Search, Distributions, Monetary and Fiscal Policy

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Abstract

I study the effects of anticipated inflation on the macroeconomic performance and whether the fiscal policy regime matters for such effects. I construct a tractable framework with competitive search that can endogenously generate dispersion of prices, wealth and income. I prove and characterize the stationary equilibrium. Findings from quantitative analysis suggest that inflation has non-trivial effects on aggregate output, price levels, price dispersion, average wealth, and inequality of wealth, income and consumption. Income taxation has its distinctive effects on the above variables, often to the opposite of the inflation effects. Moreover, the tax regime can alter the relationship between inflation and wealth dispersion. Finally, inflation can improve welfare when income taxation is imposed. The higher the tax rate, the more prominent the welfare-improving role of inflation.

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1 Introduction

Empirical evidence suggests that price dispersion, wealth and income distributions prevail in the real economies. In the presence of such distributions, monetary and fiscal policy is likely to have uneven impacts on households in an economy, which in turn can generate non-trivial effects on real activities and welfare. In this paper, I construct a tractable framework with competitive search that endogenously generates dispersion of prices, wealth and income. I investigate the long-run effects of inflation on various aspects of the macroeconomic performance, *e.g.* output, markups, price distribution, wealth dispersion, income inequality, consumption inequality and welfare. Furthermore, I study whether there is room for coordination of monetary and fiscal policies, that is, whether the fiscal policy regime affects the relationships between inflation and the aforementioned macro measures of interest.

In the model, households and firms can trade in frictionless and frictional markets. The frictional market is characterized by competitive search, where households make tradeoffs between the terms of trade and matching probabilities when choosing which submarket to participate in. Search is competitive in that both households and firms take as exogenous the terms of trade and the matching probabilities across all submarkets. In equilibrium, a submarket that requires a higher payment per transaction offers a higher quantity of goods in a transaction and also a higher probability for a buyer to be matched for a transaction. Households face uninsurable idiosyncratic shocks on labor preferences, which lead to diverse decisions on consumption, labor supply, savings and trading strategies.

Competitive search is a key feature of this model and it offers two important advantages: First, output responds to policy changes along both intensive and extensive margins. The former refers to the quantity of goods traded in a transaction and the latter is the volume of transactions. The extensive margin of output has rarely been studied in a heterogeneous-agent context. Second, competitive search significantly improves model tractability. Unlike the more commonly studied bilateral bargaining in a search environment, here individual traders cannot affect any of the submarket specifications, *i.e.* terms of trade and matching probabilities, due to the competitive nature of the search process. Taking the trading specifications as given, a household need not consider the amount of money balance that its potential trading partner might have, when making its optimal decisions. Therefore, the household decision problem is independent of the endogenous money distribution, which greatly reduces the state space and renders the model tractable.

I characterize the household optimal decisions and prove the existence of a stationary equilibrium in the version of the model with constant money supply and zero income taxation. Then I consider money growth created by lump-sum money injections, as well as proportional income taxes. I provide analytical results on direct policy effects on intensive and extensive margins. In particular, given the amount spent in a trade, inflation has a negative effect on the intensive margin and a positive effect on the extensive margin. In contrast, income taxation has the exact opposite effects, *i.e.* positive on the intensive margin and negative on the extensive margin. The amount that a household is willing to spend in a transaction is also affected by policy, which creates an indirect channel for policy to influence the two margins. I further investigate the policy effects through quantitative analysis. The model generates a rich set of results regarding the effects of long-run inflation. The results are consistent with some of the evidence from recent empirical literature, suggesting that the trading frictions can be important in helping reconcile empirical observations on the macroeconomic performance.

The key findings from the numerical exercises are the following: First, inflation has a positive effect on aggregate output, consumption inequality, average price and price dispersion. In the meanwhile, inflation reduces average wealth and income inequality. Second, income taxation has a negative effect on output, average price, and a positive effect on average wealth and income inequality. Taxation strengthens the negative relationship between inflation and income inequality. Moreover, taxation can alter the relationship between inflation and wealth inequality. At lower tax rates, inflation is a regressive wealth tax. At intermediate tax rates, there can be a hump-shape relationship between inflation and wealth dispersion. At higher tax rates, inflation is a progressive wealth tax. Finally, inflation can be welfare-improving when income taxes are imposed. The higher the tax rate, the stronger the positive welfare effect of inflation. Therefore, the model suggests that it is important to coordinate policies. The optimal inflation rate is higher if the tax rate has been raised, and vice versa.

The framework developed here is based on Lagos and Wright (2005; henceforth LW) and Menzio, Shi and Sun (2011; henceforth MSS). The LW structure features quasi-linear preferences and alternating frictional and frictionless markets. It is a tractable monetary framework because the equilibrium money distribution is degenerate.[1] However, it does not provide insights on the distributional policy effects. Menzio, Shi and Sun (2011) construct a tractable monetary environment with non-degenerate money distributions. Because of competitive search, the model has block recursivity, which refers to the model feature that

[1] Because of its tractability, the unique LW framework has prompted an exploding literature on micro-founded models of money with an emphasis on market frictions. This literature has recently been recognized as the *New Monetarist Economics* (Williamson and Wright, 2010a,b).

individual decision problems are independent of the endogenous distributions.[2] Menzio, Shi and Sun characterize the stationary equilibrium by abstracting away from money growth.

To focus on the policy effects without losing model tractability, I construct a model with key features of LW and MSS. With quasi-linear preferences, access to frictionless markets and competitive search, the model is block recursive even in the context of both monetary and fiscal policies. In this model, money is the only store of value. However, in contrast to LW and MSS, money is the required medium of exchange in any market. But unlike a representative-agent structure, households still have incentives to hold money because they may need to save the unspent balances whenever they are not matched in a trade, or because they need to self-insure against idiosyncratic shocks.

This paper is closely related to the literature on the distributional effect of monetary policy. This literature of heterogeneous-agent monetary economies can be roughly divided into two categories: with and without search frictions. In the models without search frictions, money is valued either because of the cash-in-advance constraint (Imrohorglu, 1992; Erosa and Ventura, 2002; Camera and Chien, 2011) or for precautionary purpose (Akyol, 2004; Wen, 2010; Dressler, 2011). Because agents trade in Walrasian markets, these models are not able to generate equilibrium price dispersion. The search model is a natural environment to have dispersion of prices, *e.g.* Molico (2006), Boel and Camera (2009) and Chiu and Molico (2010). A common trading arrangement in these models is bilateral bargaining. In contrast, my model features competitive search, which not only allows for the extensive margin effect but also significantly improves tractability. Finally, none of the above literature examines whether the fiscal regime is a factor in the long-run effects of inflation, which is one of the two main considerations of my paper.

The rest of this paper is organized as follows. Section 2 describes the physical model environment. Section 3 characterizes the monetary equilibrium and presents the theoretical results. Section 4 discusses theoretical policy effects. Section 5 shows numerical results. Finally, Section 6 concludes the paper.

[2]This is a concept first applied to economics by the seminal work of Shi (2009) on equilibrium wage-tenure contracts. Menzio and Shi (2010a,b; forthcoming) and Gonzalez and Shi (2010) further examine the functioning of labor markets using the notion of block recursive equilibrium.

2 A Unified Macroeconomic Framework

2.1 The environment

Time is discrete and continues forever. Each time period consists of two sub-periods. The economy is populated by a measure one of *ex ante* identical households. Each household consists of a worker and a buyer. All households consume general goods in the first sub-period and special goods in the second sub-period. There are different types of special goods. Every period a household faces a random shock on consumption preference that determines which type of special goods (other than its own production goods) it can consume for the current period. Household members share income, consumption and labor cost. The preference of a household in a time period is

$$\mathcal{U}(y, q, l) = U(y) + u(q) - \theta l, \tag{1}$$

where y is consumption of general goods, q is consumption of special goods and l is labor input in a time period. The parameter $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ measures the random disutility per unit of labor. It is *i.i.d.* across households and over time, where $0 < \underline{\theta} < \bar{\theta} < \infty$. It is drawn from the probability distribution $F(\theta)$. The value of θ is realized at the beginning of every period, before any decisions are made. The functions u and U are twice continuously differentiable and have the usual properties: $u' > 0$, $U' > 0$; $u'' < 0$, $U'' < 0$; $u(0) = U(0) = u'(\infty) = U'(\infty) = 0$; and $u'(0)$ and $U'(0)$ being large but finite. Households discount future with factor $\beta \in (0, 1)$. All goods are perfectly divisible. They are also perishable and cannot be consumed across sub-periods. There is no insurance on income risks. Nor is borrowing or lending feasible. There is a fiat object called money, which is perfectly divisible and can be stored without cost.

General goods are traded in perfectly competitive markets, called frictionless markets. Special goods are traded in frictional markets in the sense that there is random matching between buyers and sellers in such a market. The trading frictions are driven by household random demand for special goods. There is a measure one of competitive firms. All households and firms have free access to the frictionless and the frictional goods markets. Firms hire workers from households, who own equal shares of all firms. The labor market is competitive and frictionless. Labor is hired at the beginning of a period and is used in production for both general and special goods. Each firm can organize production and sales of the general goods and one particular type of special goods. Therefore, each firm only hires workers who are specialized in producing that particular type of special goods, in addition to producing general goods. A firm pays competitive wages and distributes profits to the households. In a frictional market, firms have free entry to a variety of submarkets, which differ in terms of trading protocols. A firm chooses the measure of shops to operate in each submarket. The cost of operating a shop for one period is $k > 0$ units of labor. The cost of producing q units of special goods requires $\psi(q)$ units of labor, where ψ is twice continuously differentiable with the usual properties: $\psi' > 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

In each period, trading in the frictionless goods market takes place in the first sub-period, followed by trading in the frictional market in the second sub-period.[3] The worker of a household works for a firm, while the buyer goes shopping in the goods markets. Trading in a frictional market is characterized by competitive search. Each submarket is characterized by (x, q, b, s), where (x, q) are the terms of trade and (b, s) are the respective matching probabilities for a buyer and a shop. Search is competitive in the sense that households and firms take as given the characteristics of all submarkets, and choose which submarket to participate in. Buyers and shops are randomly matched in a pair-wise manner because households and firms cannot coordinate. In equilibrium, free entry of firms is such that the characteristics of submarkets are consistent with the specified ones. The matching technology has constant returns to scale and is characterized by the matching function $s = \mu(b)$. As households and firms choose which submarket to enter, the matching probabilities in each submarket becomes functions of the terms of trade (x, q), as is shown in (4). Therefore, a submarket can be sufficiently indexed by (x, q). I impose the following assumption:

Assumption 1 *For all $b \in [0, 1]$, the matching function $\mu(b)$ satisfies: (i) $\mu(b) \in [0, 1]$, with $\mu(0) = 1$ and $\mu(1) = 0$, (ii) $\mu'(b) < 0$, and (iii) $[1/\mu(b)]$ is strictly convex, i.e., $2(\mu')^2 - \mu\mu'' > 0$.*

I focus on steady state equilibria and suppress the time index throughout the paper. The per capita money stock is fixed at M for now. I will allow it to change over time later when I analyze policy effects. I use labor as the *numeraire* of the model. In particular, let m denote the real value of a household's money balance at a particular point in time, where the label "real" means that m is measured in terms of labor units. I assume that $\bar{m}$ is the maximum real money balance that a household can carry across periods, where $0 < \bar{m} < U'^{-1}(\bar{\theta})$. Let w denote the normalized wage rate, which is the nominal wage rate divided by the money stock M. Then the dollar amount associated with a balance m is $(wM)\,m$.

