On the Coexistence of Money and Higher-Return Assets and its Social Role

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Hicks' (1935) puzzle

"The critical question arises when we look for an explanation of **the preference for holding money rather than capital goods**. For **capital goods will ordinarily yield a positive rate of return, which money does not**. What has to be explained is the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities. (...) This, as I see it, is really the central issue in the pure theory of money."

Hellwig (1993): The Challenge of Monetary Theory

Problem 1: Why does ‘worthless’ fiat money have a positive value in exchange against goods and services when there are other assets whose own rates of return in each period exceed the own rate of return on money?

Coexistence of money and higher-return assets: Still a puzzle

- DGE models impose CIA constraints or MIU
  - The "great evaders"

"Models with real balances in utility or production functions or with CIA constraints are, in several respects, direct descendants of monetary theory as it existed 100 years ago. In particular, some of the defects of that 100 year-old theory show up in these descendants in ways that are not widely acknowledged". Wallace (2005)

Coexistence of money and higher-return assets: Still a puzzle

- Modern monetary theory explains why fiat money has a positive value in exchange...
- But the money-asset margin is (largely) unresolved
  - Aruoba-Waller-Wright (2010): restrictions on the use of capital
  - Lagos-Rocheteau (2008): rate-of-return equality

Punchline of the paper

- In monetary economies with pairwise trades, whenever fiat money is essential, any optimal, incentive-feasible allocation is such that capital generates a higher rate of return than fiat money.
- Rate of return dominance is not a puzzle: it is a property of good allocations in monetary economies.

My approach

1. "Mechanism design" to identify the salient properties of good allocations in monetary economies
2. Money and capital compete as media of exchange
3. Quasi-linear environment: tractable
4. Rounds of pairwise meetings: amenable to mechanism design

Literature

1. **Mechanism design approach to monetary theory**: Kocherlakota (1998); Cavalcanti and Wallace (1999); Hu, Kennan, and Wallace (2009); Wallace (2010)
2. **Rate of return dominance**: Aiyagari, Wallace, and Wright (1996); Zhu and Wallace (2007); Li, Rocheteau, and Weill (2011).
3. **Money and capital**: Freeman (1985); Shi (1999); Molico and Zhang (2006); Lagos and Rocheteau (2008); Aruoba, Waller, and Wright (2010).

Environment

- Time: $t \in \mathbb{N}$.
- Each t divided into two stages:
  1. Decentralized market (DM) with a measure σ of pairwise meetings
  2. Centralized market (CM).
- A measure two of infinitely-lived agents
  - Divided evenly among buyers and sellers.

Preferences

- Buyers' preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\overbrace{u(q_t)}^{\text{DM}}+\overbrace{c_t-h_t}^{\text{CM}}\right],$$

  where $\beta \equiv (1+r)^{-1} \in (0,1)$.

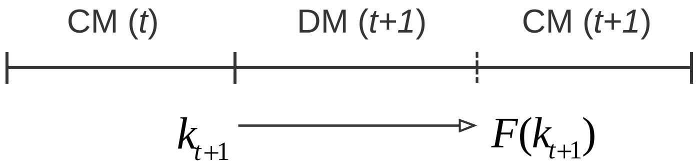
- Sellers' preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\overbrace{-\upsilon(e_t)}^{\text{DM}}+\overbrace{c_t-h_t}^{\text{CM}}\right]$$

- Linear technologies: $q=e$ and $c=h$.
- $q^* = \arg\max\left[u(q)-\upsilon(q)\right]$

Capital

- CM good transformed one-to-one into capital good.
- Capital used by sellers to produce the CM good according to $F(k)$.
- $F'(k)k$ is strictly increasing and strictly concave: e.g., $F(k) = k^{\alpha}$.
- Capital goods depreciate fully after one period.
- Rental price of capital: R_t.



Money

- Frictions:
  1. lack of commitment
  2. no enforcement
  3. no record-keeping of individual histories
- A fixed supply, M, of fiat money: intrinsically useless, perfectly divisible.
- Asset holdings are common knowledge in a match.
- I restrict sellers not to hold assets from one period to the next.

