Technical Appendixes to *Chicago Fed Letter* No. 521: Reassessing the Relationship Between Consumer Sentiment and Spending with a New Composite Index

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Appendix 1: The MF-VAR with measurement error

The model that we use to estimate the Composite Consumer Sentiment Index (C-CSI) is a mixed-frequency vector autoregression (MF-VAR) that includes the preliminary and final readings of the monthly University of Michigan Index of Consumer Sentiment and Conference Board Consumer Confidence Index, along with Morning Consult's Index of Consumer Sentiment, Rasmussen Reports' Consumer Index, Goldman Sachs' Social Media Economic Sentiment Index, and the San Francisco Fed's Daily News Sentiment Index (the final four of which are daily indexes).

The mixed frequency nature of our model is similar in spirit to Algaba et al. (2023) and their use of daily and monthly Belgian sentiment data with a mixed-frequency dynamic factor model (MF-DFM). The use of a MF-VAR instead in our case was found to provide for a more flexible characterization of the complex dynamics between the inputs of our model. In particular, the estimation of the bias term (or measurement error) in the Michigan index follows as a straightforward extension of the MF-VAR.[1]

The MF-VAR includes latent state variables representing the daily values of the Michigan (UM_t^d) and Conference Board (CB_t^d) indexes. While it is possible to observe these daily values from survey microdata, the sample sizes are typically quite small. So, instead, we make use of the twice-monthly published sample averages (preliminary and final), the published survey timelines, and related daily measures of consumer sentiment and consumer confidence to statistically infer them.

The MF-VAR is then augmented to include an additional daily latent state variable (ε_t) capturing a persistent measurement error in the Michigan index. We view this measurement error as a reduced-form way of estimating potential bias in the University of Michigan's *Surveys of Consumers* due to either its previous small sample sizes or changing methods of survey collection (i.e., phone to online) over time. Its dynamics follow a random-walk process, such that it acts very much like a time-varying intercept (or constant) for the Michigan index in our MF-VAR model.

In daily state-space form, the model used to estimate the C-CSI is given by

[1] Our MF-VAR also builds on previous work described in Brave (2022a) where a similar model was shown to produce highly accurate nowcasts of the Michigan index.

$$\begin{bmatrix} UM_t^d \\ MC_t \\ GS_t \\ SF_t \\ RM_t \\ CB_t^d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} CCSI_t \\ MC_t \\ GS_t \\ SF_t \\ RM_t \\ CB_t^d \\ \varepsilon_t \end{bmatrix},$$

$$\begin{bmatrix} CCSI_t \\ MC_t \\ GS_t \\ SF_t \\ RM_t \\ CB_t^d \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} a_c \\ a_m \\ a_g \\ a_s \\ a_r \\ a_b \\ 0 \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} & \beta_{16} & 0 \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} & \beta_{26} & 0 \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} & \beta_{35} & \beta_{36} & 0 \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & \beta_{45} & \beta_{46} & 0 \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} & \beta_{55} & \beta_{56} & 0 \\ \beta_{61} & \beta_{62} & \beta_{63} & \beta_{64} & \beta_{65} & \beta_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} CCSI_{t-1} \\ MC_{t-1} \\ GS_{t-1} \\ SF_{t-1} \\ RM_{t-1} \\ CB_{t-1}^d \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \upsilon_t \\ \zeta_t \\ \xi_t \\ \iota_t \\ \tau_t \\ \nu_t \end{bmatrix},$$

$$\begin{bmatrix} \eta_t \\ \upsilon_t \\ \zeta_t \\ \xi_t \\ \iota_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N(0, \Sigma),$$

where for ease of exposition we write the transition equation for the companion form of the VAR(p). The covariance matrix Σ is assumed to be block diagonal, imposing that the measurement error shock υ_t is uncorrelated with the remaining VAR shocks.

The temporal aggregation constraints applied to the MF-VAR then require that the monthly observations of the University of Michigan (UM_t^m) and Conference Board (CB_t^m) indexes are given by

$$UM_t^m = \frac{1}{h}\sum_{j=0}^{h} U\,M_{t-h}^d = \frac{1}{h}\sum_{j=0}^{h} (CCSI_{t-h} + \varepsilon_{t-h}),$$

$$CB_t^m = \frac{1}{k}\sum_{j=0}^{k} C\,B_{t-k}^d,$$

for known lengths h and k in each month of our sample period corresponding with the preliminary or final release survey windows of both indexes.

