Private Money and Banking Regulation*

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Abstract

We show that the regulation of the banking system is necessary for the optimal provision of private money. In an environment in which bankers cannot commit to repay their creditors, neither free banking nor a form of narrow banking in which banks hold 100% in reserves can provide the socially efficient amount of private money. If the bankers provided such an amount, then they would prefer to default on their liabilities. We show that an intervention that increases the value of the banking sector's assets (e.g., by regulating credit markets) will mitigate the commitment problem. If the return on the banking sector's assets is made sufficiently large, then it is possible to implement an efficient allocation with private money.

Keywords: Private money; banking regulation; limited commitment.

JEL classification: E40, E42, G21, G28.

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1. INTRODUCTION

The institutions composing the banking system do many things, but one of their main functions is to create liquidity. Among many forms of liquidity creation, banks issue liabilities that can be used to facilitate payments and settlement. This is private money. For example, Gorton (1999) highlights the free banking era as a period in American monetary history in which privately issued monies circulated as competing media of exchange. More contemporarily, it has been argued by many observers of the recent financial crisis that repurchase agreements are the private monies of our time (e.g., see Gorton and Metrick, 2010 and the explanations therein). Therefore, a primary concern of monetary economists should be to know whether, putting stability issues aside, a private banking system is capable of creating enough of this kind of liquidity to allow society to achieve an efficient allocation. In other words, can a private banking system provide the socially efficient amount of money? And if so, what are the characteristics of such a system? Should we leave the job to the invisible hand or should we regulate the banking system? Can narrow banking – whereby the business of lending is separated from the business of deposit-taking – provide the efficient amount of money?

To investigate these questions, we construct a general equilibrium model in which some private agents have the ability to issue liabilities that circulate as a medium of exchange. These agents then use the proceeds to make loans in the credit market, obtaining a profit from these activities after repaying their creditors. We refer to these agents as bankers. In our framework, what prevents them from supplying the efficient amount of private money? The answer is simple: Bankers cannot commit to repay creditors, and the threat of terminating their franchise may not be strong enough to induce them to always redeem their liabilities at par.[1] To ensure that bankers do not overborrow and strategically default on their liabilities, we consider a mechanism that imposes individual debt limits on each

[1] This is very much in the spirit of the hypotheses made in Gu, Mattesini, Monnet, and Wright (forthcoming), Hart and Zingales (2011), Gertler and Karadi (2011), Boissay (2011), Cavalcanti and Wallace (1999a, 1999b), and Cavalcanti, Erosa and Temzelides (1999).

banker, as in Alvarez and Jermann (2000).[2] These individual debt limits constrain the banker's portfolio choices and discipline private money creation. While these limits guarantee the solvency of each banker, they also constrain the amount of liabilities that each banker can issue.

Our contribution to the literature is to show how the degree of competition in credit markets affects the bankers' willingness to create private money. We initially characterize a free-banking regime in which lending practices are left unregulated. We show that, in this case, any equilibrium is necessarily inefficient. As lenders compete for making loans in the credit market, the return on the banking sector's assets is relatively low. As a consequence, the return that bankers are willing to pay on their liabilities cannot be too high, as otherwise they would renege on their promises. From a social standpoint, we want them to pay a sufficiently high return on their liabilities that are used as money in order to eliminate the opportunity cost of holding them (the Friedman rule). Because of the low return on their assets, bankers are unwilling to supply the socially efficient amount of private money.

This result indicates that, to achieve efficiency, it is necessary to raise the return on the banking sector's assets, given that it is socially desirable to induce bankers to pay a high return on their liquid liabilities. In particular, it rules out some forms of banking regulation such as the proposal of requiring banks to hold 100% in reserves. This form of narrow banking will not be able to provide an efficient amount of private money, as there is no clear way to increase the return on the assets of a bank that keeps 100% in reserves.

In view of the inefficient provision of private money in the absence of intervention, we characterize a regulatory framework that allows us to raise the return on the banking sector's assets. Such a framework will allow bankers to extract a bigger surplus from borrowers, permitting them to pay a higher return on their money-like liabilities. In particular, we show that it is possible to implement an efficient allocation with private money.

The role of regulation in guaranteeing a high franchise value for banks has been recognized by many experts, and, in this respect, our paper is closest to Hellmann, Murdock, and

[2] Their analysis builds on work by Kehoe and Levine (1993) and Kocherlakota (1996).

Stiglitz (2000). They consider a model of banks with moral hazard and argue that the best way to ensure a high franchise value is to put a cap on the interest rate paid on deposits. As they write it, by limiting the degree of competition in the deposit market, a deposit rate control will increase the per period profits, increasing the bank's franchise value. Their analysis does not consider the role of certificates of deposits as liquid assets. While our analysis agrees with the general finding that a high franchise value is necessary for efficiency, we show that this value should not originate from the liability side of a bank's balance sheet.

Our paper is clearly related to the large literature on the optimal creation of private liquidity. However, in this literature, the effects of competition in bank lending are usually excluded from the analysis. There are two strands in this literature. The first strand focuses on the role of liquidity as a means of payment. Cavalcanti, Erosa, and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b) study private money creation in the context of a random matching model. Azariadis, Bullard, and Smith (2001) study private and public money creation using an overlapping generations model; Kiyotaki and Moore (2001) propose a theory of inside money based on the possibility of collateralization of part of a debtor's assets; and Monnet (2006) studies the characteristics of the agent that is most able to issue money.[3] The second strand focuses on the role of liquidity as a means of funding investment opportunities. For example, Holmstrom and Tirole (2011) show that a moral hazard problem may limit the ability of firms to refinance their ongoing projects when there is aggregate uncertainty. They argue that this inefficiency can be resolved by the government issuing bonds to firms.

Other authors have focused exclusively on the study of competition in bank lending without explicitly accounting for the role of bankers as liquidity providers. These include Yanelle (1997) and Winton (1995, 1997). Our results show that the degree of competition in bank lending crucially influences the bankers' willingness to create private money. Thus, it is important to characterize the interplay between these two activities.

[3] Other papers in this literature include Williamson (1999), Kashyap, Rajan, and Stein (2002), Sun (2007), and Andolfatto and Nosal (2009).

Another paper closest to ours is Hart and Zingales (2011), who show that an unregulated private banking system creates too much money. They present an environment similar to Gu, Mattesini, Monnet, and Wright (forthcoming), to which our paper also bears a resemblance, where a lack of double coincidence of wants, a lack of commitment, and a limited pledgeability of collateral give rise to an essential role for a medium of exchange. A bank acts as a safe-keeping institution for the collateral and issues receipts that can circulate as a means of payment because the bank is able to commit to pay the bearer of a receipt on demand. Hart and Zingales uncover an interesting externality: A bank that issues more money to its customers increases the price level for all other customers as well. As a result, too much collateral is stored, and banks create too much money. We depart from their analysis in a fundamental way: While they assume that banks can commit to pay back the bearer of the receipts they issued, we assume they cannot. This suffices to overturn their result: We show that a poorly regulated banking system creates too little money.

Empirical work on bank liquidity creation is scant, and the Berger and Bouwman (2009) paper is, to the best of our knowledge, the only one that measures the amount of liquidity created by the banking system. The authors construct a measure of liquidity creation by comparing how liquid the entries on both sides of a bank's balance sheet are. According to this measure, a bank creates more liquidity the more its liabilities are liquid relative to its assets. Among other interesting things, they find that banks that create more liquidity are valued more highly by investors, as measured by the market-to-book and the price-earnings ratios.

To be clear, we are not concerned in this paper with the stability of the banking sector. This is clearly an important issue that also relates to liquidity creation. In particular, the business of liquidity transformation and the risks it entails have been highlighted most forcefully in the seminal paper by Diamond and Dybvig (1983). Their notion of liquidity is one of immediacy: Bank deposits are useful because they can be redeemed on demand when depositors have an urgency to consume. So the banking system is fragile whenever the bank cannot fulfill the demand for immediate redemption. This is the well-known problem of a bank being illiquid but solvent. However, Jacklin (1987) considers a solution to banks' inherent fragility, namely that banks issue tradeable securities. If depositors have an urge to consume, they can sell these securities instead of running to the bank. This notion of liquidity (namely the ease with which bank liabilities can be traded) is clearly related to ours.

The paper is structured as follows. In Section 2, we present the basic framework, and we discuss it in Section 3. In Section 4, we formulate and solve the planner's problem. In Section 5, we characterize equilibrium allocations in the case of an unregulated banking system. In Section 6, we discuss the role of a regulator and characterize the equilibrium allocations in the case of a regulated banking system. Section 7 concludes.

2. MODEL

Time $t = 0, 1, 2, ...$ is discrete, and the horizon is infinite. Each period is divided into two subperiods. There are three physical commodities: good 1, good 2, and an intermediate good. The intermediate good can be perfectly stored from the first to the second subperiod. It depreciates completely if stored until the following date or if used in the production process. Good 1 can be produced only in the first subperiod, and good 2 can be produced only in the second subperiod. If good 1 is not properly stored, it will depreciate completely. There exists a technology that allows people to store good 1 from one date to the next. This technology returns $\beta^{-1} > 1$ units of good 1 at date $t+1$ for each unit invested at date t. Finally, good 2 cannot be stored and must be immediately consumed.

