific features or specific features of the collateral used. In fact, multidimensional features can explain why financial institutions are not willing to take certain forms of capital as collateral or why several product lines of the financial system froze during the aftermath of the sub-prime crisis. In principle, there is no reason for volumes of intermediation to fall in the same magnitude upon a shock that affects the financial system's balance sheet.

A version ot the model with different sectors may be used to analyze how shocks to the value of collateral in one sector, such as housing for example, may end-up spilling over to manufacturing sectors by affecting the balance-sheet of FI's.

In addition, the strategy for financial institutions may also depend on the type of financial instrument employed. A version of the model that also studies different financial products, like investment banking for example, can induce spill-over effects by affecting the financial system's balance-sheet. This feature can be easily incorporated into the model by analyzing a portfolio choice problem for FI's in the interim period. In terms of the model, allowing banks to invest their equity in risky assets, is like making R^b stochastic. Thus, affecting the evolution of bank to the bank's equity.

7.2 Fire-sales

In the model, banks are forced to sell assets once they suffer losses form intermediation. So far, they sell the storage good to raise funds. The price is 1 since the bank's equity is held in the commodity that plays the role of the numeraire. The model can be altered to introduce fire-sales. For example, the bank's equity may be held in units of capital. In this case, if financial contracts are specified in unit of consumption, upon a shock, banks may be forced to sell capital at the same time. When this is the case, banks must sell capital units at a lower price, creating a negative spiral effect as in Brunnermeier and Sannikov (2010).

7.3 Outside and Convertible Equity (CoCos)

7.4 Idiosyncratic Shocks

8 Financial Stability Policies

This section discusses some potential policies aimed at restoring financial stability.

8.1 Externalities

Externality. The behavior of financial firms induces an externality. By increasing the scale of their intermediation activities, intermediaries face a larger risk of depleading their net-worth. They do this rationally considering the risks involved but fail to internalize that by reducing their own financial risk capacity, they induce market responses on prices, volumes of intermediatino and the average quality capital. This happens because in equilibrium, $p(X)$ is taken as given. If banks could coordinate, they would take into account the reduction in capacity has additional effects on the profitability of the sector. Thus, there is a coordination failure whereby there are alternative government policies may improve welfare of every agent in the economy. This externality motivates the need for policy intervention.

8.2 Dividend Taxes

Dividend taxes have two effects in the model. From a static perspective, increasing dividend taxes reduces the benefits of dividend payoffs. Intermediaries will cut-back on dividends, as long as they believe the increase is temporal. If they expect a relative increase in dividend taxes in the future, the value of bank equity will fall. Thus, the second effect of dividend taxes is to affect the value of equity, ex-ante.

These effects can be analyzed by considering the bank's dividend policy. Assume that the dividend tax is increased by Δ, for a given period. Then,

$$d > 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \leq (1 - \tau(1+\Delta)).$$

From this expression, it is clear that once and for all increase in dividend taxes reduce the incentives to payout dividends, which in turn, may be desirable to increase κ. Nevertheless, if tax increases are anticipated by markets, this would show up in the marginal value of equity:

$$\tilde{v}(X) = \beta^F \mathbb{E}\left[\min\left\{\max\left\{\mathbb{E}\left[\tilde{v}\left(X''\right)|X'\right], 1-\tau(1+\Delta)\right\}, 1\right\}|X\right].$$

It is clear that $\tilde{v}(X)$ would decrease upon a permanent increase in taxes. Perceived dividend taxes thus, have the effect of reducing the equilibrium financial risk capacity but it is not clear though, whether this policy is desirable.

8.3 Effects of capital requirements

Capital requirements are equivalent to introducing an additional term into the LLC. Assuming that the capital requirement is such that banks, are not allowed to loose more than $\theta\%$ of equity in a given period, the LLC reads:

$$-\Pi\left(X,X'\right)Q \leq \theta n'$$

Capital requirements have two effects. First, they increase the volume of intermediation in a given period, which may be desirable if banks are lending excessively (although, in general, a reduction in lending will have adverse effects on growth). The second effect is that they also reduce the profitability of the financial system which, in turn, will affect the distribution of κ. This effects show up in the conditions that determine the bank's policies:

$$\begin{aligned} e &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \geq 1 \\ d &> 0 \text{ only if } \beta^F \left[\mathbb{E}[v_2^f(X')] + \theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\} \right] \leq (1-\tau). \end{aligned}$$

The expression shows that θ reduces the marginal benefit of equity: $\theta \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X,X')]}{-\min_{\tilde{X}} \Pi\left(X,\tilde{X}\right)}, 0 \right\}$. Thus, reducing the equilibrium intervals of κ. On the other hand, the volume of financial intermediation:

$$Q = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f\left(X'\right) \Pi\left(X,X'\right) |X \right] \tilde{Q} \text{ subject to } \Pi\left(X,X'\right) Q \leq \theta n', \forall X'.$$

A social planner will face trade-off between reducing probability of a financial crises by introducing capital requirements and at the expense of a reduction in financial intermediation. An optimal government policy potentially involves a state dependent θ.

8.4 Government sponsored equity injections

Equity injections have also two counterbalancing effects. On the one hand, in times of crises, equity injections allow the government to recapitalize the financial system. This policy may prevent the negative consequences of adverse-selection. This policy has two costs. In the model, privately financed equity injections do not occur in equilibrium when expected returns to financial intermediation is low. This follows from a coordination failure by which intermediaries fail to coordinate to inject equity simultaneously all the way to increase κ to the point where the industry becomes profitable again. Government sponsored equity injections serve this purpose.

The drawback of this policy is that if is anticipated by financial institutions it would, ex-ante, induce increases in the volumes of intermediation. The reason is that this government policy would increase the marginal value of equity which would, in turn, attract more capital to financial intermediaries. With more capital on the banks balance-sheets, the volume of intermediation of financial firms would also increase. Thus, these policies would lead the system to grow beyond its *natural* size. This may or may not be efficient.

9 Conclusions

This paper provides a theory about risky financial intermediation under asymmetric information. The main message of the paper is that financial markets where asymmetric information is a relevant friction are likely to be more unstable than markets where it isn't. The source of this instability is caused by the reduction in the profitability of the financial industry as adverse-selection effects are triggered when shocks hit the balance sheets of financial institutions. The model is capable of explaining several features of financial crises financial. In particular, it explains why they lending may take a while to recover.

The paper also outlines an externality: the fact that financial firms do not internalize that their financial strategies may trigger adverse selection effects in the future. A study of optimal regulatory policies and their time consistency is a task left for future research.

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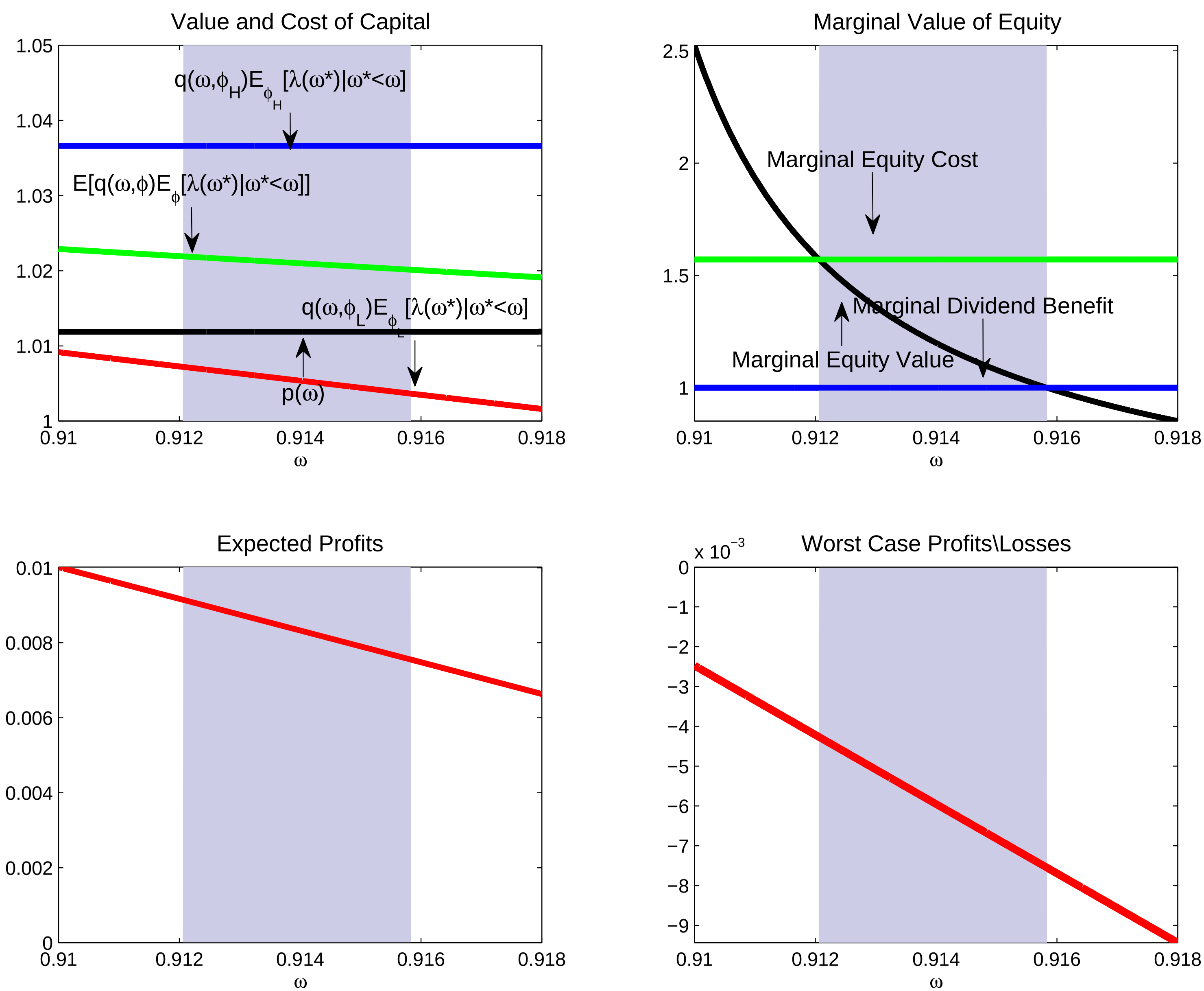

