To sell or to borrow?*

PRELIMINARY, DO NOT CITE

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Abstract

This paper studies banks' decision whether to borrow from the interbank market or to sell asset in order to cover liquidity shortage in presence of counterparty credit risk. We show that (i) there is a pecking order of liquidity sources: internal cash, interbank market, and seconadry market; (ii) there are multiple equilibria due to endogenous relative cost of liquidity between the interbank and seconadry market: in one equilibrium high quality banks sell and in other they borrow; (iii) increase in cash hoarding diminishes the return on interbank lending and results in low quality banks borrowing rather than sellling, leading to increases in credit spreads. The paper explains (i) the sudden collapse of ABCP market followed by a jump in the credit spreads on the interbank market in August 2007, and (ii) persistently high interbank borrowing costs despite liquidity injections by central banks in the aftermath of the crisis.

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1 Introduction

One of the most significant features of modern banks is their ability to turn individual illiquid loans into tradable securities and use them as a source of liquidity. At the same time these banks preserve access to interbank markets, where they can also satisfy their liquidity needs. In August 2007 such banks became the center of events, when markets for securitized assets (such as the ABCP market) and interbank markets showed signs of stress. In response to these events central banks undertook unprecedented efforts to provide liquidity to the financial system. This paper provides a theory of liquidity provision when the banks looking for liquidity have a choice between cash, borrowing from the interbank market and selling their assets.

We analyze the banks' choice between internal cash, borrowing and selling in case of a liquidity shock and credit risk. After each bank allocates its endowment between a risky asset and cash reserves, it receives a more precise but private signal about the risk of its asset and learns its liquidity need. Next the bank decides how to cover the liquidity shortage: use its cash reserves, borrow on the interbank market or sell (at least some of) its asset at a secondary market to outside investors. The bank can also sell more than enough to cover the liquidity shortfall in order to lend on the interbank market and earn return above what storage yields.

This very simple and generic setup generates powerful results. First, adverse selection implies that the banks choose liquidity sources in the following pecking order: internal cash, then interbank borrowing and finally selling. The reason is that the cost of liquidity due to adverse selection is the highest on the secondary market. Hence, the banks with high quality assets are less willing to sell than the banks with low quality assets.

Second, amount of internal cash reserves is crucial for equilibria on the interbank market: a rising credit spread in the interbank market is due to increase in cash reserves in the banking system. The reason is that amount of cash reserves held by the banks affects the return on lending and therefore the low quality banks' trade off between borrowing and selling to lend. When the high quality banks borrow, borrowing is attractive for the low quality banks because they are pooled with and subsidized by the high quality banks. However, the interbank loan rate is also affected by loan supply and demand. If the banks' cash reserves are low, the low quality banks sell and lend, because their borrowing would result in a very high borrowing cost due to excess loan demand. In such an equilibrium only the high quality banks borrow implying no credit spread in the loan rate. If the banks' cash reserves are high, the low quality banks borrow, because selling and lending by them would further increase loan supply resulting in a very low expected return on lending approaching the return on storage. In such an equilibrium the credit spread in a loan rate would be high reflecting demand for loans by the low quality banks.

Third, there are multiple equilibria in the high quality banks' choice of source for covering liquidity shortfall. The reason is the endogeneity of relative cost of liquidity between the interbank and secondary market due to self-fulfilling expectations about the banks' choice of source of liquidity. When the agents expect the high quality banks to borrow, the price of the asset and the loan rate on the interbank market fall so that the high quality banks prefer to borrow rather than sell. And vice versa, if the agents expect the high quality banks to sell, the price and the loan rate adjust so that the high quality banks prefer to sell.

The results of the paper are consistent with the sudden jumps in credit spreads on the interbank markets in August 2007 after a collapse of the ABCP market. The paper shows that change in expectations can lead to a collapse of the secondary market and jumps in the credit spreads in the interbank market without any changes in the fundamentals of the economy. For some parameters the equilibrium in which the banks primarily sell to cover liquidity shortfalls is fragile and a sudden shift in expectations can push the system into an equilibrium with frozen secondary market and an interbank market with high credit spreads. Alternative explanation could blame sudden change in the fundamentals, but the bad news about the subprime market had been piling up already since February 2007.

Moreover, the policy implication of the paper is that flooding the interbank market with liquidity at the first sign of stress signalled by a jump in the interbank loan rate will not relieve any stress and lower credit spreads. The reason is that the jump in the credit spread is already due to too much liquidity in the interbank market that attracts low quality banks to borrow instead of selling. There is too much liquidity, because the banks anticipate rationally that the secondary market will dry up leading to hoarding of cash reserves. Hence, pumping additional liquidity makes it even more attractive for the low quality banks to borrow.

This paper's results are consistent with the empirical evidence provided in Taylor and Williams (2009). First, they show that the jump in credit spreads on the interbank market following the turmoil in the ABCP markets in July 2007 is due to increase in counterparty credit risk. In our paper, this occurs because the low quality banks follow the high quality banks to the interbank market after liquidity hoarding in anticipation of collapse in the secondary market. Second, Taylor and Williams show that injections of liquidity following the observed jump in credit spreads did not ease conditions on the interbank market. This is consistent with the prediction of the model that too much liquidity on the interbank market will attract low quality banks to the borrowing side of the interbank market leading to higher credit spreads.

The contribution of this paper is to model explicitly the bank's decision whether to borrow on the interbank market or sell its assets without relying on fire sales. This assumption is very important. It distinguishes this paper from the others in the sense that it is the first one that recognizes that the banks can use both channels to generate liquidity and they are endogenized. Moreover, the paper is related to several papers about the role of asymmetric information in liquidity provision. Freixas and Holthausen (2005) show that cross-border interbank market is a result of self-fulfilling beliefs about high quality banks entering it. Heider, Hoerova and Holthausen (2011) show that liquidity hoarding and collapse of interbank market due to counterparty credit risk may also be an multiple equilibrium phenomenon if beliefs are such that high quality banks find it more profitable to sell rather than borrow on the interbank market. Malherbe (2011) shows that liquidity hoarding may lead to collapse of secondary market and may be an multiple equilibria phenomenon.

2 Setup

There are three dates, t=0,1,2, and one period. At t=0 each bank decides how to split one unit of its endowment between a long-term asset and cash reserves (called cash or reserves). The endowment belongs to the bank, i.e., we abstract from any debt except the interbank debt incurred at t=1. *[Comment: because explicit modelling of secondary market allows for more flexibility in actions of* *illiquid banks (they might be able to lend when they sell), lack of debt other than interbank market simplifies the algebra]* At t=1 each bank receives two signals: (i) about the return structure of its long term asset, and (ii) about its liquidity need. We model the liquidity need as a refinancing shock d (e.g. Rochet (2004), Tirole (2011)). If the bank does not pay d it goes out of business. With slight abuse of terminology we call banks hit by the refinancing shock illiquid, and the other banks liquid. After signals are revealed, the interbank market for banks' loans and secondary market for banks' long-term asset open. At t=2 the long−term asset's returns are realized and payments are made.

The banks. At t=0 there is a continuum of mass one of identical banks. Each bank maximizes its return at t=2 and has at t=0 a unit of endowment. Each bank can invest fraction $\lambda \in [0;1]$ of endowment in cash that returns 0 in net terms and $1-\lambda$ in a risky asset.[1] At t=1 the bank receives private signal about the return structure of its asset. With probability q the bank learns that the asset is good and returns $R_G > 1$ at t=2 with certainty (we call such a bank a "good bank"). With probability $1-q$ the bank learns that the asset is bad ("bad bank") and has the following return structure: it pays $R_B \in (1; R_G)$ at t=2 with probability $p > 0$ and 0 otherwise, where $pR_B < 1$. It holds that $qR_G + (1-q)pR_B > 1$. For the time being, we assume that $pR_B > d$, which guarantees that the bank can always pay d just by selling all of its long-term asset. This assumption simplifies the exposition of the main result. It has to be noted that it eliminates certain cases that might be interesting for two reasons. First, if $pR_B < d$ choice of λ determines banks' ability to use secondary market to repay the refinancing shock, because for some low λ selling all will not be sufficient to cover d. Second, it assumes away potential social inefficiencies that might arise in equilibrium. Although allowing for $pR_B < d$ means that for some sufficiently low λ such that $pR_B(1-\lambda) + \lambda < d$ refinancing of the bad bank is not socially efficient, such a bank might still find profitable to continue operating due to adverse selection. The reason is that its cost of refinancing might not reflect its true quality when it is pooled with the good bank. *[Comment: The breakdown of liquidity provision in a pooling equilibrium is harder to obtain because the banks are pooled together and financing in expected terms is always socially optimal. Hence, it would*

[1] Alternatively to model the short term nature of liquidity hoarding, we could assume that the bank starts out with a unit of asset and has some spare liquidity and decides what fraction of this spare liqudity to preserve and to "consume" (see Gale and Yorulmazer (2011) for identical approach).

depend on the amount of liquidity required by the illiquid banks. It is obvious that in a pooling equilibrium on the secondary market bad banks will sell all and good banks only a needed amount of asset to refinance, hence increasing the amount of bad guys]

At t=1 each bank also learns about its liquidity need. With probability π the bank is liquid. With probability $1-\pi$ the bank is illiquid. In a model with deposits taken already at t=0, this modelling can proxy for a distributional liquidity shock, where some banks are hit harder than the other. We choose refinancing shock as modelling device, because we think that explicit modelling of reason for liquidity shock does not matter for the results.

