Loss Given Default as a Function of the Default Rate

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What should be done with data?

The relationship between the default rate and LGD rate:

- **Affects the relative value of tranches of securitizations**
- **Affects the risk/return tradeoff for lenders**
- **Is part of Pillar 1, Pillar 2, CCAR…**

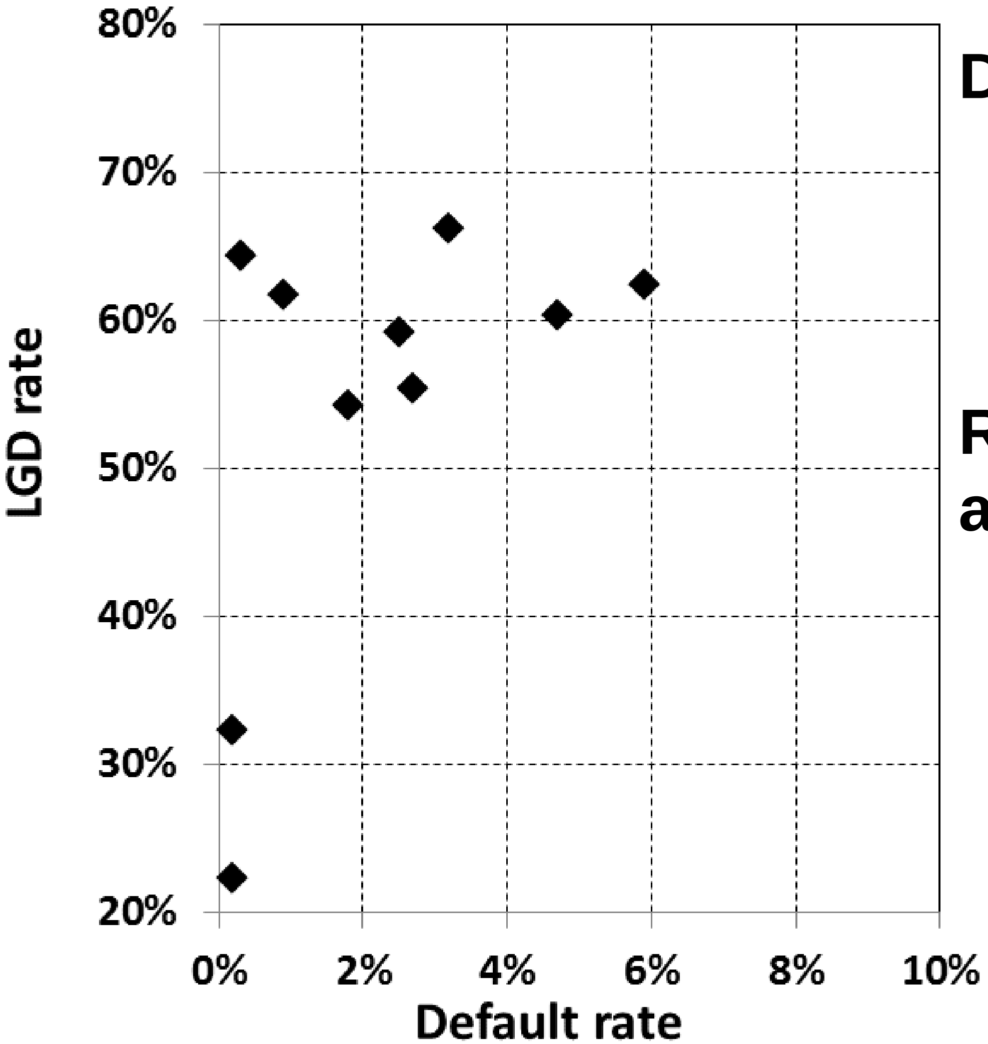
Today's presentation compares two ways to model it:

- **Linear regression (has an intercept and a slope)**
- **A newly-developed LGD function (has only one parameter)**

  The function was derived in Frye and Jacobs, J Credit Risk, Spring 2012

When simulations reflect realistic conditions, the LGD function produces lower errors on average.