[3]In this environment, one can also assume that the frictionless and frictional goods markets open simultaneously in a period. The results are similar to the sequential order of markets. Here I adopt the sequential structure for expositional convenience.

2.2 A firm's decision

In the frictionless market, a representative firm takes the general-good price as given and chooses output Y to maximize profit. It takes Y units of labor to produce Y units of general goods. Let p be the price of general goods, measured in terms of labor units. In the frictional market, the firm takes the terms of trade for each submarket, (x, q), as given and chooses the measure of shops, $dN(x, q)$, to set up in each submarket. Recall that a shop is matched by a buyer with probability $s(x, q)$. For a particular shop in the submarket, the operational cost is k units of labor and the expected cost of production is $\psi(q) s(x, q)$ units of labor. A shop's expected revenue is $xs(x, q)$, where the revenue x is measured in labor units. The firm's total profit in a period is

$$\pi = \max_{Y} \{pY - Y\} + \max_{dN(x,q)} \left\{ \int \{xs(x, q) - [k + \psi(q) s(x, q)]\} dN(x, q) \right\}. \tag{2}$$

The first item on the right-hand side denotes the firm's profit in the frictionless market and the second item its profit in the frictional market. Free entry of firms implies that the firm earns zero profit in the frictionless market and $p = 1$ in equilibrium.

The expected profit of operating a shop is

$$s(x, q)[x - \psi(q)] - k.$$

If this profit is strictly positive, the firm will choose $dN(x, q) = \infty$. However, this case will never occur in equilibrium under free entry. If this profit is strictly negative, the firm will choose $dN(x, q) = 0$. If this profit is zero, the firm is indifferent across various non-negative and finite levels of $dN(x, q)$. Thus, the optimal choice of $dN(x, q)$ satisfies:

$$s(x, q)[x - \psi(q)] \leq k \quad \text{and} \quad dN(x, q) \geq 0, \tag{3}$$

where the two inequalities hold with complementary slackness. As is common in the competitive search literature,[4] I focus on equilibria where condition (2) also holds for submarkets not visited by any buyer. This implies that the firm also earns zero profit in the frictional markets in equilibrium.

For all submarkets such that $k < x - \psi(q)$, the submarket has $dN(x, q) > 0$, and

[4] For example, Moen (1997), Acemoglu and Shimer (1999), and Menzio, Shi and Sun (2011). Given such beliefs off the equilibrium, markets are complete in the sense that a submarket is inactive only if the expected revenue of the only shop in the submarket is lower than its expected cost given that some buyers are present in the submarket. Such a restriction can be justified by a "trembling-hand" argument that an infinitely small measure of buyers appear in every submarket exogenously.

(3) holds with equality. For all submarkets such that $k \geq x - \psi(q)$, the submarket has $dN(x, q) = 0$, in which case I set $s = 1$ and $b = 0$. Putting the two cases together, the matching probability for a particular shop is given by

$$s(x, q) = \mu(b(x, q)) = \begin{cases} \frac{k}{x - \psi(q)}, & \text{if } k \leq x - \psi(q) \\ 1, & \text{if } k > x - \psi(q). \end{cases} \tag{4}$$

The free-entry condition pins down the matching probabilities in a submarket as functions of the terms of trade. Indeed, a submarket can be sufficiently indexed by the terms of trade, (x, q).

2.3 A household's decision

2.3.1 Decision in the frictionless market

Let $W(m, \theta)$ be a household's value at the beginning of a period with money balance m and the random realization θ. Given price p and the characteristics of all submarkets, a household maximizes its value by choosing consumption of general goods $y \geq 0$, labor input $l \geq 0$, the balance to spend in the frictional market $z \geq 0$, and the savings $h \geq 0$. If the household's buyer is matched with a shop in the frictional market, then the buyer spends z and the household carries h into the following period. Otherwise, the household carries a balance $z + h$ into the following period. If the balance $z + h$ contains firm IOUs, the household redeems these IOUs for money and carries it to the next period. Thus $z + h \leq \bar{m}$. The dividend Π is paid to the household at the end of a period. In equilibrium $\Pi = 0$ because firms earn zero profit.

The value $W(m, \theta)$ satisfies the following Bellman equation:

$$\begin{aligned} W(m, \theta) &= \max_{(y,l,z,h)} \{U(y) - \theta l + V(z, h)\} \\ &\quad s.t. \;\; py + z + h \leq m + l. \end{aligned} \tag{5}$$

The constraint in the above is a standard budget constraint.[5] The function $V(z, h)$ is

[5] Note that there is no money constraint in the household's optimization problem. This is because firm IOUs, as well as money, can be used as a medium of exchange in all transactions. Firm IOUs take the form of a firm's promise of wage payments at the end of a period, in terms of money. Firm IOUs are settled in a central clearinghouse at the end of a period. Such IOUs are enforcible because firms are large (in the sense that each of them owns a positive measure of shops) and thus they have deterministic revenues and costs, although the individual shops of each firm face matching risks. Firms last for one period and new ones are formed at the beginning of the next. Thus firm IOUs can be circulated for only one period. Nevertheless, personal IOUs of households are not accepted as a medium of exchange because households face idiosyncratic preference and matching risks and there is no enforcement on their IOUs.

the household's value at the beginning of the second sub-period, i.e., before the frictional market opens. Because the analysis on the decisions of frictional trading is more involved, I will postpone fully characterizing V until the next section. In Lemma 3, I show that V is differentiable and concave in z. For now, I take such information of V as given. Given $U' > 0$, the budget constraint must hold with equality and thus

$$l = y + z + h - m, \tag{6}$$

where I have incorporated $p = 1$ in equilibrium. For now I assume that the choice of l is interior, which I will prove later. Using (6) to eliminate l in the objective function yields

$$W(m, \theta) = \theta m + \max_{y} \{U(y) - \theta p y\} + \max_{z,h} \{V(z, h) - \theta(z + h)\}. \tag{7}$$

The optimal choices must satisfy the following first-order conditions:

$$U'(y) \leq p\theta, \quad \text{and} \quad y \geq 0 \tag{8}$$

$$V_z(z, h) \begin{cases} \leq \theta, & \text{and} \quad z \geq 0 \\ \geq \theta, & \text{and} \quad z \leq \bar{m} - h, \end{cases} \tag{9}$$

$$V_h(z, h) \begin{cases} \leq \theta, & \text{and} \quad h \geq 0 \\ \geq \theta, & \text{and} \quad h \leq \bar{m} - z \end{cases} \tag{10}$$

where the all sets of inequalities hold with complementary slackness. Given $0 < \bar{m} < U'^{-1}\left(\bar{\theta}\right)$, it follows that for all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$,

$$\theta \leq \bar{\theta} < U'(\bar{m}) < U'(0).$$

Given $p = 1$ in equilibrium, condition (8) implies that the choice of y is always interior and satisfies

$$U'(y) = \theta. \tag{11}$$

Clearly, the household's current money balance m does not affect these optimal choices of y, z or h. Let the policy functions be $y(\theta)$, $z(\theta)$ and $h(\theta)$. Note that $z(\theta) + h(\theta) \geq 0$ for all $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ and that $m \leq \bar{m}$. Therefore, (6) and (11) imply that $l(m, \theta) \geq U'^{-1}\left(\bar{\theta}\right) - \bar{m} > 0$ for all (m, θ). Given (7), the value function W is clearly continuous, differentiable and

Because both fiat money and firm IOUs can be used to purchase all goods, no particular type of goods are *cash goods* in this environment. This is in contrast with standard money search models, where goods traded in the frictional markets are considered cash goods. In these models, fiat money must be used as a medium of exchange to overcome the lack of double coincidence of wants and record-keeping of individual traders.

linear in m:

$$W(m,\theta) = W(0,\theta) + \theta m, \tag{12}$$

where

$$W(0,\theta) = U(y(\theta)) - \theta p y(\theta) + V(z(\theta), h(\theta)) - \theta [z(\theta) + h(\theta)]. \tag{13}$$

The preceding exposition proves the following lemma:

Lemma 1 *The value function W is continuous and differentiable in (m,θ). It is also affine in m.*

2.3.2 Decision in the frictional market

The household's decisions on frictional trading are non-trivial and deserve much attention. The household chooses whether to participate in the frictional market. If yes, then it chooses which submarket to enter and search for a trade. Given balances z and h, the household is faced with the following problem at the beginning of the second sub-period:

$$\max_{x,q} \left\{ b(x,q)\left[u(q) + \beta E\left[W(z-x+h,\theta)\right]\right] + \left[1 - b(x,q)\right]\beta E\left[W(z+h,\theta)\right]\right\}, \tag{14}$$

where $q \geq 0$, $x \leq z$ and $b(x,q)$ is determined by (4). It is convenient to use condition (4) to eliminate q in the above objective function. Given linearity of W, the problem in (14) simplifies to

$$\max_{x\leq z,\ b\in[0,1]} b\left\{u\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta)x\right\} + \beta E\left[W(z+h,\theta)\right]. \tag{15}$$

The optimal choices satisfy the following first-order conditions

$$\frac{u'\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right)}{\psi'\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right)} - \beta E(\theta) \geq 0, \quad \text{and} \quad x \leq z, \tag{16}$$

$$u\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta)x + \left[\frac{u'\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right)}{\psi'\left(\psi^{-1}\left(x - \frac{k}{\mu(b)}\right)\right)}\right]\frac{kb\mu'(b)}{[\mu(b)]^2} \leq 0, \quad \text{and} \quad b \geq 0, \tag{17}$$

where the two sets of inequalities hold with complementary slackness. It has been taken into account in condition (17) that $b = 1$ cannot be an equilibrium outcome. This is because $b = 1$ implies that $s = 0$. This further implies that firms choose $dN(z,q) = \infty$ and earn strictly positive profit, which violates free entry. Let the policy functions be $x(z)$, $b(z)$ and $q(z)$, where $q(z)$ is implied by condition (4):

$$q(z) = \psi^{-1}\left(x(z) - \frac{k}{\mu(b(z))}\right). \tag{18}$$

If $b(z) = 0$, then the choices of x and q are irrelevant. In this case, the household chooses not to participate in the frictional submarket. Without loss of generality, I impose $x(z) = z$ if $b(z) = 0$.

