

Credit Markets, Limited Commitment, and Government Debt

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- Financial crisis was about a lot of things, but some people seem to have the idea that something went “wrong” in credit markets, and that somehow government debt might fill in for some credit arrangements that have departed the scene.
- Maybe we know something about this.
- Limited commitment models: Kehoe-Levine (1993), Kocherlakota (1996), Sanches-Williamson (2010)
- Private information/productive capacity can reduce liquid asset supply: Rocheteau (2009), Lagos-Rocheteau (2008), Williamson (2011)
- Model: Lagos-Wright structure, credit with limited commitment, some limitations on punishment due to limited memory.

Principal Results

- In some equilibria the world is Ricardian - with global punishments government debt is irrelevant.
- Individual punishments change things: (i) symmetric equilibria; (ii) asymmetric equilibria with default.
- Government debt can solve an adverse selection problem and relax incentive constraints, thus improving the equilibrium allocation.

Model

- $t = 1, 2, 3, \dots$, two subperiods, centralized market (CM) followed by decentralized market (DM).
- Continuum of agents. Unit mass of buyers, each with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)]$$

- $0 < \beta < 1$, $u(\cdot)$ strictly concave, strictly increasing, $u(0) = 0$, $u'(0) = \infty$
- $u'(x^*) = 1$ and $x^{**} = \beta u(x^{**})$.
- Unit mass of sellers, each with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

- Technology: One unit labor input yields one unit output. Buyers produce in CM, sellers in DM.

- Government: lump-sum taxation of buyers in CM, and can issue 1-period government bonds in the CM - each a claim to 1 unit of goods in the next CM.
- CM timing:
 - Agents meet in centralized location, private debts repaid, taxes paid, complete access to public record.
 - Agents meet in Walrasian market, and government pays off debt and issues new debt. Only market price of government debt is observed.
- DM:
 - random matching - each buyer randomly matched with a seller.
 - take-it-or-leave-it offer by the buyer.
 - ρ meetings are *limited information*: no access to public records by participants, but actions are recorded in the public record.
 - $1 - \rho$ meetings are *full information*: full access to public records by participants.
- Limited commitment: all exchange is voluntary.

- 1 Symmetric Equilibria with Global Punishments: With and without government debt.
- 2 Symmetric Equilibria with Individual Punishments: With and without government debt.
- 3 Asymmetric Equilibria with Individual Punishments: With and without government debt.

Symmetric Equilibria, Global Punishments, No Government Debt

$$v = \max_l [u(l) - l + \beta v]$$

subject to

$$l \leq \beta(v - \hat{v}).$$

and

$$\hat{v} = 0 \text{ (autarky)}$$

- Two stationary symmetric equilibria.
 - $v = l = 0$
 - If $x^* \leq x^{**}$, then $v = \frac{u(x^*) - x^*}{1 - \beta}$ and $l = x^*$ (non-binding incentive constraint)
 - If $x^* > x^{**}$, then $v = \frac{x^{**}}{\beta}$, and $l = x^{**}$ (binding incentive constraint)

Symmetric Equilibria, Global Punishments, Government Debt

$$v = \max_{l,b} \{-qb + u(l + \beta b) - l - \beta\tau + \beta v\}$$

subject to

$$l + \beta\tau \leq \beta(v - \hat{v}),$$

- Equilibrium condition:

$$b = B,$$

- Government budget constraint:

$$\tau = B(1 - q).$$

- Government debt accomplishes nothing, as government faces the same limited commitment problem as the private sector.
- Private sector, out of equilibrium, imposes the most severe feasible punishment for defaulting.

Symmetric Equilibria, Individual Punishments, No Government Debt

$$\hat{v} = \max \left[0, \frac{-qb + \rho u(l + \beta b) + (1 - \rho)\beta b}{1 - \beta} \right]$$

- Equilibrium where incentive constraints do not bind:

$$v = \frac{u(x^*) - x^*}{1 - \beta},$$

$$\hat{v} = \frac{\rho u(x^*)}{1 - \beta},$$

and the incentive constraint must not bind, so

$$x^* \leq \beta(1 - \rho)u(x^*),$$

- Equilibrium with binding incentive constraints:

$$x = \beta(1 - \rho)u(x),$$

requiring

$$x^* > \beta(1 - \rho)u(x^*).$$

Symmetric Equilibria, Individual Punishments, Government Debt

- Construct equilibria where B is just sufficient to displace all private lending, and see what we get.
- Equilibrium where incentive constraints do not bind:
 - $\beta B = x^*$ and $q = \beta$.