Shocks to the asset's returns and liquidity are independent (Freixas and Holthausen (2005), Heider et al (2010)).

The interbank and secondary market. We impose several assumptions on how both markets function. Interbank lending is unsecured and anonymous as in Freixas and Holthausen (2005) and Heider et al (2010). In the secondary market the banks can sell their assets only to outside investors. These outside investors are competitive and are able to absorb any amount of assets that appear on the market. This assumption has several consequences. First, allowing the banks to buy assets under this assumption will not change the results. The reason is that the gross return on the banks' assets will be 1 and the same for both the banks and investors given that both types of agents have the same information about the aggregate quality of assets. Hence, in the best case the banks will be indifferent between lending or buying assets as means to invest their cash. Second, under this assumption the impact of secondary market on the illiquid banks' decision whether to borrow on the interbank market is solely due to adverse selection affecting the secondary market, because cash-in-market pricing effects in the secondary market do not exist. One caveat is that this assumption introduces an asymmetry in treatment of both markets as it allows only for cash-in-market pricing on the interbank market. (*Remark on work in progress*: two version of the model with banks buying as robustness for the results: outside investors with positive net cost of purchases or no outside investors at all)

3 Banks at t=1

We solve the model backwards. At t=1 there are four types of banks: good and liquid (GL), bad and liquid (BL), good and illiquid (GI), and bad and illiquid (BI). Illiquid banks decide how to cope with the liquidity shock. They have several possibilities: They can use their cash, borrow from other banks and/or sell at least part of the asset as well as lend any cash left after providing for the refinancing shock. Liquid banks decide how much to lend in the interbank market and how much of its asset to sell (borrowing is never optimal because there are no investment opportunities). Each bank takes the interbank market interest rate R_D and the asset price P as given. Moreover, we concentrate on the case $\lambda \leq d$, because $\lambda > d$ is never optimal given that storage of cash yields 0 in net terms.

We characterize optimal behavior of liquid and illiquid banks first, and then determine the equilibria at t=1. We discuss these two steps separately in order to explain better the intuition behind the equilibria that arise. Moreover, we also discuss separately the optimal behavior of liquid and illiquid banks, because there are some crucial differences in behavior of these two groups of banks.

3.1 Liquid banks.

Because GL and BL banks differ only in its asset quality, we can write down a general decision problem for a bank with an asset i paying R_i with probability p_i and 0 otherwise. Such a bank chooses lending l_i and selling L_i according to the following problem (we drop the index i when it is obsolete):

$$\max_{l,L} p[(1-\lambda-L)R + (LP+\lambda-l) + \widehat{p}R_D l] + (1-p)[(LP+\lambda-l) + \widehat{p}R_D l]$$

$$\text{s.t. } L \in [0; 1-\lambda],\ l \in [0; LP+\lambda].$$

The first term in the objective function of the bank is the return in case the asset succeeds with probability p. $(1-\lambda-L)R$ is the return on the long term asset after selling L units. $LP+\lambda-l$ is the cash left after the bank receives LP from selling L of the asset at price P and lending l at the interbank market. $\widehat{p}R_D l$ is the return on the interbank loans at t=2, where $\widehat{p}$ is the fraction of banks repaying these loans in equilibrium. The return on the interbank loans is deterministic because we assume that interbank loans are perfectly diversified among all borrowing banks (see also Freixas and Holthausen (2005) and Heider et al. (2011)). Hence, the share of the banks that repay their loans equals to the expected probability of asset's success $\widehat{p}$. The second term in the objective function is the return in the case of the asset's failure, which comprises of cash left after t=1 and return on the interbank loans. Because the bank lends only when $\widehat{p}R_D \geq 1$, this second term is always positive and a liquid bank has always non negative payoff if the long term asset does not pay at t=2.

The bank maximizes its profits at t=2 under two constraints. The constraint on L represents the amount of the asset available to sell. The constraint on l limits the interbank loans issued by the bank. The highest amount of loans equals to the sum of cash carried from t=0, λ, and cash raised from selling L of the asset at price P at t=1. We can prove the following result.

Result 1: *The liquid bank's lending decision is:* $l_i = LP + \lambda$ *for* $\widehat{p}R_D > 1$, $l_i \in [0; LP + \lambda]$ *for* $\widehat{p}R_D = 1$, $l_i = 0$ *for* $\widehat{p}R_D < 1$. *The liquid bank's selling decision is:* $L_i = 1 - \lambda$ *for* $P \max[\widehat{p}R_D; 1] > p_i R_i$, $L_i \in [0; 1 - \lambda]$ *for* $P \max[\widehat{p}R_D; 1] = p_i R_i$, $L_i = 0$ *for* $P \max[\widehat{p}R_D; 1] < p_i R_i$.

Proof: in the appendix.

The main observation from Result 1 is that the GL bank is less willing to sell than BL bank just because the return on the good asset is higher and the asset will trade at an adverse selection discount (the asset will never sell at $P = R_G$ because the bad bank will always want to sell for any $P > pR_B$). It is still possible to sell for GL bank despite the adverse selection if the return on investing the cash from sold assets on the interbank market compensates for the adverse selection discount.

Although not explicitly taken into account in the above result, without loss of generality we will concentrate on the cases with price $P \in [pR_B; R_G)$ and assume that the bad bank for $P = pR_B$ will sell all of its asset.

3.2 Illiquid banks.

Again we can study a general problem. The difference between the liquid and illiquid banks is that the illiquid banks can also borrow. If $b > 0$ is borrowing and $b < 0$ lending, then the illiquid bank solves the following problem:

$$\max_{b,L} \begin{cases} p[(1-\lambda-L)R + (LP+\lambda+b-d) - R_D b] + (1-p_i)\max[0;(LP+\lambda+b-d) - R_D b], \text{ if } b \geq 0 \\ p[(1-\lambda-L)R + (LP+\lambda+b-d) - \widehat{p}R_D b] + (1-p_i)[(LP+\lambda+b-d) - \widehat{p}R_D b], \text{ if } b < 0, \end{cases}$$

$$\text{s.t. } L \in [0;1-\lambda], LP+\lambda+b \geq d.$$

Because the illiquid bank borrows, there are two differences in its objective function with respect to the objective function of the liquid bank: (i) the cost of borrowing for an individual bank is R_D and different from the return on the portfolio of interbank loans, $\widehat{p}R_D$, and (ii) the illiquid bank can default on the interbank loan in case its asset defaults (hence, max-operator for $b \geq 0$).

The following result reports actions at t=1 taken by an illiquid bank for $\widehat{p}R_D \geq 1$ (we do not report the results for $\widehat{p}R_D < 1$ as they are not relevant in equilibrium).

