Technical Appendix to *Chicago Fed Letter* No. 528: The AI Revolution and Monetary Policy

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This appendix describes the model we use for our scenarios and presents results of the robustness checks described in the main article. The intended audience is advanced undergraduates as well as more experienced economists.

1 Overview

We consider a two-agent New Keynesian (TANK) economy without physical capital. The economy features Ricardian households, hand-to-mouth households, habit formation, differentiated labor with Erceg-Henderson-Levin (EHL) wage rigidity (Erceg et al., 2000), Rotemberg (1982) price adjustment costs, exogenous productivity growth, a flexible-price/flexible-wage natural allocation, and a Taylor (1993) rule. Section 5 reports the linearized equations.

Time is discrete. Aggregate productivity is denoted by A_t, and its log follows

$$a_t = a_{t-1} + g_t, \qquad g_t = g_t^{exo}. \tag{1}$$

The economy is populated by

(i) Ricardian households,

(ii) hand-to-mouth households,

(iii) a continuum of intermediate goods firms,

(iv) a competitive final-good producer,

(v) a competitive labor aggregator,

(vi) wage setters subject to EHL nominal rigidity, and

(vii) a monetary authority.

A fraction $1 - \omega$ of households is Ricardian and can trade nominal bonds; a fraction ω is hand-to-mouth and consumes current labor income. Unless stated otherwise, lowercase variables in the linear model denote log deviations from the nonstochastic steady state. Inflation and nominal interest rates are expressed as deviations from steady state.

In what follows we describe the general model, but in our baseline parameterization we assume that all households are Ricardian, with no habit formation in consumption.

2 Economy

2.1 Households

Households of type $j \in \{R, H\}$ have preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{j,t} - hC_{j,t-1})^{1-\sigma} - 1}{1-\sigma} - \chi_j \frac{N_{j,t}^{1+\varphi}}{1+\varphi} \right], \tag{2}$$

where $h \in [0, 1)$ is the habit parameter. The habit stock is taken as given by each household and equals lagged type-specific aggregate consumption in symmetric equilibrium. Define the marginal utility of current consumption by

$$U_{C,j,t} = (C_{j,t} - hC_{j,t-1})^{-\sigma}. \tag{3}$$

Because habit is type-specific in equilibrium, marginal utility of wealth that enters intertemporal and intratemporal decisions is

$$\Lambda_{j,t} = U_{C,j,t} - \beta h \mathbb{E}_t U_{C,j,t+1}. \tag{4}$$

The Dynare code normalizes the log-linear version of $\Lambda_{j,t}$ by its steady-state level, which produces the denominator $1 - \beta h$ in the linear equations.

Ricardian households trade one-period nominal bonds. Their nominal budget constraint is

$$P_t C_{R,t} + B_{R,t} = W_t N_{R,t} + R_{t-1}^n B_{R,t-1} + \mathcal{D}_{R,t} - T_{R,t}, \tag{5}$$

where R_t^n is the gross nominal interest rate, W_t is the aggregate nominal wage, and $\mathcal{D}_{R,t}$ denotes profits and other transfers. For simplicity's sake, we assume bonds are in zero net supply in equilibrium, i.e., $B_{R,t} = 0$. The Euler equation is

$$\Lambda_{R,t} = \beta \mathbb{E}_t \left[\Lambda_{R,t+1} \frac{R_t^n}{\Pi_{t+1}} \right], \qquad \Pi_{t+1} = \frac{P_{t+1}}{P_t}. \tag{6}$$

Hand-to-mouth households do not save. Their budget constraint is

$$P_t C_{H,t} = W_t N_{H,t}. \tag{7}$$

This abstracts from transfers and profit rebates to hand-to-mouth households. Their labor supply condition is static, but their marginal utility still reflects the habit-adjusted marginal utility of consumption.

2.2 Differentiated labor and wage rigidity

Labor services are differentiated across households. A competitive labor packer combines individual labor varieties according to

$$N_t = \left[\int_0^1 N_t(i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \, di \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \tag{8}$$

which implies demand

$$N_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t \tag{9}$$

and the wage index

$$W_t = \left[\int_0^1 W_t(i)^{1-\varepsilon_w} \, di \right]^{\frac{1}{1-\varepsilon_w}}. \tag{10}$$

Following Erceg-Henderson-Levin, only a fraction $1 - \theta_w$ of wage setters can reset their wage in any period. The fully flexible desired real wage for type j is the marginal rate of substitution between labor and consumption,

$$\frac{W_{j,t}^{*,flex}}{P_t} = \frac{\chi N_{j,t}^{\varphi}}{\Lambda_{j,t}}. \tag{11}$$