Now consider z such that $b(z) > 0$. It is obvious from (15) that the optimal choices are independent of z if the money constraint does not bind, i.e., $x(z) < z$. Define $\Phi(q) \equiv u'(q)/\psi'(q)$. Given $x(z) < z$, (16) holds with equality. Then conditions (16) and (18) imply

$$q^* = \Phi^{-1}[\beta E(\theta)]. \tag{19}$$

Given q^*, using (18) to eliminate x in (17) yields

$$u(q^*) - \beta E(\theta)\left[\psi(q^*) + \frac{k}{\mu(b^*)}\right] + \left[\frac{u'(q^*)}{\psi'(q^*)}\right]\frac{kb^*\mu'(b^*)}{[\mu(b^*)]^2} = 0. \tag{20}$$

It is straightforward to show that the left-hand side of (20) is strictly increasing in b^*. Moreover, $b^* > 0$ exists and is unique if $E(\theta)$ satisfies

$$u\left(\Phi^{-1}[\beta E(\theta)]\right) - \beta E(\theta)\left[\psi\left(\Phi^{-1}[\beta E(\theta)]\right) + k\right] > 0. \tag{21}$$

Given unique values of q^* and b^*, x^* is uniquely determined by

$$x^* = \psi(q^*) + \frac{k}{\mu(b^*)}. \tag{22}$$

Therefore, if condition (21) holds, then $x(z) = z$ for all $z < x^*$ and $x(z) = x^*$ for all $z \geq x^*$. If condition (21) fails to hold, $x(z) = z$ for all $z \geq 0$. Define $\hat{z}$ as the maximum value such that $x(z) = z$. Thus $\hat{z} = x^*$ if (21) holds and $\hat{z} = \infty$ otherwise.

In this environment, it is not necessary for the household to choose z higher than the amount that it plans to spend in the frictional market. Without loss of generality, I focus on the case $x(z) = z$ in the rest of the analysis. In particular, consider $z \in [0, \hat{z}]$. Given such z, the problem in (15) becomes

$$B(z) + \beta E\left[W(z+h,\theta)\right],$$

where

$$B(z) = \max_{b\in[0,1]} b\left\{u\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta) z\right\}. \tag{23}$$

The value $B(z)$ is the household's expected trade surplus. If $b > 0$, it must be the case that $q > 0$ and that the surplus from trade is strictly positive:

$$u\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta) z > 0. \tag{24}$$

The optimal choice of b satisfies condition (17) given $x = z$.

Lottery choice. It is necessary to mention that the value function $B(z)$ may not be concave in z because the objective function in (23) may not be jointly concave in its state and choice variables, (z, b). This objective function involves the product between the choice variable b itself and a function of b. Even if both of these two terms are concave, the product may not be jointly concave. Above all, it is unclear whether either of the two terms is a concave function of z, given that b is a choice variable and is yet to be determined. To make the household's value function concave, I introduce lotteries with regards to households' balances z, as is the case in Menzio, Shi and Sun (2011). In particular, lotteries are available every period immediately before trading in the frictional market takes place.

A lottery is characterized by (L_1, L_2, π_1, π_2). If a household plays this lottery, it will win the prize L_2 with probability π_2. The household loses the lottery with probability π_1, in which case it receives a payment of L_1. There is a complete set of lotteries available. Given z, a household's optimal choice of lottery solves:

$$\tilde{V}(z) = \max_{(L_1, L_2, \pi_1, \pi_2)} \left\{\pi_1 B(L_1) + \pi_2 B(L_2)\right\} \tag{25}$$

subject to

$$\begin{aligned} &\pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0; \\ &\pi_1 + \pi_2 = 1; \quad \pi_i \in [0,1] \text{ for } i = 1,2. \end{aligned}$$

Denote the policy functions as $L_i(z)$ and $\pi_i(z)$, respectively, where $i = 1, 2$. If the household is better off not playing any lottery, it is trivial to see that $L_1(z) = L_2(z) = z$.

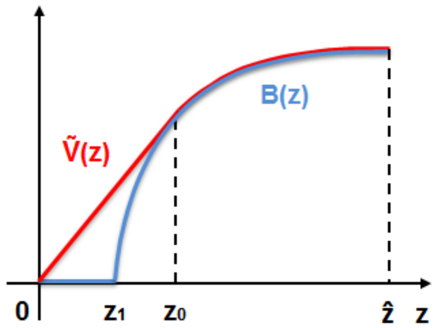


Figure 1. Lottery Choice

Figure 1 illustrates how the lottery can help make the value function $\tilde{V}(z)$ concave, even though the function $B(z)$ has some strictly convex part. It is intuitive to see that a household will choose to play a lottery if it has a very low balance. As is shown in Figure 1, for any balance $z \in (0, z_0)$, it is optimal for the household to participate in the lottery offering the prize z_0. The lottery makes $\tilde{V}(z)$ linear whenever $B(z)$ is strictly convex. The properties of z_0 are presented in part (iii) of Lemma 2. Recall the household's first-order condition (9) on the optimal choice of z. Given the lottery, the policy function $z(\theta)$ may not be unique because V has some linear segments. I focus on the symmetric equilibrium where households with the same realization of θ will choose the same value of z, whenever the optimal choice of z is not unique.

2.3.3 Properties of value and policy functions

Lemma 2 *The value function $B(z)$ is continuous and increasing in $z \in [0, \hat{z}]$. The value function $\tilde{V}(z)$ is continuous, differentiable, increasing and concave in $z \in [0, \hat{z}]$. For z such that $b(z) = 0$, the value function $B(z) = 0$. In this case, the choice of q is irrelevant. There exists $z > 0$ such that $b(z) > 0$ if and only if there exists $q > 0$ that satisfies*

$$u(q) - \beta E(\theta)[\psi(q) + k] > 0. \tag{26}$$

For z such that $b(z) > 0$, the value function $B(z)$ is differentiable, $B(z) > 0$ and $B'(z) > 0$. Moreover, the following results hold: (i) The policy functions $b(z)$ and $q(z)$ are unique and strictly increasing in z. In particular, $b(z)$ solves

$$u(q(z)) - \beta E(\theta) z + \left[\frac{u'(q(z))}{\psi'(q(z))}\right] \frac{kb(z)\mu'(b(z))}{[\mu(b(z))]^2} = 0, \tag{27}$$

where

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right). \tag{28}$$

Moreover, $b(z)$ *strictly decreases in* $E(\theta)$ *and* $q(z)$ *strictly increases in* $E(\theta)$*; (ii) There exists* $z_1 > k$ *such that* $b(z) = 0$ *for all* $z \in [0, z_1]$ *and* $b(z) > 0$ *for all* $z \in (z_1, \hat{z}]$*; (iii) There exists* $z_0 > z_1$ *such that a household with* $z < z_0$ *will play the lottery with the prize* z_0*. Moreover,* $B(z_0) = \tilde{V}(z_0) > 0$*,* $B'(z_0) = \tilde{V}'(z_0) > 0$ *and* $b(z_0) > 0$*.*

Lemma 2 summarizes the properties of the household's value and policy functions in the frictional market. According to part (i), the optimal choices of (q, b) are strictly increasing in z when the household chooses $b > 0$ to participate in frictional trading. In this case, the higher a balance the household spends, the higher a quantity it obtains and the higher the matching probability at which it trades. As a result, households endogenously sort themselves into different submarkets based on their balances to spend. For any given z, a higher value of $E(\theta)$ implies a lower matching probability for the buyer and a higher amount of goods to be purchased by the buyer. The intuition is the following: Given higher $E(\theta)$, it becomes more costly for firms to hire labor. Firms respond accordingly by setting up fewer shops in the submarkets but increasing quantity produced per trade. This helps save the fixed cost of operating shops and steer more labor into production. All else equal, more shops in a submarket lead to a higher matching probability for a shop, which tends to increase a firm's revenue. Thus the firm can afford to offer a higher quantity per trade, even though it requires a higher labor input. In this case, households face a lower matching probability for a buyer. Nevertheless, the households are compensated by an increase in the quantity per purchase.

Recall that V is the value of a household at the beginning of the second sub-period, before trading decisions are made. Given (12), (15), (23) and (25), V is given by

$$V(z, h) = \tilde{V}(z) + \beta E[W(0, \theta)] + \beta E(\theta)(z + h). \tag{29}$$

Thus V is linear in h with

$$V_h(z, h) = \beta E(\theta). \tag{30}$$

Then condition (10) implies that the optimal choice of h satisfies

$$h(\theta) \begin{cases} \geq 0, & \text{if } \theta \geq \beta E(\theta) \\ \leq \bar{m} - z(\theta), & \text{if } \theta \leq \beta E(\theta), \end{cases} \tag{31}$$

where the two sets of inequalities hold with complementary slackness. Given Lemma 1, Lemma 2 and conditions (30) and (31), it is trivial to derive the following lemma:

Lemma 3 *The function V is continuous and differentiable in (z,h). The function $V(\cdot,h)$ is also increasing and concave in $z \in [0,\hat{z}]$, with $V(z,h) \geq \beta E[W(0,\theta)] > 0$ for all z. If $\theta/E(\theta) \geq \beta$, then $V_z(z,h) \geq \beta E(\theta)$. If $\theta/E(\theta) < \beta$, then $V_z(z,h) \geq 0$. Moreover, $V(z,\cdot)$ is affine in h.*

Recall that the firm's free entry to the frictionless market implies that $p^* = 1$. Also recall that the household's optimal choice of y is given by (11). Given strict concavity of the function U and concavity of V in z, it is straightforward to obtain $y'(\theta) < 0$ and $z'(\theta) \leq 0$. Then (31) implies

$$l(m,\theta) \begin{cases} = py(\theta) + z(\theta) - m - T, & \text{if } \theta > \beta E(\theta) \\ \\ = py(\beta E(\theta)) + z(\beta E(\theta)) + h(\beta E(\theta)) - m - T, & \text{if } \theta = \beta E(\theta) \\ \\ = py(\theta) + \bar{m} - m - T, & \text{if } \theta < \beta E(\theta), \end{cases} \tag{32}$$

where $h(\beta E(\theta)) + z(\beta E(\theta)) \in (0,\bar{m})$. The above exposition leads to the following very intuitive lemma:

Lemma 4 *(i) $y'(\theta) < 0$, $z'(\theta) \leq 0$ and $h'(\theta) \leq 0$; (ii) $l_m(m,\theta) < 0$ and $l_\theta(m,\theta) < 0$.*

3 Stationary Equilibrium

Definition 1 *A stationary equilibrium consists of household values $(W,B,\tilde{V},V)$ and choices $(y,l,z,h,(q,b),(L_1,L_2,\pi_1,\pi_2))$; firm choices $(Y,dN(x,q))$; price p and wage rate w. These elements satisfy the following requirements: (i) Given the realizations of shocks, asset balances, general-good prices and the trading protocols of all frictional submarkets, a household's choices solve (7), (23), (25) and (29), which induce the value functions $W(m,\theta)$, $B(z)$, $\tilde{V}(z)$ and $V(z,h)$; (ii) Given prices and the trading protocols of all submarkets, firms maximize profit and solve (2); (iii) Free entry condition: The expected profit of a shop in each submarket is zero, and the function $s(x,q)$ satisfies (4); (iv) All labor markets, general-good markets and money markets clear; (v) Stationarity: All quantities, prices and distributions are time invariant; (vi) Symmetry: Households in the same idiosyncratic state make the same optimal decisions.*

The above definition is self-explanatory. The labor-market-clearing condition implies that the equilibrium normalized wage rate w^* is determined by

$$\begin{aligned}(w^*)^{-1} &= \int_{\underline{\theta}}^{\bar{\theta}} h(\theta)\, dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta))\left[1 - b(L_1(z(\theta)))\right] L_1(z(\theta))\, dF(\theta) \\ &\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta))\left[1 - b(L_2(z(\theta)))\right] L_2(z(\theta))\, dF(\theta). \qquad (33)\end{aligned}$$

I provide detailed formulas for the market-clearing conditions and the government transfer in Appendix D. Given the definition of the stationary equilibrium, I have the following theorem:

Theorem 2 *A stationary equilibrium exists. It is unique if and only if the lottery choices* $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ *are unique for all* $z(\theta)$. *Moreover, the following results hold: (i) The general-good consumption* $y(\theta) > 0$ *for all* θ*; (ii) If there does not exist* $q > 0$ *that satisfies condition (26), then* $z(\theta) = 0$ *for all* θ. *Otherwise,* $z(\theta) \geq 0$ *for all* θ*; (iii) The balance* $h(\theta) > 0$ *if* $\theta/E(\theta) \leq \beta$ *and* $h(\theta) = 0$ *if* $\theta/E(\theta) > \beta$.