Result 2: *If $\widehat{p}$ is the expected fraction of banks repaying interbank loans, then we have for* $p_i > \widehat{p}$

$$\begin{aligned} \textit{only borrowing} &: \quad b = d-\lambda \text{ and } L = 0, \text{ if } R_i > PR_D & (1) \\ \textit{indifferent between } b &= d-\lambda \text{ and } b = 0 \text{ for } R_i = PR_D \\ \textit{only selling} &: \quad b = 0 \text{ and } L = \frac{d-\lambda}{P}, \text{ if } \frac{\widehat{p}}{p_i}PR_D < R_i < PR_D & (2) \\ \textit{indifferent between } b &= 0 \text{ and } b = -\left[(1-\lambda)P + \lambda - d\right] \text{ for } R_i = \frac{\widehat{p}}{p_i}PR_D \\ \textit{selling and lending} &: \quad b = -\left[(1-\lambda)P + \lambda - d\right] \text{ and } L = 1-\lambda, \text{ if } R_i < \frac{\widehat{p}}{p_i}PR_D & (3) \end{aligned}$$

And for $p_i < \widehat{p}$

$$\begin{aligned}
\textit{only borrowing} \quad &: \quad b = d - \lambda \text{ and } L = 0, \text{ if } R_i > PR_D + \left(\frac{\widehat{p}}{p_i} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right) R_D \qquad (4) \\
\textit{indifferent between } b \quad &= \quad d - \lambda \text{ and } b = -\left[(1-\lambda) P + \lambda - d\right] \\
\text{for } R_i \quad &= \quad PR_D + \left(\frac{\widehat{p}}{p_i} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right) R_D \qquad (5) \\
\textit{selling and lending} \quad &: \quad b = -\left[(1-\lambda) P + \lambda - d\right] \text{ and } L = 1 - \lambda, \\
\text{if } R_i \quad &< \quad PR_D + \left(\frac{\widehat{p}}{p_i} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right) R_D. \qquad (6)
\end{aligned}$$

If $p_i = \widehat{p}$ *and* $R_i = PR_D$*, the bank is indifferent between only borrowing, only selling, and selling all and lending.*

Proof: in the appendix.

Result 2 has two consequences. First, the banks' choice between borrowing or selling to cope with the liquidity shock depends on the relative cost of liquidity between the interbank and secondary market, on the relative riskiness of the bank towards the average risk on the interbank market and on the return R_i in case of success. Generally, the higher the price and the higher the loan rate, the more attractive secondary market is. Second, the illiquid bank that is riskier than the expected risk in the credit market ($p_i < \widehat{p}$) chooses only between borrowing and selling all in order to lend. This is contrary to the illiquid bank with lower risk than the average ($p_i \geq \widehat{p}$). The reason is that selling all in order to lend is more attractive for the riskier illiquid bank, because lending in the interbank market is less risky than this bank is. This mechanism is also responsible for the pecking order in the choice of liquidity sources and therefore the structure of the equilibria:

Corollary to Result 2: *If the BI banks borrow and do not sell, the GI banks borrow and do not sell. The converse is not true.*

Proof: in the appendix.

The corollary states that the BI banks find borrowing less attractive than the GI banks (the converse is not true). We can prove a more general statement: for constant R_i the illiquid bank with $p_i < \widehat{p}$ finds borrowing *less* attractive than the illiquid bank with $p_i > \widehat{p}$ when compared to selling (observe that $PR_D + \left(\frac{\widehat{p}}{p_i} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right) > PR_D$). Finally, for constant p_i selling is more attractive for banks whose assets pay lower return in case of success. These last two results make a case with continuous support for p and R attractive, because for the time being selling becomes more attractive for the BI banks than for the GI banks due to their lower p_i and R_i. Nevertheless, we continue with the current setup that still captures the main points of the model.

3.3 Equilibria at t=1

After we characterized the liquid and illiquid banks' choices at t=1, we can turn to the structure of the equilibria at t=1. Our model is similar to the one proposed by Freixas and Holthausen (2005). The banks and the investors build beliefs about which market the banks choose and on which side of the interbank market they will be on. In turn, the banks' choices have to be optimal and consistent with the beliefs. Hence, the equilibrium concept is that of a Perfect Bayesian equilibrium, in which P and R_D are such that the banks' choice are optimal and consistent with the beliefs. We can prove the following result, in which we only describe the equilibrium choices of the illiquid banks as they are the object of our interest.

Result 3: *Assume that $d \in \left(\frac{1-q}{1-\pi}pR_B; pR_B\right)$ and $q > \pi$. There exist λ_{BS}, λ_0 and λ_1 such that the following results obtain. For $\lambda \in [0; \lambda_{BS})$ the GI banks are indifferent between borrowing and selling, and the BI banks sell all to lend. For $\lambda = \lambda_{BS}$ the GI banks borrow and the BI banks sell. For $\lambda \in (\lambda_{BS}; \lambda_0)$ the GI banks borrow and the BI banks are indifferent between borrowing and selling all to lend. For $\lambda \in [\lambda_0; d)$ the GI and BI borrow. If $d \in \left(\frac{p^2(1-q)(R_G-R_B)}{(1-p)q(1-\pi)}; pR_B\right)$ and $q > \frac{R_G-R_B}{R_G-R_B+(1-\pi)(1-p)R_B}$, then for $\lambda \in [0; \lambda_1]$ there is also an equilibrium in which the GI and BI banks sell and there is no lending on the interbank market.*

Proof: in the appendix.

First, for any $\lambda \in [0; \lambda_1]$ there are multiple equilibria because of self-fulfilling beliefs. These equilibria differ in the behavior of the GI banks, which either sell or borrow (for $\lambda \in [0; \lambda_{BS})$ they are indifferent between selling and borrowing). The beliefs about the banks' behavior are self-fulfilling because they determine the relative cost of covering the liquidity shortfall $d - \lambda$ from the interbank and secondary markets. In turn, this relative cost of liquidity between the markets determines the banks' choice of how to cope with the liquidity shortfall leading to self-fulfilling equilibria.

The crucial determinant of multiple equilibria are the expectations about the GI banks' behavior. If the agents expect the GI banks to borrow (or at least part of them), the loan rate adjusts to take into account borrowing by the GI banks and the price of the asset is low because only (or mostly) the bad banks are expected to sell. Hence, the GI banks have no incentive to sell and prefer to borrow sustaining the equilibrium with borrowing. If all GI banks are expected to sell, the price of the asset on the secondary market adjusts accordingly. Because none of the banks borrow, the interbank loan rate is undetermined on the equilibrium path. The GI banks prefer to sell rather than borrow for some sufficiently pessimistic off-the-equilibrium beliefs.[2]

The multiple equilibria exist only for sufficiently low λ. This is due to adverse selection cost of selling. Because the asset trades at an adverse selection discount to the value of good asset, the GI banks sell only the share of their asset needed to cover the liquidity shortfall, $\frac{d-\lambda}{P}$. In contrast, the bad banks sell all of their asset, $1-\lambda$, exploiting the fact that they sell at a higher price than the true value of their asset. Hence, as cash reserves λ approach d, the share of the GI banks falls to 0 and the price converges towards the expected value of the bad asset, pR_B (see also Malherbe (2011)). Hence for sufficiently high λ the GI banks will prefer to borrow even at the loan rate reflecting only risk of the bad banks (i.e. the most negative off-the-equilibrium belief about risk of borrowing banks).

Second, the structure of equilibria in which at least some GI banks borrow depends on the cash reserves from t=0 λ. As in paragraph above higher λ leads to stronger adverse selection, but the mechanism and consequences are different. In equilibrium in which all GI banks sell, adverse selection on the secondary market increases with λ because the share of good assets supplied by the GI banks decreases. In equilibria where the GI banks borrow, adverse selection on the interbank market increases with λ, because the participation of the BI banks in the interbank market is (weakly) increasing in the amount of cash hoarded at t=0, λ.

The reason for the BI banks' loan demand increasing in λ is that available cash reserves in

[2]Here one could imagine that there are also "traditional" banks that cope with their liquidity shock using only interbank market. According to some empirical evidence those banks were regarded as less risky than banks with substantial exposure to tradeable assets such as MBS and ABS. Hence, presence of the traditional banks would establish a floor for the loan rate without need to restort to off-equilibrium beliefs.

the banking system (the "cash-in-the-market" effect) affect the trade-off the BI banks face when deciding whether to borrow or sell and lend on the interbank market. First, borrowing is attractive because the BI banks can free ride on the GI banks. Second, the amount of cash reserves in the banking system, λ, and the BI banks' decision whether to borrow or sell and lend affect also return on interbank lending. If λ is low, return on lending will be so high that the BI banks will prefer to sell and lend rather than borrow. If λ is high, the BI banks prefer to borrow because high supply of loans drives down the expected return on lending to equalize it with the return on cash storage. To describe the full intuition behind this result we analyze each of the cases from Result 3.

For levels of $\lambda \in [0; \lambda_{BS}]$ the supply of loans on the interbank market by the liquid and the selling BI banks is too low to satisfy the loan demand of all good illiquid banks. The interbank market clears only for such a loan rate that the GI banks are indifferent between borrowing and selling. Of course, the BI banks sell and lend since the equilibrium price on the secondary market exceeds the value of their asset and they lend only to good banks. Given that the GI banks are indifferent between three options to cover the shortfall (the case $p = \widehat{p}$ from Result 2), there are multiple equilibria depending on beliefs about fractions of GI banks choosing these options.