$$v = \frac{u(x^*) - x^*(2 - \beta)}{1 - \beta},$$

$$\hat{v} = \frac{\rho [u(x^*) - x^*]}{1 - \beta},$$

- Necessary and sufficient condition for existence:

$$x^* \leq \frac{\beta(1 - \rho)u(x^*)}{1 - \rho\beta}$$

- Equilibrium with binding incentive constraints and bonds held by defaulting buyers with $\hat{v} > 0$:

$$q = \beta u'(x)$$

$$x [1 - \beta u'(x)] = \beta(1 - \rho)[u(x) - x],$$

Necessary and sufficient conditions for existence:

$$x^* > \frac{\beta(1 - \rho)u(x^*)}{1 - \rho\beta},$$

and

$$u'(x) \leq \frac{\rho + \beta(1 - \rho)}{\beta}$$

- Binding incentive constraints, no bonds held by defaulting buyers, $\hat{v} = 0$:

$$q = \beta u'(x)$$

$$x = \beta u(x)$$

Necessary and sufficient conditions for existence:

$$x^{**} < x^*,$$

and

$$u'(x) \geq \frac{\rho + \beta(1 - \rho)}{\beta}$$

Effects of Government Debt in Symmetric Equilibria with Individual Punishments

- In general get non-Ricardian results: government debt matters.
- Welfare:

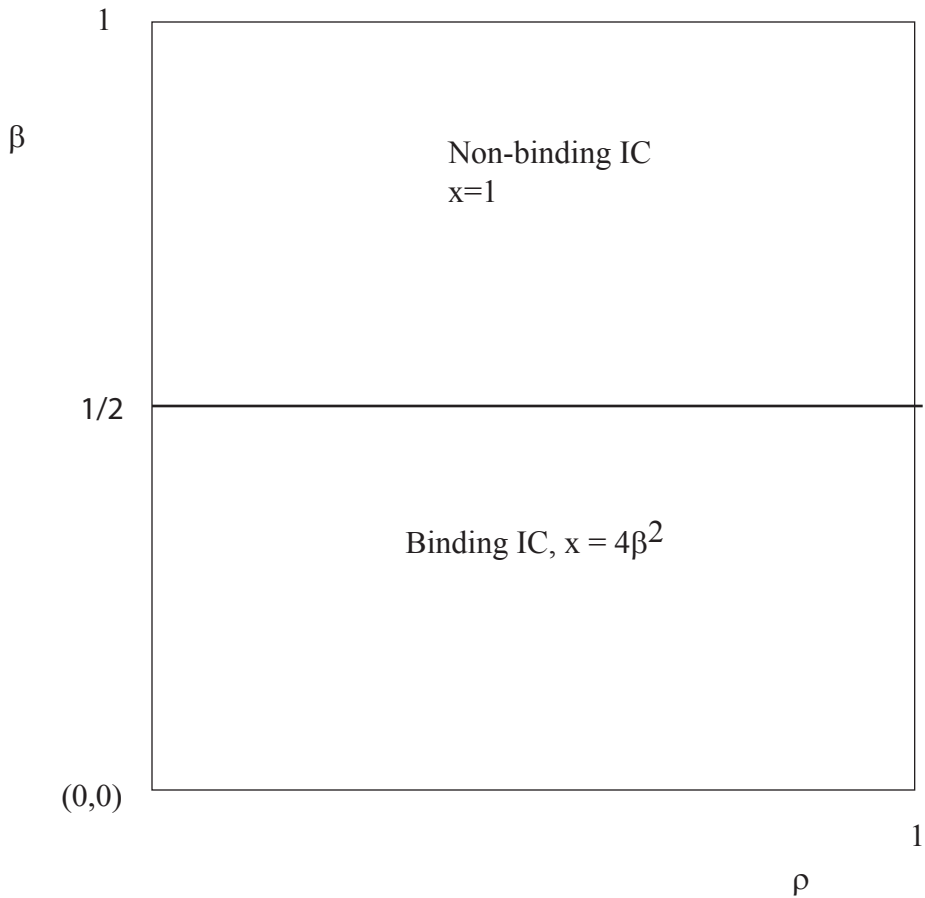
$$W = u(x) - x,$$

- x is never smaller in the equilibria with government bonds, so welfare always goes up.
- Government bonds in general relax incentive constraints: buyers who default either have to acquire bonds in order to consume, or the bonds are so expensive that the default path is autarky.