Figure 2: Equilibrium objects in the model without asymmetric information as functions of ω. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

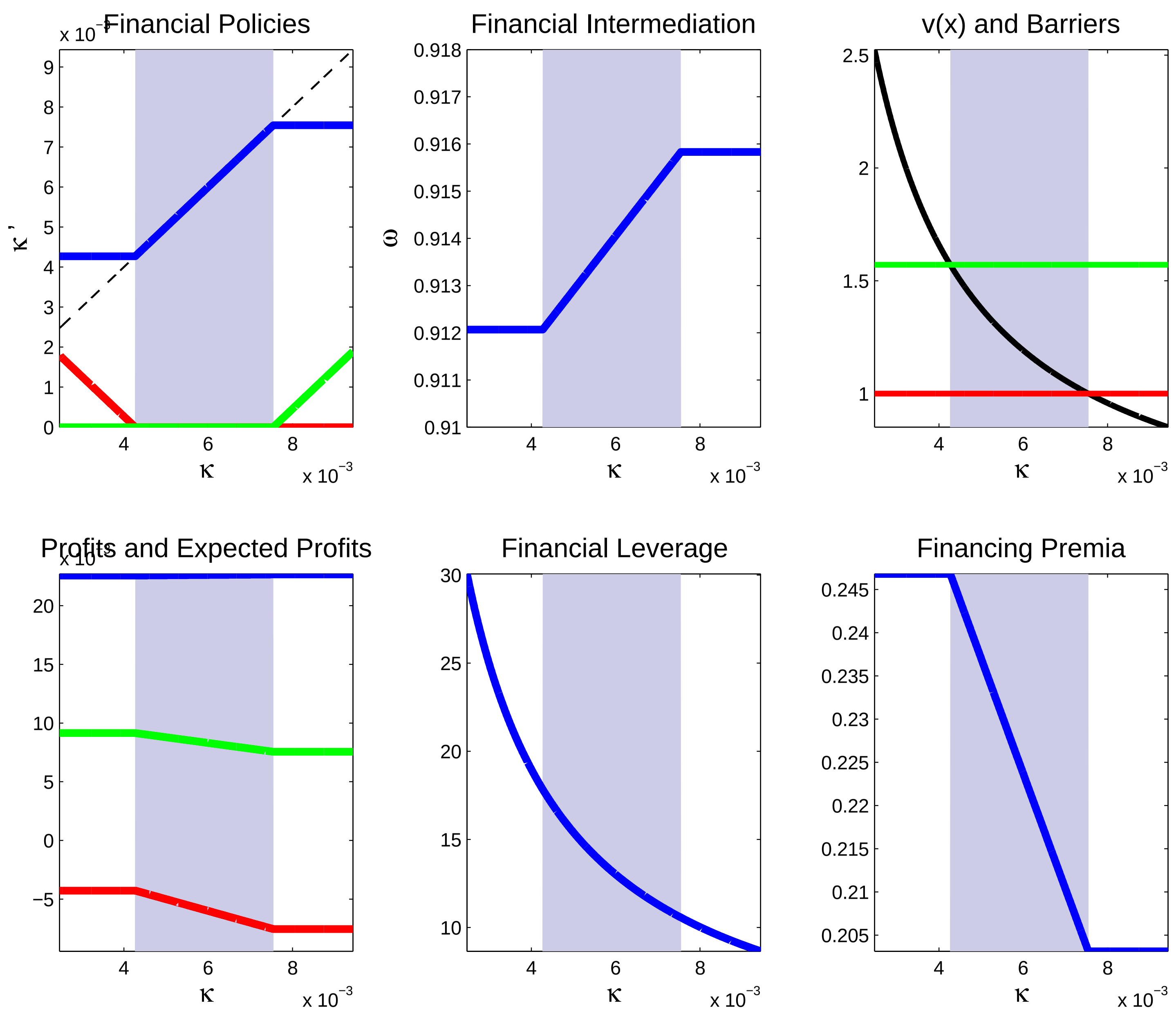

Figure 3: Equilibrium financial variables in the model without asymmetric information as functions of κ. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

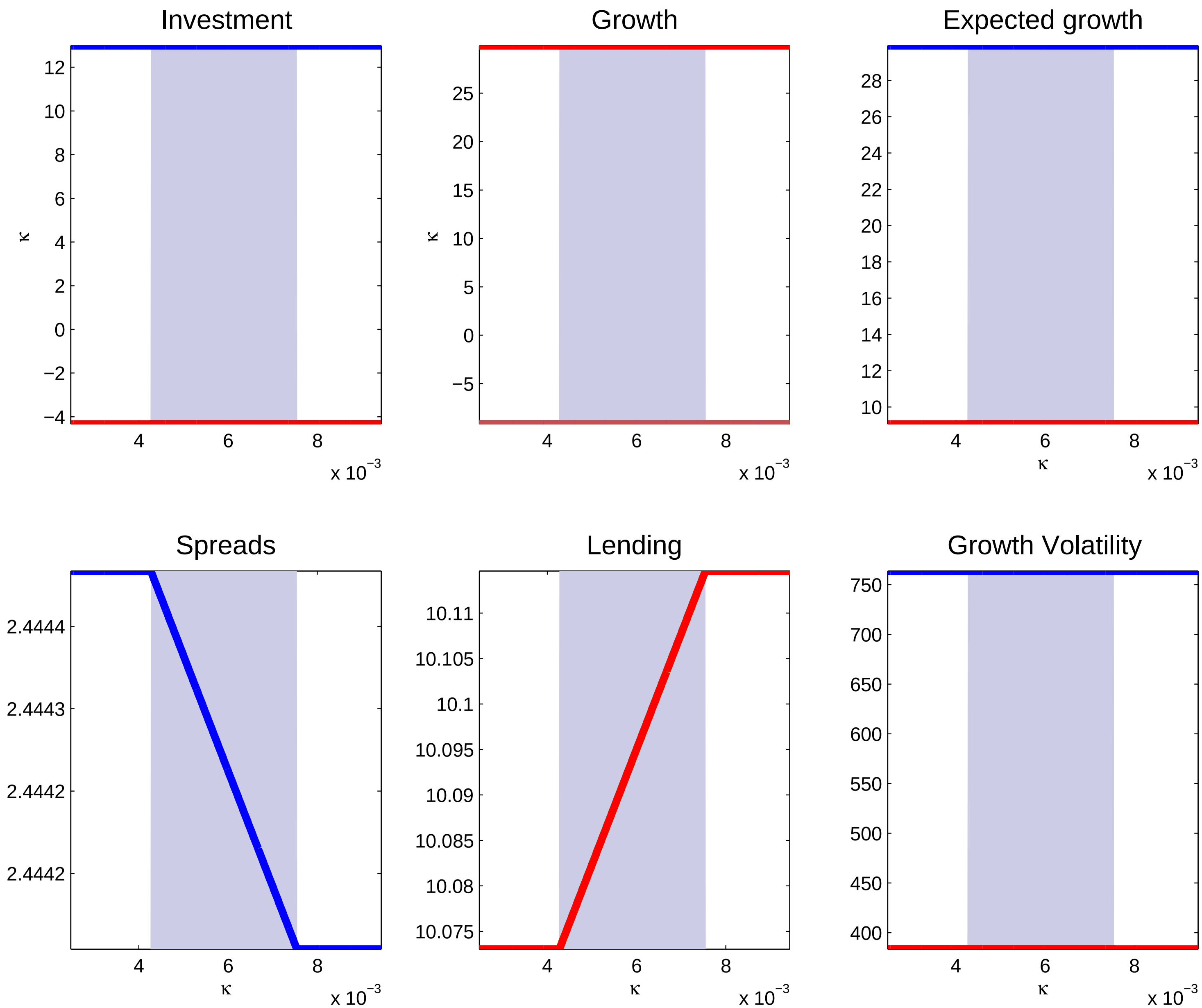

Figure 4: Equilibrium real side in the model without asymmetric information as functions of κ. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