The code imposes one common desired wage W_t^* faced by both household types. Thus, in symmetric log-linear form,

$$w_t^* = \varphi n_{R,t} - \lambda_{R,t} = \varphi n_{H,t} - \lambda_{H,t}. \tag{12}$$

The wage markup is the gap between the actual real wage and the desired real wage,

$$\mu_t^w = w_t - w_t^*. \tag{13}$$

The EHL wage Phillips curve obtained around zero steady-state wage inflation is

$$\pi_t^w = \beta \mathbb{E}_t \pi_{t+1}^w - \kappa_w \mu_t^w, \qquad \kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}. \tag{14}$$

Real wage dynamics satisfy

$$w_t = w_{t-1} + \pi_t^w - \pi_t. \tag{15}$$

2.3 Firms and price setting

A final-good firm aggregates differentiated intermediate goods:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} \, dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{16}$$

which implies demand $Y_t(z) = (P_t(z)/P_t)^{-\varepsilon} Y_t$. Intermediate firm z produces using labor only:

$$Y_t(z) = A_t N_t(z). \tag{17}$$

The real marginal cost is therefore

$$MC_t = \frac{W_t/P_t}{A_t}, \tag{18}$$

up to the reduced-form labor adjustment wedge described later.

Each intermediate firm faces Rotemberg price adjustment costs

$$\frac{\phi_P}{2}\left(\frac{P_t(z)}{P_{t-1}(z)} - 1\right)^2 Y_t. \tag{19}$$

Linearizing the optimal price-setting condition around zero inflation gives

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_p mc_t, \qquad \kappa_p = \frac{\varepsilon - 1}{\phi_P}. \tag{20}$$

The steady-state real marginal cost is

$$MC = \frac{\varepsilon - 1}{\varepsilon}. \tag{21}$$

We normalize steady-state labor symmetrically across household types by implicitly selecting type-specific labor-disutility shifters χ_R and χ_H such that

$$\bar{N}_R = \bar{N}_H = \bar{n}.$$

Hand-to-mouth households consume current labor income,

$$\bar{C}_H = \bar{W}\bar{N}_H,$$

while Ricardian households absorb the remaining aggregate resources,

$$\bar{C}_R = \frac{\bar{C} - \omega \bar{C}_H}{1 - \omega}.$$

Therefore the exact log-linear aggregation weights are

$$s_R^C = \frac{(1-\omega)\bar{C}_R}{\bar{C}}, \qquad s_H^C = \frac{\omega \bar{C}_H}{\bar{C}},$$

and

$$s_R^N = \frac{(1-\omega)\bar{N}_R}{\bar{N}}, \qquad s_H^N = \frac{\omega \bar{N}_H}{\bar{N}}.$$

The real wage equation is

$$w_t = mc_t + y_t - n_t. \tag{22}$$

Since $y_t - n_t = a_t$, this is equivalent to $w_t = mc_t + a_t$.

2.4 Aggregation and resource constraint

Aggregate consumption and labor are

$$C_t = (1-\omega)C_{R,t} + \omega C_{H,t}, \qquad N_t = (1-\omega)N_{R,t} + \omega N_{H,t}. \tag{23}$$

At first order around a zero-inflation steady state, Rotemberg costs are second order. The aggregate resource constraint used in the linear model is therefore

$$Y_t = C_t. \tag{24}$$

Production in logs is

$$y_t = a_t + n_t. \tag{25}$$

2.5 Monetary policy

Let R_t denote the deviation of the policy rate from steady state and let r_t^n denote the natural real rate. The Taylor rule is

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)\left(\phi_\pi \pi_t + \phi_x x_t + \phi_r r_t^n\right) + e_t^R, \tag{26}$$

where $x_t = y_t - y_t^n$ is the output gap. The monetary policy rule thus implicitly targets an output gap and inflation equal to zero.