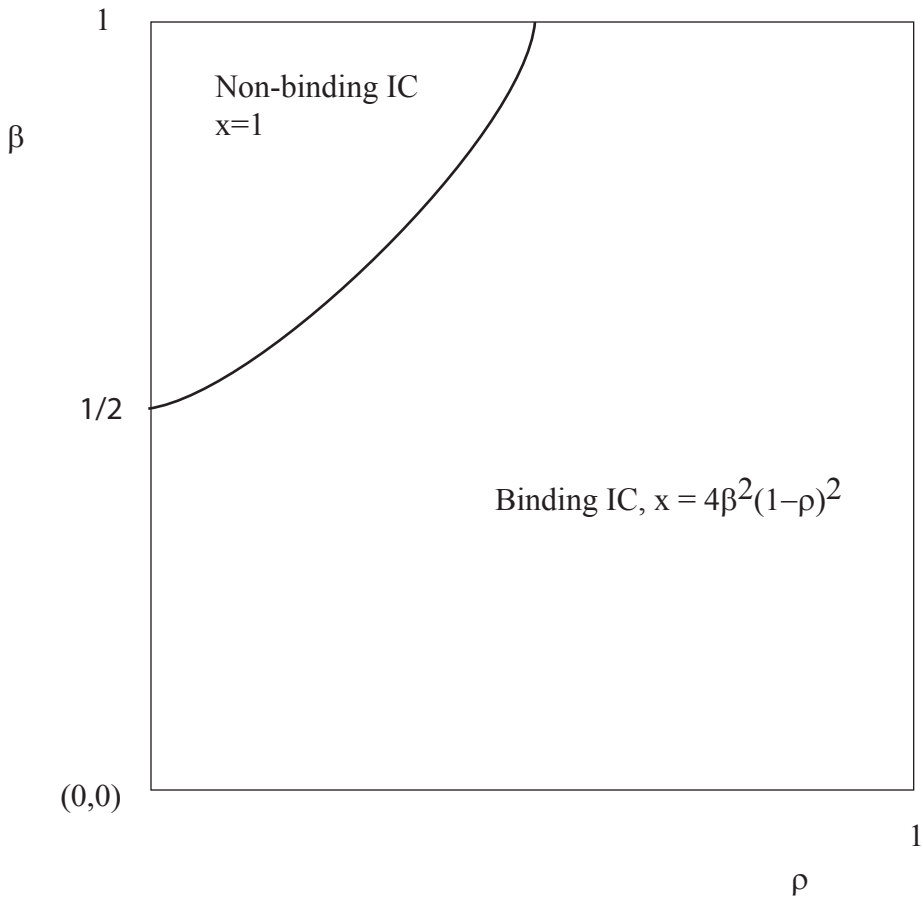
Running Example

- $u(x) = 2x^{\frac{1}{2}}$

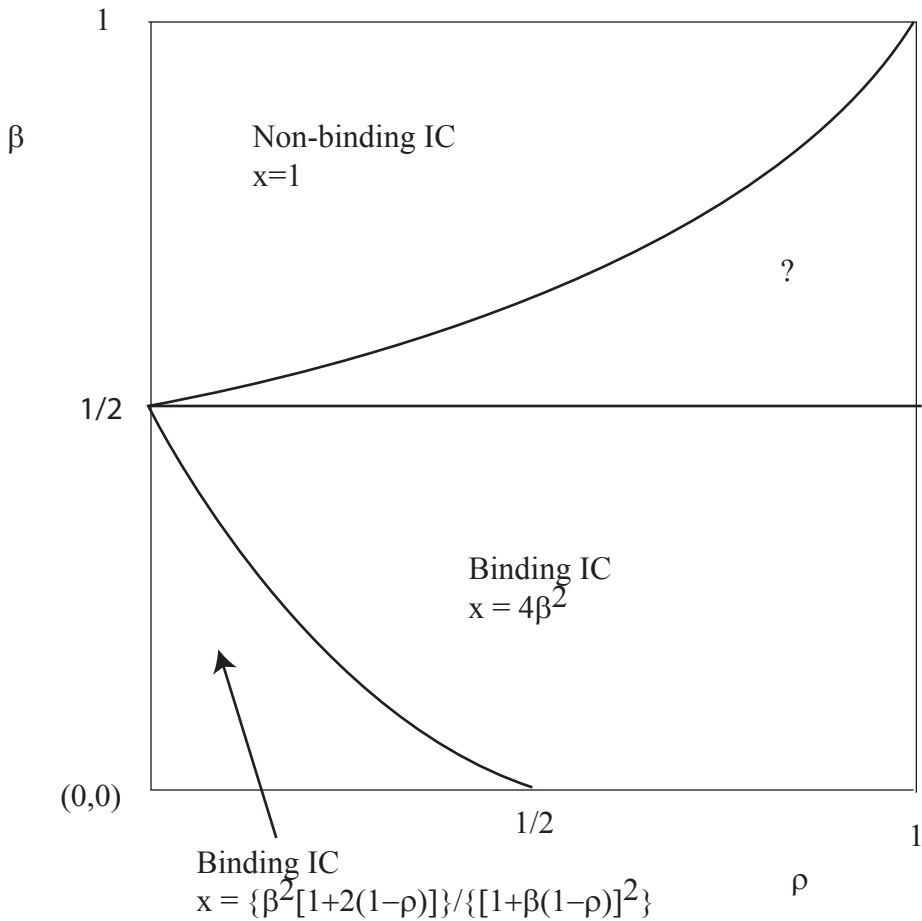
Example: Square Root Utility, Global Punishments



