

Inflation and Welfare with Search and Price Dispersion

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 - ▶ Small welfare cost of inflation and no interaction of two channels.

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- Decompose the welfare effect of inflation by different channels, identify the source of inefficiency, and understand the interaction of different channels.
 - ▶ Three channels: real balance channel, price posting channel, and search channel.
 - ▶ The coexistence of the price posting channel and the real balance channel generates an amplifying negative effect on welfare.

Related Literature

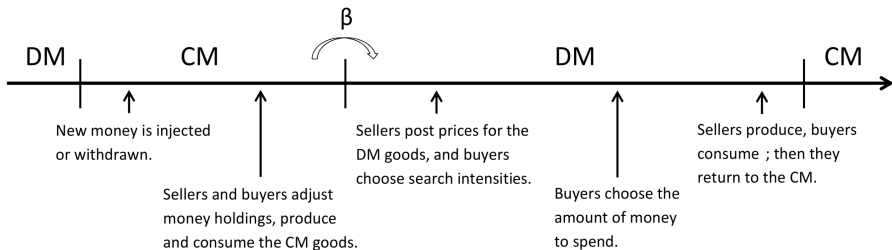
- Welfare Cost of Inflation:
 - ▶ *Money, Price Dispersion, and Search*: Head and Kumar (2005), Dutu, Julien, and King (2011)
 - ▶ *Opportunity Cost of Holding Money*: Cooley and Hansen (1989, 1991), Lucas (2000); Lagos and Wright (2005), Rocheteau and Wright (2009)
 - ▶ *Relative Price Dispersion*: Clarida, Gali, and Gertler (1999), Burstein and Hellwig (2008)
 - ▶ *Relative Price Dispersion and Search*: Benabou (1988, 1992), Diamond (1993)
- Endogenous Price Dispersion:
 - ▶ Burdett and Judd (1983), Head, Liu, Menzio, and Wright (2011)

THE MODEL

The Model

- A continuum of homogeneous buyers and sellers, each with measure 1.
- Time is discrete. Each period is divided into two subperiods:
 - ▶ 1st subperiod: bilateral trade, decentralized market (DM)
 - ▶ 2nd subperiod: Walrasian trade, centralized market (CM)
- Money is the essential medium of exchange for bilateral trades in the DM, and it is perfectly storable and divisible.
 - ▶ Money supply follows $M_{t+1} = (1 + \gamma)M_t$. New money is injected (withdrawn) equally to each buyer in a lump-sum transfer at the beginning of the CM.
 - ▶ In a stationary equilibrium, $\phi_t M_t = \phi_{t+1} M_{t+1}$ and $\phi_t / \phi_{t+1} = 1 + \gamma$, ϕ_t is the price of money in terms of the CM consumption goods in period t .

Timeline



Buyer's Problem

Centralized Market (CM)

- Buyer's value function in the CM is

$$W^b(z) = \max_{x, h, \hat{z}} \left\{ v(x) - h + \beta V^b(\hat{z}) \right\}$$
$$s.t. \quad h + z + T = x + (1 + \gamma)\hat{z}$$

- ▶ x : CM consumption; h : CM labor; z and \hat{z} : real balance of the current and the next period; $T = \gamma\phi M$ is the lump-sum transfer.

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- ▶ x : CM consumption; h : CM labor; z and \hat{z} : real balance of the current and the next period; $T = \gamma\phi M$ is the lump-sum transfer.
- The CM value function is linear in z , $W^b(z) = W^b(0) + z$.
 - ▶ The optimal choice of x and \hat{z} are independent of z ,

$$v'(x^*) = 1, \quad \beta \frac{\partial V^b(\hat{z}^*)}{\partial \hat{z}} = (1 + \gamma).$$

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Decentralized Market (DM)

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 - ▶ With probability α , the buyer samples two prices.
- The buyer's optimization problem in the DM is

$$V^b(z) = \max_{\alpha \in [0,1]} \left\{ \int_{\mathcal{Z}_F} \left[u \left(\frac{d^*(p; z)}{p} \right) + W^b [z - d^*(p; z)] \right] dG(p; \alpha) - \alpha k \right\}$$

$d^*(p; z)$ is the buyer's optimal shopping rule for the DM goods, and $G(p; \alpha)$ is the distribution of transaction prices, defined as

$$G(p; \alpha) = (1 - \alpha)F(p) + \alpha \left[1 - (1 - F(p))^2 \right]$$

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- ▶ $\alpha^* \in (0, 1)$ only if $\int_{\mathcal{Z}_F} \left[u \left(\frac{d^*(p; z)}{p} \right) - d^*(p; z) \right] [1 - 2F(p)] dF(p) = k$.