10 Years of Simulated Data

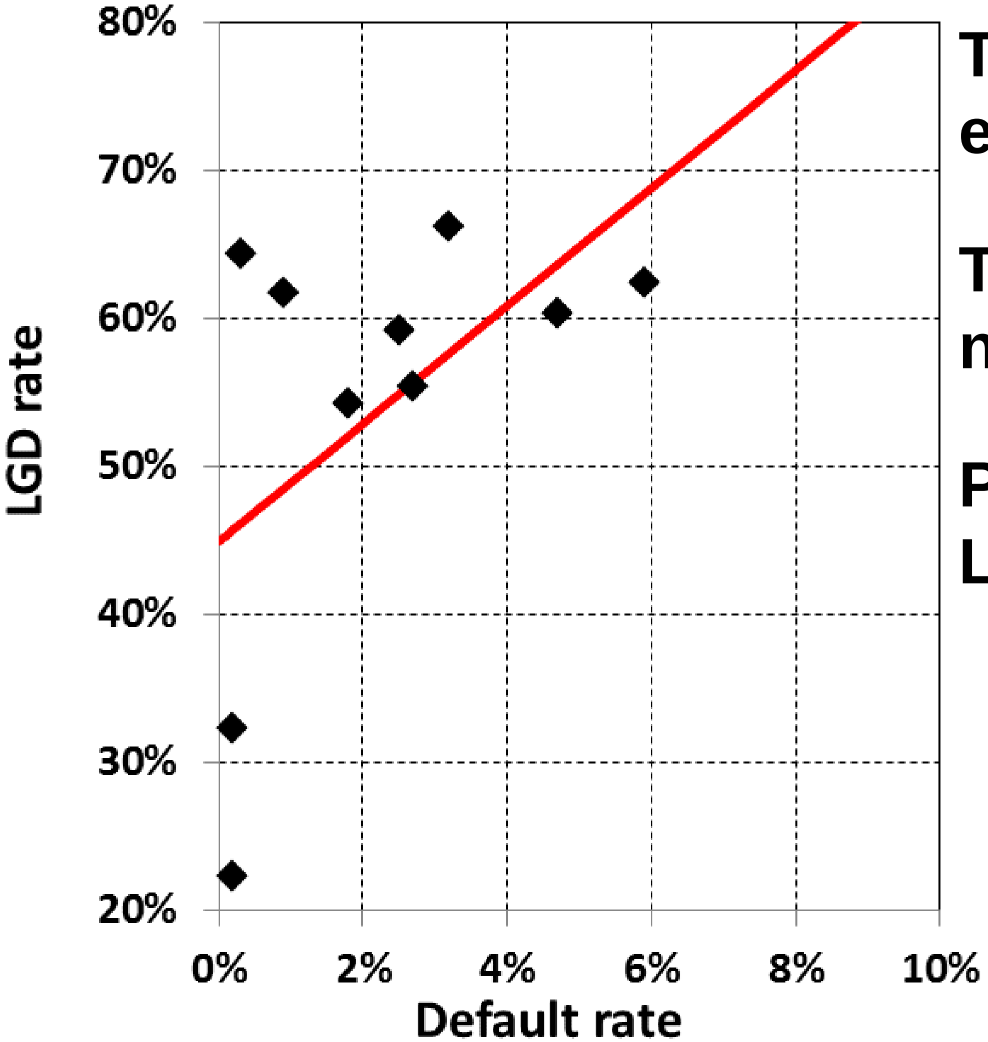


Data are intrinsically noisy.

- **This portfolio has 1,000 firms.**
- **At the left of the chart, few defaults ⇒ noisy portfolio LGD.**

Reliable statistical analysis awaits more _years_ of data.