Trading mechanism in pairwise meetings

- Indeterminacy in bilateral matches: a non-degenerate set of allocations that are (pairwise) Pareto efficient
- **Standard approach**: axiomatic bargaining solutions (e.g., Nash), strategic games (e.g., ultimatum game)
- Problems:
  1. Mechanism can generate its own inefficiencies (beyond the ones created by monetary frictions)
  2. The essentiallity of money can only be established by using mechanism design.
- **Our approach**: Search for optimal, incentive-feasible mechanisms

Trading mechanism in pairwise meetings (Cont'ed)

Terms of trade in a bilateral match where the buyer holds z real balances and k units of capital:

1. A proposed allocation: $(q, d_z, d_k) \in \mathbb{R}_+ \times [0, z] \times [0, k]$
   - q is output; d_z is a transfer of real balances; d_k is a transfer of capital goods.
   - Allocation in the pairwise core.
2. The buyer and the seller simultaneously say "yes" or "no".
   - If they both say "yes", the trade takes place.
   - Otherwise, there is no trade.

Bellman equations

- A buyer in the DM:

$$V^b(z,k) = \sigma \left\{ u\left[q(z,k)\right] + \overbrace{W^b\left[z - d_z(z,k), k - d_k(z,k)\right]}^{\text{value in CM if trade}} \right\}$$

$$+ \overbrace{(1-\sigma) W^b(z,k)}^{\text{value in CM (no trade)}}$$

The CM problem of the buyer is

$$W^b(z,k) = \max_{\hat{z}\geq 0, \hat{k}\geq 0} \left\{ \overbrace{z + Rk - \hat{z} - \hat{k}}^{=c-h} + \beta V^b(\hat{z},\hat{k}) \right\}$$

- $\hat{z}$ and $\hat{k}$ are independent of (z,k);
- $W^b(z,k) = z + Rk + W^b(0,0)$.

Stationary, symmetric allocations

- The object to implement: A 5-tuple $(q^p, d_z^p, d_k^p, z^p, k^p)$
- Two necessary conditions for incentive feasibility:

1. Buyer's participation constraint in the CM:

$$-rz^p - \overbrace{\left[\beta^{-1} - F'(k^p)\right]}^{\text{cost of capital}} k^p + \sigma \overbrace{\left[u(q^p) - d_z^p - F'(k^p) d_k^p\right]}^{\text{Buyer's DM surplus}} \geq 0, \quad (1)$$

where I used that $F'(k) = R$.

2. Seller's participation constraint in the DM:

$$\overbrace{-\upsilon(q^p) + d_z^p + F'(k^p) d_k^p}^{\text{Seller's DM surplus}} \geq 0. \quad (2)$$

Implementation

$(q^p, d_z^p, d_k^p, z^p, k^p)$ that satisfies $R = F'(k^p) \leq \beta^{-1}$, (1), and (2) can be implemented by the following mechanism:

1. $[q(z,k), d_z(z,k), d_k(z,k)] =$

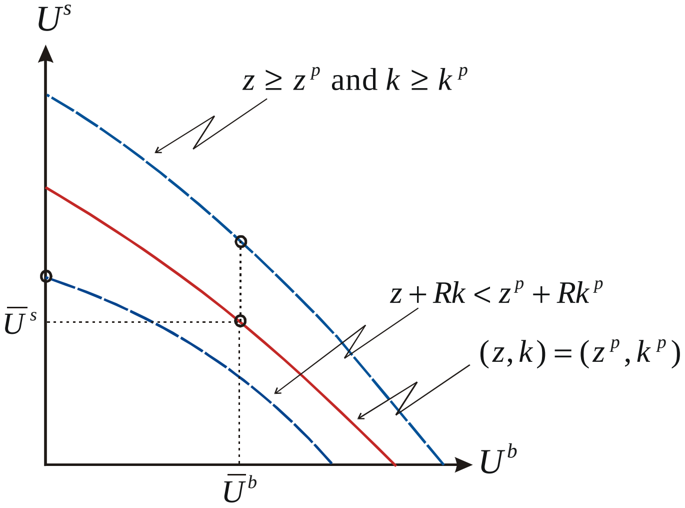
$$\arg \max_{q, d_z \leq z, d_k \leq k} \left[d_z + F'(k^p) d_k - v(q) \right]$$
$$\text{s.t.} \quad u(q) - d_z - F'(k^p) d_k \geq u(q^p) - d_z^p - F'(k^p) d_k^p$$