There are four types of agents: buyers, sellers, entrepreneurs, and bankers. There is a $[0, 1]$ continuum of each type. Buyers, sellers, and bankers are infinitely lived. Entrepreneurs live for two periods only. At each date t, entrepreneurs are born in the first subperiod and live until the second subperiod of date $t + 1$.

Buyers and sellers want to consume and are able to produce in the first subperiod. Specifically, they produce good 1 using a divisible technology that returns one unit of the good for each unit of effort they put in. Only buyers want to consume good 2, and only sellers are able to produce it. Such a technology requires k units of the intermediate good and n units of effort to produce $F(k,n)$ units of good 2. Assume that $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is twice continuously differentiable, increasing in both arguments, and strictly concave, with $F(0,n) = 0$ for all $n \geq 0$ and $F(k,0) = 0$ for all $k \geq 0$. Finally, we assume that only sellers can store the intermediate good from the first to the second subperiod.

Entrepreneurs specialize in the production of intermediate goods. Each entrepreneur is endowed with a nontradable, indivisible investment project at birth. Each project requires the investment of *exactly* e units of good 1 at date t to produce $\gamma\hat{k}$ units of intermediate goods at the beginning of date $t+1$, where $\hat{k} > 0$ is a constant. Entrepreneurs are heterogeneous with respect to their productivity levels γ. Specifically, the function $G(\gamma)$ describes the distribution of the productivity levels γ across the population of entrepreneurs. Suppose that $\gamma \in [0, \bar{\gamma}]$ and that there exists a density function $g(\gamma)$.

This means that good 1 can be either immediately consumed or used in other activities. If it is not consumed at date t, it can be converted into the intermediate good at date $t+1$ or can be stored as an inventory.

Bankers are endowed with a technology that allows them to make their actions publicly observable at no cost. This means that if a banker decides to make his actions publicly observable, other agents can keep track of his balance sheet and income statement at each date.

We now explicitly describe preferences. Let $x_t^b \in \mathbb{R}$ denote a buyer's net consumption in the first subperiod, and let $q_t^b \in \mathbb{R}_+$ denote his consumption in the second subperiod. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t^b\right) \right],$$

where $\beta \in (0,1)$. The function $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and strictly concave, with $u'(0) = \infty$. Let $x_t^s \in \mathbb{R}$ denote a seller's net consumption in the first subperiod, and let $n_t^s \in \mathbb{R}_+$ denote his effort level in the second subperiod. His preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t^s\right) \right],$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, increasing, and convex. Let $x_t \in \mathbb{R}_+$ denote a banker's consumption in the first subperiod. Each banker has preferences given by

$$\sum_{t=0}^{\infty} \beta^t x_t.$$

Finally, an entrepreneur born at date t wants to consume only at date $t+1$. Specifically, each entrepreneur born at date t derives utility x_{t+1}^e if his consumption of good 1 at date $t+1$ is $x_{t+1}^e \in \mathbb{R}_+$.

Buyers, sellers, and bankers lack any commitment, whereas entrepreneurs can fully commit to their promises. We also assume that buyers and sellers are anonymous: There is no technology to verify their identities, and their trading histories are privately observable.

In the first subperiod, there is a perfectly competitive (Walrasian) market in which agents trade good 1 and the intermediate good. In the second subperiod, only buyers and sellers trade. Following the literature, we refer to the second market as the decentralized market. For simplicity, we will use competitive pricing to determine the terms of trade in this market. Still, a medium of exchange remains essential as long as we maintain the (intertemporal) double coincidence problem and anonymity; see Rocheteau and Wright (2005) for a discussion.

Finally, note that, because buyers are anonymous and lack any commitment, they cannot credibly use goods invested in the storage technology as a means of payment in the decentralized market. Thus, the storage technology for good 1 corresponds to the concept of illiquid capital in Lagos and Rocheteau (2008).

3. DISCUSSION OF THE MODEL

In this section, we explain how the pieces of the model fit together. To generate a demand for a medium of exchange, we build on Lagos and Wright (2005).[4] In the decentralized market, the absence of commitment and recordkeeping implies that a buyer and a seller

[4] An alternative tractable framework that also creates a role for a medium of exchange is the large household model in Shi (1997).

can trade only if a medium of exchange is made available. Because bankers can make their actions publicly observable, they will be able to issue liabilities that can be used as a medium of exchange as long as people believe that they will be willing to redeem them at a future date.

Note that the bankers also lack commitment. So we need to have some sort of punishment for default to guarantee that they make good on their promises, a necessary condition for their private liabilities to circulate as money. As in Cavalcanti, Erosa, and Temzelides (1999), Cavalcanti and Wallace (1999a, 1999b), and Gu, Mattesini, Monnet, and Wright (forthcoming), we assume the existence of a mechanism that guarantees that any banker who reneges on his promises be punished. Precisely, we assume that a banker who defaults on his liabilities can no longer have his actions publicly observable. Moreover, any assets he holds when he defaults will be seized. This means that a defaulter will lose future profits because he will no longer be able to finance his business by issuing private money.

In this respect, the availability of public knowledge of the banker's actions is crucial for allowing people to identify the states of the world in which the banker will be willing to repay his creditors. In the decentralized market, a seller does not trust a buyer's IOU because he knows that the latter cannot be punished in case of default. But a seller may accept a banker's IOU because the banker can be punished if he fails to redeem his IOUs. Thus, there will be some states of the world in which the banker will be willing to redeem his notes at par, and everybody knows in which states this will happen. Figure 1 shows how a banker's note will circulate in the economy.

4. EFFICIENT ALLOCATIONS

In this section, we formulate and solve the problem of a planner who has the ability to enforce all transfers at zero cost. This means that a solution to the planner's problem will give us an unconstrained efficient allocation. We also assume that the planner treats entrepreneurs of the same generation equally. Thus, he will assign the same consumption level to each member of a given generation. Given these assumptions, an efficient allocation is obtained in the usual way: Given some minimum utility level U_t^e assigned to each entrepreneur of generation t, for all generations, and some minimum utility levels U and U^s assigned to each banker and each seller at date $t = 0$, respectively, an efficient allocation maximizes the lifetime utility of each buyer subject to the participation and resource constraints.

It should be clear that the planner will fund only the entrepreneurs who are sufficiently productive. This means that each entrepreneur whose productivity level γ is greater than or equal to a specific marginal type $\gamma_t^p \in [0, \bar{\gamma}]$ will receive e units of good 1 to undertake his project at date t, whereas the types $\gamma \in [0, \gamma_t^p)$ will not operate their projects. We refer to the type γ_t^p as the date-t marginal entrepreneur. Thus, the planner's problem consists of choosing an allocation

$$\left\{x_t^b, x_t^s, x_t, x_t^e, q_t, n_t, i_t, k_{t+1}, \gamma_t^p\right\}_{t=0}^{\infty}$$

to maximize the lifetime utility of the buyer

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^b + u\left(q_t\right)\right], \tag{1}$$

subject to the resource constraint for good 1

$$x_t^b + x_t^s + x_t^e + x_t + i_t = 0, \tag{2}$$

the resource constraint for good 2

$$q_t = F\left(k_t, n_t\right), \tag{3}$$

the law of motion for the production of intermediate goods

$$k_{t+1} = \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma, \tag{4}$$

$$i_t = e\left[1 - G\left(\gamma_t^p\right)\right], \tag{5}$$

the entrepreneurs' participation constraints

$$x_t^e \geq U_{t-1}^e, \tag{6}$$

the banker's participation constraint

$$\sum_{t=0}^{\infty} \beta^t x_t \geq U, \tag{7}$$

and the seller's participation constraint

$$\sum_{t=0}^{\infty} \beta^t \left[x_t^s - c\left(n_t\right)\right] \geq U^s,$$

taking the initial stock $k_0 = k\left(\gamma_{-1}^p\right) > 0$ and the required utility levels $\left\{U_{t-1}^e\right\}_{t=0}^{\infty}$, U, and U^s as given. Notice that any Pareto optimal allocation solves the problem described above for a particular choice of required utility levels $\left\{U_{t-1}^e\right\}_{t=0}^{\infty}$, U, and U^s, and that any solution to the problem above is a Pareto optimal allocation.

Let $k\left(\gamma_t^p\right) \equiv \hat{k} \int_{\gamma_t^p}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma$ denote the aggregate amount of intermediate goods available at the beginning of date $t+1$ as a function of the date-t marginal entrepreneur γ_t^p. The first-order conditions are given by

$$\beta u'\left[F\left(k\left(\gamma_t^p\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^p\right), n_{t+1}\right) \hat{k} \gamma_t^p = e, \tag{8}$$

$$u'\left[F\left(k\left(\gamma_{t-1}^p\right), n_t\right)\right] F_n\left(k\left(\gamma_{t-1}^p\right), n_t\right) = c'\left(n_t\right), \tag{9}$$

for all $t \geq 0$. To marginally increase each buyer's consumption at date $t+1$ without changing the effort level that each seller exerts at date $t+1$, the planner needs to give up e units of good 1 at date t at the margin to increase the amount of intermediate goods available for production at date $t+1$. The left-hand side in (8) gives the marginal benefit of an extra unit of intermediate goods at date $t+1$, whereas the right-hand side gives the marginal resource cost at date t. Similarly, to marginally increase each buyer's consumption at date t given a predetermined amount of intermediate goods at the beginning of date t, the planner needs to instruct each seller to exert more effort in the second subperiod. Condition (9) guarantees that the marginal disutility of effort equals the marginal benefit of consuming an extra unit of good 2.