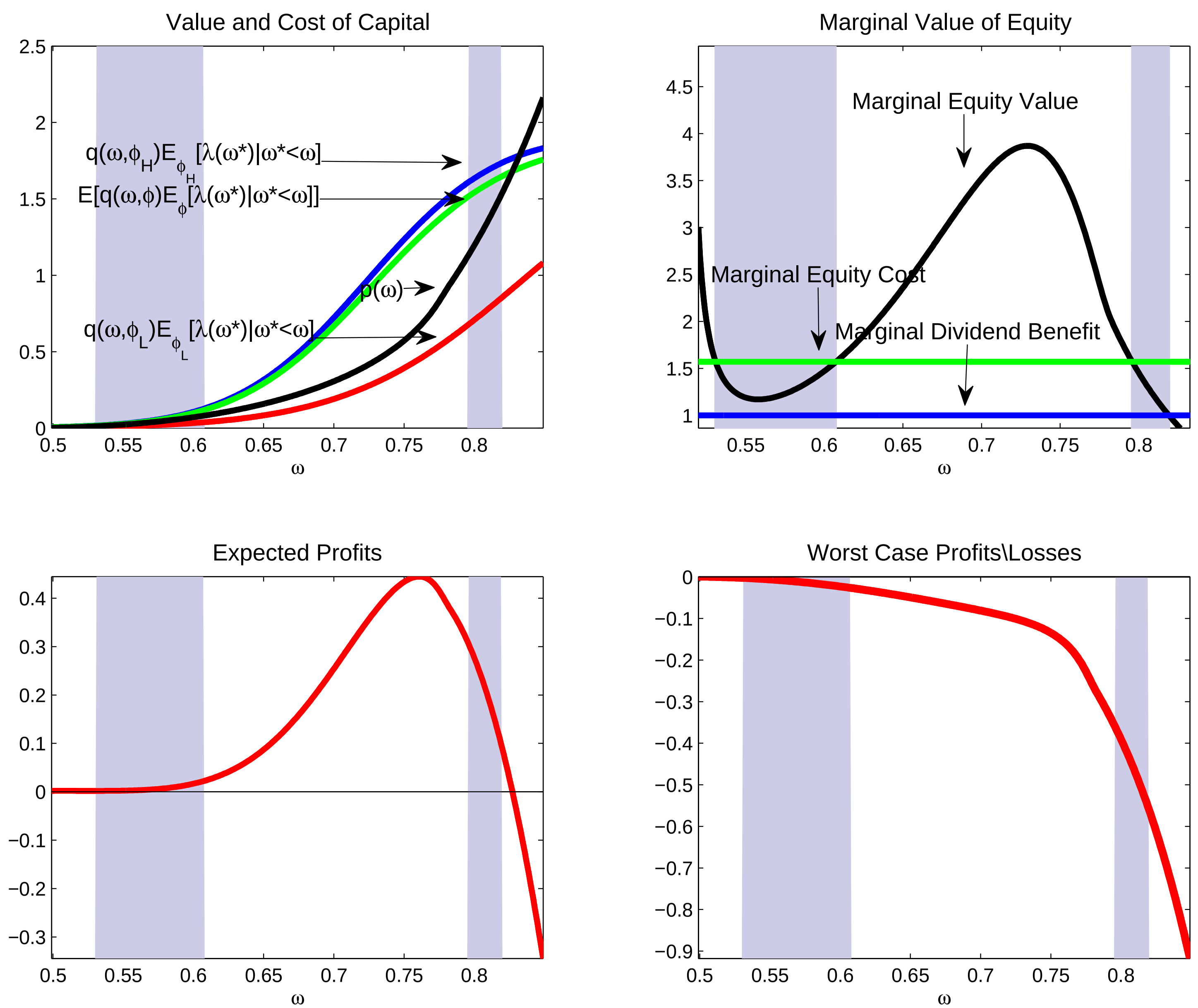

Figure 5: Equilibrium objects in the model with asymmetric information as functions of ω. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1.$.

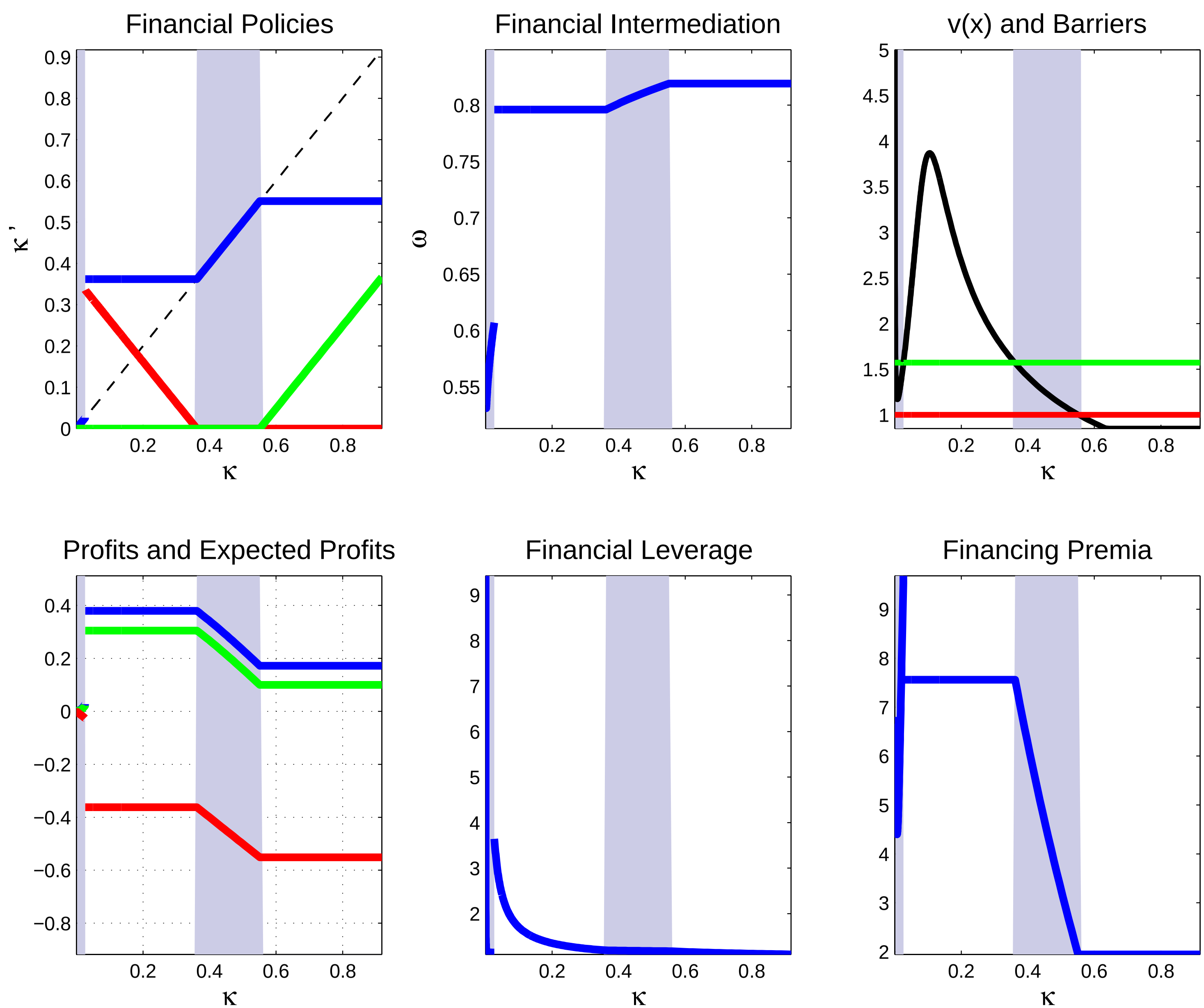

Figure 6: Equilibrium financial variables in the model with asymmetric information as functions of κ. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