We estimate this augmented MF-VAR with $p = 2$ daily lags on a sample that begins on January 2, 2018, and runs through late May 2026 using maximum likelihood methods. Our C-CSI estimate corresponds to the Kalman smoothed value of the daily latent state connected with the Michigan index in the VAR portion of the model ($CCSI_t$), while the estimated measurement error for the Michigan index is similarly constructed from the daily latent state attached to it that lies outside the VAR (ε_t). When combined with the measurement error and averaged over the known *Surveys of Consumers* survey windows, the C-CSI exactly replicates both the preliminary and final Michigan index readings.

All aspects of estimation and post-estimation were performed in MATLAB using the MFSS (mixed-frequency state-space) toolbox described in Brave et al. (2022b). The confidence intervals shown in the figures of the main text were constructed by simulating the state-space system using a simulation smoother centered around the Kalman smoothed values of the daily latent states that accounts for parameter, state, and shock uncertainty associated with their estimation. They were constructed under the typical normality assumptions for the linear state-space representation of the Kalman filter and smoother equations. To decompose the measurement error in the Michigan index into the contributions from each observable data series in the MF-VAR (as seen in figure 4, panel C of the main text), we rely upon a variable-by-variable decomposition of the Kalman gain. For further details, see Durbin and Koopman (2012).

Appendix 2: Real PCE growth forecasts

In the main text, we perform a real-time, out-of-sample forecasting exercise for six-months-ahead annual real consumer spending growth using vintage (or unrevised) data. The underlying data for this exercise were taken from the St. Louis Fed's ALFRED, the Philadelphia Fed's Real-Time Dataset for Macroeconomists, and Haver Analytics AS1REPNA database.

To produce real-time forecasts, we run the following h-step ahead regression of 12-month real personal consumption expenditures (PCE) (log) growth h steps ahead ($\Delta_{12}c_{t+h}$) on information available in real time at time t, where t corresponds to a vintage data release month for PCE (e.g., March data released in late April):

$$\Delta_{12}c_{t+h} = \alpha + \delta Covid_t + \rho\Delta_{12}c_{t-1} + \beta\Delta_{12}y_{t-1} + \gamma\Delta_{12}s_t + \lambda i_t + \theta ics_t + \varepsilon_{t+h}.$$

Included in the regression in our no-sentiment benchmark specification are one lag each of 12-month (log) growth in real PCE ($\Delta_{12}c_{t-1}$) and real disposable personal income ($\Delta_{12}y_{t-1}$); 12-month (log) Standard & Poor's (S&P) 500 returns ($\Delta_{12}s_t$); and the ten-year Treasury bond yield (i_t).

The Covid-19 pandemic presents a challenge to a real-time forecasting exercise such as this. We address it by also including a dummy variable ($Covid_t$) in the benchmark specification that first enters in March 2020 and remains active through March 2022, covering roughly two-thirds of the declared period of national emergency.

To then be able to measure consumer sentiment's marginal benefit for forecasting consumer spending growth, we run this regression both with and without including a (log) consumer sentiment index in the regression (ics_t). The timing of this regression mirrors the information set of a real-time forecaster at the end of each month t, with both real consumption and income variables lagged by one month reflecting their reporting delays, but with consumer sentiment and financial variables dated as of time t.

We estimate this regression focusing on a six-months-ahead forecast horizon (h=5, or six months from the most recent PCE release) using Bayesian methods employing regularized horseshoe priors[2] on the regression coefficients to help prevent overfitting to past data. We then roll the estimation sample forward one month at a time beginning with a forecast for April 2023 and ending with a forecast for April 2026. When evaluating these 12-month real PCE growth forecasts, we use the currently available real PCE data and drop forecast months where data releases were affected by government shutdowns or other extended reporting delays.

[2] For more information on horseshoe priors, see Carvalho et al. (2010) and Piiironen and Vehtari (2017).