A stationary solution to the planner's problem involves $\gamma_t^p = \gamma^*$ and $n_t = n^*$ for all $t \geq 0$, with γ^* and n^* satisfying

$$\beta u'\left[F\left(k\left(\gamma^*\right), n^*\right)\right] F_k\left(k\left(\gamma^*\right), n^*\right) \hat{k} \gamma^* = e, \tag{10}$$

$$u'\left[F\left(k\left(\gamma^*\right), n^*\right)\right] F_n\left(k\left(\gamma^*\right), n^*\right) = c'\left(n^*\right). \tag{11}$$

We also need the initial amount of intermediate goods to be equal to $k(\gamma^*)$. In the Appendix, we show the existence and uniqueness of a stationary solution to the planner's problem for at least some specifications of preferences and technologies.

5. FREE BANKING

In this section, we describe the equilibrium outcome of an economy without intervention in lending practices. This means that any person will be able to supply funds to entrepreneurs. Because buyers and sellers are able to produce good 1, they can supply funds if they want to. Bankers can also make loans to entrepreneurs by borrowing resources through the sale of notes or by using some of their retained earnings.

To finance his investments at date t, a banker raises funds by selling notes to buyers. Then, he uses the proceeds from the sale of notes to supply funds in the credit market or to invest in the storage technology, or both. At date $t+1$, he collects the proceeds from his investments and repays his creditors, consuming or reinvesting the remaining profits. Specifically, a note issued by a banker at date t gives him ϕ_t units of good 1 and is a promise to repay one unit of good 1 at date $t+1$ to the note holder. Each banker has a technology that allows him to create perfectly divisible notes at zero cost. Notes issued by one banker are perfectly distinguishable from those issued by any other banker so that counterfeiting is not a problem.

Throughout the paper, we restrict attention to symmetric equilibria in which all notes trade at the same price. This means that the notes issued by any pair of bankers are perfect substitutes (as long as people believe both bankers will be willing to redeem them at par). Every agent in the economy takes the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ as given when making his individual decisions.

The goal of this section is to characterize equilibrium allocations in the absence of intervention in lending practices. Because bankers cannot commit to repay creditors, we need to assume the existence of a regulator whose exclusive role will be to punish those who default on their liabilities. As in Hellmann, Murdock, and Stiglitz (2000), the regulator has access to each banker's balance sheet and can punish any banker who defaults on his liabilities by revoking his "franchise" and garnishing his assets.

5.1. Credit Market

In the first subperiod, there is a perfectly competitive market for loans in which the entrepreneurs can borrow e units of good 1 to fund their investment projects. Let R_t denote the gross interest rate that prevails in this market at date t. We claim that, in the absence of intervention, the equilibrium interest rate must be

$$R_t = \beta^{-1}. \tag{12}$$

If $R_t > \beta^{-1}$, then any buyer or seller will wish to supply an infinite amount of resources. If $R_t < \beta^{-1}$, then the supply of funds will be zero because agents have the option of using the storage technology to transfer resources from one period to the next.

Let us now consider the decision problem of a type-γ entrepreneur. This entrepreneur has a profitable project if and only if $\rho_{t+1}\hat{k}\gamma - e\beta^{-1} \geq 0$, where ρ_t denotes the price of one unit of intermediate goods in terms of good 1. Note that $\rho_{t+1}\hat{k}\gamma$ gives the value of a type-γ entrepreneur's project at date $t+1$ in terms of good 1, whereas $eR_t = e\beta^{-1}$ gives the repayment that the entrepreneur needs to make at date $t+1$. Thus, a type-γ entrepreneur has a profitable project if and only if the surplus from such a project is positive. Given the relative price of capital ρ_{t+1}, any type-γ entrepreneur for whom

$$\rho_{t+1}\hat{k}\gamma \geq e\beta^{-1} \tag{13}$$

will find it optimal to borrow at date t. Thus, given ρ_{t+1}, we can define the date-t marginal entrepreneur γ_t^m as the type satisfying

$$\gamma_t^m = \frac{e}{\beta\rho_{t+1}\hat{k}}. \tag{14}$$

This means that any entrepreneur indexed by $\gamma \in [0, \gamma_t^m]$ will find it optimal to borrow in the credit market to fund his project, whereas the types $\gamma \in [\gamma_t^m, \bar{\gamma}]$ will choose not to fund their projects. Thus, the aggregate demand for loans at date t is given by

$$\ell_t = e\left[1 - G\left(\gamma_t^m\right)\right].$$

In this case, the aggregate amount of intermediate goods available at date $t+1$ will be given by

$$k_{t+1} = \hat{k}\int_{\gamma_t^m}^{\bar{\gamma}} \gamma g\left(\gamma\right) d\gamma \equiv k\left(\gamma_t^m\right). \tag{15}$$

5.2. Buyer's Problem

Let $w_t^b\left(a, l\right)$ denote the value function for a buyer who enters the first subperiod holding $a \in \mathbb{R}_+$ notes and $l \in \mathbb{R}_+$ loans, and let $v_t^b\left(k, a, l\right)$ denote the value function for a buyer who enters the second subperiod holding a portfolio of $k \in \mathbb{R}_+$ units of intermediate goods, $a \in \mathbb{R}_+$ notes, and $l \in \mathbb{R}_+$ loans. The Bellman equation for a buyer in the first subperiod is given by

$$w_t^b\left(a, l\right) = \max_{(x,k',a',l') \in \mathbb{R}\times\mathbb{R}_+^3} \left[x + v_t^b\left(k', a', l'\right)\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' + l' = \beta^{-1} l + a.$$

Here k' denotes the amount of intermediate goods the buyer accumulates at the end of the first subperiod, a' denotes his choice of note holdings at the end of the first subperiod, and l' denotes his supply of funds in the credit market. Because of quasi-linear preferences, the value $w_t^b\left(a, l\right)$ is an affine function of the form $w_t^b\left(a, l\right) = a + \beta^{-1} l + w_t^b\left(0, 0\right)$, with the intercept $w_t^b\left(0, 0\right)$ given by

$$w_t^b\left(0, 0\right) = \max_{(k',a',l') \in \mathbb{R}_+^3} \left[-\rho_t k' - \phi_t a' - l' + v_t^b\left(k', a', l'\right)\right]. \tag{16}$$

Let p_{t+1} denote the price of one unit of good 2 at date t in terms of good 1 at date $t+1$. The Bellman equation for a buyer holding a portfolio of k' units of intermediate goods, a' notes, and l' loans in the second subperiod is given by

$$v_t^b\left(k', a', l'\right) = \max_{q \in \mathbb{R}_+} \left[u\left(q\right) + \beta w_{t+1}^b\left(a' - p_{t+1} q, l'\right)\right], \tag{17}$$

subject to the liquidity constraint

$$p_{t+1}q \leq a'. \tag{18}$$

Because there is no public record of individual loans made in the credit market, a buyer cannot credibly use his claim on entrepreneurs as a means of payment in the second market, even though entrepreneurs can fully commit to repay their creditors. On the other hand, every note issued by a banker is publicly observable and perfectly identifies him as a debtor.

Using the fact that $w_t^b(a,l)$ is an affine function, we can rewrite the Bellman equation (17) as follows:

$$v_t^b\left(k',a',l'\right) = \max_{q\in\mathbb{R}_+}\left[u\left(q\right) - \beta p_{t+1}q\right] + \beta a' + l' + \beta w_{t+1}^b\left(0,0\right).$$

First, notice that there is no benefit of accumulating intermediate goods (they fully depreciate if not properly stored). Therefore, the buyer optimally chooses $k'=0$. Because the return on funds supplied in the credit market is β^{-1}, the buyer is willing to supply any amount of resources.

The liquidity constraint (18) may either bind or not, depending on the buyer's note holdings. In particular, notice that

$$\frac{\partial v_t^b}{\partial a}\left(k',a',l'\right) = \begin{cases} \frac{1}{p_{t+1}}u'\left(\frac{a'}{p_{t+1}}\right) \text{ if } a' < p_{t+1}\hat{q}\left(p_{t+1}\right); \\ \\ \beta \text{ if } a' > p_{t+1}\hat{q}\left(p_{t+1}\right); \end{cases}$$

where $\hat{q}\left(p_{t+1}\right) = \left(u'\right)^{-1}\left(\beta p_{t+1}\right)$. If the liquidity constraint does not bind, then the marginal utility of an extra note equals β, which is simply the discounted value of the payoff of one unit of good 1 at date $t+1$. If the liquidity constraint binds, then the marginal utility of an extra note is greater than β. In this case, the notes offer a liquidity premium. Since the buyer can use the storage technology, he will hold notes if and only if he obtains a liquidity premium or the return on notes is greater than the return to storage.