According to Theorem 2, the frictionless markets are always active, while the frictional markets are not. A necessary condition for the frictional markets to be used is that condition (26) holds for some $q > 0$. This condition depends on the preferences and the production technology for special goods, the discount factor and the value of $E(\theta)$. Intuitively, if the utility derived from consuming special goods is too low, or if the production cost of special goods is too high, consumption of special goods can become too costly, especially considering the uncertainty involved in obtaining such goods. Similarly, if $E(\theta)$ is too high, then the cost of labor is high, which drives up the cost of producing special goods and suppresses the demand. These results are consistent with the findings of Camera (2000). In a model without distributional components, Camera shows that the frictional markets are used in equilibrium when households can have sufficiently high expected consumption relative to the frictionless markets.

4 Policy Effects

I now analyze the effects of monetary and fiscal policies. Consider that the money stock per capita evolves according to $M' = \gamma M$, where $\gamma \geq \beta$ is the money growth rate and M' is the money stock of the next period. Money growth is achieved by a lump-sum transfer from the government to the households, and vice versa for money contraction. The government also imposes a proportional tax rate $\tau \in (0,1)$ on wage income. The government balances its budget every period. All tax revenues are redistributed from the government to the households in a lump-sum manner, together with the transfers made for money growth purposes. Transfers are made at the beginning of each period. All tax payments and transfers are made with money. The money market opens in the second subperiod of a period.

First, it is straightforward to show that $\partial y(\theta)/\partial\tau \leq 0$, which is simply an income effect. Second, monetary and fiscal policies directly affect equilibrium trading strategies, i.e., the intensive margin $q(z)$ and the extensive margin $b(z)$ for a given balance z. Given the policies, all the results in Lemma 2 hold, except that the policy functions $b(z)$ and $q(z)$ are jointly determined by

$$u(q(z)) - \frac{\beta E(\theta)}{\gamma(1-\tau)}z + \left[\frac{u'(q(z))}{\psi'(q(z))}\right]\frac{kb(z)\mu'(b(z))}{[\mu(b(z))]^2} = 0 \tag{34}$$

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right), \tag{35}$$

instead of (27) and (28). Then follows a proposition on policy effects:

Proposition 1 *For all z such that $b(z) > 0$, (i) $\frac{\partial q(z;\tau)}{\partial\tau} > 0$ and $\frac{\partial b(z;\tau)}{\partial\tau} < 0$; (ii) $\frac{\partial q(z;\gamma)}{\partial\gamma} < 0$ and $\frac{\partial b(z;\gamma)}{\partial\gamma} > 0$.*

Part (i) summarize the effects of proportional income taxes. A higher income tax rate τ makes households frugal on spending. For any given balance, a household chooses to visit a submarket that offers a higher quantity of goods per trade, which is a positive effect on the intensive margin. In such a submarket, a firm's cost of production per trade is higher. Thus it reduces overall cost by setting up a smaller measure of shops in this submarket. This imposes a negative effect along the extensive margin. The results in part (i) are intensive and extensive margin effects of fiscal policies. These are novel results in that current literature on search-theoretic models of money rarely analyzes the effect of fiscal policy on frictional trading.

Part (ii) of Proposition 1 lists the monetary policy effects on intensive and extensive margins. In particular, the real value of a money balance over time decreases with money growth. A household responds by sending its buyer to a submarket with a higher matching probability b, in order to increase the chance of spending money in the current period. In such a submarket, the matching probability for a shop is lower, which all else equal implies a lower profit for firms. Zero profit condition requires that firms must be compensated by producing a lower quantity per trade. These results of monetary policy are standard and have been well-documented in the money search literature.

5 Numerical Results

I employ the following functional forms to simulate this economy:

$$u(c) = u_0 \frac{(c+a)^{1-\sigma} - a^{1-\sigma}}{1-\sigma}, \ U(c) = U_0 \frac{(c+a)^{1-\sigma_u} - a^{1-\sigma_u}}{1-\sigma_u},$$
$$a > 0, \ \sigma, \sigma_u > 0, \ u_0, U_0 > 0;$$

$$\psi(q) = \psi_0 q^\chi, \ \psi_0 > 0, \chi \geq 1; \quad \mu(b) = (1-b^\rho)^{1/\rho}, \ \rho > 0;$$

$$F(\theta) \text{ is uniform on } \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \tag{36}$$

I report the numerical results based on the following parameter values:

$\beta = 0.997,$	$u_0 = 1;$	$U_0 = 10^3;$	$a = 10^{-3};$	$\sigma = 2;$	$\sigma_u = 2;$
$\psi_0 = 500;$	$\chi = 1.5;$	$k = 0.01;$	$\rho = 1;$	$\bar{m} = 14;$	$\theta \in [1, 2].$

Moreover, I restrict my attention to policy parameters $\gamma \in [\beta, \beta + 0.5]$ and $\tau \in [0, 0.3]$. The above parameter values satisfy the assumption that the labor choices of all households are interior in equilibrium.

Strategy for computing the stationary equilibrium. To completely solve the equilibrium, one can first solve the optimization problems of firms and households. After obtaining the policy functions from the aforementioned decision problems, one can derive the equilibrium wage rate, aggregate labor, aggregate output and the government transfer, using the formulas presented in Appendix D.

Policy functions. Figure 2 depicts the policy functions under various monetary and fiscal policy regimes. A few observations follow immediately:

R1. The policy functions $y(\theta)$, $z(\theta)$ and $h(\theta)$ are decreasing in θ and $b(z)$ and $q(z)$ are increasing functions for z such that $b(z) > 0$. These results confirm corresponding ones in Lemmas 2 and 4;

R2. Consider a given tax rate τ. For any given value of θ, inflation (higher γ) increases the transaction balance $z(\theta)$ and decreases savings $h(\theta)$. It has no effect on $y(\theta)$. This is obvious from panels A and B for $\tau = 0$, and panels C and D for $\tau = 0.2$;

R3. Consider a given money growth rate γ. For any given value of θ, a higher tax rate τ decreases $z(\theta)$ and $y(\theta)$, but increases savings $h(\theta)$. This is obvious from panels A and C for $\gamma = 0.995$, and panels B and D for $\gamma = 1.2$;

R4. Neither the inflation rate nor the tax rate has a significant impact on the functions $b(z)$ and $q(z)$;

R5. The price of special goods in a frictional submarket, $z/q(z)$, is generally an increasing function of the amount spent z.

The results in R2-R3 are intuitive. Inflation has no effect on consumption of general goods, which is a standard result. All else equal, inflation reduces the value of real money balances over time. Hence, the household chooses to save less and spend more on special goods. Moreover, all else equal, a higher tax rate makes it more costly to supply labor. As a result, the household saves more and becomes more frugal on spending.

Intensive and extensive margins. Policy affects the intensive and extensive margins both *directly*, as summarized in Proposition 1, and *indirectly* through affecting the choice of spendings z. In particular, given θ a household's choices of b and q can be expressed by $b(z(\theta;\gamma);\gamma)$ and $q(z(\theta;\gamma);\gamma)$[6]. R4 suggests that the direct policy effects on the intensive and extensive margins can be quantitatively small. Now consider the indirect policy effects on the margins. For example, it is clear from Figure 2 that inflation increases the choice of z for every value of θ, which then leads to higher choices of b and q since both are increasing functions of z. Figure 3 reports the overall effects of inflation on the intensive and the extensive margins under various tax rates. Panel A shows the intensive margin, i.e. average quantity per trade in the frictional market, and panel B shows the extensive margin, i.e. volume of transactions in the frictional market. To summarize:

R6. Inflation has a positive overall effect on both the intensive and the extensive margins. In contrast, the tax rate has a negative overall effect on both of the margins.

[6] These expressions are valid when the lottery is not used at $z(\theta;\gamma)$. When the lottery is used, the functions are given by $b(L_i(z(\theta;\gamma));\gamma)$ and $q(L_i(z(\theta;\gamma));\gamma)$ for $i = 1, 2$. The policy effects discussed here also apply when the lottery is used.

Aggregate output and labor. Figure 4 presents the policy effects on aggregate output (Panel A) and aggregate labor (Panel B). The first column of panel A shows the effects of inflation on output given $\tau = 0.05$. Inflation has no effect on output in the general-good sector because it has no effect on $y(\theta)$. Inflation has a positive effect on output in the special-good sector. This follows from the positive effects of inflation on intensive and extensive margins. Together, inflation has a positive effect on aggregate output (and thus aggregate consumption since there is no investment in this model).[7] The first column of Panel B documents similar effects of inflation on aggregate labor. The second column of Panel A illustrates the effects of income taxation on output given $\gamma = 1.01$. Income taxation reduces output in both sectors and thus aggregate output, which is a direct result of the negative effect of taxation on intensive and extensive margins. Similarly, inflation also has a negative effect on aggregate labor, as is shown in the second column of Panel B. Panel C presents further policy effects on aggregate output in the special-good sector.

R7. Inflation has a positive effect on aggregate output/consumption and aggregate labor, while income taxation has a negative effect on both.

Equilibrium prices. Figure 5 shows the effect of inflation on the price distribution in the frictional market at various tax rates. Inflation increases both the average price level and the coefficient of variation of prices. Given $\tau = 0$, Figure 6 demonstrates the price distributions under $\gamma = 0.995$ and $\gamma = 1.2$. The horizontal axis is the price and the vertical axis is the fraction of shops offering a particular price. As inflation goes up, a smaller proportion of shops are offering the higher end of the equilibrium prices. In particular, the fraction of shops offering the highest equilibrium price is about 55% when $\gamma = 0.995$ and this number drops to 28% when $\gamma = 1.2$. This is driven by the positive extensive margin effect of inflation. All households increase their spendings z and their choices of b as inflation increases. To meet this demand, firms must set up more shops in all submarkets to ensure a higher matching probability for buyers. As is indicated in Figure 2, the function $b(z)$ is concave. As all households increase their spendings z, a low-spending household's choice of b is catching up with that of a high-spending household. That is, the differences in

[7] This is consistent with the empirical findings for the U.S. and some other countries, suggesting a positive long-run relationship between inflation and output (see King and Watson, 1992; Bullard and Keating, 1995; McCandless and Weber, 1995; Ahmed and Rogers, 2000; Rapach, 2003). Also in a model with search frictions, Molico (2006) finds a hump-shape relationship between inflation and output. The key difference between Molico (2006) and my model is that the former considers bargaining in the trading process, instead of competitive search. In my model, the competitive search mechanism provides further reinforcement on the positive inflation effect because of the endogenous response along the extensive margin. In a cash-in-advance model, Camera and Chien (2011) show a negative relationship between inflation and aggregate output. Altogether, the literature seems to indicate that the trading frictions can be important in reconciling empirical observations on macro performances.

the choice of b across households tend to shrink. As a result, the differences in the measure of shops across submarkets also wither, which explains the positive relationship between inflation and price dispersion.[8] Furthermore, income tax reduces average price for a given inflation rate. However, taxation does not have a significant effect on the coefficient of variation of prices. Figure 7 shows the policy effects on aggregate markups. Panels A and B document the aggregate markups of price over marginal cost, respectively for the both sectors and for the special-good sector alone. Panels C and D present results in a similar fashion, although using a different measure of the markup. In particular, it is the markup of price over average cost. R8 summarizes the above findings:

R8. Inflation has a positive effect on the average real price and the price dispersion in the frictional market. Income taxation has a negative effect on the average price, but does not have a significant impact on price dispersion. Inflation increases the aggregate markup of the aggregate economy, but decreases that of the frictional market.[9] Income taxes increase the aggregate markup of both the aggregate economy and the frictional market alone.