For intermediate levels of $\lambda \in (\lambda_{BS}; \lambda_0)$ the GI banks borrow and the BI banks are indifferent between selling and borrowing. The reason is that for such intermediate levels of λ the decision of the BI banks whether to borrow or sell and lend is crucial for total loan supply and demand. If all BI banks decided to borrow, the supply of loans would be so small and the loan rate so high that the BI banks would prefer to sell. However, if all BI banks decided to sell and lend, there would be excess loan supply resulting in a loan rate of 1. For such a loan rate the BI banks prefer to borrow. Hence, the interbank market clears at a loan rate for which the BI banks are indifferent between borrowing or selling.

Finally, if there is enough cash reserves $\lambda \in [\lambda_0; d)$ to cover loan demand of all illiquid banks, the BI banks prefer to borrow. For such high λ there is such an excess loan supply that the expected return on lending is the same as the return on storage, 1. Hence, if the BI banks sold and lent, they would only further increase the excess loan supply and the loan rate would still be 1 (here the lending would be riskless). Hence, borrowing for the BI banks becomes more attractive than selling because of free-riding on the GI banks resulting in loan rates that cross-subsidize the BI banks by the GI banks is more profitable than selling and lending.

From the description of the equilibria it also clear that the pecking order is the main driver of the equilibrium structure. The GI banks are the first one to switch to borrowing at $\lambda = \lambda_{BS}$, whereas the BI banks do it only for $\lambda = \lambda_0 > \lambda_{BS}$.

To close up the discussion of equilibria at t=1 we provide prices, loan rates and total lending volumes TL as functions of cash reserves.

Result 4: *For equilibria in which at least some GI banks are borrowing the following obtains:*

1. For $\lambda \in [0;\lambda_{BS})$ $P = \frac{R_G(d(1-\pi)-\lambda)}{d(1-\pi)-\lambda+(R_G-pR_B)(1-q)(1-\lambda)}$, $R_D = 1+(R_G-pR_B)\frac{(1-q)(1-\lambda)}{d(1-\pi)-\lambda}$ and $TL = (1-\pi)\,q\,(1-\gamma_S-\gamma_L)\,(d-\lambda)$. P and TL decrease and R_D increases with λ.

2. For $\lambda = \lambda_{BS}$ $P = pR_B$, $R_D \in \left[\frac{1}{1-\frac{1-p}{pR_B}\frac{d-\lambda_{BS}}{1-\lambda_{BS}}}; 1+\frac{R_G-pR_B}{pR_B}\right]$ and $TL = (1-\pi)\,q\,(d-\lambda_{bs})$.

3. For $\lambda \in (\lambda_{BS};\lambda_0)$ $P = pR_B$, $R_D = \frac{1}{\widetilde{p}-\frac{\widetilde{p}-p}{pR_B}\frac{d-\lambda}{1-\lambda}}$ and $TL = (1-\pi)\,(q+(1-q)\,(1-\beta))\,(d-\lambda)$, where the credit risk on the interbank market is $\widetilde{p} = q+(1-q)\,p+\frac{q(1-q)(1-p)\beta}{q+(1-q)(1-\beta)} \in (q+(1-q)\,p;1)$ and the fraction of the BI selling is $\beta = \frac{1-(1-q)\pi pR_B}{(1-\pi)(1-q)pR_B}\frac{\lambda_0-\lambda}{1-\lambda} \in (0;1)$. R_D is decreasing if $d \in \left(\frac{qpR_B}{1-(1-q)pR_B}; pR_B\right)$ or increasing if $d \in \left(0; \frac{qpR_B}{1-(1-q)pR_B}\right)$. When $\lambda_{BS} > 0$ ($d > \frac{1-q}{1-\pi}pR_B$), the last interval is not empty if $\frac{qpR_B}{1-(1-q)pR_B} > \frac{1-q}{1-\pi}pR_B$, which holds if $pR_B \in \left(1-\frac{q(q-\pi)}{(1-q)^2};1\right)$ and $q > \pi$. TL can be non-monotonic first increasing and then decreasing in λ.

4.For $\lambda = \lambda_0$ $P = pR_B$, $R_D = \left[\frac{1}{q+(1-q)p}; \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR_B}\frac{d-\lambda_0}{1-\lambda_0}}\right]$ and $TL = (1-\pi)\,(d-\lambda_0)$.

5. For $\lambda \in (\lambda_0;d)$ $P = pR_B$, $R_D = \frac{1}{q+(1-q)p}$ and $TL = (1-\pi)\,(d-\lambda)$. TL decreases with λ.

P and R_D as a function of λ are continuous for all $\lambda \in [0;d)$. TL is continuous for $\lambda \in [\lambda_{BS};d)$. R_D's value for $\lambda = 0$ is higher than for $\lambda \in (\lambda_0;d)$.

Proof: Proofs are straightforward.

We discuss now the behavior of the equilibrium variables. Price is different from pR_B only for $\lambda \in [0;\lambda_{BS})$. The reason is that the GI banks sell only when there is not enough liquidity on the interbank market to cover their demand for loans. The price is then decreasing in λ because less GI banks need to sell in order cover their liquidity shortfall.

The changes in loan rate are not only influenced by the cash-in-the-market effects but also by the differing behavior of the GI and BI banks as determined in Result 2. For high cash holdings

($\lambda \in (\lambda_0; d)$) the loan rate reflects only the increased credit risk, because there is enough liquidity on the interbank market for the BI banks to borrow too. For $\lambda = \lambda_0$ there is just enough liquidity on the interbank market for all illiquid banks to borrow. Hence, because the demand and supply on the interbank market are inelastic the market clears for any loan rate that makes the liquid banks lend and the bad illiquid banks borrow (see also Freixas, Martin and Skeie (2011) for a similar result). For $\lambda \in (\lambda_{BS}; \lambda_0)$ the loan rate not only reflects the expected probability of repayment $\widetilde{p}$ on the interbank market, but also the BI banks's opportunity cost of borrowing represented by the term $\frac{\widetilde{p}-p}{pR_B}\frac{d-\lambda}{1-\lambda}$. Because the BI banks are the first ones to sell when the cost of borrowing increases and only some of the illiquid banks can borrow, the interbank market clears for a loan rate that makes the BI banks indifferent between borrowing and selling, therefore reflecting their opportunity cost of borrowing. Market-clearing loan rate might be either decreasing or increasing in λ. On the one hand, as λ increases the credit risk increases because more BI banks can borrow and the loan amount decreases making borrowing more attractive. Both effect drive loan rate higher. On the other hand, as λ increases the BI banks have more cash reserves to lend out, driving the loan rate down. Which of these two countervailing effects is stronger depends on the parameters. For $\lambda = \lambda_{BS}$ there is just enough liquidity for all GI banks to borrow. Hence, market-clearing loan rate is such that the GI banks want to borrow and the BI banks want to lend. For $\lambda \in [0; \lambda_{BS})$ at least some GI banks need to sell. Hence, the interbank market clears when they are indifferent between borrowing and selling, and the loan rate reflects their opportunity cost of borrowing. Loan rate increases with λ, because for the GI banks only the adverse selection on secondary market determines the opportunity cost of borrowing. This is different from the BI banks' case as shown in Result 2, where the return on selling is also affected directly by λ. Finally, the cash-in-the-market effects can be so strong that market-clearing loan rate for $\lambda \in (\lambda_0; d)$ which reflects the highest possible credit risk is lower than the lowest loan rate for $\lambda \in [0; \lambda_{BS})$ for which only the GI banks borrow. However, it has to be noted that this result is in part driven by the fact that $R_G > R_B$.

Total lending volume is generally decreasing in λ because the shortfall $d - \lambda$ is lower. However, for $\lambda \in (\lambda_{BS}; \lambda_0)$ lending volume increases because more BI banks can borrow increasing the overall loan demand. Hence, total lending volume can be here non-monotonic.

Our results show two things: (i) the fragility due to adverse selection can result in sudden jumps in credit spreads and collapse of secondary markets, consistent with the sudden collapse of ABCP market and immediate stress on the interbank market resulting in very high loan rates in August 2007, (ii) increases in cash reserves may lead to lower return on interbank lending and encourage the banks with lower quality of assets to borrow rather than sell and lend leading to contamination of the interbank market signalled by high loan rate reflecting increased counterparty risk.