if $z \geq z^p$ **and** $k \geq k^p$,

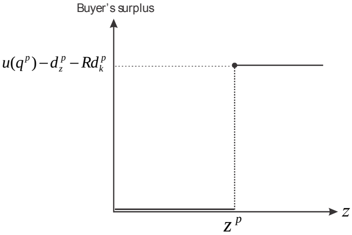
2. $[q(z,k), d_z(z,k), d_k(z,k)] =$

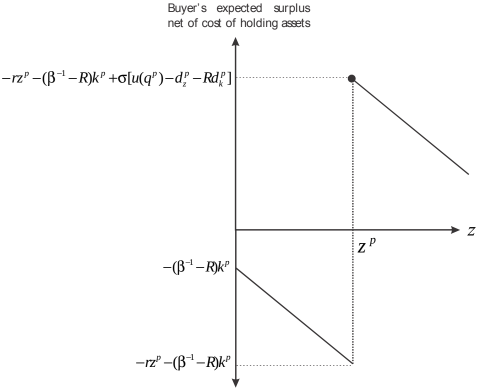
$$\arg \max_{q, d_z \leq z, d_k \leq k} \left[d_z + F'(k^p) d_k - v(q) \right]$$
$$\text{s.t.} \quad u(q) - d_z - F'(k^p) d_k = 0,$$

otherwise.



Incentive-feasible mechanism


Buyer's surplus
$u(q^p)-d_z^p-Rd_k^p$
z
z^p


Buyer's expected surplus net of cost of holding assets
$-rz^p-(\beta^{-1}-R)k^p+\sigma[u(q^p)-d_z^p-Rd_k^p]$
z
z^p
$-(\beta^{-1}-R)k^p$
$-rz^p-(\beta^{-1}-R)k^p$

Optimal, incentive-feasible allocation

- Pick the allocation that maximizes society's welfare among all implementable allocations.

$$\left(q^p, d_z^p, d_k^p, z^p, k^p\right) \in \arg\max \left\{\sigma\left[u(q) - \upsilon(q)\right] + F(k) - \beta^{-1}k\right\}$$

$$\text{s.t.} \quad -rz - \left[\beta^{-1} - F'(k)\right]k + \sigma\left[u\left(q\right) - d_z - F'(k)d_k\right] \geq 0$$

$$-\upsilon\left(q\right) + d_z + F'(k)d_k \geq 0.$$

$$\beta^{-1} - F'(k) \geq 0$$

$$d_z \in \left[0, z\right], \quad d_k \in \left[0, k\right].$$

Nonmonetary economy

- Liquidity shortage:

$$\Omega \equiv \upsilon(q^*) - (1+r)k^*.$$

- The optimal, incentive-feasible allocation is such that:

1. If $\Omega \leq 0$, then $q^p = q^*$ and $k^p = k^*$.
2. If $\Omega > 0$, then $q^p < q^*$ and $k^p > k^*$.

Monetary economy

The optimal, incentive-feasible allocation is such that:

1. If $\Omega \leq 0$, then $q^p = q^*$ and $k^p = k^*$.
2. If $0 < \Omega \leq \frac{\sigma[u(q^*)-v(q^*)]}{r}$, then $z^p = d_z^p > 0$, $q^p = q^*$ and $k^p = k^*$.
3. If $\Omega > \frac{\sigma[u(q^*)-v(q^*)]}{r}$, then $z^p = d_z^p > 0$, $q^p < q^*$ and $d_k^p = k^p$ such that $F'(k^p) \in (1, \beta^{-1}]$. Moreover, if $r + F''(k^*)k^* > 0$, then $k^p > k^*$.