Buyer's Problem

Optimal Shopping Rule in the DM

- In the DM, the buyer's optimal shopping rule $d^*(p; z)$ is the solution to

$$\begin{aligned} \max_d u\left(\frac{d}{p}\right) - d, \\ \text{s.t. } d \leq z \end{aligned}$$

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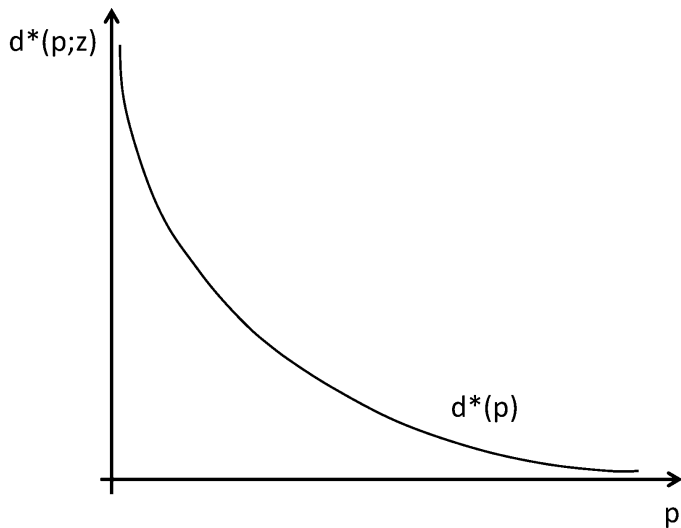
$$\begin{aligned} \max_d u\left(\frac{d}{p}\right) - d, \\ \text{s.t. } d \leq z \end{aligned}$$

- If the DM utility function $u(q)$ has the CRRA form with coefficient $\sigma < 1$, then

$$d^*(p; z) = \begin{cases} z, & \text{if } p < \hat{p} \\ d^*(p), & \text{otherwise} \end{cases}$$

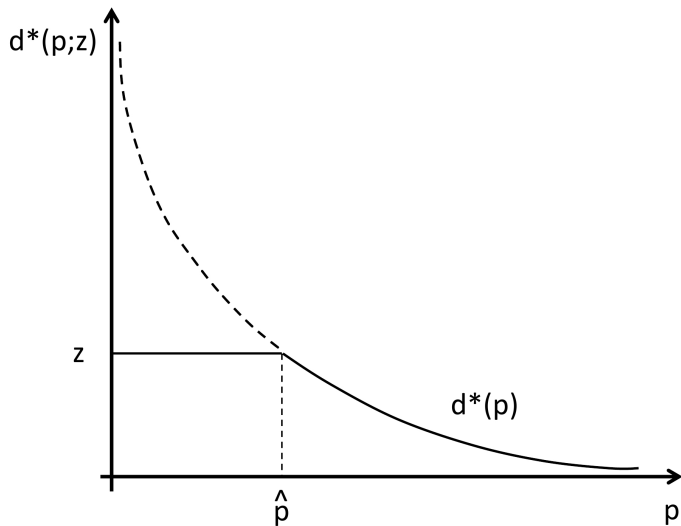
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Optimal Shopping Rule in the DM



Buyer's Problem

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Seller's Problem

Profit Function in the DM

- Given z , α , d , and F , the seller's profit function in the DM is

$$\pi(p) = \underbrace{[1 - \alpha + 2\alpha(1 - F(p))]}_{\text{expected trade volume}} \underbrace{\left(d(p; z) - c \frac{d(p; z)}{p} \right)}_{\text{profit per trade}}.$$

- ▶ The trade-off between profit per trade and expected trade volume generates endogenous price dispersion.

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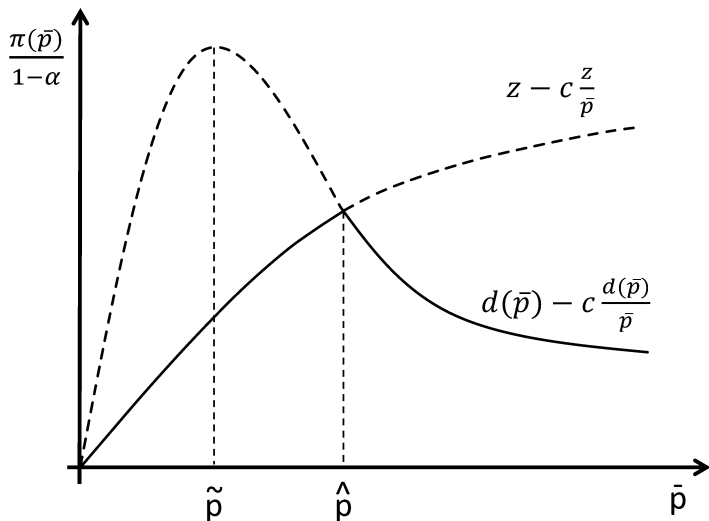
- The trade-off between profit per trade and expected trade volume generates endogenous price dispersion.
- At the upper limit \bar{p} , the profit function becomes

$$\pi(\bar{p}) = (1 - \alpha) \left[d(\bar{p}; z) - c \frac{d(\bar{p}; z)}{\bar{p}} \right].$$

- The seller chooses \bar{p} to maximize $\pi(\bar{p})$, and $\bar{p}^* = \max\{\hat{p}, \bar{p}\}$.
 - ★ Recall that \hat{p} is the cut-off price in the buyer's optimal shopping rule.