Figure 8 depicts the effect of inflation on wage rates under various tax regimes. Panel A is for the normalized wage rate (i.e. nominal wage rate divided by the aggregate money stock) and Panel B is for the real wage rate. The key observations are the following:

R9. Inflation has a positive effect on the normalized wage rate and a negative effect on the real wage rate. The former effect weakens as the tax rate arises. Income taxes have the opposite effects, *i.e.* negative effect on the normalized wage rate and positive effect on the real wage rate.

Wealth distribution. Figure 9 shows the effect of inflation on wealth distribution at various tax rates. Here a household's wealth is interpreted as its beginning-of-period real

[8] This is in line with some of the empirical evidence on price dispersion for the U.S. (*e.g.* Reinsdorf, 1994; Parsley, 1996; Debelle and Lamont, 1997) and for other countries such as Israel and Argentina (*e.g.* Van Hoomissen, 1988; Lach and Tsiddon, 1992; Tommasi, 1992). There is also a theoretical literature that studies inflation and price dispersion. Some of these models demonstrate a positive relationship between anticipated inflation and price dispersion, *e.g.* Bénabou (1988, 1992a), Diamond (1993), Peterson and Shi (2004), Head and Kumar (2005) and Wang (2011). In a search environment, Molico (2006) reports a negative effect of inflation on price dispersion. In contrast to the above literature, my model generates price dispersion through competitive search among heterogeneous agents.

[9] There is no consensus in the empirical literature regarding the relationship between inflation and markups. For example, Bénabou (1992b) finds a negative correlation between inflation and retail markups. Chirinko and Fazzari (2000) documment a positive relationship between inflation and market power in eleven U.S. industries. Banerjee and Russell (2001) document a long-run negative correlation between inflation and the markup for the aggregate U.S. economy and for twelve of the fifteen sectors. Neiss (2001) finds a positive relationship between average inflation and the markups for OECD countries.

money balances after receiving the government transfer T given by (40). Therefore, the average wealth consists of aggregate savings, aggregate unspent balances and the transfer. As is indicated by Figure 2, at a higher inflation rate households choose to save less (lower h) and spend more (higher z) at a higher rate (higher b). Thus inflation decreases aggregate savings. Its effect on aggregate unspent balances can be ambiguous. On one hand, households plan on spending more, which means the level of unspent balances held by a household is also higher. On the other hand, households also choose to trade with a higher matching probability, which reduces the chance of holding an unspent balance across periods. Figure 2 seems to suggest that the effect of matching probabilities can be quite small, suggesting that inflation is likely to increase the aggregate unspent balances. The government transfer includes the monetary component to achieve money growth and the fiscal component from taxation on labor income. Both components increase with inflation. The former is because of money injection and the latter is because aggregate labor increases as inflation rises (see panel B of Figure 4). Overall, the negative effect of inflation dominates, which indicates that the negative impact of inflation on savings can be the dominating force. Now consider the positive relationship between income taxation and average wealth. Also from Figure 2, it is clear that households choose to save more (higher h) and spend less (lower z) at a lower rate (lower b). Moreover, higher tax rates reduce aggregate labor and thus the fiscal component of government transfers. Altogether, the positive effect of income taxation dominates, which again is likely to the result of the dominating effect on household savings.

Panel B of Figure 9 shows the effect of inflation on the coefficient of variation of wealth at various tax rates. Since all households receive the same amount of government transfers, wealth dispersion critically depends on the dispersion of household savings and that of unspent balances. A rise in inflation tends to increase dispersion in household unspent balances (due to trading frictions) and decrease dispersion in household savings, which is suggested to a certain extent in Figure 2 by the changes in the functions $z(\theta)$ and $h(\theta)$ under various policy regimes. Nevertheless, as the tax rate rises, households increase savings, which allows the effect of inflation on savings to make a stronger presence. As is shown in Panel B of Figure 9, the positive effect of inflation on dispersion in household unspent balances tends to dominate when the tax rate is low (*e.g.* $\tau = 0$, $\tau = 0.01$). At some intermediate tax rates, there can be a hump-shape relationship between inflation and wealth dispersion (*e.g.* $\tau = 0.02$, $\tau = 0.03$). Then at higher tax rates, the negative effect on savings dominates (see $\tau = 0.05$, $\tau = 0.1$ and $\tau = 0.2$).

R10. Inflation has a negative effect on average wealth while income taxation has a positive effect on average wealth. At low tax rates, inflation tends to increase wealth dispersion. At intermediate tax rates, wealth dispersion first increases and then decreases with inflation. Wealth dispersion decreases with inflation at high tax rates.[10]

Income and consumption inequality. Figure 10 and Figure 11 respectively report how inflation affects the coefficients of variation of household disposable income and consumption, at various tax rates. The key findings are summarized in R11. Inflation reduces income inequality because of the redistributive effect of lump-sum transfers to sustain money growth. This negative effect strengthens with higher taxes because income taxation suppresses the incentives to supply labor. This accentuates the redistributive effect of inflation. Income taxation increases income inequality. One interpretation is that the negative impact of taxation on aggregate labor income overpowers its effect on the variation of income. Inflation increases consumption inequality because it stimulates participation in the frictional market. Income taxation has little effect on consumption inequality for the following reasons: On one hand, income taxation discourages participation in the frictional market, which reduces standard deviation of consumption. On the other hand, income taxation also reduces aggregate consumption. The two effects end up cancelling each other out, possibly due to the quasi-linear preferences.

R11. Inflation has a negative effect on income inequality.[11] This effect gets stronger as the tax rate rises. Income taxation has a positive effect on income inequality. Inflation has a positive effect on consumption inequality,[12] but income taxation does not have a significant impact on consumption inequality.

Welfare. Figure 12 illustrates the effect of inflation on welfare under various tax regimes. Welfare is defined as the weighted average of the life-time discounted value W. The key results are in R12. On one hand, inflation helps increase output. On the other hand, inflation reduces savings and increases consumption inequality. Without distortionary

[10] This is in contrast with the results from the theoretical literature that examines the distributional effect of monetary policy. For example, in a search-theoretic model with bargaining, Molico (2006) shows that dispersion in money holdings first decreases and then increases as the inflation rate rises. Moreover, Chiu and Molico (2010) establish a negative relationship between inflation and the dispersion of the money distribution. In a model where heterogeneous agents use money to self-insure against liquidity shocks, Dressler (2011) demonstrates that inflation increases dispersion in money balances. In a cash-in-advance environment with heterogeneous agents, Camera and Chien (2011) show that inflation reduces wealth dispersion when money is the only asset, but has little effect on wealth inequality when bonds are introduced. Moreover, none of the above papers consider the relevance of a fiscal policy regime.

[11] Camera and Chien (2011) also show that inflation lowers income inequality. Nevertheless, there is no consensus in the empirical literature relating inflation to income distribution. Galli and Hoeven (2001) provide an extensive review over this literature and refer to the mixed results as the "inflation-inequality puzzle".

[12] In Camera and Chien (2011), lower inflation can increase consumption inequality when agents are not allowed to borrow.

taxes, inflation does not improve welfare. However, if imposed, taxes have a negative influence on output and also make households over-accumulate assets. In this case, some inflation can help counteract the effects of distortionary taxation and thus improve welfare. The higher the tax rate, the more prominent the positive effect of inflation on welfare.

R12. The welfare-maximizing policy is the Friedman rule ($\gamma = 0.995$) and zero income taxation. Nevertheless, if income taxation is imposed ($\tau > 0$), there tends to be a hump-shape relationship between inflation and welfare. The welfare-improving role of inflation strengthens as the tax rate increases.

The results in R1-R12 are robust to variation of parameter values satisfying restrictions described in (36) and satisfying that the labor choices of all households are strictly positive. Moreover, these results are also robust to variation of the distribution $F(\theta)$, *e.g.* left-skewed and right-skewed distributions, as well as symmetric distributions other than the uniform distribution considered in (36). Results on the robustness analysis are available upon request. Note that inflation has no effect on activities in the frictionless market whatsoever. Therefore, all of the non-trivial effects of long-run inflation summarized in R6-R12 are due to trading frictions. This demonstrates that the trading frictions play an important role in reconciling empirical observations on macro performance. Furthermore, a mechanism generating real responses along both the intensive and extensive margins, such as competitive search considered in this paper, can bring extra insights on the effects of monetary and fiscal policy.

6 Conclusion

I have constructed a tractable framework of competitive search that endogenously generates dispersion of prices, income and wealth. This model is used to study the long-run effects of inflation on various aspects of the macro performance, and to study whether the fiscal regime matters for such effects. Competitive search brings two important features to the model. First, the model allows for endogenous response of output along the extensive margin, as well as the intensive margin that the literature typically focuses on. Second, competitive search significantly improves the tractability of analysis because it makes the individual decision problem independent of the endogenous wealth distribution. The key findings from quantitative analysis are the following:

First, inflation has a positive effect on aggregate output, consumption inequality, average price and price dispersion. In the meanwhile, inflation reduces average wealth and income inequality.

Second, income taxation has a negative effect on the level of output, average price, and a positive effect on the level of average wealth and income inequality. It has little effect on the respective relationships between inflation and output, average price and average wealth. Nor does income taxation have any significant impact on price dispersion or consumption inequality. Nevertheless, taxation strengthens the negative relationship between inflation and income inequality.

Third, policy effect on wealth inequality is more complex. At low tax rates, inflation tends to increase wealth dispersion. At intermediate tax rates, there can be a hump-shape relationship between inflation and wealth dispersion. At higher tax rates, wealth dispersion decreases with inflation.

Finally, inflation can improve welfare when income taxation is imposed. The higher the tax rate, the stronger the welfare-improving role of inflation. Therefore, it is important to coordinate monetary and fiscal policies. The optimal inflation rate increases if the tax rate has been raised, and vice versa.

Appendix

A Proof of Lemma 2

Given (23), it is straightforward to see that the value function $B(z)$ is continuous. Moreover, $B(z) \geq 0$ for all $z \geq 0$, where the equality holds if and only if $b = 0$. If $b = 0$, the choice of q is irrelevant. Since B is continuous on a closed interval $[0, \hat{z}]$, the lotteries in (29) make $\tilde{V}$ concave (see Appendix F in Menzio and Shi, 2010, for a proof). I prove differentiability of $\tilde{V}$ in the proof of part (iii).