4 Equilibria at t=0

At t=0 the banks choose λ rationally anticipating whether the interbank market will be inactive or not (at least some of GI banks to borrow). Because the banks rationally expect a certain equilibrium at t=0 they will maximize their profits at t=0 by choosing λ consistent with the equilibrium that will arise at t=1. The following result summarizes the optimal choice of λ at t=0.

Result 6. *There exists a threshold d^* for which the banks choose $\lambda = 0$ in anticipation of an inactive interbank market for $d \in (d^*; pR_B)$. There exist such $\underline{R}_G$, $\overline{R}_G$ and $\widetilde{R}_G$ that for $d \in \left(\frac{1-q}{1-\pi} pR_B; pR_B\right)$ and $q > \pi$ in anticipation of an active interbank market the banks choose $\lambda = \lambda_0$ for $R_G \in \left(\frac{1-(1-q)pR_B}{q}; \underline{R}_G\right]$, some $\lambda \in (\lambda_{BS}; \lambda_0)$ for $R_G \in \left(\underline{R}_G; \overline{R}_G\right)$, $\lambda = \lambda_{BS}$ for $R_G \geq \overline{R}_G$, and $\lambda = 0$ for $R_G \geq \widetilde{R}_G$ and $d \in \left(\frac{1-q}{1-\pi}; pR_B\right)$.*

Proof: In the appendix.

Result 6 has two consequences. First, for sufficiently high d there will be multiple equilibria at t=0 because the banks may anticipate rationally either active and inactive interbank market. That is crucial for the argument to that the August 2007 events could be due to a shift in expectations rather than fundamentals, and that high loan rates were due to banks with low quality assets entering the interbank market rather than lack of liquidity. Second, even if the banks anticipate active interbank market, there still might be multiple equilibria. The reason is that when the GI banks are indifferent between borrowing or selling, the interbank market can still be active for $\lambda = 0$ (we can always find such γ, γ_S and γ_L that the interbank market clears for $\lambda = 0$). However, if d is sufficiently small there is a unique equilibrium with the banks choosing $\lambda \in [\lambda_{BS}; \lambda_0]$.

5 Welfare Analysis

Welfare analysis is straightforward for two reasons. First, the only source of inefficiency is lack of coordination between the banks at t=0 and t=1. Second, social welfare at t=0 increases with lower λ. In absence of adverse selection socially optimal λ would be zero, because there are no cash-in-the-market effects on the secondary market. Hence, for parameters such that multiple equilibria are possible at t=0 and t=1 the social optimal choice of λ is 0. For parameters such that multiple equilibria do not exist ($d < \min\left[d^*; \frac{1-q}{1-\pi}\right]$) the banks' choice of λ is (constrained) socially efficient (social welfare at t=0 is the expected banks' return at t=2). Hence, in such a case hoarding of cash reserves is socially efficient.

6 Policy Implications

[work in progress]

If we are in the scenario where the shift in the expectations pushes the economy into the equilibrium with all illiquid banks borrowing, then the immediate consequence is that the interbank loan rate jumps.[3] Our explanation is that the jump in the loan rate is due to rise in the credit risk given that the riskier banks prefer to borrow when there is excess loan supply due to cash hoarding at t=0 in anticipation of illiquid banks borrowing at t=1. Hence, pumping liquidity into the system *after* cash hoarding occurred and under the premise that jump in the loan rate is due to lack of liquidity might only further increase the incentive for the riskier banks to borrow by lowering the return on lending even more.

Such a behavior is consistent with the evidence from Taylor and Williams, who show that liqduity pumping by the Fed in the aftermath of August 2007 did not have any effect on the interbank borrowing rates. They conclude that the jump in the interbank borrowing costs was due to counterparty risk as it occurs here. It is to stress that the crucial role in this model is played by private liquidity hoarding that destroys incentives for lending.

Remark: this section does not yet answer what the optimal policy would be to address the

[3] A jump is a semantic stretch given that the loan rate is undetermined in an equilibrium with liquid banks selling in the base line model.

market imperfection that is due to multiple equilibria.

7 References

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8 Proofs

Proof of Result 1: After rewriting the objective function we can just solve one general decision problem:

$$\max_{l,L}(1-\lambda-L)p_iR_i+(LP+\lambda-l)+\widehat{p}R_Dl,\ \text{s.t. } L\in[0;1-\lambda],\ l\in[0;LP+\lambda]\,.$$

The bank lends all of its cash, $l=LP+\lambda$, if lending to other banks is profitable, $\widehat{p}R_D>1$. After inserting $l=LP+\lambda$ into the bank's objective function we get

$$(1-\lambda)p_iR_i+\widehat{p}R_D\lambda+(P\widehat{p}R_D-p_iR_i)L.$$

Hence, the bank will sell all of its asset, $L = 1-\lambda$, for $P\widehat{p}R_D > p_i R_i$, be indifferent for $P\widehat{p}R_D = p_i R_i$, and keep all of it otherwise. If $\widehat{p}R_D \leq 1$, the bank is either indifferent between lending or not lending, or does not lend at all. The effect on the objective function is the same because the terms with l in the objective function vanish and the objective function reads:

$$(1-\lambda)p_i R_i + \lambda + (P - p_i R_i)L.$$

Hence, the bank will sell all of its asset, $L = 1-\lambda$, for $P > p_i R_i$, be indifferent for $P = p_i R_i$, and keep all of it otherwise. Hence, using $\max[\widehat{p}R_D; 1]$ we can summarize the rule for selling as in the Result 1.■

Proof of Result 2: The proof is pretty straightforward after realization that at optimum the illiquid bank will always exhaust the constraint $LP + \lambda + b \geq d$. If this constraint is slack then the bank either borrowed too much or sold too much or didn't lend all of its cash reserves. Given that all this is more expensive than storage (or at least cost of the same as storage), the illiquid bank prefers to exhaust this constraint. Given $LP + \lambda + b = d$ it is easy to see that each illiquid bank taking P, R_D and $\widehat{p}$ as given will have three options to cover the shortfall: $b = d - \lambda$ and $L = 0$, $b = 0$ and $L = \frac{d-\lambda}{P}$, and $b = -[(1-\lambda)P + \lambda - d]$ and $L = 1-\lambda$. Hence, for each of these solutions we can compare the value of the objective function and obtain the Result 2.

Proof of Corollary to Results 1 and 2: Proofs are straightforward. To see the claim 1, observe that the GI banks borrow and do not sell for $R_G > PR_D$, which, together with $\widehat{p} \leq 1$, implies that $R_G > \widehat{p}PR_D$, i.e. that the GI banks lend without selling. Converse statement is true only for $\widehat{p} = 1$. To see the claim 2, observe that the BI banks sell and lend for $R_D > \frac{R_B}{P + \left(\frac{\widehat{p}}{p} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right)}$, which together with $\widehat{p} \geq p$ (which is always true in equilibrium for the bad banks) and $pR_B > d$ (implying that $P > \frac{d-\lambda}{1-\lambda}$) implies that $R_D > \frac{pR_B}{\widehat{p}P}$, i.e. that the BL banks lend and sell all. Converse statement is true only for $\widehat{p} = p$. To see the claim 3, observe that the BI banks borrow and do not sell for $R_D < \frac{R_B}{P + \left(\frac{\widehat{p}}{p} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right)}$. Using $\widehat{p} \geq p$, $pR_B > d$, and $R_G > R_B$, we can show that the following holds $\frac{R_B}{P + \left(\frac{\widehat{p}}{p} - 1\right)\left(P - \frac{d-\lambda}{1-\lambda}\right)} \leq \frac{R_B}{P} < \frac{R_G}{P}$, which implies that $R_D < \frac{R_G}{P}$, i.e., the GI banks borrow do not sell. Converse is not true.

Proof of Result 3: From the previous results we know that the BI has two possible choices:

either borrow or sell all and lend out on the interbank market the rest after the shock has been accommodated. Moreover, the GI has three choices. In addition, to the two above it can sell just the amount enough to cover the shock. Observe that we can ignore the GI banks' choice of selling all and lending. This can never occur in an equilibrium, because if the GI banks sell all to lend, then the BI banks do it too and there is no bank to borrow.

We first deal with possible equilibria in which the investors expect the GI to borrow. There are two cases.