Coexistence: its social role

- Participation constraint:

$$-rz - \left[\beta^{-1} - F'(k)\right] k + \sigma \left[u(q) - d_z - F'(k) d_k\right] \geq 0$$

- Provided that $F'(k) > 1$ (rate-of-return dominance), using k instead of z relaxes participation constraint.
- But a social cost if $F'(k) < \beta^{-1}$.

Individually rational rate-of-return dominance

- Why do agents hold money if it is dominated in its rate of return?
- A mechanism that can punish or reward agents depending on the portfolio they carry
  - If buyers accumulate more than k^p units of capital, then they receive no additional surplus in the DM.
  - A feature of a non-degenerate core in pairwise meetings

Is the mechanism "realistic"?

- The buyer obtains better terms of trade if:

1. He holds enough wealth to buy large quantities
   - Discount for bulky trades
2. A fraction of his payment, $\frac{z^p}{z^p+Rk^p}$, is in terms of money.
   - Some form of reserve requirement

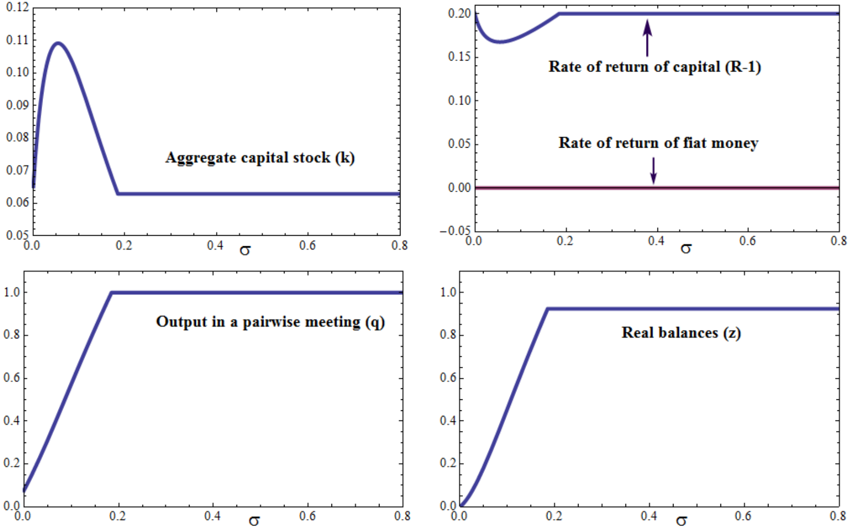


Figure: $A = 1.1$, $\alpha = 0.95$, $r = 0.2$

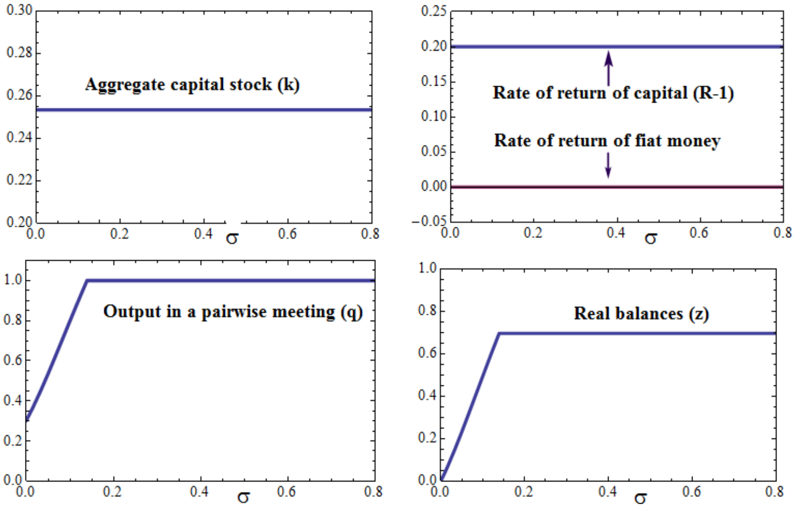


Figure: $A = 2,\ \alpha = 0.2,\ r = 0.2$

Inflation and capital

- $M_{t+1} = \gamma M_t$, where $\gamma \equiv 1 + \pi$ is constant.
- The buyer's IR constraint in the CM:

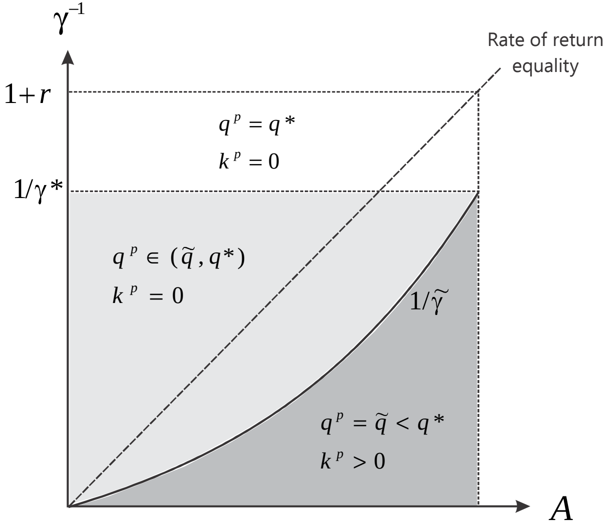
$$
\begin{gathered}
-\left(\gamma\beta^{-1} - 1\right) z^p - \left[\beta^{-1} - F'(k^p)\right] k^p \\
+\sigma\left[u\left(q^p\right) - d_z^p - F'(k^p) d_k^p\right] \geq 0.
\end{gathered}
$$

Optimal, incentive-feasible allocation

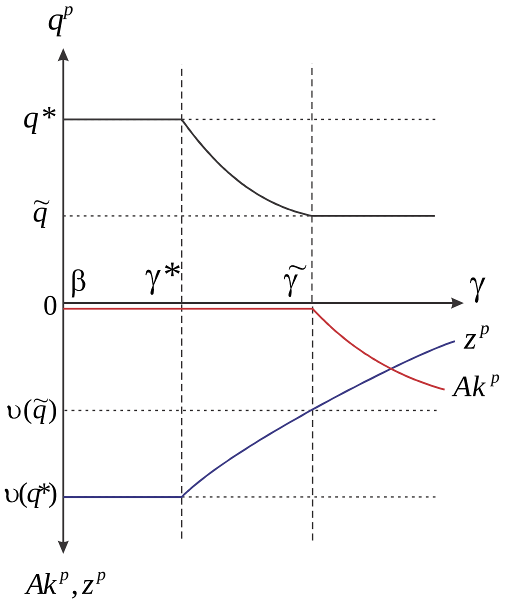
Assume $\Omega > 0$. There exists $\gamma^* \equiv \beta \left\{ 1 + \frac{\sigma[u(q^*) - v(q^*)]}{\Omega} \right\} > \beta$ such that:

1. For all $\gamma \leq \gamma^*$, $q^p = q^*$ and $k^p = k^*$.
2. For all $\gamma > \gamma^*$, $q^p < q^*$ and $F'(k^p) \in (\gamma^{-1}, \beta^{-1}]$. Moreover, if $\gamma > \frac{1}{F''(k^*)k^* + 1 + r}$, then $k^p > k^*$.

Linear technology



$F(k) = Ak$



$F(k) = Ak$, with $A < 1 + r$.

Effects of inflation

- For low inflation rates ($\gamma \leq \gamma^*$), no welfare effect
  - The welfare cost of small inflation is 0.
  - No need to implement the Friedman rule
- For intermediate inflation rates:
  - A negative effect on real balances and output
  - No effect on capital (No Tobin effect)
- For large inflation rates:
  - Agents substitute capital for real balances (Tobin effect)

Conclusion

- I applied mechanism design to an environment where money and capital compete as media of exchange.
- Coexistence of money and higher-return assets is both socially optimal and individually rational whenever money plays an essential role.
- Positive and normative implications:
  - The Friedman rule is not necessary to implement good allocations
  - For low inflation rates, there is no Tobin effect and no cost of inflation.
- What this paper does not do: explain the coexistence of fiat money and interest-bearing government bonds.
  - See Kocherlakota (2003) and Wallace (2010).