$\text{LGD} = \widehat{0.45} + \widehat{4.0}\ \text{DR}$



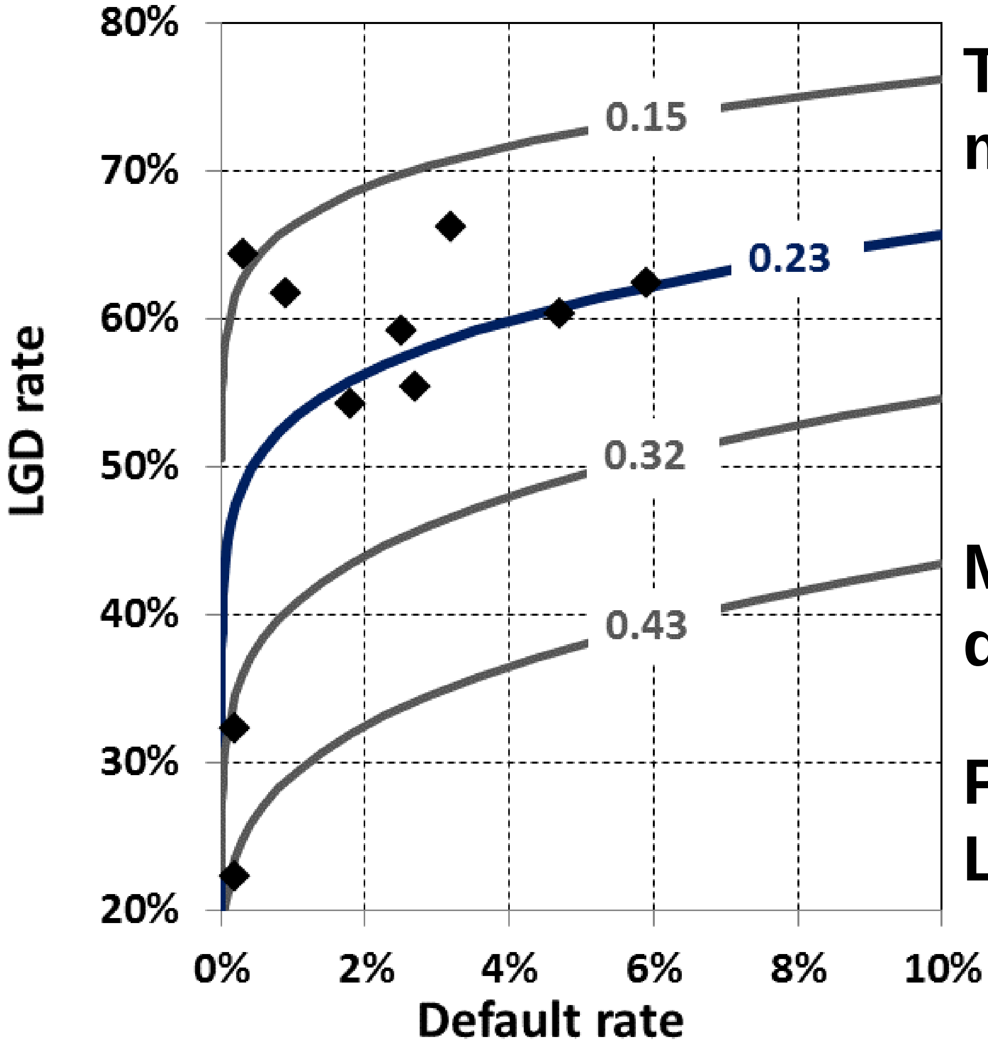
The regression line points to extreme LGD risk.

The slope is steep, but it is not statistically significant.

Prediction of 98th percentile LGD reverts to an average.

- **Default-rate-weighted average LGD equals 60%.**

$LGD = \Phi \left[\Phi^{-1}[DR] - \widehat{0.23} \right] / DR$



The parameter value depends mostly on averages.

- **The "0.23" line is above 5 points and beneath the other 5.**
- **Data dispersion matters much less than average location.**

Moderate, positive "slope" does not depend on the data.

Prediction of 98th percentile LGD equals 66%.

Verdict: The LGD function wins

True 98th percentile LGD = 72%

Regression error = 72% - 60% = 12%

LGD function error = 72% - 66% = 6%

If a risk manager cares about error, she should use the LGD function.

She should withhold credence from statistical analysis.

Do this 10,000 times

Simulate the portfolio default rate, D / n

- Draw the conditionally expected default rate, cDR, from the Vasicek Distribution [PD = 3%, ρ = 10%].
- Draw the number of defaults, D, from the Binomial[n=1000, p=cDR].