Seller's Problem

Upper Limit of Price Distribution



Seller's Problem

Price Distribution in the DM

- For any price p on the support \mathcal{Z}_F , $p \in \arg \max_p \pi(p)$, in particular,

$$\pi(p) = [1 - \alpha + 2\alpha(1 - F(p))] \left(d(p; z) - c \frac{d(p; z)}{p} \right) = \pi(\bar{p})$$

- Given the buyer's real balance z , search intensity α , and shopping rule d , $F(p)$ in the decentralized market is uniquely characterized as
 - if $\alpha = 0$, $F(p)$ is concentrated at \bar{p} .
 - if $\alpha = 1$, $F(p)$ is concentrated at c .
 - if $\alpha \in (0, 1)$, $F(p)$ is nondegenerate and continuous with a connected support $\mathcal{Z}_F = [p, \bar{p}]$, and it is given by

$$F(p) = 1 - \frac{1 - \alpha}{2\alpha} \left[\frac{d(\bar{p}; z)(1 - \frac{c}{\bar{p}})}{d(p, z)(1 - \frac{c}{p})} - 1 \right].$$

Seller's Problem

Value Function in the CM

- In the centralized market, the seller's value function is

$$W^s(z) = \max_{x, h, \hat{z}} v(x) - h + \beta [\pi^* + W^s(\hat{z})]$$
$$s.t. \quad h + z = x + (1 + \gamma)\hat{z}$$

- In equilibrium sellers do not carry any money into the decentralized market, i.e., $\hat{z}^* = 0$.

Equilibrium

A symmetric stationary monetary equilibrium (SSME) is a profile $\{F^*, z^*, x^*, h^*, d^*, \alpha^*\}$ satisfying the following conditions:

- 1 Given d^* , z^* , and α^* , F^* is consistent with the seller's optimal price posting strategy in the DM.
- 2 Given F^* , d^* , and α^* , z^* , x^* , and h^* solve the buyer's optimization problem in the CM.
- 3 Given F^* and z^* , the buyer's optimal shopping rule in the DM is characterized by d^* .
- 4 Given F^* , z^* , and d^* , α^* solves the buyer's search problem in the DM.

Equilibrium

- There exists no SSME with $\alpha = 1$ or $\alpha = 0$ if $1 + \gamma > \beta$.

Equilibrium

- There exists no SSME with $\alpha = 1$ or $\alpha = 0$ if $1 + \gamma > \beta$.
- If $1 + \gamma > \beta$ and the buyer's decentralized-market utility function has the CRRA form with coefficient $\sigma < 1$, SSME exists with nondegenerate price distribution for $k \leq \bar{k}$.

Equilibrium

Three Channels

- Real Balance Channel
 - ▶ If buyers carry a smaller real balance, welfare decreases.

- Price Posting Channel
 - ▶ If sellers post higher real prices, welfare decreases.

- Search Channel
 - ▶ If buyers search harder, welfare increases.

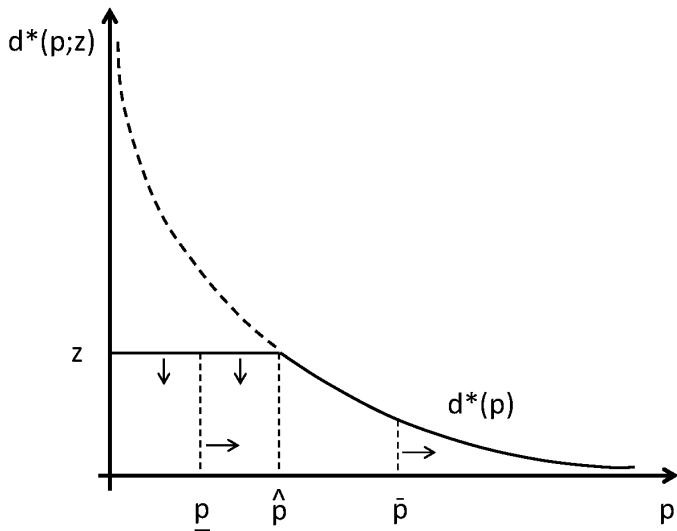
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 - ▶ If buyers carry a smaller real balance, welfare decreases.
 - ▶ If γ gets bigger, buyers bring less money, price distribution increases in the sense of first order stochastic dominance, and they will search less intensively.
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Buyer's Optimal Shopping Rule in the DM



