

### The current FRB Chicago—Gittings model

Since 1979 the Federal Reserve Bank of Chicago has been using a series of money income models developed by Thomas Gittings. These models have all been intended to capture the fundamentals of money growth's effect on the economy. The current model is a vector model. Changes in real income growth and inflation are modeled in two separate equations:

$$\dot{q} = \alpha_0^q + \sum_{i=1}^L \alpha_i^q \dot{q}_{t-i} + \sum_{j=0}^M \beta_j^q \dot{m}_{t-j} + \sum_{k=0}^N \gamma_k^q \dot{e}_{t-k}$$

$$\dot{p} = \alpha_0^p + \sum_{i=1}^L \alpha_i^p \dot{p}_{t-i} + \sum_{j=0}^M \beta_j^p \dot{m}_{t-j} + \sum_{k=0}^N \gamma_k^p \dot{e}_{t-k}$$

where  $\dot{q}$  is the growth in real income,  $\dot{p}$  is the inflation rate,  $\dot{m}$  is the growth in M1, and  $\dot{e}$  is the rate of change in real energy prices.

This model differs in a number of ways from money-income models used elsewhere. First, by separately estimating equations for real growth and inflation rather than estimating a single nominal income equation the tradeoff between real income and inflation can be directly forecasted. Benefits are also derived when the equations are not forecasting well (as all models of this type are prone to do periodically). The breakdown may occur in either the price or real income equation. By having an estimate of where the breakdown is occurring, it is easier to determine what may be causing the trouble and when it may end. Further, the FOMC may want to react to a fall in velocity differently if it is due to less than expected real growth rather than to less than expected inflation.

Second, lagged values of real growth and inflation are included. This gives the model a somewhat richer structure of time series behavior than models where no lagged endogenous variables are included,

such as the various St. Louis equations. Our research indicates that the effects of money growth on the economy, especially inflation, are much more protracted than has previously been believed. The use of lagged endogenous variables allows us to model this without using an exorbitant number of lags of money growth.

Third, the rate of change in real energy prices is included as an additional tool to minimize the effect of supply shocks. Oil price shocks and their aftereffects on the entire spectrum of energy prices have been the dominant form of supply shock since 1973. By modeling this particular type of shock directly, the model provides better estimates of the money-income relationship. Without energy prices in the model the estimates of money's effect on both real income and inflation are both smaller and slower. This reduction in size and speed is typical of econometric estimation when a large source of error has been left unmodeled. Unfortunately, oil shocks are largely unpredictable and any direct gains in terms of forecasting are limited.

Fourth, because our research has led us to the belief that a large number of lags of money growth are necessary to correctly model the money-income relationship the danger of over fitting is large. To overcome this problem we use polynomial distributed lags to force the money coefficients to follow a smooth adjustment path. This effectively reduces the number of free parameters which can create artificially good regression results.

The last and most important difference is the application of the principles of neutrality and super neutrality directly to the specification and estimation procedures. Neutrality is a fairly old concept. It states that an increase in the rate of money growth will eventually cause an equal increase in the rate of inflation and that the rate of real growth will in the long

run be unaffected. Without lagged endogenous variables this is equivalent to the statement that in the inflation equation the sum of the coefficients on money must equal one and that in the real growth equation they must sum to zero. For the case with lagged endogenous variables the constraints are slightly more complicated and can be written:

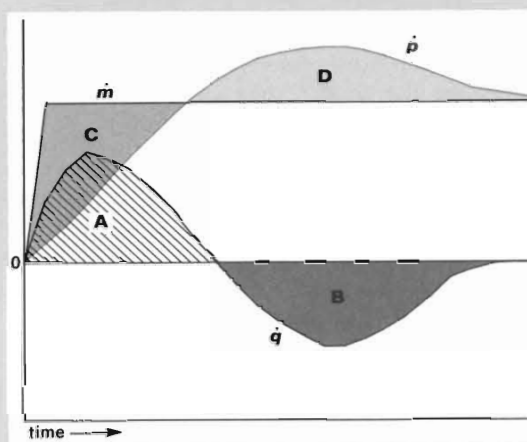
$$\sum_{j=0}^M \beta_j^q = 0, \quad \sum_{i=1}^L \alpha_i^p + \sum_{j=0}^M \beta_j^p = 1$$

Super neutrality is a generalization of neutrality from rates of growth to levels. Simply put, it says that if the money supply is doubled it will lead to a doubling of the price level but will not affect the level of real income in the long run. Super neutrality comes from the rather straightforward belief that the Federal Reserve cannot create real wealth in the long haul simply by printing more and more money. Since the equations we use are in terms of rates of growth these concepts must be translated to restrictions on growth rates. The diagram shows the impact of a one percent increase in the rate of money growth on real income growth and inflation. In the real income case, the effect on cumulative real growth must be zero so area A must equal area B. In the inflation case it is only slightly more complicated. Inflation must on average equal the rate of money growth, implying that area C must equal area D. It follows after extensive manipulation that the restrictions on the parameters must be:

$$\sum_{j=0}^M j\beta_j^q = 0, \quad \sum_{i=1}^L i\alpha_i^p + \sum_{j=0}^M j\beta_j^p = 0$$