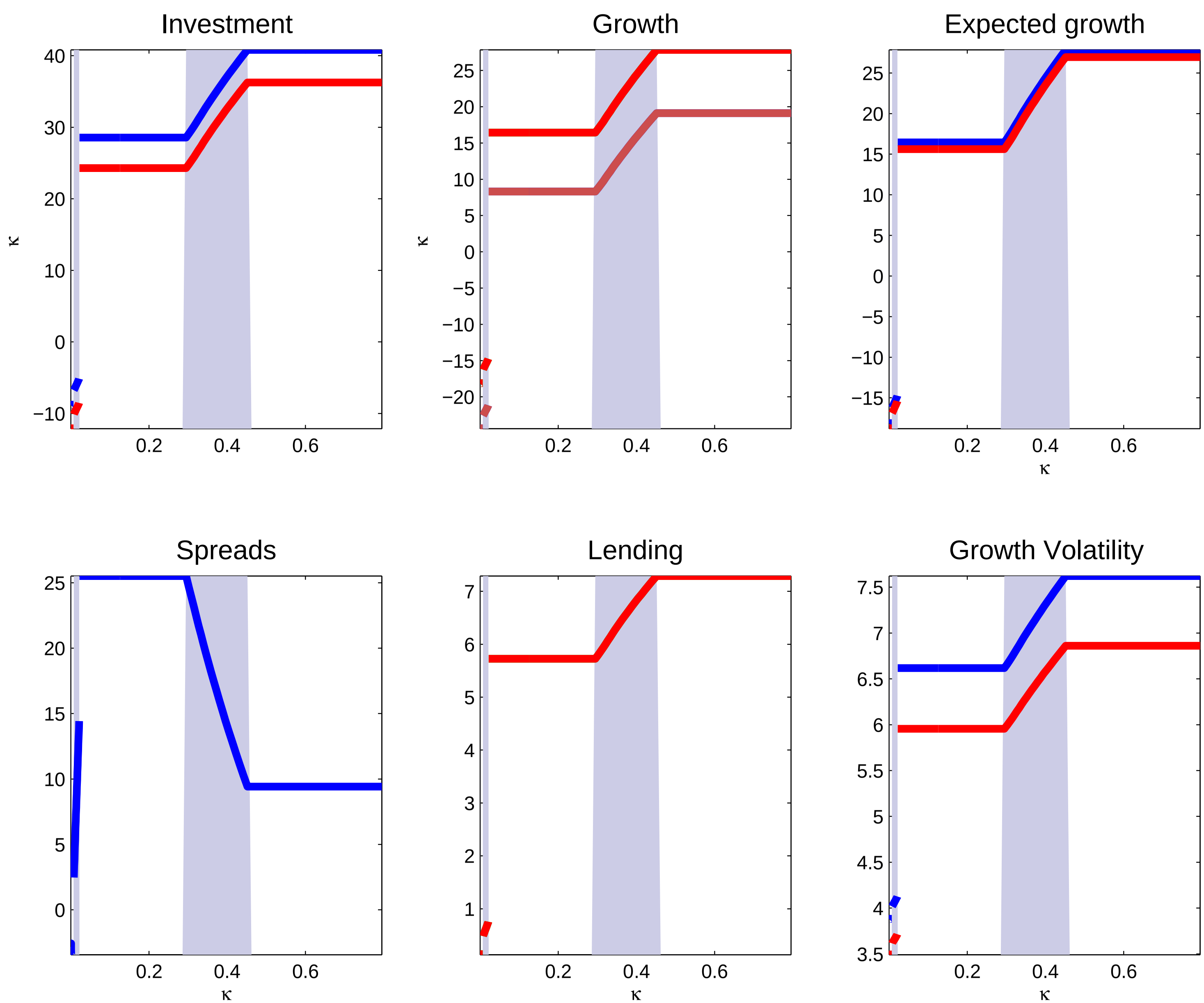

Figure 7: Equilibrium real side in the model with asymmetric information as functions of κ. The figure is constructed by setting parameters to: $\pi = 0.1$, $\beta = 0.975$, $\beta^f = 0.95$, $\tau = 0.5$ and $\rho = 1$.

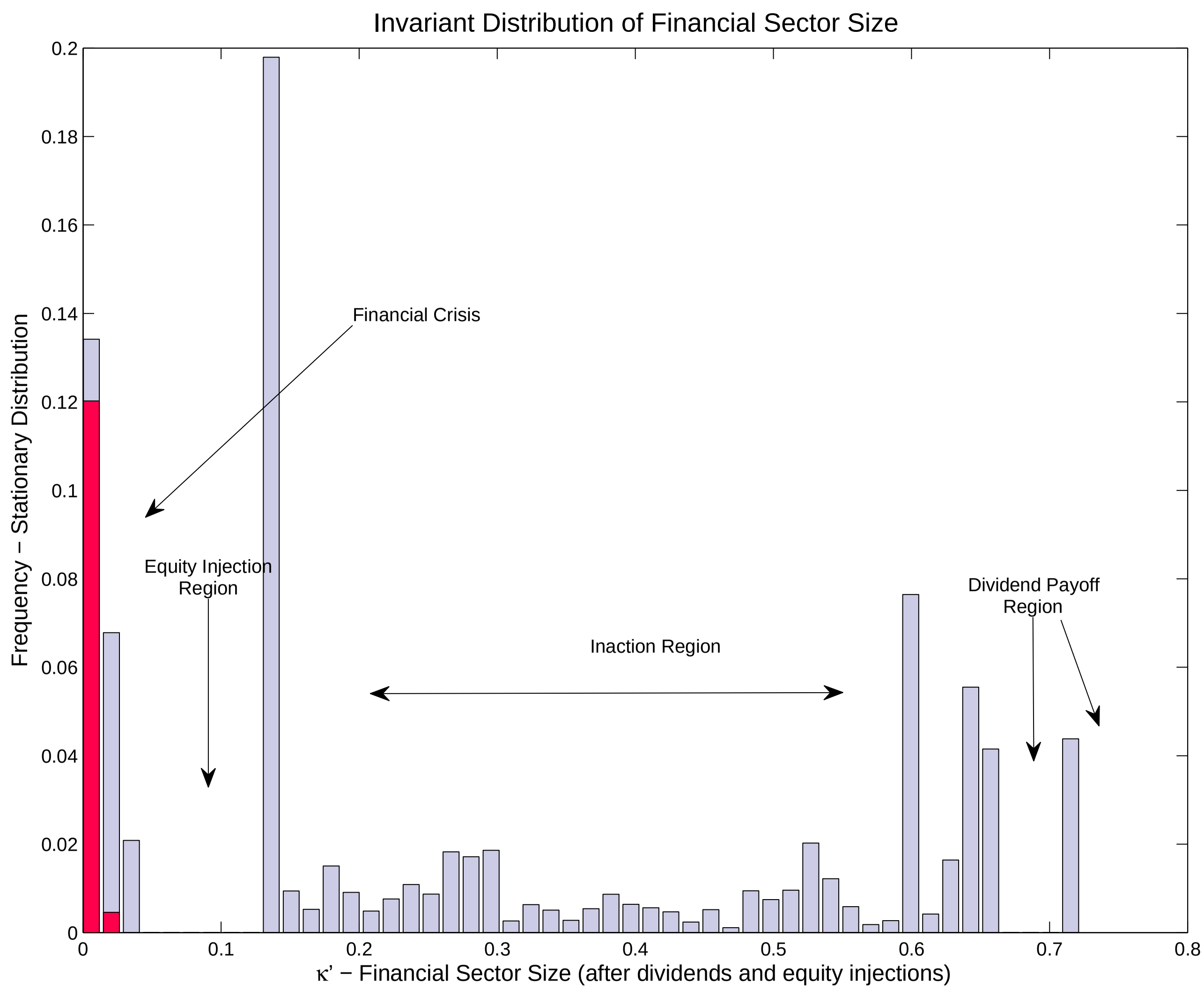

Figure 8: **Invariant distribution of κ.**

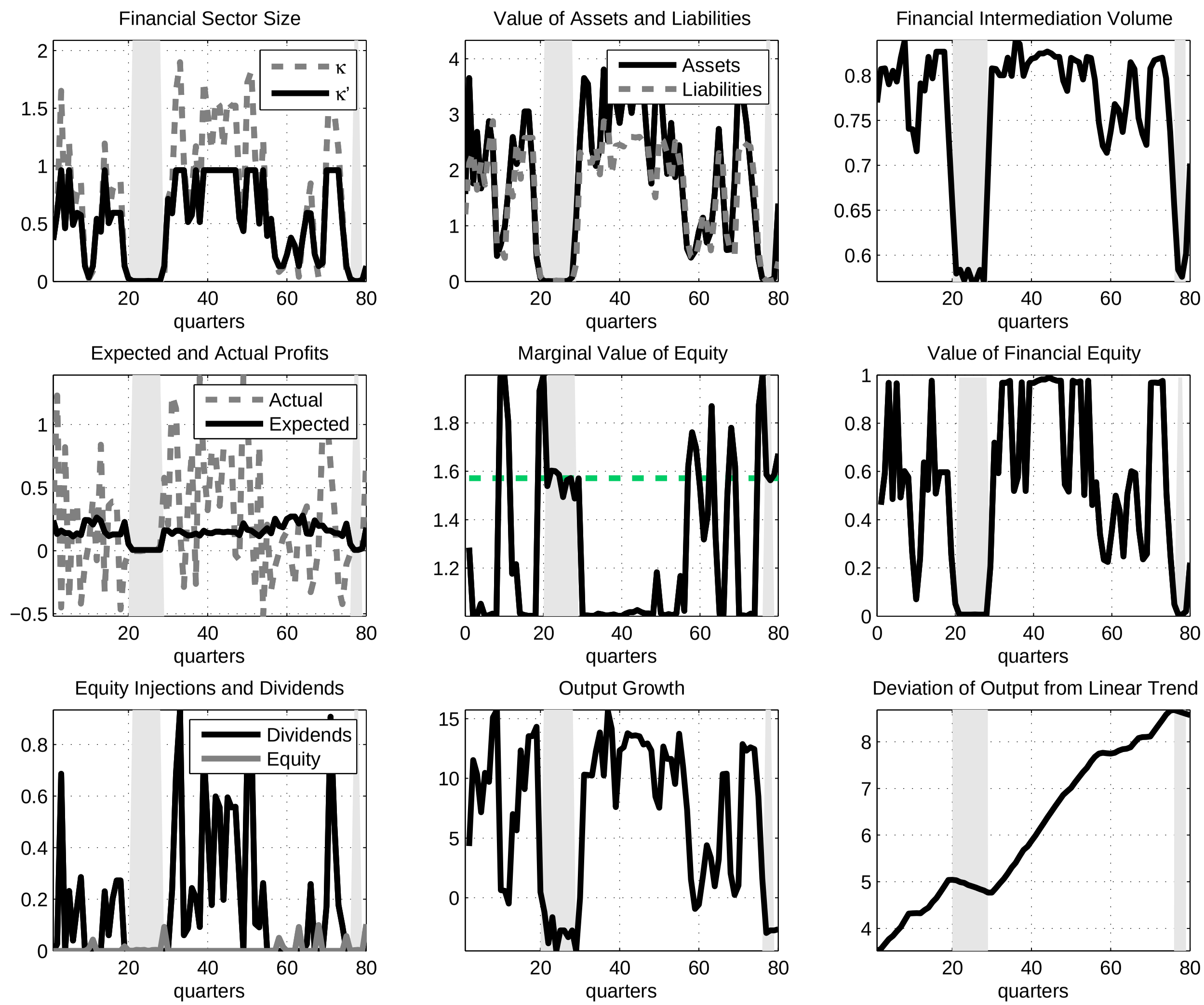

Figure 9: **Simulations.** The figure plots a simulated paths for the economy described in the paper. The shaded areas correspond to the financial crisis regimes corresponding to the definition in the paper.