Case (i): $\lambda \geq \frac{(1-\pi)d-(1-q)\pi pR_B}{1-(1-q)\pi pR_B} \equiv \lambda_0$. For $\lambda \geq \lambda_0$ the liquid banks have at least as much cash as it is needed by the illiquid banks if all illiquid banks were to borrow from the interbank market. To see this, observe that supply of cash in such a case, $\pi\left[q\lambda + (1-q)\left(\lambda + P\left(1-\lambda\right)\right)\right]$, is the sum of cash provided by the GL banks providing their reserves λ and the BL banks that provide their reserves λ and sell their assets $(1-\lambda)$ for a price P. Given that only the bad banks sell then $P = pR_B$. Demand for cash on the interbank market, $(1-\pi)(d-\lambda)$, obtains because it is the same for all illiquid banks, $d-\lambda$, whose fraction is $1-\pi$. Hence, λ_0 obtains from the inequality that supply in not lower than demand:

$$\pi\left[q\lambda + (1-q)\left(\lambda + P\left(1-\lambda\right)\right)\right] \geq (1-\pi)(d-\lambda).$$

Now we show that for $\lambda \geq \lambda_0$ there is an equilibrium in which all illiquid banks borrow. For $\lambda > \lambda_0$ the interbank market clears only if the lending banks are indifferent between lending or cash storage because there is excess supply of liquidity on the interbank market. Given that in equilibrium all illiquid banks borrow the expected risk of borrowing is $\widehat{p} = q + (1-q)\,p$ implying a loan rate of $R_D = (q + (1-q)\,p)^{-1}$ that equalizes expected return on lending with the return of 1 on cash storage. Using the conditions from Result 1 we confirm that for $P = pR_B$, $\widehat{p} = q + (1-q)\,p$ and $R_D = (q + (1-q)\,p)^{-1}$ the liquid banks are indifferent between lending and cash storage, the GL does not sell and the BL sells all of its asset. Using the Result 2 we also confirm that borrowing is optimal for the GI and BI banks ((1) for the GI and (4) for the BI banks).

It is also worthwhile to understand why for such high λ an equilibrium in which an equilibrium in which the BI banks sell and lend cannot exist. If the BI banks decide to sell, there is even more excess supply of loans than before and R_D falls to 1 given that only the GI borrows. Hence, because we showed that the BI banks borrow for $R_D > 1$, then borrowing for $R_D = 1$ is even more attractive due to free-riding on the GI banks, implying that selling and lending is not profitable for the BI banks for $\lambda > \lambda_0$.

For $\lambda = \lambda_0$ the supply and demand for loans are equal if all liquid banks borrow. Hence, the equilibrium loan rate is indeterminate because both supply and demand are inelastic for certain ranges of R_D. The liquid banks lend for any R_D such that lending in expectation is at least as profitable as cash storage, $(q + (1-q)\,p)\,R_D \geq 1$. The BI banks borrow for any R_D such that (4) holds (after rewriting (1) and (4) as inequalities for R_D it is easy to observe that if (4) holds then (1) holds too, because $\frac{R_B}{pR_B + \left(\frac{q+(1-q)p}{p} - 1\right)\left(pR_B - \frac{d-\lambda}{1-\lambda}\right)} < \frac{1}{p} < \frac{R_G}{pR_B}$, where the first inequality obtains due to $\frac{q+(1-q)p}{p} > 1$ and $pR_B > d$, and the second due to $R_G > R_B$). Hence, the interbank market clears for any $R_D \in \left[\frac{1}{q+(1-q)p}; \frac{R_B}{pR_B + \left(\frac{q+(1-q)p}{p} - 1\right)\left(pR_B - \frac{d-\lambda}{1-\lambda}\right)}\right]$. The indeterminacy of the loan rate has identical roots as in Freixas, Martin and Skeie (2011): the supply and demand of liquidity is inelastic in such a static setup, because liquid banks will lend out for any return that pays at least as storage, and illiquid banks will pay any loan rate (up to certain threshold) in order to avoid default at t=1 to cover the shortfall. At the same time again there is no equilibrium in which the BI sells for the same reasons as for $\lambda > \lambda_0$.

Case (ii): $\lambda \in [0; \lambda_0)$. In such a case the liquid banks do not have enough liquidity to accommodate borrowing needs of all illiquid banks. Hence, there is no equilibrium in which all BI banks borrow, because they are the first to drop out of the interbank market as they are more willing to sell/less willing to borrow than the GI banks.

First, we show that for $\lambda \in (\lambda_{BS}; \lambda_0)$, where $\lambda_{BS} \equiv \frac{(1-\pi)d - (1-q)pR_B}{1-(1-q)pR_B}$, there is an equilibrium in which all GI banks borrow and the BI banks are indifferent between borrowing and selling. It is easy to see why the BI have to be indifferent for $\lambda \in (\lambda_{BS}; \lambda_0)$. If the BI banks borrow, then supply of liquidity in the interbank market is too low. The market clears only for $R_D = \frac{1-\lambda}{d-\lambda} R_G$, which is the break even loan rate for the GI banks. But for such a loan rate the BI banks' payoff is zero and it is more profitable for any BI bank to deviate and to lend after selling all. If the BI banks sell and lend, then there is too much liquidity on the interbank market, which clears for $R_D = 1$ and

$\widehat{p} = 1$. Then it is easy to see that (6) does not hold meaning that each BI bank prefers to deviate and borrow. Hence, in an equilibrium with the BI banks being indifferent the following conditions have to be fulfilled in equilibrium (because still only the bad banks sell the price is $P = pR_B$).

$$\begin{aligned} & \pi\left[q\lambda + (1-q)\left(\lambda + pR_B(1-\lambda)\right)\right] + (1-\pi)(1-q)\beta\left(\lambda + pR_B(1-\lambda) - d\right) \qquad (7) \\ = \; & (1-\pi)\left(q + (1-q)(1-\beta)\right)(d-\lambda) \\ \text{or } \beta = \; & \frac{(1-\pi)d - \pi(1-q)pR_B - \lambda\left(1-(1-q)\pi pR_B\right)}{(1-\pi)(1-q)(1-\lambda)pR_B} \in (0;1) \text{ for } \lambda \in (\lambda_{BS};\lambda_0) \qquad (8) \end{aligned}$$

$$\widehat{p} = \frac{q}{q + (1-q)(1-\beta)} + \frac{(1-q)(1-\beta)}{q + (1-q)(1-\beta)}p \qquad (9)$$

$$R_D = \frac{R_B}{pR_B + \left(\frac{\widehat{p}}{p} - 1\right)\left(pR_B - \frac{d-\lambda}{1-\lambda}\right)} \qquad (10)$$

If β is the fraction of the BI banks that sell β is given by the market clearing condition (7). The left hand side of (7) is the supply of interbank loans that increases with respect to the case with $\lambda \geq \lambda_0$ by the last term representing the amount supplied by the selling BI banks. The right hand side of (7) is the demand for loans that goes down with respect to the case with $\lambda \geq \lambda_0$ by the fraction of the BI that sell. Hence, solving (7) delivers that β is given by (8). It holds that $\beta > 0$ for $\lambda < \lambda_0$ and $\beta < 1$ for $\lambda > \lambda_{BS} \equiv \frac{(1-\pi)d-(1-q)pR_B}{1-(1-q)pR_B}$, where λ_{BS} is λ for which the supply of loans provided by the liquid banks and all BI banks is equal to the demand for loans from all GI banks. (9) is the expected probability of success on the interbank market after taking into account that only $1-\beta$ of the BI banks borrow. (10) is the loan rate that makes the BI banks indifferent between borrowing and selling, which is given by (5). (10) satisfies $\widehat{p}R_D \geq 1$, meaning that the lending is profitable. Moreover, from Result 2 we know that once the BI banks are indifferent between borrowing and selling, then the GI banks prefer to borrow, and this implies that GL banks prefer to keep their asset and only lend. The requirement that the BL banks sell and lend is trivially satisfied.

For $\lambda = \lambda_{BS}$ the interbank market clears because the amount of loans supplied by all liquid banks and selling BI banks is the same as demand for loans from all GI banks. Hence, the market clears for such loan rates that the GI banks want to borrow and the BI banks want to sell,

$R_D \in \left[\frac{R_B}{pR_B+\left(\frac{1}{p}-1\right)\left(pR_B-\frac{d-\lambda}{1-\lambda}\right)}; \frac{R_G}{pR_B}\right]$, where $\widehat{p}=1$.