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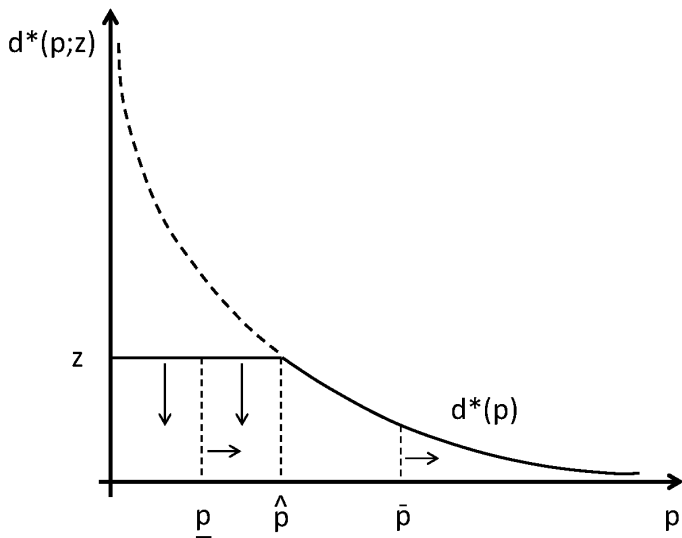
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- ▶ If buyers expect higher prices, they will carry a smaller real balance. Since price dispersion also increases, buyers will search harder.

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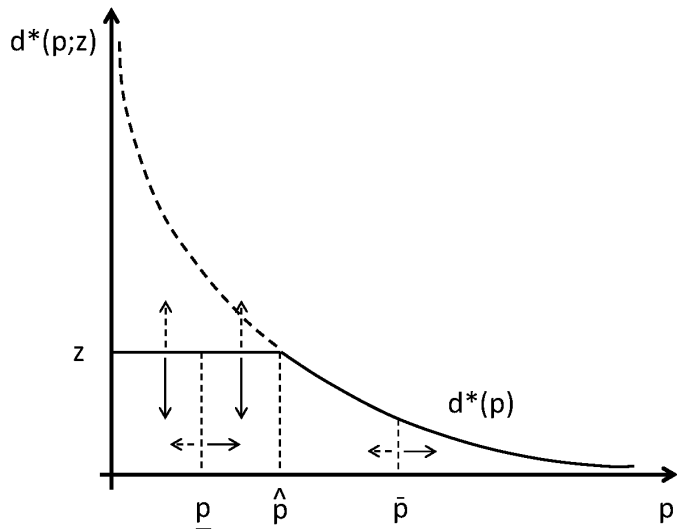
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- Search Channel

- ▶ If buyers search harder, welfare increases.
- ▶ If a higher search intensity is chosen, buyers will carry more money, and the price distribution decreases in the sense of first order stochastic dominance.

Equilibrium

Buyer's Optimal Shopping Rule in the DM



CALIBRATION

Calibration

Functional Forms and Calibration Strategy

Centralized Market (CM): $v(x) = A \log x - h$

Decentralized Market (DM): $u(q) = \frac{q^{1-\sigma}}{1-\sigma}$ and $c(q) = q$

Parameter	Target
β discount factor	annual real interest rate 4%
k search cost	magnitude of price dispersion
A CM preference	money demand
σ elasticity of demand	money demand

Calibration Targets

Money Demand

- Model-generated money demand function

$$L(i) = \frac{M/P}{Y} = \frac{z}{2A + \int_{\underline{p}}^{\bar{p}} \frac{d(p;z)}{p} dG(p; \alpha)},$$

where i is nominal interest rate.

- U.S. annual data on nominal GDP, M1, and short-term (6-month) commercial paper rate from 1900 to 2000. (Lucas 2000, Lagos & Wright 2005) [▶ More](#)

Calibration Targets

Magnitude of Price Dispersion

- Relative Price Variability (RPV)

$$RPV = \left[\int_{\underline{p}}^{\bar{p}} (R_i - \bar{R})^2 dF \right]^{\frac{1}{2}}$$

where $R_i = \log(p_i / \bar{p})$.

- Targets from empirical studies on inflation and price dispersion based on annual U.S. price data of the retail sector. [▶ More](#)
 - ▶ Average RPV 0.035 at annual inflation rate of 4.3% (DeBelle & Lamont, 1997)
 - ▶ Average RPV 0.0923 at 5.3% (Parsley, 1996)

Calibration

Results and Model Fitness

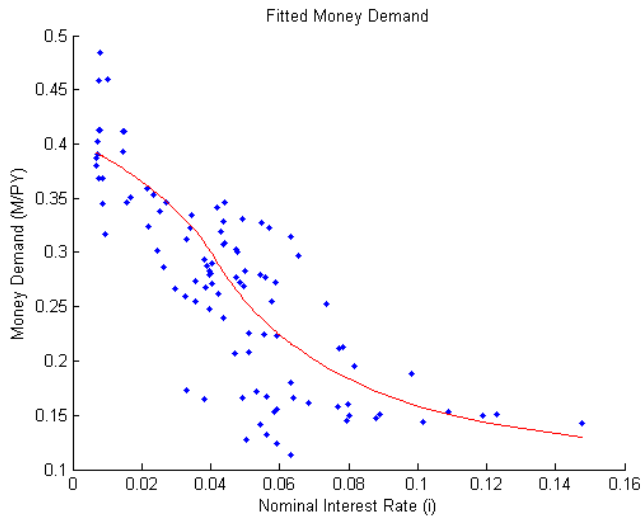
- Benchmark: match RPV target 0.035 and money demand.