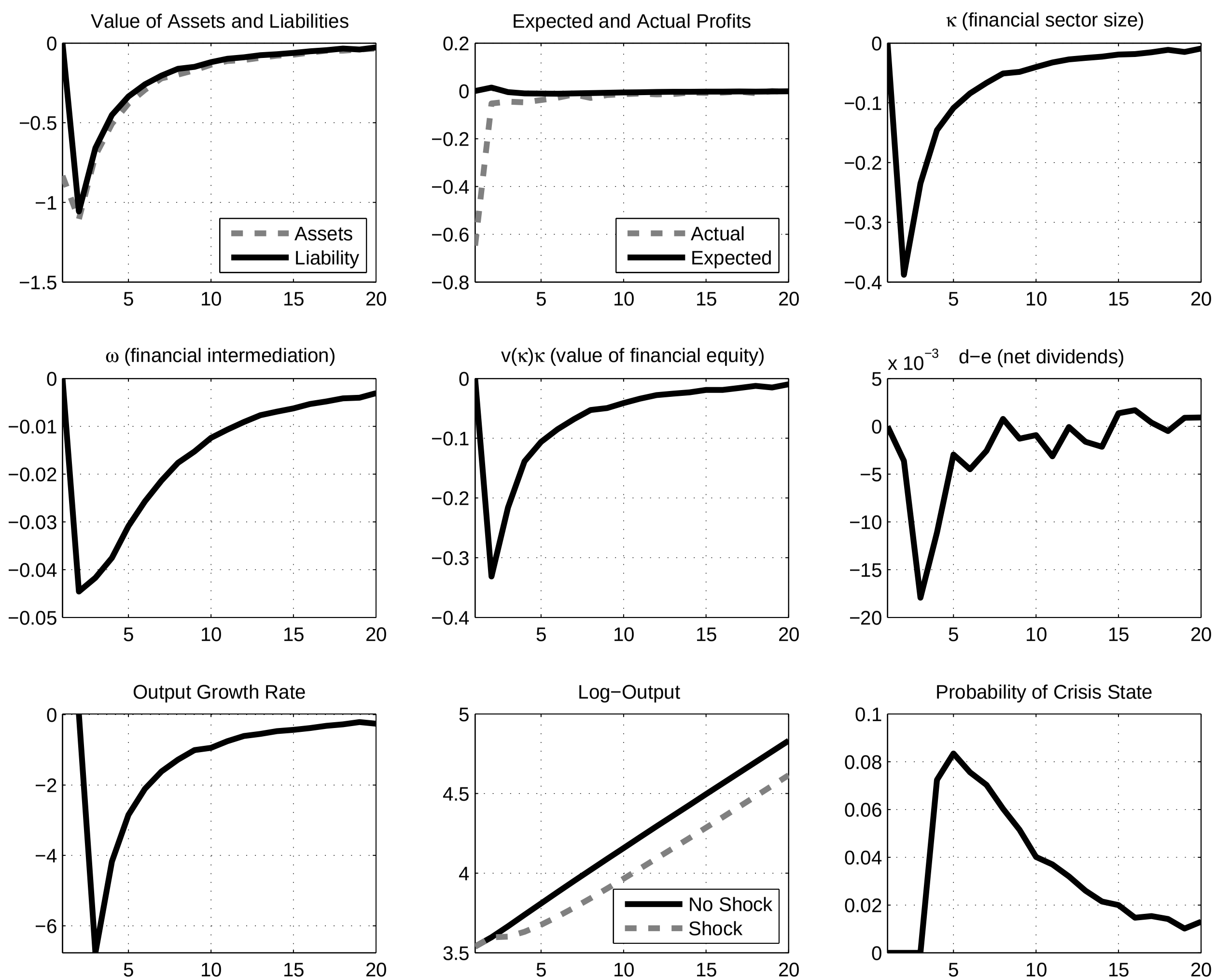

Figure 10: **Impulse response function.** The figure plots the response to an increase in dispersion.

	Model	**Data**	**Reference**
Occupation Times			
Efficient Region	0%		
Inefficient Region	100%		
Financial Crisis	12% (quarters)	13% (years)	Reinhart and Rogoff [2009]
Average Duration - Financial Crisis	9 quarters		
Growth			
Average Growth	4.2%	2.5%	FRED
Average Growth - Financial Crisis	-2.5%	-8%	Cerra and Saxena [2008]
Growth contribution of A	0%		
Growth contribution A	2%		
Variance Decomposition of Growth			
Volatility of GDP Growth	0.14%		
Volatility of GDP Growth - Financial Crisis	0.025%		
Volatility explained ϕ	97%		
Volatility explained ϕ - Financial Crisis	82%		
Volume			
Financial Intermediation	73%		Flow of Funds
Average Volume - Financial Crisis	58%		Flow of Funds
Financial Sector			
Financial Sector GDP	20%	7%	Phillipon [2009]
Financial Sector GDP - Financial Crisis	0.7%		
κ	40%		
κ - Financial Crisis	0.16%		

Table 1: **Model moments and reference statistics.**

	Model	Data	Reference
Volume			
Average Volume	40%		Flow of Funds
Average Volume - Financial Crisis	0.1%		Flow of Funds
Financial Premia			
Financial Premia	20.1% (annualized)	6%	Gorton and Metrick [2010] average corporate spread
Financial Premia - Financial Crisis	35.92% (annualized)	40%	Gorton and Metrick [2010] Repo haircuts
FI Leverage			
Leverage	1.1	9	Gorton and Metrick [2010]
Leverage - Financial Crisis	1.5		
Return on Equity			
Bank ROE	52%	10%	FRED
Banks ROE - Financial Crisis	61%		
Return on Assets			
Bank ROA	9.7%		Gorton and Metrick [2010]
Banks ROA - Financial Crisis	0.45%		
Reduction in Bank Equity			
Reduction in Bank Equity in crisis	99%		

Table 2: **Model moments of financial variables and reference statistics.**

A Algorithm

The following algorithm was used to compute equilibria in the model.

Algorithm 1 *1. Discretize the state space of $A \times \phi$ and transition function χ.*

- *I use a grid of 20 points for each variable.*

2. Discretize the unit interval for the volumes of intermediation.

- *I use a grid size of 1000 points.*

3. For all possible realizations $A \times \phi$ and ω on the grid solve $\{\mathbf{p}(\omega, \mathbb{A}, \phi), \mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi), \mathbf{\Pi}(\omega, \mathbf{A}, \phi)\}$.

- *$\mathbf{p}(\omega, \mathbf{A}, \phi)$ is discretizing the space of all possible values $p_L = \lambda(0) \min_\phi f_\phi(0)$ and $p_H = \lambda(1) \max_\phi f_\phi(1)$ and solving the optimal portfolio problem assuming $\mathbf{q}^c(\omega, \mathbf{A}, \phi) = 1$.*
- *Using this $\mathbf{p}$, one finds $\mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi)$, $\mathbf{\Pi}(\omega, \mathbf{p}, \mathbf{A}, \phi)$ and checks whether $\mathbf{q}(\omega, \mathbf{p}, \mathbf{A}, \phi) \geq 1$.*
- *For those values of condition fails, one solves $\mathbf{p}(\omega, \mathbf{A}, \phi)$, $\mathbf{q}(\omega, \mathbf{A}, \phi)$ jointly and then finds $\mathbf{\Pi}(\omega, \mathbf{A}, \phi)$.*

4. Guess a candidate function for $\tilde{v}$.

- *I use an initial guess of $\tilde{v} = 1$, corresponding to the case when $\rho = 1$.*

5. Compute the set ω^o using the candidate function $\tilde{v}$.

- *To do this I interpolate over the upper countour of $\mathbb{E}[\tilde{v}(X') \mathbf{\Pi}(\omega, \mathbf{p}, \phi) | X]$.*

6. Compute the set ω^κ.

- *This is done by computing for ω in the grid, $\kappa = \min_\phi \mathbf{\Pi}(\omega, \mathbf{p}, \phi) \omega$, and then findig*

7. Compute $\omega^*(X)$ for this iteration. Then, define $p(X) = \mathbf{p}(\omega^*(X), \mathbf{A}, \phi)$, $\Pi(X) = \mathbf{\Pi}(\omega^*(X), \mathbf{A}, \phi)$ and $q(X) = \mathbf{q}(\omega^*(X), \mathbf{p}(\omega^*(X), \mathbf{A}, \phi), \mathbf{A}, \phi)$.