The first-order condition for the optimal choice of note holdings on the right-hand side of (16) is given by

$$-\phi_t + \frac{\partial v_t^b}{\partial a}\left(k',a',l'\right) \leq 0,$$

with equality if $a' > 0$. If $\phi_t > \beta$, then the optimal choice of note holdings will be given by

$$u'\left(\frac{a'}{p_{t+1}}\right) = \phi_t p_{t+1}, \tag{19}$$

so that notes offer a liquidity premium. Because of quasi-linear preferences, all buyers choose to hold the same quantity of notes at the end of the first market. Thus, condition (19) gives the aggregate demand for notes as a function of the relative price of good 2 p_{t+1} and the price of notes ϕ_t. A higher price for notes reduces the amount of notes demanded. The effect of the relative price p_{t+1} on the demand for notes depends on the curvature of the utility function $u(q)$. If $-[u''(q)q]/u'(q) < 1$, then an increase in p_{t+1} reduces the demand for notes, holding ϕ_t constant. If $-[u''(q)q]/u'(q) > 1$, then an increase in p_{t+1} results in a higher demand for notes.

5.3. Seller's Problem

Let $w_t^s(a, l)$ denote the value function for a seller who enters the first subperiod holding $a \in \mathbb{R}_+$ notes and $l \in \mathbb{R}_+$ loans, and let $v_t^s(k, a, l)$ denote the value function for a seller who enters the second subperiod holding $k \in \mathbb{R}_+$ units of intermediate goods, $a \in \mathbb{R}_+$ notes, and $l \in \mathbb{R}_+$ loans. The Bellman equation for a seller in the first subperiod is given by

$$w_t^s(a, l) = \max_{(x,k',a',l') \in \mathbb{R} \times \mathbb{R}_+^3} \left[x + v_t^s\left(k', a', l'\right)\right],$$

subject to the budget constraint

$$x + \rho_t k' + \phi_t a' + l' = \beta^{-1} l + a.$$

Here k' denotes the amount of intermediate goods the seller accumulates at the end of the first subperiod, a' denotes his choice of note holdings at the end of the first subperiod, and l' denotes his supply of funds in the credit market. Similarly, the value $w_t^s(a, l)$ is an affine function, $w_t^s(a, l) = a + \beta^{-1} l + w_t^s(0, 0)$, with the intercept $w_t^s(0, 0)$ given by

$$w_t^s(0, 0) = \max_{(k',a',l') \in \mathbb{R}_+^3} \left[-\rho_t k' - \phi_t a' - l' + v_t^s\left(k', a', l'\right)\right]. \tag{20}$$

The Bellman equation for a seller with a portfolio of k' units of intermediate goods, a' notes, and l' loans in the second subperiod is given by

$$v_t^s\left(k', a', l'\right) = \max_{n \in \mathbb{R}_+} \left[-c\left(n\right) + \beta w_{t+1}^s\left(p_{t+1} F\left(k', n\right) + a', l'\right)\right]. \tag{21}$$

Using the fact that $w_t^s\left(a, l\right)$ is an affine function, we can rewrite the right-hand side of (21) as follows:

$$\max_{n \in \mathbb{R}_+} \left[-c\left(n\right) + \beta p_{t+1} F\left(k', n\right)\right] + \beta a' + l' + \beta w_{t+1}^s\left(0, 0\right).$$

The first-order condition for the optimal choice of effort in the second subperiod is given by

$$c'\left(n\right) = \beta p_{t+1} F_n\left(k', n\right). \tag{22}$$

Because $\left(\partial v_t^s / \partial k\right)\left(k', a', l'\right) = \beta p_{t+1} F_k\left(k', n\right)$, the first-order condition for the optimal choice of intermediate goods on the right-hand side of (20) is given by

$$\rho_t = \beta p_{t+1} F_k\left(k', n\right). \tag{23}$$

Thus, conditions (22) and (23) determine the demand for intermediate goods and the effort decision as a function of the relative price of the good 2 p_{t+1} and the relative price of intermediate goods ρ_t. Combining (22) with (23), we obtain the following condition:

$$\frac{\rho_t}{c'\left(n\right)} = \frac{F_k\left(k', n\right)}{F_n\left(k', n\right)}. \tag{24}$$

Because the return on funds supplied in the credit market is β^{-1}, the seller is willing to supply any amount of resources. Finally, the first-order condition for the optimal choice of note holdings is given by

$$-\phi_t + \beta \leq 0,$$

with equality if $a' > 0$. This means that the seller does not hold notes if $\phi_t > \beta$.

5.4. Banker's Problem

Now we describe the decision problem of a banker. Let $w_t\left(b_{t-1}, i_{t-1}\right)$ denote the value function for a banker with debt b_{t-1} and assets i_{t-1} at the beginning of date t. The banker's assets at the beginning of date t consist of loans made at date $t-1$ and units invested in the storage technology at date $t-1$, whereas the banker's debt refers to the amount of notes issued at date $t-1$. As we have seen, the marginal return on the banker's assets is given by β^{-1} in the absence of intervention, whether he invests in the credit market or in the storage technology. Thus, the banker's decision problem can be formulated as follows:

$$w_t\left(b_{t-1}, i_{t-1}\right) = \max_{\left(x_t, i_t, b_t\right) \in \mathbb{R}_+^3} \left[x_t + \beta w_{t+1}\left(b_t, i_t\right)\right] \tag{25}$$

subject to the budget constraint

$$i_t + x_t + b_{t-1} = \beta^{-1} i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

Here i_t denotes the amount of resources (units of good 1) that the banker decides to invest at date t (i.e., his assets at the beginning of date $t+1$). When making his investment decisions, the banker takes as given the sequence of debt limits $\left\{\bar{B}_t\right\}_{t=0}^{\infty}$, the marginal return on his assets β^{-1}, and the sequence of prices $\left\{\phi_t\right\}_{t=0}^{\infty}$.

If $\phi_t > \beta$, then the banker finds it optimal to borrow up to his debt limit, i.e., he will choose $b_t = \bar{B}_t$. Because the return paid on his notes (his cost of funds) is lower than the return on his assets, he makes a profit by borrowing and investing the proceeds in the storage technology. Note also that, because the return on his assets equals his rate of time preference, he is indifferent between immediately consuming and reinvesting the proceeds from his previous profits (his retained earnings). Therefore, a solution to the banker's optimization problem is $i_t = \phi_t \bar{B}_t$, which means that the banker invests all funds he has borrowed at date t but does not invest his own funds. Thus, the balance sheet of a typical banker will have no equity, only debt. In this case, the banker's consumption at date t is simply given by

$$x_t = \bar{B}_{t-1}\left(\beta^{-1}\phi_{t-1} - 1\right).$$

We refer to the franchise value as the lifetime utility associated with a particular choice of the return on the banker's assets, the sequence of debt limits, and the sequence of prices for the banker's liabilities. At each date t, the franchise value is given by

$$w_t\left(\bar{B}_{t-1}, \phi_{t-1}\bar{B}_{t-1}\right) = \sum_{\tau=t}^{\infty} \beta^{\tau-t}\bar{B}_{\tau-1}\left(\beta^{-1}\phi_{\tau-1} - 1\right).$$

Perfect competition in the credit market implies that the return on the banker's assets is the smallest possible, β^{-1} at each date, which lowers the franchise value. As we will see, the introduction of banking regulation will play a crucial role in increasing the return on the banker's assets.

5.5. Aggregate Note Holdings

Let a_t denote the date-t aggregate note holdings. For any price $\phi_t > \beta$, the liquidity constraint (18) is binding, in which case the value of the notes in circulation must equal the value of the aggregate production in the second market,

$$a_t = p_{t+1}F\left(k\left(\gamma_{t-1}^m\right), n_t\right). \tag{26}$$

Note that the aggregate production depends on the total amount of intermediate goods and the effort level that each seller is willing to exert to produce good 2. Combining (19) with (26), we obtain

$$u'\left[F\left(k\left(\gamma_{t-1}^m\right), n_t\right)\right] = \phi_t p_{t+1}.$$

Using (22) to substitute for p_{t+1}, we get the following equilibrium condition:

$$u'\left[F\left(k\left(\gamma_{t-1}^m\right), n_t\right)\right] = \frac{\phi_t}{\beta}\frac{c'\left(n_t\right)}{F_n\left(k\left(\gamma_{t-1}^m\right), n_t\right)}. \tag{27}$$

This condition determines the equilibrium effort decision, given the predetermined stock of intermediate goods. The price of notes ϕ_t influences this decision in the following way: A lower price for notes increases their return and, consequently, the buyer's expenditure decision, raising the relative price p_{t+1} and inducing each seller to exert more effort.

As we have seen, the choice of the date-t marginal entrepreneur is given by (14). Using (24) to substitute for ρ_{t+1}, we obtain the following equilibrium condition:

$$\beta u'\left[F\left(k\left(\gamma_t^m\right), n_{t+1}\right)\right] F_k\left(k\left(\gamma_t^m\right), n_{t+1}\right)\hat{k}\gamma_t^m = e\frac{\phi_{t+1}}{\beta}. \tag{28}$$

This condition determines the equilibrium amount of intermediate goods at date t given the effort decision at date $t+1$. Notice that a lower anticipated value for ϕ_{t+1} results in a larger amount of intermediate goods at date $t+1$, holding n_{t+1} constant.