β	k	A	σ	μ_{DM}	μ
0.9615	0.0043	0.4916	0.1181	9.72%	2.4%

- ▶ μ_{DM} and μ stand for markup in the decentralized market and average markup, respectively.
- Model Fitness:
 - ▶ Model-generated money demand function vs. money demand data.

Calibration

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- ▶ μ_{DM} and μ stand for markup in the decentralized market and average markup, respectively.
- Model Fitness:
 - ▶ Model-generated money demand function vs. money demand data.
 - ▶ Price dispersion is regressed against inflation.
 - ★ In the model, the coefficient of inflation is 0.2784.
 - ★ In Debelle and Lamont (1997) the range of coefficients is (0.12, 0.39) with an average of 0.21.

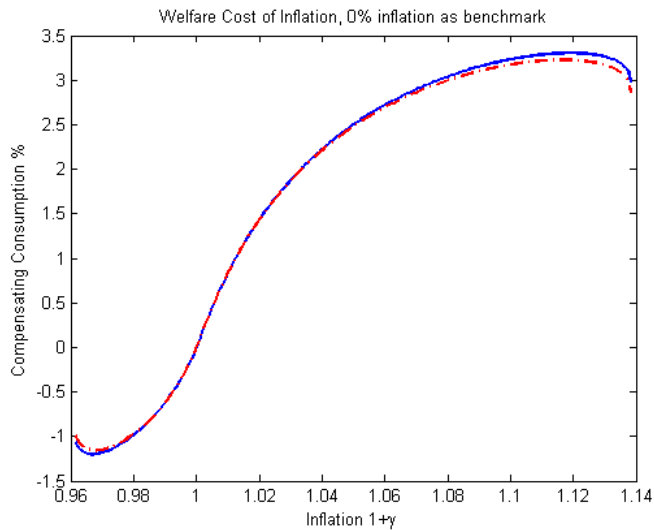
WELFARE ANALYSIS

Welfare Cost of Inflation

- Social welfare is the total surplus from trade in the centralized market and the decentralized market minus the cost of search.
- Welfare cost of inflation is measured by compensated consumption
 - ▶ How much agents would be willing to increase or decrease their consumption in the benchmark equilibrium with zero inflation in order to be indifferent to the economy with τ percent inflation?
- The welfare cost of 10% annual inflation is worth 3.23% of consumption.
- Compare with previous studies:

Model	Cooley & Hansen	Lucas	Lagos & Wright	Burstein & Hellwig	Aruoba & Schorfheide
3.23%	0.52%	<1%	4.6%	1.31%	0.6%

Welfare Cost of Inflation

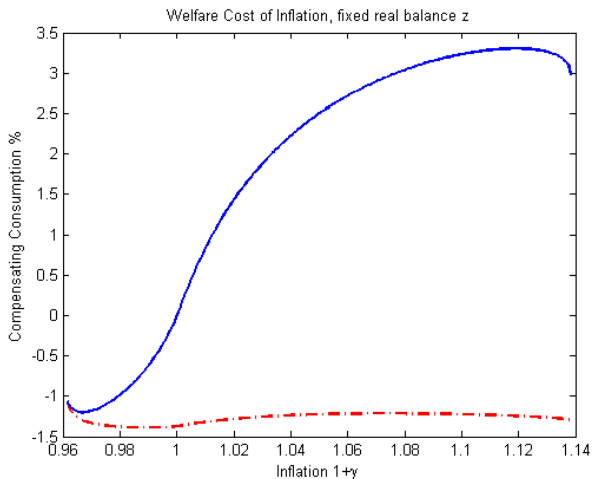


Welfare Cost Decomposition

- 1 Solve the equilibrium real balance, search intensity, and price distribution of the model at different levels of inflation.
- 2 Keep two endogenous objects (channels) at their equilibrium levels, and change the values of the third object (channel) by holding it constant at a benchmark level.
- 3 Using the welfare of the original economy at zero inflation as the benchmark, calculate the welfare cost of the artificial economy at different levels of inflation.
- 4 Compare the new welfare cost of inflation with the original value, and the difference represents the contribution of the third channel.

Welfare Cost Decomposition

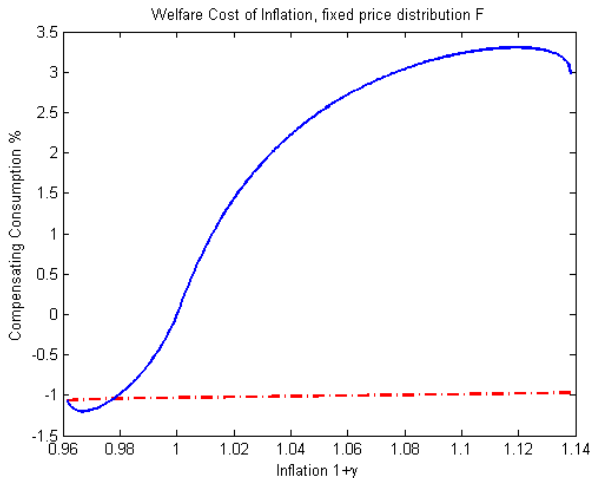
Constant Real Balance



The welfare cost of 10% annual inflation decreases to 0.04% of consumption.