For $\lambda \in [0; \lambda_{BS})$ the supply of loans from all liquid and BI banks is lower than the demand for loans from all GI banks. Hence, the market clearing is achieved when some of the GI banks decide to sell. Because only some of the GI banks will borrow, per Result 2 we get that for $\widehat{p}=1=p_G$ the GI bank is not only indifferent between borrowing and selling, but also to the way it sells: selling only the needed amount to accommodate the shock or all and lend out (for $\widehat{p}=1=p_G$ the GI banks get the same payoff from all three actions when $R_G = PR_D$). In addition, $R_G = PR_D$ implies that the GL banks are also indifferent between keeping and selling all. Denote γ_S the fraction of the GI banks selling only $\frac{d-\lambda}{P}$ and γ_L the fraction of the GI banks selling all, and γ the fraction of the GL banks selling all. Then the equilibrium conditions are

$$\begin{aligned}
& \pi\left[q\left[(1-\gamma)\lambda+\gamma(\lambda+P(1-\lambda))\right]+(1-q)(\lambda+P(1-\lambda))\right] \\
& +(1-\pi)(1-q)(\lambda+P(1-\lambda)-d) \\
& +(1-\pi)q\gamma_L(\lambda+P(1-\lambda)-d) \\
= \; & (1-\pi)q(1-\gamma_S-\gamma_L)(d-\lambda)
\end{aligned} \tag{11}$$

$$\begin{aligned}
P = \; & \frac{\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_L(1-\lambda)\right]}{(1-q)(1-\lambda)+\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_L(1-\lambda)\right]}R_G \\
& +\frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda)+\pi q\gamma(1-\lambda)+(1-\pi)q\left[\gamma_S\frac{d-\lambda}{P}+\gamma_L(1-\lambda)\right]}pR_B,
\end{aligned} \tag{12}$$

$$R_D = \frac{R_G}{P} \tag{13}$$

(11) is the market clearing condition for the interbank market taking into account that fraction $1-\gamma$ of the GL banks lends only cash reserves, fraction γ of the GL banks sells all and lends, all bad banks sell, fraction γ_L of the GI banks lends out what is left after selling all of their assets and covering the shortfall $d-\lambda$, and only the fraction $1-\gamma_S-\gamma_L$ of the GI banks borrows. (12) is the equilibrium price paid by the competitive investors who take into account that all bad banks sell all of their assets, a fraction γ of the GL banks sells all, a fraction γ_S of the GI banks sell only

$\frac{d-\lambda}{P}$ and fraction γ_L sell all. (13) is the loan rate that guarantees that the GI banks are indifferent between borrowing and selling and the GL banks between keeping and selling all. Moreover, it is easy to check (6) to see that the BI banks prefer to sell all and lend out under (13). The expression derives when we compare (1), (2) and (3) for $1 = \widehat{p}$. There will be multiple equilibria, because we have three equations, (11), (12) and (13), and 5 unknowns, R_D, P, γ, γ_S and γ_L.

Finally, we can show that there is an equilibrium in which all GI banks sell and the interbank market is inactive. In such a case the investors set expectations in such a way that

$$P = \frac{(1-\pi)\, q \frac{d-\lambda}{P}}{(1-q)(1-\lambda) + (1-\pi)\, q \frac{d-\lambda}{P}} R_G + \frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda) + (1-\pi)\, q \frac{d-\lambda}{P}} pR_B, \tag{14}$$

where it is taken into account that all bad banks sell all of their assets and the GI banks sell only the amount they need to cover the shortfall $d - \lambda$. Hence, the aggregate amount of bad assets on the market is $(1-q)(1-\lambda)$ and of good assets $(1-\pi)\, q \frac{d-\lambda}{P}$. The only condition that has to be checked is the condition for the GI banks. The off-equilibrium loan rate on the interbank market is $1/p$ which reflects the most pessimistic belief of the banks that have cash to lend that the borrowing bank is of bad type. Hence, using (2) combined with $R_D = \frac{1}{p}$ and $\widehat{p} = p$ we get that the GI banks prefer to sell $\frac{d-\lambda}{P}$ if $R_D P = \frac{P}{p} > R_G \Leftrightarrow \lambda \in [0; \lambda_1)$ (it is easy to see that the GI banks will never sell all to lend as $R_G > \frac{\widehat{p}}{pG} P R_D = \frac{p}{1} P \frac{1}{p} = P$). It is easy to see from (14) that the price is decreasing in λ and equals pR_B for $\lambda = d$. The equilibrium price is given by one of the solutions to an equation defined by (14) and satisfying the condition that it is not lower than pR_B:

$$P = \frac{1}{2}\left(pR_B - z + \sqrt{4zR_G + (pR_B - z)^2}\right), \text{ where } z \equiv \frac{(1-\pi)\, q\, (d-\lambda)}{(1-q)(1-\lambda)}. \tag{15}$$

■

Proof of Result 5: The solution is such that the bank at t=0 chooses λ anticipating rationally wether the interbank market will be inactive or not (at least some of GI banks to borrow or only sell) (similar problem is solved in Freixas and Holthausen (2005)). First we derive the optimal choice of λ when the bank anticipates an inactive interbank market, and then proceed to optimal choice of λ under anticipation of active interbank market.

First, when the bank anticipates that the interbank market will be inactive at t=1, it takes into account that the only way to cover the shortfall is to sell (the solution is similar to the one presented in Malherbe (2012)). Hence, when it takes the price at t=1 as given it maximizes its expected return at t=2 as

$$\max_{\lambda\in[0;d]} \pi\left[q\left[R_G\left(1-\lambda\right)+\lambda\right]+\left(1-q\right)\left(\left(1-\lambda\right)P+\lambda\right)\right]+$$
$$\left(1-\pi\right)\left[qR_G\left(1-\lambda-\frac{d-\lambda}{P}\right)+\left(1-q\right)\left(\left(1-\lambda\right)P+\lambda-d\right)\right].$$

The objective function takes into account the optimal behavior of each bank at t=1 (the GL bank does not sell at a discount, the illiquid banks sell all, and the GI bank sell only enough to cover the shortfall $d-\lambda$). The derivative of the objective function with respect to λ reads

$$\pi\left[q\left(1-R_G\right)+\left(1-q\right)\left(1-P\right)\right]+\left(1-\pi\right)\left[qR_G\left(\frac{1}{P}-1\right)+\left(1-q\right)\left(1-P\right)\right]. \tag{16}$$

If (16) is negative, the optimal $\lambda=0$. If (16) is zero, any λ is optimal. If (16) is positive, the optimal $\lambda=d$. Solving for P when (16) is equal to zero, delivers two solutions from which only one is negative and for which it holds that (when optimal λ in such an equilibrium is denoted as λ_S): $\lambda_S=0$ for $P>P_{ind}$, $\lambda_S\in[0;d]$ for $P=P_{ind}$, and $\lambda_S=d$ for $P<P_{ind}$, where

$$P_{ind}=\frac{1-q\left(1-\pi+R_G\right)+\sqrt{\left[q\left(R_G-1+\pi\right)+1\right]^2-4\pi qR_G}}{2\left(1-q\right)}>0.$$

This optimal choice of λ translates into the following possible equilibrium prices at t=1 in accordance with (15). If the bank chooses $\lambda_S=0$ at t=0, then the equilibrium price at t=1 would read

$$\frac{1}{2}\left(pR_B-\frac{\left(1-\pi\right)qd}{1-q}+\sqrt{\frac{4\left(1-\pi\right)qd}{1-q}R_G+\left(pR_B-\frac{\left(1-\pi\right)qd}{1-q}\right)^2}\right).$$

If bank chooses $\lambda_S=d$ at t=0, then the equilibrium price at t=1 is pR_B. If the bank is indifferent, then the equilibrium would be P_{ind}. However, there are two conditions that have be fulfilled for a given price and corresponding choice of λ to constitute an equilibrium. First, the GI banks have to find selling better than borrowing for the most negative beliefs as in proof of Result 3. Hence, the equilibrium price P has to fulfill that $P > pR_G$. That eliminates $\lambda = d$ as equilibrium. Second, the equilibrium price for $\lambda = 0$ has to be higher than P_{ind} to guarantee that it's the optimal choice at t=0 as well. Hence, for $\lambda = 0$ the price has to be higher than the higher of two numbers pR_B and P_{ind}. We disregard P_{ind} as the equilibrium price (the reason is that such an equilibrium would be unstable: any slight deviation from the equilibrium price would make the system converge to neighboring equilibria). $P > \max\left[pR_G; P_{ind}\right]$ holds for $d > d^* \equiv \max\left[\frac{p^2(1-q)(R_G-R_B)}{(1-p)q(1-\pi)}; d_{ind}\right]$, where d_{ind} is solution to $P = P_{ind}$.