We can use (27) and (28) to implicitly define the functions $\gamma_{t-1}^m = \gamma^m(\phi_t)$ and $n_t = n(\phi_t)$. Using these functions, we can define the aggregate production of good 2 by $q(\phi_t) = F[k(\gamma^m(\phi_t)), n(\phi_t)]$. Then, the aggregate note holdings as a function of the price ϕ_t are given by

$$a(\phi_t) = \frac{u'[q(\phi_t)]\,q(\phi_t)}{\phi_t}. \tag{29}$$

5.6. Equilibrium

To define an equilibrium, we need to specify the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$ in such a way that the bankers are willing to supply the amount of notes other agents demand and are willing to fully repay their creditors. We take two steps to define a sequence of debt limits satisfying these two conditions. First, for any given sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, we set

$$\bar{B}_t = a(\phi_t) \tag{30}$$

at each date t. This condition guarantees that each banker is willing to supply the amount of notes in (29) at the price ϕ_t. Then, given this choice for the individual debt limits, we need to verify whether a particular choice for the price sequence $\{\phi_t\}_{t=0}^{\infty}$ implies that each banker does not want to renege on his liabilities at any date. As we have seen, a banker who reneges on his liabilities will lose his franchise, in which case he will no longer be able to issue notes. Thus, a particular price sequence $\{\phi_t\}_{t=0}^{\infty}$ is consistent with the solvency of each banker if and only if

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} a(\phi_{\tau-1})\left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq a(\phi_{t-1})\left(\beta^{-1}\phi_{t-1} - 1\right) + \phi_t a(\phi_t)$$

holds at each date t. As in Alvarez and Jermann (2000), these solvency constraints allow the banker to borrow as much as possible without inducing him to default on his liabilities. The left-hand side gives the franchise value. The right-hand side gives the current payoff the banker gets if he decides *not* to invest the resources he has borrowed at date t. In this case, he can increase his current consumption by the amount $a(\phi_t)\phi_t$, but he will permanently lose his franchise at date $t+1$. We can rewrite the solvency constraints above as follows:

$$-\phi_t a(\phi_t) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} a\left(\phi_{\tau-1}\right)\left(\beta^{-1}\phi_{\tau-1} - 1\right) \geq 0. \tag{31}$$

As in Alvarez and Jermann, we want to allow the bankers to borrow as much as they can and, at the same time, make sure that they do not want to default (i.e., debt limits that are not too tight).

Definition 1 *An equilibrium is an array* $\left\{\gamma_t^m, n_t, a_t, \bar{B}_t, \phi_t, R_t\right\}_{t=0}^{\infty}$ *satisfying (12), (27), (28), (29), (30), and (31) with equality at each date t, given the initial stock of intermediate goods.*

5.7. Welfare Properties

Now we want to show an important property of any equilibrium allocation in the absence of intervention (even though we have not shown existence yet). If we compare equations (27) and (28) with the solution to the planner's problem, given by equations (8) and (9), we realize that setting $\phi_t = \beta$ at each date $t \geq 0$ makes the choices of the marginal entrepreneur and the effort level exactly the same as those in the planner's solution. Thus, $\phi_t = \beta$ for all $t \geq 0$ is a necessary condition for efficiency so that the optimal return on notes at each date should be given by β^{-1}. But condition (31) implies that the banker's solvency constraints are necessarily violated in this case, so we cannot have an equilibrium with $\phi_t = \beta$ for all $t \geq 0$. This means that any allocation that can be implemented in the absence of intervention is necessarily *not* Pareto optimal. We summarize these findings in the following proposition.

Proposition 2 *Any equilibrium allocation in the absence of intervention in lending practices is inefficient.*

Why are the bankers unwilling to supply the socially efficient amount of money? As we have seen, the return on the banker's assets is the same as the return to storage and the rate of time preference. Because of perfect competition in credit markets, there is no markup over the return to storage. When lenders compete for borrowers, the return that each one of them gets is lower, which in turn restricts the bankers' ability to pay a higher return on their notes.

To implement the optimal return on notes, we must drive the franchise value to zero, which is inconsistent with the solvency constraints. Because the return on the banking sector's assets is relatively low, there exists an upper bound on the return the bankers are willing to offer on their liabilities without inducing them to default. Any return above this bound makes the banker prefer to default on his liabilities.

The previous result says that any kind of regulation that seeks to restrict competition on the liability side of banks' balance sheets, such as the interest rate cap proposed by Hellmann, Murdock, and Stiglitz (2000), will result in an inefficient amount of private money, regardless of the kind of intervention that is carried out on the asset side. Regulation Q in the U.S. is an example of a regulatory measure aimed at restricting the return that banks are allowed to pay to their depositors. Our analysis thus predicts that these measures necessarily lead to an inefficient amount of bank liquidity creation.

5.8. Existence

To show existence, we will restrict attention to stationary equilibria for which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we discuss the characterization of non-stationary equilibria. In the case of stationary allocations, we have $\phi_t = \phi$, $\gamma_t^m = \gamma^m$, $n_{t+1} = n$, $\bar{B}_t = \bar{B}$, $a_t = a$, and $R_t = \beta^{-1}$ for all $t \geq 0$.

Note that we can use (27) and (28) to define the choices of the marginal entrepreneur γ^m and the effort level n as a function of the price ϕ and then define the aggregate note holdings a in the same way. Finally, any stationary equilibrium must also satisfy the solvency constraints (31). In particular, a stationary solution satisfies these constraints if and only if

$$-\phi a(\phi) + \frac{\beta}{1-\beta} a(\phi) \left(\beta^{-1}\phi - 1\right) \geq 0. \tag{32}$$

Because $a(\phi) > 0$ for any $\phi > \beta$, condition (32) holds if and only if

$$\phi \geq 1.$$

This means that the bankers are willing to supply any amount of notes for which the return on these notes is nonpositive. In other words, in the case of perfect competition in credit markets, the bankers need to charge for their liquidity services in order to be individually rational for them to redeem their notes at par. As we have seen, this result has a crucial implication for the welfare properties of equilibrium allocations. In particular, the nonpositive-return-on-notes property arises in the case of stationary allocations and implies that any stationary equilibrium is necessarily inefficient.

The following proposition establishes existence and uniqueness for some specifications of preferences and technologies.

Proposition 3 *Suppose that* $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$*, with* $0 < \sigma < 1$*,* $c(n) = n$*, and* $F(k,n) = k^{\alpha}n^{1-\alpha}$*, with* $0 < \alpha < 1$*. Suppose also that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. Then, there exists a unique non-autarkic stationary equilibrium for which* $\phi_t = 1$ *for all* $t \geq 0$*.*

Under these specifications of preferences and technologies, it is straightforward to show that the aggregate amount of notes $a(\phi)$ is strictly decreasing in ϕ. This means that the lack of intervention results in an inefficiently small amount of private money, in which case the price of notes will be too high to allow society to achieve a Pareto optimal allocation. This result suggests that we can mitigate the commitment problem only by increasing the return on the banker's assets, which can be accomplished through the creation of banking regulation.

6. REGULATED BANKING

We have shown that a free-banking regime fails to deliver an efficient allocation. Our results have also suggested that the way to achieve efficiency is by raising the return on the banking sector's assets, which will allow us to "relax" the bankers' solvency constraints. In this section, we consider the existence of a regulatory mechanism that sets the terms of trade in the credit market by means of interest rate controls. Because the entrepreneurs' types are publicly observable, the regulator can set different interest rates for different types of borrowers. Let $r_t(\gamma)$ denote the interest rate offered to a type-γ entrepreneur, which is the interest rate that will prevail in the submarket for real loans to type-γ entrepreneurs. So, the goal of the regulator is to find the minimum interest rates $r_t(\gamma)$ that imply a sufficiently high return on the bankers' assets to allow them to supply the socially efficient amount of notes. We start by describing a regulatory mechanism.

6.1. A Regulatory Mechanism

Here we describe the regulatory mechanism (or just mechanism for short) in the credit market. The mechanism announces a return function $\hat{R}_{t+1}(i_t)$ that promises to deliver $i_t\hat{R}_{t+1}(i_t)$ units of good 1 at date $t+1$ if the banker decides to invest i_t units of good 1 at date t. Then, the mechanism collects all funds raised from the bankers and allocates these resources to fund entrepreneurs and invest in the storage technology. Because of the possibility of using the storage technology, the banker's participation constraint is given by

$$\hat{R}_{t+1}(i_t) \geq \beta^{-1}. \tag{33}$$

We will restrict attention to return functions of the form:

$$\hat{R}_{t+1}(i_t) = \begin{cases} \beta^{-1} + \mu_t \text{ if } i_t < e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \\ \\ \beta^{-1} \text{ if } i_t \geq e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right], \end{cases} \tag{34}$$

where $\mu_t \geq 0$ denotes the date-t markup over the return to storage. At each date t, the mechanism chooses a portfolio that devotes the amount $e\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right]$ to fund entrepreneurs and invests the remaining resources in the storage technology. In this way, we can guarantee that any entrepreneur whose project has a positive surplus, given the price ϕ_{t+1}, will be able to get funding provided that the bankers (the suppliers of those funds) have enough resources (raised from the sale of notes and their own retained earnings).