3 Flexible-price and flexible-wage natural allocation

The natural allocation is the allocation that would prevail with flexible prices and flexible wages. It is represented by a parallel block of variables with suffix n. Flexible prices imply

$$mc_t^n = 0. \tag{27}$$

Flexible wages imply that the real wage equals each type's desired wage:

$$w_t^n = \varphi n_{R,t}^n - \lambda_{R,t}^n = \varphi n_{H,t}^n - \lambda_{H,t}^n. \tag{28}$$

The natural real rate is the intertemporal marginal-rate-of-substitution of Ricardian households:

$$r_t^n = \lambda_{R,t}^n - \mathbb{E}_t \lambda_{R,t+1}^n. \tag{29}$$

The natural output gap in the sticky economy is

$$x_t = y_t - y_t^n. \tag{30}$$

The natural real rate can be expressed as a function of the current and expected productivity path. The real wage equation (22), together with equations (27) and (28), implies the following for the representative household economy (without hand-to-mouth agents, $\omega = 0.0$):

$$\begin{aligned}
a_t &= w_t^n = \varphi n_t^n - \lambda_{R,t}^n, \\
a_t &= \varphi n_t^n - \frac{-\sigma}{1-h}(c_t^n - h c_{t-1}^n), \\
a_t &= \varphi(c_t^n - a_t) + \frac{\sigma}{1-h}(c_t^n - h c_{t-1}^n), \\
a_t + \varphi a_t + \frac{\sigma h}{1-h} c_{t-1}^n &= (\varphi + \frac{\sigma}{1-h}) c_t^n, \\
c_t^n &= \left[a_t(1+\varphi) + \frac{\sigma h}{1-h} c_{t-1}^n \right] \frac{1}{\varphi + \frac{\sigma}{1-h}}, \\
c_t^n &= \frac{1+\varphi}{\varphi + \frac{\sigma}{1-h}} \sum_{k=0}^{\infty} \left(\frac{\frac{\sigma h}{1-h}}{\varphi + \frac{\sigma}{1-h}} \right)^k a_{t-k}.
\end{aligned} \tag{31}$$

In the $h = 0$ case with no habit in consumption,

$$c_t^n = \frac{1+\varphi}{\varphi+\sigma} a_t. \tag{32}$$

Equation (29) then implies:

$$\begin{aligned}
r_t^n &= \lambda_{R,t}^n - \mathbb{E}_t \lambda_{R,t+1}^n \\
&= \sigma \left(\mathbb{E}_t c_{t+1}^n - c_t^n \right) \\
&= \sigma \left(\mathbb{E}_t a_{t+1} - a_t \right) \frac{1+\varphi}{\varphi+\sigma}.
\end{aligned} \tag{33}$$

4 Steady state

Steady-state objects for the case with $\omega = 0.0$ are

$$MC = \frac{\varepsilon - 1}{\varepsilon}, \qquad Y = C = \bar{n}, \qquad W = MC, \qquad \pi = 0, \tag{34}$$

and the habit-adjusted steady-state marginal utility of wealth is

$$\Lambda = (1 - \beta h) \left[C(1-h) \right]^{-\sigma}. \tag{35}$$

5 Linearized model matching the Dynare code

This section lists the linear equations in the same order and with the same variable names as the Dynare file. See section 7 for an explanation of variable names as they appear in this section. Conditional expectations are implicit.

5.1 Exogenous productivity growth

$$g_t = g_t^{exo}, \tag{36}$$

$$a_t = a_{t-1} + g_t. \tag{37}$$

5.2 Sticky-price, sticky-wage TANK economy

Ricardian household.

$$ucR_t = -\frac{\sigma}{1-h}\left(cR_t - hcR_{t-1}\right), \tag{38}$$

$$\lambda_t^R = \frac{ucR_t - \beta h\, ucR_{t+1}}{1-\beta h}, \tag{39}$$

$$uc_t = ucR_t, \qquad \lambda_t = \lambda_t^R, \tag{40}$$

$$\lambda_t^R = \lambda_{t+1}^R + R_t - \pi_{t+1}. \tag{41}$$

Hand-to-mouth household.

$$ucH_t = -\frac{\sigma}{1-h}\left(cH_t - hcH_{t-1}\right), \tag{42}$$

$$\lambda_t^H = \frac{ucH_t - \beta h\, ucH_{t+1}}{1-\beta h}. \tag{43}$$

Aggregation and production.

$$c_t = s_R^C c_{R,t} + s_H^C c_{H,t}, \tag{44}$$

$$n_t = s_R^N n_{R,t} + s_H^N n_{H,t}, \tag{45}$$

$$y_t = a_t + n_t. \tag{46}$$

Real wage and wage setting.