Second, when the bank anticipates active interbank market, it chooses optimal λ anticipating its and other banks' optimal behavior as one of 4 possible types of banks at t=1 as a function of λ as determined in the Result 3. Hence, the bank solves the following taking as given P, R_D and $\widehat{p}$:

$$\max_{\lambda\in[0;d]} \begin{cases} \pi q\left[(1-\gamma)\left[R_G(1-\lambda)+\widehat{p}R_D\lambda\right]+\gamma\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ \pi(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)+ \\ (1-\pi)q\left[(1-\gamma_S-\gamma_L)\left(R_G(1-\lambda)-R_D(d-\lambda)\right)+\gamma_S R_G\left(1-\lambda-\frac{d-\lambda}{P}\right)\right]+ \\ +(1-\pi)q\gamma_L\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right)+(1-\pi)(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right), \text{ for } \lambda\in[0;\lambda_{BS}) \\ \\ \pi\left[q\left[R_G(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R_G(1-\lambda)-R_D(d-\lambda)\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right)\right], \text{ for } \lambda=\lambda_{BS} \\ \\ \pi\left[q\left[R_G(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R_G(1-\lambda)-R_D(d-\lambda)\right]+(1-q)(1-\beta)p[(1-\lambda)R_B-R_D(d-\lambda)]\right]+ \\ (1-\pi)\beta(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda-d\right), \text{ for } \lambda\in(\lambda_{BS};\lambda_0) \\ \\ \pi\left[q\left[R_G(1-\lambda)+\widehat{p}R_D\lambda\right]+(1-q)\widehat{p}R_D\left((1-\lambda)P+\lambda\right)\right]+ \\ (1-\pi)\left[q\left[R_G(1-\lambda)-R_D(d-\lambda)\right]+(1-q)p[(1-\lambda)R_B-R_D(d-\lambda)]\right], \text{ for } \lambda\in[\lambda_0;d] \end{cases}$$

The bank will choose λ at most λ_0, because taking more than λ_0 would be waste of resources.

For $\lambda = \lambda_0$ in equilibrium R_D at t=0 has to be such that the first order condition with respect to λ holds. Otherwise, R_D would be such that the bank would either prefer to set $\lambda = 0$ or $\lambda = d$. Hence, such equilibrium R_D at $t = 0$ is $\frac{qR_G+(1-\pi)(1-q)pR_B}{(q+(1-q)p)(1-\pi(1-q)pR_B)}$. Under such a loan rate the anticipated banks' choices are at t=1 optimal if the loan rate $\frac{qR_G+(1-\pi)(1-q)pR_B}{(q+(1-q)p)(1-\pi(1-q)pR_B)}$ is in the interval $\left[\frac{1}{q+(1-q)p}; \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR_B}\frac{d-\lambda_0}{1-\lambda_0}}\right]$ as determined in the proof of the Result 3. We can see that is always holds that $\frac{qR_G+(1-\pi)(1-q)pR_B}{(q+(1-q)p)(1-\pi(1-q)pR_B)} > \frac{1}{q+(1-q)p}$ and that $\frac{qR_G+(1-\pi)(1-q)pR_B}{(q+(1-q)p)(1-\pi(1-q)pR_B)} \leq \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR_B}\frac{d-\lambda_0}{1-\lambda_0}} \Leftrightarrow$ $R_G \leq \underline{R_G}$, where $\underline{R_G}$ is given by solving the last inequality with equality sign. Hence, the bank chooses $\lambda = \lambda_0$ for $R_G \leq \underline{R_G}$.

For $\lambda \in (\lambda_{BS}; \lambda_0)$ we have a similar procedure. For given P, R_D, β and $\widehat{p}$ we derive R_D such that the first order condition holds. Such an equation together with $P = pR_B$, (8), (9), and (10) gives the equilibrium values of λ, R_D, β and $\widehat{p}$. All four values are complicated objects, so for saving of space we refrain from providing them (the reader is more than welcome to ask the author for the Mathematica code that offers the solutions). The solutions constitute an equilibrium if the chosen λ is between $(\lambda_{BS}; \lambda_0)$. This occurs for $R_G \in \left(\underline{R_G}; \overline{R_G}\right)$, where $\overline{R_G}$ is the solution of an equation in which λ_{BS} is equal to the chosen λ. Hence, the bank chooses optimally some $\lambda \in (\lambda_{BS}; \lambda_0)$ for $R_G \in \left(\underline{R_G}; \overline{R_G}\right)$.

For $\lambda = \lambda_{BS}$ again we apply the same procedure. The bank at t=0 is indifferent between any choice of λ for $R_D = \frac{qR_G}{1-(1-q)pR_B}$, which has to be in the interval $\left[\frac{1}{1-\frac{1-p}{pR_B}\frac{d-\lambda_{bs}}{1-\lambda_{bs}}}; 1 + \frac{R_G-pR_B}{pR_B}\right]$. We have that it always holds that $\frac{qR_G}{1-(1-q)pR_B} \leq 1 + \frac{R_G-pR_B}{pR_B}$, and that $\frac{qR_G}{1-(1-q)pR_B} \geq \frac{1}{1-\frac{1-p}{pR_B}\frac{d-\lambda_{bs}}{1-\lambda_{bs}}} \Leftrightarrow R_G \geq \overline{R_G}$. Hence, the bank chooses optimal $\lambda = \lambda_{BS}$ for any $R_G \geq \overline{R_G}$.

For $\lambda \in [0; \lambda_{BS})$ the matters get a little bit more complicated, because P and R_D have to be determined jointly (this was not the case in other cases where $P = pR_B$). The derivative of the objective function with respect to λ reads

$$\begin{aligned}
&(R_D - R_G)\, q\, (1 - \pi\gamma - (1-\pi)(\gamma_S + \gamma_L)) + \\
&(1-P)\left[R_D\,(1 - q + \pi q\gamma + (1-\pi)\, q\gamma_L) + \frac{R_G}{P}(1-\pi)\, q\gamma_S\right].
\end{aligned}$$

Using $R_D = \frac{R_G}{P}$ which has to hold if the GI banks are indifferent between borrowing and selling at t=1 the above condition boils down to

$$\frac{R_G}{P}(1-P).$$

First, if the highest possible price at t=1, which obtains for $\lambda = 0$, is lower than 1, then the bank will never choose any $\lambda \in [0; \lambda_{BS})$, because the objective function is increasing in λ for all $\lambda \in [0; \lambda_{BS})$. This occurs for the price (taken from Result 5) $P = \frac{R_G(d(1-\pi)-\lambda)}{d(1-\pi)-\lambda+(R_G-pR_B)(1-q)(1-\lambda)}$ for $\lambda = 0$, i.e., $P(\lambda = 0) = \frac{R_G d(1-\pi)}{d(1-\pi)+(R_G-pR_B)(1-q)}$, that is lower than 1. That holds for any R_G if $d \in \left(\frac{1-q}{1-\pi}pR_B; \frac{1-q}{1-\pi}\right]$ or for $R_G \in \left(1; \widetilde{R}_G\right)$ if $d \in \left(\frac{1-q}{1-\pi}; pR_B\right)$, where $\widetilde{R}_G \equiv \frac{d(1-\pi)-(1-q)pR_B}{d(1-\pi)-(1-q)}$

Second, when $R_G \geq \widetilde{R}_G$ and $d \in \left(\frac{1-q}{1-\pi}; pR_B\right)$, $\lambda = 0$ can be an equilibrium choice for the bank. The reason is that we have that $P(\lambda = 0) > 1$, which then makes the derivative of the objective function negative and for any given $\lambda \in [0; \lambda_{BS})$ we can always find such γ, γ_S and γ_L such that the interbank market clears. As in case of the inactive interbank market we can show that $P = 1$ that nullifies the derivative of the objective function is an unstable equilibrium. The reason is that if we take any arbitrarily small perturbation from the equilibrium price of 1 the bank would prefer to set either $\lambda = 0$, which is another possible equilibrium or the bank would prefer to have a higher λ, and we showed that there are other equilibria with higher λ for any R_G. This also means that again here we can have multiple equilibria at t=1, because the bank can either choose $\lambda = 0$ or any other optimal $\lambda \in [\lambda_{BS}; \lambda_0]$ depending on the position of $\widetilde{R}_G$ relative to $\underline{R_G}$ and $\overline{R_G}$.