The mechanism also requires that the interest rate $r_t(\gamma)$ that the regulator wants to implement in each active submarket $\gamma \in [0, \bar{\gamma}]$ satisfies the type-γ entrepreneur's participation constraint:

$$\rho_{t+1}\hat{k}\gamma - [1 + r_t(\gamma)]\, e \geq 0. \tag{35}$$

Also, we must have that, given the interest rates $r_t(\gamma)$, the announced return function $\hat{R}_{t+1}(i_t)$ satisfies

$$\left(\beta^{-1} + \mu_t\right)\left[1 - G\left(\gamma^m\left(\phi_{t+1}\right)\right)\right] \leq \int_{\gamma^m(\phi_{t+1})}^{\bar{\gamma}} [1 + r_t(\gamma)]\, g(\gamma)\, d\gamma. \tag{36}$$

The left-hand side gives the amount of resources that the mechanism promised at date t to deliver at date $t+1$, and the right-hand side gives the total repayment received at date $t+1$ from the entrepreneurs who were funded at date t. Thus, condition (36) guarantees that the announced return function $\hat{R}_{t+1}(i_t)$ is feasible. Finally, the participation constraints (33) and (35) imply that the interest rates $r_t(\gamma)$ can neither be too large nor too small:

$$\beta^{-1} \leq 1 + r_t(\gamma) \leq \beta^{-1}\frac{\gamma}{\gamma^m\left(\phi_{t+1}\right)} \tag{37}$$

for each type $\gamma \geq \gamma^m\left(\phi_{t+1}\right)$.

Definition 4 *Given a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$, a mechanism consists of a sequence of markups $\{\mu_t\}_{t=0}^{\infty}$ and a sequence of interest rate functions $\{r_t(\gamma)\}_{t=0}^{\infty}$ satisfying (36) and (37) at each date.*

The mechanism specifies a sequence of markups and interest rates as a function of the sequence of prices $\{\phi_t\}_{t=0}^{\infty}$. We can think of this mechanism as a regulated mutual fund in which all bankers invest their resources. The rules of the fund then determine the amount of resources that will be devoted to finance entrepreneurs and to invest in the storage technology. The regulator will prohibit bankers to make loans on their own. This means that bankers can either invest in the fund or in the storage technology. The choice of a particular mechanism then determines the marginal return on each unit invested in the fund. Different choices for the interest rates $r_t(\gamma)$ by the regulator will imply different values for the markup μ_t, determining the profitability of the fund.

Note that setting $\mu_t = 0$ at each date gives us the competitive solution that we have analyzed in the previous section. We now characterize equilibria for which the markup μ_t is positive at each date so that the bankers will be able to extract some of the surplus from the entrepreneurs. As a result, the average return on the bankers' assets will be higher.

In order to effectively implement the promised return function $\hat{R}_{t+1}(i_t)$, the regulator will restrict entry in the credit market, excluding buyers and sellers from lending. Despite this restriction, the buyer's optimal choice of note holdings continues to be given by (19), and the seller's optimal choices of effort and intermediate goods continue to be given by (22) and (23).

6.2. Banker's Problem

The banker's decision problem can now be formulated as follows:

$$w_t(b_{t-1}, i_{t-1}) = \max_{(x_t, i_t, b_t) \in \mathbb{R}^3_+} [x_t + \beta w_{t+1}(b_t, i_t)]$$

subject to the budget constraint

$$i_t + x_t + b_{t-1} = \hat{R}_t(i_{t-1}) i_{t-1} + \phi_t b_t$$

and the debt limit

$$b_t \leq \bar{B}_t.$$

The return function $\hat{R}_{t+1}(i_t)$ is given by (34) with $\mu_t > 0$. The banker takes the announced return functions as given when making his decisions, as well as the sequence of debt limits $\{\bar{B}_t\}_{t=0}^{\infty}$ and prices $\{\phi_t\}_{t=0}^{\infty}$.

As before, if $\phi_t > \beta$, then we have $b_t = \bar{B}_t$ at the optimum, so the banker finds it optimal to borrow up to his debt limit. If the banker has enough funds at date t, then the optimal choice for i_t is such that $i_t \geq e[1 - G(\gamma^m(\phi_{t+1}))]$ because $\mu_t > 0$. If the investment amount i_t is lower than $e[1 - G(\gamma^m(\phi_{t+1}))]$, then the return to each incremental amount invested at date t is greater than the rate of time preference. In this case, the banker would be better off if he increased his investment at date t. If the investment at date t exceeds

$e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]$, then the return to each extra unit invested at date t equals the rate of time preference. In this case, the banker is indifferent between immediately consuming and investing one extra unit. This means that $i_t = \phi_t \bar{B}_t$ is part of a solution to the banker's problem provided that $\phi_t \bar{B}_t \geq e\left[1-G\left(\gamma^{m}\left(\phi_{t+1}\right)\right)\right]$. We will later show that this will be the case in equilibrium.

6.5. Equilibrium

To construct an equilibrium, we follow the same steps as in the previous section. We need to find a sequence of debt limits that guarantees that the bankers are willing to supply the amount of notes other people demand and are willing to fully repay their creditors. The banker's solvency constraints are now given by

$$-\phi_t a\left(\phi_t\right)+\sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\left[\Pi\left(\phi_{\tau-1}, \phi_\tau, \mu_{\tau-1}\right)-a\left(\phi_{\tau-1}\right)\right] \geq 0 \tag{38}$$

at each date $t \geq 0$, where the date-t revenue $\Pi\left(\phi_{t-1}, \phi_t, \mu_{t-1}\right)$ is given by

$$\Pi\left(\phi_{t-1}, \phi_t, \mu_{t-1}\right) \equiv \mu_{t-1} e\left[1-G\left(\gamma^{m}\left(\phi_t\right)\right)\right]+\beta^{-1} \phi_{t-1} a\left(\phi_{t-1}\right).$$

The solvency constraints (38) are similar to those that we have obtained in the previous section, except that now the banker's date-t revenue has increased by the amount $\mu_{t-1} e\left[1-G\left(\gamma^{m}\left(\phi_t\right)\right)\right]$. The definition of an equilibrium is now straightforward.

Definition 5 *An equilibrium is an array* $\left\{\gamma_t^{m}, n_t, a_t, \bar{B}_t, \phi_t, r_t(\gamma), \mu_t\right\}_{t=0}^{\infty}$ *satisfying (27), (28), (29), (30), (36), (37), and (38) with equality at each date* t*, given the initial stock of intermediate goods.*

6.6. Existence

To show existence, we will restrict attention to stationary equilibria in which the aggregate amount of notes issued at each date is constant over time. In the Appendix, we characterize non-stationary equilibria. First, consider a solution to the banker's decision problem when $\phi_t = \phi$ and $\bar{B}_t = a(\phi)$ at each date $t \geq 0$. In this case, we have $b_t = a(\phi)$ and $i_t = \phi a(\phi)$.

Second, the following result guarantees that, at any given price ϕ, the bankers will be able to raise enough resources from the sale of notes to finance all entrepreneurs whose projects have a positive surplus.

Lemma 6 *For any given $\phi > \beta$, we have $\phi a(\phi) > e[1 - G(\gamma^m(\phi))]$.*

Finally, we need to find the set of stationary prices ϕ for which the solvency constraints hold. Given a stationary markup $\mu > 0$, any price ϕ satisfying

$$-\phi a(\phi) + \frac{\beta}{1-\beta}\left[\hat{\Pi}(\phi,\mu) - a(\phi)\right] \geq 0 \tag{39}$$

implies that the repayment of creditors is individually rational for each banker. Here the value $\hat{\Pi}(\phi,\mu)$ is defined by

$$\hat{\Pi}(\phi,\mu) \equiv \mu e[1 - G(\gamma^m(\phi))] + \phi a(\phi)\beta^{-1},$$

which gives the banker's revenue at each date as a function of the price ϕ and the markup μ.

The markup μ must satisfy the following condition:

$$\beta^{-1} < \beta^{-1} + \mu \leq \beta^{-1}\frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} \gamma g(\gamma)\, d\gamma}{\gamma^m(\phi)[1 - G(\gamma^m(\phi))]} \equiv \hat{R}(\phi). \tag{40}$$

Here $\hat{R}(\phi)$ gives the average return on the banking sector's loan portfolio for the case in which the mechanism is such that, for each type $\gamma \geq \gamma^m(\phi)$, the interest rate $r(\gamma)$ makes the type-γ entrepreneur's participation constraint hold with equality. This interest rate is given by

$$1 + r(\gamma) = \beta^{-1}\frac{\gamma}{\gamma^m(\phi)}. \tag{41}$$

This means that the average return $\beta^{-1} + \mu$ on the banking sector's loan portfolio can range from the competitive return β^{-1} to the monopolist return $\hat{R}(\phi)$, depending on the regulatory mechanism.

Indeed, given a particular choice of the interest rates $\{r(\gamma)\}_{\gamma \geq \gamma^m(\phi)}$, the markup μ will be given by

$$\mu = \frac{\int_{\gamma^m(\phi)}^{\bar{\gamma}} [1 + r(\gamma)]\, g(\gamma)\, d\gamma}{[1 - G(\gamma^m(\phi))]} - \beta^{-1}. \tag{42}$$

Thus, given any price ϕ, a stationary mechanism consists of a stationary markup μ and an interest rate function $\{r(\gamma)\}_{\gamma \geq \gamma^m(\phi)}$ satisfying (42) and

$$\beta^{-1} \leq 1 + r(\gamma) \leq \beta^{-1} \frac{\gamma}{\gamma^m(\phi)} \tag{43}$$

for each $\gamma \geq \gamma^m(\phi)$.