$$w_t = mc_t + y_t - n_t - \psi_n(n_t - n_{t-1}) + \beta\psi_n(n_{t+1} - n_t), \tag{47}$$

$$w_t^* = \varphi nR_t - \lambda_t^R, \tag{48}$$

$$w_t^* = \varphi nH_t - \lambda_t^H, \tag{49}$$

$$\mu_t^w = w_t - w_t^*, \tag{50}$$

$$\pi_t^w = \beta\pi_{t+1}^w - \kappa_w\mu_t^w, \tag{51}$$

$$w_t = w_{t-1} + \pi_t^w - \pi_t. \tag{52}$$

Hand-to-mouth budget, resource constraint, and price Phillips curve.

$$cH_t = w_t + nH_t, \tag{53}$$

$$y_t = c_t, \tag{54}$$

$$\pi_t = \beta\pi_{t+1} + \kappa_p mc_t. \tag{55}$$

Output gap and monetary policy.

$$ygap_t = y_t - yn_t, \tag{56}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left(\phi_\pi \pi_t + \phi_x ygap_t + \texttt{use\_nat}\,\phi_r rn_t\right) + eR_t. \tag{57}$$

Here `use_nat` is a boolean parameter specifying whether or not the monetary policy rule incorporates the natural rate.

5.3 Natural TANK economy

Ricardian natural household.

$$ucRn_t = -\frac{\sigma}{1-h}\left(cRn_t - hcRn_{t-1}\right), \tag{58}$$

$$\lambda_t^{Rn} = \frac{ucRn_t - \beta h\, ucRn_{t+1}}{1-\beta h}, \tag{59}$$

$$ucn_t = ucRn_t, \qquad lambdan_t = \lambda_t^{Rn}, \tag{60}$$

$$rn_t = \lambda_t^{Rn} - \lambda_{t+1}^{Rn}. \tag{61}$$

Hand-to-mouth natural household.

$$ucHn_t = -\frac{\sigma}{1-h}\left(cHn_t - hcHn_{t-1}\right), \tag{62}$$

$$\lambda_t^{Hn} = \frac{ucHn_t - \beta h\, ucHn_{t+1}}{1-\beta h}. \tag{63}$$

Natural aggregation, production, and factor prices.

$$cn_t = s_R^C cRn_t + s_H^C cHn_t, \tag{64}$$

$$nn_t = s_R^N nRn_t + s_H^N nHn_t, \tag{65}$$

$$yn_t = a_t + nn_t, \tag{66}$$

$$mcn_t = 0, \tag{67}$$

$$wn_t = mcn_t + yn_t - nn_t - \psi_n(nn_t - nn_{t-1}) + \beta\psi_n(nn_{t+1} - nn_t). \tag{68}$$

Flexible-wage conditions and natural resource constraint.

$$wn_t = \varphi nRn_t - \lambda_t^{Rn}, \tag{69}$$

$$wn_t = \varphi nHn_t - \lambda_t^{Hn}, \tag{70}$$

$$cHn_t = wn_t + nHn_t, \tag{71}$$

$$yn_t = cn_t. \tag{72}$$

6 Parameters summary

Figure A1 describes the parameters used to generate the figures in the main article:

A1. Parameter calibration

Parameter	Symbol	Value
Discount factor	β	0.99
Risk aversion	σ	1.0
Inverse Frisch elasticity	φ	1.0
Habit parameter	h	0.0
Steady-state hours	$\bar{n}$	0.33
Goods elasticity of substitution	ϵ	6.0
Rotemberg price adjustment cost	ϕ_P	100.0
Calvo wage stickiness target	θ_w	0.75
Labor elasticity of substitution	ϵ_w	6.0
Hand-to-mouth share	ω	0.00
Interest rate smoothing	ρ_R	0.0
Inflation response	ϕ_π	1.5
Output gap response	ϕ_x	0.5
Natural rate coefficient	ϕ_r	1.00
Labor adjustment wedge	ψ_n	0.0

7 Mapping to Dynare variables

A2. Dynare variables

Dynare variable	Interpretation	Notes
c, n, y	Aggregate consumption, labor, output	Sticky economy
cR, cH	Type-specific consumption	Ricardian and hand-to-mouth
nR, nH	Type-specific labor	Ricardian and hand-to-mouth
ucR, ucH	Habit-adjusted consumption utility components	Type-specific
λ^R, λ^H	Marginal utility of wealth	Type-specific
uc, λ	Legacy aliases	Set equal to Ricardian objects
a, g	Productivity level and growth	$a_t = a_{t-1} + g_t$
mc, w, w^*, μ^w	Marginal cost, real wage, desired wage, wage markup	Sticky economy
π, π^w, R	Price inflation, wage inflation, policy rate	Deviations from steady state
cn, nn, yn	Natural aggregate consumption, labor, output	Flexible allocation
cRn, cHn nRn, nHn	Natural type-specific quantities	Flexible allocation
rn	Natural real rate	Ricardian intertemporal marginal rate of substitution
$ygap$	Output gap	$y_t - yn_t$