Simulate the portfolio LGD rate

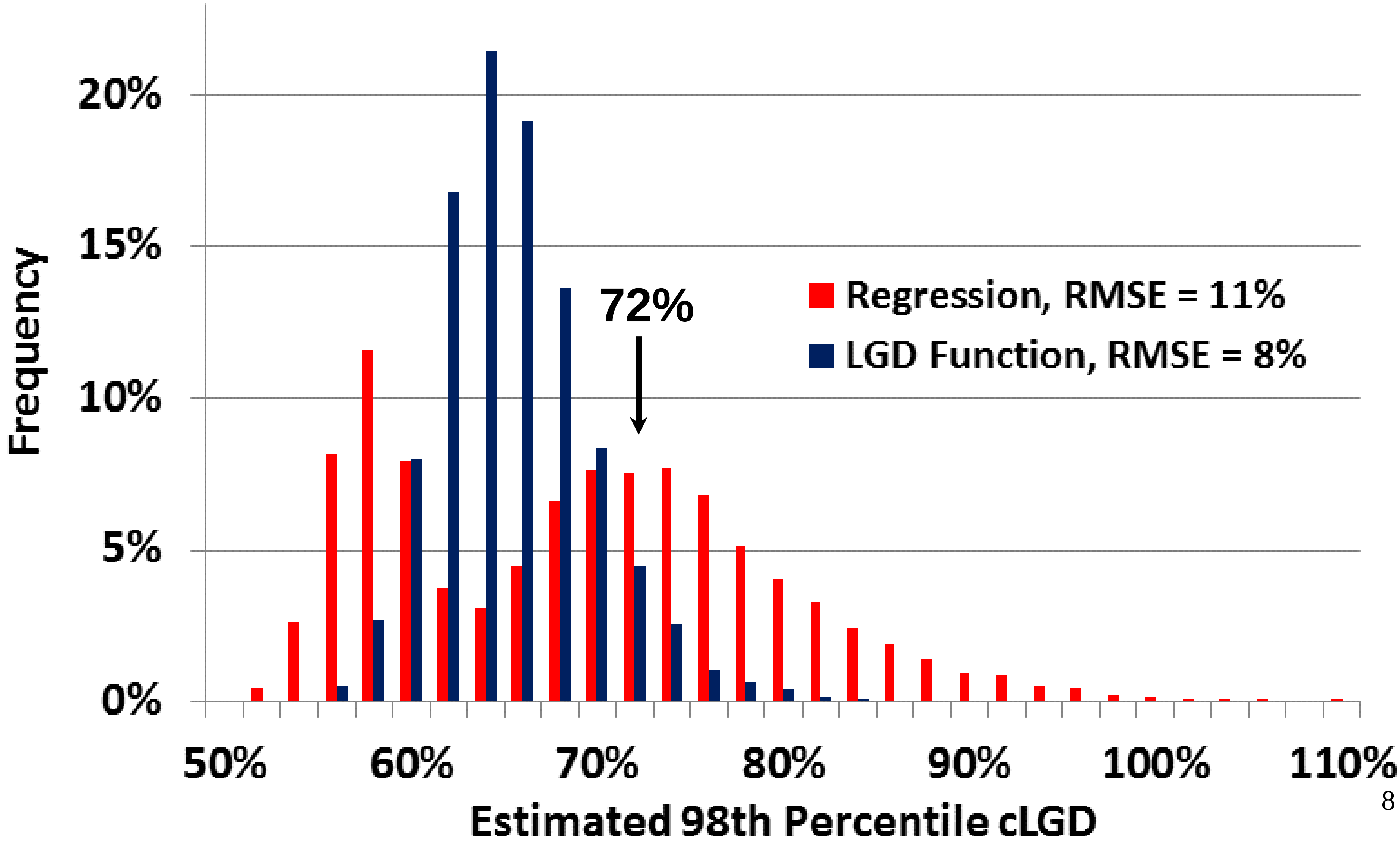
- Infer the conditionally expected LGD rate from cLGD = .5 + 2.3 cDR

  Philosophically, linear regression would be the right model to analyze data that is produced by this linear model. Still, the LGD function produces lower RMSE.
- Draw the portfolio LGD rate from N [cLGD, σ^2 / D]; σ = 20%.