Welfare Cost Decomposition

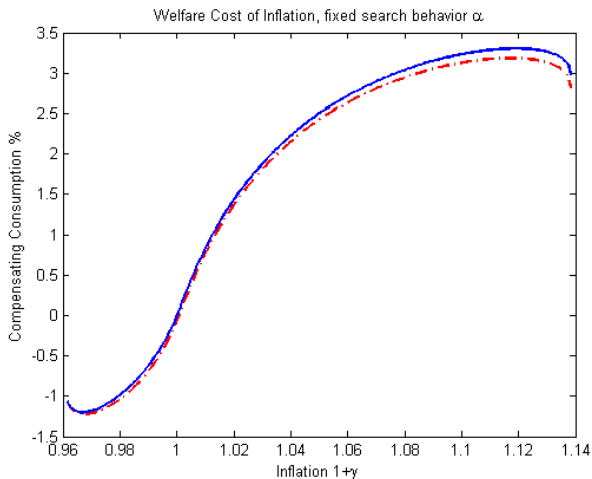
Constant Price Distribution



The welfare cost of 10% annual inflation decreases to 0.15% of consumption.

Welfare Cost Decomposition

Constant Search Intensity



The welfare cost of 10% annual inflation decreases by 0.1% of consumption.

Welfare Cost Decomposition

Definition of Social Welfare: Review

- Social welfare is the total surplus from trade in the centralized market and in the decentralized market minus the cost of search

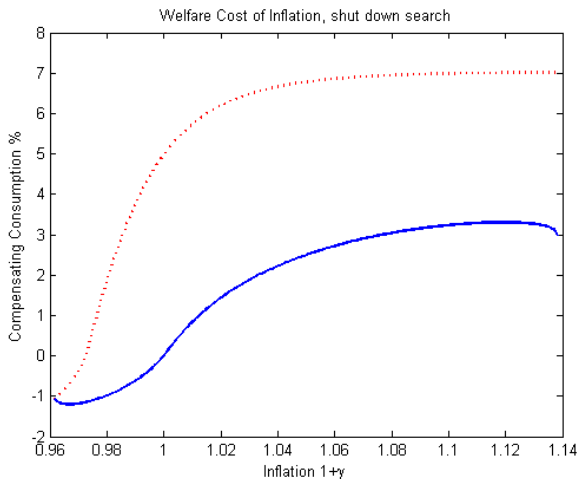
$$(1 - \beta)\mathcal{W}(\tau) = \int_{\underline{p}}^{\bar{p}} \left[u \left(\frac{d^*(p; z^*)}{p} \right) - c \frac{d^*(p; z^*)}{p} \right] dG(p; \alpha^*) - \alpha^* k + 2 [v(x^*) - x^*]$$

and $G(p; \alpha^*)$, the distribution of transaction prices is

$$G(p; \alpha^*) = (1 - \alpha^*)F(p) + \alpha^* \left[1 - (1 - F(p))^2 \right].$$

Welfare Cost Decomposition

Shut Down Search Channel



The welfare cost at 10% annual inflation increases to 6.99% of consumption.

Welfare Cost Decomposition

Summary

- If either the real balance or the price posting channel is held constant, the welfare cost significantly decreases from 3.23% to less than 0.15% of consumption.
- The main source of inefficiency resides in the interaction of the real balance channel and the price posting channel.
 - ▶ The coexistence of these two channels generates an amplifying effect.
 - ▶ The total negative effect on welfare exceeds the positive effect due to the search channel.
 - ▶ The search channel mostly affects welfare indirectly by changing the distribution of posted prices.
 - ▶ The search cost alone generates a negligible welfare loss.

Conclusion

- In this paper, I develop a general equilibrium model with money, price dispersion, and consumer search to study the welfare cost of inflation.
 - ▶ Search friction is the only driving force for price dispersion and the circulation of money.
- The welfare cost of 10% annual inflation is worth 3.23% of consumption, and it decreases to less than 0.15% of consumption if either real balance or price distribution is held constant.
 - ▶ Calibration exercises with alternative targets and different sample periods generate similar results.
 - ▶ Welfare cost is positively correlated with the value of the price dispersion target.
- Inflation has a non-monotonic effect on social welfare, and the optimal monetary policy is above the Friedman rule.

Conclusion

- In this paper, I develop a general equilibrium model with money, price dispersion, and consumer search to study the welfare cost of inflation.
 - ▶ Search friction is the only driving force for price dispersion and the circulation of money.
- The welfare cost of 10% annual inflation is worth 3.23% of consumption, and it decreases to less than 0.15% of consumption if either real balance or price distribution is held constant.
 - ▶ The price posting channel amplifies the welfare-diminishing effect of the real balance channel, and the aggregated negative effect exceeds the positive effect due to the search channel.
 - ▶ The search cost alone generates a negligible welfare loss.
 - ▶ Calibration exercises with alternative targets and different sample periods generate similar results.
- Inflation has a non-monotonic effect on social welfare, and the optimal monetary policy is above the Friedman rule.