8. Compute the transition function for X.

9. Update the $\tilde{v}(X)$.

- *This is done by iterating the Bellman equation for $\tilde{v}(X)$.*

10. Iterate steps 4-9 until convergence.

- *I used a tolerance of 0.001% for the value of equity $\tilde{v}(X)$.*

11. Compute $v(X), d(X), e(X)$.

Computation time. The algorithm runs in 5 minutes in Matlab.

B Proofs

B.1 Proof of Propositions 1, 2 and 3

The proof of propositions 1, 2 and 3 is presented jointly. The idea of the proof is to transform the entrepreneur's problem into a consumption-savings problem with log-preferences and linear constraints. For this, one has to deal with the asymmetric information problem and the randomization across entrepreneur types first. Once this is done, one can use the dynamic programming arguments for homogenous objectives in **?** to argue that all the Bellman equations here have unique solutions. I thus, proceed by guess and verify to find these solutions. A similar proofs appear in Bigio [2009].

Define $W^p \equiv w^p k \equiv \left(A + q\bar{\lambda}(X)\right) k$ and $W^i \equiv w^i k \equiv (p\omega^* + q^c \mathbb{E}_{\phi}[\lambda(\omega)|\omega < \omega^*])k$ as in the main text. The guess for the p-entrepreneur's policy function is $k^{p,\prime} = \beta \frac{W^p}{q}$ and $c^p = (1-\beta) W^p$ and that his value function is of the form $V_2^p = \psi^p(X) + \frac{1}{(1-\beta)} \log W^p$ where $\psi^p(X)$ is a function of the aggregate state. For i-entrepreneurs the guess is that $k^{i,\prime} = \beta \frac{W^i}{q^c}$ and $c^i = (1-\beta) W^i$ and that their value function is of the form $V_2^i = \psi^i(X) + \frac{1}{(1-\beta)} \log W^i$ where $\psi^i(X)$ is, again, a function of the aggregate state.

Consider the i-entrepreneur's problem during the first stage then. Substituting the guess for V_2^i and his constraints yields:

$$\begin{aligned} V_1^i &= \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\psi^i(X) + \log\left(\left(p\int_0^1 \mathbb{I}(\omega)\, d\omega \ + q^c \mathbb{E}_{\phi'}[\lambda(\omega)|\omega < \omega^*(X)]\right) k\right) |X\right] \\ &= \psi^i(X) + \log k + \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[\log\left(\left(p\int_0^1 \mathbb{I}(\omega)\, d\omega \ + q^c \mathbb{E}_{\phi'}[\lambda(\omega)|\omega < \omega^*(X)]\right)\right) |X\right] \end{aligned}$$

From this expression, we observe that choosing $\mathbb{I}(\omega)$ is identical to choosing a cut-off ω^* under which all units of quality lower than this cut-off are sold. This follows from the fact that a solution to the problem above can be attained by an optimal $\mathbb{I}^*(\omega)$ which is monotone decreasing. Suppose not and assume the optimal plan is given by some $\mathbb{I}'(\omega)$ whose value is cannot attained by any monotone decreasing policy. It is enough to show that the entrepreneur can find another $\mathbb{I}(\omega)$ that integrates to the same number, that is monotone decreasing and that makes his value weakly greater.

Since $\mathbb{I}'(\omega)$ and $\mathbb{I}(\omega)$ integrate to the same number, the amount of IOUs obtained by the i-entrepreneur during the first stage, is the same: $p\int_0^1 \mathbb{I}(\omega)\, d\omega = p\int_0^1 \mathbb{I}'(\omega)\, d\omega$. Now, since $\mathbb{I}(\omega)$ is monotone decreasing and $\lambda(\omega)$ is monotone increasing,

$$\int \lambda(\omega)\left[1 - \mathbb{I}'(\omega)\right] f_{\phi'}(\omega)\, d\omega \leq \int \lambda(\omega)\left[1 - \mathbb{I}(\omega)\right] f_{\phi'}(\omega)\, d\omega$$

implying that any optimal can be attained by some $\mathbb{I}^*(\omega)$ monotone decreasing. This implies that, as in the textbook lemons problem, solving for $\mathbb{I}^*(\omega)$ is equivalent to choosing a threshold ω^*. Substituting this threshold into the objective yields and expression for the optimal cutoff rule:

$$\omega^*(X) = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log\left[p\tilde{\omega} \ + q^c\left(X, X'\right)\int_{\tilde{\omega}}^1 \lambda(\omega)\, f_{\phi'}(\omega)\, d\omega\right] |X\right]. \tag{7}$$

This proofs Proposition 3. I now return to the second stage problems. Taking the solution to (7) as given, we know that the optimal plan for an i-entrepreneurs sets $x = p(X)\,\omega^*(X)\,k$. Using the optimal policy for ω^* and these definitions, one can write the second stage Bellman equation without reference to the first stage. To do this, one can substitute in for x and k in the second stage Bellman equation to rewrite the i-entrepreneur's problem as,

$$\max_{c\geq 0, i, k^b \geq 0} \log(c) + \beta\mathbb{E}\left[\pi V_1^i + (1-\pi)\,V_1^p | X\right]$$
$$c + i + qk^b = p\omega^*(X)\,k \text{ and } k' = k^b + i + k\int_{\omega^*(X)}^{1} \lambda(\omega)\, f_{\phi'}(\omega)\, d\omega.$$

Now, since $\left(V_1^i, V_1^p\right)$ are increasing in k', and k^b and i are perfect substitutes, an optimal solution will set $i > 0$ only if $q \geq 1$ and $k^b > 0$ only if $q \leq 1$. This implies that substituting the i-entrepreneur's capital accumulation equation into his budget constraint simplifies his problem to:

$$\max_{c\geq 0, k'} \log(c) + \beta\mathbb{E}\left[\pi V_1^i\left(k', X'\right) + (1-\pi)\,V_1^p\left(k', X'\right) | X\right] \text{ s.t. } c + q^c k' = w^i k.$$

The same steps allow one to write the p-entrepreneur's problem as,

$$\max_{c\geq 0, k'} \log(c) + \beta\mathbb{E}\left[\pi V_1^i + (1-\pi)\,V_1^p | X\right] \text{ s.t. } c + qk' = w^p k.$$

Replacing the definitions of V_1^i and V_1^p into the objective above, and substituting our guess yields V_2^i and V_2^p, we obtain:

$$\max_{c\geq 0, k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^i(X) \text{ s.t. } c + q^c k' = w^i k$$

and

$$\max_{c\geq 0, k'} \log c + \frac{\beta}{(1-\beta)} \log k' + \tilde{\psi}^p(X) \text{ s.t. } c + qk' = w^p k.$$

respectively. In this expressions, $\tilde{\psi}^i(X)$ and $\tilde{\psi}^p(X)$ are functions of X and don't depend on the current periods choice. Taking first order conditions for (k', c) in both problems leads to:

$$c^i = (1-\beta)\, w^i(\omega^*, X)\, k \text{ and } k^{i,\prime} = \frac{\beta}{q^c} w^i(\omega^*, X)\, k$$
$$c^p = (1-\beta)\, w^i(X)\, k \text{ and } k^{p,\prime} = \frac{\beta}{q} w^p(X)\, k$$

These solutions are consistent with the statement of Propositions 1 and 2. To verify that the guess for our value functions is the correct one, we substitute in the optimal policies:

$$\log(1-\beta)\, w^i(\omega^*, X)\, k + \frac{\beta}{(1-\beta)} \log \frac{\beta}{q^c} w^i(\omega^*, X)\, k + \tilde{\psi}^i(X)$$
$$= \frac{\log w^i(\omega^*, X)\, k}{(1-\beta)} + \psi^i(X) = \frac{\log W^i(k, \omega^*, X)}{(1-\beta)} + \psi^i(X)$$

for some function $\psi^i(X)$. The same steps lead to a similar expression for p-entrepreneurs. This verifies the initial guess.

B.2 Proof of Lemma 1 and Proposition 4

Lemma 1 and Proposition 4 are proven jointly here. We begin by guessing that that $V_1^f(n,X) = v_1^f(X)\,n$, and $V_2^f(n,X) = v_2^f(X)\,n$ where $v_2^f(X) = \beta^F \mathbb{E}\left[v_1^f(X)\,R^b\right]$ if the intermediary remains alive and $v_2^f(X) = \beta^F R^b n$ if he dies.