With 10 years of simulated data, predict 98th percentile cLGD.

The LGD function outperforms regression, which can point anywhere.

The LGD function has lower MSE

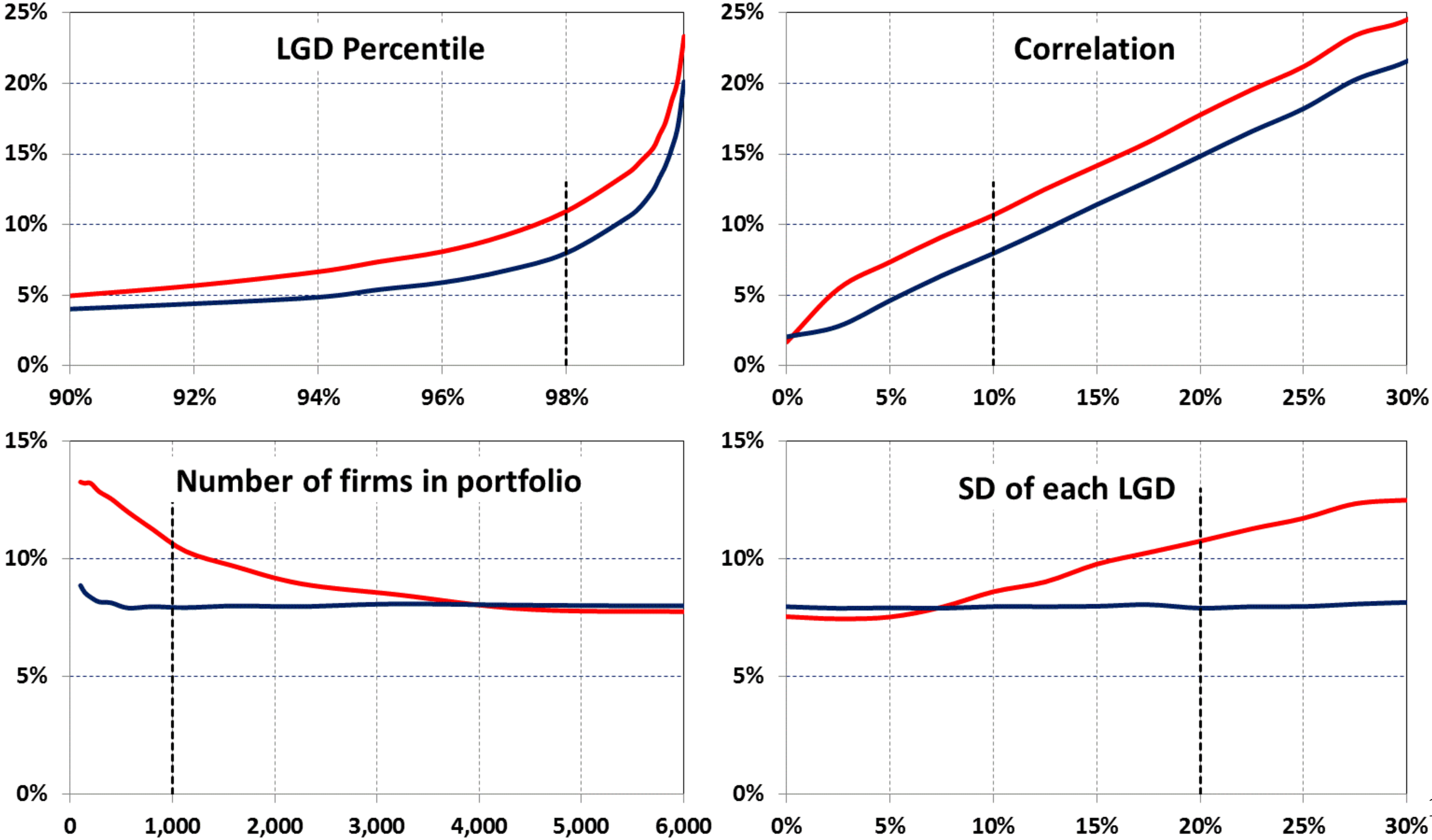


Robustness Checks

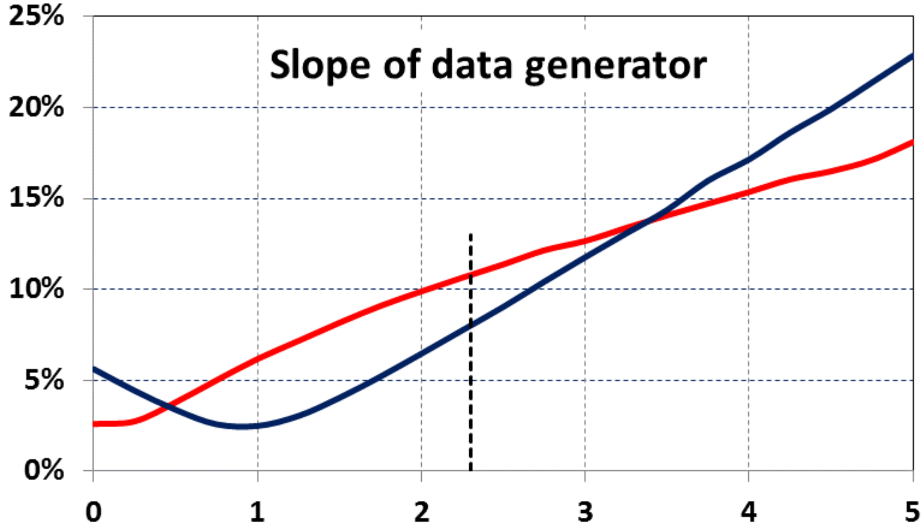
Allow each control variable to take a range of values:

- **Four variables have little effect on the verdict.**
- **Two variables have decisive importance.**
- **The results are robust with respect to different values of PD and different ranges of LGDs produced. (See paper.)**

Four variables have little effect

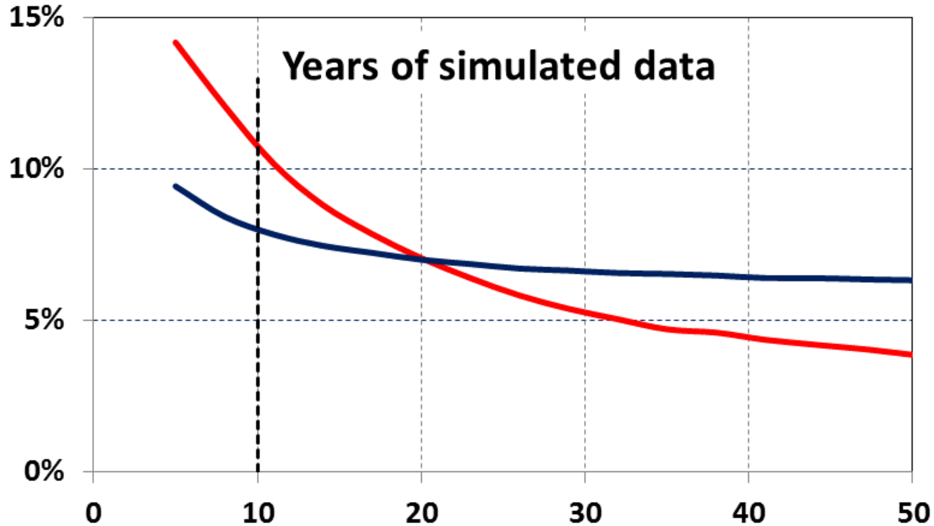


Two variables are decisive



The function outperforms for a range of true sensitivities.

Regression is better only if true sensitivity is zero or is much greater than people think.



Eventually, regression wins.

But the real cross-over point is later than 20 years because real data are serially dependent.

The cross-over happens much slower if the true slope ≈ 1.0.

Summary

The LGD function outperforms statistical analysis

under the realistic conditions that

- **the data set is short and**
- **systematic LGD risk is neither zero nor extreme.**

This holds even if the statistical analyst uses the true model.

The gold standard remains the rigorous statistical hypothesis test as performed in JCR Spring 2012.