For part (i), define the left-hand side of (17) as $LHS(b)$ and impose $x = z$:

$$LHS(b) \equiv u(q) - \beta E(\theta) z + \left[\frac{u'(q)}{\psi'(q)}\right] \frac{kb\mu'(b)}{[\mu(b)]^2}, \tag{37}$$

where q is given by (18) with $x = z$. It is straightforward to derive that

$$\begin{aligned} LHS(b=0) &= u\left(\psi^{-1}(z-k)\right) - \beta E(\theta) z, \\ &= u(q) - \beta E(\theta)[\psi(q) + k], \end{aligned}$$

where (18) yields $q = \psi^{-1}(z-k)$ given $b = 0$. Thus the above implies that $LHS(b=0) > 0$ if and only if there exists $q > 0$ such that condition (26) holds. Moreover, one can further derive $LHS(b=1) = -\infty$, and

$$\begin{aligned} LHS'(b) &= u'(q) q'(b) + \frac{u''(q)\psi'(q) - u'(q)\psi''(q)}{[\psi'(q)]^2} q'(b) \left(\frac{kb\mu'(b)}{[\mu(b)]^2}\right) \\ &\quad + k\left[\frac{u'(q)}{\psi'(q)}\right] \frac{\mu(b)[\mu'(b) + b\mu''(b)] - 2b[\mu'(b)]^2}{[\mu(b)]^3} < 0. \end{aligned}$$

Given all the above results, condition (26) implies that there exists $z > 0$ such that $b > 0$. Furthermore, the above results imply that the policy function $b(z)$ is unique, which further implies that $q(z)$ is also unique given (18). Given $x = z$, (16) implies

$$\frac{u'(q)}{\psi'(q)} - \beta E(\theta) > 0. \tag{38}$$

Therefore, for z such that $b > 0$,

$$\frac{\partial LHS(b;z)}{\partial z} = \frac{u'(q)}{\psi'(q)} - \beta E(\theta) + \frac{kb\mu'(b)[u''(q)\psi'(q) - u'(q)\psi''(q)]}{[\mu(b)]^2[\psi'(q)]^3} > 0.$$

This implies that an increase of z shifts the entire function $LHS(b)$ upwards. Because $LHS'(b) < 0$, it follows that $b'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, (17) holds with equality. Total differentiating (17) by z yields

$$\begin{aligned} 0 &= u'(q)\,q'(z) - \beta E(\theta) + \frac{kb\mu'(b)\left[u''(q)\,\psi'(q) - u'(q)\,\psi''(q)\right]}{[\mu(b)]^2\,[\psi'(q)]^3}\,q'(z) \\ &\quad + k\,\frac{u'(q)}{\psi'(q)}\left[\frac{\mu(b)\left[\mu'(b) + b\mu''(b)\right] - 2b\left[\mu'(b)\right]^2}{[\mu(b)]^3}\right] b'(z)\,. \end{aligned}$$

Given $b'(z) > 0$ and Assumption 1, rearranging the above yields $q'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, one can derive that

$$B'(z) = b'(z)\left[u(q(z)) - \beta E(\theta)\,z\right] + b(z)\left[\frac{u'(q(z))}{\psi'(q(z))} - \beta E(\theta)\right] > 0.$$

This is because $b'(z) > 0$ and the trade surplus, $u(q(z)) - \beta z E(\theta)$, is strictly positive given $b > 0$, and also condition (38). Obviously, $b(z)$ is strictly decreasing in $E(\theta)$, given the results about $LHS(b)$ in part (ii). Then (28) implies that $q(z)$ is strictly increasing in $E(\theta)$.

For part (ii), note that the previous proof has established that $b(z)$ is continuous and increasing in all $z \in [0, \hat{z}]$. In particular, $b(z)$ is strictly increasing in z if $b > 0$. It is obvious from (18) that $b(z) = 0$ for all $z \in [0, k]$. Continuity of $b(z)$ implies that there exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$.

I now prove part (iii) and the differentiability of $\tilde{V}$ together. If $b(z) = 0$ for all z, then obviously $\tilde{V}(z)$ is differentiable. Now consider the case where there exists z such that $b(z) > 0$, i.e., condition (26) holds. It is obvious that $B(z)$ is differentiable for all z such that $b(z) > 0$. Consider z such that $b(z) > 0$. Recall that a concave function has both left-hand and right-hand derivatives (see Royden, 1988, pp113-114). Let $\tilde{V}'(z^-)$ and $\tilde{V}'(z^+)$ be the left-hand and right-hand derivatives, respectively. Suppose $\tilde{V}'(z^-) > \tilde{V}'(z^+)$ for some z such that $b(z) > 0$. Then $\tilde{V}$ is strictly concave at such z, which implies $\tilde{V}(z) = B(z)$. It follows that $B'(z^-) \geq \tilde{V}'(z^-) > \tilde{V}'(z^+) \geq B'(z^+)$, where the first and the last inequalities follow from the construction of lotteries. However, $B'(z^-) > B'(z^+)$ contradicts the differentiability of B. Therefore, the value function $\tilde{V}(z)$ is differentiable for all z such that $b(z) > 0$. Part (ii) has established that there exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$. This has two implications: First, $B'\left(z_1^-\right) = 0$ because $b(z) = B(z) = 0$ all $z \in [0, z_1]$. Second, $B'\left(z_1^+\right) > 0$ because $b(z) > 0$ in the right neighborhood of z_1. Therefore, B is strictly convex but not differentiable at z_1 because $0 = B'(z^-) < B'(z^+)$. Strict convexity of B at z_1 implies that there is a lottery over the region $z \in [0, z_1]$. Let the winning prize of this lottery be z_0. Then all households with $z \in (0, z_0)$ will play this lottery and receive zero payment if they lose. Moreover, it must be the case that $z_0 > z_1$, $b(z_0) > b(z_1) > 0$ and $B(z_0) = \tilde{V}(z_0) > 0$. Given $b(z_0) > 0$, both value functions are differentiable at z_0 and $B'(z_0) = \tilde{V}'(z_0) > 0$. Therefore, $\tilde{V}$ is differentiable for all $z \in [0, z_0]$ because of the lottery. Moreover, $\tilde{V}$ is also differentiable for all $z > z_0$ because $b > 0$ for all $z > z_0$. **QED**

B Proof of Theorem 2

Recall the normalized wage rate w^* as given in (33). Note that all the policy functions in the right-hand side of (33) are independent of w^*. It is obvious that $w^* > 0$ exists. Therefore, a stationary equilibrium exists and is characterized by w^*. It is unique if and only if the lottery choices $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ are unique for all $z(\theta)$. Part (i) follows from (11). Part (ii) follows condition (31). For part (iii), recall from Lemma 2 that there exists $z > 0$ such that the policy function $b(z) > 0$ if any only if condition (26) holds for some $q > 0$. Therefore, if (26) does not hold, then $b(z) = 0$ for all z. Moreover, $B(z) = B'(z) = \tilde{V}(z) = \tilde{V}'(z) = 0$ for all z. In this case, the household does not trade in the frictional market. Therefore, there is no need to hold a positive balance for transaction purposes, i.e., $z(\theta) = 0$ for all θ. Consider the case where condition (26) holds for some $q > 0$. In this case, there exists $z > 0$ such that the policy function $b(z) > 0$, according to Lemma 2. Note that condition (9) implies that $z(\theta) > 0$ if $V_z(0, h) \geq \theta$. If $V_z(0, h) < \theta$, then $z(\theta) = 0$ is optimal. If $z(\theta) > 0$, $b(L_2(z(\theta))) > 0$ follows from construction of the lottery. **QED**

C Proof of Proposition 1

Substituting (35) into the left-hand side of (34) yields

$$LHSP(b) \equiv u(q) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z + \left(\frac{u'(q)}{\psi'(q)}\right) \frac{kb\mu'(b)}{[\mu(b)]^2}. \tag{39}$$

Similar to the proof of Lemma 2, one can show that $LHSP'(b) < 0$. Given $b > 0$, all results in this proposition follow trivially. **QED**

D Government Transfers and Market Clearing

In this Appendix, I further characterize the market-clearing conditions and the formula for the government transfer. The analysis in this Appendix is carried out with the monetary and fiscal policies. For the benchmark case without any policy, one can simply apply $\gamma = 1$ and $\tau = 0$ to all the derivations in what follows.

With policies, the definition of a stationary equilibrium must satisfy one more condition that the government balances its budget every period. Therefore, the total dollar amount of transfers that a household receives in a period consists of the transfer for monetary policy purposes and the transfers for fiscal policy purposes. For money growth, the household receives a dollar amount of $(\gamma - 1) M$, which is equivalent to $(\gamma - 1) M / (wM') = (\gamma - 1) / (w\gamma)$ units of labor. For income taxation, the amount of the government transfer in terms of labor units is τLS. Altogether, the total real transfer is given by

$$T^* = \frac{\gamma - 1}{w^* \gamma} + \frac{\tau}{\gamma} LS. \tag{40}$$

For part (iv) of the equilibrium definition, the market-clearing condition for the general-good market is

$$Y = \int_{\underline{\theta}}^{\bar{\theta}} y(\theta)\, dF(\theta)\,. \tag{41}$$

The market-clearing condition for the labor market is aggregate demand for labor, LD, is equal to aggregate supply of labor, LS. Consider LD first. A household's realization of θ determines the money balance $z(\theta)$. Given this money balance, the resulted money balance after lotteries is $L_i(z(\theta))$, $i = 1, 2$, which takes place with probability $\pi_i(z(\theta))$. Thus the measure of such households is $N_b = \pi_i(z(\theta))\, dF(\theta)$. The measure of shops corresponding to the households holding $L_i(z(\theta))$ is given by

$$N_s = \pi_i(z(\theta))\, dF(\theta)\, b(L_i(z(\theta))) / [\mu(b(L_i(z(\theta))))]\,,$$

which is derived from $b/\mu(b) = N_s/N_b$ given the constant-return-to-scale matching technology. Then for each shop, the expected labor demand is $k + \psi(q)\mu(b)$, which is used to compute the aggregate demand for labor in the frictional markets. Note that such calculation is also valid for cases when some households do not use lotteries and when $L_i(z(\theta))$

is the same for some different realizations of θ. Thus, LD is given by

$$
\begin{aligned}
LD &= Y + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\pi_1 (z(\theta)) b(L_1(z(\theta)))}{\mu(b(L_1(z(\theta))))} [k + \psi(q(L_1(z(\theta))))\mu(b(L_1(z(\theta))))] dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} \frac{\pi_2 (z(\theta)) b(L_2(z(\theta)))}{\mu(b(L_2(z(\theta))))} [k + \psi(q(L_2(z(\theta))))\mu(b(L_2(z(\theta))))] dF(\theta).
\end{aligned} \tag{42}
$$

The firm's zero-profit condition (3) implies that for $i = 1, 2$,

$$
k + \psi(q(L_i(z(\theta))))\mu(b(L_i(z(\theta)))) = L_i(z(\theta))\mu(b(L_i(z(\theta)))).
$$

Then (42) can be transformed to

$$
\begin{aligned}
LD &= \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) b(L_1(z(\theta))) L_1(z(\theta)) dF(\theta) \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) b(L_2(z(\theta))) L_2(z(\theta)) dF(\theta).
\end{aligned} \tag{43}
$$