Welfare Cost of Inflation

Welfare Cost and Price Dispersion

- Recalibrate the model to match different targets of relative price variability (RPV) at 4.3% inflation:

RPV	k	A	σ	μ	$1 - \Delta_0$
0.035	0.0043	0.4916	0.1181	2.47%	3.2%
0.06	0.0072	0.5396	0.3256	10.65%	12.7%
0.1	0.0362	1.0373	0.6091	12.38%	15.4%

- Welfare cost gets higher with a bigger target of RPV, which implies a higher search cost and less competition in the decentralized market.

Alternative Calibration Targets

- Target for search cost:
 - ▶ Average RPV 0.0923 at the annual inflation rate of 5.3%. (Parsley, 1996)
 - ▶ DM markup 30% at an annual inflation of 5.46%. (Faig and Jerez 2005)
- Target for elasticity of demand: interest elasticity of money demand

$$\log z_t = b_0 + b_1 \log i_t + b_2 \log y_t + b_3 \log z_{t-1} + u_t$$

- ▶ assume $u_t = \rho u_{t-1} + \varepsilon_t$ and apply Cochrane-Orcutt procedure to correct first-order serial correlation. (Goldfeld and Sichel 1990; Aruoba, Waller, and Wright 2010)

Alternative Calibration Targets

Calibration Results and Welfare Cost

	Baseline	Target 2	Target 3	Target 4
k	0.0043	0.0328	0.0057	0.003
A	0.4916	1.0134	0.5064	0.3309
σ	0.1181	0.5326	0.2111	0.1005
μ_{DM}	9.72%	47.13%	25.64%	12.21%
μ	2.4%	8.75%	5.52%	2.95%
$1 - \Delta_0$	3.23%	8.31%	7.23%	3.25%

Shorter Sample Period

1959-2000

	Baseline	Target 2	Target 3	Target 4
k	0.0089	0.033	0.0061	0.0062
A	1.1841	1.5488	0.6175	0.8702
σ	0.1861	0.5262	0.2200	0.1441
μ_{DM}	8.34%	45.14%	35.32%	9.96%
μ	1.42%	5.78%	4.79%	1.67%
$1 - \Delta_0$	2.34%	5.08%	6.68%	2.69%

▶ Conclude

Buyer's Problem

Optimal Shopping Rule in the DM

- If $\sigma > 1$, the buyer's optimal spending rule is

$$d^*(p; z) = \begin{cases} d^*(p), & \text{if } p < \hat{p} \\ z, & \text{if } \hat{p} \leq p \leq p^R \\ 0, & \text{otherwise} \end{cases}$$

where p^R satisfies $u(d^*(p^R; z)/p^R) = d^*(p^R; z)$, $\partial p^R / \partial z > 0$.

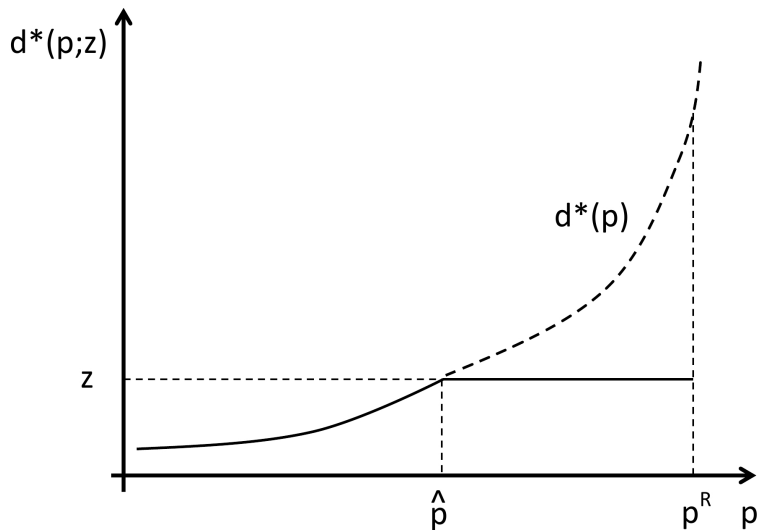
- If $\sigma = 1$, the buyer's optimal spending rule is

$$d^*(p; z) = \begin{cases} \min\{\tilde{d}, z\}, & \text{if } p \leq p^R \\ 0, & \text{otherwise} \end{cases}$$

where \tilde{d} is a constant satisfying $u'(\frac{\tilde{d}}{p}) = p$.