Plugging this guess into the intermediaries problem yields:

$$\begin{aligned}
& \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\,d - e + \mathbb{E}\left[v_2^f(X')\left(\Pi\left(X,X^{'}\right)Q + n'\right)|X\right] \\
= & \max_{Q\geq 0, e\in[0,\bar{e}], d\in[0,1]} (1-\tau)\,d - e + \beta^F R^b \mathbb{E}\left[v_1^f(X)\left(\Pi\left(X,X^{'}\right)Q + n'\right)|X\right]
\end{aligned}$$

subject to,

$$\begin{aligned}
-\min_{X'} \Pi\left(X,X^{'}\right)Q & \leq n' \\
n' & = n + e - d
\end{aligned}$$

Assume that the optimal solution to this problem is characterized by some $e^*(n,X)$ and $d^*(n,X)$ to be determined. In equilibrium, $\Pi\left(X,X^{'}\right)$ is finite. Hence, $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)\right]$ is also finite, provided that the problem has a finite solution. If $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)\right] > 0$ and $-\min_{X'}\Pi\left(X,X^{'}\right) \leq 0$, the intermediary would set $Q^* = \infty$. But this would imply that in equilibrium $\Pi\left(X,X^{'}\right) \leq 0$ for any X' because there cannot be a future state where firms provide infinite intermediation and there are positive profits. Hence, it is the case that if $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)\right] > 0 \to -\min_{X'}\Pi\left(X,X^{'}\right) > 0$. Now if this is the case,

$$Q^* = \frac{n'}{-\min_{X'}\Pi(X,X')} > 0. \tag{8}$$

If $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)\right] < 0$, the entrepreneur optimally sets Q^*. If $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)\right] = 0$, Q^* is indeterminate (but finite). Thus, in either case, $\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)Q^*\right] = 0$.

This implies that for any optimal policy,

$$\mathbb{E}\left[v_2^f(X')\,\Pi\left(X,X^{'}\right)Q^*\right] = \max\left\{\frac{v_2^f(X')\,\Pi\left(X,X^{'}\right)n'}{-\min_{X'}\Pi(X,X')}, 0\right\}.$$

Thus, one can substitute this expression into the objective of the firm and express it without reference to Q :

$$\begin{aligned} & \max_{e\in[0,\bar{e}],d\in[0,n]} (1-\tau)\,d - e + n'\beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right] \\ = & \max_{e\in[0,\bar{e}],d\in[0,n]} (1-\tau)\,d - e + (n+e-d)\beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right] \end{aligned}$$

where the second line uses the definition of n'. Now, it is clear from this expression that any optimal financial policy satisfies,

$$\begin{aligned} e &> 0 \text{ only if } \beta^F\left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right] \geq 1 \\ d &> 0 \text{ only if } \beta^F\left[\mathbb{E}[v_2^f(X')] + \max\left\{\frac{\mathbb{E}[v_2^f(X')\Pi\left(X,X'\right)]}{-\min_{\tilde{X}}\Pi\left(X,\tilde{X}\right)},0\right\}\right] \leq (1-\tau). \end{aligned}$$

If the inequalities are strict, it is clear that, $e=\bar{e}$ and $d=n$. By linearity, e and d are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied. By assumption $e=\bar{e}$ is never binding. If $d=n$, for some state, d is linear in n. This implies that $e\left(n,X\right)=e^*\left(X\right)n$, $d\left(n,X\right)=d^*\left(X\right)n$ are solutions to the entrepreneur's problem.

We now use this results to show that the value function is linear in n. Plugging in the optimal policies into the objective we obtain:

$$\left[(1-\tau)\,d^*\left(X\right) - e^*\left(X\right) + \left(1+e^*\left(X\right)-d^*\left(X\right)\right)\beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + \max\left\{\frac{v_2^f\left(X'\right)\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right]\right] n$$

which is a linear function of n.

Returning back to the optimal quantity decision, then it is clear that (**??**) can be written as,

$$Q = \frac{1+e^*\left(X\right)-d^*\left(X\right)}{-\min_{X'}\Pi\left(X,X'\right)}n =: Q^*\left(X\right)n.$$

and clearly, $Q^*\left(X\right) = \arg\max_{\tilde{Q}}\mathbb{E}\left[v_2^f\left(X'\right)\Pi\left(X,X'\right)|X\right]\tilde{Q}$ subject to $\Pi\left(X,X'\right)\tilde{Q} \leq n'$. This proves, Proposition 6.

We are ready to show that $v_1^f\left(X\right)$ solves a functional equation. Define

$$\tilde{v}\left(X\right) = \beta^F R^b \mathbb{E}\left[v_2^f\left(X'\right) + v_2^f\left(X'\right)\max\left\{\frac{\Pi\left(X,X'\right)}{-\min_{X'}\Pi\left(X,X'\right)},0\right\}|X\right]$$

as the marginal value of equity in the bank and note that:

$$v_1^f(X) = \max_{d^*(X)\in[0,1],e\geq 0} (1-\tau)\, d^*(X) - e^*(X) + (1 + e^*(X) - d^*(X))\, \tilde{v}(X).$$

If $\tilde{v}(X) \in ((1-\tau), 1)$, then $v_1^f(X) = \tilde{v}(X)$ because $(d^*(X), e^*(X)) = 0$. If $\tilde{v}(X) \leq (1-\tau)$, then $\mathrm{e}^*(X) = 0$ and we have that,

$$(1-\tau)\, d^*(X) + (1 - d^*(X))\, \tilde{v}(X) = (1-\tau).$$

Finally, if $\tilde{v}(X) = 1$, then, $v_1^f(X) = 1$. This information is summarized in the following functional equation for $v_1^f(X)$:

$$v_1^f(X) = \min\left\{\max\left\{\beta^F R^b \mathbb{E}\left[v_2^f(X')\left\{1 + \max\left\{\frac{\Pi\left(X, X'\right)}{-\min_{X'} \Pi(X, X')}, 0\right\}\right\} |X\right], (1-\tau)\right\}, 1\right\}$$

which equals,

$$\min\left\{\max\left\{\mathbb{E}\left[\left(\rho + (1-\rho)\, v_1^f(X'')\right)\beta^F R^b \left\{1 + \max\left\{\frac{\Pi\left(X, X'\right)}{-\min_{X'} \Pi(X, X')}, 0\right\}\right\} |X\right], (1-\tau)\right\}, 1\right\}.$$

This functional equation determines the slope of the intermediaries value function, $v_1^f(X)$. It can be shown that the solution to this functional equation is unique. Assumptions 9.18-9.20 of Stokey et al. [1989] are satisfied by this problem. It remains to show that Assumption 9.5 (part a) is also satisfied. By assumption, X is compact so the only piece left is that X is countable. Because the transition function for the state is an endogenous object, as it depends on an aggregate state, κ. It will be shown that although (d,e) are not uniquely defined, there is unique mapping from ϕ to κ'. By exercise 9.10 in Stokey et al. [1989], together, these assumptions ensures that there is a unique solution to the this functional equation.

B.3 Proof of Proposition Increasing $p(x)$

Let $q(X, X')$, is the market price that solves the market clearing condition given a price under asymmetric information of p. In equilibrium, this price is a function of the previous state, $p(X)$. Thus, through the equilibrium price $p(X)$, $q(X, X')$ defines an equilibrium q, implicitly, as function of the current state $X\prime$ and the previous state X: $q(X', X) \equiv \tilde{}\,(p(X), X')$. Given, X, and the law of motion for $X\prime$, $\tilde{q}(X', X)$ determines the profits for the financial sector given and amount of trade.

PROOF: To characterize the key objects $Q(X), p(X)$ and $q(X)$. we need some we need to define some objects. The supply for financial contracts is $Q^s(p, X) = \tilde{\omega}(p, X)\,\kappa$ and $Q^d = \{[]$ and $\tilde{\Pi}(p, X') = \tilde{q}(p, X')\, \mathbb{E}_{\phi'}[\lambda(\omega)\,|\omega > \bar{\omega}(p, X)] - p$. I provide some further characterization of this supply.

$$Q_p^s > 0, Q^s(0, X) = 0$$

and

$$Q^s\left(\mathbb{E}\left[q^c\left(X'\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\right]\right],X\right)=k.$$

Thus, the supply schedule is invertible and bounded, and thus, we define:

$$P\left(Q\right)\equiv p \text{ such that } Q^s\left(p,X\right)=Q$$

and

$$P_Q\left(Q\right)>0, P\left(0\right)=0 \text{ and } P\left(X\right)=\mathbb{E}\left[q^c\left(X'\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)\right]\right]$$

This proposition establishes the existence of a well behaved supply function: $P\left(Q\right)$ is bounded, differentiable and increasing.