One immediate consequence of the existence of a positive markup is that the average return on the banker's assets is higher than the average return he gets in the case of perfect competition. Specifically, for any given ϕ, the banking sector's revenue exceeds the revenue obtained in the case of perfect competition by the amount $\mu e [1 - G(\gamma^m(\phi))]$. As a consequence, the set of stationary prices satisfying the solvency constraints must be larger than the one we obtain in the case of unregulated lending because a higher return on assets essentially relaxes the solvency constraints. The following proposition establishes existence and uniqueness of a non-autarkic stationary equilibrium in the presence of regulation.

Proposition 7 *Suppose that* $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$*, with* $0 < \sigma < 1$*,* $c(n) = n$*, and* $F(k, n) = k^{\alpha} n^{1-\alpha}$*, with* $0 < \alpha < 1$*. Suppose also that* $g(\gamma) = 1$ *for all* $0 \leq \gamma \leq 1$ *and* $g(\gamma) = 0$ *otherwise. Then, there exists a unique non-autarkic stationary equilibrium for which* $\phi_t = \bar{\phi}$ *for all* $t \geq 0$*, where* $\bar{\phi} < 1$*.*

With a positive markup, it is possible to have an equilibrium in which the return on notes is strictly positive. As should be expected, a positive markup raises the return on the banking sector's assets, mitigating the commitment problem associated with the note-issuing privileges. Thus, there exists an equilibrium in which the price of bank liabilities is lower, and the aggregate supply of these liabilities is larger than those that we have obtained in the absence of regulation.

6.7. Welfare Properties

Now we turn to the welfare implications of having a regulated credit market. In particular, we want to know whether the regulation of credit markets will allow us to implement the optimal return on notes.

Proposition 8 *If β is sufficiently close to one, then an equilibrium with $\phi_t = \beta$ for all $t \geq 0$ exists.*

For any μ sufficiently close to the upper bound, given by the monopolist markup ($\mu = \hat{R}(\phi) - \beta^{-1}$), it is possible to have an equilibrium in which the return on notes equals the rate of time preference. In this case, we eliminate the opportunity cost of holding money, maximizing the surplus from trade in the decentralized market. Because any other allocation that makes at least one entrepreneur better off necessarily makes a banker worse off, we conclude that setting $\phi_t = \beta$ for all $t \geq 0$ is both necessary and sufficient for efficiency.

The regulatory mechanism has a crucial impact on the welfare properties of an equilibrium allocation. In the absence of intervention, bankers compete on the asset side of their balance sheets and can only get a positive franchise value if they offer a low return on their liabilities. We have shown that the role of the regulator is to increase the return on the banking sector's assets by regulating the credit market. This is important because such an intervention will allow bankers to increase the return on their liabilities, thus favouring the provision of liquidity. As we have shown, bankers are willing to supply the optimum quantity of money only if the average return on their assets is sufficiently close to the return that a monopolist banker would obtain.

It is important to notice that a monopolist banker would not choose an efficient allocation because he would certainly not choose the price of his liabilities to be $\phi_t = \beta$ at each date. We have to keep in mind that we have assumed a perfectly competitive market for the bankers' notes, which is crucial for the efficiency of the system. The fact that a monopolist would obtain a high return on his assets does not mean that he would be willing to offer the socially efficient return on his liabilities. To obtain efficiency, a monopolist banker would have to be regulated as well.

An important corollary that follows immediately is that narrow banking (a system in which banks hold 100% in reserves) cannot provide the efficient amount of liquidity. Indeed, narrow banking does not offer any means to increase the return on the banking sector's assets.

7. CONCLUSION

We have shown that an unregulated banking system is unable to supply an efficient amount of private money. In the absence of intervention, the return on the bankers' assets will be relatively low because of competition in credit markets. This makes the option of defaulting on their liabilities relatively more attractive. Thus, the bankers will be willing to offer to pay only a low return on their liabilities, creating a cost for their liability holders (that they are willing to bear because these liabilities provide them with a transaction service). For this reason, any equilibrium allocation in the absence of intervention is necessarily inefficient.

In view of this inefficiency, we have considered the possibility of regulating the banking system. In particular, we have characterized an optimal intervention. The way to induce bankers to supply an efficient amount of private money is to sufficiently raise the return on their assets. The regulator's goal is to ensure that the bankers get some of the surplus from the borrowers. This can be achieved by restricting entry in the credit market and by imposing direct interest rate controls.

So far, we have left aside the role of banks as risk transformers, whereby banks undertake risky investments but issue relatively safe debt, or alternatively whereby banks' assets are information sensitive while they issue information-insensitive liabilities (an idea that dates back to Gorton and Pennacchi, 1990, but has regained some traction recently; see Gorton, 2010). This is clearly an important issue that will impact the optimal provision of liquidity, and we leave it for future work.

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APPENDIX

A.1. Existence of a Unique Stationary Solution to the Planner's Problem

Here we show the existence of a unique stationary solution to the planner's problem for some specifications of preferences and technologies. In particular, we assume that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma}-1\right)$, with $0<\sigma<1$, $c(n)=n$, and $F(k,n)=k^{\alpha}n^{1-\alpha}$, with $0<\alpha<1$. We also assume that $g(\gamma)=1$ for any $0\leq\gamma\leq 1$ and $g(\gamma)=0$ otherwise. In this case, conditions (10) and (11) become

$$n=\chi\left[\gamma^{-1}\left(1-\gamma^{2}\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}}\equiv Z(\gamma), \tag{44}$$

$$n=\lambda\left(1-\gamma^{2}\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\equiv H(\gamma), \tag{45}$$

respectively, where the constants χ and λ are defined as

$$\chi\equiv\left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}}\left[\frac{e}{\alpha\beta\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}}, \tag{46}$$

$$\lambda\equiv\left[(1-\alpha)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}. \tag{47}$$

Notice that $Z'(\gamma)<0$ for all $\gamma\in(0,1)$. Also, we have that $\lim_{\gamma\to 0}Z(\gamma)=+\infty$ and $\lim_{\gamma\to 1}Z(\gamma)=0$. This means that the function $Z(\gamma)$ is strictly decreasing in the open interval $(0,1)$. With respect to the function $H(\gamma)$, we have that $H'(\gamma)<0$ and $H''(\gamma)<0$ for all $\gamma\in(0,1)$. Also, we have that $\lim_{\gamma\to 0}H(\gamma)=\lambda$ and $\lim_{\gamma\to 1}H(\gamma)=0$. This means that the function $H(\gamma)$ is strictly decreasing and concave in the open interval $(0,1)$. This means that a unique interior solution exists.

A.2. Proof of Proposition 3

Suppose that $u(q)=(1-\sigma)^{-1}\left(q^{1-\sigma}-1\right)$, with $0<\sigma<1$, $c(n)=n$, and $F(k,n)=k^{\alpha}n^{1-\alpha}$, with $0<\alpha<1$. Suppose also that $g(\gamma)=1$ for any $0\leq\gamma\leq 1$ and $g(\gamma)=0$

otherwise. In this case, conditions (27) and (28) become

$$n = \chi^e(\phi)\left[\gamma^{-1}\left(1-\gamma^2\right)^{1-\alpha+\alpha\sigma}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \equiv Z^e(\gamma,\phi), \tag{48}$$

$$n = \lambda^e(\phi)\left(1-\gamma^2\right)^{\frac{\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}} \equiv H^e(\gamma,\phi), \tag{49}$$

respectively, where the functions $\chi^e(\phi)$ and $\lambda^e(\phi)$ are given by

$$\chi^e(\phi) = \left(\frac{1}{2}\right)^{\frac{1-\alpha+\alpha\sigma}{(1-\alpha)(1-\sigma)}}\left[\frac{e\phi}{\alpha\beta^2\hat{k}^{\alpha(1-\sigma)}}\right]^{\frac{1}{(1-\alpha)(1-\sigma)}},$$

$$\lambda^e(\phi) = \left[(1-\alpha)\frac{\beta}{\phi}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\right]^{\frac{1}{\alpha+\sigma(1-\alpha)}}.$$

Notice that $d\chi^e/d\phi > 0$, whereas $d\lambda^e/d\phi < 0$. Also, we have that $\chi^e(\beta) = \chi$ and $\lambda^e(\beta) = \lambda$, where χ and λ are given by (46) and (47), respectively. For any fixed $\phi > \beta$, we have that $\partial Z^e/\partial\gamma < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} Z^e(\gamma,\phi) = +\infty$, and $\lim_{\gamma\to 1} Z^e(\gamma,\phi) = 0$. For any fixed $\phi > \beta$, we also have that $\partial H^e/\partial\gamma < 0$ and $\partial^2 H^e/\partial\gamma^2 < 0$ for all $\gamma \in (0,1)$, $\lim_{\gamma\to 0} H^e(\gamma,\phi) = \lambda^e(\phi)$, and $\lim_{\gamma\to 1} H^e(\gamma,\phi) = 0$. Thus, for any fixed $\phi > \beta$, a unique interior solution exists. Moreover, the Implicit Function Theorem implies that $d\gamma^m/d\phi > 0$ and $dn/d\phi < 0$.