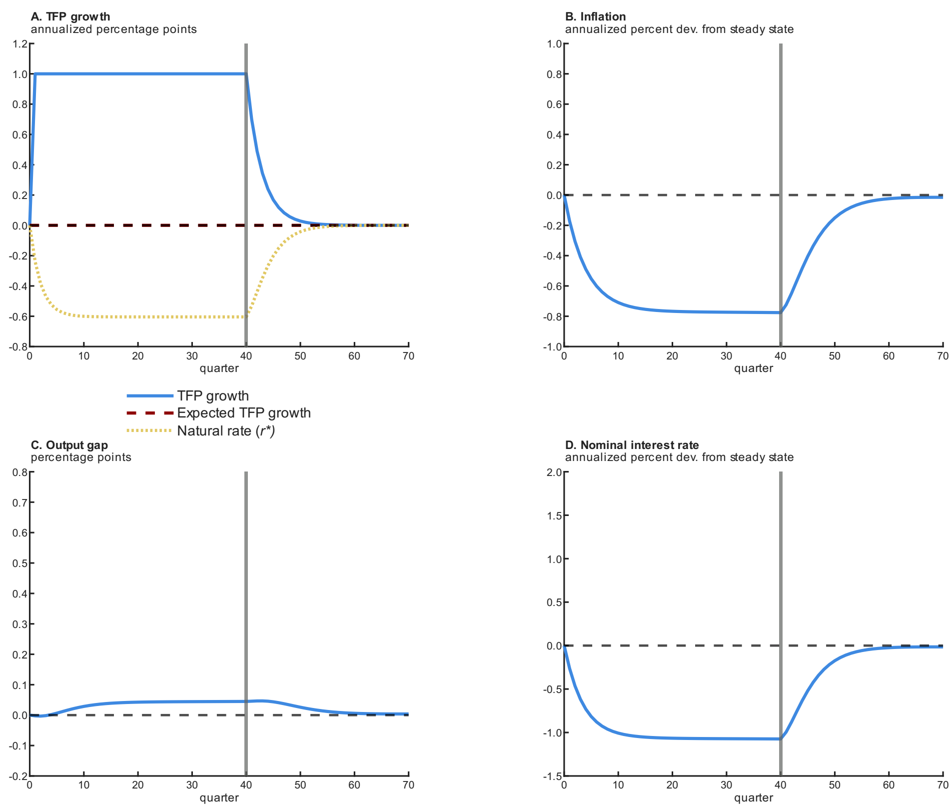
8 Robustness exercises

In this section we show the results of two robustness exercises discussed in the main article. These exercises explore whether our results are dampened by 1) habit persistence in consumption behavior or 2) credit constraints for some households.

8.1 Baseline calibration with habit persistence

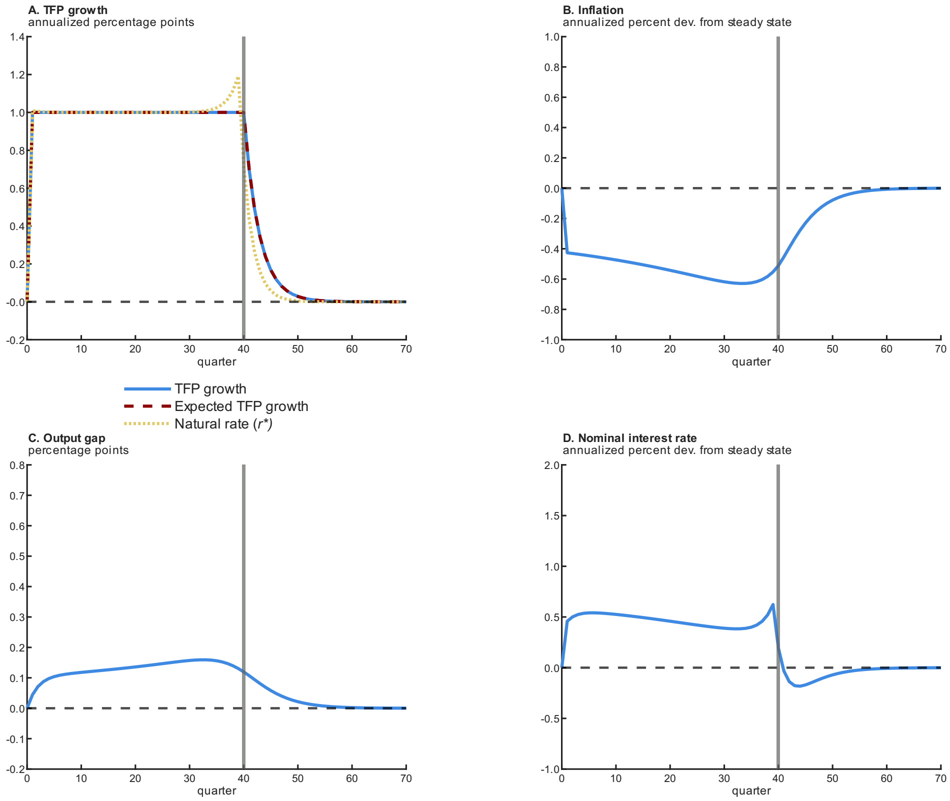
This subsection presents model results under the baseline calibration with habit set to $h = 0.7$.