The aggregate labor supply is given by

$$
LS = \int_{\underline{\theta}}^{\bar{\theta}} \int l(m, \theta) dG_a(m) dF(\theta),
$$

where $G_a(m)$ is the money distribution at the beginning of a period. Recall $l(m, \theta)$ from (32) given $t_g = 0$. Thus,

$$
LS = \int_{\underline{\theta}}^{\bar{\theta}} \int \frac{1}{1-\tau} [py(\theta) + z(\theta) + h(\theta) - m - T^*] dF(\theta) dG_a(m).
$$

Use (40) to substitute for T^* in the above. Also recall the constraint for the household's lottery choice, $\pi_1(z(\theta)) L_1(z(\theta)) + \pi_2(z(\theta)) L_2(z(\theta)) = z(\theta)$. It follows that

$$
\begin{aligned}
\left(1 - \tau + \frac{\tau}{\gamma}\right) LS &= \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} h(\theta) dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} z(\theta) dF(\theta) \\
&\quad - \int m dG_a(m) - \frac{\gamma - 1}{w^* \gamma}.
\end{aligned} \tag{44}
$$

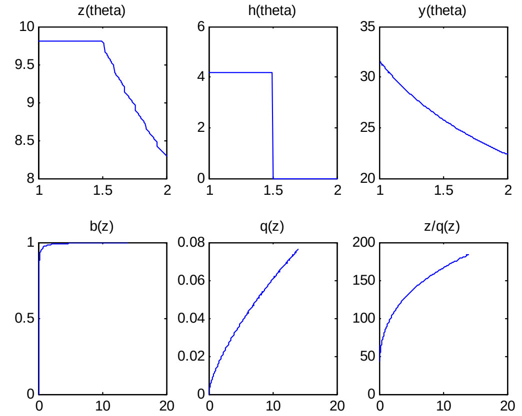
Because m is a household's money balance at the beginning of a period, it consists of the money balance carried over for savings purposes and if any, the transactional balance unspent due to matching frictions. Thus,

$$\int m dG_a(m) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{h(\theta)}{\gamma} dF(\theta) + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) \left[1 - b(L_1(z(\theta)))\right] \frac{L_1(z(\theta))}{\gamma} dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) \left[1 - b(L_2(z(\theta)))\right] \frac{L_2(z(\theta))}{\gamma} dF(\theta). \quad (45)$$

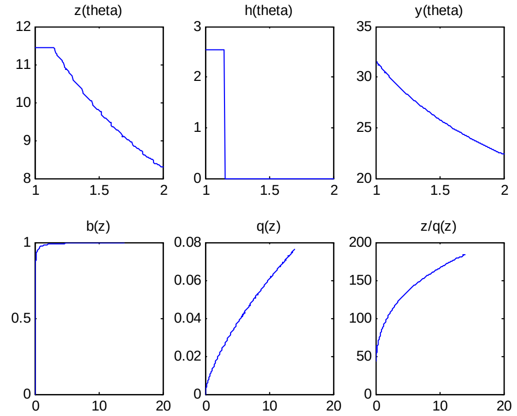
The labor-market clearing requires $LD = LS$. Thus (43), (44) and (45) together solve for the normalized wage rate in the steady state:

$$\begin{aligned}(w^*)^{-1} = & \int_{\underline{\theta}}^{\bar{\theta}} h(\theta) dF(\theta) + \tau \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) dF(\theta) \\ & + \tau \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) b(L_1(z(\theta))) L_1(z(\theta)) dF(\theta) \\ & + \tau \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) b(L_2(z(\theta))) L_2(z(\theta)) dF(\theta) \\ & + \int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta)) \left[1 - b(L_1(z(\theta)))\right] L_1(z(\theta)) dF(\theta) \\ & + \int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta)) \left[1 - b(L_2(z(\theta)))\right] L_2(z(\theta)) dF(\theta). \end{aligned} \quad (46)$$

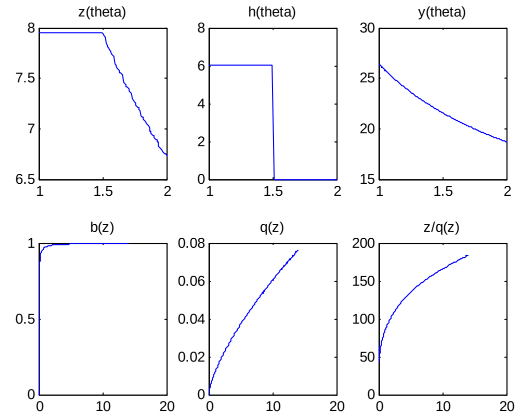
Note that the formula in (33) is clearly given by setting $\tau = 0$ in the above equation. Given that the labor market clears, the money market clears by Walras' law. Note that $(w^*)^{-1}$ is essentially the normalized price of money in terms of labor.



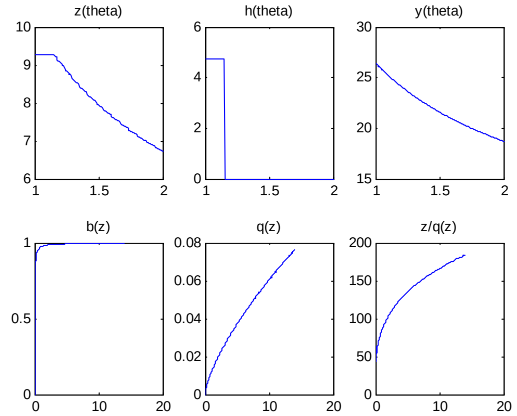
A. $\gamma = 0.997, \tau = 0$



B. $\gamma = 1.3, \tau = 0$

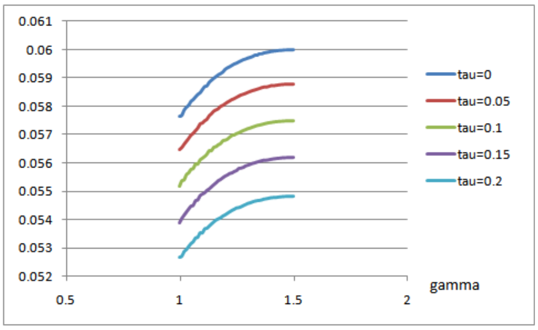


C. $\gamma = 0.997, \tau = 0.3$

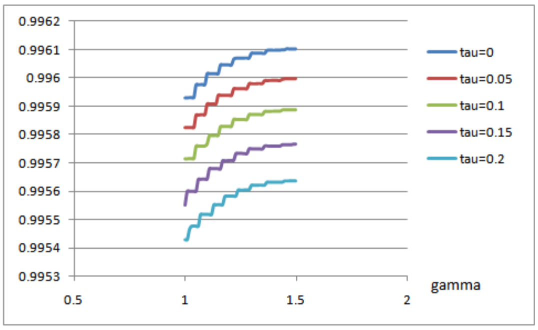


D. $\gamma = 1.3, \tau = 0.3$

Figure 2. Policy functions

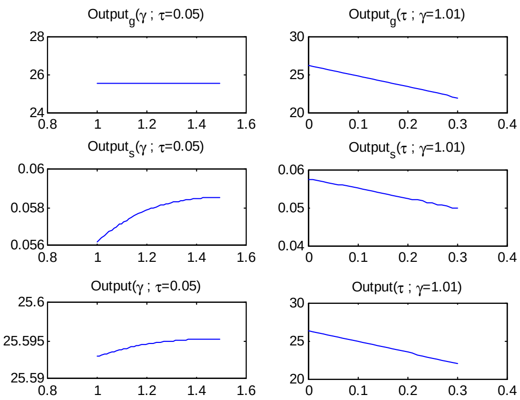


A. Average quantity per trade

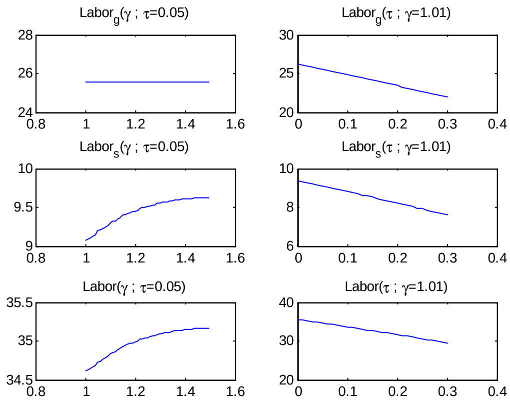


B. Volume of transactions

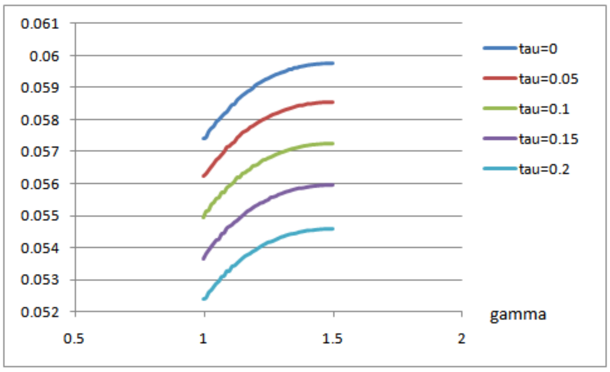
Figure 3. Inflation, intensive margin and extensive margin



A. Aggregate output

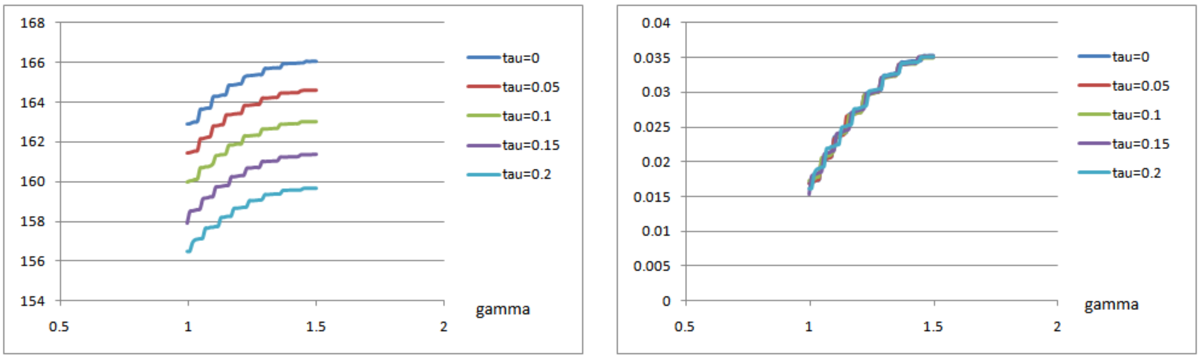


B. Aggregate labor



C. Aggregate output in the frictional market

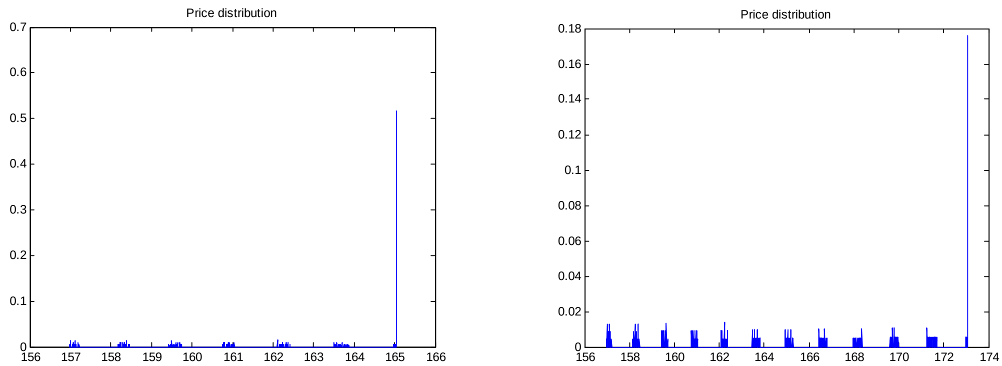
Figure 4. Policy effect on output and labor