**Example: Square Root Utility, Individual Punishments,
No Government Debt**



Example: Square Root Utility, Individual Punishments, Government Debt



Asymmetric Equilibria with Individual Punishments

- α buyers (the *good buyers*) never default, but a fraction $1 - \alpha$ (*bad buyers*) will default on their debts if anyone chooses to lend to them.

$$v = \max_{l_1, l_2, b} \left\{ \begin{array}{l} -qb + \rho u(l_1 + \beta b) + (1 - \rho)u(l_2 + \beta b) \\ -\rho \frac{l_1}{\alpha} - (1 - \rho)l_2 - \beta\tau + \beta v \end{array} \right\}$$

subject to

$$\frac{l_1}{\alpha} + \beta\tau \leq \beta(v - \hat{v}),$$

$$l_2 + \beta\tau \leq \beta(v - \hat{v})$$

with

$$\hat{v} = \max \left[0, \frac{-qb + \rho u(l_1 + \beta b) + (1 - \rho)\beta b}{1 - \beta} \right],$$

- Government budget constraint:

$$\tau = \frac{B(1 - q)}{\alpha}.$$

- Bond market clears (bad buyers mimic goods buyers):

$$b = B,$$

- Bond market clears (bad buyers choose autarky):

$$\alpha b = B,$$

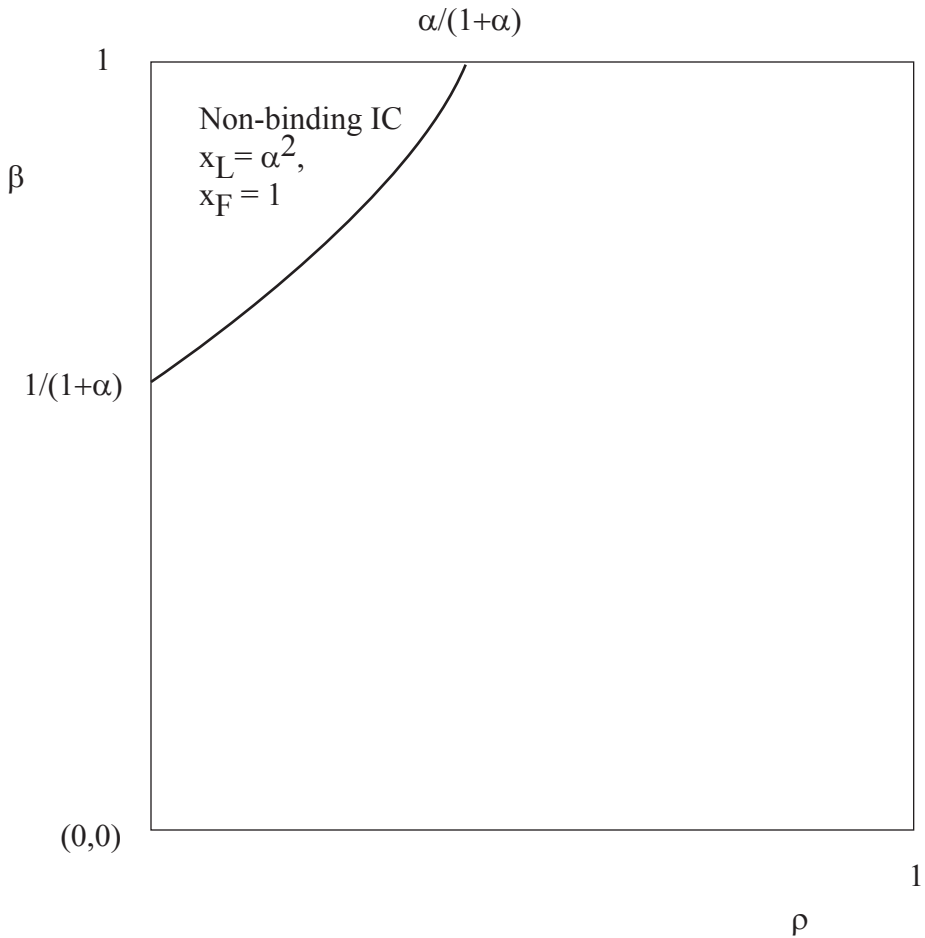
Asymmetric Equilibria, Individual Punishments, No Government Debt