A3. Ten-year increase in productivity growth with agents repeatedly surprised, with habit persistence



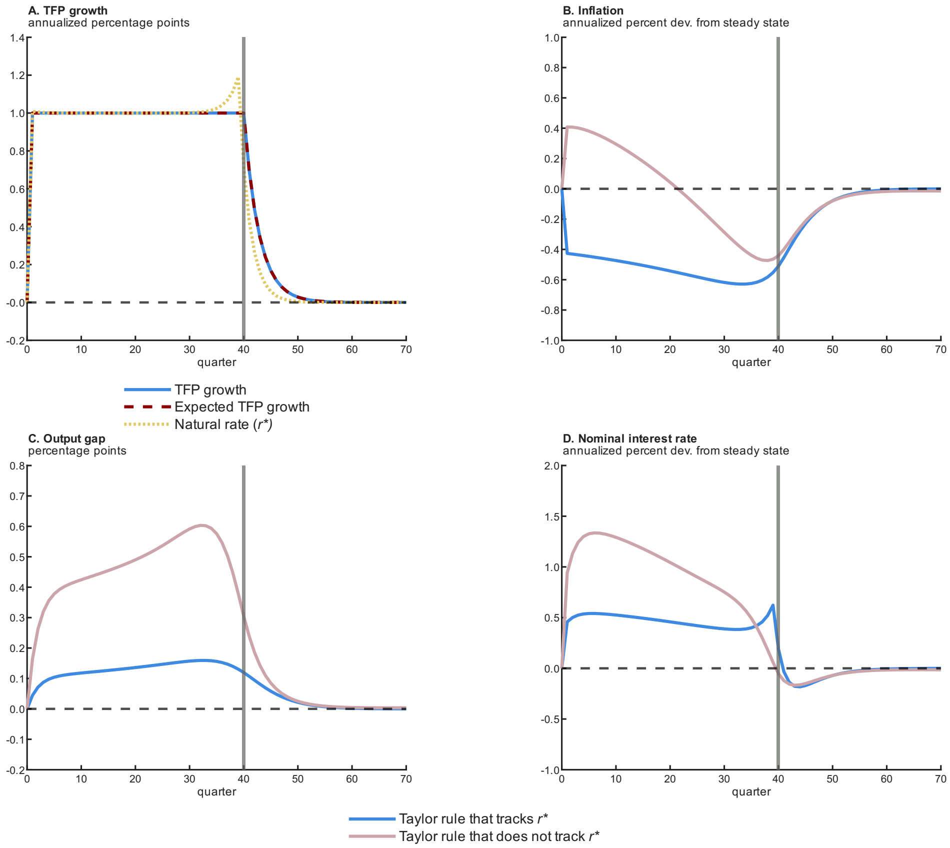
Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations, the central bank's Taylor rule tracks the natural rate of interest (r^*).
Source: Authors' calculations.

A4. Ten-year increase in productivity growth with perfect foresight and habit persistence



Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations, the central bank's Taylor rule tracks the natural rate of interest ($r*$).
Source: Authors' calculations.