Buyer's Problem

Optimal Shopping Rule in the DM



Price Dispersion Data

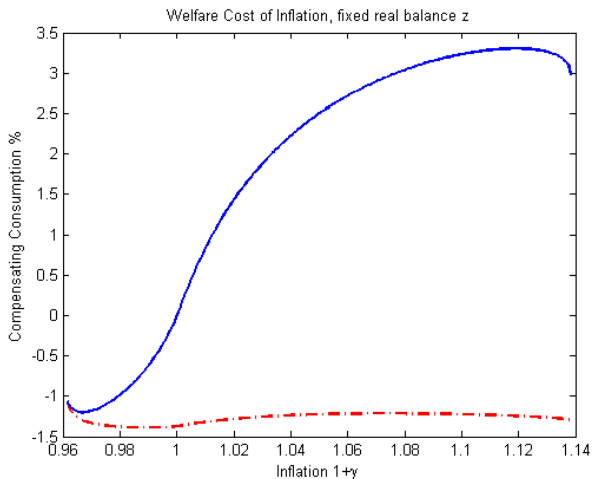
- DeBelle and Lamont (1997): two balanced panel of CPI data from BLS, 1954-1986 for 19 cities and 14 kinds of goods and services, 1977-1986 for 24 cities and 18 kinds of goods and services, mostly from the retail sector.
- Parsley (1996): quarterly survey data from Cost of Living Index published by the American Chamber of Commerce Researchers Association, a panel from 1975 to 1992 with 48 cities and 32 kinds of goods and services, mostly from the retail sector. [▶ Back](#)

Money Demand Data

- Nominal GDP is taken from the Historical Statistics of the United States, Colonial Times to Present (1970) and the GDPA series from the Citibase database.
- Money supply is M1, as of December of each year, and is not seasonally adjusted. It is from the Historical Statistics of the United States (1960), Friedman and Schwartz (1963), and the FRED II database of the Federal Reserve Bank of St. Louis.
- The interest rate is the short-term commercial paper rate. From 1900 to 1997, it is taken from Friedman and Schwartz (1982), Economic Report of the President (1996), and Economic Report of the President (2003). From 1998 to 2000 it is the short-term 90-day AA credit rate from the Federal Reserve Board. [▶ Back](#)

Welfare Cost Decomposition

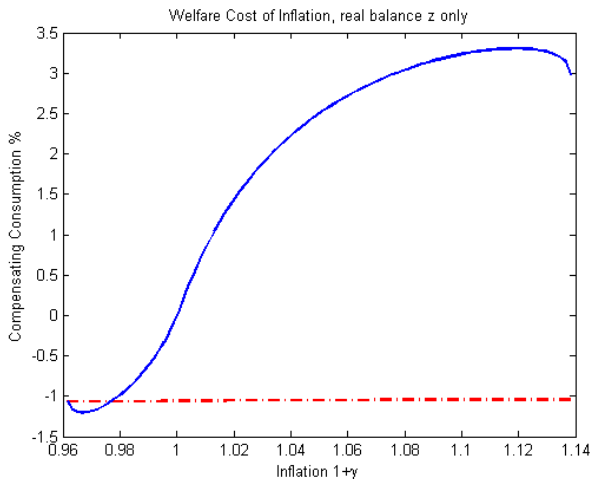
Constant Real Balance



The welfare cost of 10% annual inflation decreases to 0.04% of consumption.

Welfare Cost Decomposition

Real Balance Channel Only



The welfare cost of 10% annual inflation is equal to 0.02% of consumption. [▶ Back](#)

Buyer's Problem: An Alternative Setup

- In the CM, buyers first choose whether they will sample one price or two prices in the next DM

$$W^b(z) = \max \left\{ W_1^b(z), W_2^b(z) \right\},$$

and

$$W_i^b(z) = \max_{x, h, \hat{z}_i} v(x) - h + \beta V_i^b(\hat{z}_i) - ik$$
$$s.t. \quad h + z + T = x + (1 + \gamma)\hat{z}_i$$

for $i = 1$ or 2 .

- The buyer's value functions in the DM are

$$V_1^b(z_1) = \int_{\mathcal{Z}_F} \left[u \left(\frac{d^*(p; z_1)}{p} \right) + W^b [z_1 - d^*(p; z_1)] \right] dF(p; \alpha)$$
$$V_2^b(z_2) = \int_{\mathcal{Z}_F} \left[u \left(\frac{d^*(p; z_2)}{p} \right) + W^b [z_2 - d^*(p; z_2)] \right] dG(p; \alpha)$$

where α is the fraction of total buyers that sample two prices in the DM.

Seller's Problem: An Alternative Setup

- If buyers first choose the number of prices to sample in DM, the seller's profit function in the DM now becomes

$$\pi(p) = (1 - \alpha) \left[d(p; z_1) - c \frac{d(p; z_1)}{p} \right] + 2\alpha(1 - F(p)) \left[d(p; z_2) - c \frac{d(p; z_2)}{p} \right]$$

- For $\alpha \in (0, 1)$, the nondegenerate price distribution in the DM is characterized as

$$F(p) = 1 - \frac{(1 - \alpha) \left[\left(d(\bar{p}; z_1) - c \frac{d(\bar{p}; z_1)}{p} \right) - \left(d(p; z_1) - c \frac{d(p; z_1)}{p} \right) \right]}{2\alpha \left[d(p; z_2) - c \frac{d(p; z_2)}{p} \right]}$$

and $\bar{p} = \max \{ \hat{p}(z_1), \tilde{p} \}$.