B.4 Proof of Proposition 6

To pin down q, fix any sequence of states $\left(X,X'\right)$, and let $\bar{\omega}=\omega\left(X\right)$. We begin the proof assuming $q>1$ so that $D^i=0$. Market clearing in stage 2 requires $D^p\left(X,X'\right)=S\left(X\right)=\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega\leq\bar{\omega}\right]\bar{\omega}\pi K$. By Proposition 1, we can integrate acrros the p-entrepreneur's policy functions to obtain an expression for $D^p\left(X,X'\right)$ as a function of q:

$$\beta\int\left[\frac{W^p\left(k,x,X\right)}{q}-\bar{\lambda}k\right]\Gamma\left(dk,p\right)=\beta\frac{A+q\bar{\lambda}}{q}\left(1-\pi\right)K$$

By market clearing, q be such that:

$$\left[\beta\frac{A+q\bar{\lambda}}{q}-\bar{\lambda}\right]\left(1-\pi\right)K=\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega\leq\bar{\omega}\right]\bar{\omega}\pi K.$$

Manipulating this expression leads to the value of q that satisfies market clearing:

$$q=\frac{\beta A\left(1-\pi\right)}{\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega\leq\bar{\omega}\right]\bar{\omega}\pi+\left(1-\pi\right)\left(1-\beta\right)\bar{\lambda}}$$

Recall now that this expression is valid only when $q>1$, because investor's are not participating in the market. Thus, the expression is only true for value of

$$\beta A\left[\frac{\pi}{\left(1-\pi\right)}\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega<\bar{\omega}\right]\bar{\omega}+\left(1-\beta\right)\lambda\right]^{-1}>1. \tag{9}$$

If $q=1$, then it must be the case that the total demand for capital must be larger than the supply provided by financial firms. $D^i\left(X,X'\right)$ in this case is obtained also by integrating accross the demand for capital of i-entrepreneur's given in Proposition 1. Thus, for a stage one price p, this demand is given by

$$D^i+I=\beta p\bar{\omega}\pi K-\left(1-\beta\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\lambda\omega>\bar{\omega}\right]\left(1-\bar{\omega}\right)\pi K \text{ for } q=1$$

The corresponding condition is that,

$$\beta p\bar{\omega}-\left(1-\beta\right)\mathbb{E}_{\phi'}\left[\lambda\left(\omega\right)|\omega>\bar{\omega}\right]\left(1-\bar{\omega}\right)\pi+\left[\beta A-\left(1-\beta\right)\bar{\lambda}\right]\left(1-\pi\right)\geq\pi E_{\phi'}\left[\lambda\left(\omega\right)|\omega<\bar{\omega}\right]\bar{\omega} \tag{10}$$

where, the aggregate capital stock has been canceled from both sides. If the condition is satisfied, then $q = 1$, and

$$D^{i}(q,p) = \pi E_{\phi'}\left[\lambda(\omega)|\omega < \bar{\omega}\right]\bar{\omega} - \left[\beta R - (1-\beta)\lambda\right](1-\pi)$$

and

$$I = \left[\beta p^{i}\bar{\omega} - (1-\beta)\mathbb{E}_{\phi'}\left[\lambda(\omega)|\omega > \bar{\omega}\right](1-\bar{\omega})\right]\pi - D^{i}(q,p)$$

If (9) and (10) are violated, this implies $q < 1$ and that $I = 0$. The corresponding market clearing condition is obtained by solving q from:

$$\begin{aligned}
&\left[\frac{\beta p\bar{\omega}}{q} - (1-\beta)\mathbb{E}_{\phi'}\left[\lambda(\omega)|\omega > \bar{\omega}\right](1-\bar{\omega})\right]\pi + \left[\frac{\beta A}{q} - (1-\beta)\bar{\lambda}\right](1-\pi) \\
\geq\ & \pi E_{\phi'}\left[\lambda(\omega)|\omega < \bar{\omega}\right]\bar{\omega}.
\end{aligned}$$

We can collect the terms where q shows in the denominator to obtain,

$$\frac{\beta(p\bar{\omega}\pi + A)}{q} = \pi E_{\phi'}\left[\lambda(\omega)|\omega < \bar{\omega}\right]\bar{\omega} + (1-\beta)\left[-\mathbb{E}_{\phi'}\left[\lambda(\omega)|\omega < \bar{\omega}\right]\bar{\omega}\pi + \bar{\lambda}\right]$$

The solution is given by:

$$q = \frac{\beta\left(\pi p^{i}\bar{\omega} + A\right)}{\left(\beta E\left[\lambda(\omega)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}\pi + (1-\beta)\bar{\lambda}\right)}$$

The formula in Proposition 6 corresponds. . Moreover, the demand function is weakly decreasing so for each p, X there will be a unique q satisfying the market clearing condition.

From (**??**) we observe that the shock affects the $\tilde{q}(X', X)$ and $\mathbb{E}\left[\lambda(\omega)|\omega < \bar{\omega}\right]$. From Proposition (**??**), we can express the profit function in the following way:

$$\Pi\left(X, X'\right) = \max\left\{(1-\pi)\beta A\frac{\pi\mathbb{E}_{\phi'}\left[\lambda(\omega)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}}{\pi\mathbb{E}_{\phi'}\left[\lambda(\omega)|\bar{\omega} < \bar{\omega}\right]\bar{\omega} + (1-\beta)\lambda}, \tilde{\pi}(X, X')\right\}$$

where

$$\tilde{\Pi}(X, X') = \min\left\{1, \left(\pi p^{i} + A(1-\pi)\right)\frac{\pi\mathbb{E}_{\phi'}\left[\lambda(\omega)|\bar{\omega} < \bar{\omega}\right]\bar{\omega}}{\left(\frac{(1-\beta)\lambda}{\beta} + \pi\bar{\omega}\mathbb{E}_{\phi'}\left[\lambda(\omega)|\bar{\omega} < \bar{\omega}\right]\right)}\right\}$$

Since both functions are increasing in $\mathbb{E}_{\phi'}\left[\lambda(\omega)|\omega < \bar{\omega}\right]$ in the conditional expectation, we know by Assumption **A1,** that this functions are decreasing in the shock ϕ'. Thus, $\Pi(X, X')$ is decreasing in ϕ'.

B.5 Proof of Lemma ??

Observe that $\mathbb{E}\left[\Pi(X', p) - p\right] > 0$, is continuous in p.Thus, there is a sufficiently small $\varepsilon > 0$ increase in p such that the inequality still holds. Since the inequality implies that capacity constraints bind in equilibrium, then, market clearing implies that there exists some $\epsilon(\varepsilon)$, such that $p(X) + \varepsilon = P\left(\frac{s(\kappa+\epsilon(\varepsilon))}{\pi\left(p(X)+\varepsilon, X'_{\min}\right)}\right)$. In an un-improvable, it must be the case that $\pi\left(p(X) + \varepsilon, X'_{\min}\right)$ is decreasing, because otherwise there would have existed a larger equilibrium with a higher price. Thus, $\epsilon(\varepsilon)$ must increase. This, implies that for a given κ, we can find a small enough increase in κ, so that the equilibrium price increase.

If $\mathbb{E}[\Pi(X', p) - p] = 0$ and constraint does not bind, then, there is always a small enough increase in κ, such that capacity constraints don't bind, and therefore, the equilibrium price remains constant. If $\mathbb{E}[\Pi(X', p) - p] = 0$ and capacity constraints bind, then increase in κ will relax the binding constraint. Either Q remains the same or increases. Thus, p must increase.

B.6 Proof of Efficiency Proposition

Proof. The necessary condition: $(1 - \tau_d) \leq \beta^F R^b$. Suppose not, then for any state $\Pi(X', X) = 0$, then, it is convenient to pay dividends.

$$
\begin{aligned}
\tilde{v}(X) &= \beta^F R^b \left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max \left\{ \frac{\mathbb{E}[\tilde{v}(X')\,\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&< \beta^F R^b \left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max \left\{ \frac{\mathbb{E}[\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&= \beta^F R^b \mathbb{E}[\tilde{v}\left(X'\right)] \\
&< \beta^F R^b
\end{aligned}
$$

where the first line follows from the definition of $\tilde{v}(X)$ and v_2^f, the second follows from the fact that $\tilde{v}(X') \leq 1$ and the third from the assumption that an efficient equilibrium satisfies $\mathbb{E}[\Pi(X, X')] = 0$ and the last inequality uses $\tilde{v}(X') \leq 1$ once more. Then, if the condition is not satisfied, $\tilde{v}(X) < \beta^F R^b < (1 - \tau_d)$. This fact in turn implies that any state with financial risk capacity κ consistent with efficient intermediation is reflected to another state with $\mathbb{E}[\Pi(X, X')] > 0$.

The sufficient condition is obtained reversing the equalities.

$$
\begin{aligned}
\tilde{v}(X) &= \beta^F R^b \left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max \left\{ \frac{\mathbb{E}[\tilde{v}(X')\,\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&> \beta^F R^b \left[\mathbb{E}[\tilde{v}\left(X'\right)] + \max \left\{ \frac{(1 - \tau_d)\,\mathbb{E}[\Pi(X, X')]}{-\min_{\tilde{X}} \Pi\left(X, \tilde{X}\right)}, 0 \right\} \right] \\
&> \beta^F R^b \mathbb{E}[\tilde{v}\left(X'\right)]
\end{aligned}
$$

The first line follow from the definition of $\tilde{v}(X)$. The second uses that $(1 - \tau_d)\,\tilde{v}(X)$ has an upper bound. The third uses The hypothesis that $\mathbb{E}[\Pi(X, X')] = 0$. With this inequality, it is enough to argue that there will always exist some κ such that $\mathbb{E}[\tilde{v}(X')]$ is sufficiently above $(1 - \tau_d)$ such that that state is not reflected. □

C Microfoundation for constraints

D Screening Devices