A5. Ten-year increase in productivity growth with perfect foresight and habit persistence: Implications of tracking r^*

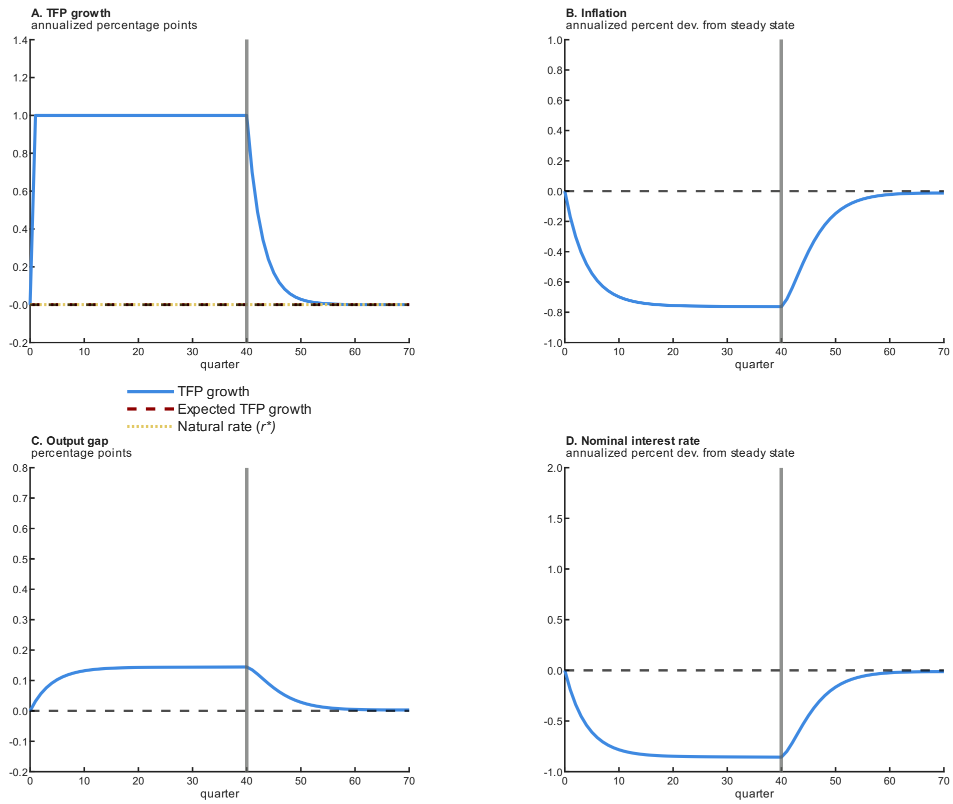


Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations in panels B–D, the central bank's Taylor rule tracks the natural rate of interest (r^*) for the impulse responses shown as solid blue lines, but it does not track the natural rate for those shown as solid pink lines.
Source: Authors' calculations.

8.2 Baseline calibration with hand-to-mouth agents

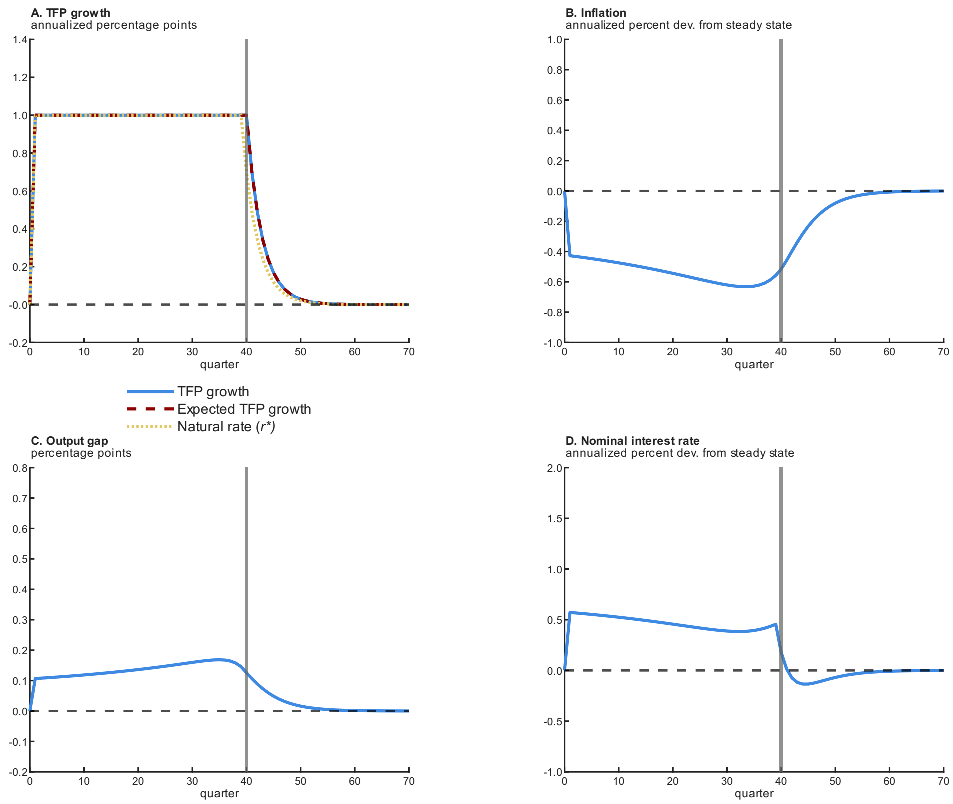
This subsection presents model results under the baseline calibration with the share of hand-to-mouth agents set to $\omega = 0.3$.