A. Average price

B. Coefficient of variation of prices

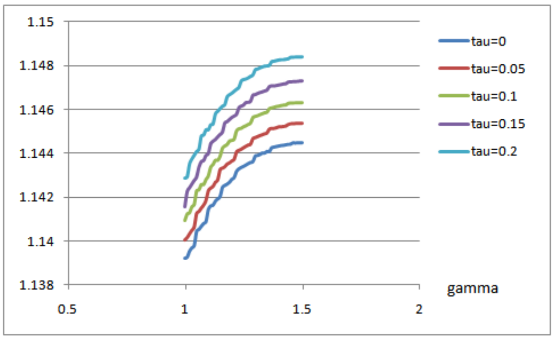
Figure 5. Inflation and real prices in the frictional market



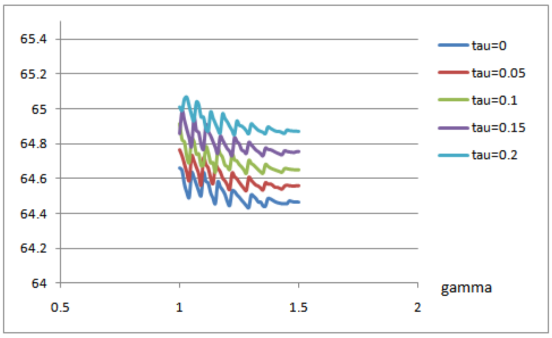
A. $\gamma = 0.997, \tau = 0$

B. $\gamma = 1.3, \tau = 0$

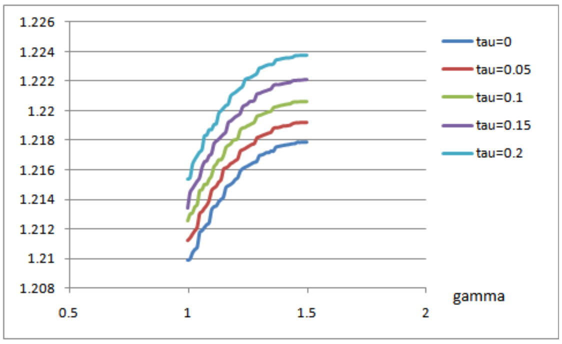
Figure 6. Price distribution in the frictional market



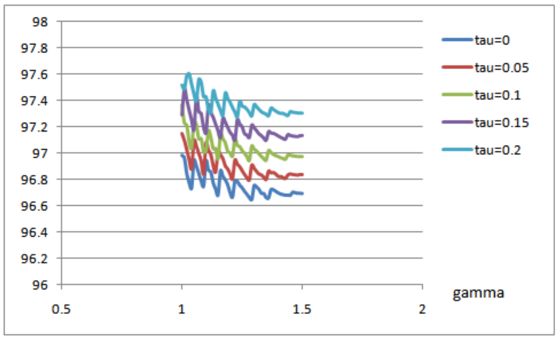
A. Aggregate markup
(price over marginal cost)



B. Aggregate markup of the frictional market (price over marginal cost)[13]

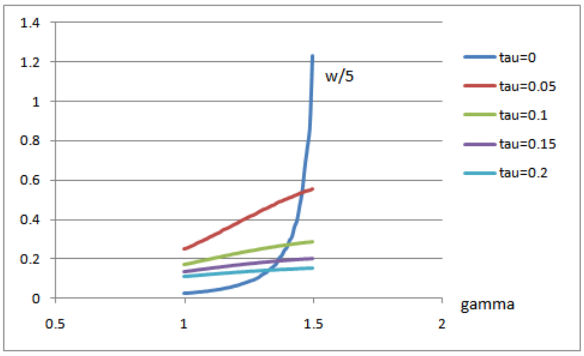


C. Aggregate markup
(price over average cost)

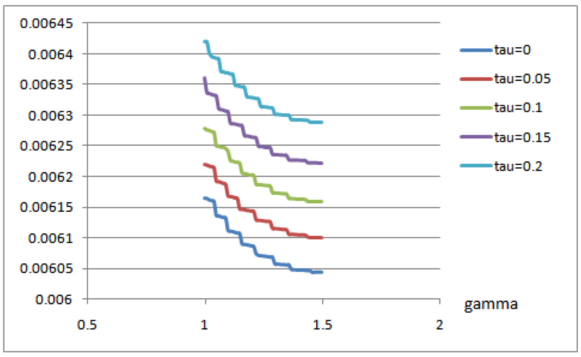


D. Aggregate markup of the frictional market (price over average cost)

Figure 7. Inflation and aggregate markups



A. Normalized wage rate[14]

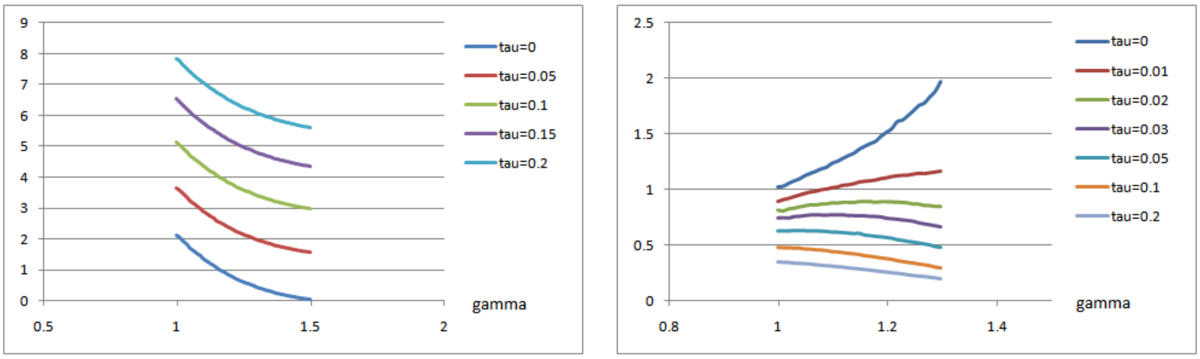


B. Real wage rate

Figure 8. Inflation and wage rates

[13] There appears to be considerable fluctuations in this graph. However, such "fluctuations" are only generated by computational imprecision due to gridding and do not bear real economic meaning. This also applies other graphs in this section that display some irregularities.

[14] For $\tau = 0$, the graph shows the values of the normalized wage rate divided by 5.



A. Average wealth B. Coefficient of variation of wealth

Figure 9. Inflation and wealth distribution

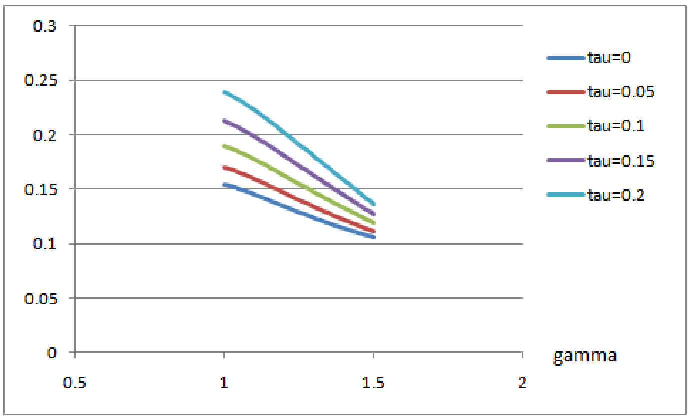


Figure 10. Inflation and income inequality

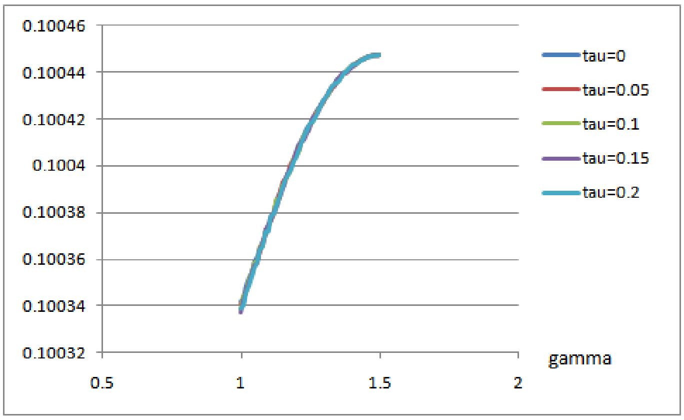


Figure 11. Inflation and consumption inequality

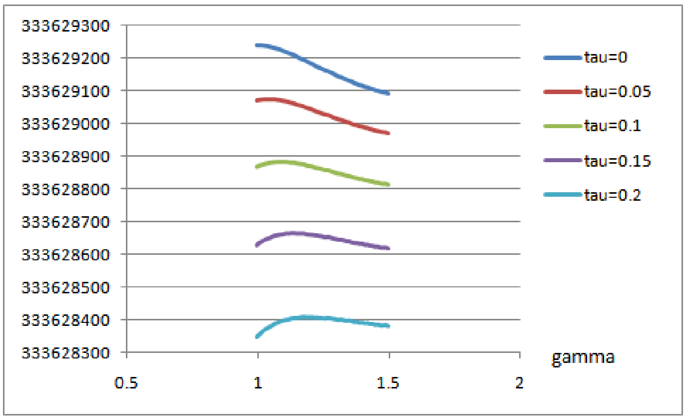


Figure 12. Inflation and welfare

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A List of the Notation

β: discount factor;
$U(y)$: a household's utility of consuming y units of general goods;
$u(q)$: a household's utility of consuming q units of special goods;
l: a household's wage income in a period;
z: a household's balance to spend in the frictional market;
h: a household's balance for savings;
$\psi(q)$: labor input needed for producing q units of special goods;
θ: a household's random disutility per unit of labor;
$F(\theta)$: CDF of the random shock θ;
k: a firm's cost of operating a shop in a frictional submarket, measured in labor units;
N_s, N_b: numbers of shops and buyers, respectively, in a submarket;
$\mathcal{M}(N_b, N_s)$: aggregate number of matches in a submarket with N_b buyers and N_s shops;
$s = \mu(b)$: primitive matching function;
M: aggregate stock of money per capita in a period;
p: price of general goods in terms of labor;
w: normalized wage rate; nominal wage rate divided by the aggregate money stock;
m: a household's real money balance, measured in terms of labor;
x: money spending in a frictional trade, measured in labor;
$W(m, \theta)$: a household's value at the beginning of the first sub-period;
$V(z, h)$: a household's value at the beginning of the second sub-period;
$\tilde{V}(z)$: a household's value of the lottery choice;
$B(z)$: a household's value immediately after the lottery takes place but before trading in the frictional submarket;
L_i: the realization in a lottery;
π_i: the probability with which L_i is realized in the lottery;
z_0: the prize in a lottery participated by the households with low balances of z;
γ: money growth rate;
τ: proportional income tax rate;
T: lump-sum government transfer.