As we have seen, condition (32) holds if and only if $\phi \geq 1$. In particular, it holds with equality if and only if $\phi = 1$. Thus, there exists a unique non-autarkic stationary equilibrium for which $\gamma^m = \gamma^m(1)$, $n = n(1)$, $a = a(1)$, where $a(1)$ is given by

$$a(1) = \left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1-\gamma^m(1)^2\right]^{\alpha(1-\sigma)} n(1)^{(1-\alpha)(1-\sigma)}. \tag{50}$$

Q.E.D.

A.3. Proof of Lemma 6

Note that we can rewrite the expression for the aggregate note holdings as follows:

$$\phi a(\phi) = \frac{e\phi F\left[k(\gamma^m(\phi)), n(\phi)\right]}{\beta^2\hat{k}\gamma^m(\phi) F_k\left[k(\gamma^m(\phi)), n(\phi)\right]}.$$

For any price $\phi > \beta$, we have

$$\begin{aligned}
\frac{e\phi F\left[k\left(\gamma^m\left(\phi\right)\right),n\left(\phi\right)\right]}{\beta^2\hat{k}\gamma^m\left(\phi\right)F_k\left[k\left(\gamma^m\left(\phi\right)\right),n\left(\phi\right)\right]} &> \frac{e\phi k\left(\gamma^m\left(\phi\right)\right)}{\beta^2\hat{k}\gamma^m\left(\phi\right)} \\
&> \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\beta\hat{k}\gamma^m\left(\phi\right)} \\
&> \frac{ek\left(\gamma^m\left(\phi\right)\right)}{\hat{k}\gamma^m\left(\phi\right)} \\
&= \frac{e\int_{\gamma^m(\phi)}^{\bar{\gamma}}\gamma g\left(\gamma\right)d\gamma}{\gamma^m\left(\phi\right)} \\
&> e\left[1-G\left(\gamma^m\left(\phi\right)\right)\right].
\end{aligned}$$

Q.E.D.

A.4. Proof of Proposition 7

Suppose that $u\left(q\right) = \left(1-\sigma\right)^{-1}\left(q^{1-\sigma}-1\right)$, with $0 < \sigma < 1$, $c\left(n\right) = n$, and $F\left(k,n\right) = k^{\alpha}n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g\left(\gamma\right) = 1$ for any $0 \leq \gamma \leq 1$ and $g\left(\gamma\right) = 0$ otherwise. Note that we can rewrite (39) as follows:

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta}a\left(\phi\right)\left(\phi\beta^{-1}-1\right) + \frac{\beta}{1-\beta}\mu\left[1-\gamma^m\left(\phi\right)\right] \geq 0, \tag{51}$$

where

$$a\left(\phi\right) = \phi^{-1}\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1-\gamma^m\left(\phi\right)^2\right]^{\alpha(1-\sigma)}n\left(\phi\right)^{(1-\alpha)(1-\sigma)}.$$

We have already shown that

$$-\phi a\left(\phi\right) + \frac{\beta}{1-\beta}a\left(\phi\right)\left(\phi\beta^{-1}-1\right) \geq 0$$

if and only if $\phi \geq 1$. This means that there exists $\bar{\phi} < 1$ such that, for $\phi = \bar{\phi}$, there exist $\{r\left(\gamma\right)\}_{\gamma\geq\gamma^m(\bar{\phi})}$ and $\mu > 0$ satisfying (42), (43), and (51). **Q.E.D.**

A.5. Proof of Proposition 8

Suppose now that $r\left(\gamma\right)$ is given by (41) for any given ϕ. Then, (39) can be written as

$$\frac{e}{\beta}\left[\frac{1}{2\gamma^m\left(\phi\right)} + \frac{\gamma^m\left(\phi\right)}{2} - 1\right] - \left(\frac{1-\phi}{\phi}\right)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1-\gamma^m\left(\phi\right)^2\right]^{\alpha(1-\sigma)}n\left(\phi\right)^{(1-\alpha)(1-\sigma)} \geq 0.$$

Taking the limit as $\phi \to \beta$ from above, the left-hand side of this expression converges to

$$e\left(\frac{1}{2\gamma_\beta^*}+\frac{\gamma_\beta^*}{2}-1\right)-(1-\beta)\left(\frac{\hat{k}}{2}\right)^{\alpha(1-\sigma)}\left[1-\left(\gamma_\beta^*\right)^2\right]^{\alpha(1-\sigma)}\left(n_\beta^*\right)^{(1-\alpha)(1-\sigma)},$$

where $\left(\gamma_\beta^*, n_\beta^*\right)$ denotes the solution to the planner's problem [i.e., the unique interior solution to the system (44)-(45)] for any given discount factor $\beta < 1$. As $\beta \to 1$ from below, we have that $0 < \lim_{\beta \to 1} \gamma_\beta^* < 1$. This means that there exists $\beta < 1$ sufficiently close to one such that the expression above is strictly positive. Therefore, we have constructed an equilibrium in which $\phi_t = \beta$,

$$1 + r_t(\gamma) = \beta^{-1}\frac{\gamma}{\gamma_\beta^*},$$

for each $\gamma \geq \gamma_\beta^*$, and

$$\mu_t = \frac{\beta^{-1}}{2}\left(\frac{1}{\gamma_\beta^*}-1\right).$$

for all $t \geq 0$. **Q.E.D.**

A.6. Non-Stationary Equilibria

In this subsection, we consider the existence of non-stationary equilibria. Consider first the case of unregulated lending. Let $\hat{w}_t$ denote the banker's discounted lifetime utility at the beginning of date t. Then, the equations defining the equilibrium dynamics of $\hat{w}_t$ and ϕ_t are given by

$$\hat{w}_t = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1}-1\right)+\beta\hat{w}_{t+1} \tag{52}$$

and

$$\phi_t a\left(\phi_t\right) = \beta\hat{w}_{t+1}, \tag{53}$$

where $\hat{w}_t = w_t\left(a\left(\phi_{t-1}\right), \phi_{t-1}a\left(\phi_{t-1}\right)\right)$. Combining these two conditions, we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$\phi_t a\left(\phi_t\right) = a\left(\phi_{t-1}\right), \tag{54}$$

given an initial condition $\phi_0 > 0$. The initial price of notes must be such that it guarantees market clearing at date $t = 0$, given the predetermined stock of intermediate goods available for the production of good 2 at date $t = 0$.

Note that there exists at least one stationary solution: $\phi_{t-1} = \phi_t = 1$. Suppose now that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k,n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Using the Implicit Function Theorem, we find that

$$\frac{d\phi_t}{d\phi_{t-1}} = \frac{a'\left(\phi_{t-1}\right)}{\phi_t a'(\phi_t) + a(\phi_t)} > 0.$$

In particular, we have

$$\left.\frac{d\phi_t}{d\phi_{t-1}}\right|_{\phi_{t-1}=\phi_t=1} = \frac{a'(1)}{a'(1) + a(1)} > 1.$$

If $\phi_{t-1} = \phi_t = 1$ is the unique non-autarkic stationary solution, then we have that, for any initial value $\phi_0 > 1$, the equilibrium price trajectory is strictly increasing and unbounded, so the equilibrium allocation approaches the autarkic allocation as $t \to \infty$. Along this equilibrium path, the debt limits, given by $\bar{B}_t = a(\phi_t)$, shrink over time and converge to zero, similar to the analysis in Gu and Wright (2011). This means that liquidity becomes scarcer and more expensive over time, and consumers are able to trade smaller amounts of goods in the decentralized market.

Consider now the case of regulated lending. Suppose that $r_t(\gamma)$ is given by (41). In this case, the equations defining the equilibrium dynamics of $\hat{w}_t$ and ϕ_t are given by (53) and

$$\hat{w}_t = a\left(\phi_{t-1}\right)\left(\beta^{-1}\phi_{t-1} - 1\right) + e\beta^{-1}\frac{\left[1 - \gamma^m(\phi_t)\right]^2}{2\gamma^m(\phi_t)} + \beta\hat{w}_{t+1}. \tag{55}$$

Combining (53) with (55), we can define an equilibrium as a sequence of prices $\{\phi_t\}_{t=0}^{\infty}$ satisfying

$$a\left(\phi_{t-1}\right) = e\beta^{-1}\frac{\left[1 - \gamma^m(\phi_t)\right]^2}{2\gamma^m(\phi_t)} + \phi_t a(\phi_t), \tag{56}$$

given an initial condition $\phi_0 > 0$. Suppose that $u(q) = (1-\sigma)^{-1}\left(q^{1-\sigma} - 1\right)$, with $0 < \sigma < 1$, $c(n) = n$, and $F(k,n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. Suppose also that $g(\gamma) = 1$ for any $0 \leq \gamma \leq 1$ and $g(\gamma) = 0$ otherwise. Notice that, for β sufficiently close to one, $\phi_{t-1} = \phi_t = \beta$ is a stationary solution. Again, if this is the unique non-autarkic stationary solution, for any initial condition $\phi_0 > \beta$, the debt limits shrink over time and the price of liquid assets grows unbounded as the economy approaches autarky.

Figure 1 – Circulation of Bank Notes