A6. Ten-year increase in productivity growth with agents repeatedly surprised, with hand-to-mouth agents



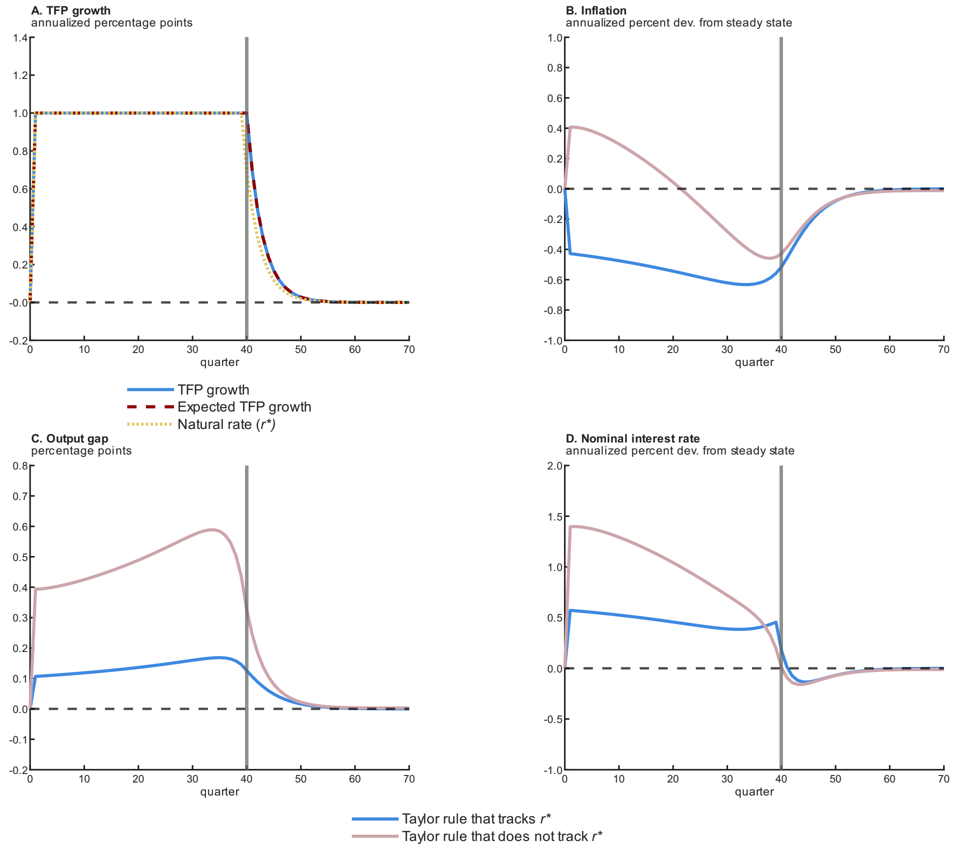
Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations, the central bank's Taylor rule tracks the natural rate of interest ($r*$).
Source: Authors' calculations.

A7. Ten-year increase in productivity growth with perfect foresight and hand-to-mouth agents



Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations, the central bank's Taylor rule tracks the natural rate of interest ($r*$).
Source: Authors' calculations.

A8. Ten-year increase in productivity growth with perfect foresight and hand-to-mouth agents: Implications of tracking r^*



Notes: TFP stands for total factor productivity; percent dev. is shorthand for percent deviation. In all four panels, a vertical solid dark gray line at 40 quarters designates the point at which TFP growth begins falling back toward 0.0 percentage points. In the simulations in panels B–D, the central bank's Taylor rule tracks the natural rate of interest (r^*) for the impulse responses shown as solid blue lines, but it does not track the natural rate for those shown as solid pink lines.
Source: Authors' calculations.