By imposing the neutrality and super neutrality restrictions, we guarantee that the model will not imply that the Federal Reserve can create unlimited wealth by supplying greater and greater quantities of

Responses to changes in the rate of monetary growth



Coefficients of Gittings model equations

	Real GNP	GNP deflator	Nominal GNP
Intercept			
$\alpha_0$	4.012	-0.243	4.402
Dependent variable			
$\alpha_1$	-0.122	0.180	-0.145
$\alpha_2$	0.014	0.218	-0.110
$\alpha_3$	-0.102	0.074	-0.172
Money			
$\beta_0$	0.395	0.110	0.590
$\beta_1$	0.242	0.103	0.439
$\beta_2$	0.116	0.095	0.312
$\beta_3$	0.014	0.087	0.206
$\beta_4$	-0.064	0.077	0.119
$\beta_5$	-0.121	0.068	0.051
$\beta_6$	-0.159	0.058	-0.001
$\beta_7$	-0.180	0.047	-0.038
$\beta_8$	-0.186	0.037	-0.062
$\beta_9$	-0.179	0.027	-0.075
$\beta_{10}$	-0.162	0.017	-0.078
$\beta_{11}$	-0.135	0.008	-0.073
$\beta_{12}$	-0.102	-0.001	-0.062
$\beta_{13}$	-0.064	-0.009	-0.046
$\beta_{14}$	-0.023	-0.016	-0.028
$\beta_{15}$	0.018	-0.022	-0.008
$\beta_{16}$	0.058	-0.027	0.011
$\beta_{17}$	0.094	-0.031	0.028
$\beta_{18}$	0.125	-0.033	0.042
$\beta_{19}$	0.149	-0.033	0.049
$\beta_{20}$	0.163	-0.032	0.050
Energy			
$\gamma_1$	0.029	-0.010	0.010
$\gamma_2$	-0.090	0.075	0.016
R <sup>2</sup>	0.386	0.688	0.305
F ratio	5.66	19.82	3.94
Neutrality Lagrange multiplier t-ratio	0.573	-0.065	0.642
Super neutrality Lagrange multiplier t-ratio	1.536	-1.563	-1.187

money, an implication of many money-income models that do not use such restrictions.

The restrictions can also be used to help determine how many lags should be included in the model. As can be seen from the diagram, if the lags are cut off too soon an unrestricted estimation of the equations would violate neutrality and super neutrality. Thus, we need to include enough lags so that the data is consistent with the restrictions. Use of this principle has led to longer lags than are typically used elsewhere. We believe many studies that have rejected super neutrality did so because they included too few lags. For instance, the current St. Louis equation uses 10 lags while our equations use 20 lags.

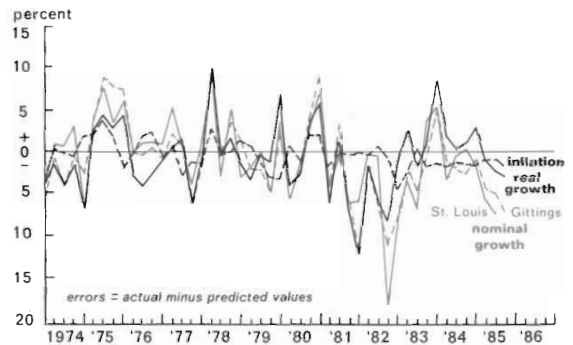
In order to to make easy comparisons with other reduced form models of nominal income, we also maintain a pure nominal income model, which is estimated with the same constraints as the inflation equation. A set of estimated equations for both the vector and single equation models are shown in the table. The sample was restricted to 64:Q2-81:Q4 in order to avoid the questions about the definition of money which have undermined the usefulness of the money-income relationship in the 80s, as the accompanying article documents. Current research is emphasizing techniques to forecast the breakdowns in the money income relationship so that we will have a better idea of when these relationships are useful for policy and when they are not.

—Thomas Gittings and Steven Strongin

overpredicts inflation from 1983 through 1985. The cumulative real growth errors fall rapidly in 1981 and 1982, remain fairly level throughout most of 1983, increase steadily from late 1983 through 1984, and then fall again in 1985.

The breakdown observed in the money/income relationship from 1981 through 1985 thus reflects the breakdowns of the real growth and inflation components at different points. The steep fall in the cumulative nominal growth errors is set off in 1981 by the

Figure 5  
One-quarter-ahead forecast errors—  
Gittings and St. Louis models



overprediction of real growth. Overprediction of inflation starts to contribute to the nominal breakdown in 1983 just as the real growth equation begins to forecast fairly accurately for several quarters. The apparent stability of the nominal money/income relationship in 1984 actually results from offsetting errors in the inflation and real growth equations. Real growth is consistently underpredicted while inflation is consistently overpredicted during that year. But in 1985, negative inflation and real growth forecast errors reinforce each other, resulting in persistent overprediction of nominal growth.

The different patterns and timing of the cumulative inflation and real growth forecast errors suggest that the two nominal income components could be deviating from their past

Figure 6  
Cumulative one-quarter-ahead  
forecast errors—Gittings model