- Four cases: (i) Neither incentive constraint binds; (ii) Only LI constraint binds; (iii) Only FI constraint binds; (iv) Both constraints bind.
- As in symmetric equilibrium case, individual punishments tend to tighten incentive constraints and reduce the welfare of good types.
- Additional problem: Adverse selection in LI meetings, with good types paying a default premium. $x_L < x^*$ even when incentive constraints do not bind.

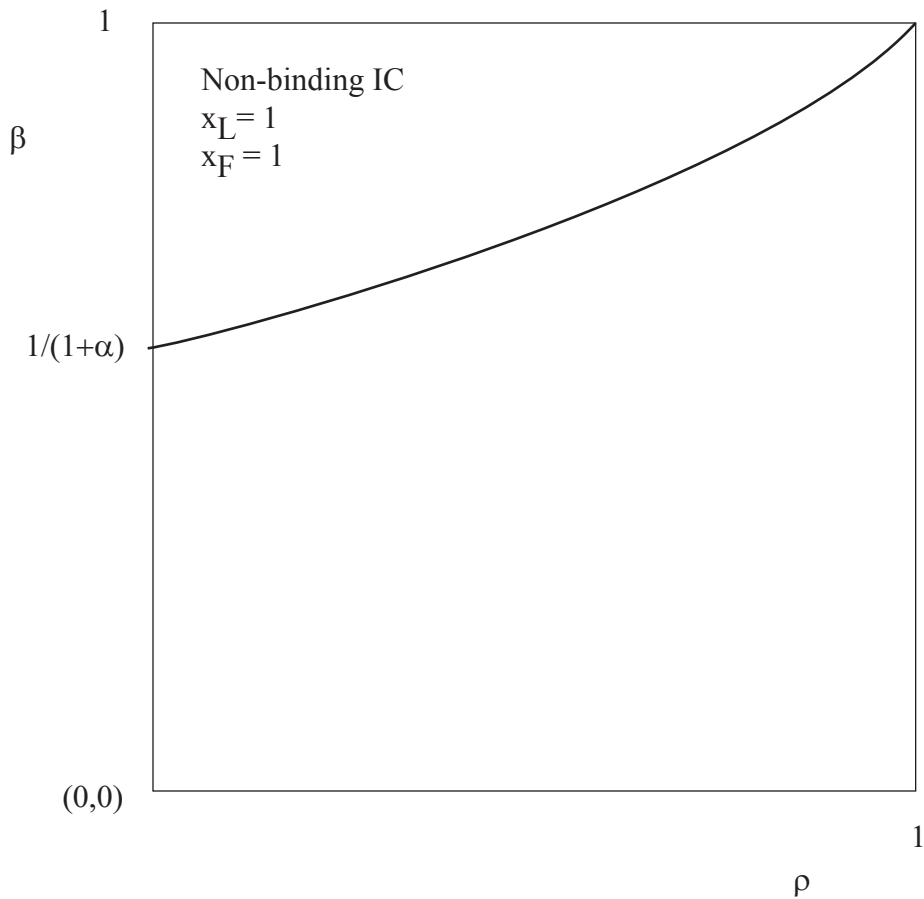
Asymmetric Equilibria, Individual Punishments, Government Debt

- Given α , welfare is higher in general with government debt.
- Additional effect here: Government debt solves the adverse selection problem. Consume $x = x^*$ when incentive constraints do not bind.

Asymmetric Equilibria, No Government Debt



Asymmetric Equilibria, Government Debt



Conclusions

- Global punishments imply Ricardian results. Incentive constraints may bind in credit markets, but the size of the government's debt is irrelevant.
- With individual punishments, government debt matters. If there is enough government debt to displace private credit, this is always optimal, but government debt generally does not stand in for global punishments.
- There are asymmetric equilibria where some fraction of borrowers always defaults. More of these borrowers in the population makes everyone worse off, due to an adverse selection problem.
- In asymmetric equilibria government debt generally improves things by solving the adverse selection problem and relaxing incentive constraints. However the government cannot make the bad borrowers behave.